Decoding exotic hadrons in few-body calculations

11th workshop on the XYZ particles
Guizhou Minzu University

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Based on: 2310.14597, 2401.14899, 2409.03373, 2508.11161, 2602.12010

Together with: Wei-Lin Wu, Lu Meng, Yao Ma, Jun-Zhang Wang, Shi-Lin Zhu

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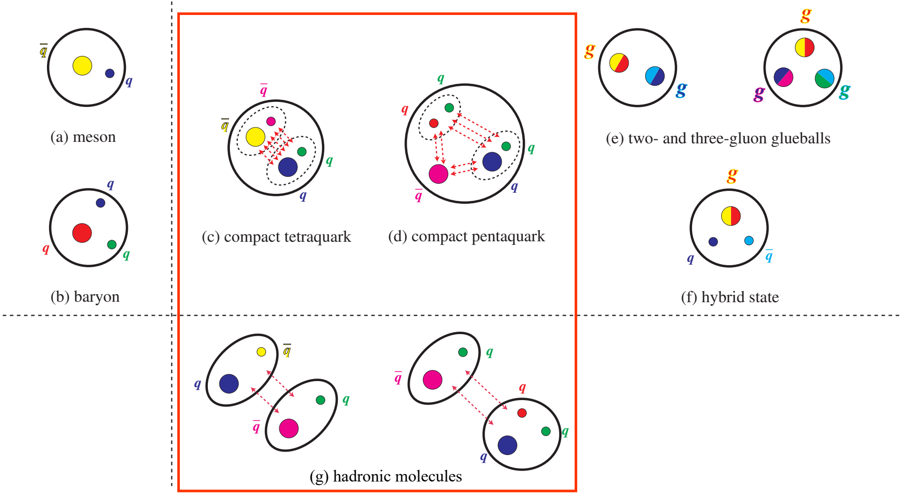
Background

Exotic hadrons: states beyond conventional mesons ($q\bar{q}$) and baryons (qqq):

[H. X. Chen, W. Chen, X. Liu, Y. R. Liu, S. L. Zhu, *Rept.Prog.Phys.* **86**, 026201 (2023)]

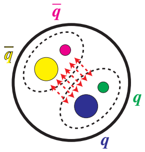


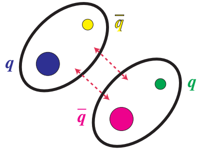
Recently, more and more hadrons composed of at least four quarks (manifestly exotic) were observed:

$[cd\bar{c}\bar{u}]$	$[cc\bar{c}\bar{c}]$	$[cs\bar{u}\bar{d}]$	$[cs\bar{c}\bar{u}]$	$[cc\bar{u}\bar{d}]$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$
$Z_c(3900)$	$X(6900)$	$T_{cs1}(2900)$	$Z_{cs}(3985)$	$T_{cc}(3875)^+$	$T_{c\bar{s}0}(2900)^{++}$
$Z_c(4020)$	$X(7200)$	$T_{cs0}(2900)$	$Z_{cs}(4000)$		$T_{c\bar{s}0}(2900)^0$
BESIII:2013ris	LHCb:2020bwg	LHCb:2020bls	BESIII:2020qkh	LHCb:2021vvq	LHCb:2022sfr
BESIII:2013ouc	ATLAS:2023bft	LHCb:2020pxc	LHCb:2021uow	LHCb:2021auc	LHCb:2022lzp
	CMS:2023owd				

- Advancing hadron spectroscopy
- An excellent platform for studying the non-perturbative properties of QCD

Theoretical interpretations: compact state VS molecular





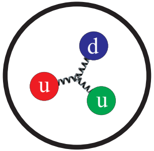
- Unified description
- Dynamic calculations that treat the molecular and compact state equally
- $\Rightarrow$ Constituent quark model + 4-body Schrödinger equation

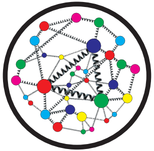
Contents

Constituent quark model (CQM)

CQM: a naive model

[H. X. Chen, W. Chen, X. Liu, Y. R. Liu, S. L. Zhu, *Rept.Prog.Phys.* **86**, 026201 (2023)]





- but widely and successfully used
- the valence quarks and antiquarks well describe the internal symmetry of hadrons

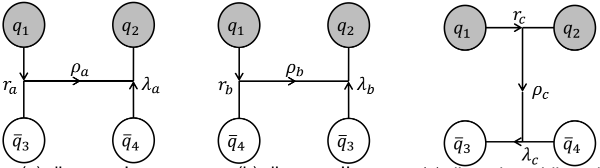
AL1 quark potential model: B. Silvestre-Brac, *Few Body Syst.* **20** 1 (1996)

$$H=\sum_{j=}\frac{p_j^2}{2m_j}+\sum_{i\neq j}V_{ij}+\sum_j m_j,\quad V_{ij}(r)=\left[-\frac{\kappa}{r}+\lambda r-\Lambda+\frac{2\pi}{3m_im_j}\kappa'\frac{1}{\pi^{3/2}r_0^3}e^{\left(-r^2/r_0^2\right)}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\lambda_i\cdot\lambda_j,$$

- One-gluon-exchange + Linear confinement.
- Parameters are fitted by the ground state spectra of mesons and baryons.
- Theoretical uncertainties $\sim$ tens of MeV
- We do not add or fine-tune any parameters.

Solving 4-body Schrödinger equation

Gaussian Expansion Method (GEM) E. Hiyama, Y. Kino M. Kamimura, *Prog.Part.Nucl.Phys.* **51** 223 (2003)



(a) dimeson I (b) dimeson II (c) diquark-antidiquark

$$\psi = \sum_{\alpha} \sum_{\beta=1}^{3} \sum_{n_1=1}^{n_{\max}} \sum_{n_1=2}^{n_{\max}} \sum_{n_3=1}^{n_{\max}} C_{\alpha,\beta,n_\beta} \chi_{sc}^{(\alpha)} \exp\left[-\frac{r_\beta^2}{r_{n_1,\beta}} - \frac{\lambda_\beta^2}{r_{n_2,\beta}} - \frac{\rho_\beta^2}{r_{n_3,\beta}} \right], \quad r_{n_r,\beta} = r_{1,\beta} \left(\frac{r_{n_{\max},\beta}}{r_{1,\beta}} \right)^{\frac{n_r-1}{n_{\max}-1}}$$

- Include both meson-meson and diquark-antidiquark correlations
- Embed both long-range and short-range correlations

  $\Rightarrow$ Dynamic calculations that treat the molecular and compact state equally.
- Analytical expression for matrix elements.
- $N_\alpha \times N_\beta \times (n_{\max})^{(n_{\text{particle}}-1)} \sim 10^4$ bases in 4-body systems ($n_{\max} \sim 15 - 20$)
- DMC/QMC: advantages will become apparent with a large particle number.

