



Axions in Chiral Effective Field Theory

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2406.11948 [Phys.Rev.Lett. 134, 081803 (2025)],
2505.24822 [JHEP 02 (2026) 117]

第十一届手征有效场论研讨会
2026-08-12 @ 兰州大学

The Strong CP problem

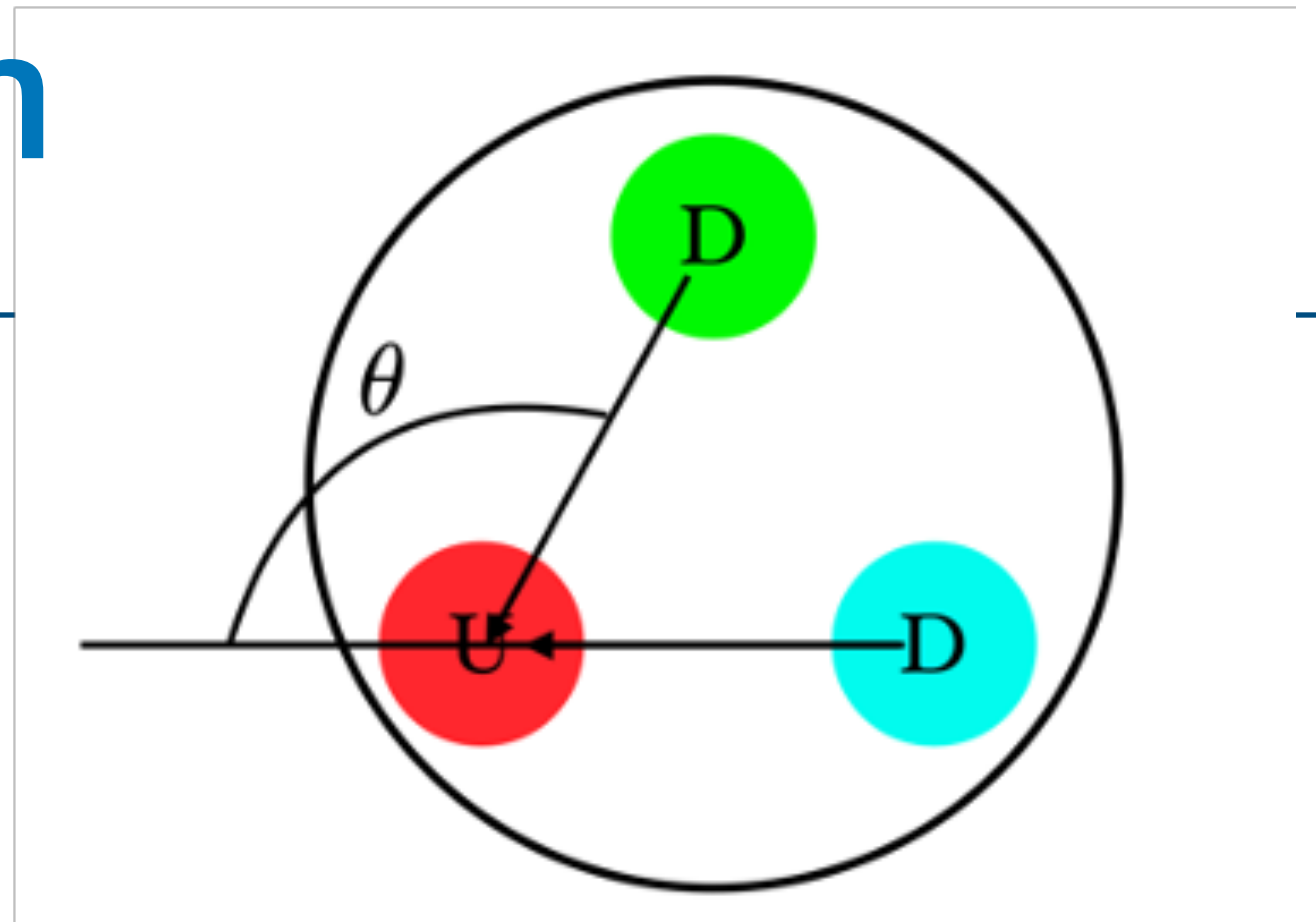
- Neutron charge compositions: $n = u + d + d$
- Multipole expansion:

- $Q_n = -\frac{2}{3} + \frac{1}{3} + \frac{1}{3} = 0$

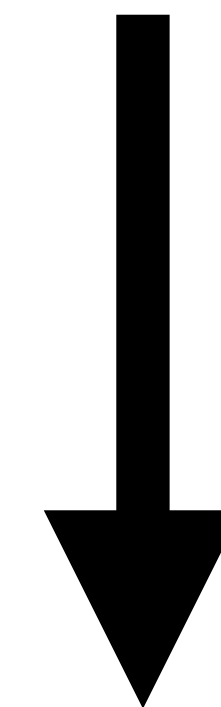
- Electric Dipole Moment:

$$d_n = -\frac{2}{3}r_u + \frac{1}{3}r_{d_1} + \frac{1}{3}r_{d_2} \approx \mathcal{O}(r_n \cdot e) \approx \text{fm} \cdot e$$

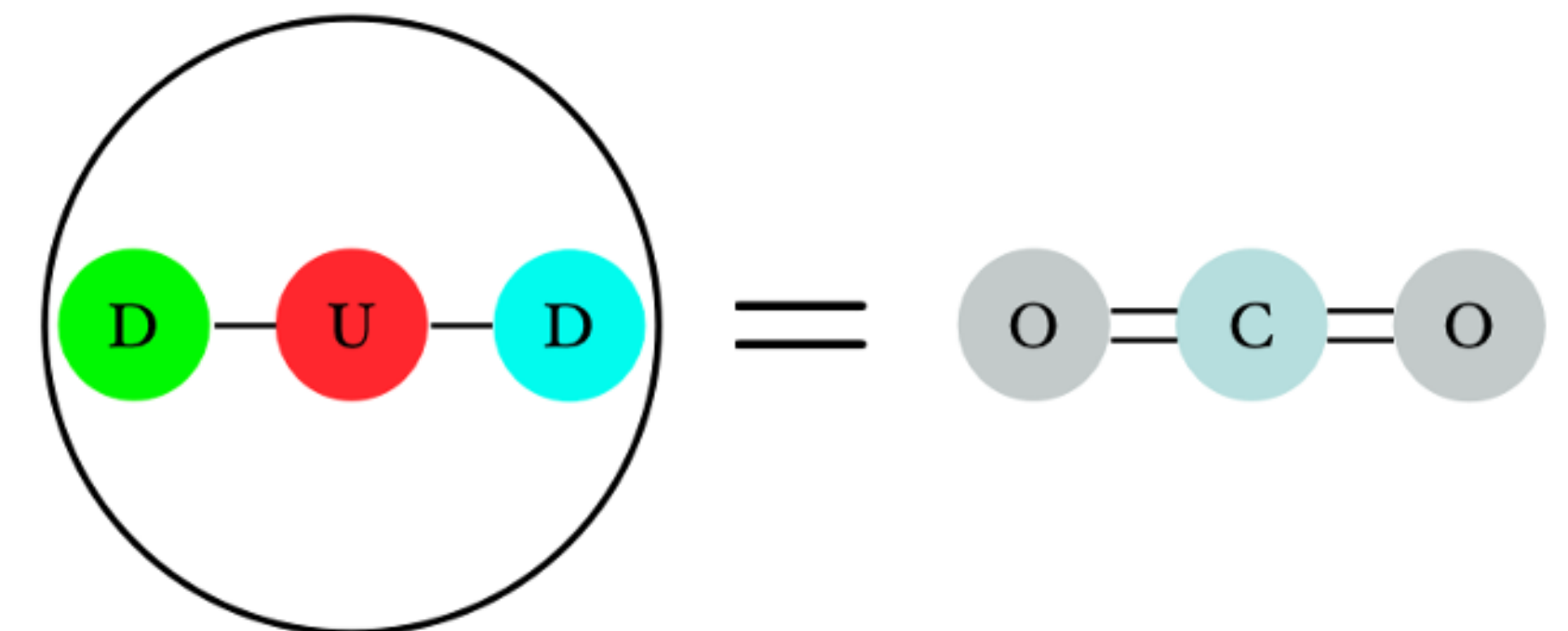
- Neutron is more complex than Atom e-cloud
- **But Exp tells:** $d_{\text{exp}}^n < 10^{-26} \text{ e cm}$
- Naive estimate is off by 10 orders
- Fine tuning without **anthropic principle** excuse



$$|d_n| \approx 10^{-13} \sqrt{1 - \cos\theta} \text{ e cm}$$



$$\theta \lesssim 10^{-10}$$



The Strong CP problem

$$\mathcal{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - (\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \text{h.c.})$$

- The CKM matrix from $M_{u,d}$
 - CP violating phase $\theta_{\text{CP}} \sim 1.2$ radian

- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg [\det [M_u M_d]]$$

- $\bar{\theta}$ is invariant under quark chiral rotation
- According to neutron EDM experiment

$$\bar{\theta} \lesssim 1.3 \times 10^{-10} \text{ radian}$$

A total derivative, but is allowed by non-trivial QCD vacuum

$$G_{\mu\nu}^a \tilde{G}^{a\mu\nu} = \partial_\mu K^\mu,$$

$$K^\mu = \epsilon^{\mu\nu\alpha\beta} \left(G_\nu^a \partial_\alpha G_\beta^a + \frac{g}{3} f^{abc} G_\nu^a G_\alpha^b G_\beta^c \right)$$

$$d_{\text{EDM}}^n \sim \bar{\theta} \times 10^{-16} \text{ e cm}$$

$$d_{\text{exp}}^n < 10^{-26} \text{ e cm}$$

Why?

