# Prediction of fully charmed bound states $X(6400)\eta_c$ and $X(6600)J/\psi$

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# Outline

# 1.4 Fully charmed family

- Charmonium $c\bar{c}$
  - Eldest $J/\psi$, Ting and Richter, 1974
  - Other members, $\psi(2s)$, $\psi(3s)$, $\eta_c$, $\eta_c(2s)$, $\eta_c(3s)$...
- Fully charmed tetra-quark $c^2\bar{c}^2$
  - Theoretical exploration, Iwasaki, 1975
  - Experimental discovery, X(6900), LHCb, 2020
    - X(6600), X(6900), X(7100), $J^{PC}=2^{++}$, n=1,2,3 or 2, 3, 4, CMS, arXiv: 2602.02252
  - X(6400), ATLAS, unconfirmed, $J^{PC}=0^{++}$?
- Is there fully charmed hexa-qaurk state $c^3\bar{c}^3$?
  - Mass spectrum, inner structure..., unknown
- We want to do
  - Search for possible bound $c^3\bar{c}^3$ based on $c\bar{c}$ and $cc\bar{c}\bar{c}$
  - Explore their natures, structure and forming mechanism.

# 2.1 Model Hamiltonian

- Without light quark, no meson-exchange, no mixing like $X(3872)$
- One-gluon-exchange interaction

$$V_{ij}^{oge} = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j}\right)$$

- Quark confinement potential

$$V_{ij}^{con} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c r_{ij}^2$$

- Full model Hamiltonian

$$H_n = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_c + \sum_{i>j}^{n}\left(V_{ij}^{oge} + V_{ij}^{con}\right)$$

- WF of heavy meson

$$\Phi_{IJ} = \chi_c \otimes \eta_i \otimes \psi_s \otimes \phi_{l_r m_r}(\mathbf{r})$$

- Color part

$$\chi_c = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

- Flavor part

$$\eta_i = Q\bar{q},\ \ Q\bar{Q},\ \ where\ \ Q = c,\ \ b\ and\ \ q = u, q, s$$

- Spin part

$$S = 0: \quad \psi_s = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow); \quad S = 1: \quad \psi_s = \downarrow\downarrow, \quad \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad \uparrow\uparrow$$

- Orbit part, Gaussian expansion method

$$\phi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}),$$

# 2.3 Model parameters

- Solving Schrodinger equation and fitting spectrum

Table 1: Model parameters, quark mass and $\Lambda_0$ unit in MeV, $a_c$ unit in MeV$\cdot$fm$^{-2}$, $r_0$ unit in MeV$\cdot$fm and $\alpha_0$ is dimensionless.

| Parameter | $m_{u,d}$ | $m_s$ | $m_c$ | $m_b$ | $a_c$ | $\alpha_0$ | $\Lambda_0$ | $r_0$ |
|---|---|---|---|---|---|---|---|---|
| Value. | 313 | 494 | 1664 | 5006 | $-150$ | 4.25 | 40.85 | 119.3 |

Table 2: Heavy-meson spectra, mass unit in MeV and $\langle r^2\rangle^{\frac{1}{2}}$ unit in fm.

| State | $D$ | $D^*$ | $D_s$ | $D_s^*$ | $\eta_c$ | $J/\psi$ | $B$ |
|---|---|---|---|---|---|---|---|
| Theory | 1886 | 2000 | 1982 | 2109 | 2965 | 3103 | 5261 |
| PDG | 1869 | 2007 | 1968 | 2112 | 2980 | 3097 | 5280 |
| $\langle \mathbf{r}^2\rangle^{\frac{1}{2}}$ | 0.50 | 0.53 | 0.43 | 0.47 | 0.31 | 0.35 | 0.50 |
| State | $B^*$ | $B_s$ | $B_s^*$ | $B_c$ | $B_c^*$ | $\eta_b$ | $\Upsilon(1S)$ |
| Theory | 5305 | 5346 | 5399 | 6244 | 6336 | 9376 | 9486 |
| PDG | 5325 | 5366 | 5416 | 6277 | ... | 9391 | 9460 |
| $\langle \mathbf{r}^2\rangle^{\frac{1}{2}}$ | 0.51 | 0.42 | 0.44 | 0.26 | 0.29 | 0.17 | 0.19 |

# 3.1 Configurations of $c^2\bar{c}^2$

- Configurations of $c^2\bar{c}^2$.

