Direct Lattice QCD calculation of the CP-violating pion-nucleon coupling

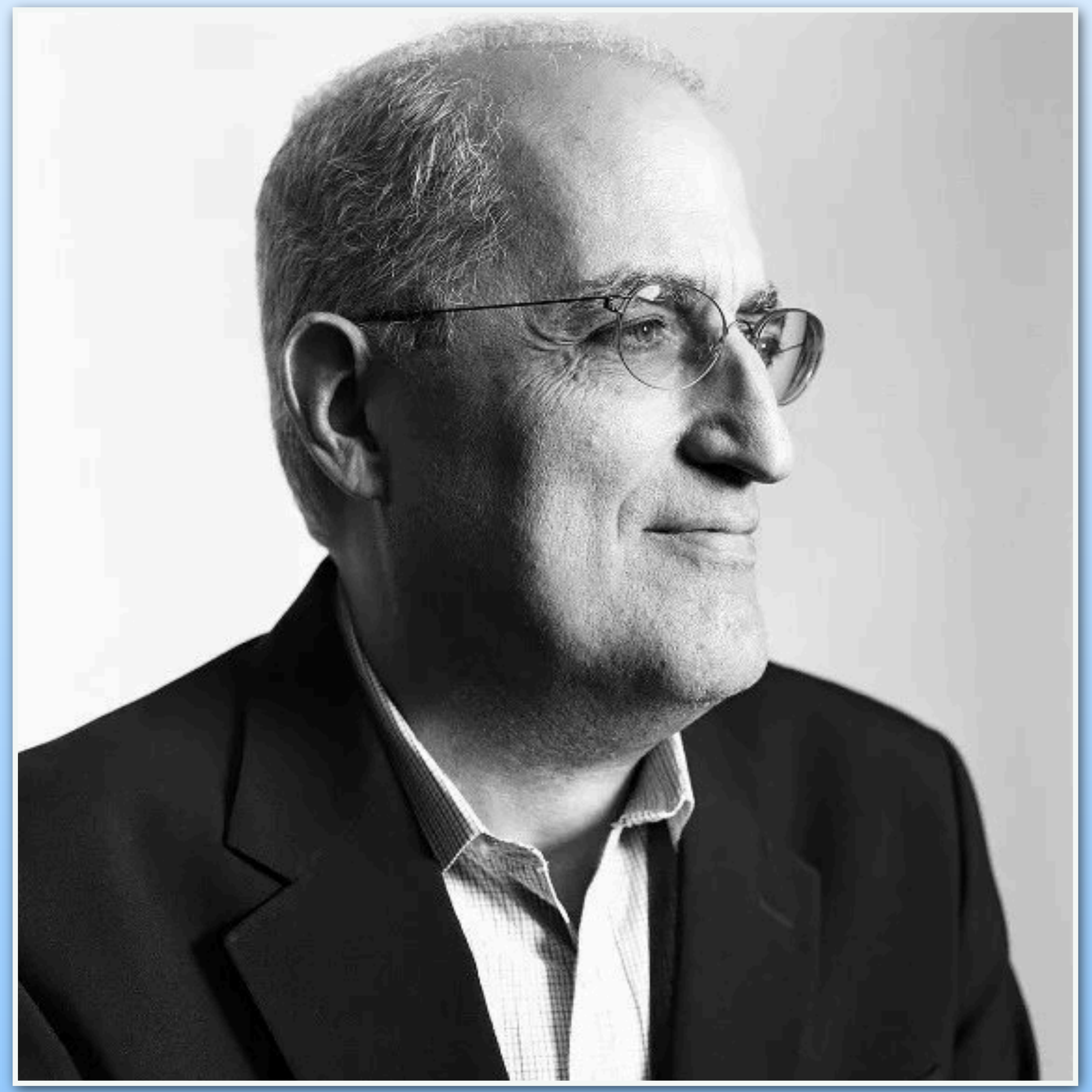
梁 剑

华南师范大学 量子物质研究院

08/12/2026 @ 第十一届手征有效场论研讨会 · 兰州大学

Motivation and Background

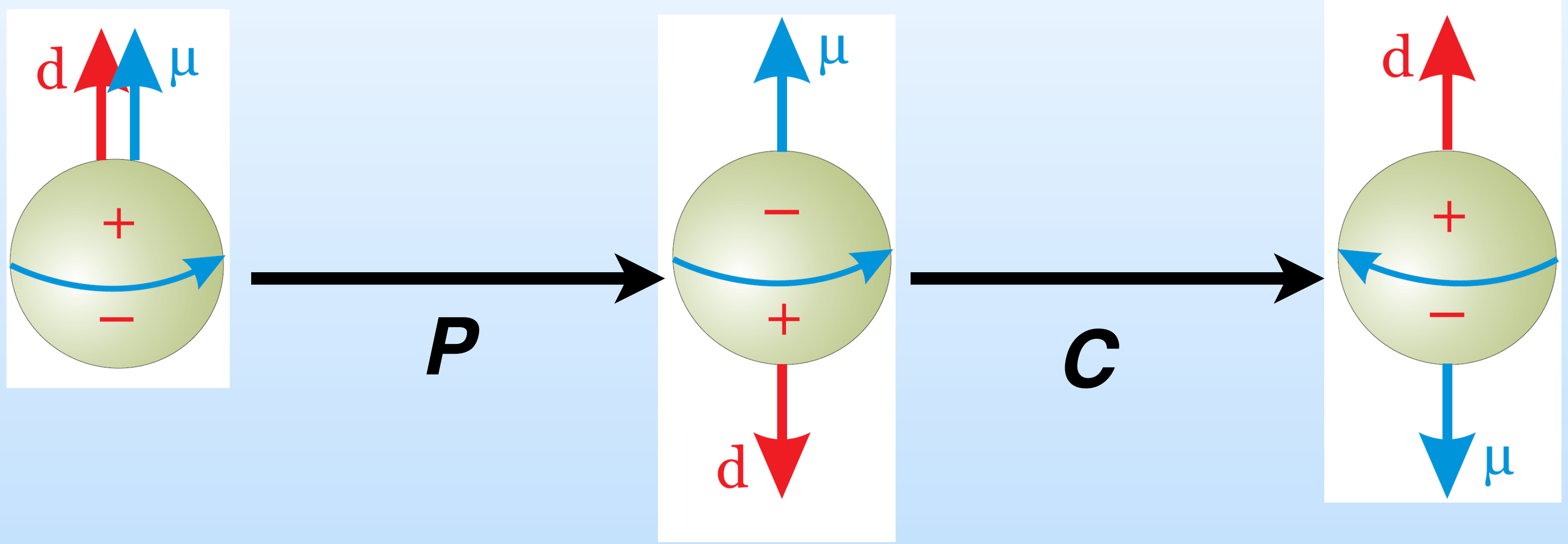
Nucleon EDM

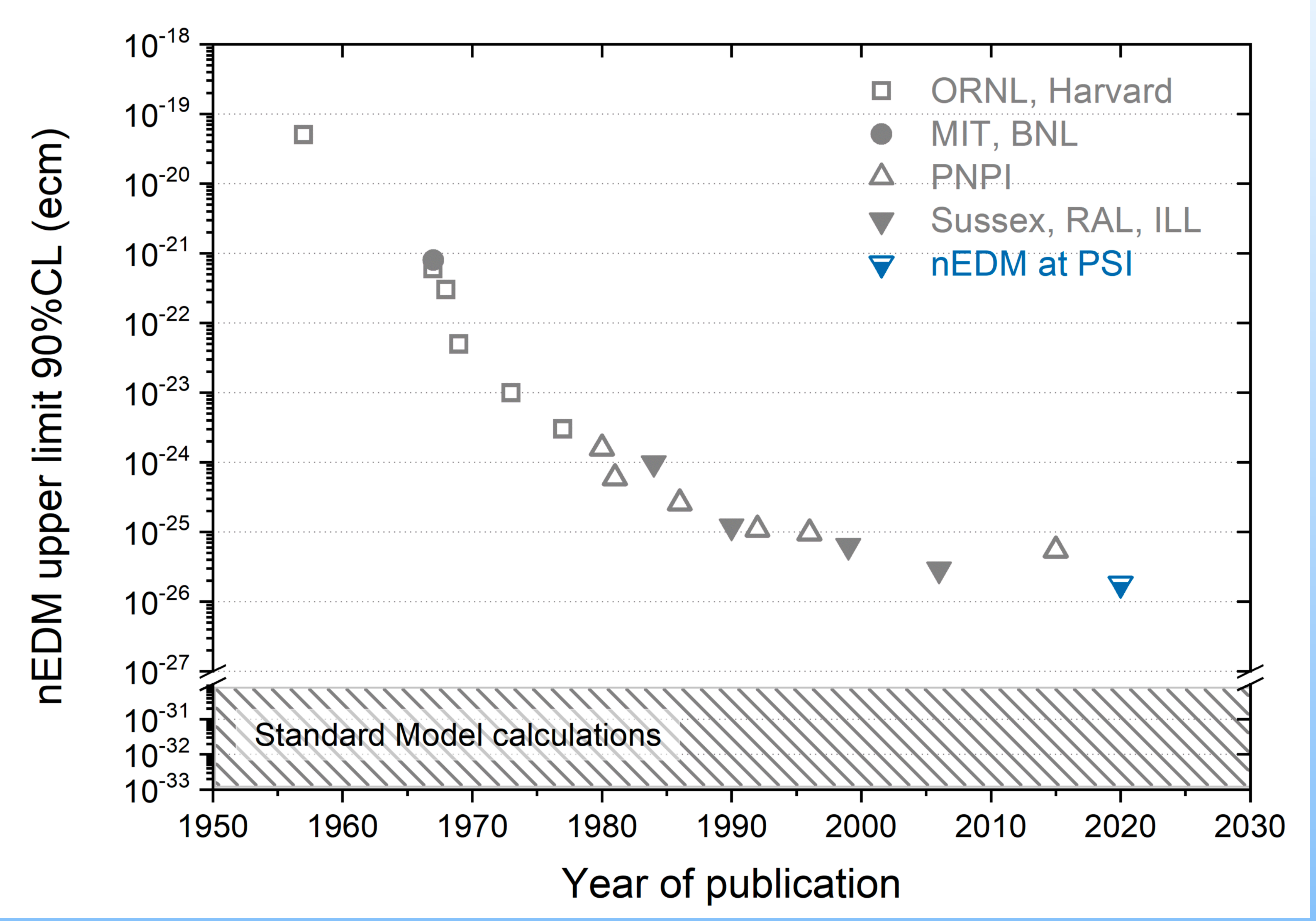


… It is also important to look for electron or neutron electric dipole moments, which may give a new physical signal, and under the energy scales we have already observed…

