格点QCD强子谱学研究进展

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In collaboration with
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第十一届手征有效场论研讨会
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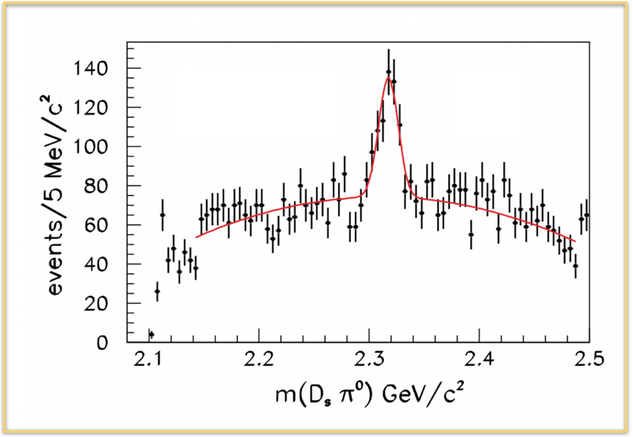
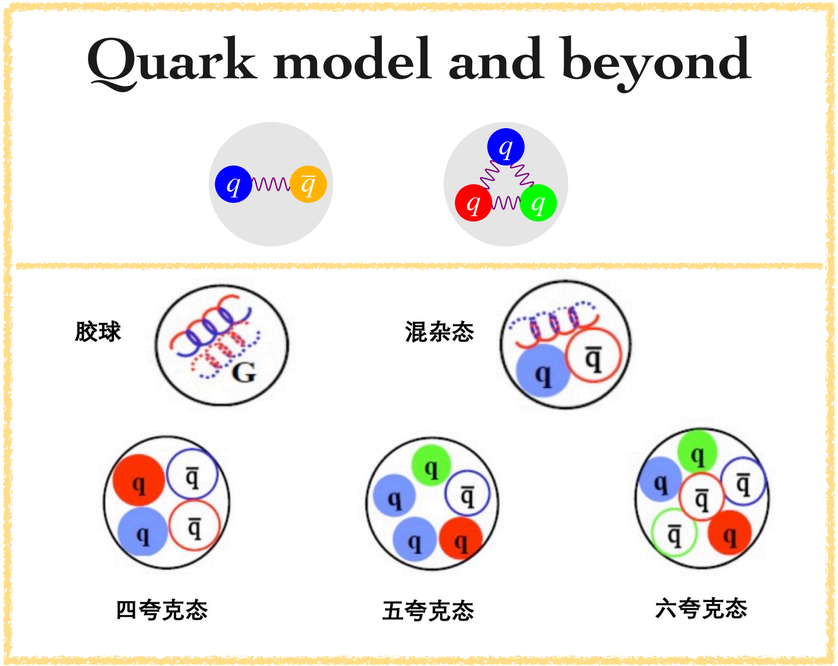
Hadron spectroscopy

✦ Spectroscopy:

- Most of the hadrons observed in experiments are unstable.
- Understand how hadrons are built from quarks and gluons.



✦ New experimental discoveries provide opportunities and challenges for lattice studies.

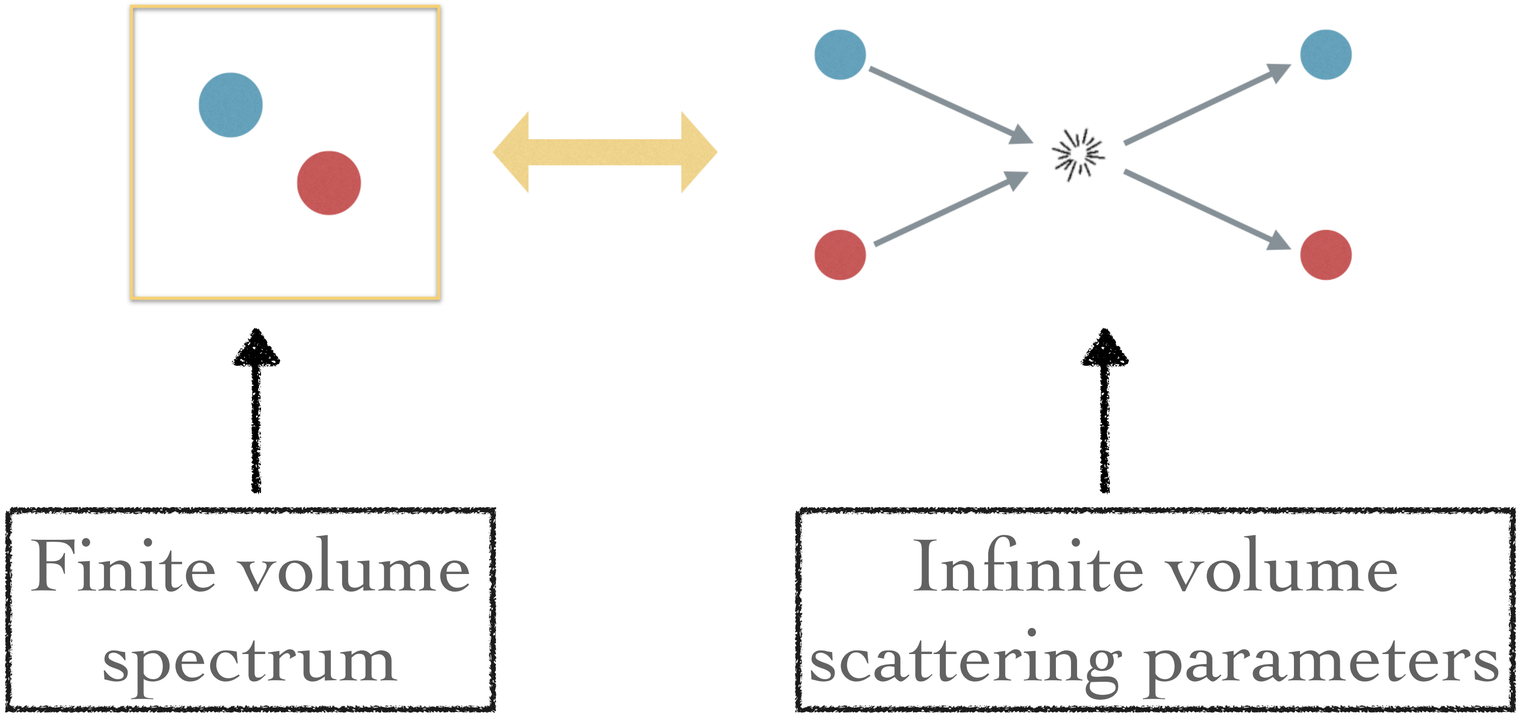


Outline

- Hadron scattering on Lattice
- Results
  - Meson-meson scattering: $\rho, K^*, D_0^*(2300)$
  - Meson-baryon scattering: pentaquarks
  - Baryon-baryon scattering: NN
  - Three-body scattering: $\pi\pi\pi$ $I = 0,1$
- Summary and discussions

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Scattering on lattice

Excited states:

- build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = <0|\mathcal{O}_i\mathcal{O}_j^{\dagger}|0> = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- Solve the generalized eigenvalue problem(GEVP):

$$C_{ij}v_j^n(t) = \lambda_n(t)C_{ij}^0 v_j^n(t)$$

- Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

- Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

Scattering on lattice

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors

$$\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$$

Infinite-volume scattering matrix

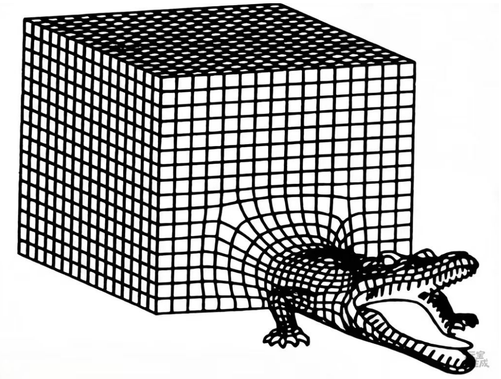
Finite volume information

$$M(E_{cm}, L)$$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

CLQCD ensembles

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
0.105 fm	$24^3 \times 72$	292	1000
	$32^3 \times 64$		1000
	$48^3 \times 96$		800
	$64^3 \times 128$		70
	$32^3 \times 64$	225	450
	$48^3 \times 96$		700
	$48^3 \times 96$	135	700
	$64^3 \times 128$		200
0.090 fm	$32^3 \times 64$	350	900
0.077 fm	$24^3 \times 72$	300	250
	$32^3 \times 96$		480
	$48^3 \times 96$		200
	$32^3 \times 64$	210	460
	$48^3 \times 96$		200
	$64^3 \times 128$	135	180
0.069 fm	$36^3 \times 108$	300	700
0.052 fm	$48^3 \times 144$	317	1000
	$64^3 \times 128$		100



- Unphysical light quark mass
- Finite lattice spacing
- Finite volume

Z.-C.Hu el al., (CLQCD), PRD109(2024)5,054507

Meson-meson scattering

- $\pi\pi$ P-wave scattering and the ρ resonance
- $K\pi$ P-wave scattering and the K^* resonnace
- $D\pi$ scattering and $D_0^*(2300)$

