B Decays in the Precision Era:

examples and implications

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Outline

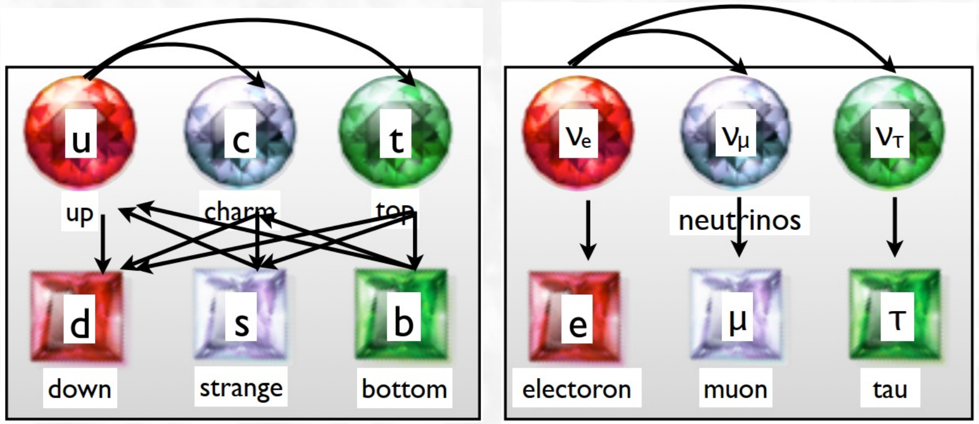
- **Introduction**
- **Theoretical framework**
- **Rare decays:** $B_s \to \mu^+\mu^-$, $B \to X_s\gamma$, $b \to s\ell^+\ell^-$, $b \to s\nu\bar{\nu}$
- **Lepton flavor universality violation in** $b \to c\tau\nu$
- **Class-I two-body B decays:** $b \to c\bar{u}d(s)$ **decays**
- **Summary**

Flavor physics

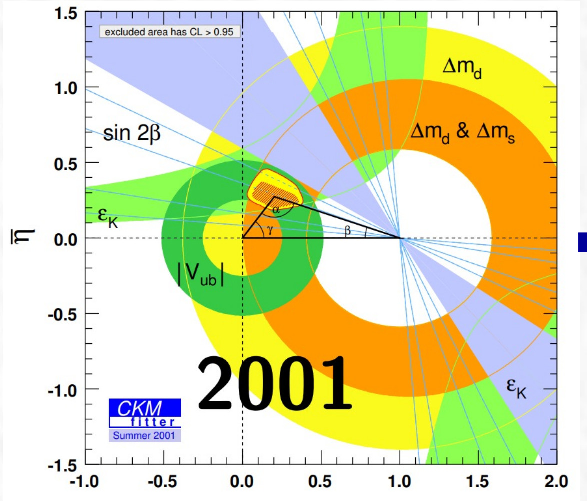
- **An important branch of particle physics: most free parameters come from flavor sector**

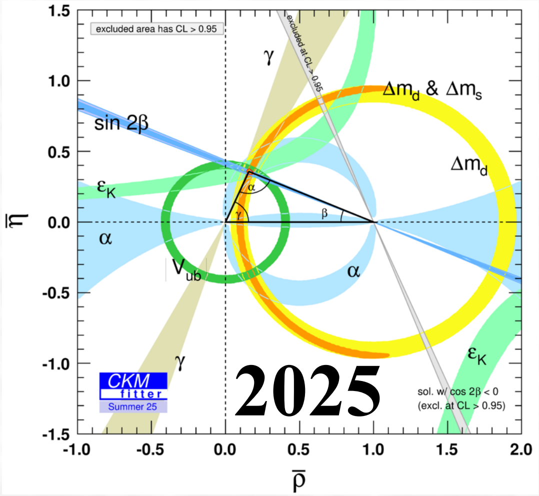
gauge sector	Higgs sector	flavor sector
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi$	$+ \lvert D_\mu \phi \rvert^2 - V(\phi)$	$+ \psi_i y_{ij} \psi_j \phi + h.c.$
describes the gauge interactions of the quarks and leptons	breaks electro-weak symmetry and gives mass to the $W^\pm$ and Z bosons	leads to masses and mixings of the quarks and leptons
parametrized by 3 gauge couplings g_1, g_2, g_3	2 free parameters Higgs mass Higgs vev	$\boxed{\text{22 free parameters}}$ to describe the masses and mixings of the quarks and leptons

- **Flavor physics:** transitions between different species



- **Much exp. & theo. progress achieved, now entering a precision flavor era**



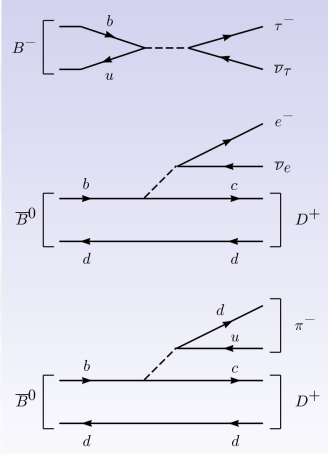


- **With current LHCb, Belle-II & future STCF, CEPC, FCC ➡ bright prospects expected**

Why B physics

- ◻ **b-quark massive enough to have many decay modes & various observables available**

$$b \to \begin{cases} c \\ u \end{cases} + W^{*-} \to \begin{cases} c \\ u \end{cases} + \begin{cases} \bar{u} + d \\ \bar{c} + s \\ \bar{u} + s \\ \bar{c} + d \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{cases}$$



purely leptonic

semi-leptonic

hadronic

branching ratios,

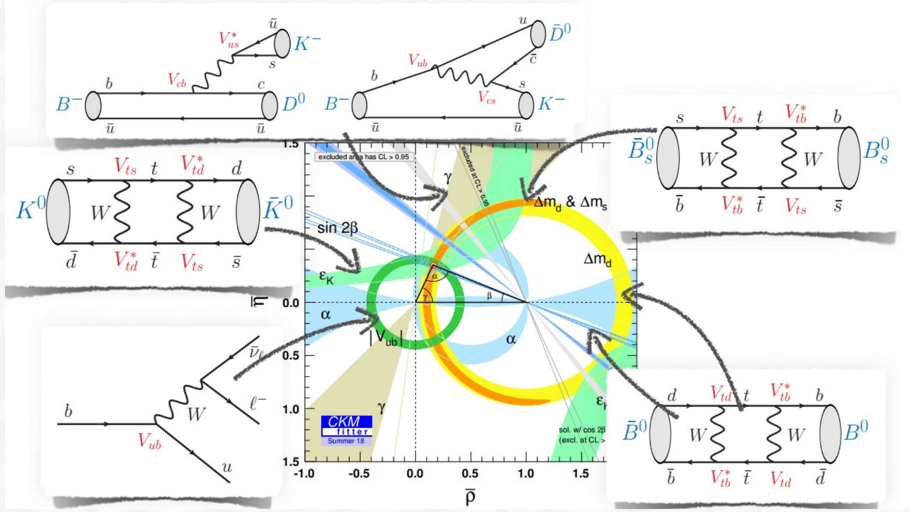
CP asymmetries,

polarizations,

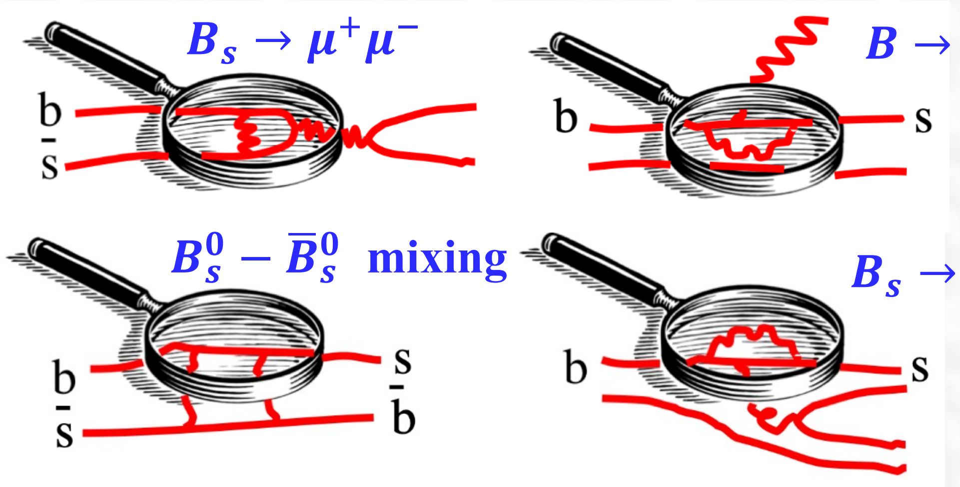
Dalitz distributions,

. . .

- ◻ **Test the SM: quark mixing & KM mechanism**



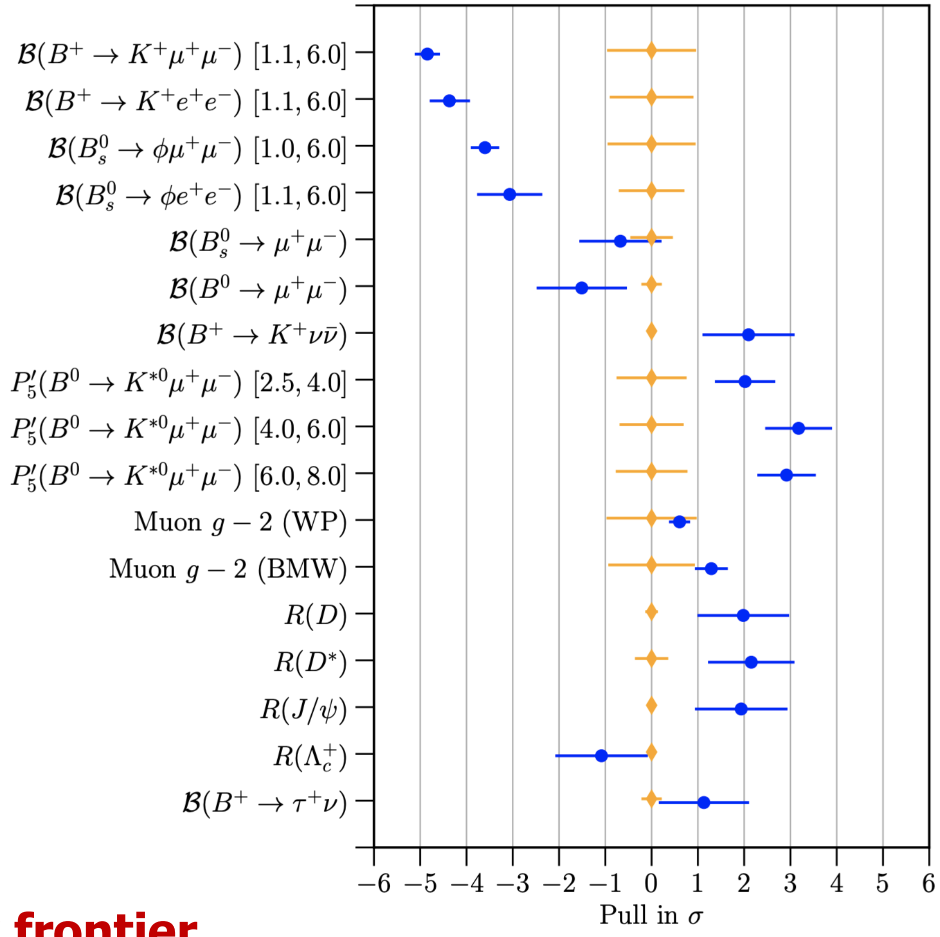
- ◻ **Efficient indirect probe of various NP**



B anomalies/puzzles

https://www.nikhef.nl/~pkoppenb/anomalies.html

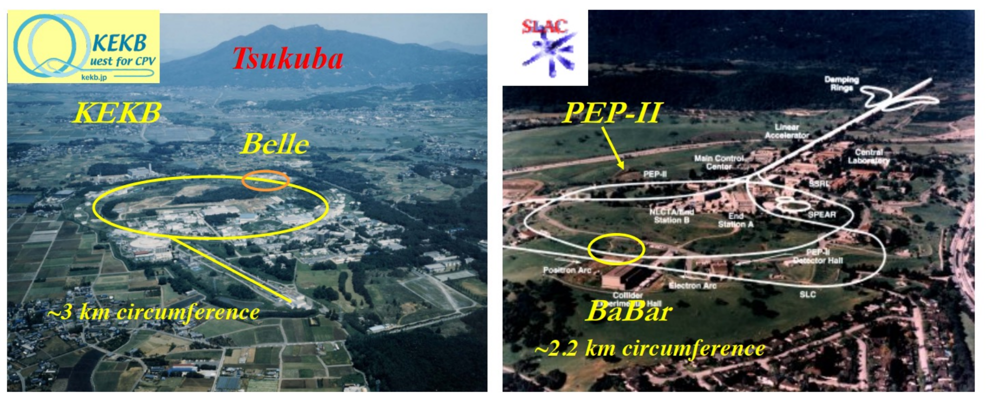
- **CKM UT: most predictions consistent with data**
- **Several discrepancies observed in B physics:**
  - $R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}l\nu_l)}$: lepton flavor university violation?
  - $\mathcal{B}(B^+ \to K^{(*)+}\mu^+\mu^-), P_5'(B^0 \to K^{*0}\mu^+\mu^-), \mathcal{B}(B^+ \to K^+\nu\bar{\nu})$: insufficient QCD estimation or NP in $b \to s$ FCNC decays?
  - $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-) = (11.0 \pm 1.2)\%$: pert. vs non-pert. QCD effects before discussing any NP?



- **However, no clear evidence of NP from high-energy frontier**
  - **Improve calculation precision within the SM**: higher-order & higher-power QCD/EW/QED corrections
  - **If NP existed, they must have specific flavor structures**: any new flavor- & CP-violating sources?

B physics experiments

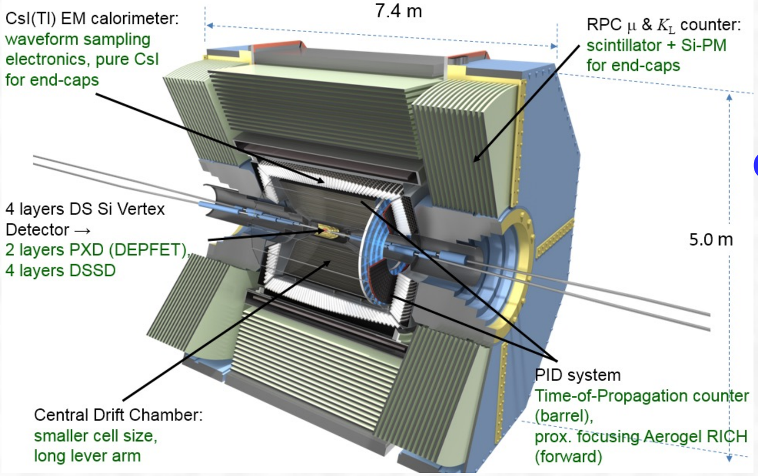
□ B-factories (e^+e^-): Belle & BaBar



3.5 GeV e^+ 8 GeV e^- 3.1 GeV e^+ 9 GeV e^-

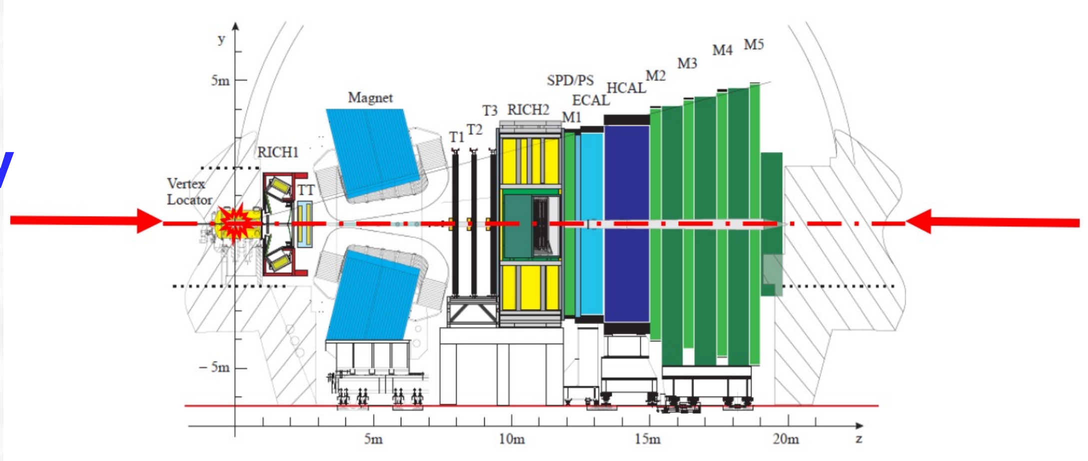
□ Hadron colliders ($p\bar{p}$): CDF & D0 @ Tevatron

□ Super B-factories (e^+e^-): Belle-II @ KEK



complementary to each other!