— *Edward Witten: Where is the New Physics Lurking? (cern courier)*

Nucleon EDM





- ✦ A non-zero intrinsic electric dipole moment (EDM) of a fundamental particle violates the CP (T) symmetry

- ✦ Nucleon EDM (NEDM) is sensitive probe of BSM/ Strong CPV: the contribution to the nEDM from the weak CPV phase ~10^{-31} e·cm, 10^{-5} of the current experimental limit: $0.0(1.1)(0.2) \times 10^{-26}$ e·cm

C. Abel et al., PRL124:081803 (2020)

- ✦ Experiments are aiming at improving the limit down to 10^{-28} e·cm in the next ~10 years (?)

- ✦ Still a long way to trek, room for the strong CPV and/or BSM physics

CPV Pion-Nucleon Coupling

$$D_n = g_{\pi NN}\bar{g}_{\pi NN} \ln\left(M_N/m_\pi\right)/4\pi^2 M_N$$

Crewther, Di Vecchia, Veneziano, Witten, PLB 88:123 (1979)

$$d_0^\theta = \bar{d}_0^\theta - \frac{eg_A\bar{g}_0}{16\pi F_\pi}\frac{3m_\pi}{M_N},$$

$$d_1^\theta = \bar{d}_1^\theta(\mu) + \delta_1(\mu) + \frac{eg_A\bar{g}_0}{4\pi^2 F_\pi}\left[\ln\frac{m_\pi^2}{m_N^2} - \frac{5\pi m_\pi}{4m_N}\right]$$

A. Shindler, EPJA57: 128 (2021)

- ✦ In effective theories, nucleon EDM is determined by the CPV pion-nucleon coupling
- ✦ EDMs of systems involving more than one nucleon also depends on the CPV nucleon-nucleon interactions
- ✦ Mainly the one-pion-exchange contributions (CPV pion-nucleon vertices) according to the chiral power counting
- ✦ Multi-nucleon contributions often dominate nuclear and atomic EDMs compared to the EDMs of constituent nucleons
- ✦ No direct first-principles calculations

J. de Vries et al., PLB766:254 (2017)

J. de Vries et al., PRC92:045201 (2015)

CPV Pion-Nucleon Coupling

- To leading order the coupling is related to the octet baryon masses

$$\bar{g}_{\pi\mathrm{NN}} = -\theta \frac{(M_\Xi - M_N)m_u m_d}{F_\pi (m_u + m_d)(2m_s - m_u - m_d)}$$

$$|\bar{g}_{\pi\mathrm{NN}}| \approx 0.038|\theta|$$

Crewther, Di Vecchia, Veneziano, Witten, PLB 88:123 (1979)

- A more recent work shows that this coupling can be achieved using the proton-neutron mass difference, which receives no SU(3)-flavor breaking corrections up to NNLO in chiral expansion

$$\bar{g}_0 = \delta m_N \frac{m_* \bar{\theta}}{\bar{m}\varepsilon}$$

$$m_* = \frac{m_u m_d m_s}{m_s(m_u + m_d) + m_u m_d} = \frac{\bar{m}(1-\varepsilon^2)}{2 + \frac{\bar{m}}{m_s}(1-\varepsilon^2)}$$

$$\frac{\bar{g}_0}{2F_\pi} = (15.5 \pm 2.0 \pm 1.6) \times 10^{-3} \bar{\theta}$$

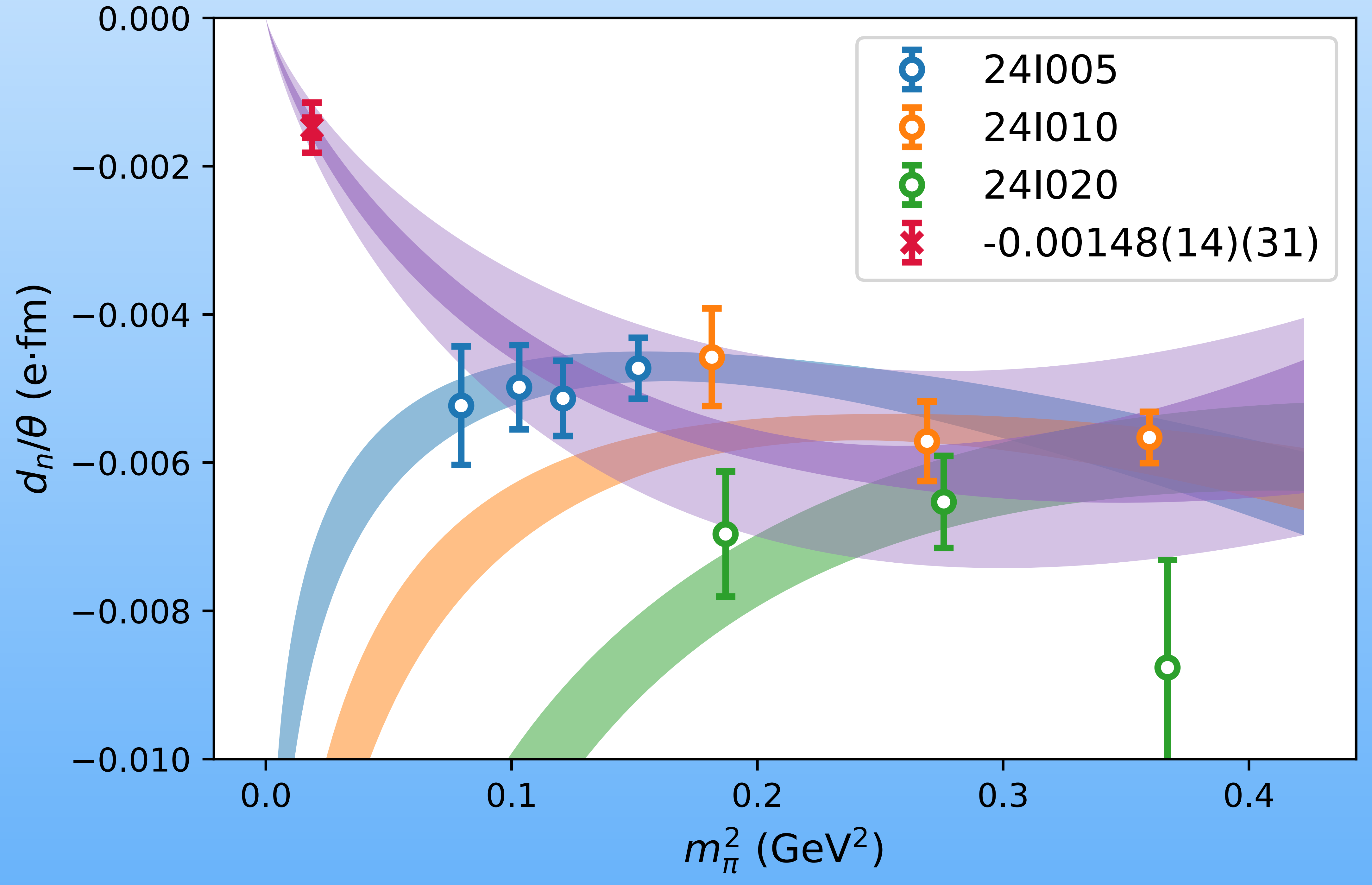
$$2\bar{m} = m_u + m_d \qquad \varepsilon = (m_d - m_u)/(m_d + m_u)$$

J. de Vries et al., PRC92:045201 (2015)

Nucleon EDM from Lattice QCD

$$d_n^{(PQ)} = \frac{e\,\bar{\theta}\,m_s}{4\pi^2 f^2}\left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \bar{\theta}\,\frac{e}{\Lambda_\chi^2}\left[\frac{m_s}{2}c(\mu) + \underline{d\,(m_s - m_v)} + fq_{jl}\,\underline{(m_s - m_v)}\right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)



***Liang** and et. al., PRD108 094512 (2023)*

$$g_{\pi\mathrm{NN}} = \frac{g_A M_N}{f}$$

$$D_n = g_{\pi\mathrm{NN}}\bar{g}_{\pi\mathrm{NN}} \ln\left(M_N/m_\pi\right)/4\pi^2 M_N \sim -\frac{g_A \bar{g}_{\pi NN}}{4\pi^2 f}\log\frac{m_\pi}{M_N}$$

$$\bar{g}_{\pi NN}\ (m_\pi \sim 0.135\ \mathrm{GeV}) \sim -0.0156(15)(33)$$

✦ Higher order effects?