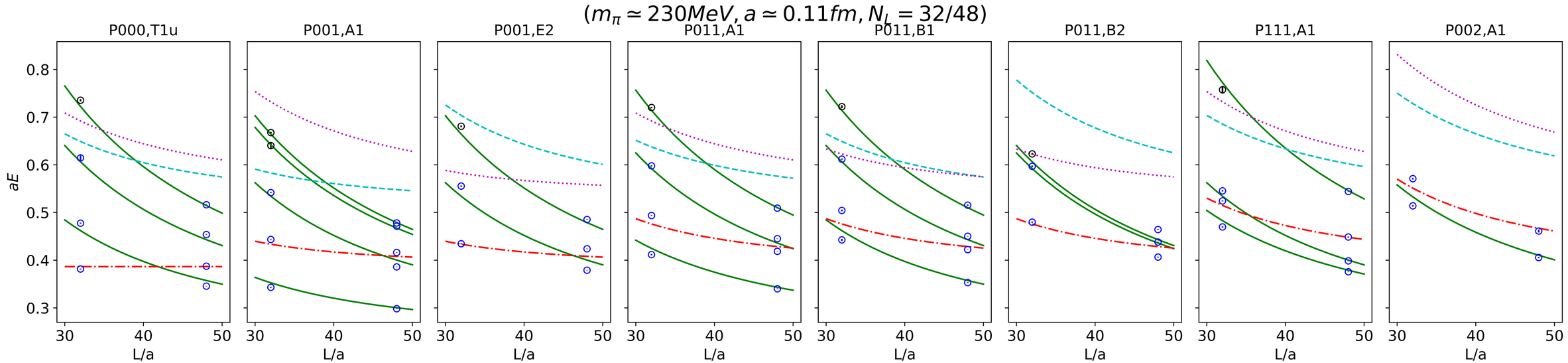
$\pi\pi$ P-wave scattering and the ρ resonance

Zhengli Wang et al., JHEP08(2025)064

Interpolating operators:

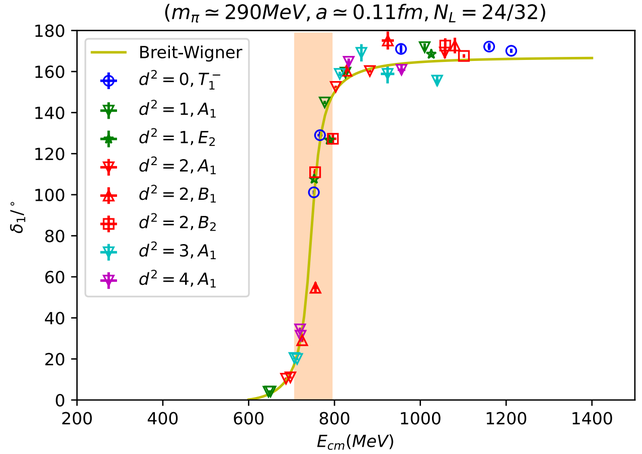
$$\mathcal{O}_{\rho,i}\left(\vec{x},t\right)=\frac{1}{\sqrt{2}}\left(\bar{u}\gamma_i u\left(\vec{x},t\right)-\bar{d}\gamma_i d\left(\vec{x},t\right)\right)$$

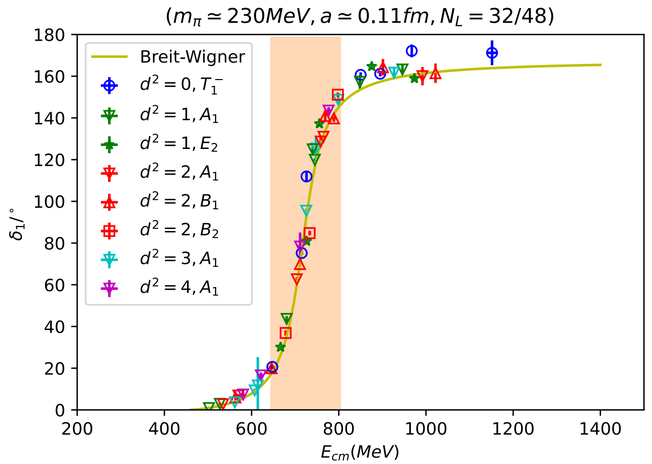
$$(\pi\pi)^{[\vec{k}_1,\vec{k}_2]}_{\vec{P},\Lambda,\mu}=\sum_{\substack{\vec{k}_1\in\{\vec{k}_1\}^\star\\ \vec{k}_2\in\{\vec{k}_2\}^\star\\ \vec{k}_1+\vec{k}_2=\vec{P}}}\mathcal{C}\left(\vec{P},\Lambda,\mu;\vec{k}_1;\vec{k}_2\right)\times\left(\pi^+(\vec{k}_1)\pi^-(\vec{k}_2)-\pi^-(\vec{k}_1)\pi^+(\vec{k}_2)\right)$$

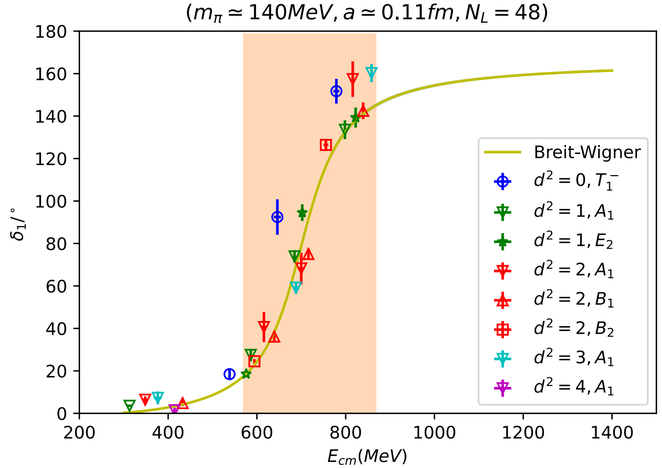


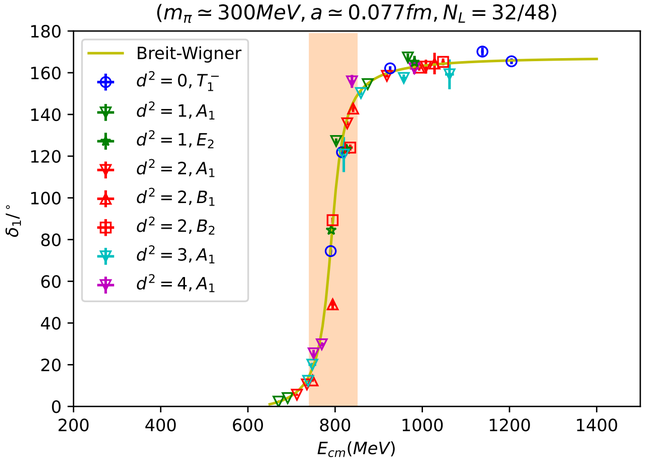
$\pi\pi$ P-wave scattering and the ρ resonance