L. Meng, Y. K. Chen, Y. Ma, S. L. Zhu, 2310.13354 PRD

Identify resonances

Many experimental states lie above meson-meson thresholds and can decay strongly.
However, when calculate the mass spectrum in a finite number of bases:

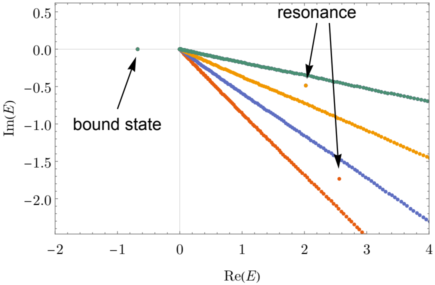
- All the eigenvalues including those for the continuum states are discrete
- The wave functions of resonances $\psi_R \sim e^{i\kappa_r r+\gamma_r r}$ have divergent behavior

Complex Scaling Method (CSM)

J. Aguilar, J. M. Combes, *Commun.Math.Phys.* **22** 269 (1971) & E. Balslev, J. M. Combes, *Commun.Math.Phys.* **22** 280 (1971)

$$r \to re^{i\theta}, \qquad p \to pe^{-i\theta},$$

$$H(\theta) = \sum_{j=1}^{4} \frac{p_j^2 e^{-2i\theta}}{2m_j} + \sum_{i<j=1}^{4} V_{ij}\left(r_{ij}e^{i\theta}\right) + \sum_j m_j$$

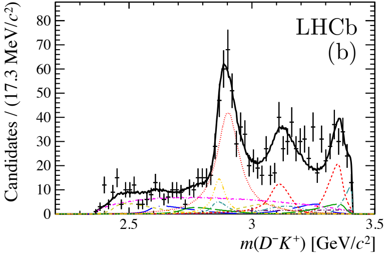


- ☞ when $\theta > \arctan\left(\gamma_r/\kappa_r\right)$, $\psi_R^\theta \sim e^{(-\kappa_r \sin\theta+\gamma_r\cos\theta)r} \cdot e^{i(\kappa_r\cos\theta+\gamma_r\sin\theta)r}$ become normalizable
- ☞ E_B, E_R do not shift as θ changes
- GEM: Easy to perform analytic continuation $\Rightarrow$ solving $10^4 \times 10^4$ non-Hermitian matrices

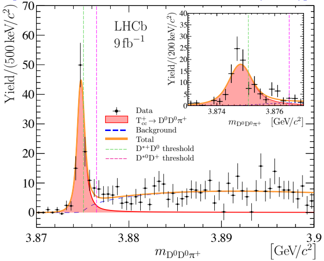
Singly, Doubly and Fully Heavy Tetraquarks

Exciting experimental progress:

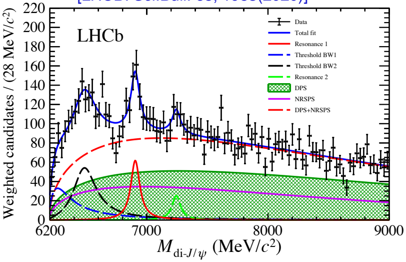
[LHCB: *Phys.Rev.Lett.* **125**, 242001(2020)]
[LHCB: *Phys.Rev.D.* **102**, 112003(2020)]



[LHCB: *Nature Phys.* **18**, 751(2022)]
[LHCB: *Nature Commun.* **13**, 3351(2022)]



[LHCB: *Sci.Bull.* **65**, 1983(2020)]



Our AL1 model + GEM + CSM calculations:

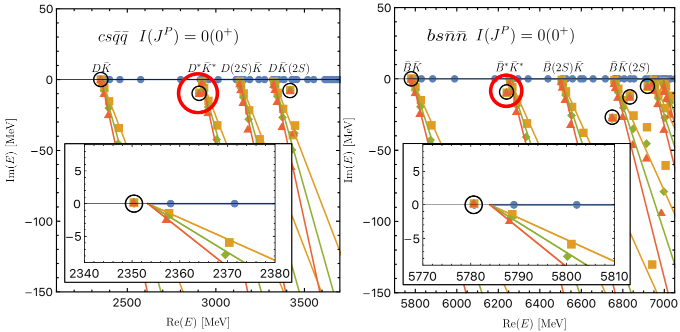
- $[Qs\bar{n}\bar{n}]$, $[QQ\bar{q}\bar{q}]$, $[QQ\bar{Q}\bar{Q}]$
- Do not account for the widths of the conventional meson
- Only consider the constituent two-body decays

  $\Rightarrow$Theoretical widths of the resonances are expected to be smaller than the experimental values.

  $\Rightarrow$ Results below $M(2S)M$thresholds are more reliable.

$Qs\bar{n}\bar{n} \quad I(J^P) = 0(0^+)$

Y. K. Chen, W. L. Wu, L. Meng, S. L. Zhu, 2310.14597 PRD



States	$M-i\Gamma/2$	ΔM	$r_{c\bar{q}}^{\rm rms}$	$r_{s\bar{q}}^{\rm rms}$	$r_{cs}^{\rm rms}$	$r_{\bar{q}\bar{q}}^{\rm rms}$	Type
$D\bar{K}$	2353		0.61	0.59			S.
$D^*\bar{K}^*$	2920		0.70	0.81			S.
$0(0^+)$	2350	-3	0.61	0.59	2.45	2.52	M.
	$2906-10i$	-14	0.74	0.86	1.12	1.26	M.
	$3419-7i$		0.91	1.09	0.87	1.22	C.

States	E	ΔM	$r_{b\bar{q}}^{\rm rms}$	$r_{s\bar{q}}^{\rm rms}$	$r_{bs}^{\rm rms}$	$r_{\bar{q}\bar{q}}^{\rm rms}$	Type
$\bar{B}\bar{K}$	5784		0.62	0.59			S.
$\bar{B}^*\bar{K}^*$	6254		0.66	0.81			S.
$0(0^+)$	5781	-3	0.62	0.59	2.11	2.21	M.
	$6240-9i$	-14	0.69	0.86	1.04	1.21	M.
	$6748-28i$		1.12	1.19	0.74	1.09	C.
	$6834-13i$		0.77	1.22	1.06	1.23	C.
	$6920-5i$		0.72	1.33	0.98	1.33	C.