Strong CPV on the lattice

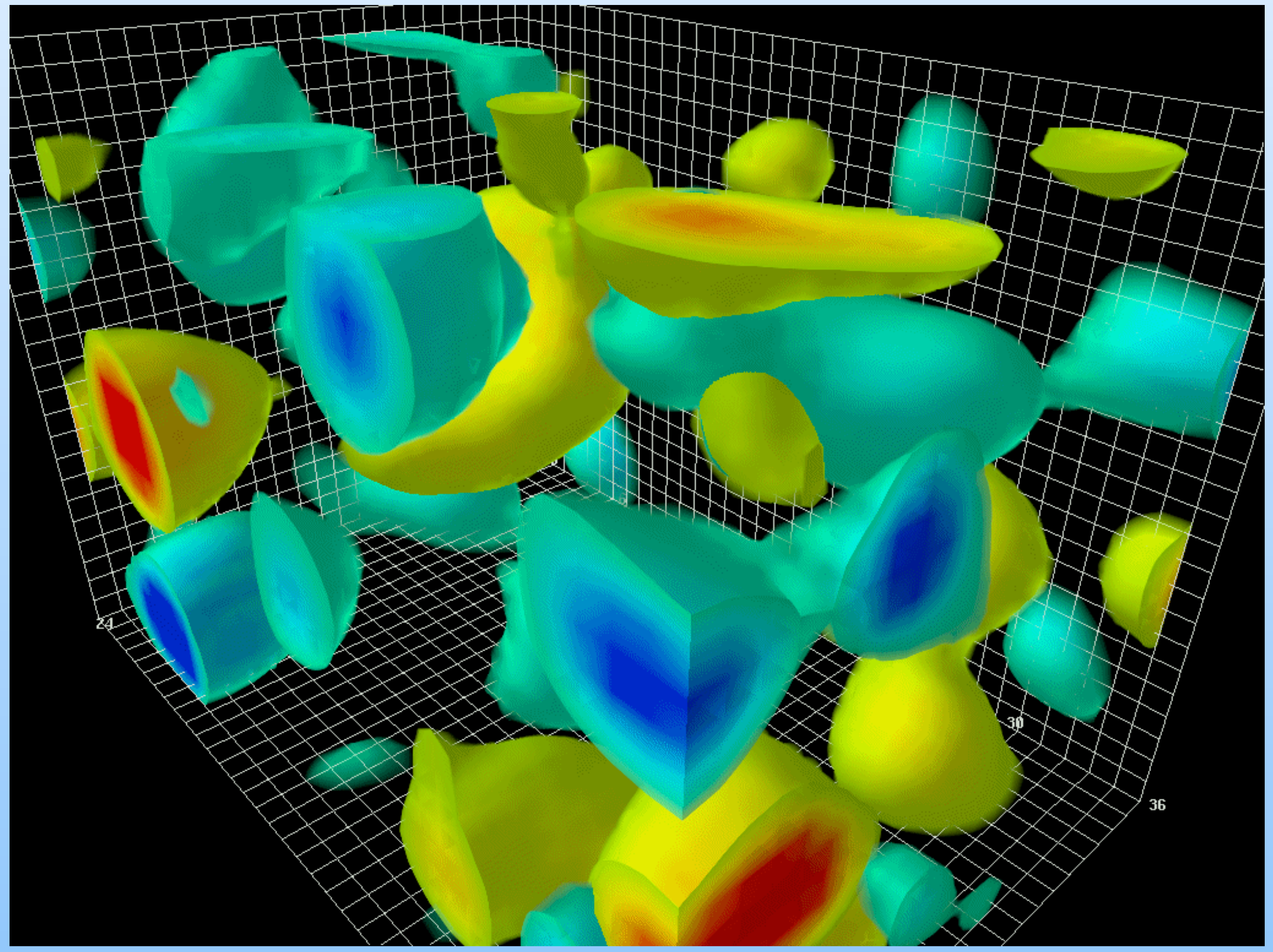
Strong CP-Violation

$$\mathscr{L}^E + \mathscr{L}^E_\theta = \bar{\psi}\left(\not{D}^E + m\right)\psi + \frac{1}{2}\mathrm{Tr}\left[F^E_{\mu\nu}F^E_{\mu\nu}\right] - i\bar{\theta}\frac{g^2}{16\pi^2}\mathrm{Tr}\left[F^E_{\mu\nu}\tilde{F}^E_{\mu\nu}\right]$$

- ✦ Additional term (the θ-term) can be introduced into the QCD Lagrangian
- ✦ Related to the phase of quark mass matrix by chiral rotations
- ✦ Contributes as a total divergence (surface term) to the QCD action
  - not affecting the classical equations of motion
  - not participating in perturbative expansions
  - influencing the QCD vacuum structure, non-perturbative observable CP-violating effects
- ✦ No direct experimental evidence of strong CP violation: θ is either extremely small or strictly zero
  - naturalness issue known as the "fine-tuning problem" or the strong CP problem

Topological Charges

$$\mathscr{L}_\theta^E = -i\bar{\theta}\frac{g^2}{16\pi^2}\mathrm{Tr}\left[F_{\mu\nu}^E\tilde{F}_{\mu\nu}^E\right] \equiv -i\bar{\theta}q(x) \qquad Q = \int d^4x q(x)$$



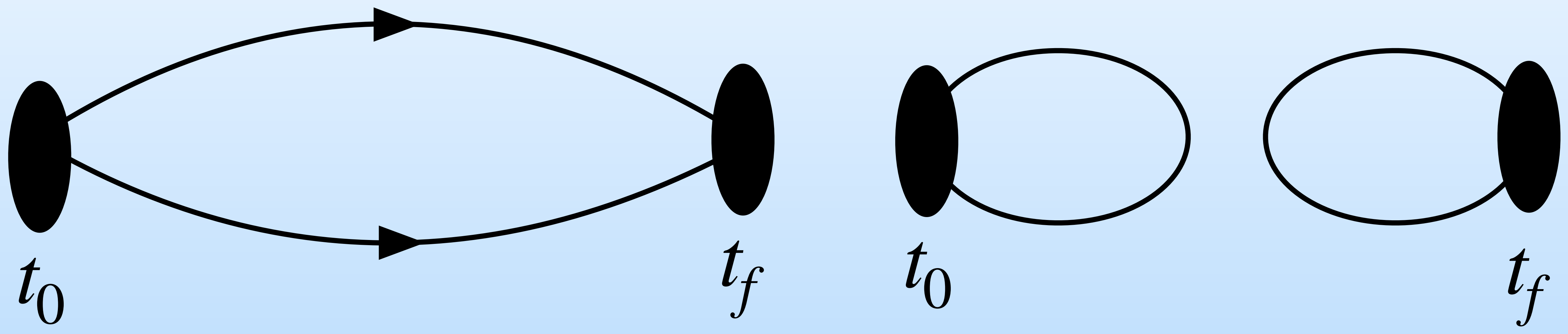
Leinweber/VisualQCD

- ✦ Q is the topological charge of the gauge field
- ✦ Q should be integer, according to the index theorem

$$Q = n_- - n_+$$

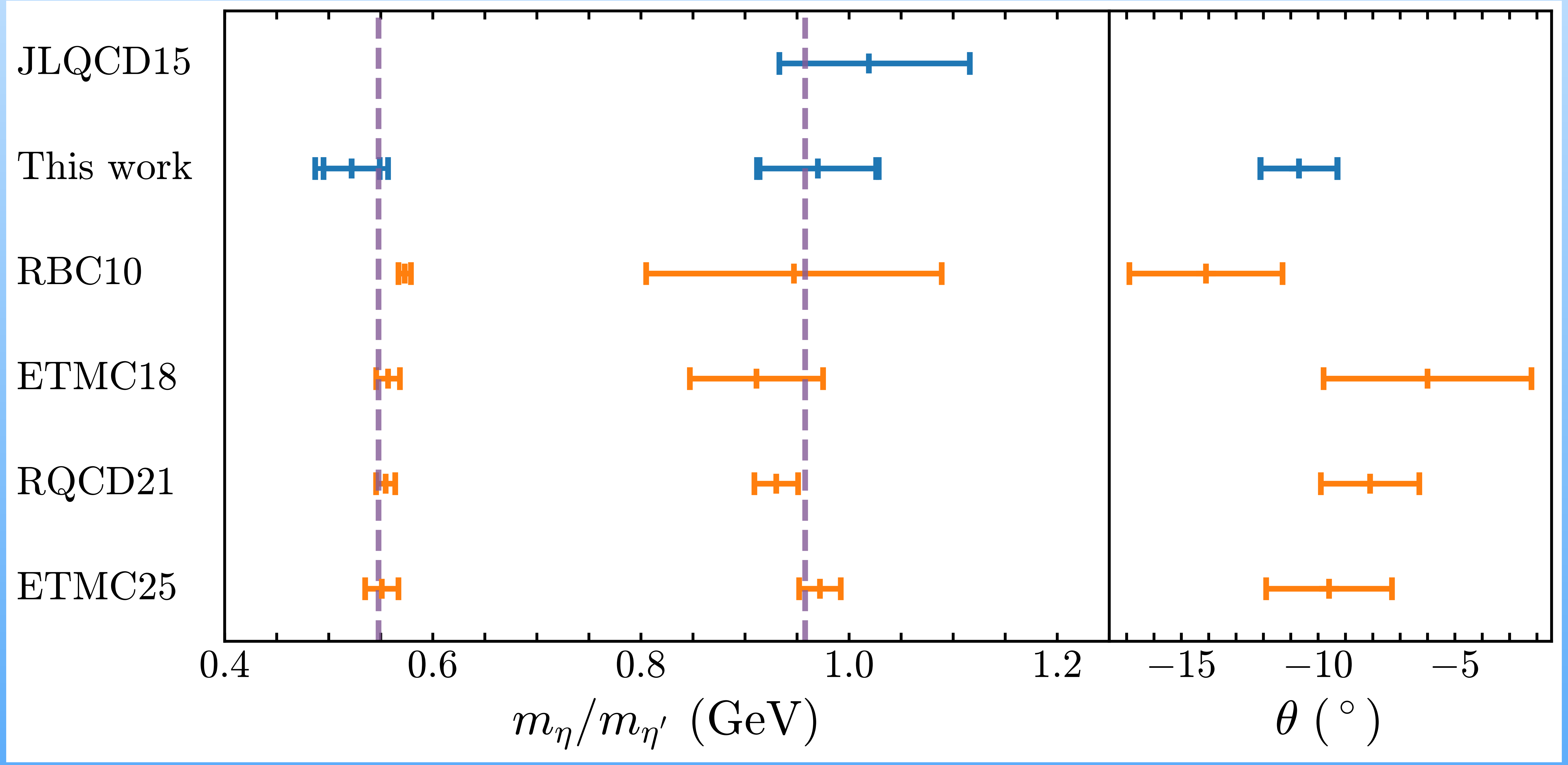
$$q(x) = \frac{1}{2}\mathrm{Tr}\left[\gamma_5 D\right] = -\mathrm{Tr}\left[\gamma_5\left(1 - \frac{D}{2}\right)\right]$$