□ Hadron colliders (pp): LHCb @ LHC

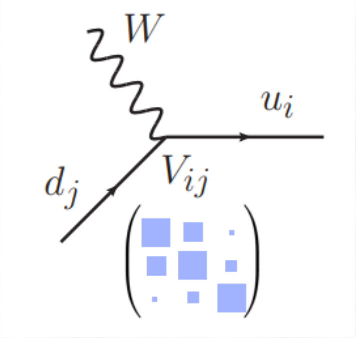


B decays in full SM theory

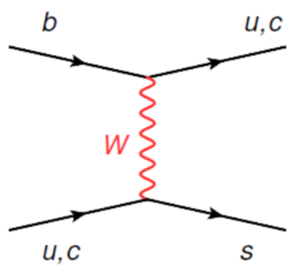
At the quark level: all processes mediated by flavor-changing charged-current transitions

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^{\mu}_{\rm CC} W^{\dagger}_{\mu} + \text{h.c.}$$

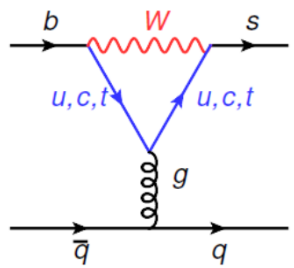
$$J^{\mu}_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\mu \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix} + (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\,\gamma^\mu\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

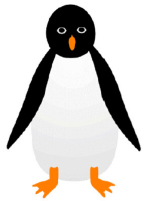


charged current

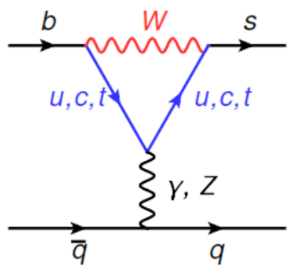


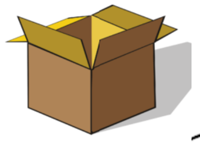
QCD-penguin

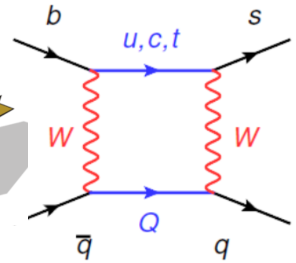




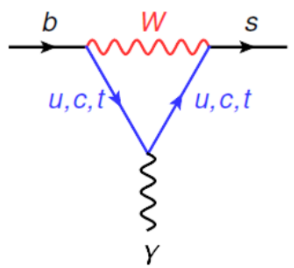
EW-penguin

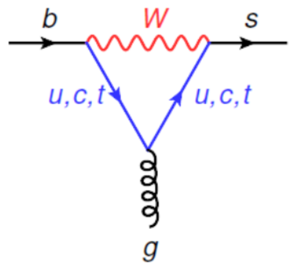






electro- & chromo-mgn





Multi-scale involved with large hierarchies:

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

$$\boxed{P(M_W, m_b) = 1 + \alpha_s\left(\#\ln\frac{M_W}{m_b} + *\right) + \alpha_s^2\left(\#\ln^2\frac{M_W}{m_b} + *\right) + \ldots}$$

Issue: direct pert. calculations encounter large logs of $\ln M_W/m_b$, and thus spoil pert. convergence

RG-improved perturbative theory

- **Solution:** re-organize the pert. series and make all these large logs $(\alpha_s \ln\frac{M_W}{m_b})^n$ re-summed

- **Final result for $P(M_W, m_b)$:**

$$\boxed{P(M_W,m_b) = \underbrace{C(M_W,\mu_{\rm high})U(\mu_{\rm high},\mu_{\rm low})}_{C_{\rm RGimproved}(M_W,\mu_{\rm low})} D(m_b,\mu_{\rm low})}$$

$\Rightarrow$ $U(\mu_{\rm high}, \mu_{\rm low})$ always takes an exponential form, and thus re-sums large logs $(\alpha_s \ln\frac{\mu_{\rm high}}{\mu_{\rm low}})^n$

- **Effective weak Hamiltonian for B decays:**

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\sum_{i=1}^{2} C_i(\mu)\left(V_{ub}V_{us}^* \mathcal{O}_i^{(u)} + V_{cb}V_{cs}^* \mathcal{O}_i^{(c)}\right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(\sum_{i=3}^{10} C_i(\mu, x_t)\,\mathcal{O}_i + C_8(\mu, x_t)\,\mathcal{O}_8^g\right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(\sum_{i=9}^{10} C_i(\mu, x_t)\,\mathcal{O}_i^{\ell\ell} + C_7(\mu, x_t)\,\mathcal{O}_7^\gamma\right)$$

$$\mathcal{O}_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i=1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i=3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i=7, & C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i=8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L\gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i=9,10 & |C_i(m_b)| \sim 4 \end{cases}$$

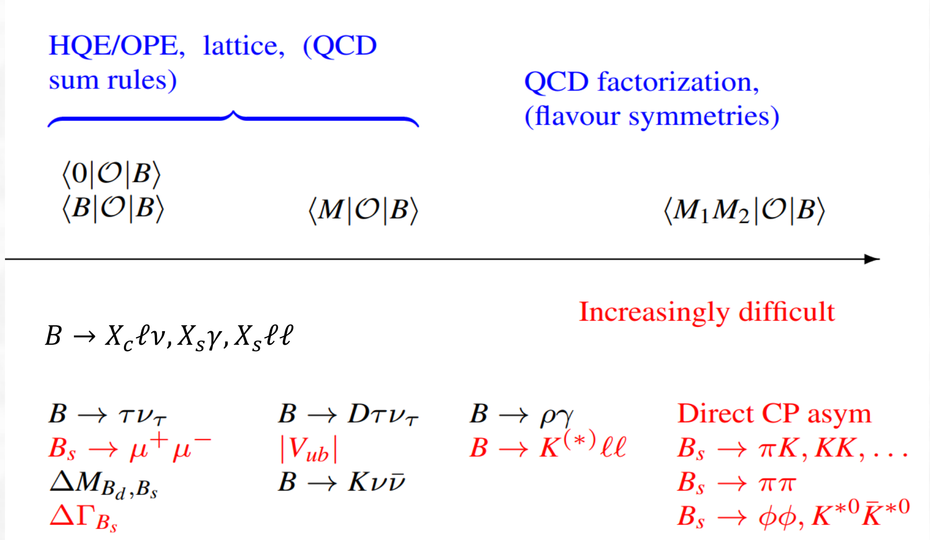
- **WC C_i:** perturbatively calculable and now the NNLL calculation complete [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

- **Main task: how to evaluate precisely and reliably hadronic matrix elements $\langle f|\mathcal{O}_i|\bar{B}\rangle$?**

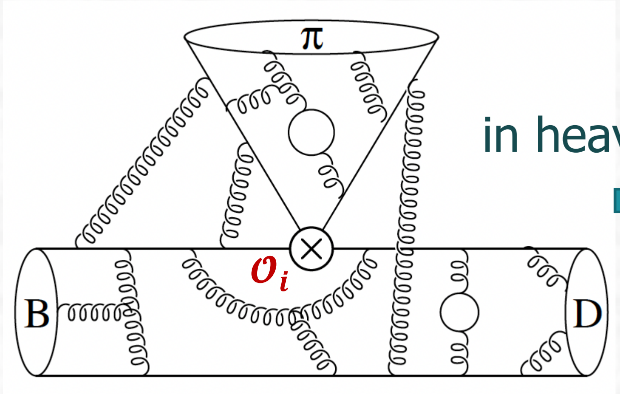
Hadronic matrix elements

- $\langle f|\mathcal{O}_i|\bar{B}\rangle_{\mathrm{QCD,QED}}$: involve complicated QCD & QED effects, and needs combination of various methods

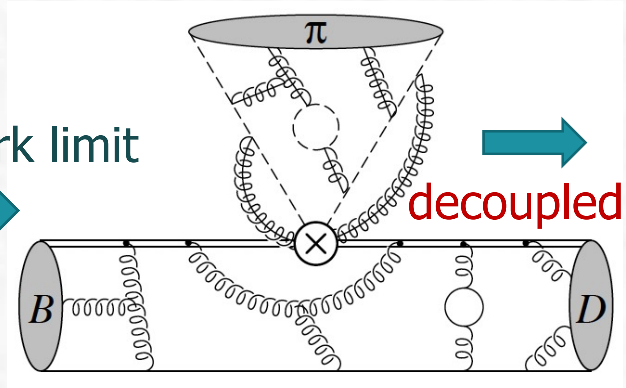
  - inclusive processes: HQE and OPE
  - leptonic decays: $\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle = i f_B p^\mu$
  - semi-leptonic decays: $\langle \pi(p')|\bar{u}\gamma^\mu b|\bar{B}(p)\rangle \propto f_\pm(q^2)$
  - hadronic decays: most complicated, and rely on various EFTs of QCD and factorization theorems



- **QCDF/SCET:** systematic framework from QCD, valid to all orders in α_s, limited by $\frac{\Lambda_{\mathrm{QCD}}}{m_b}$ corrections



in heavy-quark limit

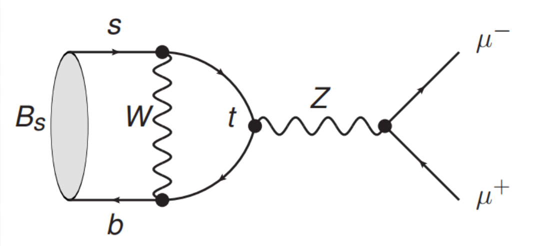


$$\langle D_q^{(*)+}L^-|\,\mathcal{Q}_i\,|\bar{B}_q^0\rangle = \sum_j F_j^{\bar{B}_q\to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, \boxed{T_{ij}(u)}\,\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

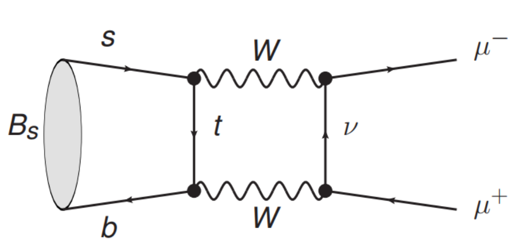
$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Purely leptonic $B_s \to \mu^+\mu^-$ decay

- **Has simplest hadronic structure:** all QCD dynamics encoded in $f_{B_s} = (230.3 \pm 1.3)\mathrm{MeV}$

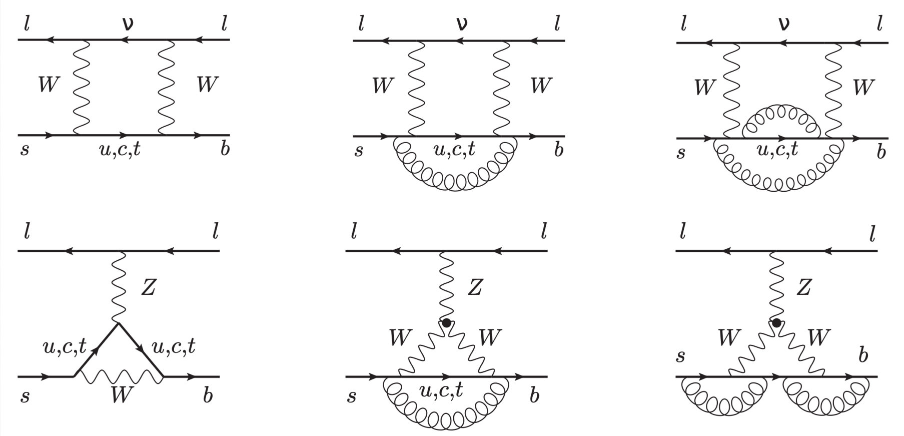


FCNC

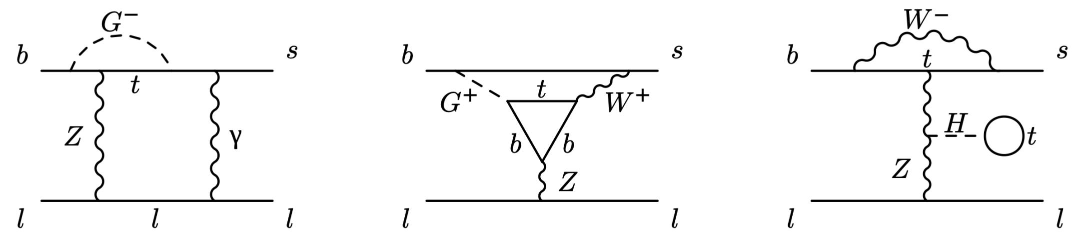


$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 b(0)|\bar{B}(p)\rangle = i f_B p^\mu$$

- **Much progress achieved via multi-loop techniques** [T. Hermann, M. Misiak & M. Steinhauser, '13]



$\mathcal{O}(\alpha_s^0)$ $\mathcal{O}(\alpha_s^1)$ $\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_e)$ NLO EW correction

$$\mathcal{L}_{\Delta B=1} = \mathcal{N}_{\Delta B=1}\left[\sum_{i=1}^{10} C_i(\mu_b)\,Q_i + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*}\sum_{i=1}^{2} C_i(\mu_b)\left(Q_i^u - Q_i^c\right)\right]$$

$$\begin{aligned} Q_9 &= \frac{\alpha_{em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell) \\ Q_{10} &= \frac{\alpha_{em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell) \\ Q_7 &= \frac{e}{16\pi^2} m_b\,(\bar{q}\sigma^{\mu\nu}P_R b)F_{\mu\nu} \end{aligned}$$

$$\boxed{\mathcal{B}^{\mathrm{SM}}(B_s \to \mu^+\mu^-) = (3.64 \pm 0.12)\times 10^{-9} \quad \text{vs} \quad \mathcal{B}^{\mathrm{exp}}(B_s \to \mu^+\mu^-) = (3.34 \pm 0.27)\times 10^{-9}}$$

$B_s \to \mu^+\mu^-$: ideal probe of NP beyond SM

- **Rare FCNC decays:** suitable for indirect probe of NP and NP scale can be much higher $\Lambda_{NP} \gg \Lambda_{\exp}$

decay constant

Effect of lifetime difference of Bs and Bs-bar

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}$$

triply suppressed

helicity suppression

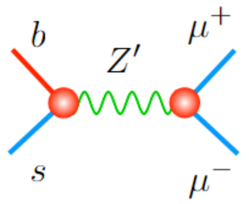
loop suppression

CKM suppression

$$\mu_{B_s \to \mu^+\mu^-} = \frac{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)}{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\rm SM}} = 0.92 \pm 0.08$$