Description for the experimental $T_{cs0}(2870)$: $M_{\rm The.} = 2906{\rm MeV}$, $\Gamma_{\rm The.} = 20$ MeV

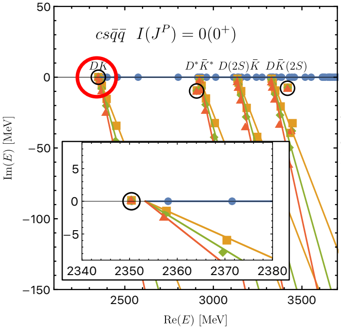
- $M_{\rm Exp.} = 2874 \pm 11$ MeV, $\Gamma_{\rm Exp.} = 68 \pm 17$ MeV.
- $D^*\bar{K}^*$ molecular quasi-bound (-14 MeV) state (Feshbach resonance).
- Cross-verification channel: $B^+ \to (D^+K^-)\pi^+$. Y. K. Chen, J. J. Han, Q. F. Lü, J. P. Wang, F. S. Yu, 2009.01182 EPJC

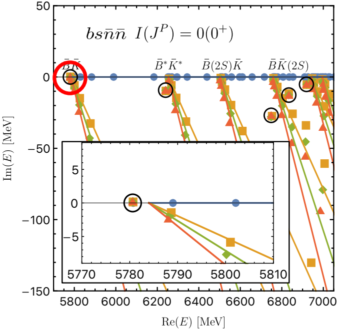
The bottom partner: $M_{\rm The.} = 6240{\rm MeV}$, $\Gamma_{\rm The.} = 18$ MeV

- B^*K^* molecular quasi-bound (-14 MeV) state.
- Decay channel: $T_{bs} \to BK, BK\pi\pi$

$Qs\bar{n}\bar{n}$ $\quad I(J^P) = 0(0^+)$

Y. K. Chen, W. L. Wu, L. Meng, S. L. Zhu, 2310.14597 PRD





States	$M-i\Gamma/2$	ΔM	$r^{\rm rms}_{c\bar{q}}$	$r^{\rm rms}_{s\bar{q}}$	$r^{\rm rms}_{cs}$	$r^{\rm rms}_{\bar{q}\bar{q}}$	Type
$D\bar{K}$	2353		0.61	0.59			S.
$D^*\bar{K}^*$	2920		0.70	0.81			S.
$0(0^+)$	2350	−3	0.61	0.59	2.45	2.52	M.
	$2906-10i$	−14	0.74	0.86	1.12	1.26	M.
	$3419-7i$		0.91	1.09	0.87	1.22	C.

States	E	ΔM	$r^{\rm rms}_{b\bar{q}}$	$r^{\rm rms}_{s\bar{q}}$	$r^{\rm rms}_{bs}$	$r^{\rm rms}_{\bar{q}\bar{q}}$	Type
$\bar{B}\bar{K}$	5784		0.62	0.59			S.
$\bar{B}^*\bar{K}^*$	6254		0.66	0.81			S.
$0(0^+)$	5781	−3	0.62	0.59	2.11	2.21	M.
	$6240-9i$	−14	0.69	0.86	1.04	1.21	M.
	$6748-28i$		1.12	1.19	0.74	1.09	C.
	$6834-13i$		0.77	1.22	1.06	1.23	C.
	$6920-5i$		0.72	1.33	0.98	1.33	C.

$cs\bar{n}\bar{n}\,0(0^+)$ bound state: $M_{\rm The.} = 2350{\rm MeV}$

- $D\bar{K}$ molecular bound (-3 MeV) state.
- Decay channel $B^+ \to D^+(T_{cs} \to K^-K^-\pi^+\pi^+)$ F. S. Yu, *Eur.Phys.J.C* **82**, 641 (2022)

bottom partner: $M_{\rm The.} = 5781{\rm MeV}$

- BK molecular bound (-3 MeV) state.
- Decay channel: $T_{bs} \to J/\Psi K^-K^-\pi^+,\ D^+K^-\pi^-$ F. S. Yu, *Eur.Phys.J.C* **82**, 641 (2022)

$Qs\bar{n}\bar{n}$ with other $I(J^P)$ numbers

Y. K. Chen, W. L. Wu, L. Meng, S. L. Zhu, 2310.14597 PRD

States	$M - i\Gamma/2$	ΔM	$r^{\rm rms}_{c\bar{q}}$	$r^{\rm rms}_{s\bar{q}}$	$r^{\rm rms}_{cs}$	$r^{\rm rms}_{\bar{q}\bar{q}}$	Type
$D\bar{K}$	2353		0.61	0.59			S.
$D^*\bar{K}$	2507		0.70	0.59			S.
$D\bar{K}^*$	2766		0.61	0.81			S.
$D^*\bar{K}^*$	2920		0.70	0.81			S.
$0(1^+)$	$3431-24i$		1.11	1.26	0.57	0.79	C.
$0(2^+)$	$3607-2i$		0.83	1.30	1.10	1.50	C.
$1(0^+)$	$3563-6i$		0.89	1.15	0.97	1.33	C.
	$3578-16i$		0.99	1.11	0.95	1.24	C.
$1(2^+)$	$3605-3i$		1.23	1.17	0.99	1.29	C.

States	$M - i\Gamma/2$	ΔM	$r^{\rm rms}_{b\bar{q}}$	$r^{\rm rms}_{s\bar{q}}$	$r^{\rm rms}_{bs}$	$r^{\rm rms}_{\bar{q}\bar{q}}$	Type
$\bar{B}\bar{K}$	5784		0.62	0.59			S.
$\bar{B}^*\bar{K}$	5841		0.66	0.59			S.
$\bar{B}\bar{K}^*$	6197		0.62	0.81			S.
$\bar{B}^*\bar{K}^*$	6254		0.66	0.81			S.
$0(1^+)$	5840	-1	0.66	0.59	2.66	2.75	M.
	$6753-24i$		1.11	1.18	0.82	1.12	C.
$1(0^+)$	$6909-26i$		1.04	0.96	0.72	1.18	C.
$1(2^+)$	$6916-2i$		1.18	1.17	0.79	1.16	C.

$bs\bar{n}\bar{n}\,0(1^+)$ bound state: $M_{\rm The.} = 5840{\rm MeV}$

- B^*K molecular bound (-1 MeV) state.
- Decay channel: Weakly, $T_{bs} \to BK\gamma$.

Other compact resonances above the $D(2S)\bar{K}$ or $B(2S)K$ threshold.

Doubly heavy tetraquarks $QQ^{(\prime)}\bar{q}\bar{q}$

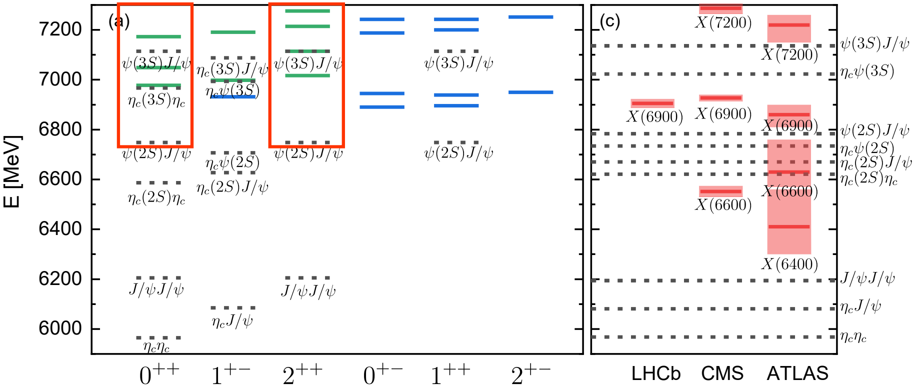
W. L. Wu, Y. Ma, Y. K. Chen, L. Meng, S. L. Zhu, 2409.03373 PRD