The Peccei-Quinn solution to Strong CP problem

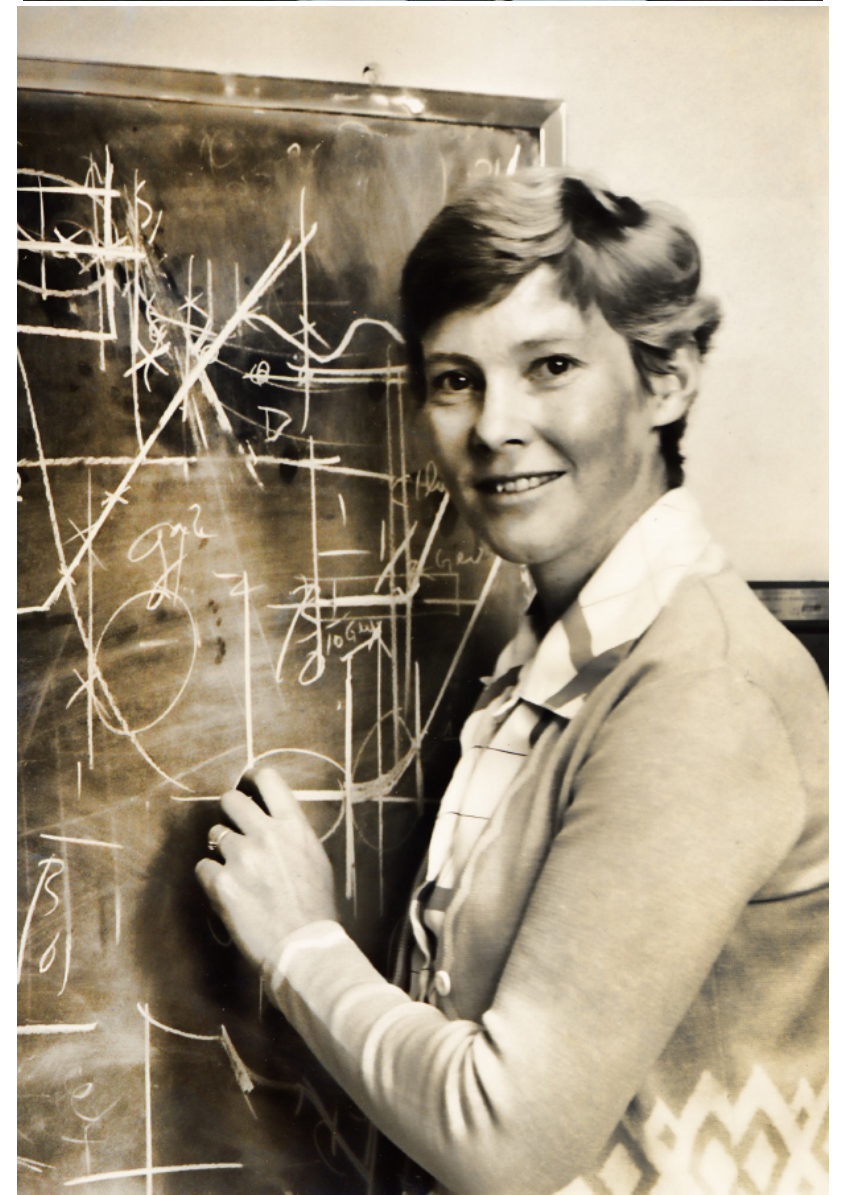
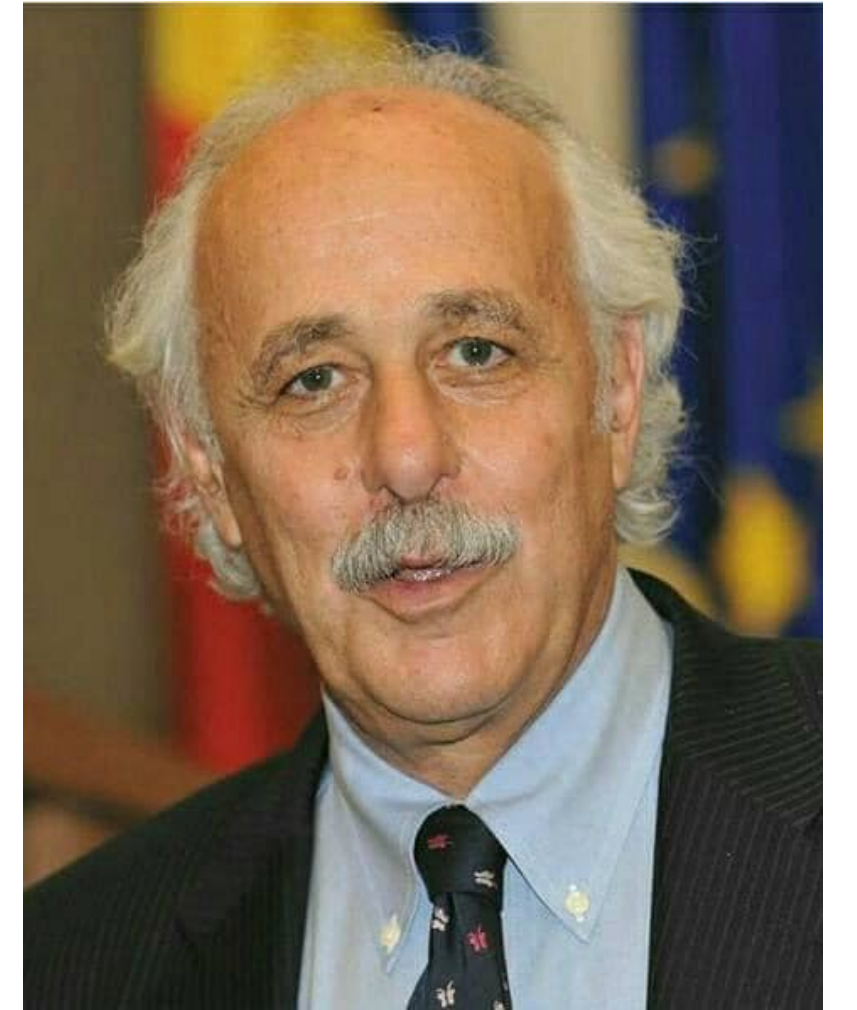
- Experiment requires

$$\bar{\theta} = \theta + \arg \left[\det [M_u M_d] \right] \lesssim 10^{-1} \text{rad}$$

- PQ: promote the constant $\bar{\theta}$ to a dynamical field, $a(x)$
- Introduce a *global* PQ-symmetry $U(1)_{\text{PQ}}$, *anomalous* under the QCD

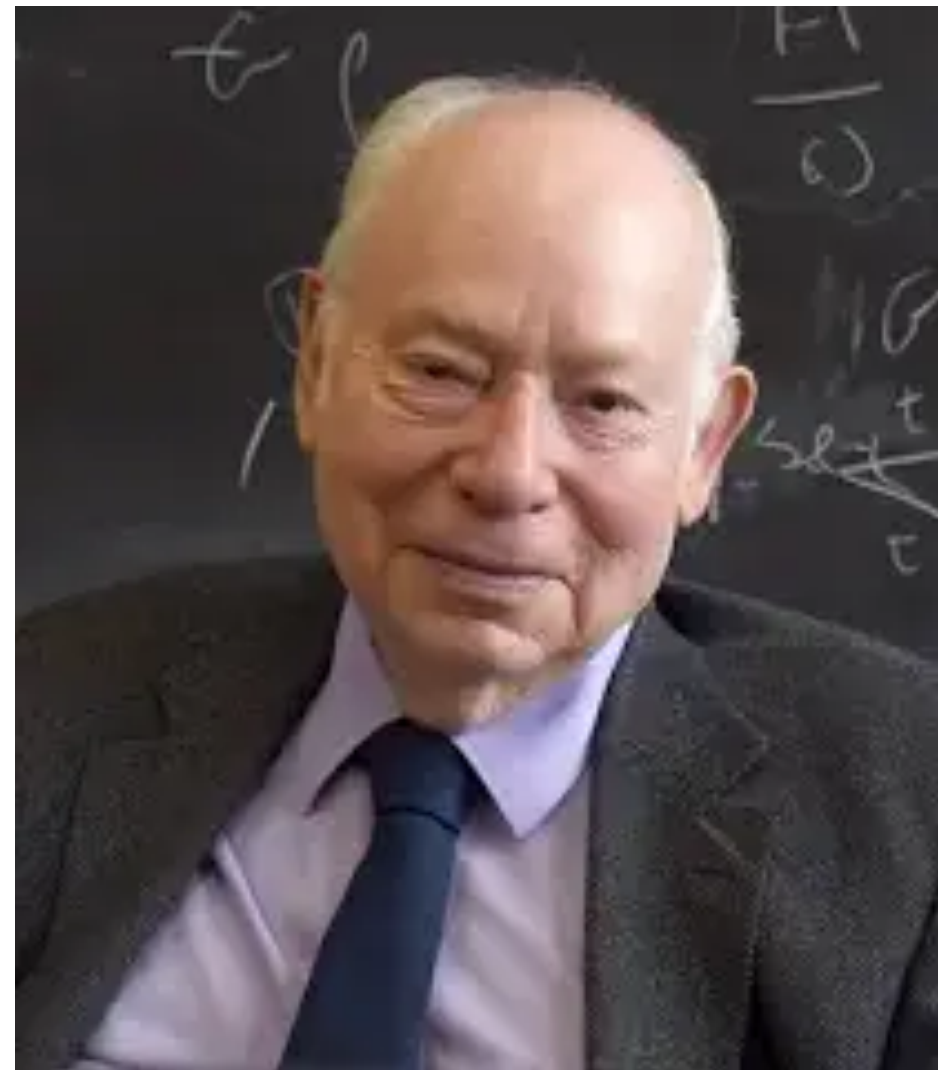
- $a \rightarrow a + \kappa f_a \Rightarrow \mathcal{S} \rightarrow \mathcal{S} + \frac{\kappa}{32\pi^2} \int d^4x G\tilde{G}$, cancels $\bar{\theta}$

- Vafa-Witten theorem: vector-like theory (QCD) has ground state $\langle \bar{\theta} \rangle = 0$



PQWW Axion

- In 1978, Weinberg and Wilzeck realize there is an light particle



Axion!!!