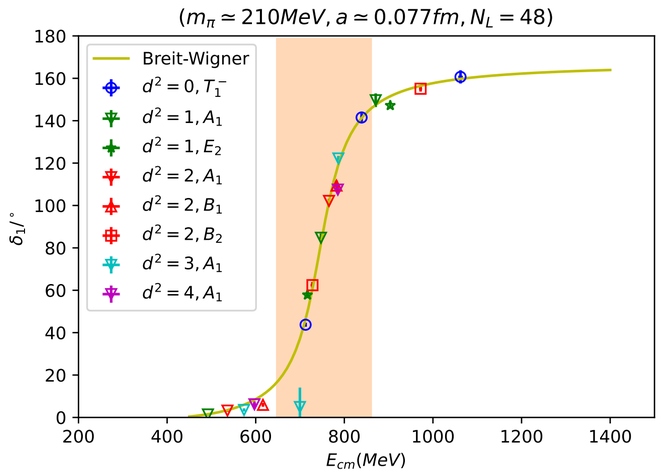
Zhengli Wang et al., JHEP08(2025)064

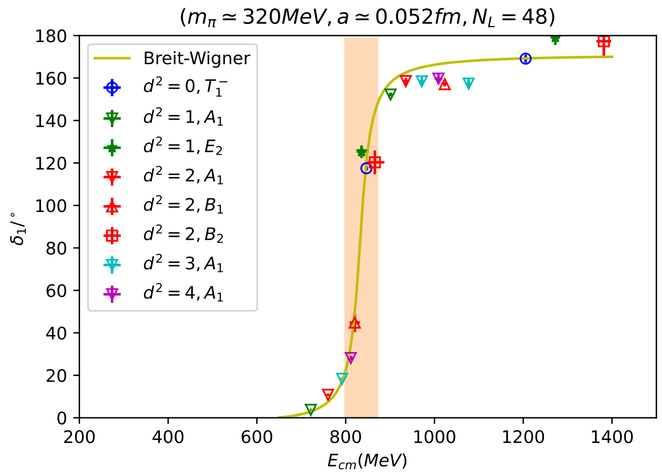








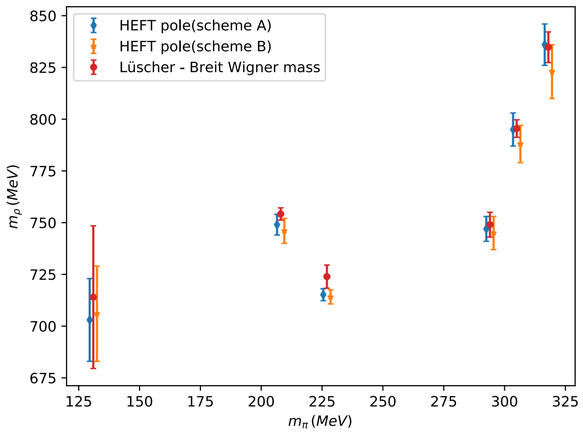


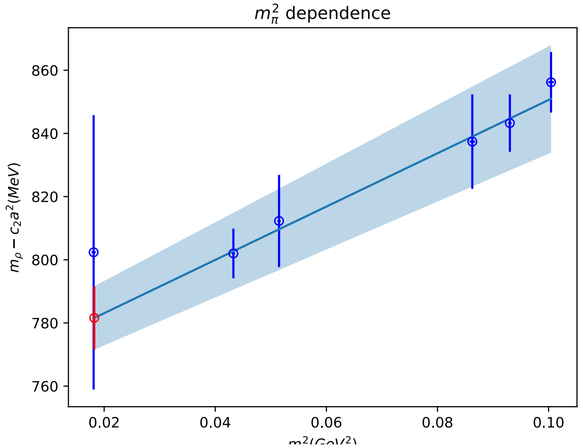


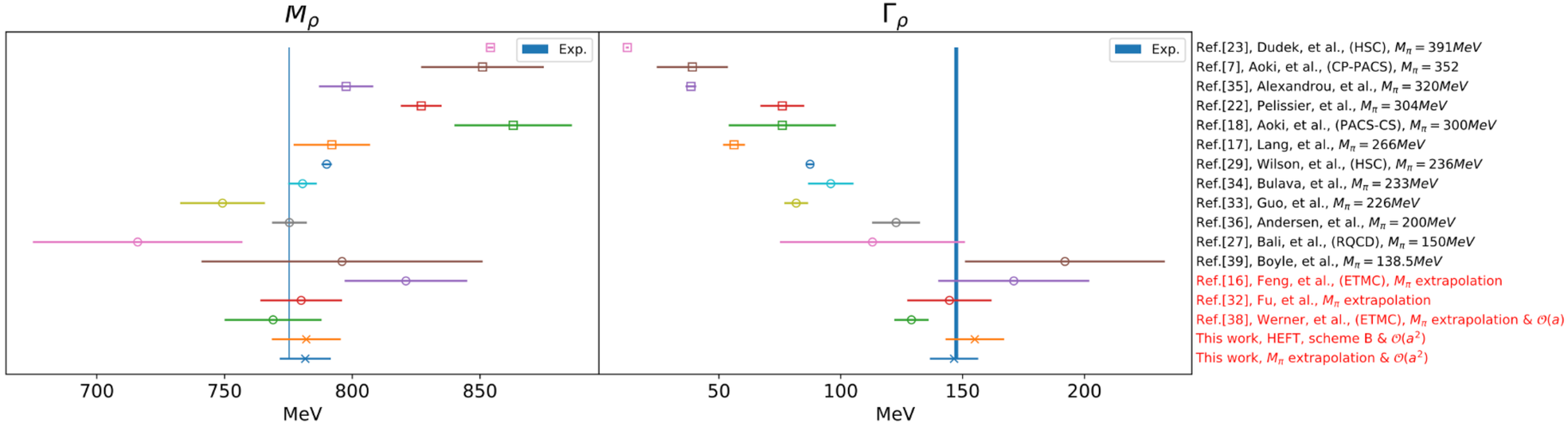
$\pi\pi$ P-wave scattering and the ρ resonance

Zhengli Wang et al., JHEP08(2025)064

Pion mass and continuum extrapolation: $m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2$





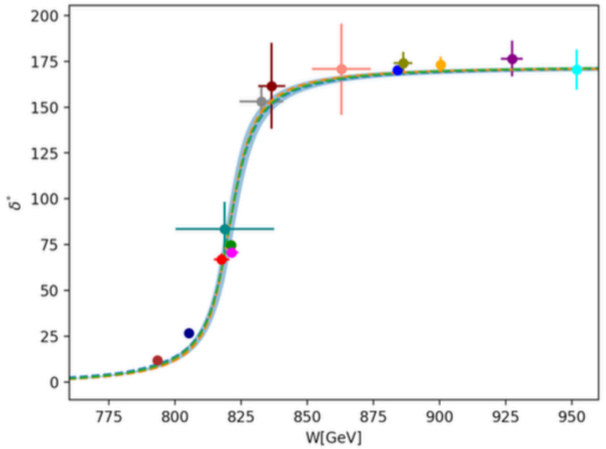


Ref.[23], Dudek, et al., (HSC), $M_\pi = 391 MeV$
Ref.[7], Aoki, et al., (CP-PACS), $M_\pi = 352$
Ref.[35], Alexandrou, et al., $M_\pi = 320 MeV$
Ref.[22], Pelissier, et al., $M_\pi = 304 MeV$
Ref.[18], Aoki, et al., (PACS-CS), $M_\pi = 300 MeV$
Ref.[17], Lang, et al., $M_\pi = 266 MeV$
Ref.[29], Wilson, et al., (HSC), $M_\pi = 236 MeV$
Ref.[34], Bulava, et al., $M_\pi = 233 MeV$
Ref.[33], Guo, et al., $M_\pi = 226 MeV$
Ref.[36], Andersen, et al., $M_\pi = 200 MeV$
Ref.[27], Bali, et al., (RQCD), $M_\pi = 150 MeV$
Ref.[39], Boyle, et al., $M_\pi = 138.5 MeV$
Ref.[16], Feng, et al., (ETMC), M_π extrapolation
Ref.[32], Fu, et al., M_π extrapolation
Ref.[38], Werner, et al., (ETMC), M_π extrapolation & $\mathcal{O}(a)$
This work, HEFT, scheme B & $\mathcal{O}(a^2)$
This work, M_π extrapolation & $\mathcal{O}(a^2)$

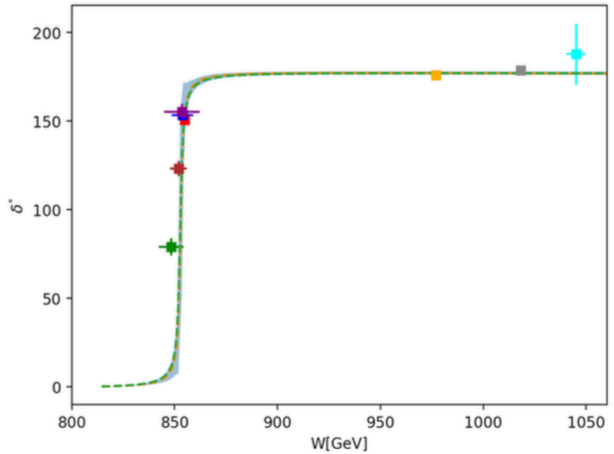
$K\pi$ P-wave scattering and the ρ resonance

Qu-Zhi Li et al., arXiv:2603.16266

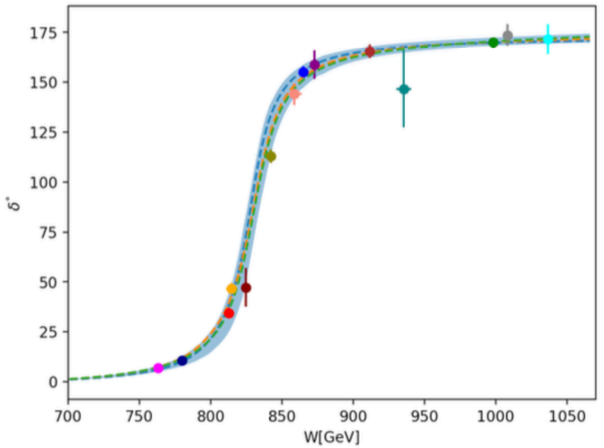
$m_\pi = 230\text{MeV}, a = 0.105\text{fm}$



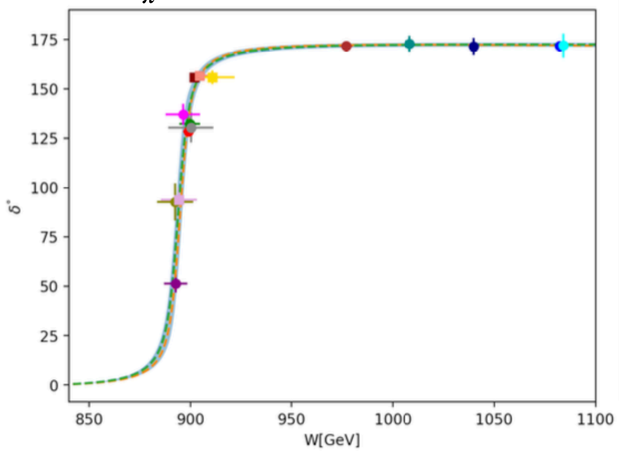
$m_\pi = 290\text{MeV}, a = 0.105\text{fm}$



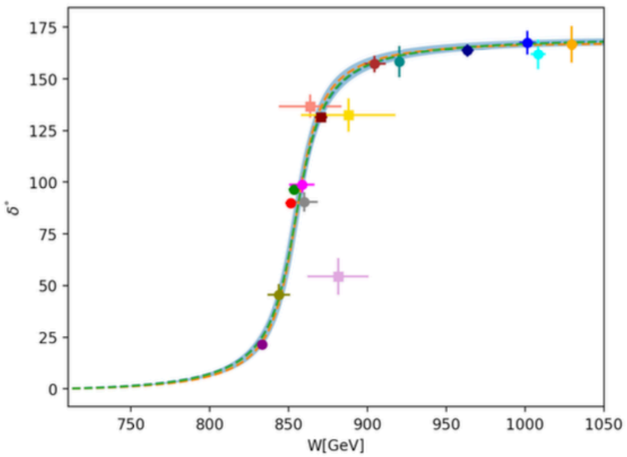
$m_\pi = 135\text{MeV}, a = 0.105\text{fm}$