System	$I(J^P)$	$M - i\Gamma/2$ [MeV]	ΔE [MeV]	Configuration
$cc\bar{q}\bar{q}$	$0(1^+)$	3864	-14	(D^*D) molecule
		$4031 - 27i$		(D^*D^*) molecule ?
$bb\bar{q}\bar{q}$	$0(1^+)$	10491	-153	compact
		10642	-1	$(\bar{B}^*\bar{B})$ molecule
		$10700 - 1i$		$(\bar{B}^*\bar{B}^*)$ molecule
	$1(1^+)$	$10685 - 7i$		compact
	$1(2^+)$	$10715 - 2i$		compact
$bc\bar{q}\bar{q}$	$0(0^+)$	7129	-26	compact
	$0(1^+)$	7185	-27	compact
	$0(2^+)$	7363	-3	$(\bar{B}^*D^*)$ molecule
	$1(2^+)$	$7430 - 28i$		compact

- DD^* molecule with $\Delta E = -14$ MeV agrees with experimental $T_{cc}^+(3875)$.
- T_{cc}^* resonance near D^*D^* threshold.
- T_{bb}: deeply bound compact state, (BB^*) molecular bound state, (B^*B^*) molecular resonance.
- T_{bc}: compact bound states and resonances, $(\bar{B}^*D^*)\,0(2^+)$ molecular resonance.

Fully heavy tetraquarks

W. L. Wu, Y. K. Chen, L. Meng, S. L. Zhu, 2401.14899 PRD



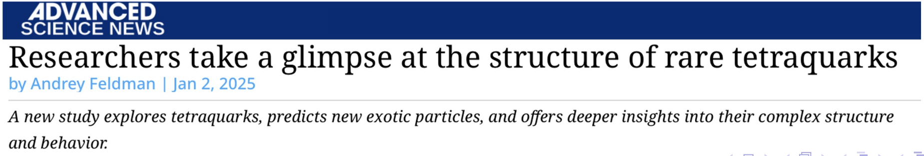
- Good candidates of experimental states $X(6900)$ and $X(7200)$ in $0^{++}, 2^{++}$ channels.
- All $cc\bar{c}\bar{c}$ resonances are compact tetraquark states.
- However, no signal of resonant state below 6.9 GeV is found. Possible explanations:
  - $X(6600)$ is too broad
  - Hints of modifying interactions (confining mechanism) in the model.

G. J. Wang, M. Oka, D. Jido, Phys.Rev.D **108** L071501 (2023)

Mini-recap:

- Unified description of the molecular and compact tetraquark states in the quark potential model.
- Consistently describe experimental observed without fine-tuning any parameters
  - ☞ $T_{cs0}(2870)$: $D^* \bar{K}^*$ molecular quasi-bound state (Feshbach resonance).
  - ☞ $T_{cc}^{+}(3875)$: DD^* molecular bound state.
  - ☞ $X(6900)$, $X(7200)$ compact resonances.
- Forward-looking predictions for future experiments:

Content	State	$I(J^{PC})$	$M - i\Gamma/2$ [MeV]	Type	Decays
$[cs\bar{n}\bar{n}]$	T_{cs}	$0(0^+)$	2350	$D\bar{K}(-3)$ M.	$B^+ \to D^+(T_{cs} \to K^-K^-\pi^+\pi^+)$
$[bs\bar{n}\bar{n}]$	T_{bs}	$0(0^+)$	5781	$BK(-3)$ M.	$T_{bs} \to J/\Psi K^-K^-\pi^+$
					$T_{bs} \to D^+K^-\pi^-$
	T_{bs}	$0(0^+)$	$6240 - 9i$	$B^*K^*(-14)$ M.	$T_{bs} \to BK$, $BK\pi\pi$
	T_{bs}	$0(1^+)$	5840	$B^*K(-1)$ M.	Weakly, $T_{bs} \to BK\gamma$
$cc\bar{n}\bar{n}$	T_{cc}	$0(1^+)$	$4031 - 27i$	$D^*D^*(-1)$ M.?	$T_{cc} \to DD\pi$, $DD\pi\pi$


ADVANCED SCIENCE NEWS

Researchers take a glimpse at the structure of rare tetraquarks

by Andrey Feldman | Jan 2, 2025

A new study explores tetraquarks, predicts new exotic particles, and offers deeper insights into their complex structure and behavior.

Mini-recap:

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	T_{bs}	$0(1^+)$	5840	$B^*K(-1)$ M.	Weakly, $T_{bs} \to BK\gamma$
$cc\bar{n}\bar{n}$	T_{cc}	$0(1^+)$	$4031 - 27i$	$D^*D^*(-1)$ M.?	$T_{cc} \to DD\pi$, $DD\pi\pi$

- Other tetraquark systems:
  Fully strange $[ss\bar{s}\bar{s}]$: Y. Ma, W. L. Wu, L. Meng, Y. K. Chen, S. L. Zhu 2408.00503 PRD
  Triply heavy $[QQ\bar{Q}\bar{q}]$ H. M. Yang, Y. Ma, W. L. Wu, S. L. Zhu, 2502.10798 PRD

Contents

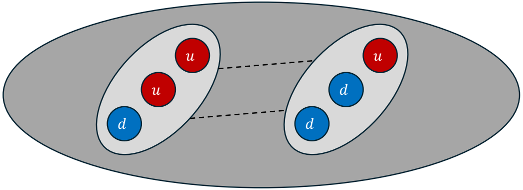
Hadronic molecules

Some exotic candidates stand out as the “star”examples:

- It is natural to interpret them as S-wave molecular bound states of the corresponding meson pairs.
- Four-body QM calculations also support molecular descriptions.

State	$I(J^{PC})$	M [MeV]	Γ [MeV]	S-wave threshold	ΔE [MeV]
$X(3872)$	$0(1^{++})$	3871.64 ± 0.06	1.19 ± 0.21	$\left[D\bar{D}^*/\bar{D}D^*\right]^{C=+1}$	-4.09 ± 0.21
$T_{cc}(3875)$	$0(1^+)^\star$	3874.74 ± 0.10	$0.048^{+0.0020}_{-0.0141}$	D^*D	-0.99 ± 0.23
$T_{cs0}(2870)$	$0(0^+)^\star$	2872 ± 16	67 ± 24	$D^*\bar{K}^*$	-30 ± 16

Hadronic molecules: Indeed, the deuteron stands as the archetype of a two-body hadronic molecular state.



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$T_{cc}(3875)$	$0(1^{+})^{\star}$	3874.74 ± 0.10	$0.048^{+0.0020}_{-0.0141}$	D^*D	-0.99 ± 0.23
$T_{cs0}(2870)$	$0(0^{+})^{\star}$	2872 ± 16	67 ± 24	$D^*\bar{K}^*$	-30 ± 16

Hadronic molecules: Indeed, the deuteron stands as the archetype of a two-body hadronic molecular state.