➡ ***strong constraints on NP beyond SM***

- **Z' boson with generic tree-level FCNC:**

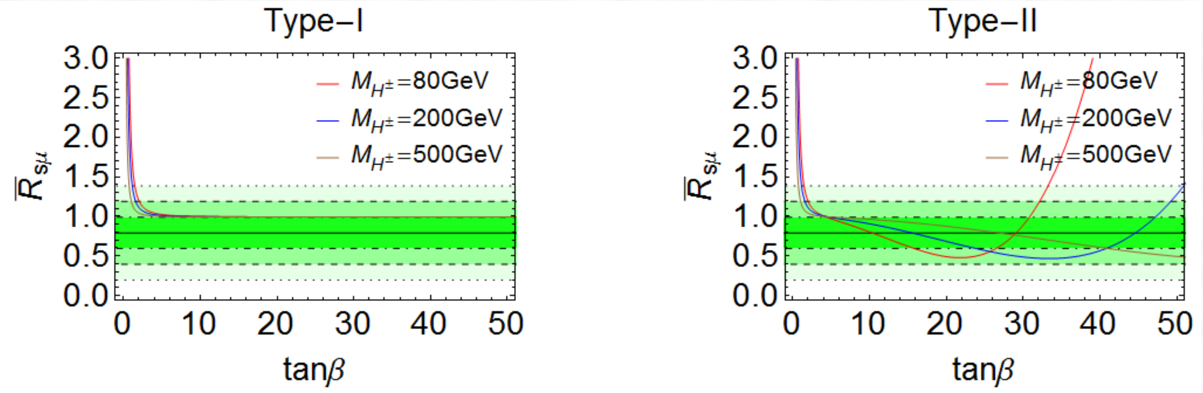


$$\mu_{B_s \to \mu^+\mu^-} \simeq 1 \pm \frac{4\pi}{g^2|V_{tb}^* V_{ts}|^2} \frac{v^2}{\Lambda^2},$$

➡ $$\boxed{\Lambda \geq \frac{v}{\sqrt{0.2}} \times \frac{\sqrt{4\pi}}{g|V_{tb}^* V_{ts}|} \simeq 50\ \mathrm{TeV}}$$

[U. Haisch, arXiv:1510.03341]

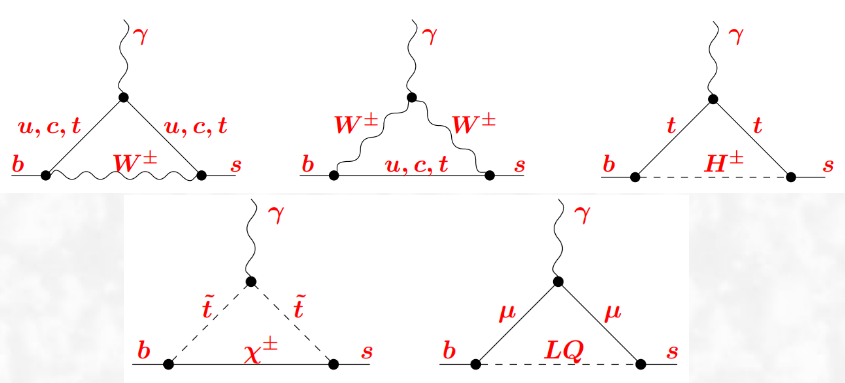
- **Z_2-symmetric 2HDMs:** [XQLI, J. Lu, A. Pich, '14]



➡ $$\boxed{\tan\beta = v_2/v_1 > 1.6}$$

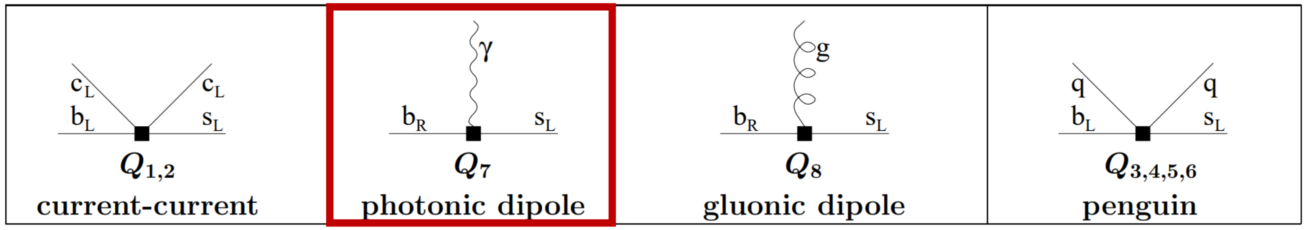
$B \to X_s\gamma$: ideal probe of charged Higgs

- Within the SM, it is a FCNC process dominated by γ-penguin diagrams



➡ effective weak Lagrangian by integrating out $W^{\pm}, Z^0, t, H^{\pm}$

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\left(\begin{array}{c}\text{quarks} \neq t \\ \text{\& leptons}\end{array}\right) + \sum_n C_n(\mu)\, Q_n$$



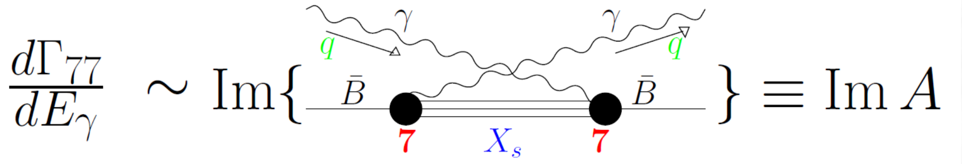
$$Q_7 = \frac{e}{16\pi^2} m_b \left(\bar{q}\sigma^{\mu\nu} P_R b\right) F_{\mu\nu}$$

- **Inclusive decay:** based on HQE/OPE,

  its decay rate well approximated by the b-quark decay rate

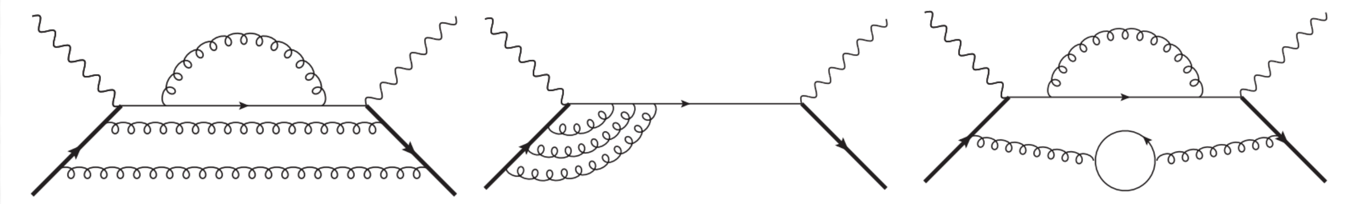
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \left(\begin{array}{c}\text{non-perturbative effects} \\ \sim (2 \pm 5)\% \\ \text{Benzke et al., arXiv:1003.5012}\end{array}\right)$$

optical theorem



- **Has been calculated up to N³LO**

  [M. Czakon, M. Fael, T. Huber, M. Misiak, and M. Steinhauser, '13; '20; '24; '25]



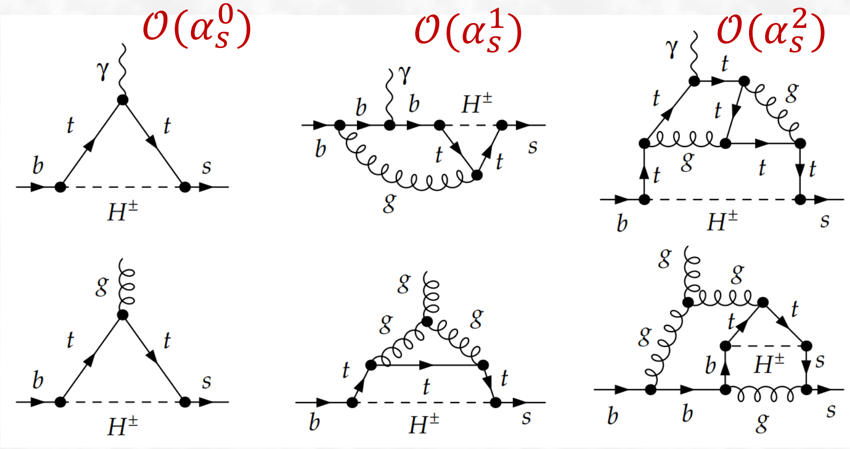
$B \to X_s\gamma$: ideal probe of charged Higgs

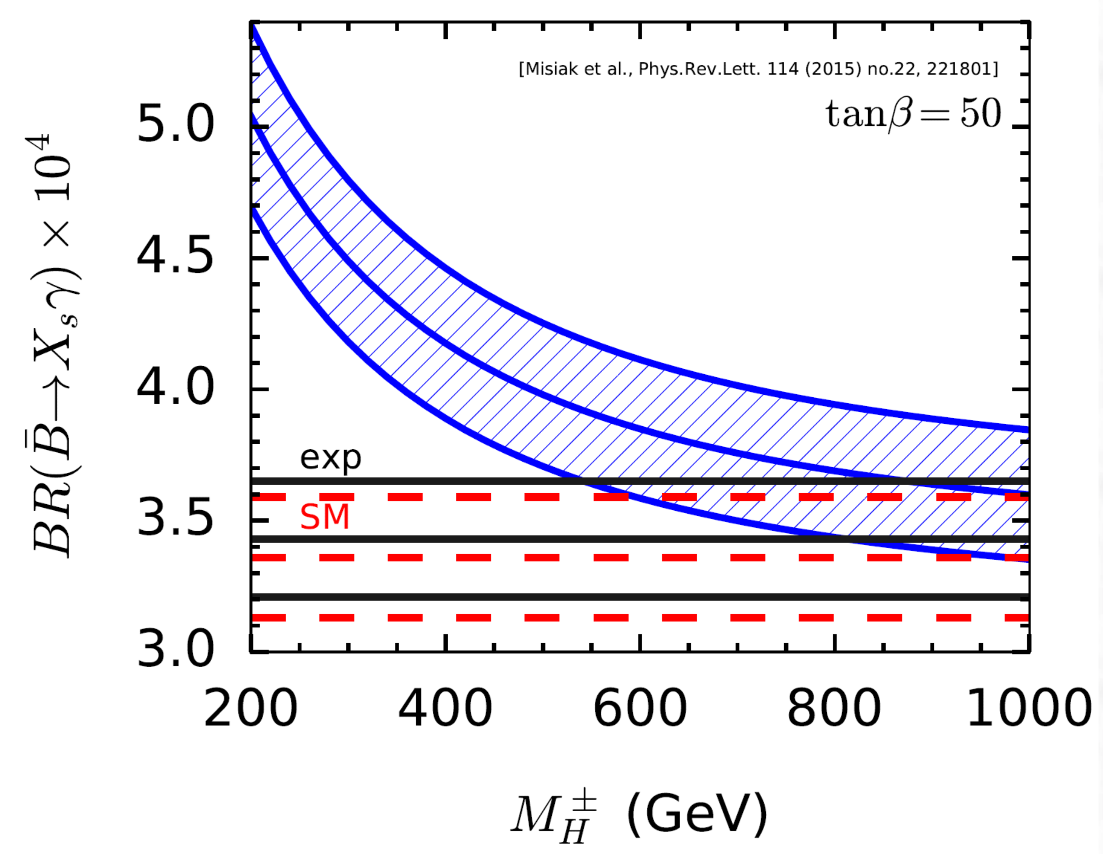
- **Status:** theo. & exp. well consistent, and provides high-precision test of SM & indirect probe of NP

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm SM}_{E_\gamma>1.6\rm GeV} = (3.54 \pm 0.14)\times 10^{-4} \quad \text{vs} \quad \mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma>1.6\rm GeV} = (3.49 \pm 0.19)\times 10^{-4}$$

- **Useful probe of $H^\pm$ in 2HDM:** [Hermann, Misiak, Steinhauser, '12; Rehman, Misiak, M. Steinhauser, '20]

$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^{3} \overline{u}_i \left(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R\right) d_j H^+ + h.c.$$





$\Rightarrow$ $M_{H^\pm} > 580$ GeV at 95% C.L. for type-II 2HDM

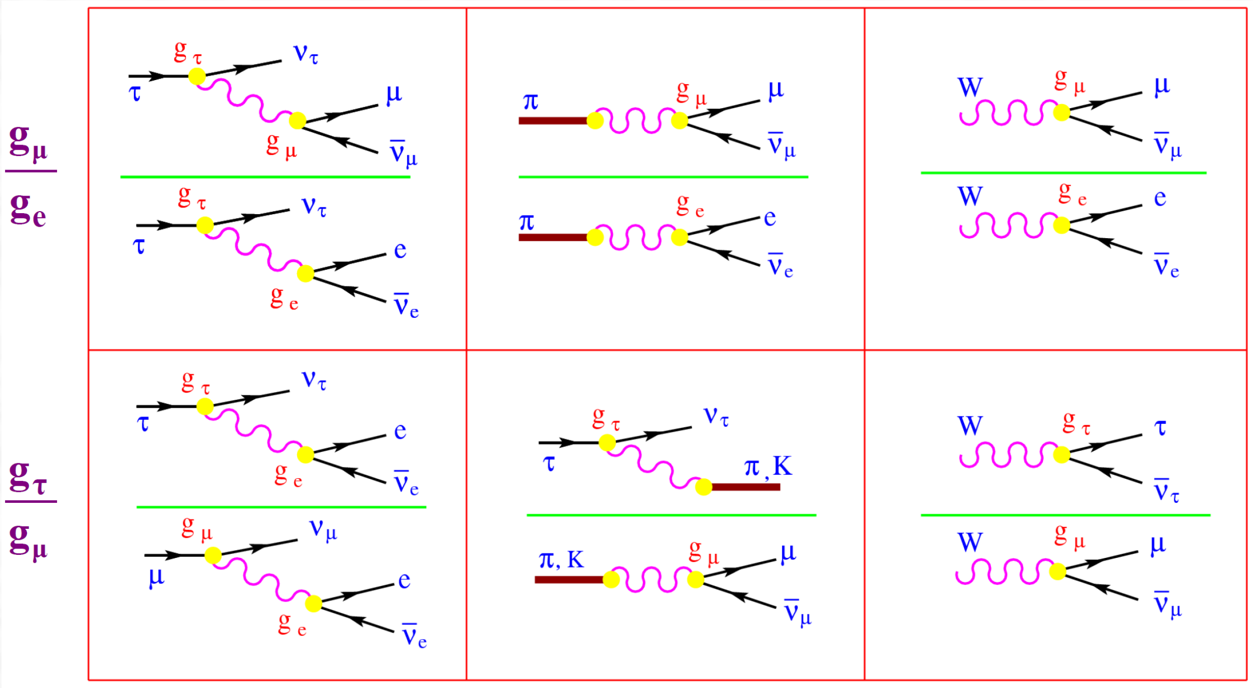
Lepton flavor university (LFU)

- **Within the SM, three families of leptons couple with $W^{\pm}, Z^0$ & γ with equal strengths**

$$\boxed{\mathcal{L}_{\rm cc}^{\ell} = g_W \sum_{i=1,2,3} \bar{\nu}^i{}_L \gamma_\mu \left(V_{\rm PMNS}^{ie} e_L + V_{\rm PMNS}^{i\mu} \mu_L + V_{\rm PMNS}^{i\tau} \tau_L\right) W^{+\mu} + {\rm h.c.}}$$

$$\Rightarrow \quad \left|\mathcal{M}_j\right|^2 \propto \sum_{i=1,2,3} \left|V_{\rm PMNS}^{ij}\right|^2 = 1 \ \forall j$$

$$\boxed{\mathcal{L}_{\rm nc}^{\ell} = \left(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu \mu + \bar{\tau}\gamma_\mu \tau\right)\left(g_\gamma A^\mu + g_Z Z^\mu\right)}$$