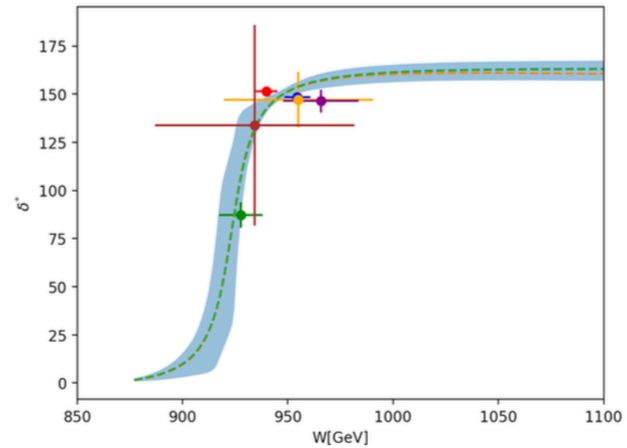
$m_\pi = 300\text{MeV}, a = 0.077\text{fm}$



$m_\pi = 210\text{MeV}, a = 0.077\text{fm}$



$m_\pi = 320\text{MeV}, a = 0.052\text{fm}$

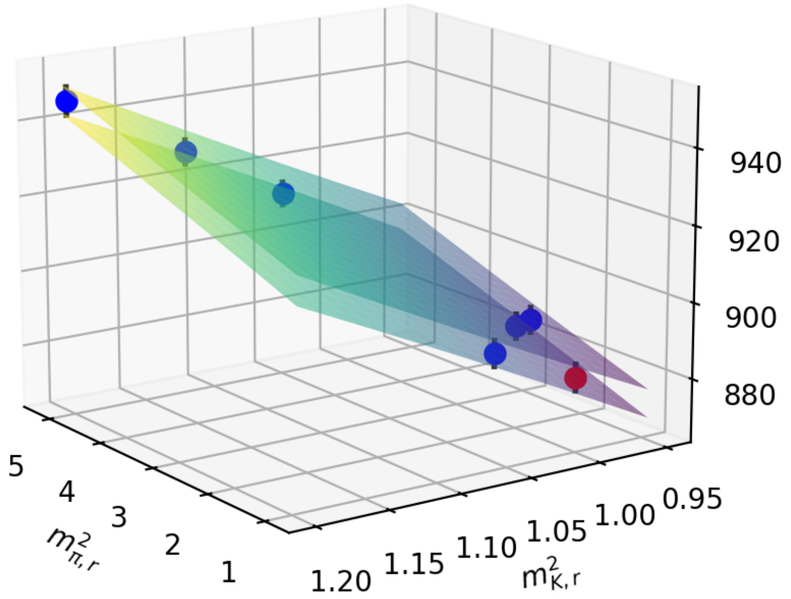


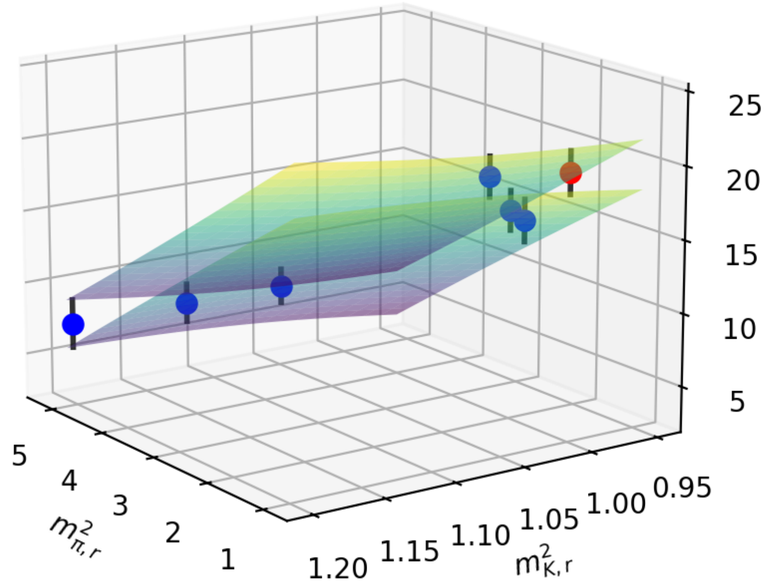
$K\pi$ P-wave scattering and the ρ resonance

Qu-Zhi Li et al., arXiv:2603.16266

Pion mass and continuum extrapolation:

$$\mathrm{Re}\left(\sqrt{s_0}\right) = b_0^r + b_1^r m_\pi^2 + b_2^r m_K^2 + b_3^r a^2$$
$$\mathrm{Im}\left(\sqrt{s_0}\right) = b_0^i + b_1^i m_\pi^2 + b_2^i m_K^2 + b_3^i a^2$$





	pole position (MeV)
This work	$883.0(3.4) - i20.1(1.4)$
PDG [77]	$890(14) - i26(6)$
Boyle et al. [30]	$893(2)_{\mathrm{stat}}(54)_{\mathrm{sys}} - i26(1)_{\mathrm{stat}}(6)_{\mathrm{sys}}$

Scalar charmed mesons $D_0^*(2300)$ & $D_{s0}*(2317)$

Charm-light

$D_0^*(2300)$ $I(J^P) = \frac{1}{2}(0^+)$

Mass $m = 2343 \pm 10$ MeV

Width $\Gamma = 229 \pm 16$ MeV

Charm-strange

$D_{s0}^*(2317)$ $I(J^P) = 0(0^+)$

Mass $m = 2317.8 \pm 0.5$ MeV

Width $\Gamma < 3.8$ MeV

- Mass ordering puzzle: $c\bar{l}$ is heavier than $c\bar{s}$
- $D_{s0}^*(2317)$ is much lighter than quark model predictions
- Very different widths of $D_0^*(2300)$ and $D_{s0}^*(2317)$

$D_0^*(2300)$

- D. Mohler, S. Prelovsek and R. Woloshyn, Phys. Rev. D 87, 034501(2013)

  $D\pi(I=1/2)$ scattering at $m_\pi = 266\text{MeV}$, resonance pole.

- G. Moir et. al., JHEP10(2016)011

  $D\pi, D\eta, D_s\bar{K}$ coupled channel scattering at $m_\pi = 391\text{MeV}$, bound state pole.

- L. Gayer et. al., JHEP 07 (2021) 123

  $D\pi(I=1/2)$ single channel scattering at $m_\pi = 239\text{MeV}$, resonance pole .

- The pole positions do not agree with the experimental values, and do not show a monotonic movement with the change of m_π .

$D_0^*(2300)$

H. Yan, C. Liu, L. Liu, Y. Meng and H. Xing, PRD111(2025)1,014503

- Pion mass depencence of the pole position of $D_0^*(2300)$
- Six ensembles with four different pion masses and three lattice spacings.
- Large number of interpolating operators are constructed to reliably extract the full spectrum.

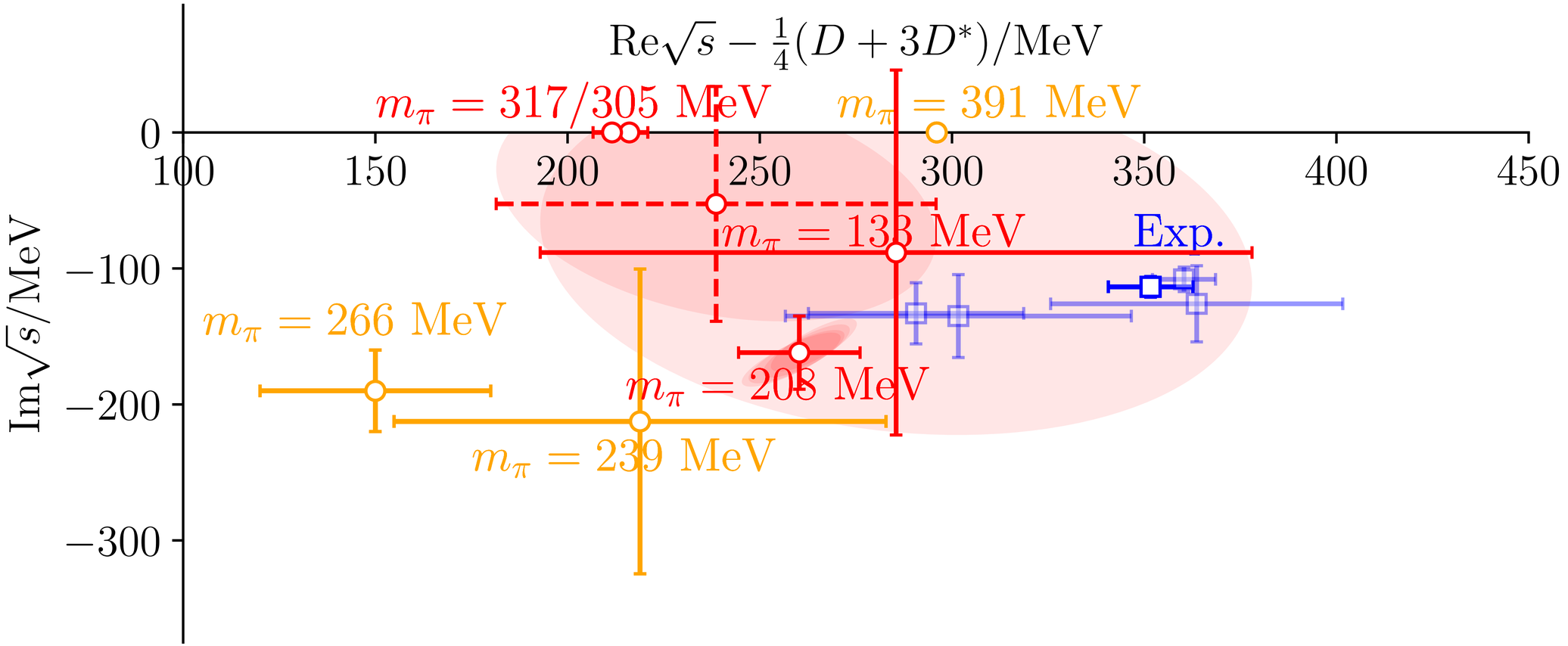
Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.105fm	$48^3 \times 96$	135	259
~0.077fm	$32^3 \times 96$	305	566
	$48^3 \times 96$		200
	$32^3 \times 64$	208	460
	$48^3 \times 96$		250
~0.052fm	$48^3 \times 144$	317	270