η and η' Mesons from Topological Charge Operators



$$C_2(t) = \sum_{\vec{x},\vec{y}} \langle q(\vec{y},t)\ q(\vec{x},0)\rangle$$

$$C_2(r) = \frac{1}{VN_r} \sum_{y\in\{|y-x|=r|\}} \sum_x \langle q(y)\ q(x)\rangle$$



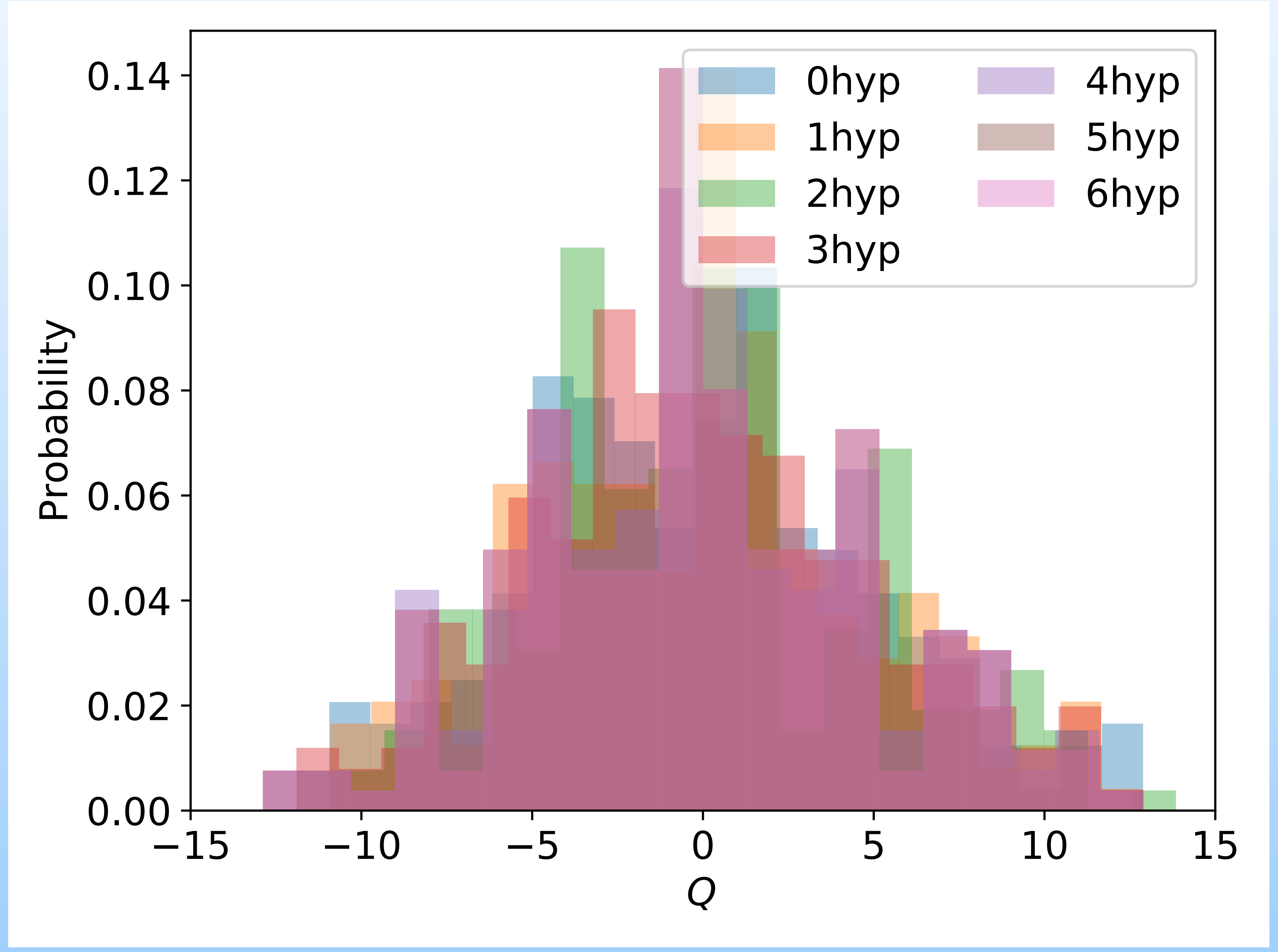
YS, NW, LG, JH, **JL**, and JS, ArXiv:2505.19775

Strong CP-Violation on the Lattice

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle = \frac{1}{Z}\int \mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}A_\mu O_1(\psi,\bar{\psi},A_\mu)O_2(\psi,\bar{\psi},A_\mu)\ldots e^{i\mathcal{S}_{QCD}}$$

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E = \frac{1}{Z^E}\int \mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}A_\mu O_1(\psi,\bar{\psi},A_\mu)O_2(\psi,\bar{\psi},A_\mu)\ldots e^{-\mathcal{S}^E_{QCD}}$$

$$\int \mathscr{D}\psi = \prod_x \int d\psi(x) \qquad \int \mathscr{D}\psi = \prod_{n\in L}\int d\psi(n)$$



$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E_\theta \sim \int \mathscr{D}\ldots O_1O_2\ldots e^{-\mathcal{S}^E_{QCD}+i\bar{\theta}Q} \sim \int \mathscr{D}\ldots O_1O_2\ldots(1+i\bar{\theta}Q+\ldots)e^{-\mathcal{S}^E_{QCD}}$$

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E_\theta \sim \langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E + i\bar{\theta}\langle T[\hat{O}_1\hat{O}_2,\ldots]Q\rangle^E$$

- ✦ Sign problem and the "signal-to-noise" problem are essentially the same one
- ✦ Operators or vacuum (action)? different points of views

Direct $\tilde{g}_{\pi NN}$ calculation

Detailed Formulas of Direct Calculation

$$G^{(3)Q}(t_f,\tau) = \sum_{\vec{x}\vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \langle \chi(x)\ Q\ P(y)\ \bar{\chi}(0)\rangle$$

$$\begin{aligned} R_3(\Gamma_e,\gamma_5) &= \frac{\mathrm{Tr}\left[\Gamma_e G^{(3)Q}(\gamma_5)\right]}{\mathrm{Tr}\left[\Gamma_e G^{(2)}\left(\vec{p}=0\right)\right]} \\ &= \frac{\mathrm{Tr}\left[\frac{1+\gamma_4}{2}\left[\alpha^1\gamma_5 G_p \gamma_5 \frac{1+\gamma_4}{2} + \frac{1+\gamma_4}{2} G_p \gamma_5 \alpha^1 \gamma_5 + \frac{1+\gamma_4}{2}\tilde{G}_P' \frac{1+\gamma_4}{2}\right]\right]}{2} \\ &= 2\alpha^1 G_p + \tilde{G}_p' \end{aligned}$$