- **LFU checked @ permile level in W, Z, τ & π, K, D (semi-)leptonic decays:**

$\lvert g_\mu/g_e \rvert$	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$ 1.000 (2)	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$ 1.0002 (11)	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$ 1.0010 (9)	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$ 0.9978 (18)	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$ 1.0009 (18)
$\lvert g_\tau/g_\mu \rvert$	$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$ 1.001 (10)	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$ 1.0016 (14)	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$ 0.996 (4)	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$ 0.986 (8)	
$\lvert g_\tau/g_e \rvert$	$\Gamma_{W\to\tau}/\Gamma_{W\to e}$ 1.007 (10)	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$ 1.0018 (14)			

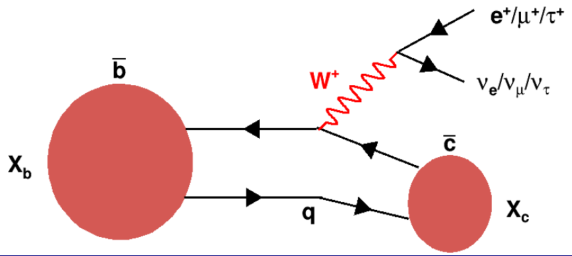
A. Pich, 2511.04587

LFU in semi-leptonic B-hadron decays

- **LFU can also be probed in semi-leptonic $b \to cl\nu$ decays:**

$$R(X_c) = \frac{\mathcal{B}(X_b \to X_c \tau^+ \nu_\tau)}{\mathcal{B}(X_b \to X_c \ell^+ \nu_\ell)}$$

$$X_b = B^0, B^+_{(c)}, B^0_s, \Lambda_b, \ldots \quad X_c = D, D^*, D_s, \Lambda_c, \ldots$$

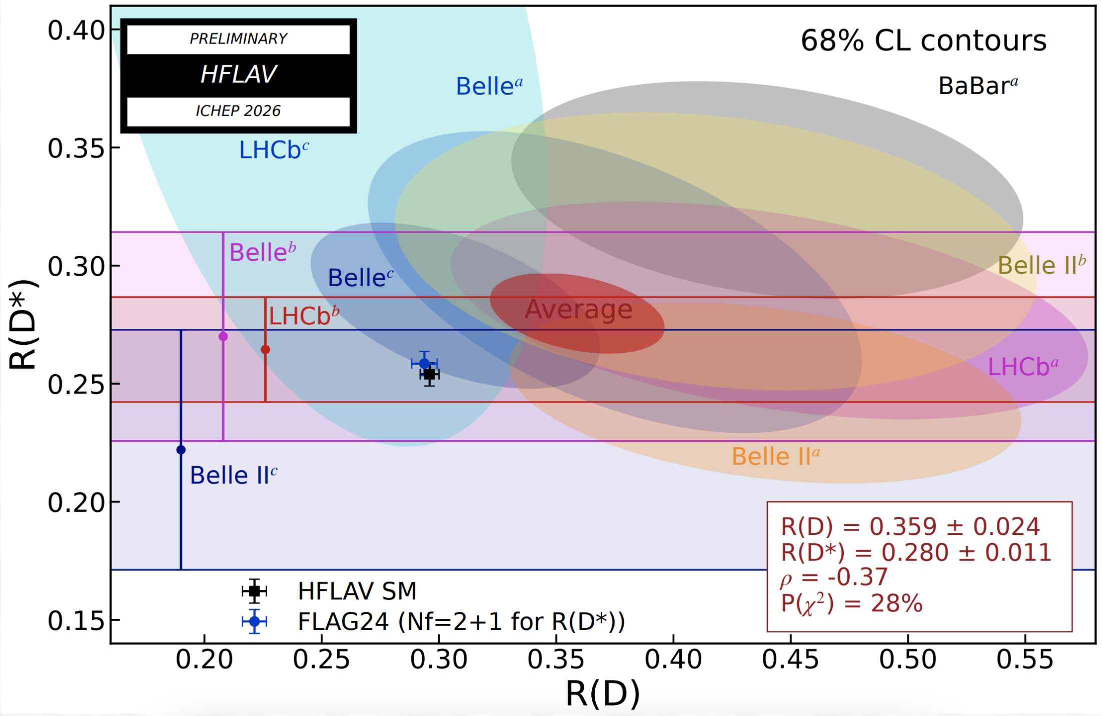


- **These ratios are featured by**
  - dependence on $|V_{cb}|$ completely cancelled
  - hadronic uncertainties from FFs mostly cancelled
  - exp. systematic uncertainties also cancelled

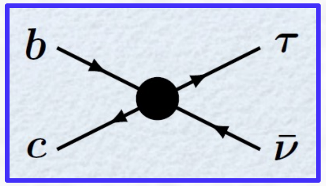
➡ **They can be predicted & measured precisely**

- **However, $\sim 3.73\sigma$ deviation still observed**

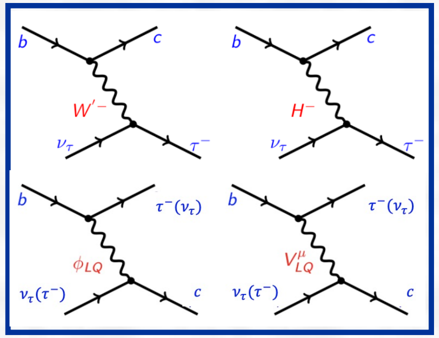
➡ ***need to understand dynamics behind it!***



Model-independent analysis in LEFT



- **Most general low-energy effective field theory (LEFT) for $b \to c\tau\nu_\tau$**



$m_R \gg m_b$

$$\mathcal{L}_{\rm SM}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{cb}\,\mathcal{O}_{V_L} + \text{H.c.}, \quad (V-A)\otimes(V-A)$$

$$\mathcal{L}_{\rm NP}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{cb}\left(C_{V_L}\mathcal{O}_{V_L} + C_{V_R}\mathcal{O}_{V_R} + C_{S_L}\mathcal{O}_{S_L} + C_{S_R}\mathcal{O}_{S_R} + C_T\mathcal{O}_T\right) + \text{H.c.}$$

$$\mathcal{O}_{V_{L(R)}} = (\bar{c}\gamma^\mu P_{L(R)} b)(\bar{\tau}\gamma_\mu P_L \nu_\tau),$$
$$\mathcal{O}_{S_{L(R)}} = (\bar{c} P_{L(R)} b)(\bar{\tau} P_L \nu_\tau),$$
$$\mathcal{O}_T = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\tau).$$

FF inputs from

Lattice (2014+2016+2017) + LCSR + Belle (2017 + 2018)

FF parameterization

general HQET up to 1/mc^2 (See 2004.10208)

HFLAV average: RD = 0.342(26) RD* = 0.287(12) corr. = -0.39

Private average: FLD* = 0.49(5) Belle 2019 + LHCb 2023

- **Latest global fits with a single operator:**

  ✓ $\mathcal{O}_{V_L}^\tau$ gives the best Pull

  ✓ $\mathcal{O}_{S_R}^\tau$ gives the worst Pull

  ✓ $\mathcal{O}_{V_R}^\tau$ & $\mathcal{O}_{S_L}^\tau$ need very large WCs → disfavored by collider bounds

  ✓ $\mathcal{O}_T^\tau$ also a viable solution

$(\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu)$ ✓	$C_{V_L}^\tau \approx 0.08$	$\text{Pull} \equiv \sqrt{\chi^2_{\rm SM} - \chi^{2,\rm best}_{\rm NP}} \approx 4.8$ $\left(\sqrt{\chi^2_{\rm SM}} \approx 4.8\right)$
$(\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu)$	$C_{V_R}^\tau \approx 0.01 \pm i0.41$	$\text{Pull} \approx 4.4$
$(\bar{c} P_L b)(\bar{\ell} P_L \nu)$	$C_{S_L}^\tau \approx -0.79 \pm i0.86$	$\text{Pull} \approx 4.3$
$(\bar{c} P_R b)(\bar{\ell} P_L \nu)$	$C_{S_R}^\tau \approx 0.18$	$\text{Pull} \approx 3.9$
$(\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu} P_L \nu)$ ✓	$C_T^\tau \approx 0.02 \pm i0.13$	$\text{Pull} \approx 4.3$

[S. Iguro, T. Kitahara, R. Watanabe, 2405.06062]

$\Lambda_b \to \Lambda_c l\nu$ decays mediated by $b \to cl\nu$

- **$R(\Lambda_c) = \mathrm{Br}(\Lambda_b \to \Lambda_c \tau\nu_\tau)/\mathrm{Br}(\Lambda_b \to \Lambda_c \ell\nu_\ell)$:**
  - at the quark level, also mediated by $b \to cl\nu$ transitions
  - first result reported by LHCb with large error [arXiv:2201.03497]

$$R_{\Lambda_c}^{\mathrm{LHCb}} = 0.242 \pm 0.026 \pm 0.04 \pm 0.059$$
$$\Updownarrow$$
$$R_{\Lambda_c}^{\mathrm{SM}} = 0.324 \pm 0.004$$

exp. central value lower than SM

- **General expressions for $R(D), R(D^*)$ & $R(\Lambda_c)$ in LEFT:**

$$\mathcal{H}_{\rm eff} = 2\sqrt{2}G_F V_{qb}\left[\left(1+C_{V_L}^{ql}\right)\mathcal{O}_{V_L}^{ql} + C_{V_R}^{ql}\mathcal{O}_{V_R}^{ql} + C_{S_L}^{ql}\mathcal{O}_{S_L}^{ql} + C_{S_R}^{ql}\mathcal{O}_{S_R}^{ql} + C_T^{ql}\mathcal{O}_T^{ql}\right]$$

$$\mathcal{O}_{V_L}^{ql} = (\bar q\gamma^\mu P_L b)\left(\bar l\gamma_\mu P_L \nu_l\right), \qquad \mathcal{O}_{V_R}^{ql} = (\bar q\gamma^\mu P_R b)\left(\bar l\gamma_\mu P_L \nu_l\right),$$
$$\mathcal{O}_{S_L}^{ql} = (\bar q P_L b)\left(\bar l P_L \nu_l\right), \qquad \mathcal{O}_{S_R}^{ql} = (\bar q P_R b)\left(\bar l P_L \nu_l\right),$$
$$\mathcal{O}_T^{ql} = (\bar q\sigma^{\mu\nu} P_L b)\left(\bar l\sigma_{\mu\nu} P_L \nu_l\right)$$

LEFT

$$\frac{R_P}{R_P^{\rm SM}} = |1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}|^2 + a_P^{SS}|C_{S_L}^{q\tau} + C_{S_R}^{q\tau}|^2 + a_P^{TT}|C_T^{q\tau}|^2$$
$$+ a_P^{VS}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} + C_{S_R}^{q\tau*}\right)\right] + a_P^{VT}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right)C_T^{q\tau*}\right],$$

$$\frac{R_V}{R_V^{\rm SM}} = |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_V^{SS}|C_{S_L}^{q\tau} - C_{S_R}^{q\tau}|^2 + a_V^{TT}|C_T^{q\tau}|^2$$
$$+ a_V^{V_LV_R}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] + a_V^{VS}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} - C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} - C_{S_R}^{q\tau*}\right)\right]$$
$$+ a_V^{V_LT}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_V^{V_RT}\mathrm{Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right],$$

$$\frac{R_H}{R_H^{\rm SM}} = |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_H^{SS}\left[|C_{S_L}^{q\tau}|^2 + |C_{S_R}^{q\tau}|^2\right] + a_H^{TT}|C_T^{q\tau}|^2$$
$$+ a_H^{V_LV_R}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] + a_H^{VS_1}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{S_L}^{q\tau*} + C_{V_R}^{q\tau}C_{S_R}^{q\tau*}\right]$$
$$+ a_H^{VS_2}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{S_R}^{q\tau*} + C_{V_R}^{q\tau}C_{S_L}^{q\tau*}\right] + a_H^{S_LS_R}\mathrm{Re}\left[C_{S_L}^{q\tau}C_{S_R}^{q\tau*}\right]$$
$$+ a_H^{V_LT}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_H^{V_RT}\mathrm{Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right],$$

- ✓ assuming only τ modes affected by NP
- ✓ light neutrinos always left-handed
- ✓ a_X^{ij} calculable with FF inputs together with uncertainties inherited from them

Sum Rule for $b \to c$ Sector

$|1+\cancel{C_{V_L}^{q\tau}}|^2$

$\mathrm{Re}[(1+\cancel{C_{V_L}^{q\tau}})C_{S_L}^{q\tau*}]$

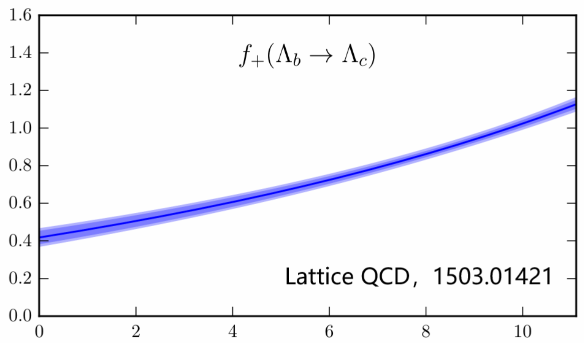
□ Sum rule for $R(D)$, $R(D^*)$ & $R(\Lambda_c)$: [M. Blanke et al., 1811.09603, 1905.08253]

$$\boxed{\frac{R_H}{R_H^{\rm SM}} = b\frac{R_P}{R_P^{\rm SM}} + c\frac{R_V}{R_V^{\rm SMM}} + \delta_H(C_i)}$$

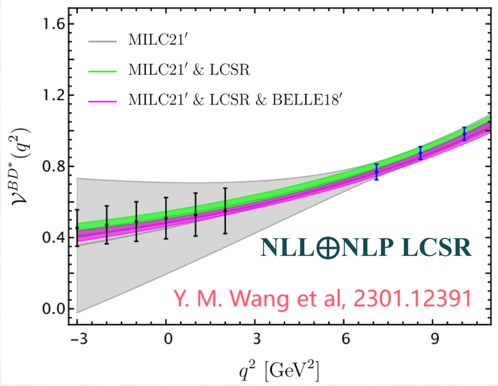
→ $b + c = 1$ & $a_P^{VS} b + a_V^{VS} c = a_H^{VS_1}$, so that $\delta_H(C_i)$ small

→ model-indep. & holds for any tau-philic NP

□ State-of-the-art prediction: [W. F. Duan, S. Iguro, XQLI, R. Watanabe, Y. D. Yang, 2410.21384]



	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM
$B \to D$	Refs. [85, 86]	no data	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$B \to D^*$	Refs. [87–89]	no data (*)	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$\Lambda_b \to \Lambda_c$	Ref. [80]	Ref. [92]	no data	no data	–



$$\boxed{\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}} = \underset{1/4}{(0.272 \pm 0.015)}\frac{R_D}{R_D^{\rm SM}} + \underset{3/4}{(0.728 \mp 0.015)}\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{\Lambda_c}}$$

$$\begin{aligned}\delta_{\Lambda_c} = &(-0.001 \pm 0.005)\left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) + (-0.007 \pm 0.005)\,\mathrm{Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau*}\right) \\ &+ \underline{(-2.681 \pm 6.907)}\,|C_T^{c\tau}|^2 + \underline{(-0.561 \pm 1.439)}\,\mathrm{Re}\left(C_{V_R}^{c\tau} C_T^{c\tau*}\right) \\ &+ \mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{(0.041 \pm 0.034)C_{V_R}^{c\tau*} + \underline{(0.594 \pm 1.274)}C_T^{c\tau*}\right\}\right] \\ &+ (-0.002 \pm 0.009)\mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)C_{S_R}^{c\tau*} + C_{S_L}^{c\tau} C_{V_R}^{c\tau*}\right]\end{aligned}$$