$$O_{D\pi}^{I=\frac{1}{2}}(P) = \sqrt{2}D^0(p_1)\pi^+(p_2) - D^+(p_1)\pi^0(p_2), (P = p_1 + p_2, \quad P^2 = 0,1,2,3,4)$$

$$\mathcal{O}_1(P) = \bar{d}\Gamma c(P)$$

$D_0^*(2300)$

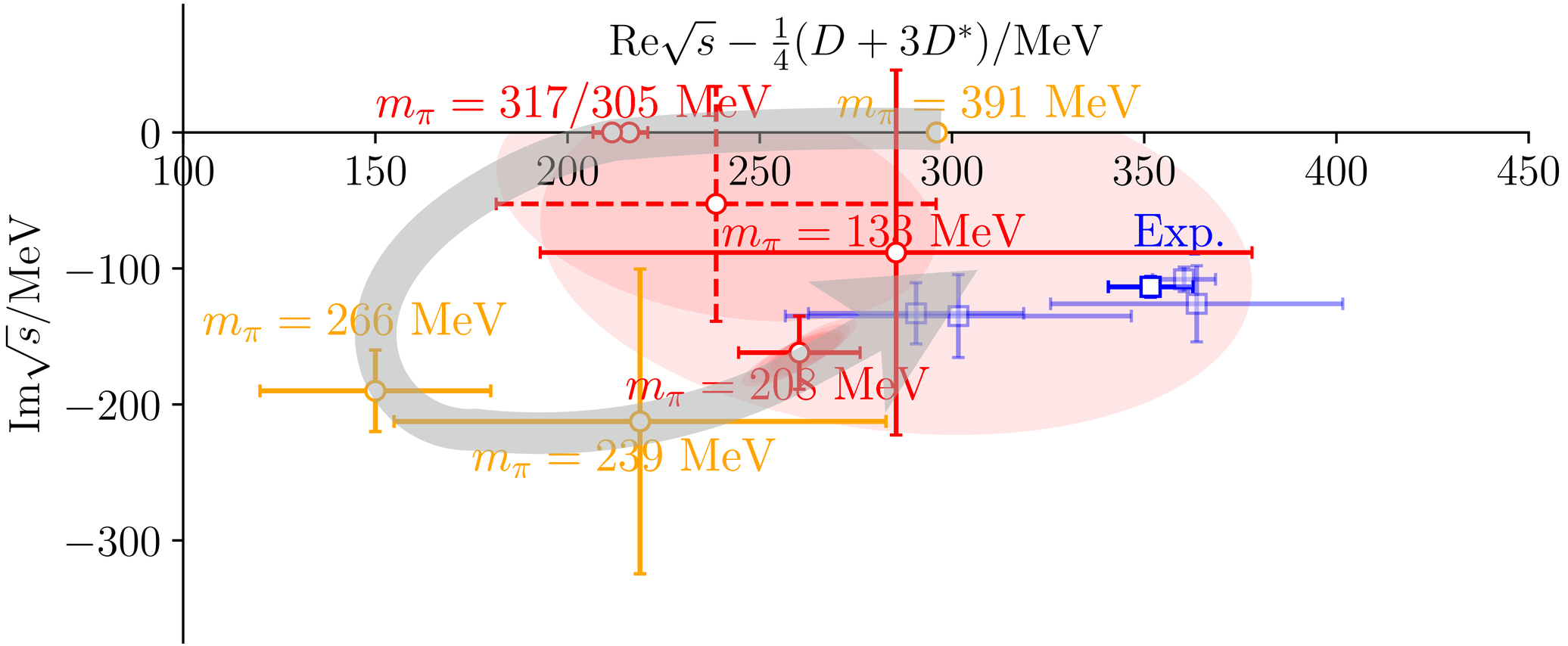
Pole position



- At $m_\pi \sim 300$MeV, there is a virtual state pole. When pion mass decreases, it becomes a resonance and the pole position gets closer to the experimental value.

$D_0^*(2300)$

Pole position

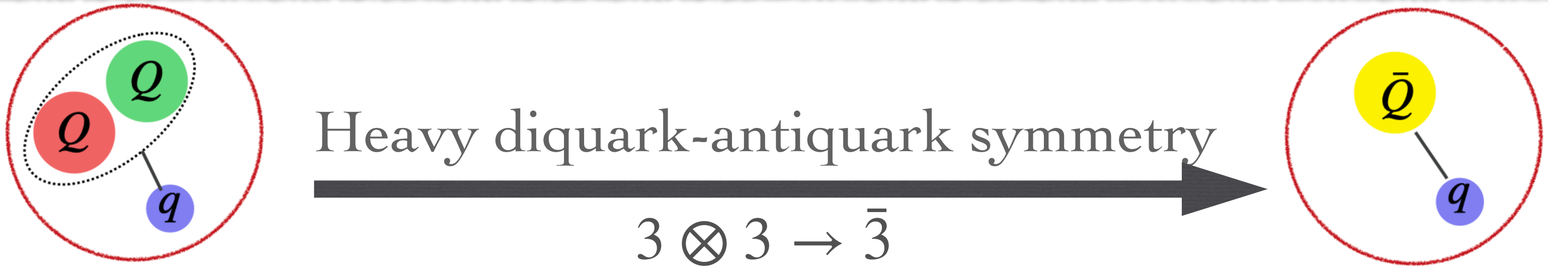


- At $m_\pi \sim 300$MeV, there is a virtual state pole. When pion mass decreases, it becomes a resonance and the pole position gets closer to the experimental value.

Meson-baryon scattering

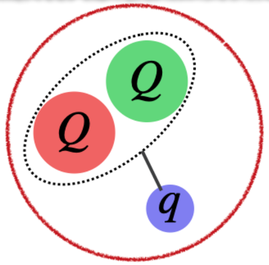
- Double-charm pentaquarks: $(\Xi_{cc}, \Omega_{cc}) - (\pi, K)$ scattering

Doubly charmed Pentaquarks



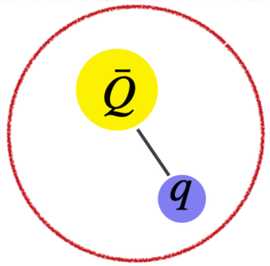
$I = 1/2$	$\Omega_{cc}\bar{K}$	$D_s K$
$I = 1$	$\Xi_{cc} K$	$D\bar{K}$
$I = 0$	$\Xi_{cc} K$	$D\bar{K}$
$I = 3/2$	$\Xi_{cc}\pi$	$D\pi$
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$	DK, $D_s\eta$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,	DK, $D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$	$D\pi$, $D\eta$, $D_s\bar{K}$