- It can wash out the unwanted strong CP phase
- QCD axion : $m_a^2 f_a^2 \approx \Lambda_{\text{QCD}}^4$; Neutrino Seesaw: $m_\nu m_{N_R} \approx (y v_h)^2$
- Light particles can probe high scale physics!

PQWW Axion

- PQWW axion assumes breaking scale $f_a \sim v_{EW}$ in a **2HDM setup**.
- Axion mass from $100 \text{ keV} \sim 1 \text{ MeV}$, and the coupling strength is large $1/f_a$
- Eight scalars:
 - Neutral: CP even h^0, H^0
 - Charged: $H^\pm$;
 - Massless goldstones (eaten):

$$G^\pm, G^0 = \frac{v_1 \eta_1 + v_2 \eta_2}{v}$$
 - Massless goldstone for PQ:

$$A^0 = \frac{-v_2 \eta_1 + v_1 \eta_2}{v} \text{ (Axion)}$$

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix}$$

Field	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$U(1)_{PQ}$
Q_L	$(3, 2, +\frac{1}{6})$	0
u_R	$(3, 1, +\frac{2}{3})$	+1
d_R	$(3, 1, -\frac{1}{3})$	+1
L_L	$(1, 2, -\frac{1}{2})$	0
e_R	$(1, 1, -1)$	+1
Φ_1	$(1, 2, +\frac{1}{2})$	+1
Φ_2	$(1, 2, +\frac{1}{2})$	-1

Yukawa interactions: global PQ symmetry

$$\mathcal{L}_Y = y_u \bar{Q}_L \tilde{\Phi}_1 u_R + y_d \bar{Q}_L \Phi_2 d_R + y_e \bar{L}_L \Phi_2 e_R + \text{h.c.}$$

Scalar interactions:

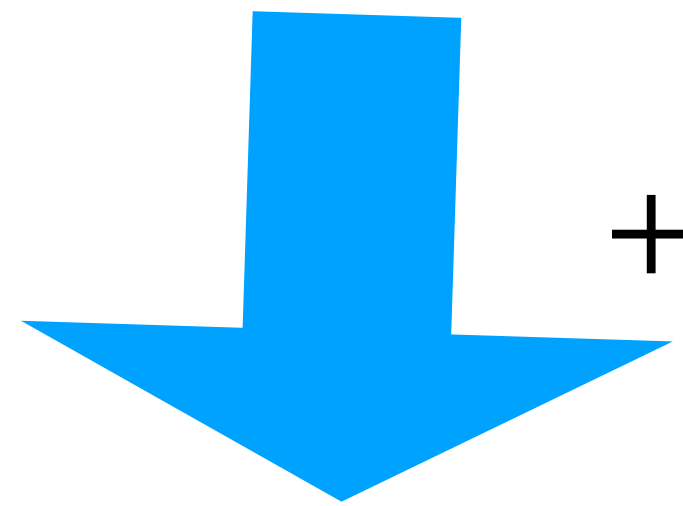
$$V(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1).$$

PQWW axion to Invisible QCD axion

- PQWW axion assumes breaking scale $f_a \sim v_{EW}$
- Axion mass from $100 \text{ keV} \sim 1 \text{ MeV}$, and the coupling strength is large $1/f_a$
- PQWW axion is quickly ruled out by
 - Lab constraints: $K^\pm \rightarrow \pi^\pm + a$, $J/\Psi \rightarrow \gamma + a$, and $\Upsilon \rightarrow \gamma + a$
 - Astrophysical constraints: Red giant and Supernovae
- (*Invisible*) QCD axion: the leading axion models are KSVZ/DFSZ model with $f_a \gg v_{EW}$

The consistent ChPT axion Lagrangian at meson level

$$\mathcal{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$



$$+ g_{a\gamma} \frac{a}{f_a} F\tilde{F} + \frac{\partial_\mu a}{f_a} (\bar{q}_L \mathbf{k}_L(a) \gamma^\mu q_L + \bar{q}_R \mathbf{k}_R(a) \gamma^\mu q_R + \dots)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a \tau^a]$$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \left[(D^\mu U)(D_\mu U)^\dagger \right] + \frac{f_\pi^2}{4} B_0 \text{Tr} \left[\mathbf{m}_q(a) U^\dagger + h.c. \right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma} \frac{a}{f_a} F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \rightarrow D^\mu U - i \frac{\partial^\mu a}{f_a} (\mathbf{k}_L U - U \mathbf{k}_R)$$

Why consistent interactions are important?

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp \left[-i(\delta_{q,0} + \kappa_{q,0}\gamma_5) c_{gg} \frac{a(x)}{f_a} \right] q(x)$$

- The most important channel $\text{BR}(K \rightarrow \pi a)$ is off by a factor of 37 for 35 years

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \rightarrow \pi a$ and $\pi^- \rightarrow e^- \bar{\nu}_e a$ have been obtained for all axion couplings, only in 2021

Bauer et al, PRL 127 (2021), 081803

Why consistent interactions are important?

- Prediction for thermal axion and its near future test by CMB observation

- Thermal axion production (high T): $q\bar{q} \rightarrow ga, qg \rightarrow qa$

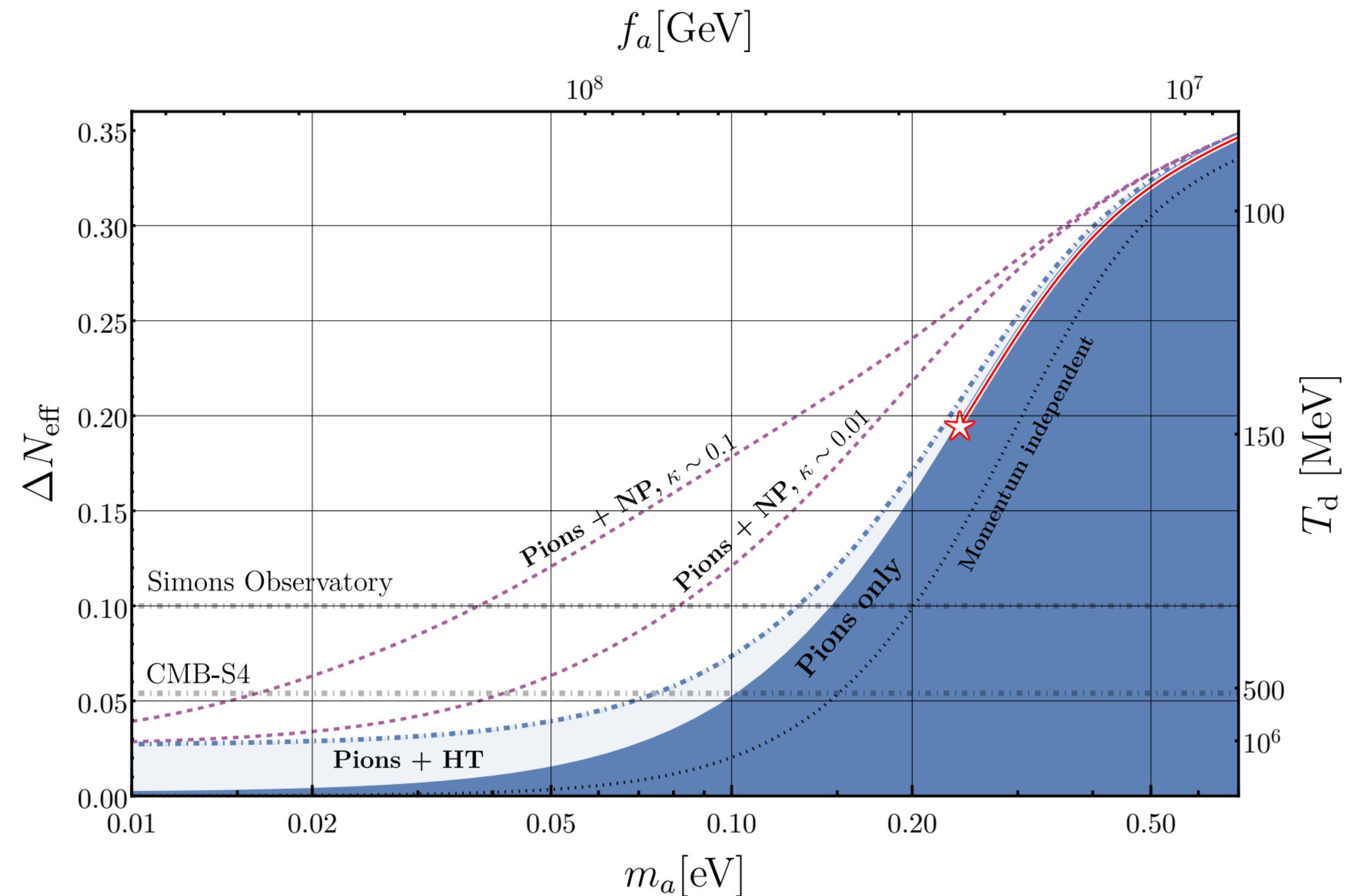
Ferreira, Notari PRL 120 (2018)191301

- QCD phase transition:

D'Eramo, Hajkarim, Yun PRL 128 (2022)152001

- Improved axion-pion scattering production: $\pi\pi \leftrightarrow \pi a$

Notari, Rompineve, Villadoro PRL 131 (2023)011004



QCD axion: $m_a < 0.24$ eV

Axion couplings to other mesons/baryons/EFT

- Axion couplings to other mesons , e.g. K , K^* , η , η' etc

Wang, Guo, Zhou PRD Letter 2025; Gao, Guo, Oller, Zhou JHEP04(2023)022; Wang, Guo, Zhou PRD 109(2024)075030; Wang, Guo Lu, Zhou 2403.16064; Cao, Guo, 2408.15825;

- Axion couplings to baryons

Vonk, Guo, Meissner JHEP03(2020)138, Lu, Du, Guo, Meissner, Vonk JHEP05(2020)001, Vonk Guo, Meissner JHEP08(2021)024 ...