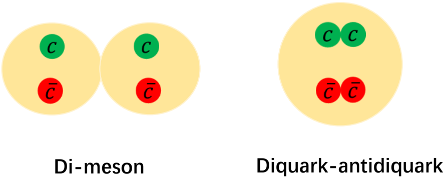


Figure 1: Two configurations of $c^2\bar{c}^2$

- Di-meson configuration: unbound in quark models.
- Diquark configuration: compact, color force.

# 3.2 Natures of diquark

- S-wave $[cc]$ and $[\bar{c}\bar{c}]$ in the model
  - Diquarks are compound objects with various color-flavor-spin-space combinations, Pauli principle.
  - "Good" diquark $[cc]$: spin s=1, color $\bar{\mathbf{3}}$;
    'Good" antidiquark $(\bar{c}\bar{c})$: spin s=1, color $\mathbf{3}$;
    Strong attraction, Coulomb interaction dominant.
  - "Bad" diquark $[cc]$: spin s=0, color $\mathbf{6}$;
    "Bad" antidiquark $(\bar{c}\bar{c})$: spin s=0, color $\bar{\mathbf{6}}$;
    Strong repulsion, Coulomb interaction dominant.

- Total WF of $[cc][\bar{c}\bar{c}]$

$$\Phi^{J^{PC}}_{[cc][\bar{c}\bar{c}]} = \sum_{\alpha} c_{\alpha} \left[\Phi_{[cc]}\Phi_{[\bar{c}\bar{c}]}\phi_{00}(\boldsymbol{\lambda})\right]$$

  - $\alpha = \{color, spin\}$ and $c_{\alpha}$ can be determined by solving Schrodinger eq.
  - $\phi_{00}(\boldsymbol{\lambda})$ is the relative WF between $[cc]$ and $[\bar{c}\bar{c}]$.

# 3.3 WF of diquark picture

- Color of $[cc][\bar{c}\bar{c}]$

$$(\bar{\mathbf{3}} \oplus \mathbf{6}) \otimes (\mathbf{3} \oplus \bar{\mathbf{6}}) = \underbrace{(\bar{\mathbf{3}} \otimes \mathbf{3})}_{\mathbf{1}\oplus\mathbf{8}} \oplus \underbrace{(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}})}_{\mathbf{8}\oplus\overline{\mathbf{10}}} \oplus \underbrace{(\mathbf{6} \otimes \mathbf{3})}_{\mathbf{8}\oplus\mathbf{10}} \oplus \underbrace{(\mathbf{6} \otimes \bar{\mathbf{6}})}_{\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27}}$$

- Spin of $[cc][\bar{c}\bar{c}]$

$$s = \begin{cases} 0{=}1 \oplus 1 \quad \text{or} \quad 0 \oplus 0 \\ 1{=}1 \oplus 1; \quad 2{=}1 \oplus 1 \end{cases}$$

- Orbit of $[cc][\bar{c}\bar{c}]$
  - Jacobi ordinates: $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \ \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4, \ \boldsymbol{\lambda} = \frac{\mathbf{r}_1+\mathbf{r}_2}{2} - \frac{\mathbf{r}_3+\mathbf{r}_4}{2}$
  - Also, $\phi(\mathbf{r})$, $\phi(\mathbf{R})$, and $\phi(\boldsymbol{\lambda})$ are expressed by Gaussian expansion method.

- $J^{PC}$ of $[cc][\bar{c}\bar{c}]$

$$0^{++}: \left[[cc]^{1}_{\bar{3}}[\bar{c}\bar{c}]^{1}_{3}\right]^{0}_{\mathbf{1}}, \ \left[[cc]^{0}_{6}[\bar{c}\bar{c}]^{0}_{\bar{6}}\right]^{0}_{\mathbf{1}}; \quad 1^{+-}: \left[[cc]^{1}_{\bar{3}}[\bar{c}\bar{c}]^{1}_{3}\right]^{1}_{\mathbf{1}}; \quad 2^{++}: \left[[cc]^{1}_{\bar{3}}[\bar{c}\bar{c}]^{1}_{3}\right]^{2}_{\mathbf{1}}$$

# 3.4 Natures of $[cc][\bar{c}\bar{c}]$

- Eigenenergy $E_4$

$$(H_4 - E_4)\Phi^{J^{PC}}_{[cc][\bar{c}\bar{c}]} = 0$$

- Average distance between two particles

$$\sqrt{\langle \mathbf{r}^2_{ij} \rangle} = \sqrt{\langle \Phi^{J^{PC}}_{[cc][\bar{c}\bar{c}]} | \mathbf{r}^2_{ij} | \Phi^{J^{PC}}_{[cc][\bar{c}\bar{c}]} \rangle}$$