Doubly charmed Pentaquarks



Heavy diquark-antiquark symmetry

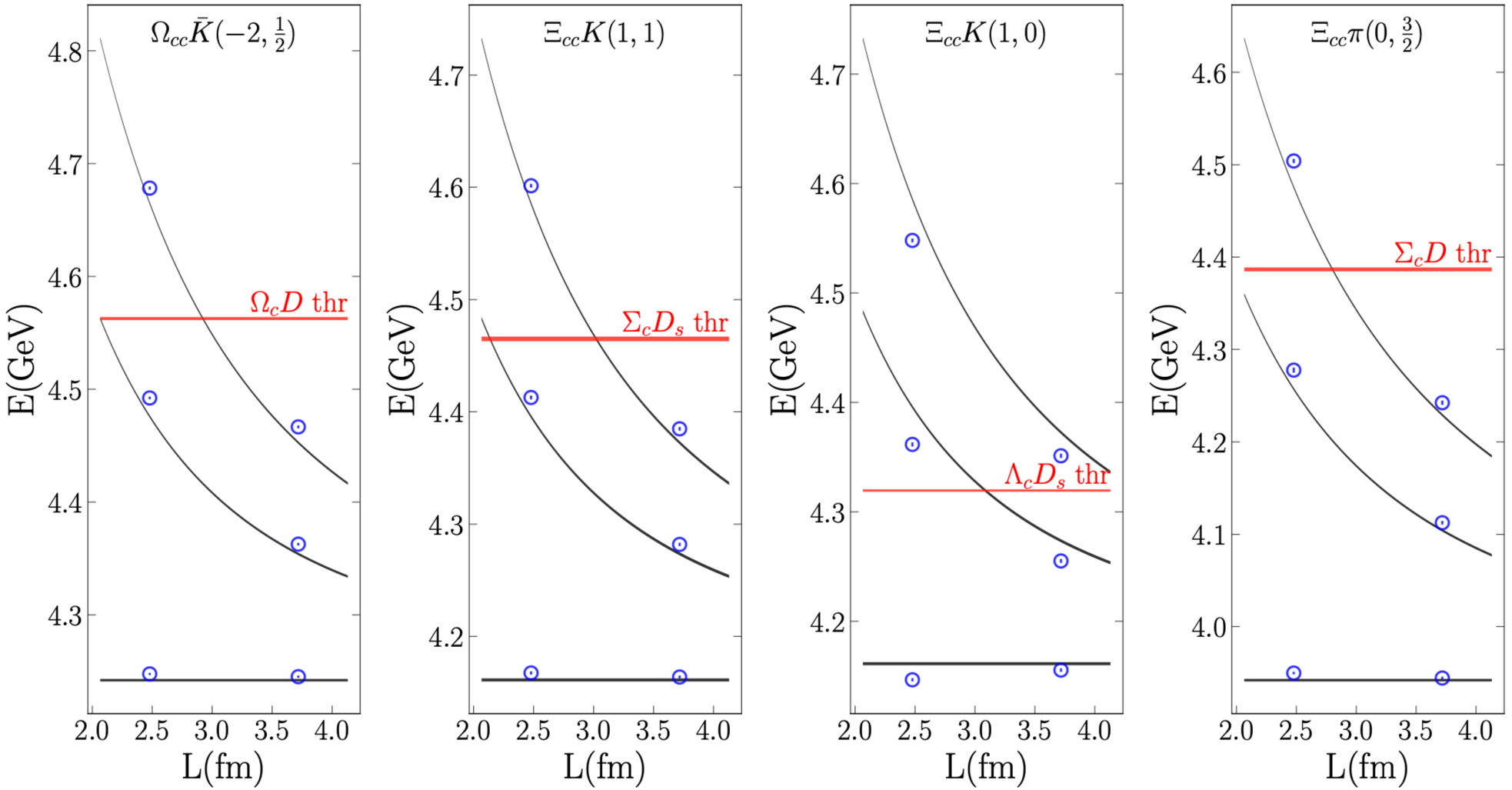
$3 \otimes 3 \to \bar{3}$



$I = 1/2$	$\Omega_{cc}\bar{K}$	$D_s K$
$I = 1$	$\Xi_{cc} K$	$D\bar{K}$
$I = 0$	$\Xi_{cc} K$	$D\bar{K}$
$I = 3/2$	$\Xi_{cc}\pi$	$D\pi$
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$	DK, $D_s\eta$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,	DK, $D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$	$D\pi$, $D\eta$, $D_s\bar{K}$

Doubly charmed Pentaquarks

J.-Y. Yi, Z.-R. Liang, L. Liu and D.-L. Yao, arXiv:2511.12611, JHEP accepted



Double-charm Pentaquarks

- Scattering lengths from lattice QCD and BChPT

(S, I)	Processes	$M_\pi \sim 300$ MeV	$M_\pi \sim 210$ MeV	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	$-0.161(20)$	$-0.136(12)$	$-0.09^{+0.12}_{-0.13}$	-0.20(1)
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	$-0.177(23)$	$-0.212(14)$	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	$0.63(10)$	$0.694(90)$	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	$-0.140(15)$	$-0.143(24)$	-0.16 ± 0.02	$-0.10(2)$

Double-charm Pentaquarks

- Scattering lengths from lattice QCD and BChPT

(S, I)	Processes	$M_\pi \sim 300$ MeV	$M_\pi \sim 210$ MeV	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	$-0.161(20)$	$-0.136(12)$	$-0.09^{+0.12}_{-0.13}$	-0.20(1)
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	$-0.177(23)$	$-0.212(14)$	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.63(10)	0.694(90)	1.03 ± 0.19	0.92(2)
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	$-0.140(15)$	$-0.143(24)$	-0.16 ± 0.02	$-0.10(2)$

Double-charm Pentaquarks

Scattering amplitude:

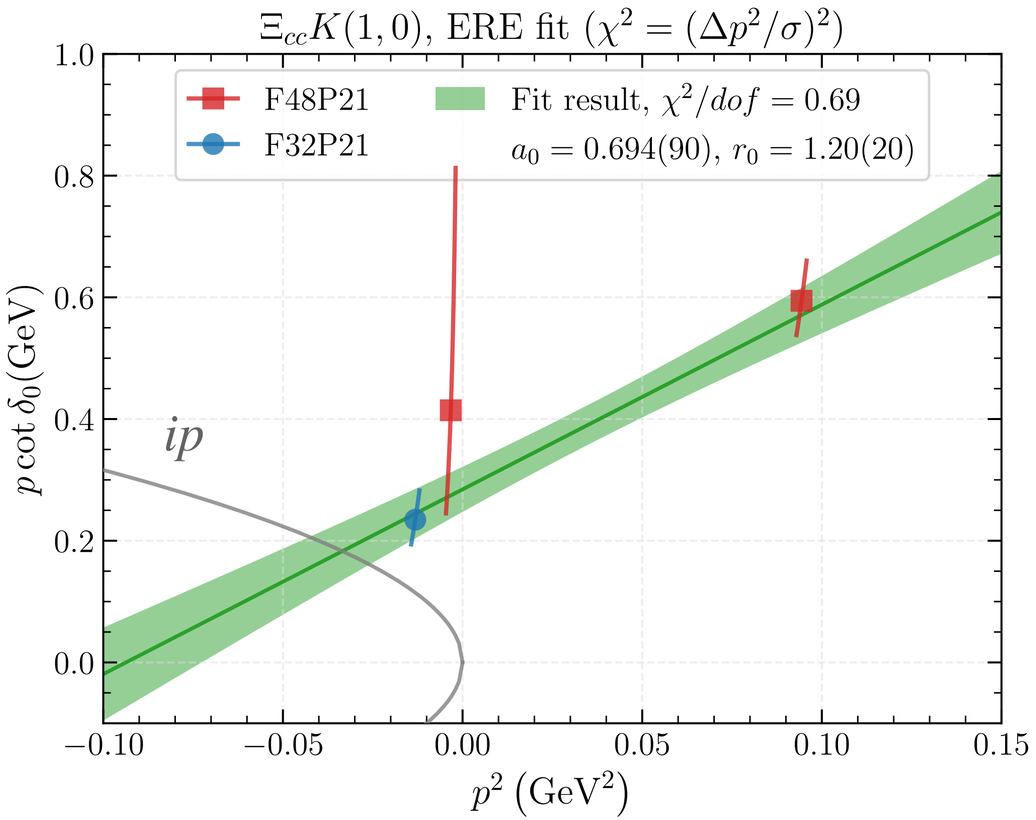
$$T \sim \frac{1}{pcot\delta - ip}$$

Effective range expansion:

$$pcot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \cdots$$

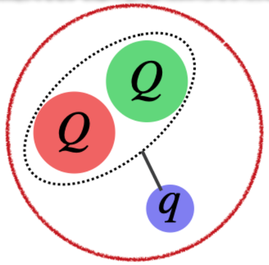
Luscher's formula:

$$pcot\delta(p) = \frac{2Z_{00}(1; (\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$



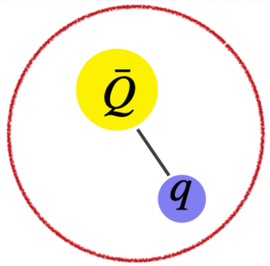
$m_\pi = 210\,\mathrm{MeV}$

Doubly charmed Pentaquarks



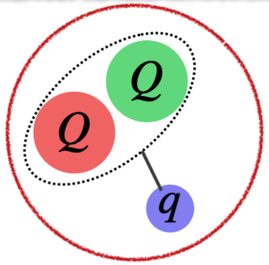
Heavy diquark-antiquark symmetry

$3 \otimes 3 \to \bar{3}$



$I = 1/2$	$\Omega_{cc}\bar{K}$	Repulsive	$D_s K$
$I = 1$	$\Xi_{cc} K$	Repulsive	$D\bar{K}$
$I = 0$	$\Xi_{cc} K$	Attractive, virtual pole	$D\bar{K}$
$I = 3/2$	$\Xi_{cc}\pi$	Repulsive	$D\pi$
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$		DK, $D_s\eta$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,		DK, $D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$		$D\pi$, $D\eta$, $D_s\bar{K}$

Doubly charmed Pentaquarks



Heavy diquark-antiquark symmetry

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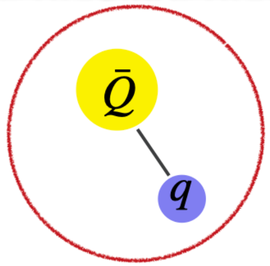

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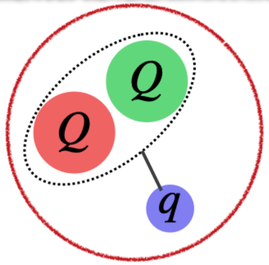
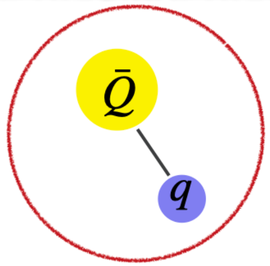
$I = 1$	$\Xi_{cc} K$	Repulsive	$D\bar{K}$	
$I = 0$	$\Xi_{cc} K$	Attractive, virtual pole	$D\bar{K}$	
$I = 3/2$	$\Xi_{cc}\pi$	Repulsive	$D\pi$	
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$??		DK, $D_s\eta$	$D_{s0}^*(2317)$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,		DK, $D_s\pi$	
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$??		$D\pi$, $D\eta$, $D_s\bar{K}$	$D_0^*(2300)$