- $^2\mathrm{H} \Rightarrow\ ^3\mathrm{H},\ ^3\mathrm{He}$
- heavy di-hadron molecules $\overset{?}{\Rightarrow}$ heavy three-body molecules
  - $D^*D^*\bar{K}^*$ N. Ikeno, M. Bayar, E. Oset, *Phys.Rev.D* **107** 034006 (2023)
  - $D\bar{D}K$ T. C. Wu, L. S. Geng, *Phys.Rev.D* **108** 014015 (2023)
  - $D^{(*)}D^*K$ Z. Zhang, X. Y. Hu, G. He, J. Liu, J. A. Shi et al., *Phys.Rev.D* **111** 036002 (2025)
  - $D^{(*)}D^*D^*$ S. Q. Luo, T. W. Wu, M. Z. Liu, L. S. Geng, X. Liu *Phys.Rev.D* **105** 074033 (2022)
    H. L. Fu, Y. H. Lin, F. K. Guo, H. W. Hammer, U. G. Meißner et al. *JHEP* **07** 081 (2025)
  - $\bar{D}_sDK/\bar{D}^*D\eta$ Y. W. Pan, M. Z. Liu, L. S. Geng, *Phys.Rev.Lett.* **136** 151901 (2026)
  - $\cdots$

Hadronic molecules

Some exotic candidates stand out as the “star”examples:

- It is natural to interpret them as S-wave molecular bound states of the corresponding meson pairs.
- Four-body QM calculations also support molecular descriptions.

State	$I(J^{PC})$	M [MeV]	Γ [MeV]	S-wave threshold	ΔE [MeV]
$X(3872)$	$0(1^{++})$	3871.64 ± 0.06	1.19 ± 0.21	$\left[D\bar{D}^*/\bar{D}D^*\right]^{C=+1}$	-4.09 ± 0.21
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Hadronic molecules: Indeed, the deuteron stands as the archetype of a two-body hadronic molecular state.

- $DD^*\bar{K}^*$ three-body system: $(D[D^*)\bar{K}^*] \Rightarrow \frac{1}{2}(0^-)$ three-body bound state?
  - ☞ one-boson-exchange (OBE) model : has achieved significant success
    - N. A. Tornqvist, *Phys.Rev.Lett.* **67** 556 (1991)
    - N. Li, Z. F. Sun, X. Liu, S. L. Zhu, *Phys.Rev.D* **88** 114008 (2013)
    - X. K. Dong, F. K. Guo, B. S. Zou, *Progr.Phys.* **41** 65 (2021)
    - Z. Y. Lin, J. Z. Wang, J. B. Cheng, L. Meng, S. L. Zhu, *Phys.Rev.Lett.* **133** 241903 (2024)
  - ☞ In this work, we focus on $DD^*\bar{K}^*$ system within OBE model

OBE Lagrangians

The Lagrangians in the OBE model J. Z. Wang, Z. Y. Lin, B. Wang, L. Meng, S. L. Zhu, *Phys.Rev.D* **110** 114003 (2024)

- constructed according to the heavy quark symmetry, chiral symmetry and the SU(2) flavor symmetry
- considered the exchange of mesons such as π, η, ρ, ω, and σ

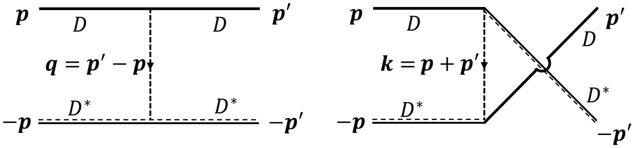
$$\begin{aligned}
\mathcal{L}_{D^{(*)}D^{(*)}\sigma} &= -2g_s D_b^{\dagger} D_b \sigma + 2g_s D_b^{*} \cdot D_b^{*\dagger}\sigma, \\
\mathcal{L}_{D^{(*)}D^{(*)}P} &= +\frac{2g_a}{f_\pi}(D_b D_{a\lambda}^{*\dagger} + D_{b\lambda}^{*} D_a^{\dagger})\partial^\lambda P_{ba} + i\frac{2g_a}{f_\pi} \\
&\quad \times v^\alpha \varepsilon_{\alpha\mu\nu\lambda} D_b^{*\mu} D_a^{*\lambda\dagger}\partial^\nu P_{ba}, \\
\mathcal{L}_{D^{(*)}D^{(*)}V} &= +\sqrt{2}\beta g_V D_b D_a^{\dagger} v \cdot V_{ba} - 2\sqrt{2}\lambda g_V \\
&\quad \times \epsilon_{\lambda\mu\alpha\beta} v^\lambda (D_b D_a^{*\mu\dagger} + D_b^{*\mu} D_a^{\dagger})(\partial^\alpha V_{ba}^\beta) \\
&\quad -\sqrt{2}\beta g_V D_b^{*} \cdot D_a^{*\dagger} v \cdot V_{ba} \\
&\quad -i2\sqrt{2}\lambda g_V D_b^{*\mu} D_a^{*\nu\dagger}\left(\partial_\mu V_\nu - \partial_\nu V_\mu\right)_{ba},
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{K^{(*)}K^{(*)}\sigma} &= -2g_s' K_b^{\dagger} K_b \sigma + 2g_s' K_b^{*} \cdot K_b^{*\dagger}\sigma, \\
\mathcal{L}_{K^{(*)}K^{(*)}P} &= +\frac{2g_a'}{f_\pi}(K_b K_{a\lambda}^{*\dagger} + K_{b\lambda}^{*} K_a^{\dagger})\partial^\lambda P_{ba} + i\frac{2g_a'}{f_\pi} \\
&\quad \times v'^\alpha \varepsilon_{\alpha\mu\nu\lambda} K_b^{*\mu} K_a^{*\lambda\dagger}\partial^\nu P_{ba}, \\
\mathcal{L}_{K^{(*)}K^{(*)}V} &= +\sqrt{2}\beta' g_V' K_b K_a^{\dagger} v' \cdot V_{ba} - 2\sqrt{2}\lambda' g_V' \\
&\quad \times \epsilon_{\lambda\mu\alpha\beta} v'^\lambda (K_b K_a^{*\mu\dagger} + K_b^{*\mu} K_a^{\dagger})(\partial^\alpha V_{ba}^\beta) \\
&\quad -\sqrt{2}\beta' g_V' K_b^{*} \cdot K_a^{*\dagger} v' \cdot V_{ba} \\
&\quad -i2\sqrt{2}\lambda' g_V' K_b^{*\mu} K_a^{*\nu\dagger}\left(\partial_\mu V_\nu - \partial_\nu V_\mu\right)_{ba},
\end{aligned}$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \end{pmatrix}, \qquad V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}$$