- Axion coupling to DM

Cheng, Bian, Zhou, Phys.Rev.D 104 (2021) 6, 063010;

Yang, Feng, Wu, J. Phys. G: Nucl. Part. Phys. 51 065201(2024);

- Axion EFT

Hu, Jiang, Li, Xiao, Yu, PRD 103(2021)095025; Song, Sun, Yu JHEP01(2024)161;

- Etc ...

Wess-Zumino-Witten Interactions in QCD

- WZW terms can describe anomalies in QCD, ensuring gauge invariance and completing chiral L
- Low-energy dynamics of mesons:
e.g. multiple mesons and photons interactions
 $\pi_0/\eta/\eta' \rightarrow \gamma\gamma, \eta' \rightarrow 4\pi, \gamma^* \rightarrow 3\pi, 5\pi$
- Axion should be involved in WZW interactions systematically,
not only in $a - \gamma - \gamma$ interactions
 - Raised by Harvey Hill Hill in [PRL 99 (2007) 261601], but not solved in previous study

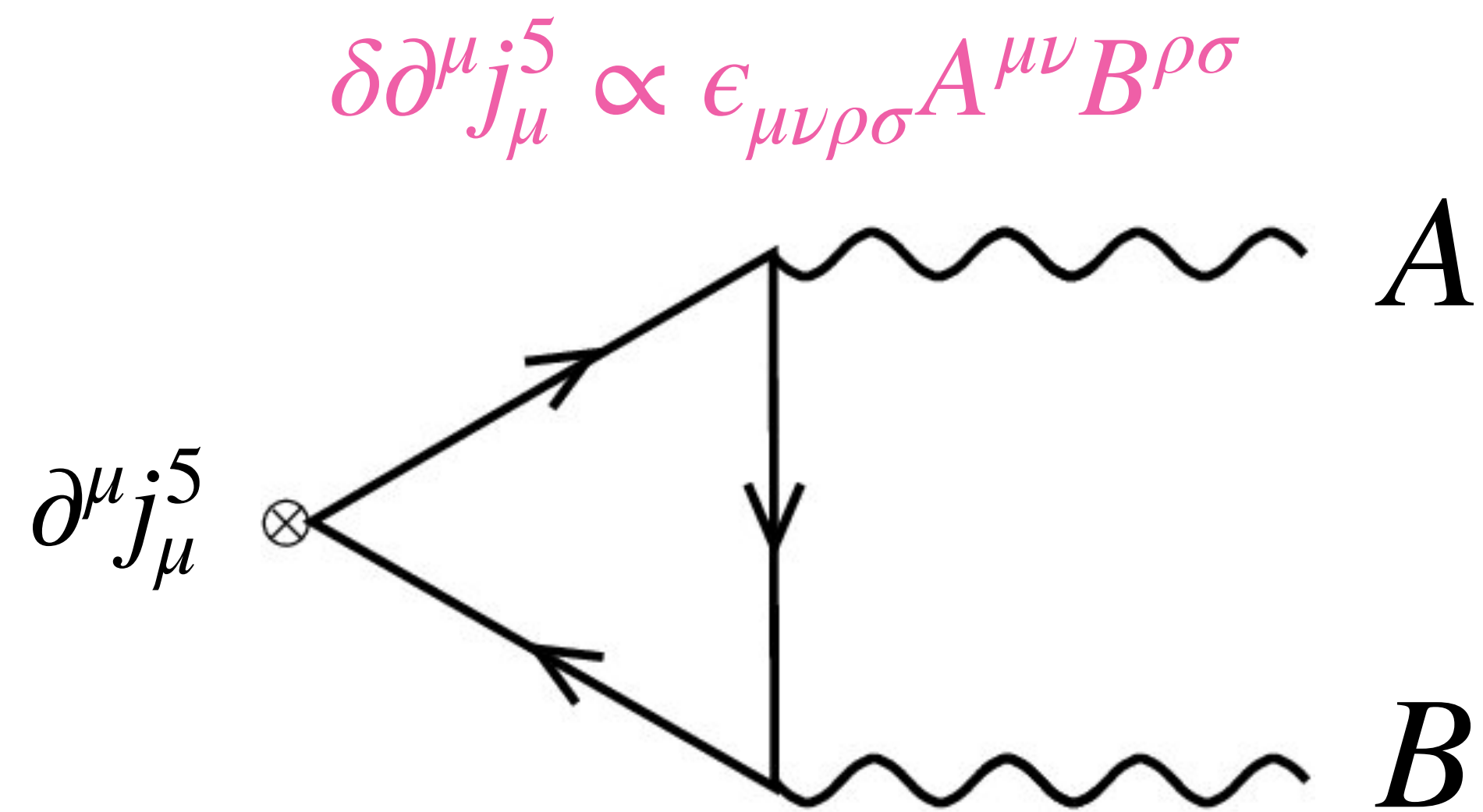
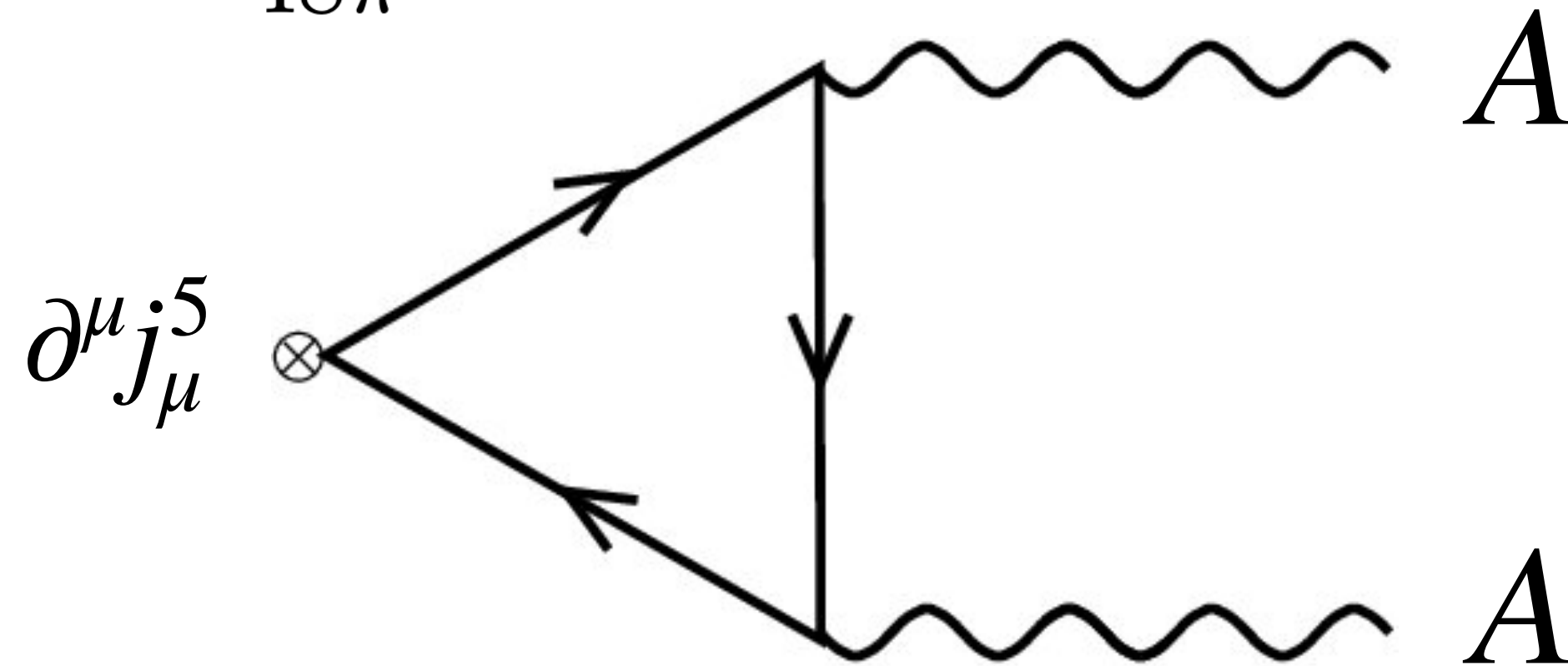
Challenges in axion-meson interactions

- 1. Physics should not depend on the choice of chiral basis
- 2. How to systematically include axion interactions into WZW terms
- 3. Global symmetry (e.g. PQ, $U(1)_B$) will induce mixed anomaly to be dealt with

Global currents and background vector fields

- Background fields can couple to currents of $\mathcal{L}_{\chi\text{PT}}$
 - Baryon currents $U(1)_B$ in neutron star, ω meson
 - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j_\mu^5 = \frac{1}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
- Vector fields in 1-form: $\mathcal{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
Similar to Hidden Local Symmetry

$$\mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathcal{A}_L, \mathcal{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathcal{C} \int \text{Tr} \left[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L) \mathbb{B}_L + \frac{1}{2} \mathbb{A}_L (\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i \mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i \mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2} \mathbb{A}_L \mathbb{B}_L^3 \right] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

Axion treatment in three flavor

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{q}i\not{D}q - (\bar{q}_L \mathbf{m}_q q_R + \text{H.c.}) + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{m_a^2}{2}a^2$$

$$+ \frac{\partial^\mu a}{f_a} (\bar{q}_L \gamma^\mu \mathbf{k}_L q_L + \bar{q}_R \gamma^\mu \mathbf{k}_R q_R) + c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{a}{f_a} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu} + \mathcal{L}_c ,$$