Doubly charmed Pentaquarks



Heavy diquark-antiquark symmetry

$3 \otimes 3 \to \bar{3}$



$I = 1/2$	$\Omega_{cc}\bar{K}$	Repulsive	$D_s K$	
$I = 1$	$\Xi_{cc} K$	Repulsive	$D\bar{K}$	
$I = 0$	$\Xi_{cc} K$	Attractive, virtual pole	$D\bar{K}$	
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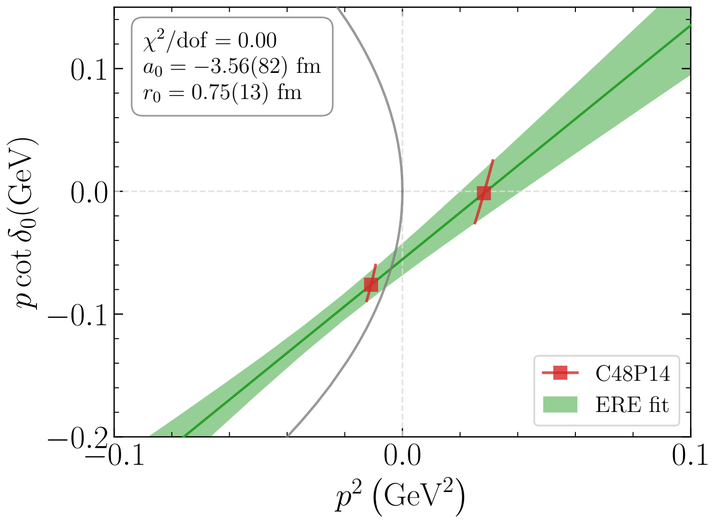
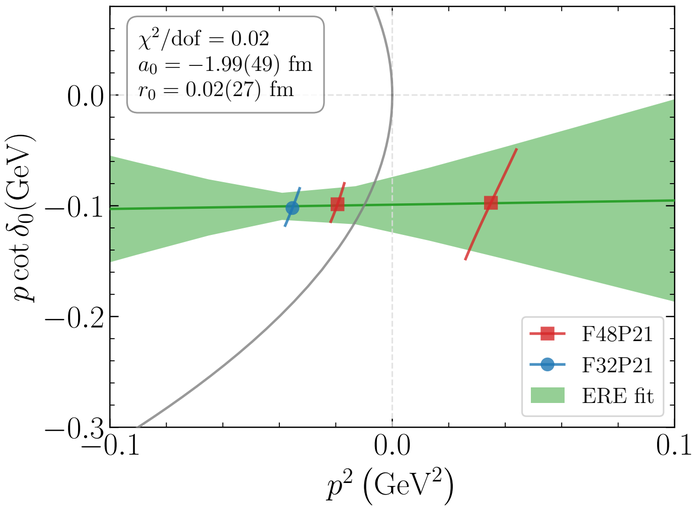
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$??		DK, $D_s\eta$	$D_{s0}^*(2317)$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,		DK, $D_s\pi$	
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$??		$D\pi$, $D\eta$, $D_s\bar{K}$	$D_0^*(2300)$

Double-charm Pentaquarks

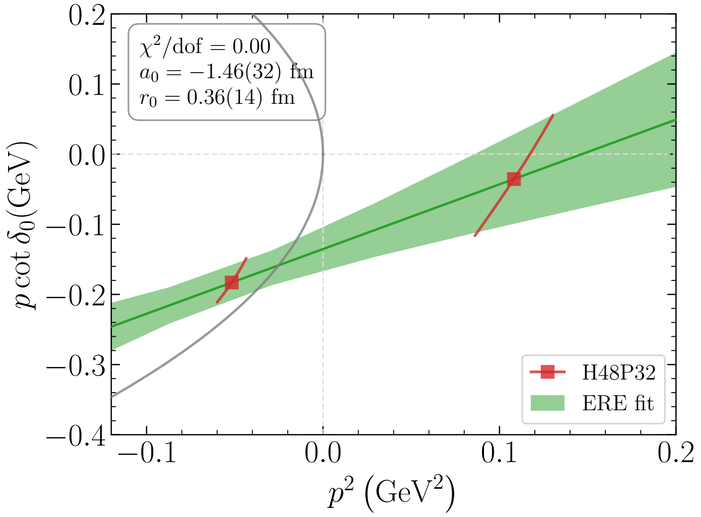
$m_\pi = 135\text{MeV}, \text{a} = 0.105\text{fm}$



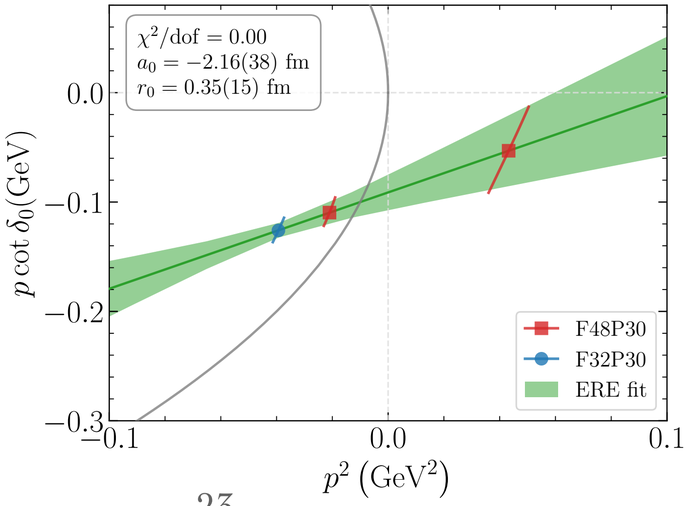
$m_\pi = 300\text{MeV}, \text{a} = 0.077\text{fm}$



$m_\pi = 320\text{MeV}, \text{a} = 0.052\text{fm}$



$m_\pi = 210\text{MeV}, \text{a} = 0.077\text{fm}$



$I = 0\ \ \Xi_{cc}\bar{K}$

A bound state pole is found at various pion masses and lattice spacings.

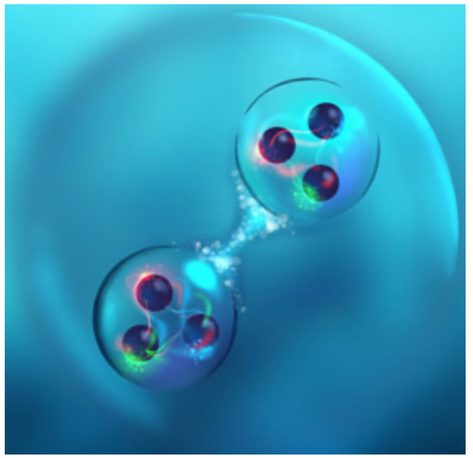
Baryon-baryon scattering

- Deuteron: *NN* scattering

Kuan Zhang et al., arXiv:2605.16977 (accepted by PRD)

NN scattering

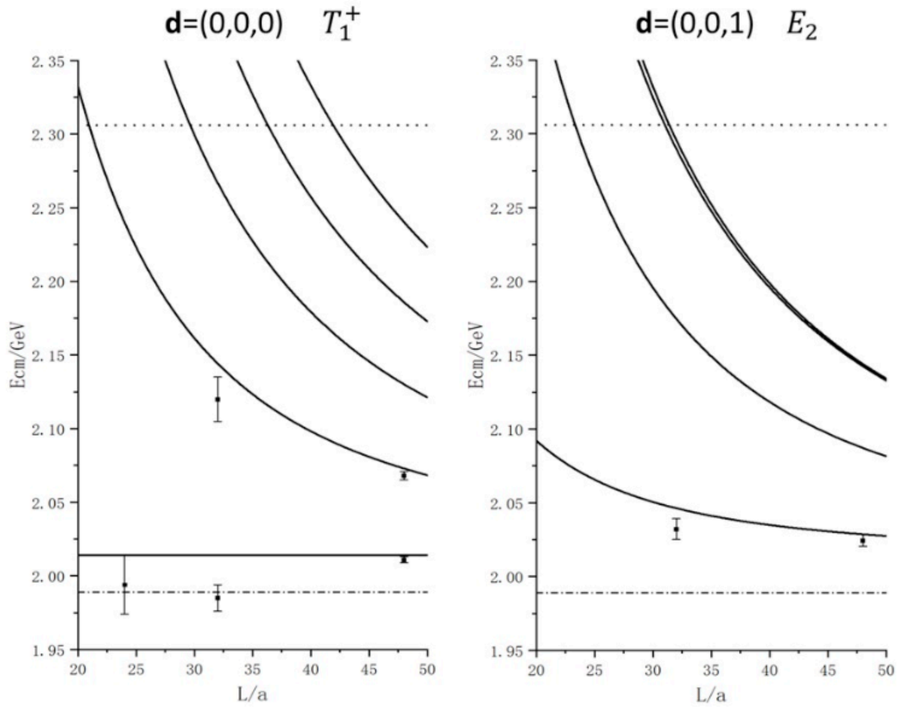
- The only known stable dibaryon — Deuteron.
- Lattice calculation is difficult due to poor signal.
- Most of the previous lattice studies are performed at very large pion masses and the results are controversial.



ID	$L^3 \times T$	a(fm)	m_π(MeV)	$m_\pi L$	m_N(MeV)	$N_{\rm cfg}$
C24P29	$24^3 \times 72$	0.1053	292	3.75	1026(4)	838
C32P29	$32^3 \times 64$	0.1053	292	5.01	1005(3)	981
C48P29	$48^3 \times 96$	0.1053	292	7.52	1007(1)	731