OBE potentials

The OBE potentials can be derived from two-body scattering Feynman diagrams



Propagator:

$$-\frac{1}{k_0^2-\boldsymbol{k}^2-m^2} \approx \frac{1}{\boldsymbol{k}^2+[m^2-(m_f-m_i)^2]} = \frac{1}{\boldsymbol{k}^2+u^2}$$

$$V_{\sigma}^{DD^*} = -g_s^2 \frac{1}{\boldsymbol{q}^2+u_\sigma^2}\mathcal{O}_\sigma, \quad V_{\pi/\eta}^{DD^*} = \frac{g_a^2}{f_\pi^2}\frac{(\boldsymbol{k}\cdot\boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{k}\cdot\boldsymbol{\epsilon}_2)}{\boldsymbol{k}^2+u_{\pi/\eta}^2}\mathcal{O}_{\pi/\eta}^C,$$

$$V_{\rho/\omega}^{DD^*} = \frac{\beta^2 g_v^2}{2}\frac{(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)}{\boldsymbol{q}^2+u_{\rho/\omega}^2}\mathcal{O}_{\rho/\omega} + 2\lambda^2 g_v^2 \frac{\boldsymbol{k}^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)-(\boldsymbol{k}\cdot\boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{k}\cdot\boldsymbol{\epsilon}_2)}{\boldsymbol{k}^2+u_{\rho/\omega}^2}\mathcal{O}_{\rho/\omega}^C$$

Additionally, we introduces a regulator to suppress contributions from the high-momentum region

$$V(\boldsymbol{k},u_{\text{exc}}) \to V(\boldsymbol{k},u_{\text{exc}})F^2\left(u_{\text{exc}},\Lambda,\boldsymbol{k}^2\right), \quad F\left(u_{\text{exc}},\Lambda,\boldsymbol{k}^2\right) = \frac{\Lambda^2-u_{\text{exc}}^2}{\Lambda^2+\boldsymbol{k}^2}$$

Parameters

Hadron masses and coupling constants [ParticleDataGroup:2024cfk]:

Mass	m_D	m_{D^*}	m_π	m_η	m_ρ	m_ω	m_σ
	1.867	2.009	0.137	0.548	0.775	0.783	0.600
Coupling	f_π	g_a	g'_a	$g_V = g'_V$	$\beta = \beta'$	$\lambda = \lambda'$	$g_s = g'_s$
	0.13025	0.57	0.88	5.8	0.9	0.56	0.76

- pseudoscalar sector : $f_\pi, \ g_a^{(\prime)}$
  - ☞ f_π is a well-known quantity, $g_a^{(\prime)}$ can be accurately extracted from the $D^*(K^*)$ width
- vector and scalar sectors: $g_s, \ g_V, \ \beta, \ \lambda$
  - ☞ g_V is an overall factor in the vector sector
  - ☞ $\beta, \ \lambda, \ g_s$: can only be determined through model-dependent methods

Introduce three ratio factors H. X. Zhu, L. Meng, Y. Ma, N. Li, W. Chen, S. L. Zhu, *Phys.Rev.D* **111** 094022 (2025)

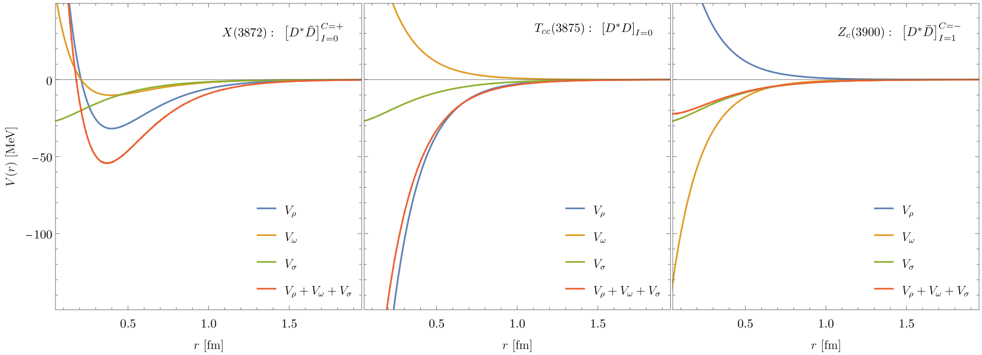
$$\lambda \to \lambda R_\lambda, \quad \beta \to \beta R_\beta, \quad g_s \to g_s R_s$$

to explore the impact of varying coupling constants in the scalar and vector sectors

- For each cutoff Λ, we use the pole positions of $X(3872)$, $T_{cc}(3875)$ and $Z_c(3900)$ to determine R_i

Fix the coupling constants

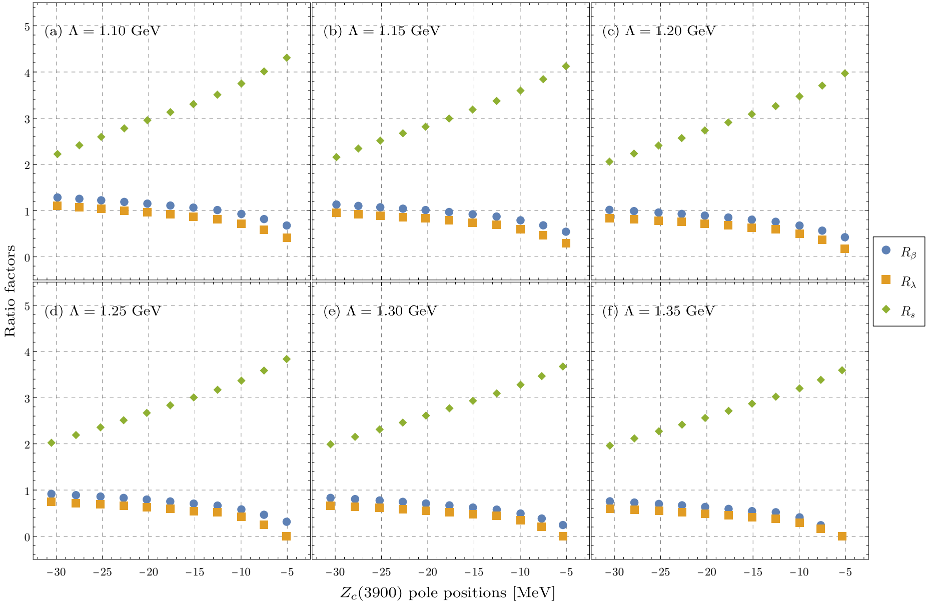
Scalar- and vector-meson-exchange interactions: $R_\beta = R_\lambda = R_s = 1$ for illustration



- $Z_c(3900)$: less definite and primarily sensitive to R_s
  - ☞ One truly free parameter: $Z_c(3900)$ virtual state pole position $\Leftrightarrow$ R_s
  - ☞ R_β, R_λ: fixed by $X(3872)$ and $T_{cc}(3875)$