- ➢ important to properly consider correlations among FF para.s
- ➢ δ_{Λ_c} mostly sensitive to tensor operator $\mathcal{O}_T = (\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\tau}\sigma_{\mu\nu}P_L\nu_\tau)$

Sum rule for $b \to c$ sector

- **Sum-rule prediction for $R(\Lambda_c)$:**

$$\boxed{\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}} = (0.272 \pm 0.015)\frac{R_D}{R_D^{\rm SM}} + (0.728 \mp 0.015)\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{\Lambda_c}}$$

➢ provide a unique prediction of $R(\Lambda_c)$ model-independently

HFLAG 2024: $R_D^{\rm exp} = 0.342 \pm 0.026$, $R_{D^*}^{\rm exp} = 0.287 \pm 0.012$

$$\boxed{R_{\Lambda_c}^{\rm SR} = 0.372 \pm 0.017} \quad \text{vs} \quad R_{\Lambda_c}^{\rm LHCb} = 0.242 \pm 0.076$$

the sum rule relation without the shift factor $\delta_{\Lambda_c}(C_X)$ (i.e. in SM) seems to be violated by current data

➢ with $\delta_{\Lambda_c}(C_T = 0.02 \pm i\, 0.13) \simeq -0.035 \pm 0.096$ included, the sum rule deviation can still be explained

$$\delta_{\rm SR/\,exp} = \frac{R_{\Lambda_c}^{\rm exp}}{R_{\Lambda_c}^{\rm SM}} - \left(0.272\frac{R_D^{\rm exp}}{R_D^{\rm SM}} + 0.728\frac{R_{D^*}^{\rm exp}}{R_{D^*}^{\rm SM}}\right)$$
$$= -0.39 \pm 0.23$$

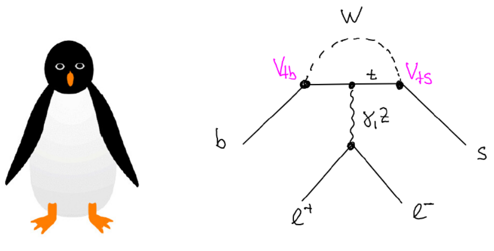
Jie Wu PhD thesis:

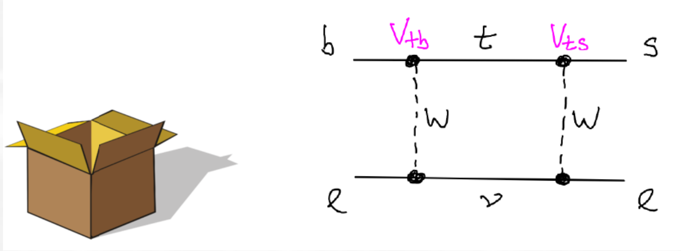
$R(\Lambda_c^+) = 0.164 \pm 0.070(\text{stat}) \pm 0.014(\text{syst})$

- **This exercise encourages a more precise measurement of $\Lambda_b \to \Lambda_c \tau\nu$ decays from LHCb**

Rare $b \to s\ell^+\ell^-$ decays

☐ Why $b \to s\ell^+\ell^-$ processes:





- occur firstly at 1-loop; suppressed by loop factor
- proportional to $|V_{tb}V_{ts}^*|$; $\mathrm{Br}(b \to s\ell\ell) \sim 10^{-6}$
- sensitive to NP beyond the SM

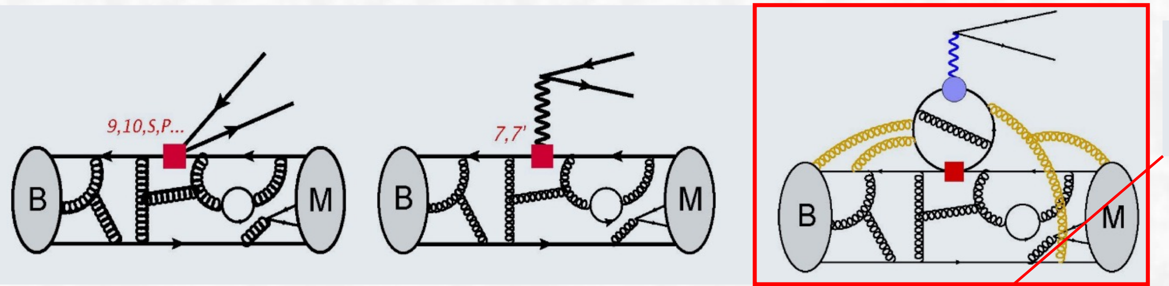
$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \cdots$$

$$Q_9 = \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell)$$

$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}\sigma^{\mu\nu} P_R b) F_{\mu\nu}$$

☐ Local & non-local hadronic matrix elements: charming loop



Non-local form-factors:

$$\mathcal{H}_\lambda(k,q) = i\int d^4x\, e^{iq\cdot x}\mathcal{P}_\lambda^\mu \langle \bar{M}(k)|T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2) \right] \right\}$$

- $B \to K^{(*)}\mu\mu$
- $B_s \to \varphi\,\mu\mu$, ...

BSZ parametrization
LQCD & LCSR

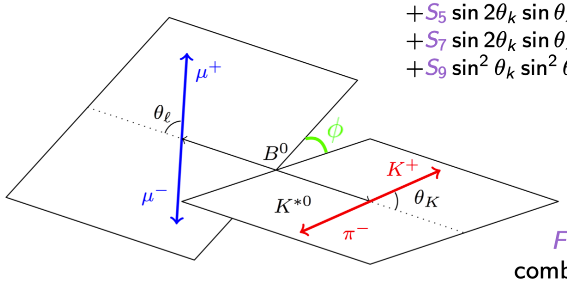
Local form-factors,
involves e.g. $\mathcal{F}_\lambda(k,q) = \mathcal{P}_\lambda^\mu \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$

- **dominated by charming loops from $\mathcal{O}_{1,2}^c$**
- **they can easily mimic NP by a shift of C_9**
- **with a simple analytic structure, charming loops are small** [Gubernari 2022; LHCb 2024]

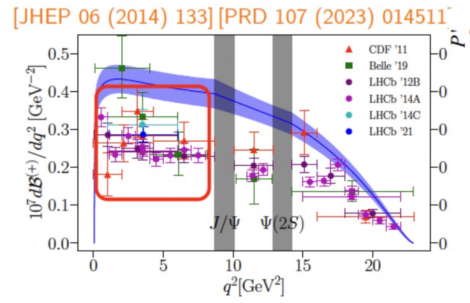
Rare $B \to K^*(\to K\pi)\ell^+\ell^-$ decays

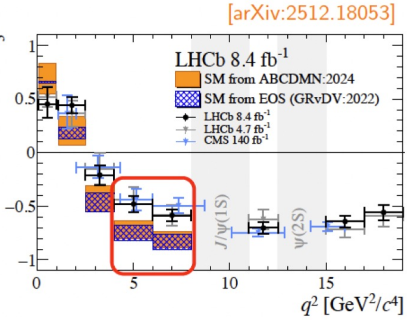
☐ Featured by many observables:

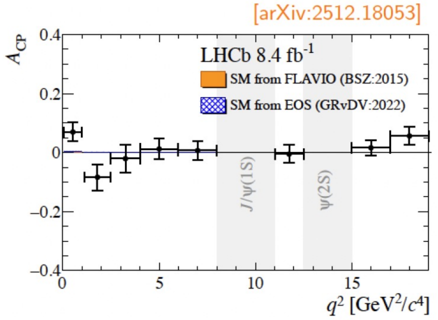
$$
\frac{1}{d(\Gamma+\bar\Gamma)/dq^2}\frac{d^4(\Gamma+\bar\Gamma)}{d\vec\Omega dq^2} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_L)\sin^2\theta_k + F_L\cos^2\theta_k + \tfrac{1}{4}(1-F_L)\sin^2\theta_k\cos 2\theta_\ell - F_L\cos^2\theta_k\cos 2\theta_\ell + S_3\sin^2\theta_k\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_k\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_k\sin\theta_\ell\cos\phi + \tfrac{4}{3}A_{FB}\sin^2\theta_k\cos\theta_\ell + S_7\sin 2\theta_k\sin\theta_\ell\sin\phi + S_8\sin 2\theta_k\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_k\sin^2\theta_\ell\sin 2\phi\Big],
$$



angular observables
F_L, A_{FB}, $S_i = f(C_7, C_9, C_{10})$,
combinations of K^{*0} decay amplitudes







Branching fractions
affected by form-factors and $c\bar c$-loop

Angular observables
affected by $c\bar c$-loop

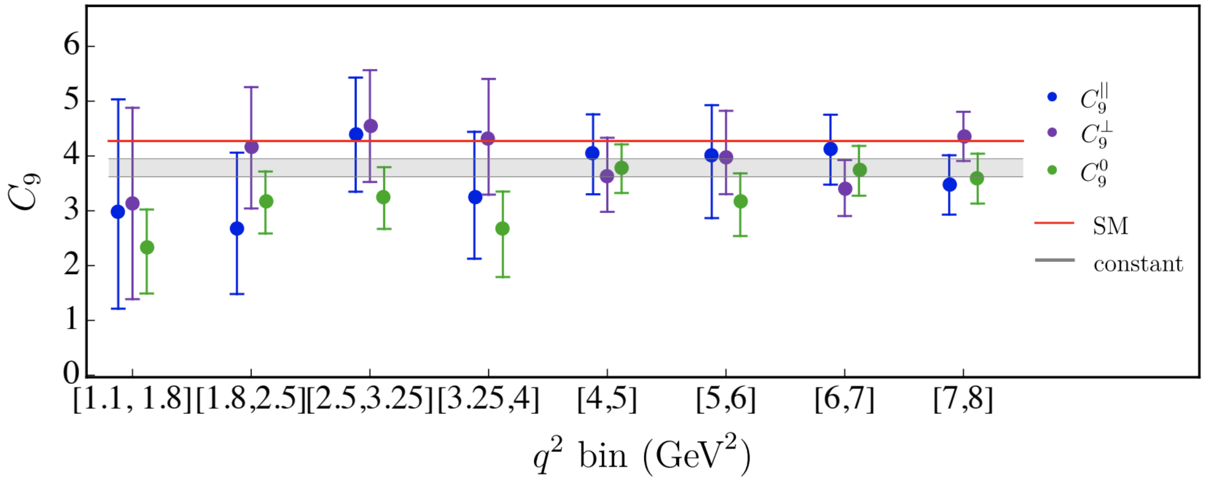
CP-asymmetries
clean

☐ NP in C_9 required and no visible q^2 & helicity dependences for C_9: [Bordone etal 2026]

$$
\Delta\mathcal{H}(q^2) = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda}\frac{(q^2-q_0^2)}{(m_V^2-q_0^2)}A_V^{\rm res}(q^2)
$$

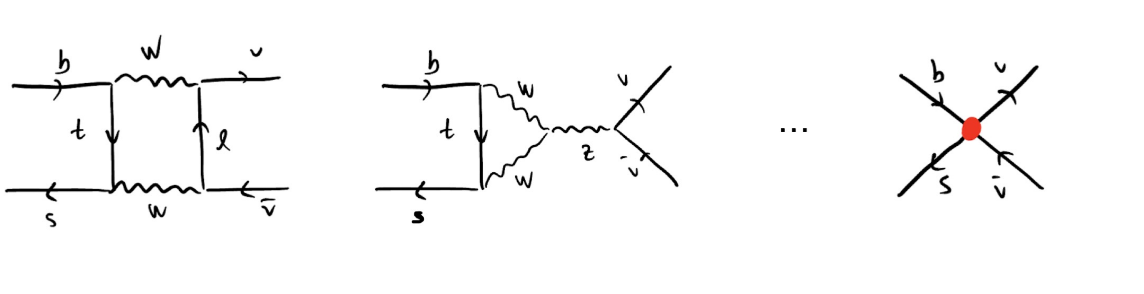
dispersive description

$$
A_V^{\rm res}(q^2) = \frac{m_V\Gamma_V}{m_V^2 - q^2 - i m_V\Gamma_V}\,.
$$



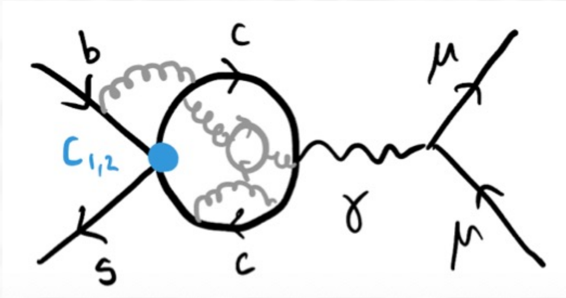
Rare $b \to s\nu\bar{\nu}$ decays

- **$b \to s\nu\bar{\nu}$ decays:** theoretically cleaner than $b \to s\ell\ell$ decays due to absence of long-distance $c\bar{c}$ loop



$$O_{exp} = O_{SM}\left(1 + \# \frac{C}{\Lambda^2} + \dots\right)$$

$$\propto \frac{|V_{tb}V_{ts}^*|^2}{(16\pi^2)^2}$$



State-of-the-art SM prediction

- **Effective Hamiltonian** in the SM:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi}\sum_i C_L^{\rm SM}\underline{(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Li})} + \text{h.c.}\,,$$

$\lambda_t = V_{tb}V_{ts}^*$

only operator in SM

- **Short-distance** contributions known to **good precision**:

$$C_L^{\rm SM} = -X_t/\sin^2\theta_W$$
$$= -6.32(7)$$

Including NLO QCD and two-loop EW contributions:

$$X_t = 1.462(17)(2)$$

[Buchala et al. '93, '99], [Misiak et al. '99], [Brod et al. '10]

Main theo. uncertainties:

$$\langle K^{(*)}|\bar{s}_L\gamma^\mu b_L|B\rangle = \sum_a K_a^\mu\,\mathcal{F}_a(q^2)$$

Form-factors (e.g., LQCD)

$$|V_{tb}V_{ts}^*| = |V_{cb}|\,(1+\mathcal{O}(\lambda^2))$$

Rare $B \to K^{(*)}\nu\bar{\nu}$ decays

- **$B \to K^{(*)}$ form factors from LQCD + LCSR:**

$$\mathcal{B}(B \to K\nu\bar{\nu})^{\rm SM}/|\lambda_t|^2 = \begin{cases} (1.33 \pm 0.04)_{K_S} \times 10^{-3} \\ (2.87 \pm 0.10)_{K^+} \times 10^{-3} \end{cases}$$

[≈ 3 % uncertainty]

Becirevic et al., 2301.06990

$$\mathcal{B}(B \to K^*\nu\bar{\nu})^{\rm SM}/|\lambda_t|^2 = \begin{cases} (5.9 \pm 0.8)_{K^{*0}} \times 10^{-3} \\ (6.4 \pm 0.9)_{K^{*+}} \times 10^{-3} \end{cases}$$

[≈ 15 % uncertainty]