- Model predictions for $[cc][\bar{c}\bar{c}]$ in 2020 Phys.Rev.D 103, 014001 (2021)

Table 3: The $[cc][\bar{c}\bar{c}]$ ground states in the model, energies in MeV.

| $J^{PC}$ | $[cc]^1_{\bar{\mathbf{3}}} \otimes [\bar{c}\bar{c}]^1_{\mathbf{3}}$ | $[cc]^0_{\mathbf{6}} \otimes [\bar{c}\bar{c}]^0_{\bar{\mathbf{6}}}$ | Mixing | $\sqrt{\langle \mathbf{r}^2_{ij} \rangle}$ (fm) | Candidate |
|---|---|---|---|---|---|
| $0^{++}$ | 6573, 36% | 6537, 64% | 6491 | $\in (0.40, 0.50)$ | $X(6400)$ |
| $1^{+-}$ | 6580 | ... | ... | $\in (0.40, 0.50)$ | ... |
| $2^{++}$ | 6605 | ... | ... | $\in (0.40, 0.50)$ | $X(6600)$ |

# 3.4 Natures of $[cc][\bar{c}\bar{c}]$

- $[cc]^0_{\mathbf{6}} \otimes [\bar{c}\bar{c}]^0_{\bar{\mathbf{6}}}$ is lower, strong attraction between $[cc]^0_{\mathbf{6}}$ and $[\bar{c}\bar{c}]^0_{\bar{\mathbf{6}}}$.

Table 4: Color matrix elements, $\hat{\mathbf{O}}^c_{ij} = \boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j$.

| Configuration | $\langle \hat{\mathbf{O}}^c_{12} \rangle$ | $\langle \hat{\mathbf{O}}^c_{34} \rangle$ | $\langle \hat{\mathbf{O}}^c_{13} \rangle$ | $\langle \hat{\mathbf{O}}^c_{24} \rangle$ | $\langle \hat{\mathbf{O}}^c_{14} \rangle$ | $\langle \hat{\mathbf{O}}^c_{23} \rangle$ |
|---|---|---|---|---|---|---|
| $[cc]^1_{\bar{\mathbf{3}}} \otimes [\bar{c}\bar{c}]^1_{\mathbf{3}}$ | $-\frac{8}{3}$ | $-\frac{8}{3}$ | $-\frac{4}{3}$ | $-\frac{4}{3}$ | $-\frac{4}{3}$ | $-\frac{4}{3}$ |
| $[cc]^0_{\mathbf{6}} \otimes [\bar{c}\bar{c}]^0_{\bar{\mathbf{6}}}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $-\frac{10}{3}$ | $-\frac{10}{3}$ | $-\frac{10}{3}$ | $-\frac{10}{3}$ |

- Matching the experimental data.
  - $X(6600)$, $J^{PC} = 2^{++}$, n=1, compact diquark configuration
  - $X(6400)$, $J^{PC} = 0^{++}$, scalar partner of $X(6400)$

- Configurations of $c^3\bar{c}^3$.

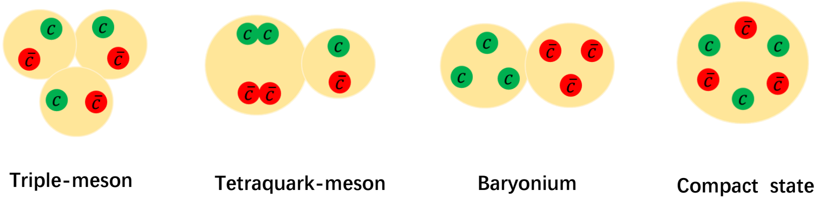


Figure 2: Four configurations

- Several colorless subclusters: tri-meson, tetraquark-meson, baryonium.
- Overall colorless (hidden color): compact state.

- Tetraquark-meson configuration $[cc][\bar{c}\bar{c}][c\bar{c}]$.