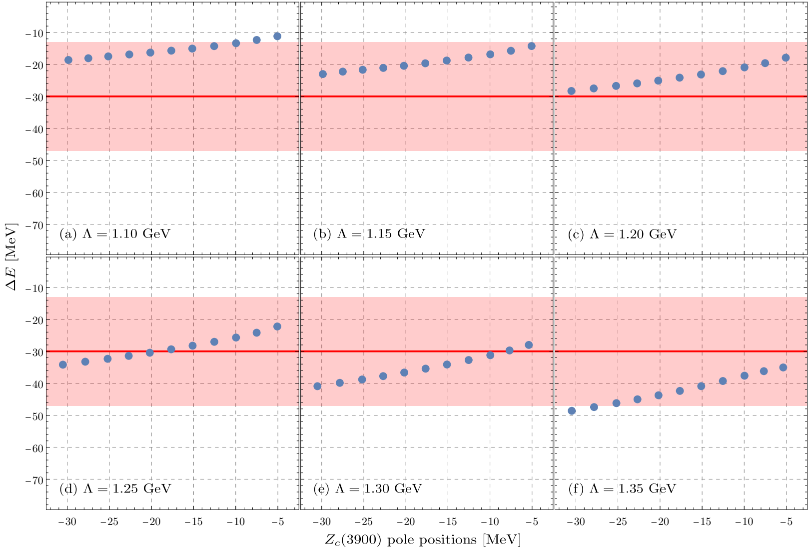
Fix the coupling constants

Y. K. Chen, L. Meng, J. Z. Wang, S. L. Zhu, 2602.12010 PRD



$T_{cs0}(2870)$ pole positions

Y. K. Chen, L. Meng, J. Z. Wang, S. L. Zhu, 2602.12010 PRD



- $X(3872)$, $T_{cc}(3875)$, $Z_c(3900)$, and $T_{cs0}(2870)$ can be uniformly described using each set of parameters.

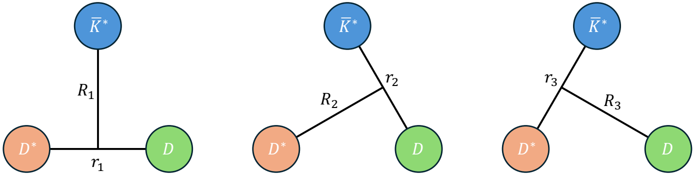
Three-body calculation

- three-body nonrelativistic Hamiltonian:

$$H = \sum_{i=1}^{3}\left(m_i + \frac{p_i^2}{2m_i}\right) + \sum_{i<j} V_{ij}, \quad V_{ij}: \text{OBE pair interaction}$$

the three-body force is neglected

- three-body wave function is addressed in GEM E. Hiyama, Y. Kino M. Kamimura, *Prog.Part.Nucl.Phys.* **51** 223 (2003)

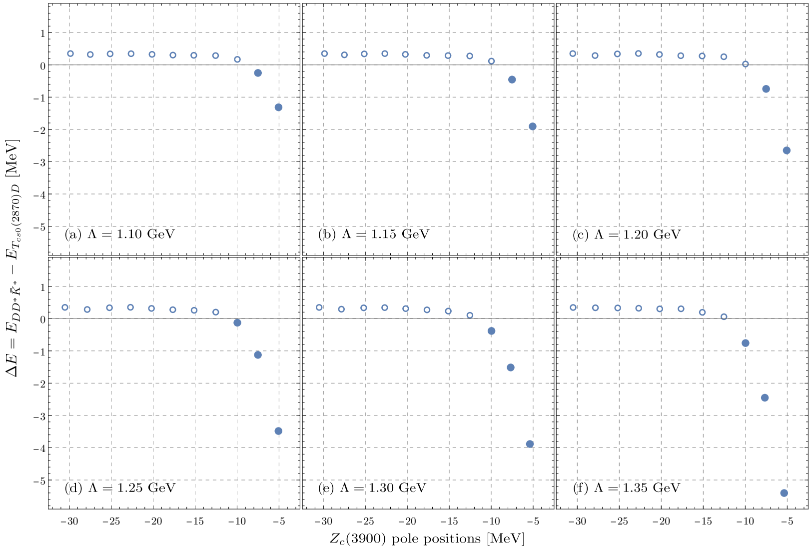


$$\psi = \sum_{\alpha}\sum_{\text{jac}=1}^{3}\sum_{n_1,n2=1}^{n_{\max}} C^{(\alpha)}_{\text{jac},n_{\text{jac}}}\chi^{(\alpha)}_{S,I}\exp\left[-\frac{r_{\text{jac}}^2}{r_{n_1,\text{jac}}} - \frac{R_{\text{jac}}^2}{r_{n_2,\text{jac}}}\right], \quad r_{n_i,\text{jac}} = r_{1,\text{jac}}\left(\frac{r_{n_{\max},\text{jac}}}{r_{1,\text{jac}}}\right)^{\frac{n_i-1}{n_{\max}-1}}$$

$r_1 = 0.01$ fm, $r_{\max} = 10$ fm, $n_{\max} = 20$

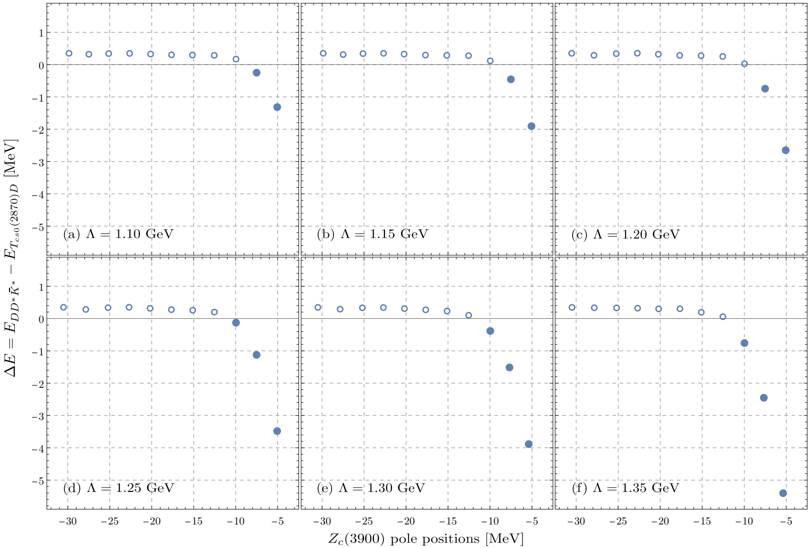
- identify resonances by CSM: $r \to re^{i\theta}, \quad p \to pe^{-i\theta}$

Bound state results: $\frac{1}{2}(0^-)$ three-body binding energy Y. K. Chen, L. Meng, J. Z. Wang, S. L. Zhu, 2602.12010 PRD