- **Belle-II first evidence of $B^+ \to K^+\nu\bar{\nu}$ decay:**

exp. data $\simeq 2.7\sigma$ above the SM prediction

Evidence for $B^+ \to K^+\nu\bar{\nu}$ decays

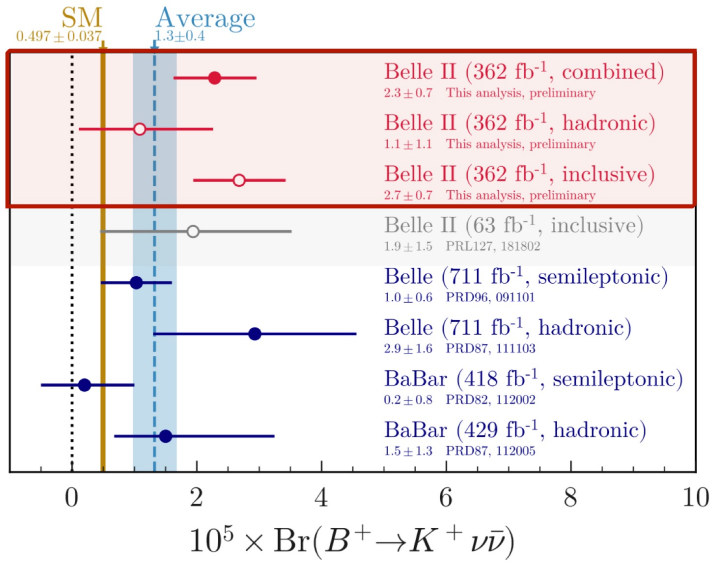
Belle-II Collaboration • I. Adachi et al. (Nov 24, 2023)

Published in: *Phys.Rev.D* 109 (2024) 11, 112006 • e-Print: 2311.14647

pdf DOI cite datasets claim

reference search 264 citations

[Belle-II, 2311.14647]



New Belle-II results

First Belle-II result

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})^{\rm exp} = \left[2.3 \pm 0.5\,({\rm stat})^{+0.5}_{-0.4}\,({\rm syst})\right] \times 10^{-5}$$

- **NP signals via new heavy mediators in loop or new light invisible particles in final state?**

Rare $B \to K^{(*)} \nu\bar{\nu}$ decays

☐ SMEFT analysis: $d = 6$ four-fermion contributions at tree level; [Buchmuller & Wyler, '85; Gradkowski et al., '10]

- ψ^4 operators invariant under $\boxed{SU(2)_L \times U(1)_Y}$:

$$[\mathcal{O}_{lq}^{(1)}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{Q}_k\gamma_\mu Q_l)$$
$$[\mathcal{O}_{lq}^{(3)}]_{ijkl} = (\overline{L}_i\gamma^\mu \tau^I L_j)(\overline{Q}_k\tau^I\gamma_\mu Q_l)$$
$$[\mathcal{O}_{ld}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{d}_k\gamma_\mu d_l)$$
$$[\mathcal{O}_{eq}]_{ijkl} = (\overline{e}_i\gamma^\mu e_j)(\overline{Q}_k\gamma_\mu Q_l)$$
$$[\mathcal{O}_{ed}]_{ijkl} = (\overline{e}_i\gamma^\mu e_j)(\overline{d}_k\gamma_\mu d_l)$$

$b \to s\ell\ell$ $\qquad$ $b \to s\nu\bar{\nu}$

$$[\mathcal{O}_{lq}^{(1)}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{Q}_k\gamma_\mu Q_l) = (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + \ldots$$

$$[\mathcal{O}_{lq}^{(3)}]_{ijkl} = (\overline{L}_i\gamma^\mu\tau^I L_j)(\overline{Q}_k\gamma_\mu\tau^I Q_l) = (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) - (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + \ldots$$

$$[\mathcal{O}_{ld}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{d}_k\gamma_\mu d_l) = (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Rk}\gamma_\mu d_{Rl}) + (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Rk}\gamma_\mu d_{Rl})$$

- Correlations for concrete mediators:

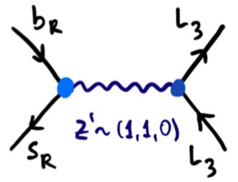
- $Z' \sim (\mathbf{1}, \mathbf{1}, 0)$	:	$\mathcal{C}_{lq}^{(1)}$, $\mathcal{C}_{ld}$
- $V \sim (\mathbf{1}, \mathbf{3}, 0)$	:	$\mathcal{C}_{lq}^{(3)}$
- $U_1 \sim (\mathbf{3}, \mathbf{1}, 2/3)$	:	$\mathcal{C}_{lq}^{(1)} = \mathcal{C}_{lq}^{(3)}$
- $S_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	:	$\mathcal{C}_{lq}^{(1)} = 3\,\mathcal{C}_{lq}^{(3)}$
- $\widetilde{R}_2 = (\mathbf{3}, \mathbf{2}, 1/6)$	:	$\mathcal{C}_{ld}$

...

$(SU(3)_c, SU(2)_L, U(1)_Y)$

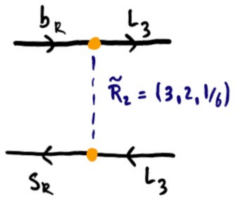
☐ Strong correlations among observables arise in concrete models:

- Z':

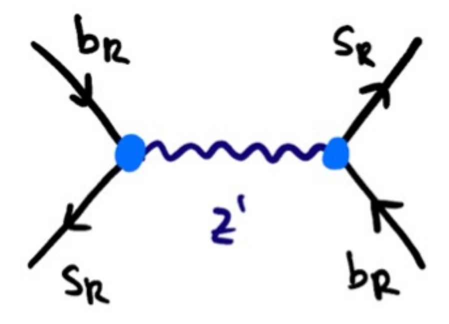


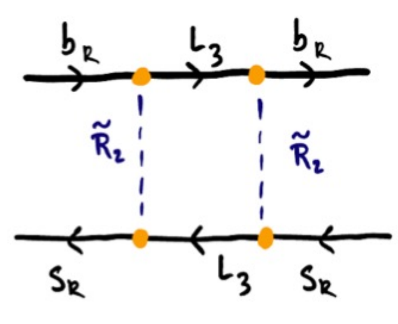
$$\mathcal{L}_{Z'} \supset g_{ij}^{\psi}\,(\bar{\psi}_i\gamma^\mu\psi_j)Z'_\mu$$

- LQs:



$$\mathcal{L}_{\widetilde{R}_2} \supset y_{ij}^{R}\,(\bar{d}_{Ri}\widetilde{R}_2 i\tau_2 L_j) + \text{h.c.}$$





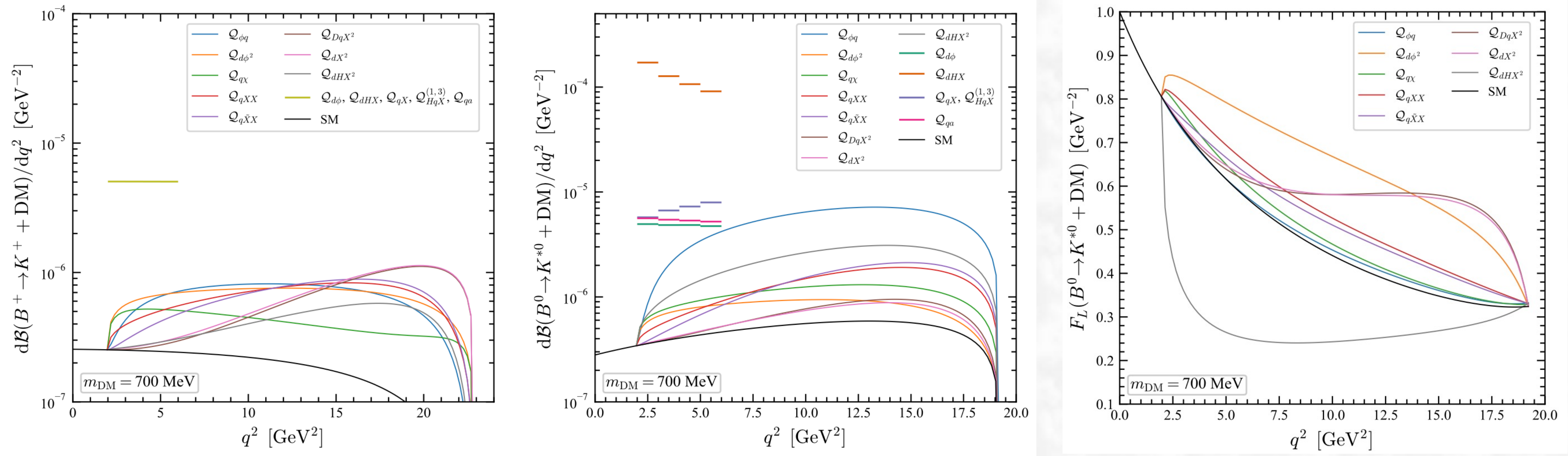
difficult to explain Belle-II excess, but possible in certain models

Rare $B \to K^{(*)}\nu\bar{\nu}$ decays

- **The Belle-II excess can also be mimicked by $B \to K + \text{invisible}$:** [Hou, XQLI, Shen, Yang and Yuan, '24]

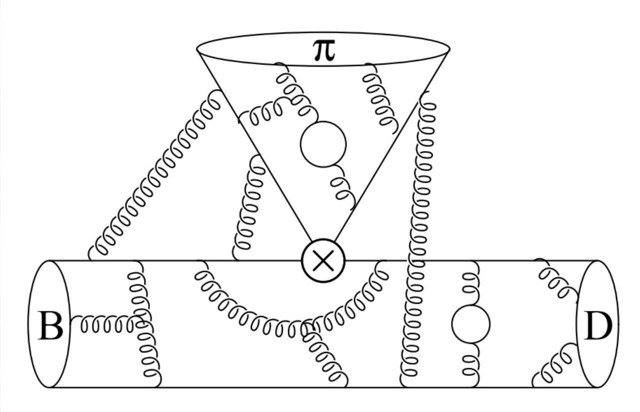
$$\mathcal{L}_{\rm DSMEFT} \supset \frac{1}{\Lambda}\sum_i \mathcal{C}_i \mathcal{Q}_i^{(5)} + \frac{1}{\Lambda^2}\sum_j \mathcal{C}_j \mathcal{Q}_j^{(6)},$$

$$\mathcal{L}_{\rm DLEFT} \supset \frac{1}{\Lambda}\sum_i L_i \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_j L_j \mathcal{O}_j^{(6)},$$



22 DSMEFT operators can explain Belle-II excess, and they can be distinguished from each other by more dedicated measurements of differential decay rates & $F_L(q^2)$

Class-I $B_q^0 \to D_q^{(*)-} L^+$ decays



- **At the quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

  all four flavors different from each other,

  no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- **For class-I decays:** QCDF formula much simpler,

  and only the form-factor term at leading power

  [Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

i) only color-allowed tree topology $T = a_1$

ii) spectator & annihilation power-suppressed

iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etc.

theoretically simpler & cleaner, and can be used to test various factorization theorems

- **Hard kernel T:** both NLO & NNLO results known

  [Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, XQLI '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Non-leptonic/semi-leptonic ratios

Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

$$R_{(s)L}^{(*)} \equiv \frac{\Gamma(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} L^-)}{d\Gamma(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 \, f_L^2 \, |a_1(D_{(s)}^{(*)+} L^-)|^2 \, X_L^{(*)}$$

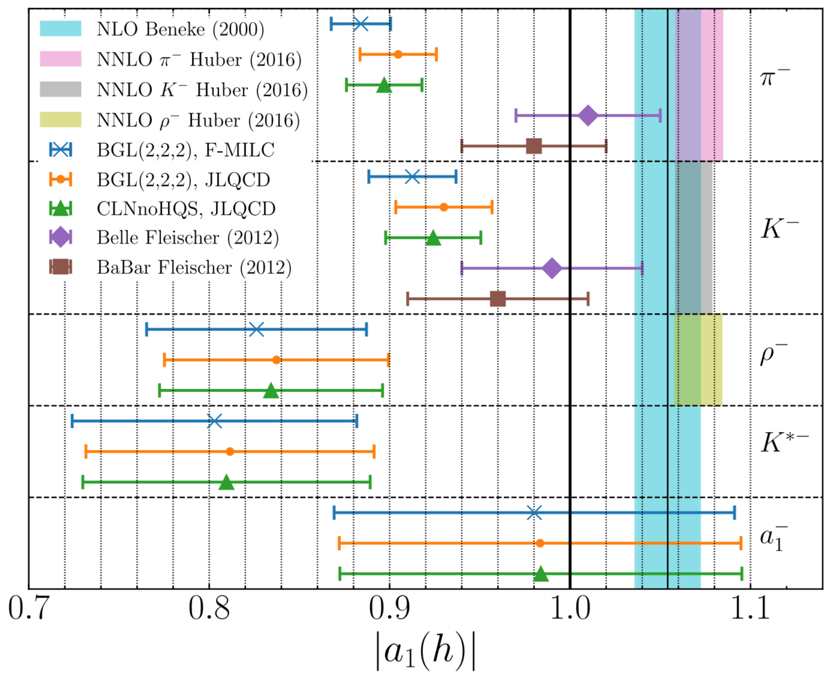
free from the uncertainties from

V_{cb} & $B_{d,s} \to D_{d,s}^{(*)}$ form factors

Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R_{(s)L}^{(*)}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R_π^*	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K ($\bar{B}_d \to D^+ K^-$)	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R_K^*	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$ ($\bar{B}_s \to D_s^+ \pi^-$)	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

Confirmed by Belle: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016 \; [1.071^{+0.020}_{-0.016}]$

15% lower than SM

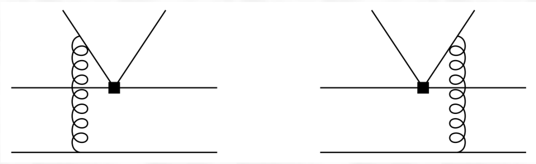
$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013 \; [1.069^{+0.020}_{-0.016}]$

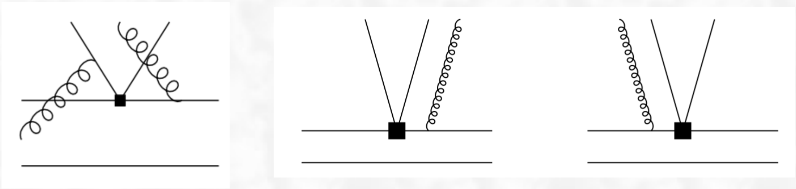
SM or NP in these class-I decays?