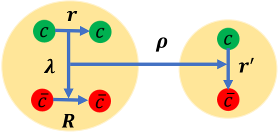


Figure 3: Jacobian coordinates

- WF of $[cc][\bar{c}\bar{c}][c\bar{c}]$

$$\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]} = \sum_{J_1,J_2} c_{J_1J_2}\mathcal{A}\left\{\left[\Phi^{J_1^{P_1C_1}}_{[cc][\bar{c}\bar{c}]}\Phi^{J_2^{P_2C_2}}_{[c\bar{c}]}\right]\phi_{00}(\boldsymbol{\rho})\right\}$$

  - Quantum numbers: $J = J_1 \oplus J_2$, $P = P_1P_2$, $C = C_1C_2$.
  - Antisymmetrization operator: $\mathcal{A} = (1 - P_{15} - P_{25})(1 - P_{36} - P_{46})$.
  - Coefficient $C_{J_1J_2}$ can be determined by solving six schrodinger eq.

- Eigenenergy $E_6$

$$(H_6 - E_6)\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]} = 0$$

- Binding energy

$$E_b = E_6 - E_{[cc][\bar{c}\bar{c}]} - E_{[c\bar{c}]}$$

- Binding mechanism

$$\langle V^{\chi}\rangle = \langle\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]}|V^{\chi}|\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]}\rangle - \langle\Phi^{J_1^{P_1C_1}}_{[cc][\bar{c}\bar{c}]}|V^{\chi}|\Phi^{J_1^{P_1C_1}}_{[cc][\bar{c}\bar{c}]}\rangle - \langle\Phi^{J_2^{P_2C_2}}_{[c\bar{c}]}|V^{\chi}|\Phi^{J_2^{P_2C_2}}_{[c\bar{c}]}\rangle$$

- Separation between $[cc][\bar{c}\bar{c}]$ and $[c\bar{c}]$

$$\sqrt{\langle\boldsymbol{\rho}^2\rangle} = \sqrt{\langle\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]}|\boldsymbol{\rho}^2|\Psi^{J^{PC}}_{[cc][\bar{c}\bar{c}][c\bar{c}]}\rangle}$$

# 4.3 Natures of $[cc][\bar{c}\bar{c}][c\bar{c}]$

- Stability

Table 5: Stability of $X(6400)\eta_c$, $X(6400)J/\psi$, $X(6600)\eta_c$, and $X(6600)J/\psi$.

| | | | |
|---|---|---|---|
| Unbound states | $X(6400)J/\psi$, $1^{--}$ | $X(6600)J/\psi$, $1^{--}$ | Mixing |
| | $X(6600)\eta_c$, $2^{-+}$ | $X(6600)J/\psi$, $2^{--}$ | |
| Bound states | $X(6400)\eta_c$, $0^{-+}$ | $X(6600)J/\psi$, $3^{--}$ | |

- Bound states

Table 6: Binding energy and binding mechanism, all energies in MeV.

| $J^{PC}$ | State | $E_b$ | $\langle V^{\mathrm{cp}}\rangle$ | $\langle V^{\mathrm{cc}}\rangle$ | $\langle V^{\mathrm{cm}}\rangle$ | $\langle T\rangle$ | $\sqrt{\langle \rho^2\rangle}$ (fm) |
|---|---|---|---|---|---|---|---|
| $0^{-+}$ | $X(6400)\eta_c$ | $-59.0$ | $-1.2$ | $-80.6$ | $-20.7$ | 43.5 | 0.68 |
| $3^{--}$ | $X(6600)J/\psi$ | $-15.6$ | $-1.3$ | $-25.1$ | 1.9 | 8.6 | 0.91 |

# 4.3 Natures of $[cc][\bar{c}\bar{c}][c\bar{c}]$

- Bound state $X(6400)\eta_c$ with $J^{PC} = 0^{-+}$.
  - Binding energy: $-59.0$ MeV, deeply bound state.
  - Binding mechanisms
    - chromocoulomb>chromomagnetic>confinement potential
  - Overlapped spatial configuration: compact hexaquark state,
    - Separation: $\sqrt{\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}}} = 0.68$ fm.
- Bound state $X(6600)/J\psi$ with $J^{PC} = 3^{--}$.
  - Binding energy: $-15.6$ MeV, shallow bound state.
  - Binding mechanisms
    - chromocoulomb>confinement potential
  - Well-separated configuration: deuteron-like molecules,
    - Separation: $\sqrt{\langle \boldsymbol{\rho}^2 \rangle^{\frac{1}{2}}} = 0.91$ fm

# 4.4 Natures of other configurations

- Similar calculations for other configurations.