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \rightarrow D_\mu + i \frac{\partial_\mu a}{f_a} (\mathbf{k}_L P_L + \mathbf{k}_R P_R)$
- Hints from ChPT L: $D^\mu U \rightarrow D^\mu U - i \frac{\partial^\mu a}{f_a} (\mathbf{k}_L U - U \mathbf{k}_R)$

Pseudoscalars in three flavor

- U matrix for mesons in three flavor

$$U = \exp \left[(\sqrt{2}i/f_\pi) \pi^a \mathbf{t}^a \right] \equiv \exp \left[(\sqrt{2}i/f_\pi) \mathbf{\Phi} \right]$$

$$\mathbf{\Phi} = \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

- η' mass from instanton effect $\bar{\theta} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \rightarrow -\frac{\tau}{2} (-i \log \det U - \bar{\theta})^2$

**Axionized
Lagrangian:**

$$\mathcal{L}_{\chi\text{PT}} \supset -\frac{\tau}{2} \left(-i \log \det U - 2c_{gg} \frac{a}{f_a} \right)^2 = -\frac{m_0^2}{2} \left(\eta_0 - \frac{c_{gg}}{\sqrt{3}} \frac{f_\pi}{f_a} a \right)^2$$

Axion treatment in three flavor

- Vector fields in 1-form: $\mathcal{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$ $\mathbb{A}_L = \frac{e}{s_w} W^i \mathbf{T}_i + \frac{e}{c_w} W^0 \mathbf{Y}_Q$, $\mathbb{A}_R = \frac{e}{c_w} W^0 \mathbf{Y}_q$
Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields:

$$\mathbb{B}_{L/R} \rightarrow \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \frac{da}{f_a}$$

- 3-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K_+^* \\ \sqrt{2}\rho^- & -\rho_0 + \omega & \sqrt{2}K_0^* \\ \sqrt{2}K_-^* & \sqrt{2}\bar{K}_0^* & \sqrt{2}\phi \end{pmatrix} + (\mathbf{k}_L + \mathbf{k}_R) \frac{da}{f_a},$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 + f_1 & \sqrt{2}a^+ & \sqrt{2}K_{A+}^* \\ \sqrt{2}a^- & -a_1 + f_1 & \sqrt{2}K_{A0}^* \\ \sqrt{2}K_{A-}^* & \sqrt{2}\bar{K}_{A0}^* & \sqrt{2}f_s \end{pmatrix} + (\mathbf{k}_L - \mathbf{k}_R) \frac{da}{f_a}$$

The consistent full axion-meson Lagrangian at Λ_{QCD}

$$D^\mu = \partial^\mu - i \sum_A (\mathcal{A}_L^\mu P_L + \mathcal{A}_R^\mu P_R)$$

- ChPT: $\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr} \left[(D^\mu U) (D_\mu U)^\dagger \right] + \frac{f_\pi^2}{4} B_0 \text{Tr} [\mathbf{m}_q U^\dagger + \text{H.c.}] - \frac{m_0^2}{2} \left(\eta_0 - \frac{c_{gg}}{\sqrt{3}} \frac{f_\pi}{f_a} a \right)^2$
 $+ \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{m_a^2}{2} a^2 + \frac{a}{f_a} \sum_{A_1, 2} c_{A_1 A_2} F_{A_1 \mu \nu} \tilde{F}_{A_2}^{\mu \nu},$
- Full WZW: $\mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathcal{A}_L, \mathcal{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$
- Full $\mathcal{L}$: $\mathcal{L}_{\text{axion}}^{\text{full}} \equiv \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] \left(U, \mathbf{m}_q(a), \mathcal{A}_{L/R} + \mathbf{k}_{L/R}(a) da/f_a \right)$

Bai, Chen, **JL**, Ma 2406.11948 (PRL)

Bai, Chen, **JL**, Ma 2505.24822 (JHEP)

Matching between $\mathcal{L}_{\text{eff}}$ and $\mathcal{L}_{\text{axion}}^{\text{full}}$

$$\begin{array}{ccc}
 \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{\text{手征基变换}} & \left\{ \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_a^{\text{ano}} \right\} \\
 \supset c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G \tilde{G} & q \rightarrow \exp \left[-i \left(\delta_q + \kappa_q \gamma_5 \right) \frac{c_{gg} a}{f_a} \right] q & \supset c_{gg} [1 - \text{Tr}(\kappa_q)] \frac{\alpha_s}{4\pi} \frac{a}{f_a} G \tilde{G}
 \end{array}$$

低能匹配
matching

matching
低能匹配

$$\begin{array}{ccc}
 \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{\text{手征基变换}} & \left\{ \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_{\text{WZW}}^{\text{ano}} \right\} \\
 \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2c_{gg} a/f_a \right)^2 & U \rightarrow U_L U U_R^\dagger & \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2c_{gg} [1 - \text{Tr}(\kappa_q)] a/f_a \right)^2
 \end{array}$$

- Important: consistency for any κ_q rotation

Comparing 2-quark flavor to 3-quark flavor

$$\begin{array}{ccc}
 \mathcal{L}_{\text{eff},0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0}) & & \\
 \downarrow q_0 = \exp\left(-i c_{gg} \boldsymbol{\kappa}_{q,0} \gamma_5 \frac{a}{f}\right) q & & \\
 \mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) & \xrightarrow{q' = \exp\left[i\left(\delta_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{a}{f}\right] q} & \mathcal{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathcal{L}_a^{\text{ano}} \\
 \downarrow \text{matching} & & \downarrow \text{matching} \\
 \mathcal{L}_{\text{axion}}^{\text{full}} \equiv \mathcal{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da) & \xrightarrow{U' = U_L^\dagger U U_R} & \mathcal{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da) \\
 + \mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da) & & + \mathcal{L}_{\text{WZW}}^{\text{full}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da) \\
 & & + \delta \mathcal{L}_{\text{WZW}}^{\text{ano}}
 \end{array}$$

- Typically, people eliminate GG term by **chiral rotation first**
- In our 2-quark flavor, only prove quark chiral basis invariant without re-introducing GG term

Bai, Chen, **JL**, Ma 2406.11948 (PRL)

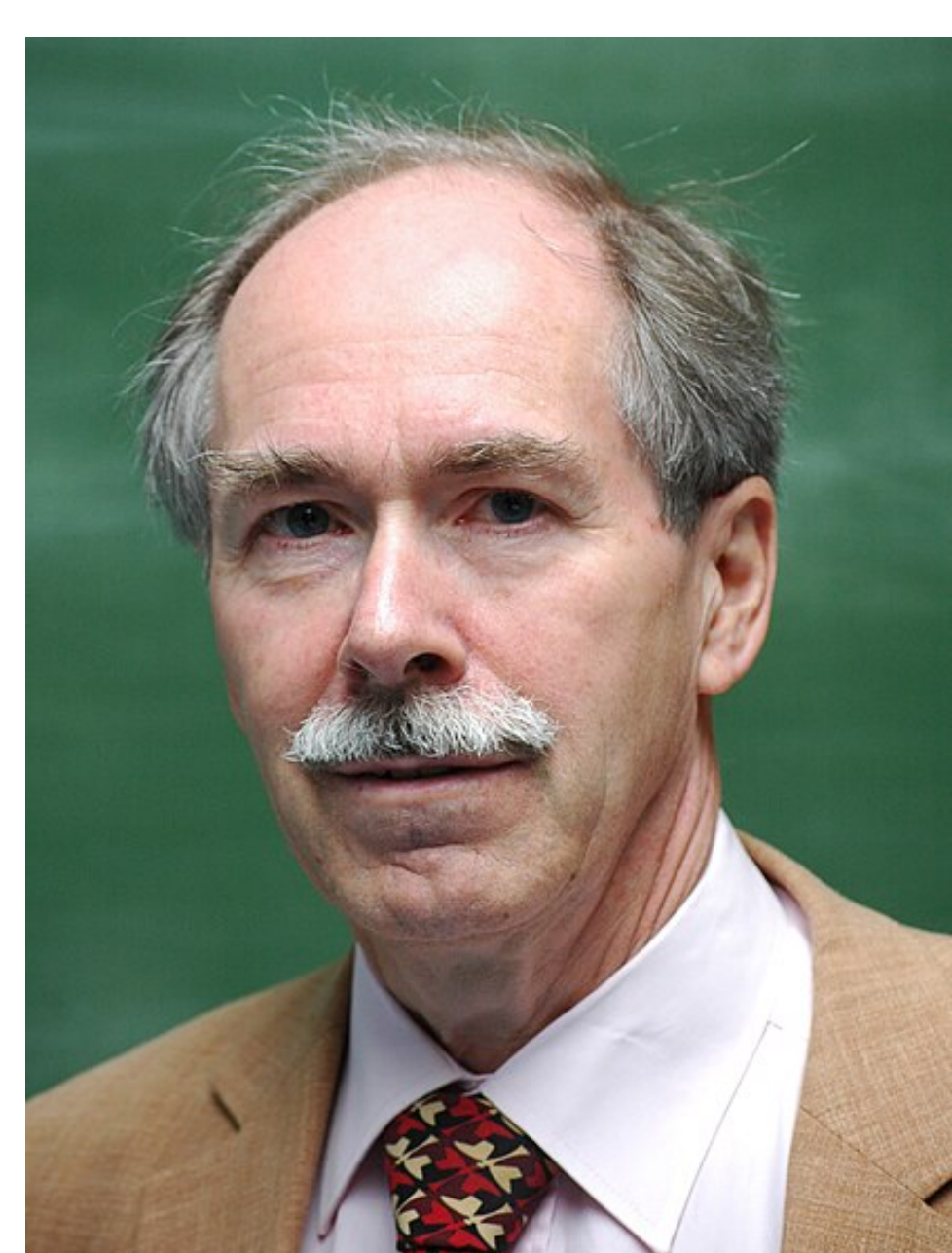
$$\begin{array}{ccc}
 \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{q \rightarrow \exp\left[-i\left(\delta_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{c_{gg} a}{f_a}\right] q} & \left\{ \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_a^{\text{ano}} \right\} \\
 \supset c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G\tilde{G} & & \supset c_{gg} [1 - \text{Tr}(\boldsymbol{\kappa}_q)] \frac{\alpha_s}{4\pi} \frac{a}{f_a} G\tilde{G} \\
 \downarrow \text{matching} & & \downarrow \text{matching} \\
 \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{U \rightarrow U_L U U_R^\dagger} & \left\{ \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_{\text{WZW}}^{\text{ano}} \right\} \\
 \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2 c_{gg} a / f_a \right)^2 & & \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2 c_{gg} [1 - \text{Tr}(\boldsymbol{\kappa}_q)] a / f_a \right)^2
 \end{array}$$