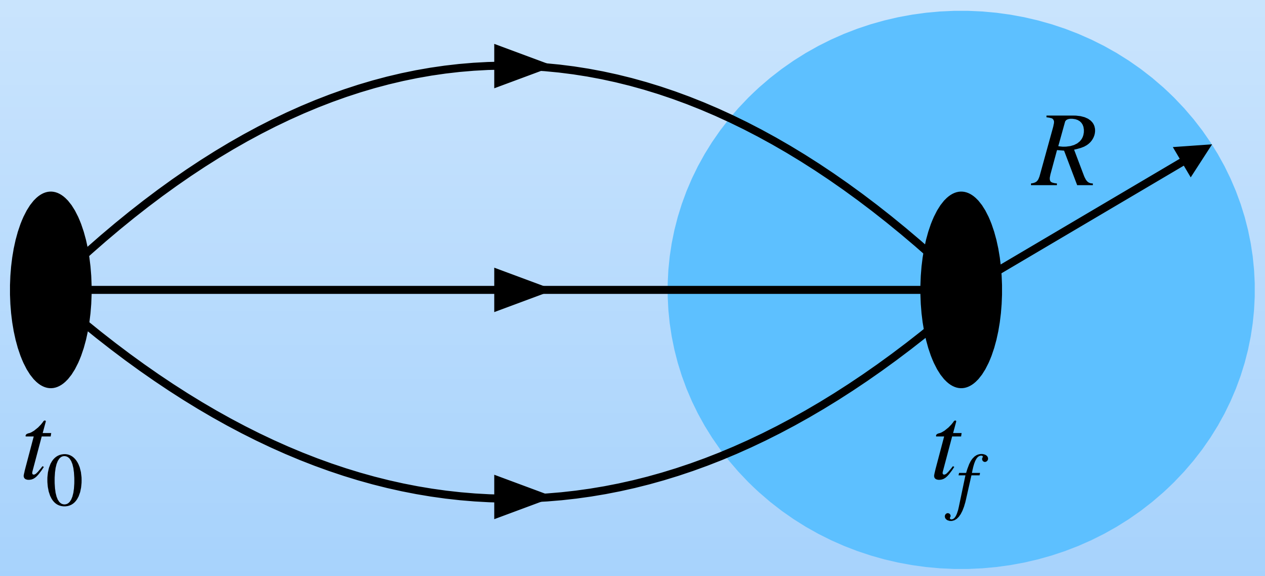
$$2i\alpha_1\theta G_p + i\theta\tilde{G}_P' = i\theta\left(2\alpha_1 G_P + \tilde{G}_P'\right) \equiv i\theta\tilde{G}_P$$

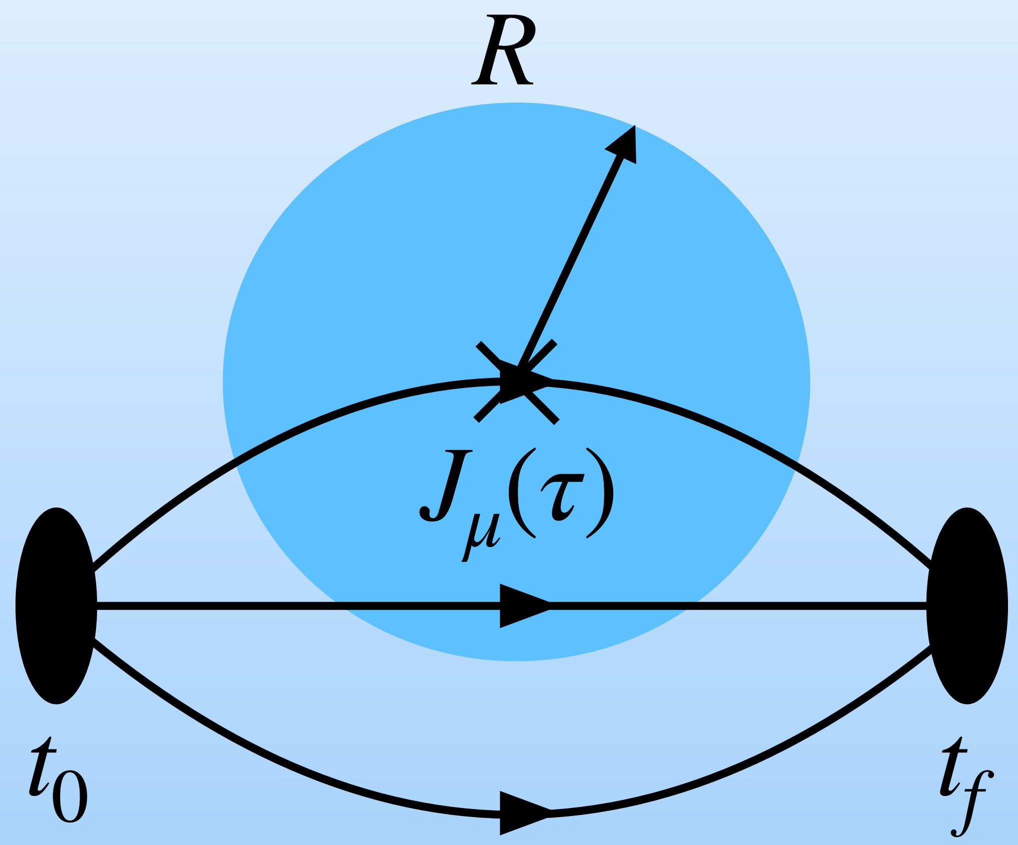
$$m_q\tilde{G}_P = \frac{m_\pi^2 f_\pi}{m_\pi^2 - q^2}\tilde{g}_{\pi NN}$$

✦ chiral fermions, renormalization free

CDER

- ✦ cluster decomposition error reduction (CDER)





$$G^{(2)Q}(t_f) \sim \sum_{\vec{x}} \left\langle Q\, \chi(x)\, \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$

$$\sim \sum_{\vec{x}} \left\langle \sum_{r}^{|r|<R} q_t(x+r)\, \chi(x)\, \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$

$$G^{(3)Q} \sim \sum_{\vec{x}\vec{y}} e^{-i\vec{q}(\vec{x}-\vec{y})} \left\langle \chi(x)\ Q\ P(y)\, \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$

$$\sim \sum_{\vec{x}\vec{y}} e^{-i\vec{q}(\vec{x}-\vec{y})} \left\langle \chi(x) \sum_{r}^{|r|<R} q_t(y+r)\ P(y)\, \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$