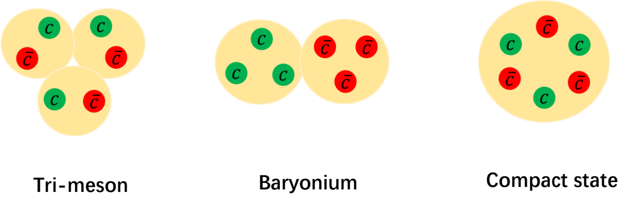


Figure 4: Other configurations

- Unbound tri-meson $[c\bar{c}][c\bar{c}][c\bar{c}]$
  - Unbound di-meson $[c\bar{c}][c\bar{c}]$ in the quark model.
  - Unbound tri-meson $[c\bar{c}][c\bar{c}][c\bar{c}]$ for all possible $J^{PC}$.
- Unbound baryonium $[ccc][\bar{c}\bar{c}\bar{c}]$
  - No exchange term, color-dependent interactions vanish, noninteracting baryonium, model prediction $m_{\Omega_{ccc}} = 4895$ MeV.

# 4.4 Natures of other configurations

- Compact state $[ccc\bar{c}\bar{c}\bar{c}]$
  - Hidden color, complex color configurations.

$$[[ccc]_{\mathbf{8}}[\bar{c}\bar{c}\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}, [[cc\bar{c}]_{\mathbf{3}}[\bar{c}\bar{c}c]_{\bar{\mathbf{3}}}]_{\mathbf{1}}, [[cc\bar{c}]_{\bar{\mathbf{6}}}[\bar{c}\bar{c}c]_{\mathbf{6}}]_{\mathbf{1}},$$

$$[[c\bar{c}]_{\mathbf{8}}[c\bar{c}]_{\mathbf{8}}[c\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}, [[cc]_{\bar{\mathbf{3}}}[\bar{c}\bar{c}]_{\mathbf{3}}[c\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}, [[cc]_{\bar{\mathbf{3}}}[\bar{c}\bar{c}]_{\bar{\mathbf{6}}}[c\bar{c}]_{\mathbf{8}}]_{\mathbf{1}},$$

$$[[cc]_{\mathbf{6}}[\bar{c}\bar{c}]_{\mathbf{3}}[c\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}, [[cc]_{\mathbf{6}}[\bar{c}\bar{c}]_{\bar{\mathbf{6}}}[c\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}, \ldots\ldots$$

- Similar energy for all color configurations.
  - Taking $[[ccc]_{\mathbf{8}}[\bar{c}\bar{c}\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}$ as an example,
    $J^{PC} = 0^{-+}$, 10016 MeV, $\sqrt{\langle \mathbf{r}^2 \rangle} \simeq 0.50$ fm
    $J^{PC} = 1^{--}$, 10051 MeV, $\sqrt{\langle \mathbf{r}^2 \rangle} \simeq 0.50$ fm
  - Higher than $[c\bar{c}][c\bar{c}][c\bar{c}]$, $[cc][\bar{c}\bar{c}][c\bar{c}]$, and $[ccc][\bar{c}\bar{c}\bar{c}]$.

- Energy ordering for four configurations.
  - $[c\bar{c}][c\bar{c}][c\bar{c}] < [cc][\bar{c}\bar{c}][c\bar{c}] < [ccc][\bar{c}\bar{c}\bar{c}] < [ccc\bar{c}\bar{c}\bar{c}]$
  - Decay into triple-charmonium via quark rearrangement.
- $[cc][\bar{c}\bar{c}]$-cluster, similar to $\alpha$-cluster in nuclear physics?

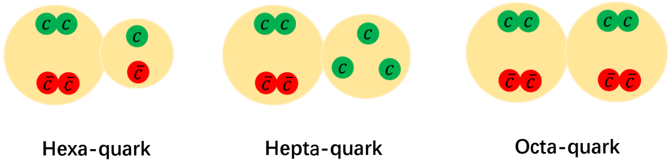


Figure 5: $[cc][\bar{c}\bar{c}]$-cluster in hexa-, hepta-, and octa-quark states

# 5. Summary

- Matching the experimental data well.
  - $X(6600)$, $J^{PC}=2^{++}$, compact diquark configuration
  - $X(6400)$, $J^{PC}=0^{++}$, scalar partner of $X(6600)$
- Deuteron-like molecule $X(6600)/J\psi$ with $J^{PC}=3^{--}$.
  - Binding energy: $-15.6$ MeV, shallow bound state.
  - Binding mechanisms: chromocoulomb interaction
- Compact bound state $X(6400)\eta_c$ with $J^{PC}=0^{-+}$.
  - Binding energy: $-59.0$ MeV, deeply bound state.
  - Binding mechanisms: chromocoulomb, chromomagnetic interaction

Thank you for your attention!