- Important: consistency for any $\boldsymbol{\kappa}_q$ rotation
- Topological charge density $\langle G\tilde{G} \rangle$ integrated out
- $aG\tilde{G}$ term can be matched directly to η' mass term

Bai, Chen, **JL**, Ma 2505.24822 (JHEP)

Gerard 't Hooft UV-IR anomaly matching condition

- The anomalies of global symmetries must match between the ultraviolet (UV) and infrared (IR) descriptions of a QFT



Gerard 't Hooft

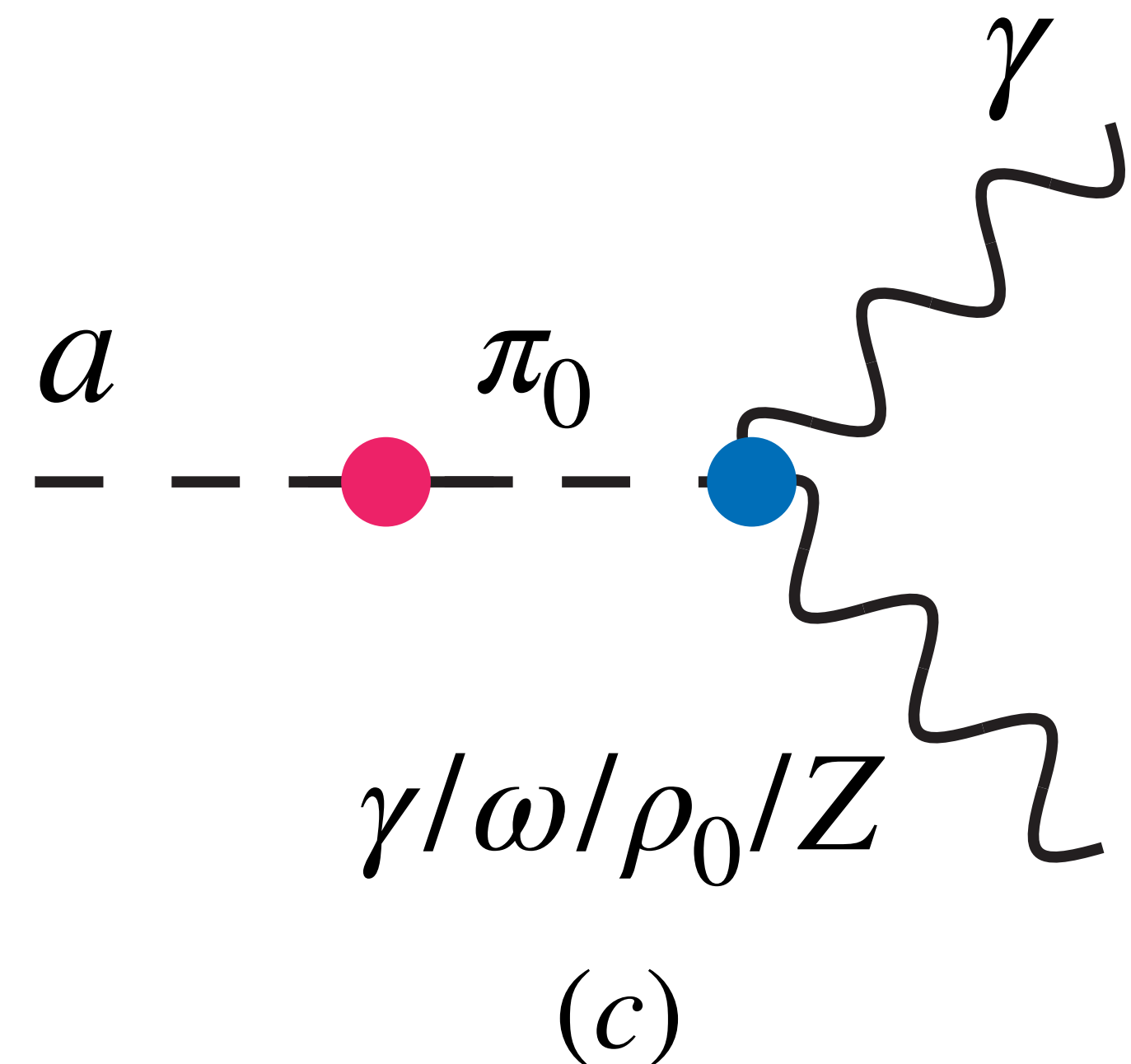
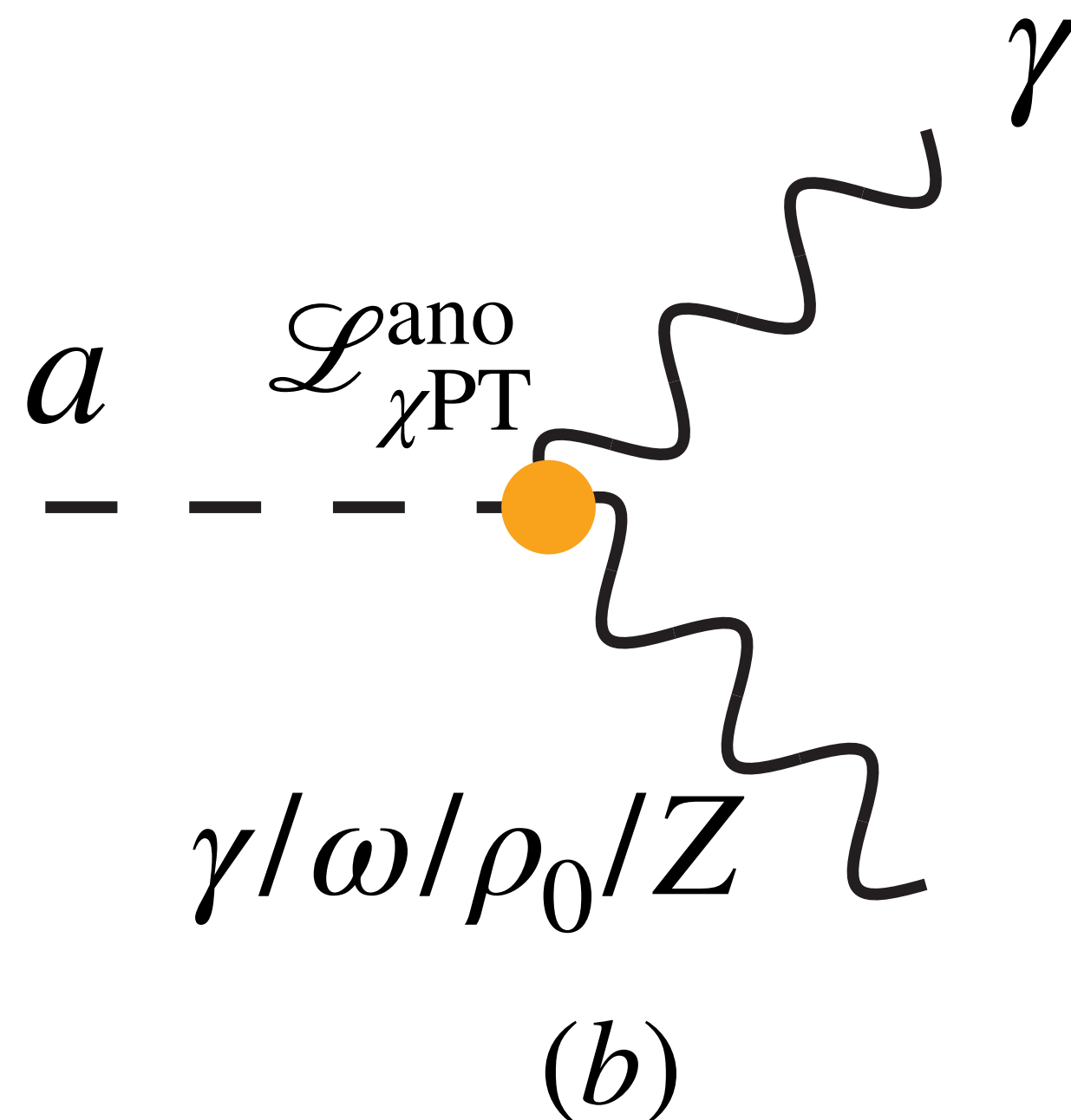
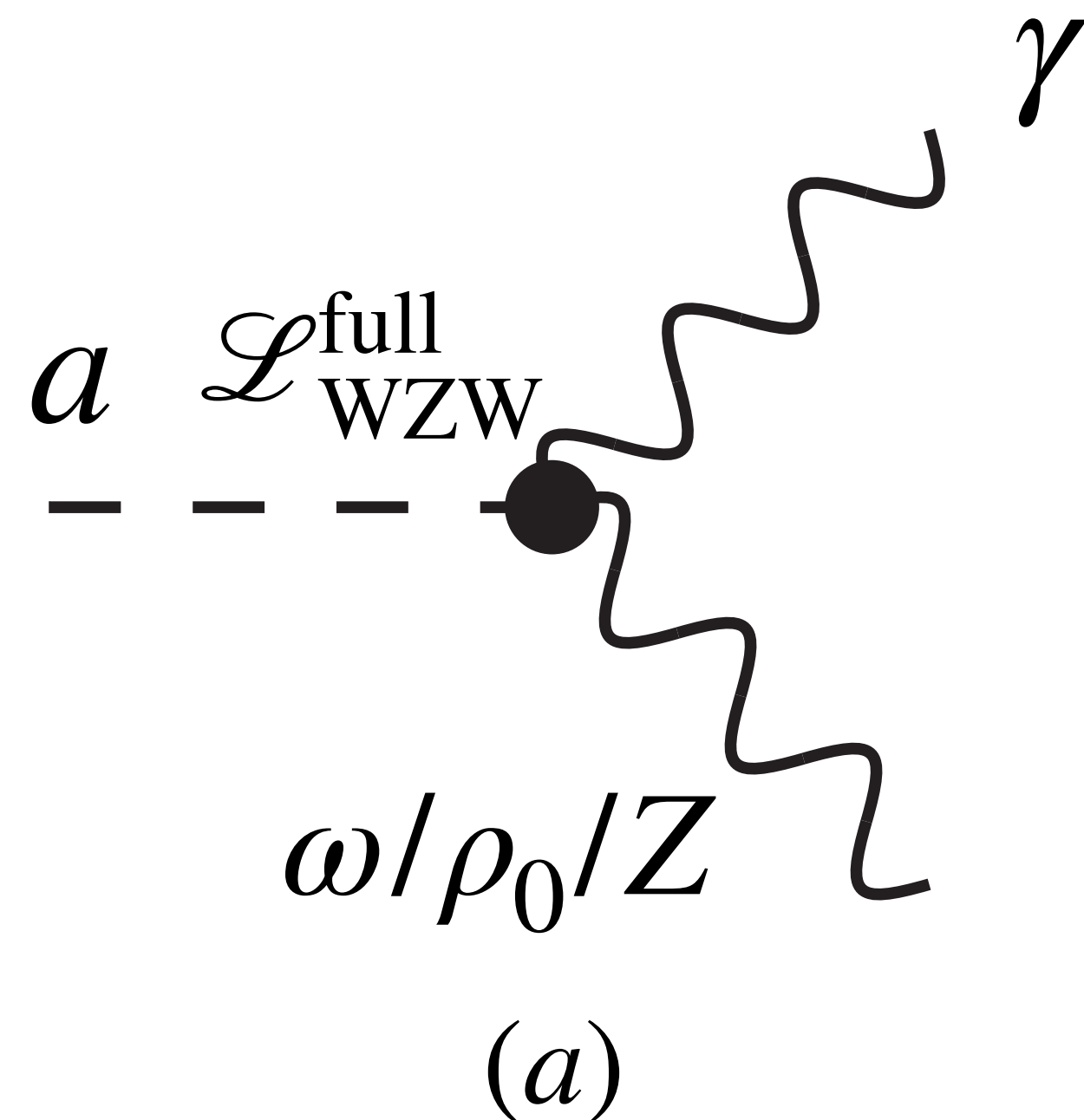
$$\begin{array}{ccc}
 \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{q \rightarrow \exp\left[-i\left(\delta_q + \kappa_q \gamma_5\right) \frac{c_{gg} a}{f_a}\right] q} & \left\{ \mathcal{L}_{\text{eff}}(q, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_a^{\text{ano}} \right\} \\
 \supset c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G\tilde{G} & & \supset c_{gg} [1 - \text{Tr}(\kappa_q)] \frac{\alpha_s}{4\pi} \frac{a}{f_a} G\tilde{G} \\
 \downarrow \text{matching} & & \downarrow \text{matching} \\
 \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}_q, \mathbf{k}_{L/R}, c_{gg}) & \xrightarrow{U \rightarrow U_L U U_R^\dagger} & \left\{ \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] (U, \mathcal{A}_{L/R} | \mathbf{m}'_q, \mathbf{k}'_{L/R}, c'_{gg}) + \delta \mathcal{L}_{\text{WZW}}^{\text{ano}} \right\} \\
 \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2c_{gg} a/f_a \right)^2 & & \supset -\frac{\tau}{2} \left(-i \log[\det U] - 2c_{gg} [1 - \text{Tr}(\kappa_q)] a/f_a \right)^2
 \end{array}$$