*Liu, **Liang** and Yang, PRD97:034507 (2018)*

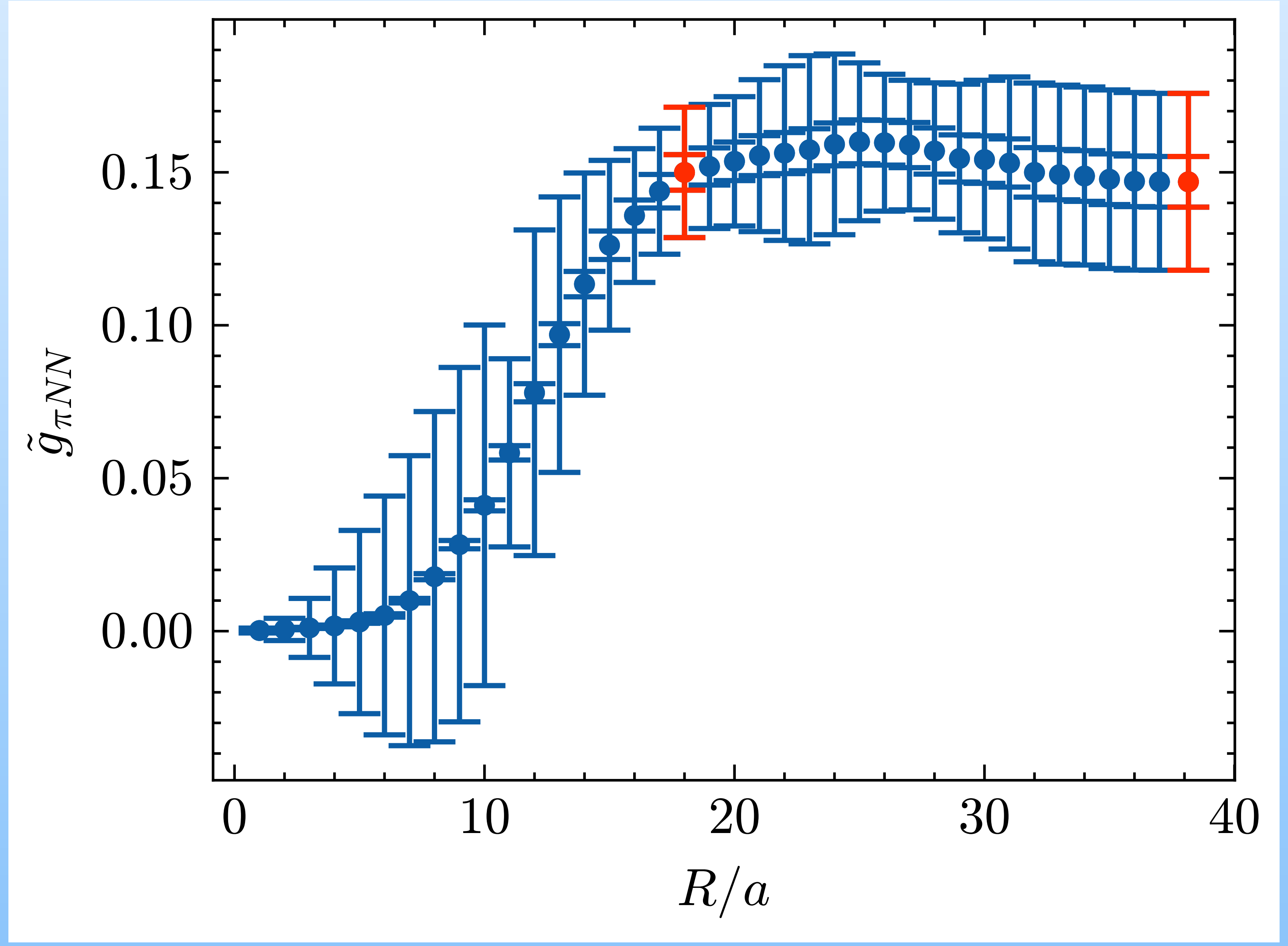
Matrix Elements

339 MeV / 321 MeV

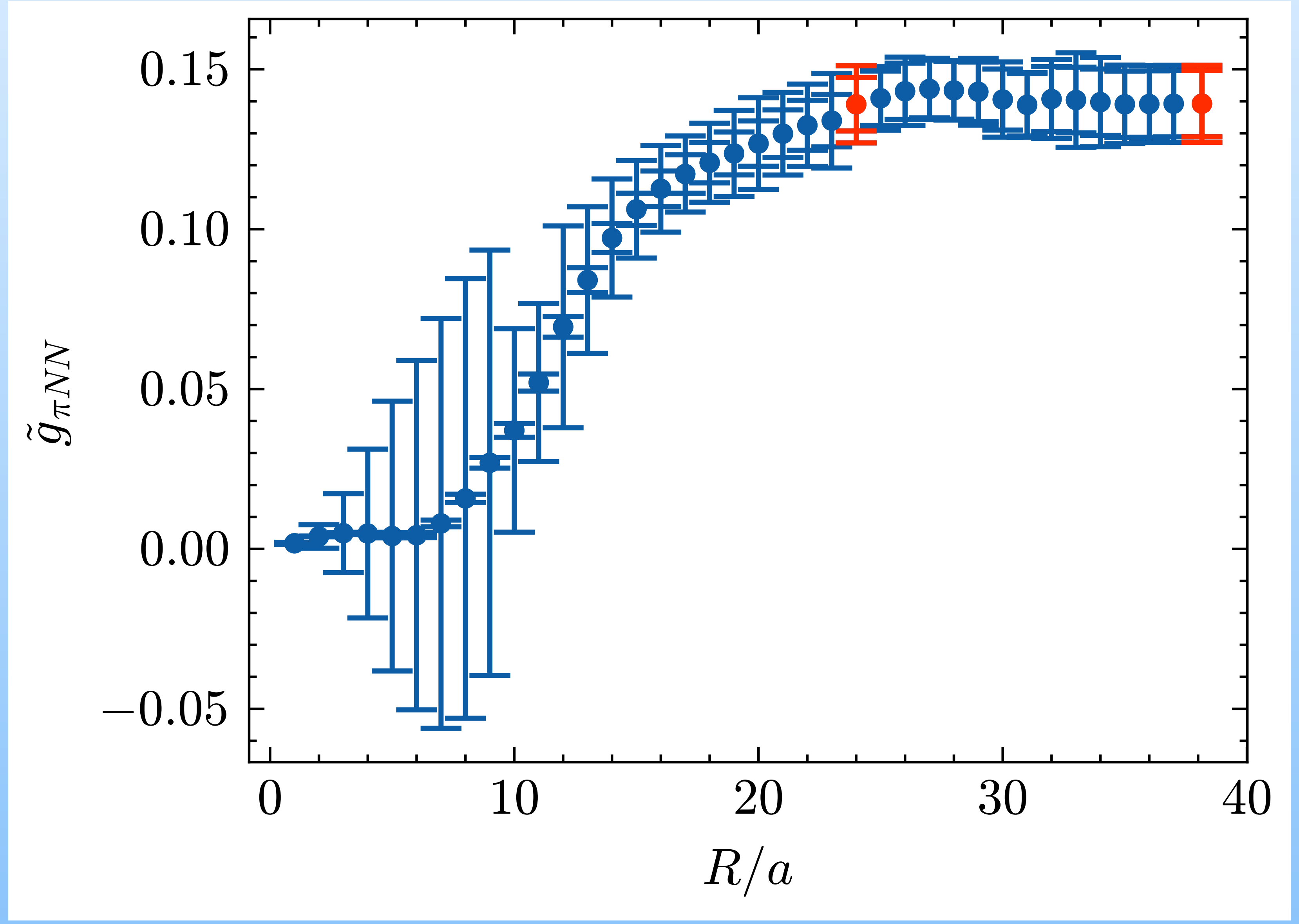
432 MeV / 519 MeV

560 MeV / 525 MeV

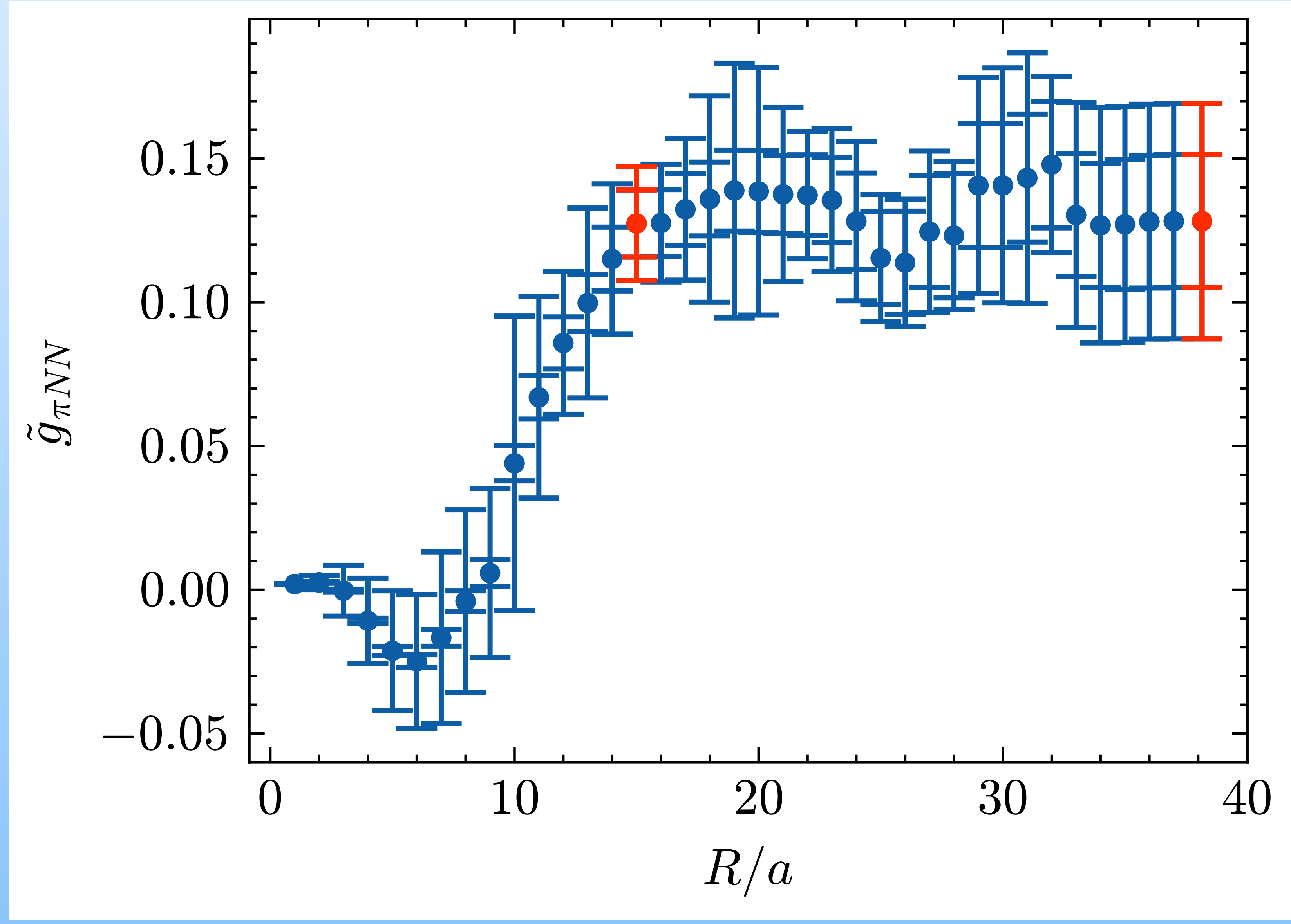
CDER Cut Dependence



339 MeV / 321 MeV

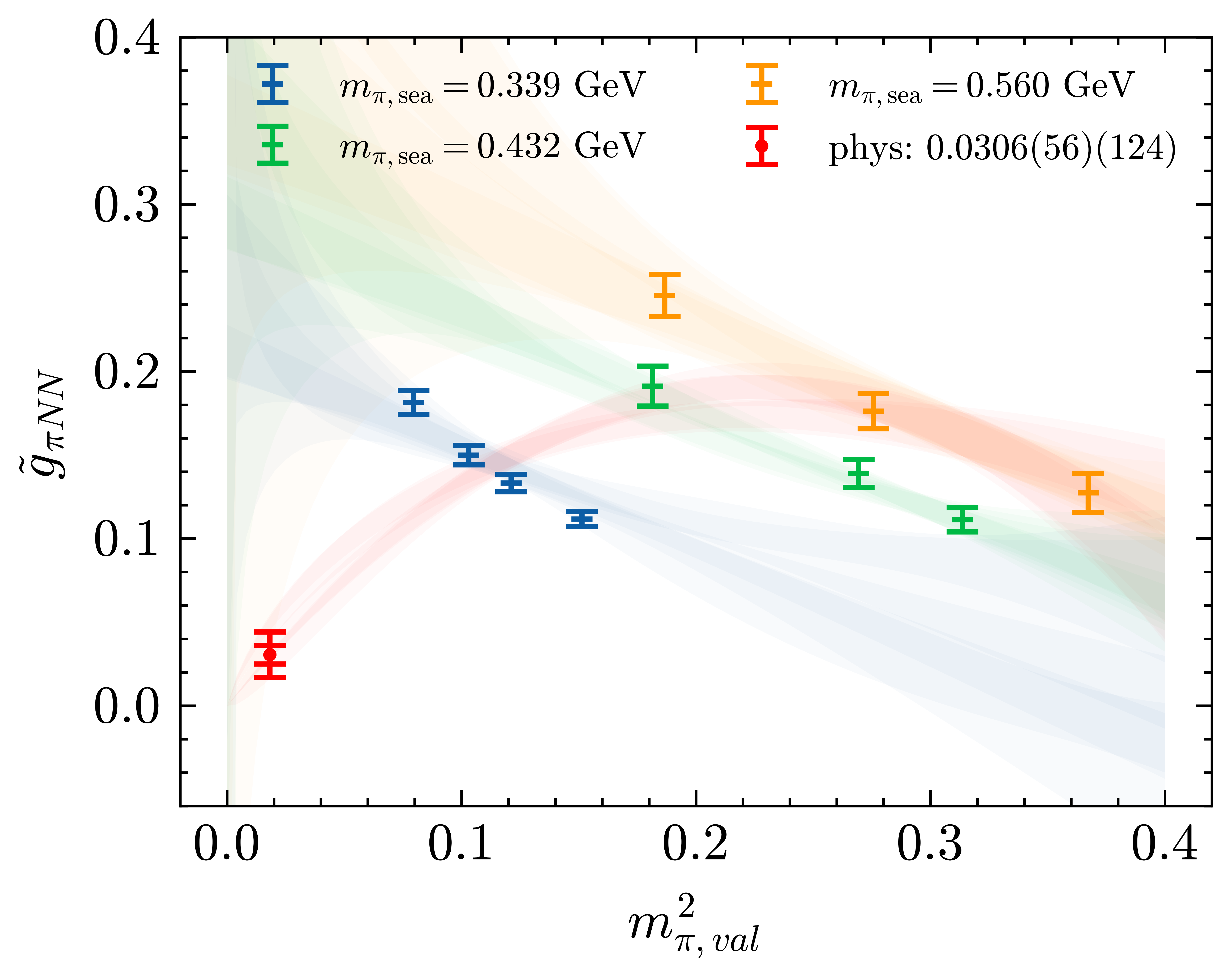


432 MeV / 519 MeV



560 MeV / 606 MeV

Chiral Extrapolation



$am^2_{\pi,v} + bm^2_{\pi,s}$

$am^2_{\pi,v} + bm^2_{\pi,s} + cm^4_{\pi,s}$

$am^2_{\pi,v} + bm^4_{\pi,v} + cm^2_{\pi,s}$

$am^2_{\pi,v} + bm^4_{\pi,v} + cm^2_{\pi,s} + dm^4_{\pi,s}$