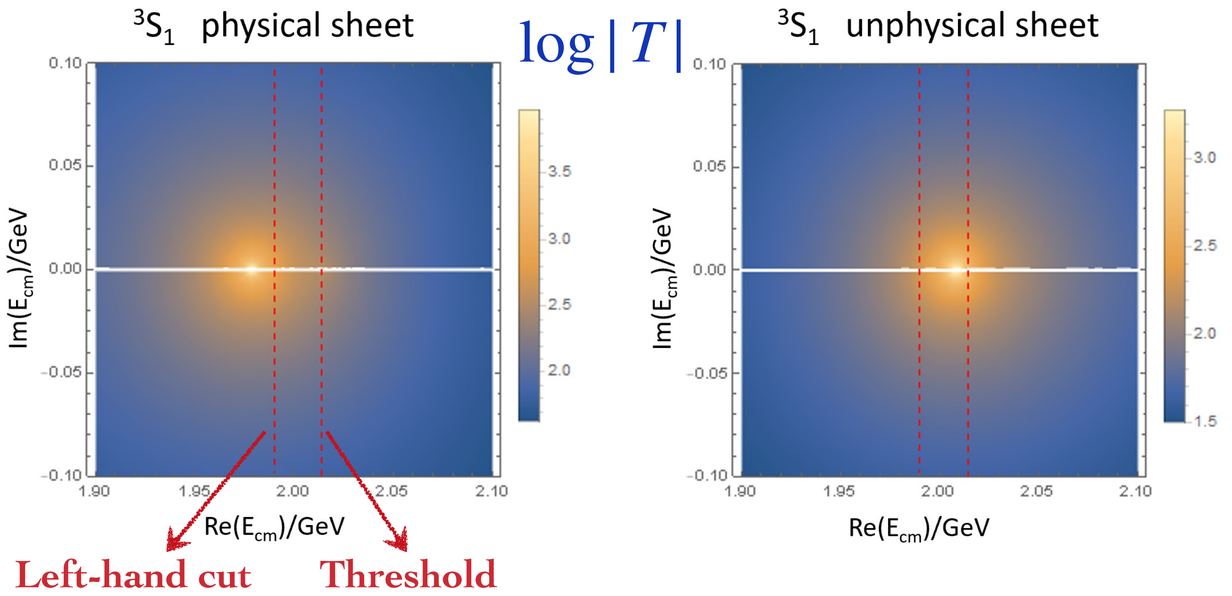
NN scattering

Finite-volume spectrum



Parametrize the scattering amplitude as:

$$T^{-1} = c_0 + c_1 s - i\rho$$



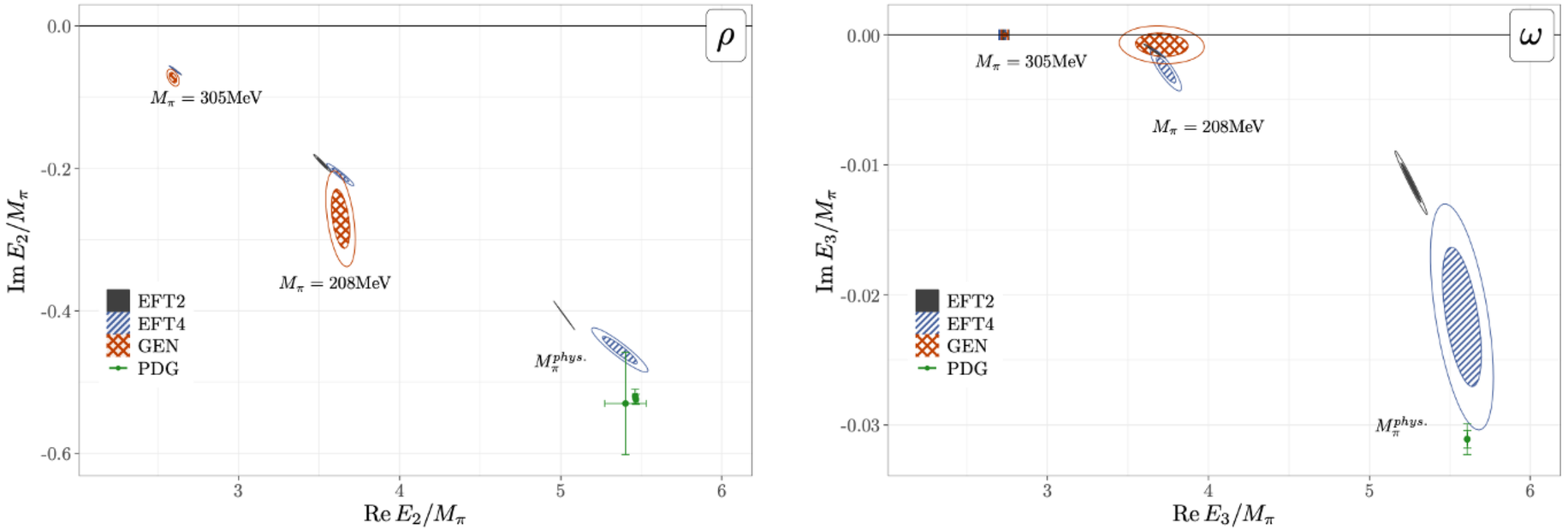
- Virtual pole is found at both $^3S_1$ and $^1S_0$ channels, $E_B(^3S_1) = 6^{+5}_{-3}$, $E_B(^1S_0) = 11^{+6}_{-5}$.
- Finite-volume Hamiltonian method considering the left-hand cut effects gives the same conclusions.

Three body scattering

- ω meson: $\pi\pi\pi$ $I = 0$ scattering
- $\pi(1300)$: $\pi\pi\pi$ $I = 1$ scattering

$\pi\pi\pi$ $I = 0$ scattering

Haobo Yan et al., PRL133(2024)21,211906 (Editor's suggestion)

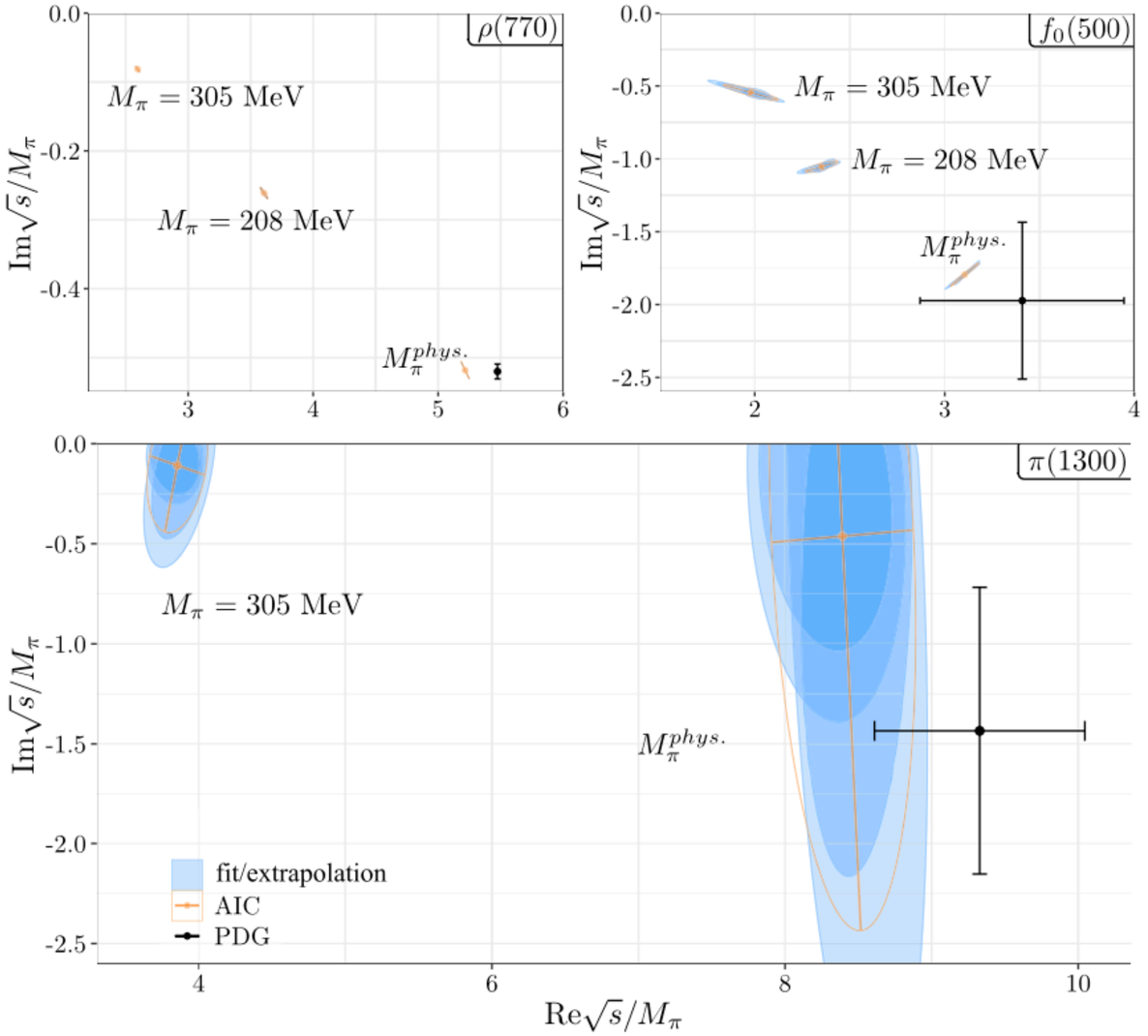


- First determination of ω-meson resonance from three-body scattering in lattice QCD

$$\sqrt{s_\rho} = (748.9(10.0) - i\,63.5(1.8))\ \text{MeV}$$
$$\sqrt{s_\omega} = (778.0(11.2) - i\,3.0(5))\ \text{MeV}$$

$\pi\pi\pi$ $I = 1$ scattering

Haobo Yan et al., PRL136(2026)14,141901



Summary and discussions

✦ Meson-meson scattering:

- The resonance ρ and K^* are determined with full control of systematics.
- The pion mass dependence of $D_0^*(2300)$ has a non-monotonic behavior.

✦ Meson-baryon scattering:

- The scattering of the $(\Xi_{cc}, \Omega_{cc}) - (\pi, K, \bar{K})$ has similar behaviour as $(D, D_s) - (\pi, K, \bar{K})$.
- A double-charm pentaquark is predicted as the $I = 0$ $\Xi_{cc}\bar{K}$ channel.

✦ Baryon-baryon scattering:

- Deuteron is not bound at $m_\pi \sim 290$MeV

✦ Three-body scattering:

- First determination of the ω meson and $\pi(1300)$ from $\pi\pi\pi$ scattering.
- Other three-body scattering process: $DD\pi, DDK$

Thanks!