- **Difficult to explain data with power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \boxed{\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)}$$

  - non-factorizable spectator interactions
  - non-leading higher Fock-state contributions





- **Possible NP four-quark operators with different Dirac structures:** [Buras, Misiak, Urban '00]

There are totally 20 linearly independent operators

$$\mathcal{L}_{\rm WET} = -\frac{4G_F}{\sqrt{2}} V_{cb} V_{uq}^* [\mathcal{C}_1^{SM}(\mu) \mathcal{Q}_1^{SM} + \mathcal{C}_2^{SM}(\mu) \mathcal{Q}_2^{SM} + \sum_{\substack{i=1,2;\\ j=1,2,3,4.}} (\mathcal{C}_i^{VLL} \mathcal{Q}_i^{VLL} + \mathcal{C}_i^{VLR} \mathcal{Q}_i^{VLR} + \mathcal{C}_i^{SLR} \mathcal{Q}_i^{SLR} + \mathcal{C}_j^{SLL} \mathcal{Q}_j^{SLL})] + L \leftrightarrow R$$

$$\begin{aligned}
\mathcal{Q}_1^{VLL} &= (\bar{c}_\alpha \gamma^\mu P_L b_\beta)(\bar{q}_\beta \gamma_\mu P_L u_\alpha) & \mathcal{Q}_1^{VLR} &= (\bar{c}_\alpha \gamma^\mu P_L b_\beta)(\bar{q}_\beta \gamma_\mu P_R u_\alpha) \\
\mathcal{Q}_2^{VLL} &= (\bar{c}_\alpha \gamma^\mu P_L b_\alpha)(\bar{q}_\beta \gamma_\mu P_L u_\beta) & \mathcal{Q}_2^{VLR} &= (\bar{c}_\alpha \gamma^\mu P_L b_\alpha)(\bar{q}_\beta \gamma_\mu P_R u_\beta) \\
\mathcal{Q}_1^{SLL} &= (\bar{c}_\alpha P_L b_\beta)(\bar{q}_\beta P_L u_\alpha) & \mathcal{Q}_1^{SLR} &= (\bar{c}_\alpha P_L b_\beta)(\bar{q}_\beta P_R u_\alpha) \\
\mathcal{Q}_2^{SLL} &= (\bar{c}_\alpha P_L b_\alpha)(\bar{q}_\beta P_L u_\beta) & \mathcal{Q}_2^{SLR} &= (\bar{c}_\alpha P_L b_\alpha)(\bar{q}_\beta P_R u_\beta) \\
\mathcal{Q}_3^{SLL} &= (\bar{c}_\alpha \sigma^{\mu\nu} P_L b_\beta)(\bar{q}_\beta \sigma_{\mu\nu} P_L u_\alpha) \\
\mathcal{Q}_4^{SLL} &= (\bar{c}_\alpha \sigma^{\mu\nu} P_L b_\alpha)(\bar{q}_\beta \sigma_{\mu\nu} P_L u_\beta)
\end{aligned}$$

Analysis at m_b scale

- With only one NP C_i^{NP} in each time, NP four-quark operators with **three Dirac structures**

C.L.\Obs. / NP Coeff.	C.L.	R_π	R_π^*	R_ρ	R_K	R_K^*	R_{K^*}	$R_{s\pi}$	R_{sK}	Combined
C_1^{VLL}	1σ	[-1.40,-0.847]	[-1.18,-0.626]	[-1.50,-0.267]	[-1.18,-0.662]	[-1.54,-0.145]	[-1.05,0.392]	[-1.57,-0.835]	[-2.12,-1.31]	$\varnothing$
	2σ	[-1.63,-0.656]	[-1.41,-0.426]	[-2.06,0.135]	[-1.42,-0.462]	[-2.41,0.402]	[-1.70,0.856]	[-1.92,-0.567]	[-2.55,-1.02]	[-1.41,-1.02]
C_2^{VLL}	1σ	[-0.237,-0.148]	[-0.205,-0.111]	[-0.254,-0.047]	[-0.198,-0.116]	[-0.261,-0.026]	[-0.183,0.070]	[-0.264,-0.146]	[-0.345,-0.226]	$\varnothing$
	2σ	[-0.273,-0.115]	[-0.244,-0.075]	[-0.340,0.024]	[-0.237,-0.081]	[-0.401,0.071]	[-0.288,0.155]	[-0.318,-0.099]	[-0.406,-0.176]	[-0.237,-0.176]
C_1^{SRR}	1σ	[-0.748,-0.418]	[-1.03,-0.502]	$\varnothing$	[-0.711,-0.368]	[-1.50,-0.133]	R	[-0.839,-0.412]	[-1.25,-0.712]	$\varnothing$
	2σ	[-0.867,-0.326]	[-1.23,-0.344]	R	[-0.854,-0.259]	[-2.32,0.395]	R	[-1.02,-0.283]	[-1.48,-0.556]	[-0.854,-0.556]
C_2^{SRR}	1σ	[-0.249,-0.139]	[-0.343,-0.167]	$\varnothing$	[-0.237,-0.123]	[-0.500,-0.044]	R	[-0.280,-0.137]	[-0.417,-0.237]	$\varnothing$
	2σ	[-0.289,-0.109]	[-0.410,-0.115]	R	[-0.285,-0.086]	[-0.773,0.132]	R	[-0.339,-0.094]	[-0.492,-0.185]	[-0.285,-0.185]
C_1^{SRL}	1σ	[0.487,0.873]	[0.585,1.20]	$\varnothing$	[0.429,0.829]	[0.155,1.75]	R	[0.480,0.979]	[0.830,1.46]	$\varnothing$
	2σ	[0.381,1.01]	[0.401,1.44]	R	[0.302,0.996]	[-0.460,2.71]	R	[0.330,1.18]	[0.648,1.72]	[0.648,0.996]
C_2^{SRL}	1σ	[0.139,0.249]	[0.167,0.343]	$\varnothing$	[0.123,0.237]	[0.044,0.500]	R	[0.137,0.280]	[0.237,0.416]	$\varnothing$
	2σ	[0.109,0.289]	[0.115,0.410]	R	[0.086,0.285]	[-0.132,0.773]	R	[0.094,0.339]	[0.185,0.492]	[0.185,0.285]

$$\mathcal{Q}_{1,2}^{VLL} = \bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_{\beta(\alpha)}\, \bar{q}_\beta \gamma^\mu (1-\gamma_5) u_{\alpha(\beta)}$$

$(V-A)\otimes(V-A)$

$$\mathcal{Q}_{1,2}^{SRR} = \bar{c}_\alpha (1+\gamma_5) b_{\beta(\alpha)}\, \bar{q}_\beta (1+\gamma_5) u_{\alpha(\beta)}$$

$(S+P)\otimes(S+P)$

$$\mathcal{Q}_{1,2}^{SRL} = \bar{c}_\alpha (1+\gamma_5) b_{\beta(\alpha)}\, \bar{q}_\beta (1-\gamma_5) u_{\alpha(\beta)}$$

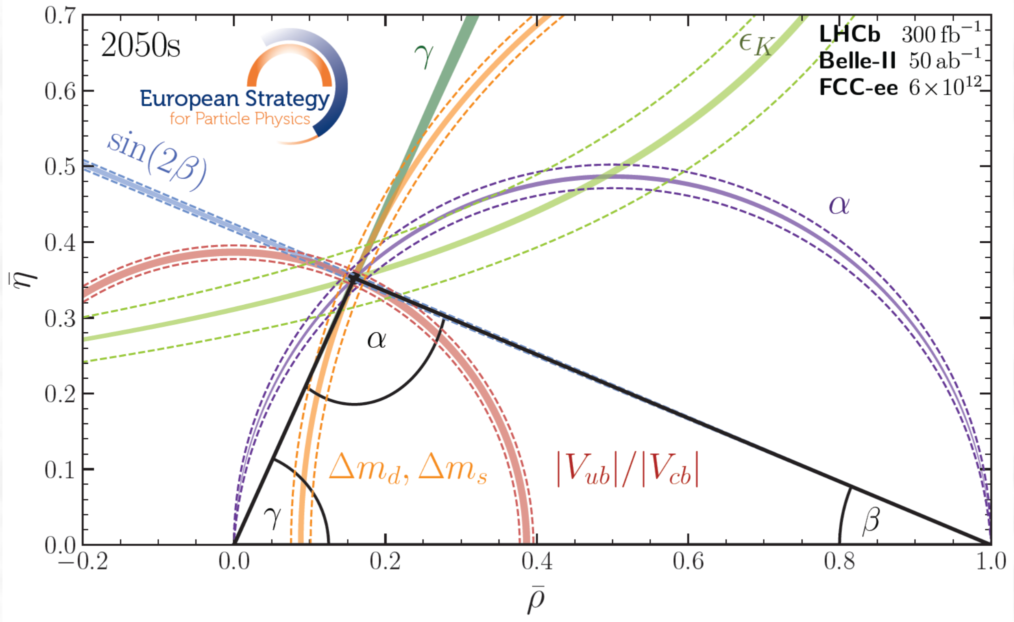
$(S+P)\otimes(S-P)$

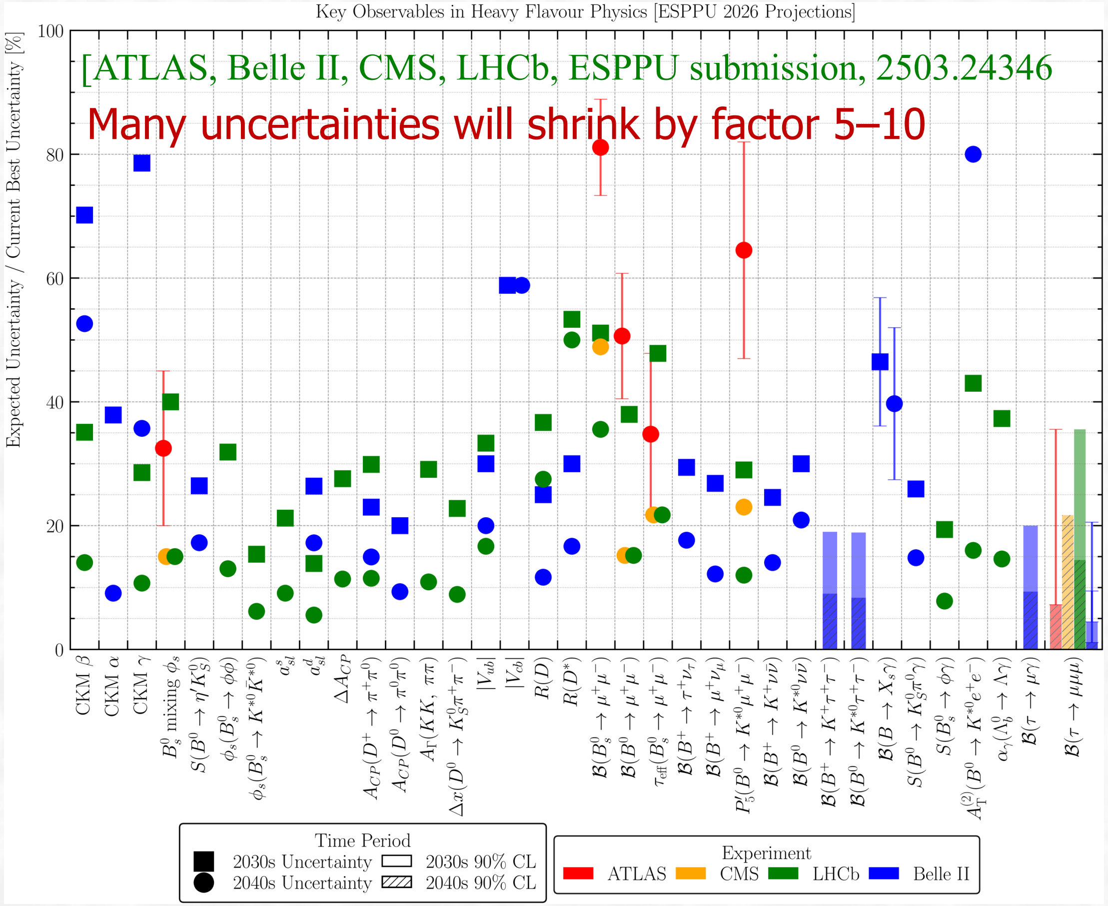
- Useful for **UV model buildings** and can be checked by colliders & other flavor processes

dijet resonance searches @ LHC; constraints from B-meson lifetime; $B_c^- \to J/\psi(\eta_c)L^-$ decays

Summary

- With **exp. & theo. progress**, we are now entering a **precision era for flavor physics**
- Most exp. data consistent with SM but also several deviations observed ➡ **NP signals?**
- More precise exp. data & more precise theo. predictions, & LQCD inputs needed





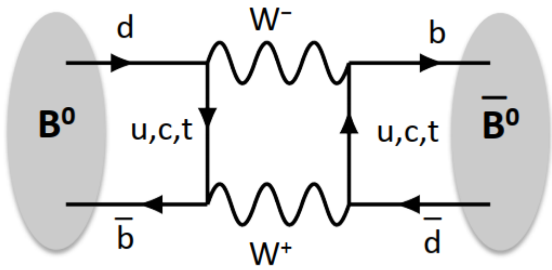
➡ ***bright opportunities in B physics!***

back-up

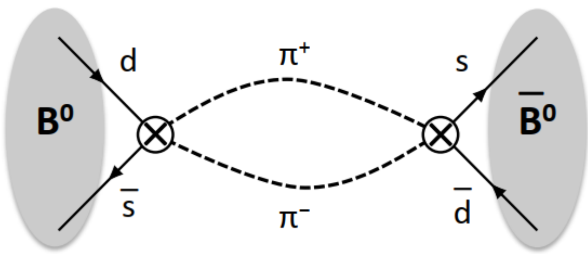
Neutral B-meson mixings

- **For neutral B_q^0 mesons:** flavor eigenstates ≠ mass eigenstates ⇒ mix with each other via box diagrams

$$i\frac{d}{dt}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}=\left(\hat{M}-\frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$$



"short-distance"
(=virtual particle exchange)



"long-distance"
(=real particle exchange)

- **Three observables for B mixings**

  - **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
    $|M_{12}|$: heavy internal particles: t, SUSY, ...

  - **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi$ **(on-shell)**
    $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

  - **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to X l \nu$ (*semi-leptonic*)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\overline{B}_q(t)\to f)-\Gamma(B_q(t)\to \overline{f})}{\Gamma(\overline{B}_q(t)\to f)+\Gamma(B_q(t)\to \overline{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right|\sin\phi$$

$$M_{12}=\frac{G_F^2}{12\pi^2}\left(V_{tq}^*V_{tb}\right)^2M_W^2S_0(x_t)B_{B_q}f_{B_q}^2M_{B_q}\hat{\eta}_B$$

† 1-loop calculation $S_0\left(x_t=m_t^2/M_W^2\right)$

† 2-loop perturbative QCD corrections $\hat{\eta}_B$

† $\frac{8}{3}B_{B_q}f_{B_q}^2M_{B_q}=\left\langle\overline{B}_q\middle|(\bar{b}q)_{V-A}(\bar{b}q)_{V-A}\middle|B_q\right\rangle$

➡

- ✓ M_{12}: dispersive (off-shell) part of the box diagram
- ✓ Γ_{12}: absorptive (on-shell) part of the box diagram
- ✓ $\phi=\arg(-M12/\Gamma12)$: relative phase between them

$$\Gamma_{12}=\left(\frac{\Lambda}{m_b}\right)^3\left(\Gamma_3^{(0)}+\frac{\alpha_s}{4\pi}\Gamma_3^{(1)}+\ldots\right)$$ **HQE**

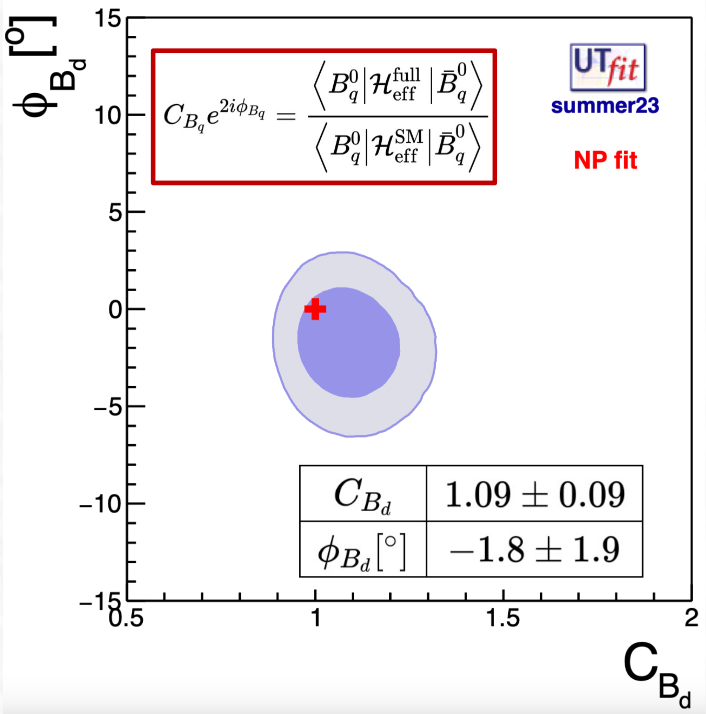
$$+\left(\frac{\Lambda}{m_b}\right)^4\left(\Gamma_4^{(0)}+\ldots\right)+\left(\frac{\Lambda}{m_b}\right)^5\left(\Gamma_5^{(0)}+\ldots\right)+\ldots$$