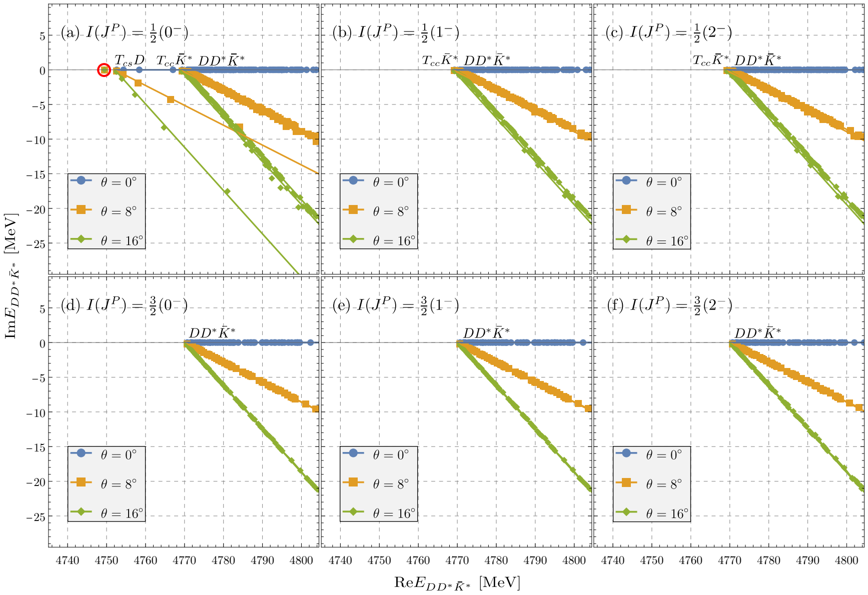
- three-body molecular bound state exists when Z_c is a near-threshold pole (within about 10 MeV)
- varying the cutoff would not alter the results qualitatively

Bound state results: $\frac{1}{2}(0^-)$ three-body binding energy Y. K. Chen, L. Meng, J. Z. Wang, S. L. Zhu, 2602.12010 PRD



- possible decay channels: $H_{DD^*\bar{K}^*} \to DD\bar{K}\pi\pi$, $H_{DD^*\bar{K}^*} \to DD\bar{K}$
- no bound state was found in other $I(J^P)$

CSM results: $\Lambda = 1.20$ GeV, $E_{Z_c} = -5$ MeV for illustration Y. K. Chen, L. Meng, J. Z. Wang, S. L. Zhu, 2602.12010 PRD



- no resonance was found in any of the channels

Contents

Summary and Outlook

- Quark level: A unified picture
  - Unified treatment of compact and molecular tetraquarks.
  - Experimental connections: $T_{cs0}(2870)$, $T_{cc}(3875)$, $X(6900)$, $X(7200)$
  - Predictions
- Hadron-level: Three-body molecular states
  - Recalibrated OBE parameters
    unified description of the $X(3872)$, $T_{cc}(3875)$, $Z_c(3900)$ and $T_{cs0}(2870)$
  - Potential existence of the $DD^*\bar{K}^*$ molecular state
  - no resonance was found in any of the channels
- Outlook:
  - Further development of quark potential model
  - Unquenched effects
  - Fully dynamic Five- or Six-body calculations
  - DNNs/MC + CSM

Thanks for your attention

Back up (1)

- AL1 parameters

$$m_u = m_d = 0.315\text{GeV}; \quad m_s = 0.577\text{GeV}; \quad m_c = 1.836\text{GeV}; \quad m_b = 5.227\text{GeV}$$

$$\kappa = 0.5069; \quad \kappa' = 1.8609; \quad \lambda = 0.1653\text{GeV}^2; \quad \Lambda = 0.8321\text{GeV}$$

$$r_0 = A\left(\frac{2m_i m_j}{m_i + m_j}\right)^{-B}, \quad B = 0.2204; \quad A = 1.6553\text{GeV}^{B-1}$$

- meson spectrum

Mesons	$I(J^P)$	$AL1$	Exp.[PDG]
π	$1(0^-)$	138	137.28
K	$\frac{1}{2}(0^-)$	491	495.64
D	$\frac{1}{2}(0^-)$	1862	1867.25
D_s	$0(0^-)$	1963	1968.35
η_c	$0(0^-)$	3006	2983.90
B	$\frac{1}{2}(0^-)$	5294	5279.50
B_s	$0(0^-)$	5361	5366.88
B_c	$0(0^-)$	6293	6274.90
η_b	$0(0^-)$	9434	9398.70

Mesons	$I(J^P)$	$AL1$	Exp.[PDG]
ρ	$1(1^-)$	770	775.26
K^*	$\frac{1}{2}(1^-)$	904	893.61
D^*	$\frac{1}{2}(0^-)$	2016	2008.55
D_s^*	$0(1^-)$	2102	2112.20
J/Ψ	$0(1^-)$	3102	3096.90
B^*	$\frac{1}{2}(1^-)$	5351	5324.70
B_s^*	$0(1^-)$	5418	5415.40
B_c^*	$0(1^-)$	6345	–
Υ	$0(1^-)$	9472	9460.30

Back up (2)

When the mesons $(Q\bar{q})$ and $(s\bar{q})$ form a molecular state, the wave function satisfying the Pauli principle is

$$|\Psi_{\mathcal{A}}\rangle = |(Q\bar{q}_1)(s\bar{q}_2)\rangle - |(Q\bar{q}_2)(s\bar{q}_1)\rangle$$

- each light quark belongs to both mesons simultaneously
- neither $\left\langle \Psi_{\mathcal{A}} \left| r^2_{Q\bar{q}} \right| \Psi_{\mathcal{A}} \right\rangle$ nor $\left\langle \Psi_{\mathcal{A}} \left| r^2_{s\bar{q}} \right| \Psi_{\mathcal{A}} \right\rangle$ can reflect the size of the constituent mesons.

To avoid such ambiguities, we uniquely decompose the resulting antisymmetric wave function

$$\begin{aligned}\Psi_{\mathcal{A}}(\theta) &= \left|\{Q\bar{q}_1\}_{1_c}\{s\bar{q}_2\}_{1_c}\right\rangle \otimes \left|\psi_1^\theta\right) + \left|\{Q\bar{q}_2\}_{1_c}\{s\bar{q}_1\}_{1_c}\right\rangle \otimes \left|\psi_2^\theta\right) \\ &= \mathcal{A}\left[\left|\{Q\bar{q}_1\}_{1_c}\{s\bar{q}_2\}_{1_c}\right\rangle \otimes \left|\psi_1^\theta\right)\right]\end{aligned}$$

we only use $\left|\psi_1^\theta\right\rangle$ to define the rms radius

$$r_{ij}^{\mathrm{rms}} \equiv \mathrm{Re}\left[\sqrt{\frac{\left(\psi_1^\theta \left| r_{ij}^2 e^{2i\theta} \right| \psi_1^\theta\right)}{\left(\psi_1^\theta \mid \psi_1^\theta\right)}}\right]$$