Anomaly Matching

$$\delta \mathcal{L}_a^{\text{ano}} = -\delta \left[\mathcal{L}_{\text{WZW}} + \mathcal{L}_c \right] (\theta_L, \theta_R) = \delta \mathcal{L}_{\text{WZW}}^{\text{ano}}$$

Consistent physical amplitudes

- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations
- Full WZW interactions are important for a-A-B amplitudes



Pseudoscalar mixing in three flavor

- Mass/flavor eigenstates

$$a = a_{\text{phys}} - \sum_{P^0=\pi^0,\eta,\eta'} h(P^0, m_{P^0}) P_{\text{phys}}^0 ,$$

$$P^0 = P_{\text{phys}}^0 - \sum_{P^{0'} \neq P^0} \frac{M_{P^0 P^{0'}}^2}{m_{P^0}^2 - m_{P^{0'}}^2} P_{\text{phys}}^{0'} + h(P^0, m_a) a_{\text{phys}} ,$$

$$h(P^0, m_X) = \frac{1}{m_a^2 - m_{P^0}^2} \left[M_{aP^0}^2 - m_X^2 K_{aP^0} + \sum_{P^{0'} \neq P^0} M_{P^0 P^{0'}}^2 \frac{M_{aP^{0'}}^2 - m_X^2 K_{aP^{0'}}}{m_X^2 - m_{P^{0'}}^2} \right]$$

- Mixing angles

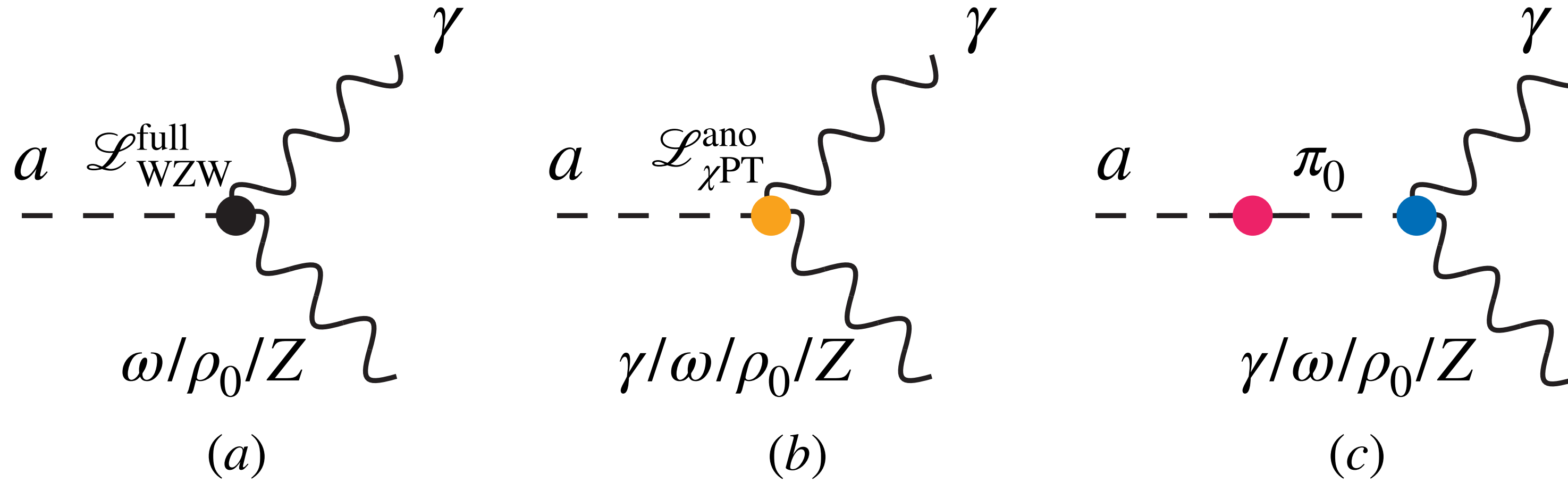
$$\theta_{\pi^0 a} = \frac{\epsilon}{2\sqrt{2}(m_\pi^2 - m_a^2)} \left\{ m_a^2 [c_d^L - c_u^L - c_d^R + c_u^R - (2\kappa_d - 2\kappa_u)c_{gg}] \right. \\ \left. + 2m_\pi^2 c_{gg} [(1 + \delta)\kappa_d + (-1 + \delta)\kappa_u] \right\} ,$$

$$\theta_{\eta a} = -\frac{\epsilon}{2\sqrt{3}(m_\eta^2 - m_a^2)} \left\{ m_a^2 [c_d^L - c_s^L + c_u^L - c_d^R + c_s^R - c_u^R - (2\kappa_d - 2\kappa_s + 2\kappa_u)c_{gg}] \right\} \\ - \frac{2\epsilon}{2\sqrt{3}(m_\eta^2 - m_a^2)} \left\{ m_\pi^2 [1 + \delta(\kappa_d - \kappa_u)] c_{gg} + m_\eta^2 (-1 + \kappa_d - \kappa_s + \kappa_u) c_{gg} \right\} ,$$

$$\theta_{\eta' a} = -\frac{\epsilon}{2\sqrt{6}(m_{\eta'}^2 - m_a^2)} \left\{ m_a^2 [c_d^L + 2c_s^L + c_u^L - c_d^R - 2c_s^R - c_u^R - (2\kappa_d + 4\kappa_s + 2\kappa_u)c_{gg}] \right\} \\ - \frac{2\epsilon}{2\sqrt{6}(m_{\eta'}^2 - m_a^2)} \left\{ (m_{\eta'}^2 + 3m_\pi^2)(-1 + \kappa_d + 2\kappa_s + \kappa_u) c_{gg} \right. \\ \left. - m_\pi^2 [-4 - (-3 + \delta)\kappa_d + 6\kappa_s + 3\kappa_u + \delta\kappa_u] c_{gg} \right\} . \quad (;$$