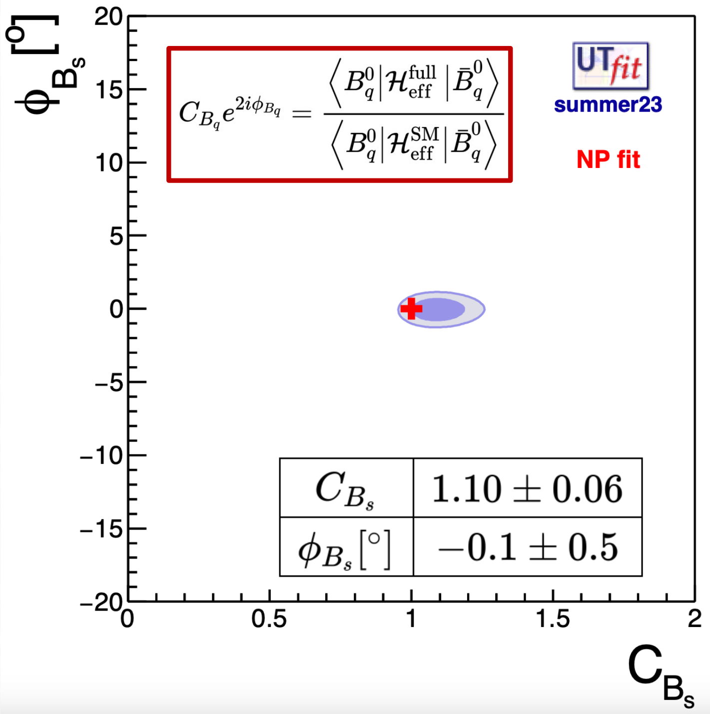
NP constraints from neutral B mixings

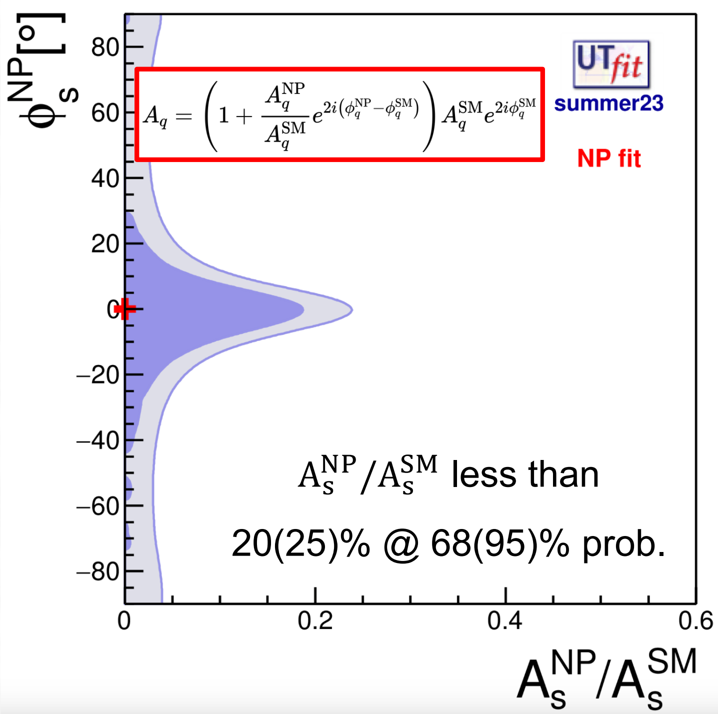
- **Exp. observables related to SM and NP parameters:**

$$\Delta M_d^{exp} = C_{B_d} \Delta M_d^{SM}\,, \qquad \sin 2\beta^{exp} = \sin\left(2\beta^{SM} + 2\phi_{B_d}\right)$$
$$\Delta M_s^{exp} = C_{B_s} \Delta M_s^{SM}\,, \qquad \phi_s^{exp} = \left(\beta_s^{SM} - \phi_{B_s}\right)$$

- **Latest fit results by UTfit group:**



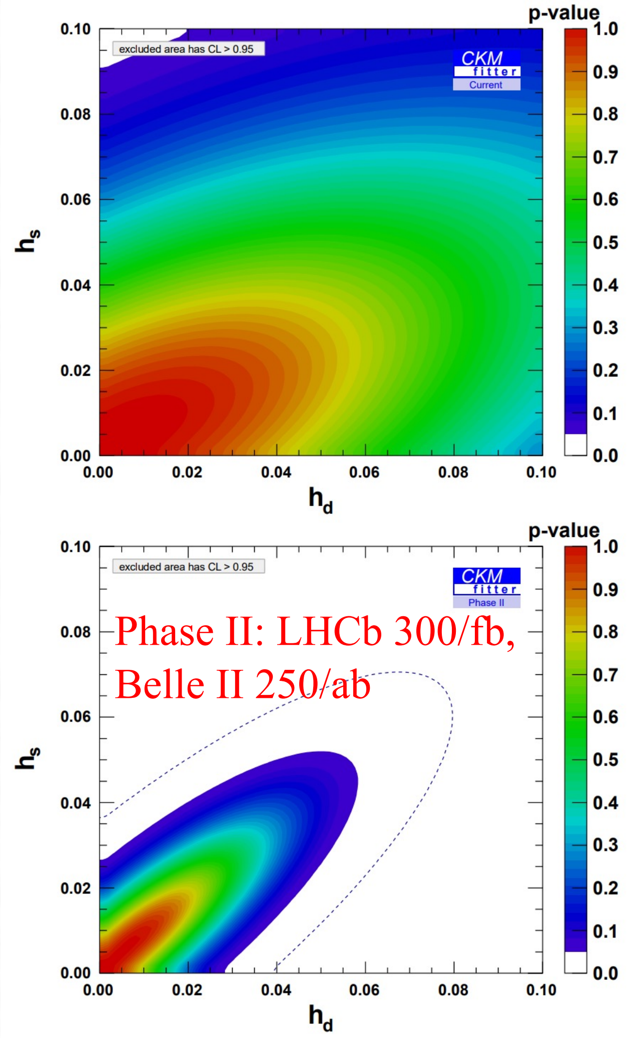




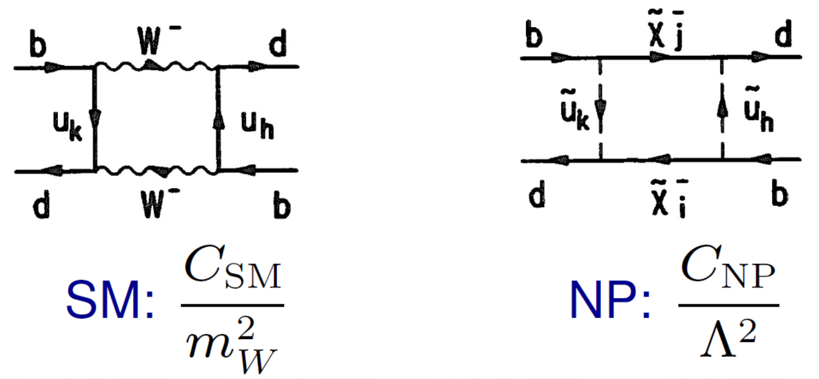
consistency between data & SM for B mixing observables puts stringent constraint on NP

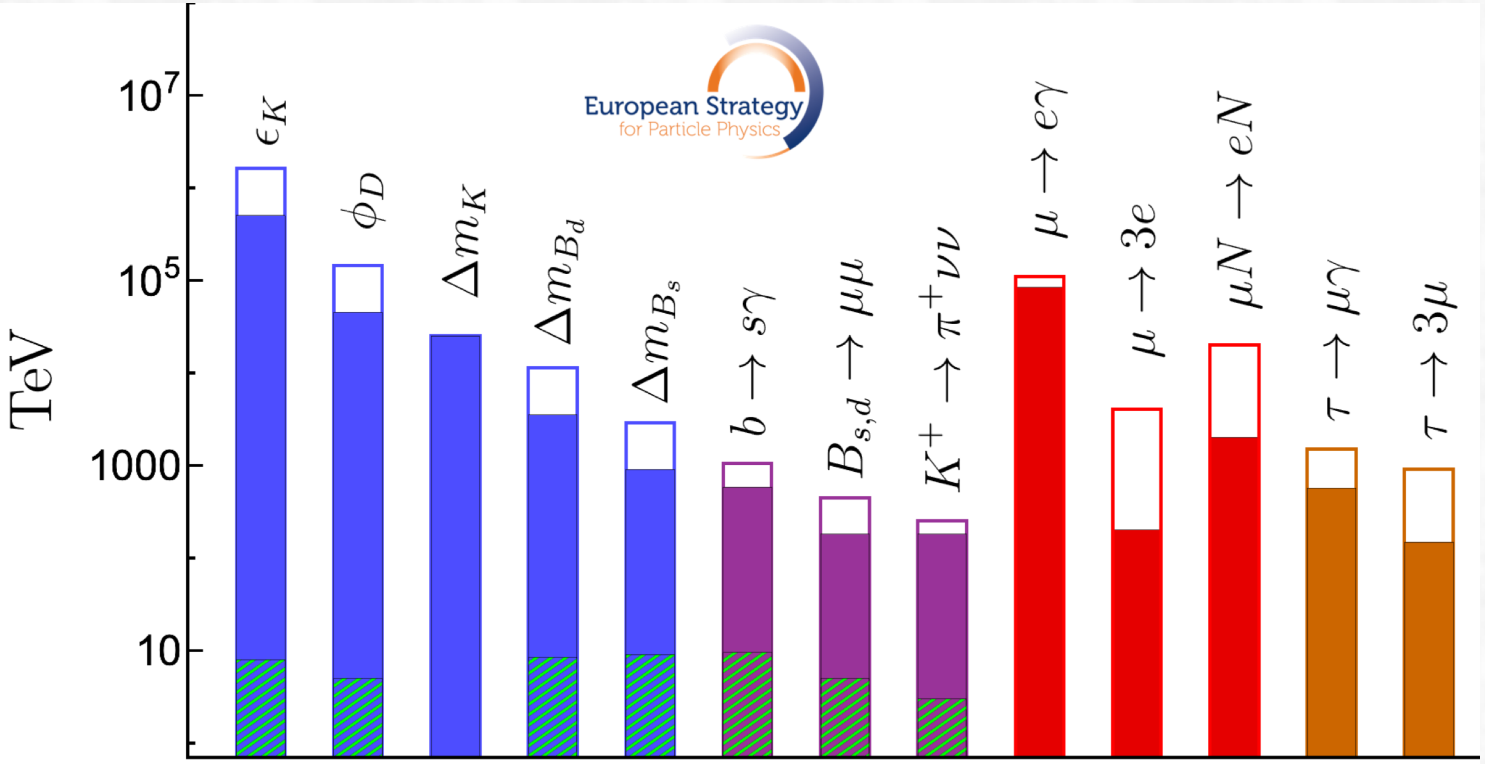
NP constraints from neutral B mixings

- **Future prospects:** $M_{12} = (M_{12})_{\rm SM} \times \left(1 + h_{d,s} e^{2i\sigma_{d,s}}\right)$



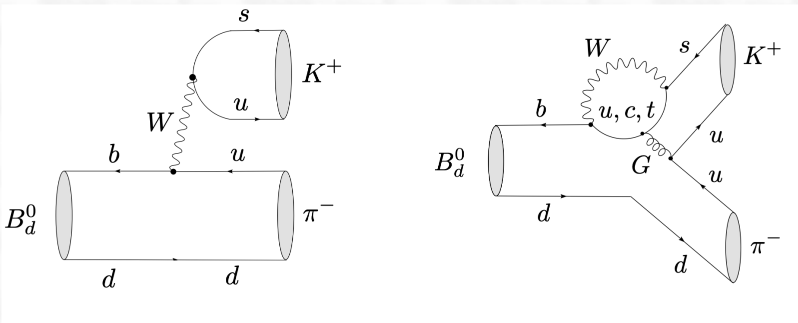
$$\boxed{\frac{C_{ij}}{\Lambda^2} = \frac{1}{\Lambda^2}}$$





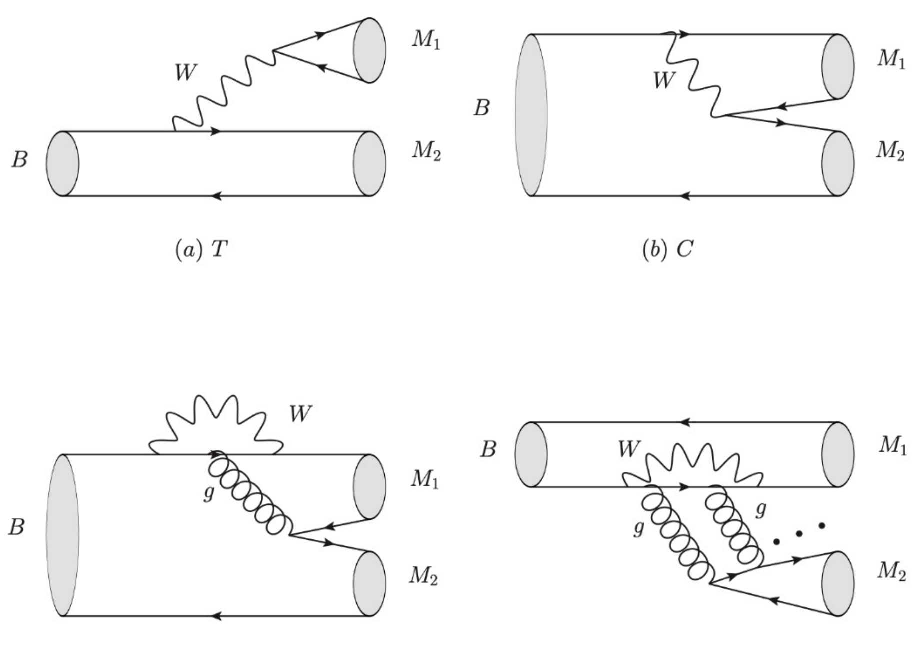
$B \to \pi K$ puzzle

- $B \to \pi K$ **decays dominated by QCD penguin diagrams**
- **For direct CPV:** color-suppress tree & EW penguin also crucial



$\propto A\lambda^4 R_b e^{i\gamma}$ $\propto A\lambda^2$

$\lambda^2 R_b = \mathcal{O}(0.02) \Rightarrow$ QCD penguins *dominate*



(a) T (b) C (c) $P,\ P_{\rm EW}^C$ (d) $S,\ P_{\rm EW}$

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\big] + A_{\bar K\pi}\big[\delta_{pu}\alpha_2+\delta_{pc}\tfrac{3}{2}\alpha^c_{3,\rm EW}\big],$$
$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\big],$$

$$\Delta A_{CP}(\pi K) = A_{CP}(\pi^0K^\pm) - A_{CP}(\pi^\mp K^\pm)$$
$$= -2\sin\gamma\big(\mathrm{Im}\,r_C - \mathrm{Im}(r_T r_{EW})\big) + \cdots$$
$$= (11.0 \pm 1.2)\%$$ **differs from 0 by ~9σ**

$\Delta A_{CP}(\pi K)$ puzzle

some mechanism or sub-leading power corrections

or even NP to enhance $C = \alpha_2$ or $P_{\rm EW} = \alpha^p_{3,\rm EW}$?