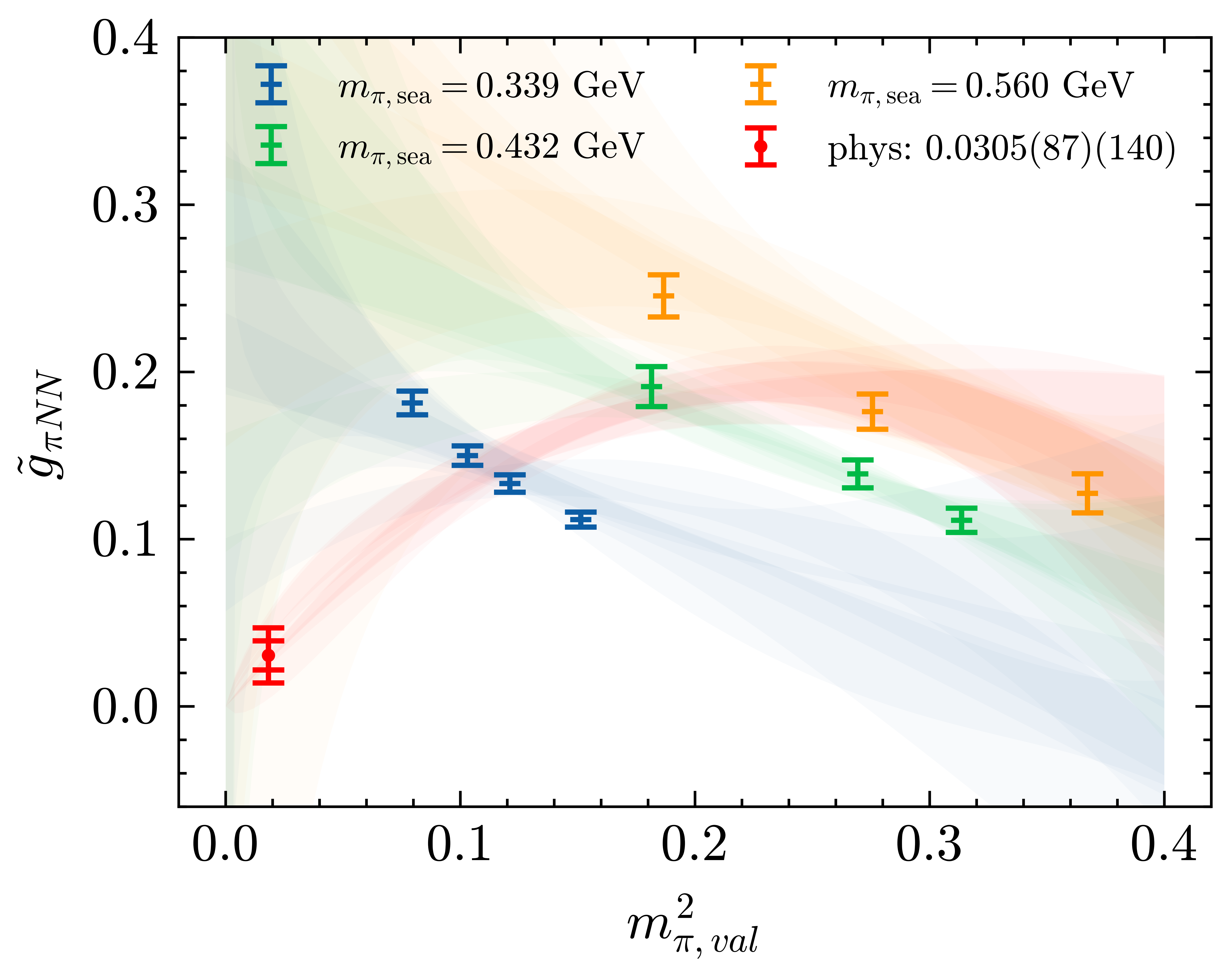
$am^2_{\pi,v} + bm^4_{\pi,v} + cm^2_{\pi,s} + dm^2_{\pi,s}\log\frac{m^2_{\pi,v}}{\Lambda^2}$

$am^2_{\pi,v} + bm^2_{\pi,s} + cm^4_{\pi,s} + dm^2_{\pi,s}\log\frac{m^2_{\pi,v}}{\Lambda^2}$

$am^2_{\pi,v} + bm^2_{\pi,s} + cm^2_{\pi,s}\log\frac{m^2_{\pi,v}}{\Lambda^2} + dm^2_{\pi,s}\log\frac{m^2_{\pi,s}}{\Lambda^2}$

✦ AIC to average different models

Chiral Extrapolation



$\tilde{g}_{\pi NN} = 0.0306(56)(124)$

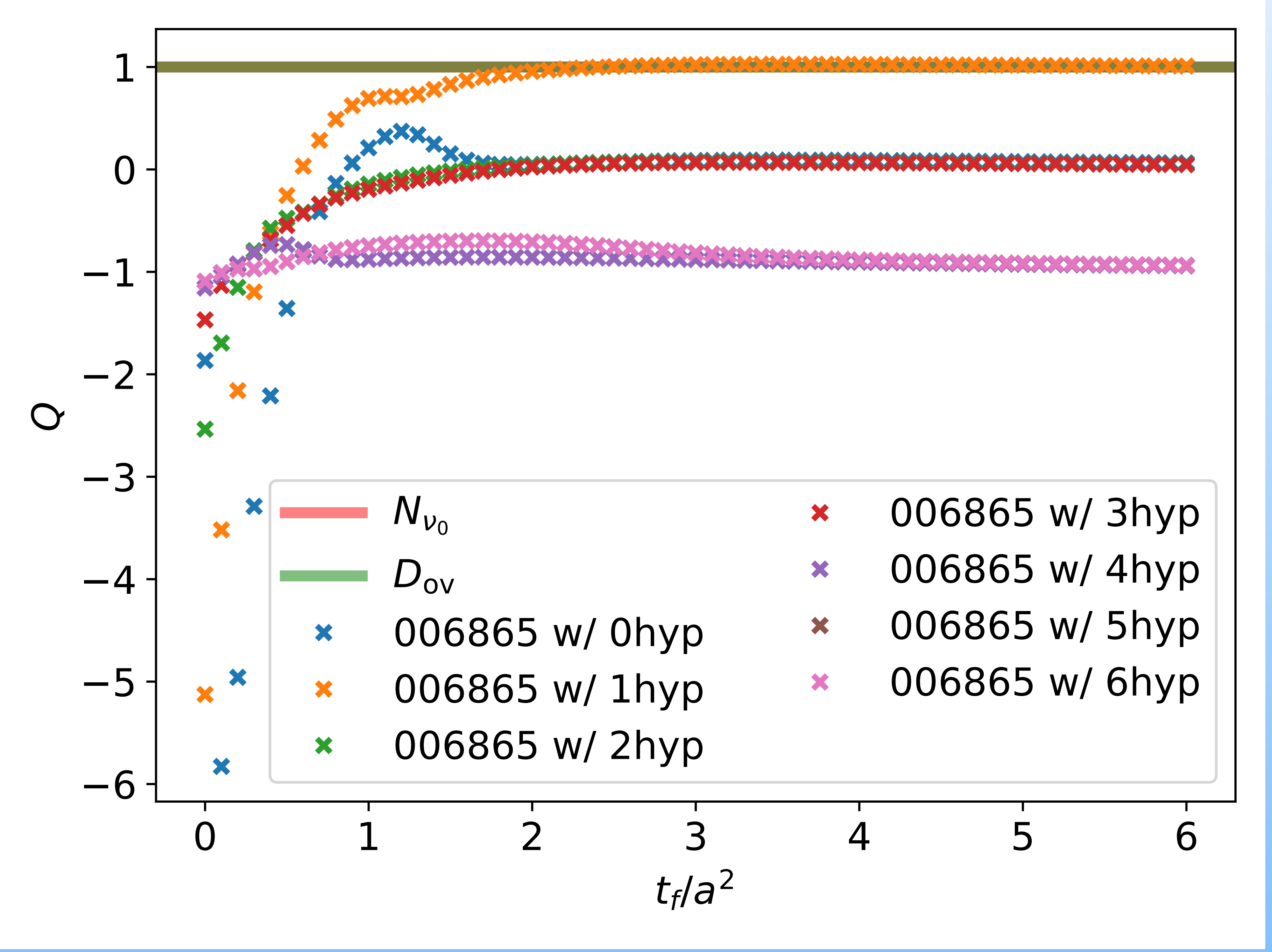
- ✦ Chiral extrapolation
- ✦ Excited-states effects
- ✦ finite-lattice-spacing effects