Consistent physical amplitudes

- Auxiliary rotations are cancelled $c_{\omega\gamma}^{\text{eff}} = c_{\text{WZW}} + c_{\text{ano}} + \sum_{=\pi^0, \eta, \eta'} c_p \theta_{pa}$



$$c_{\text{WZW}} = \frac{egN_c(c_d^L Q_d - c_d^R Q_d + c_u^L Q_u - c_u^R Q_u - 2Q_d c_{gg} \kappa_d - 2Q_u c_{gg} \kappa_u)}{16\pi^2 f_a},$$

$$c_{\text{ano}} = -\frac{egN_c(Q_d \kappa_d c_{gg} + Q_u \kappa_u c_{gg})}{8\pi^2 f_a},$$

$$c_{\pi^0} = \frac{egN_c(Q_d - Q_u)}{4\sqrt{2}f_\pi\pi^2}, \quad c_{\eta_8} = -\frac{egN_c(Q_d + Q_u)}{4\sqrt{6}f_\pi\pi^2}, \quad c_{\eta_0} = -\frac{egN_c(Q_d + Q_u)}{4\sqrt{3}f_\pi\pi^2},$$

$$c_\eta = \frac{2\sqrt{2}}{3}c_{\eta_8} + \frac{1}{3}c_{\eta_0}, \quad c_{\eta'} = -\frac{1}{3}c_{\eta_8} + \frac{2\sqrt{2}}{3}c_{\eta_0}.$$

Consistent physical amplitudes for $a \rightarrow \gamma \rightarrow \omega$

• 3-flavor results

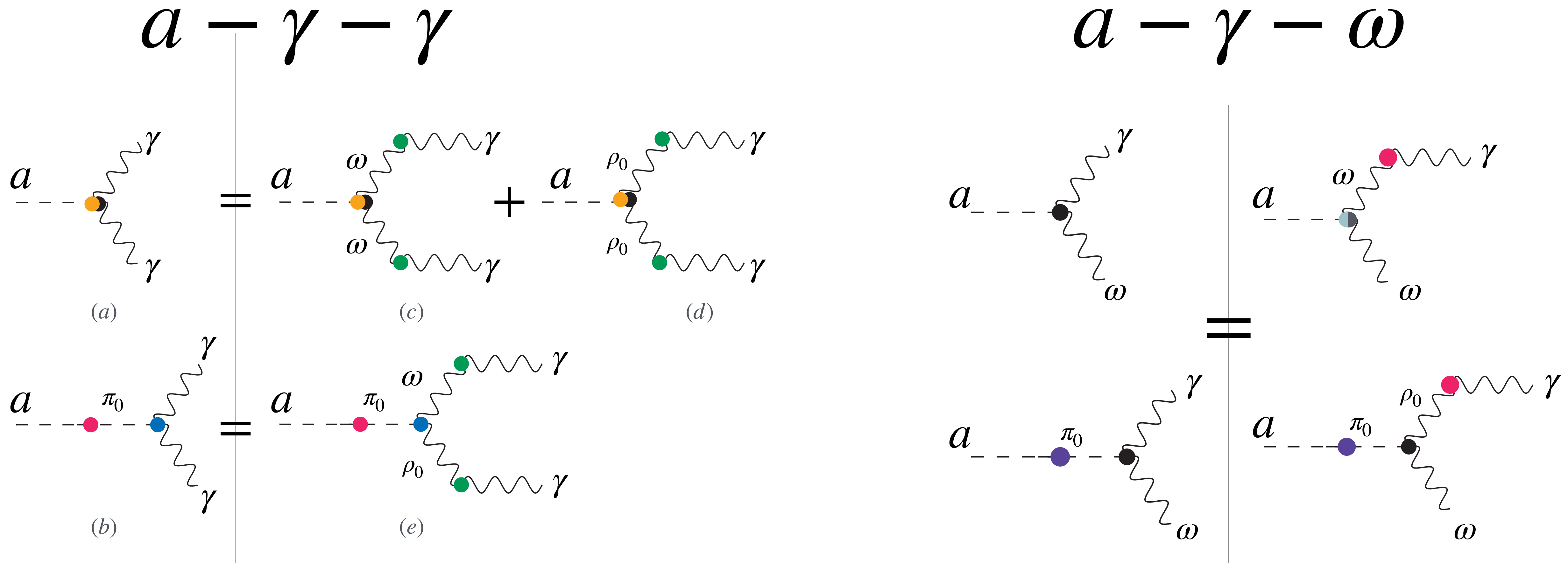
$$\begin{aligned}
 c_{\omega\gamma}^{\text{eff}} = & \frac{c_{gg}egN_c(Q_d + Q_u)}{24\pi^2 f_a(m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \left\{ m_a^2(2m_\eta^2 + m_{\eta'}^2 - 3m_\pi^2) \right. \\
 & \left. + m_\eta^2(m_\pi^2 - 3m_{\eta'}^2) + 2m_{\eta'}^2 m_\pi^2 \right\} \\
 & + \frac{c_{gg}egN_c(Q_d - Q_u)\delta m_\pi^2}{24\pi^2 f_a(m_a^2 - m_\pi^2)(m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \left\{ m_a^2(2m_\eta^2 + m_{\eta'}^2 - 3m_\pi^2) \right. \\
 & \left. + m_\eta^2(m_\pi^2 - 3m_{\eta'}^2) + 2m_{\eta'}^2 m_\pi^2 \right\} \\
 & + \frac{egN_c}{48\pi^2 f_a} \left\{ 3Q_d(c_d^L - c_d^R) + 3Q_u(c_u^L - c_u^R) - \frac{3m_a^2(Q_d - Q_u)(c_d^L - c_d^R - c_u^L + c_u^R)}{m_a^2 - m_\pi^2} \right. \\
 & \frac{2m_a^2(Q_d + Q_u)(c_u^L - c_u^R + c_d^L - c_d^R + c_s^R - c_s^L)}{m_\eta^2 - m_a^2} \\
 & \left. \frac{m_a^2(Q_d + Q_u)(c_u^L - c_u^R + c_d^L - c_d^R + 2c_s^L - 2c_s^R)}{m_{\eta'}^2 - m_a^2} \right\} \\
 & - \frac{egN_c\delta m_a^2 m_\pi^2}{24\pi^2 f_a(m_a^2 - m_\pi^2)(m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \left\{ (-3m_a^2 + m_\eta^2 + 2m_{\eta'}^2) [Q_u(c_u^L - c_u^R) \right. \\
 & \left. - Q_d(c_d^L - c_d^R)] - (m_\eta^2 - m_{\eta'}^2)(Q_d - Q_u) \right\} .
 \end{aligned}$$

$m_\eta, m_{\eta'} \rightarrow \infty$ • 2-flavor results



$$c_{\omega\gamma}^{\text{eff}} = eg' \left\{ \frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f} \left[\frac{m_a^2}{m_\pi^2 - m_a^2} \left(\frac{c_u - c_d}{2} \right) + c_{gg} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right] + \frac{1}{16\pi^2 f} (c_d + c_Q - 2c_u) \right\}$$

Vector Meson Dominance for axion-gluon couplings



- For g_{agg} , the complete axion WZW effective theory automatically realizes VMD
- “Pion-shift” + Hidden Local Symmetry formalism, see
Ovchinnikov, Zaporozhchenko, 2501.04525 (PRD) [Need pre-treatment of aGG term]

Complete vertex in Mathematica notebooks

$$\begin{aligned}\mathcal{L}_{\text{eff},0} = & \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 \\ & + g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \dots)\end{aligned}$$

<https://github.com/nun3366/Axion-WZW-3>

README MIT license

WZW Interactions of Axions

What is it?

In this work, we implement the routines to derive the relevant axion WZW interactions with the ground-state (axial-)vector mesons and electroweak gauge bosons and calculate the relevant hadronic decay widths derived in [2406.11948](#) and [2505.24822](#). We also provide the routines to plot the hadronic axion decay widths under different parameter schemes of the user's choice.

These notebooks are based on the open-source Mathematica notebooks written by Maksym Ovchynnikov and Andrii Zaporozhchenko introduced in [2310.03524](#) and [2501.04525](#).

Dependencies

To run the routines, one must first install [FeynCalc](#) and [xAct-xTeror](#).

Repository structure

The main routines are implemented in `main.nb`, with the basis modules defined in the notebooks stored in the folder `notebooks`. In addition to the axion/ALP mass m_a and axion decay constant f_a , the user should specify seven other parameters: the axion couplings to gluons c_{gg} and left/right-handed u/d/s-quarks $c_{L/R}^{u/d/s}$. The major contribution of this work, the details of auxiliary parameter cancellation in the effective axion WZW couplings, is implemented in `WZW_axion_interactions.nb`.

Summary

- A full chiral axion Lagrangian for axion and pseudoscalar/vector mesons
- $\mathcal{L}_{\text{axion}}^{\text{full}} \equiv \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] \left(U, \mathbf{m}_q(a), \mathcal{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a \right)$
 - 1. Wess-Zumino-Witten counter term is included for gauge invariance
 - 2. t'Hooft UV-IR anomaly matching is achieved
 - 3. Consistent physical amplitudes without auxiliary rotation parameters
- Three light quarks scheme: consistent treatment with η'
 - 4. Demonstrates two ways of resolving $aG\tilde{G}$ are consistent. Work for any chiral-rotation.
- Important for Axion/ALP searches via mesons.
- All calculation machinery provided as Mathematica code in GitHub.

Thank you!