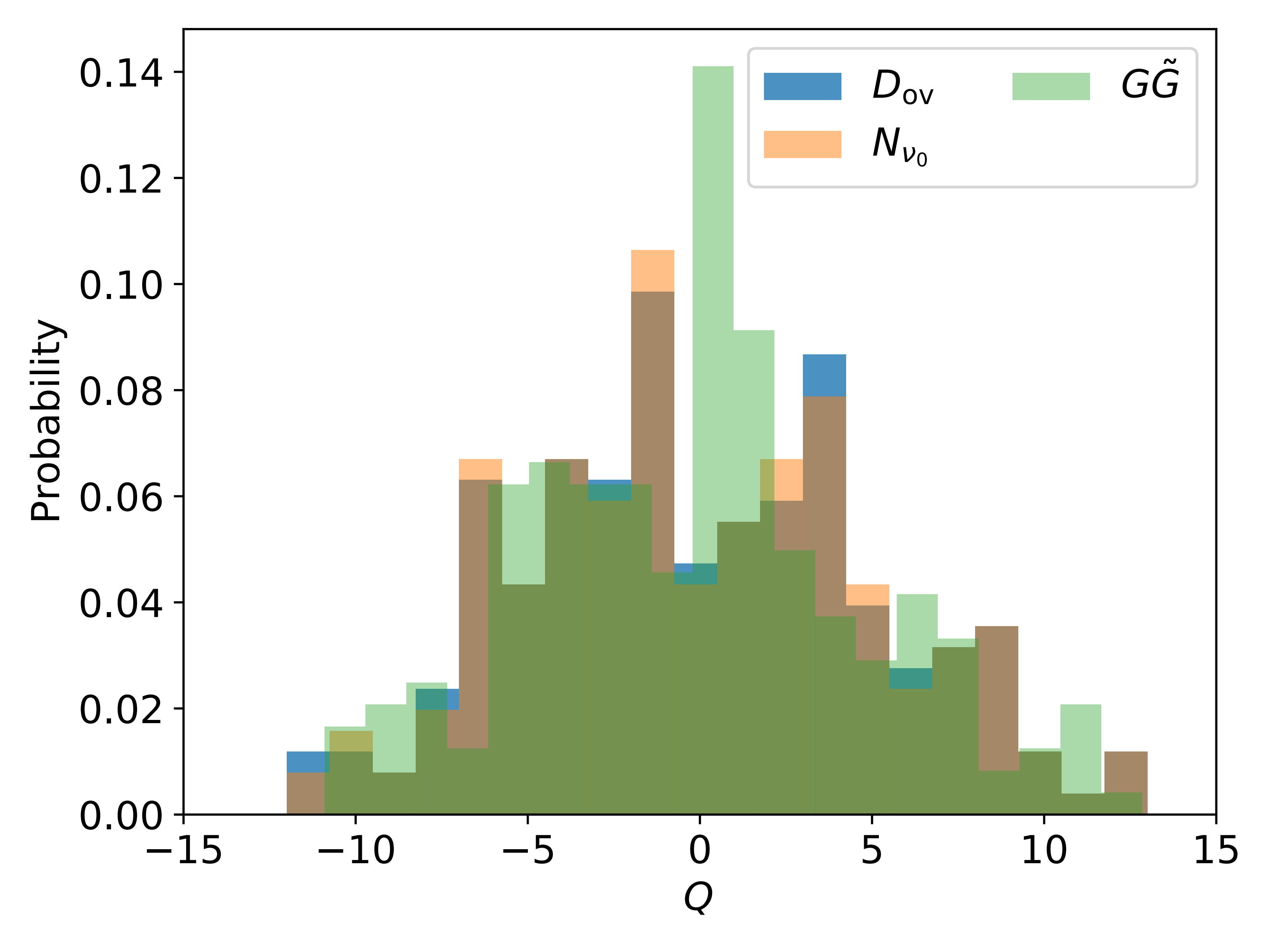
Summary and Outlook

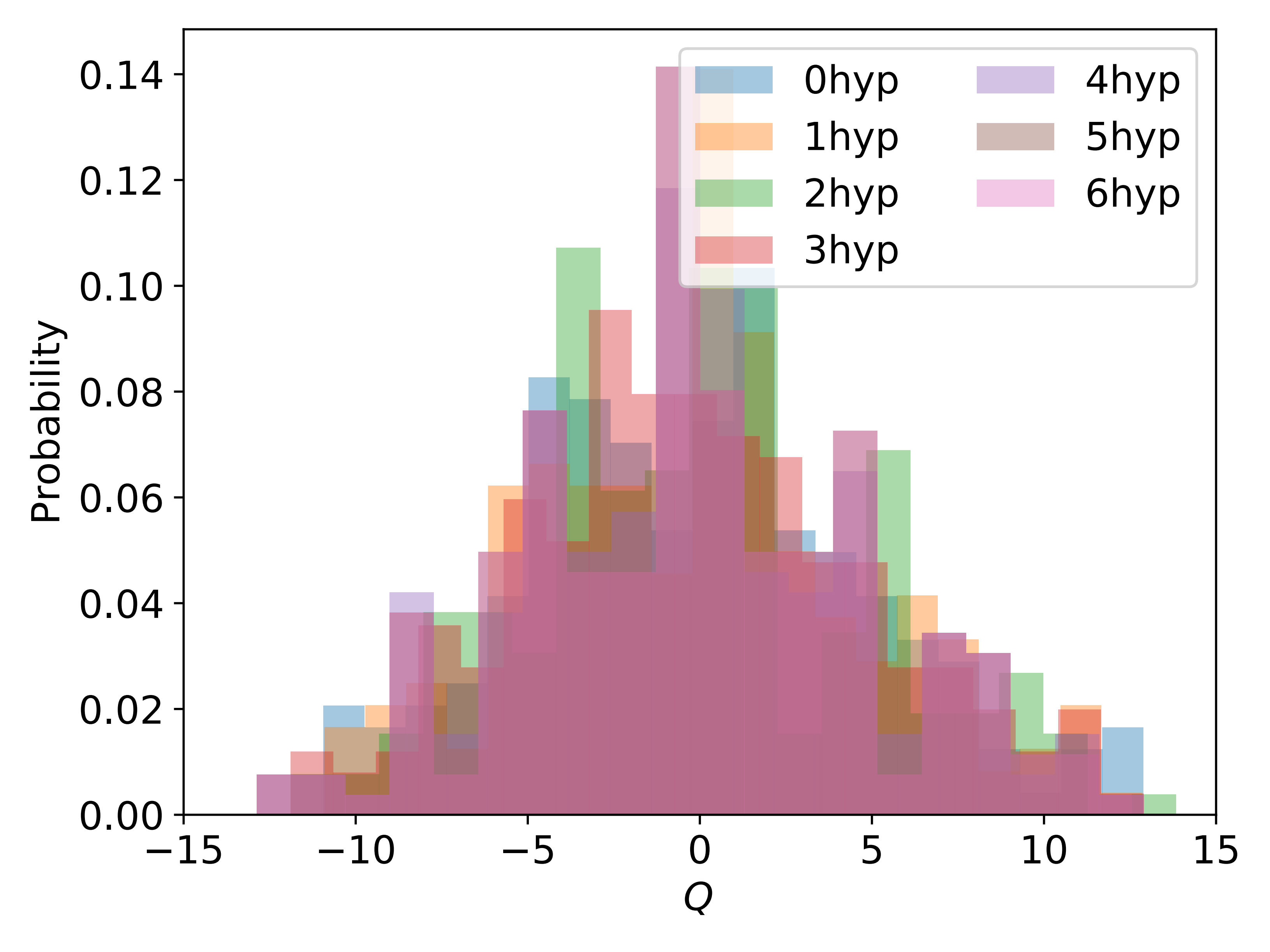
- We carried out the first direct lattice calculation of the θ-induced CP-violating pion-nucleon coupling
- Detailed (partially-quenched) chiral extrapolation forms help to reduce systematic uncertainties, and calculations with lighter (physical) pion masses are on going

THANK YOU

Topological Charges on the Lattice



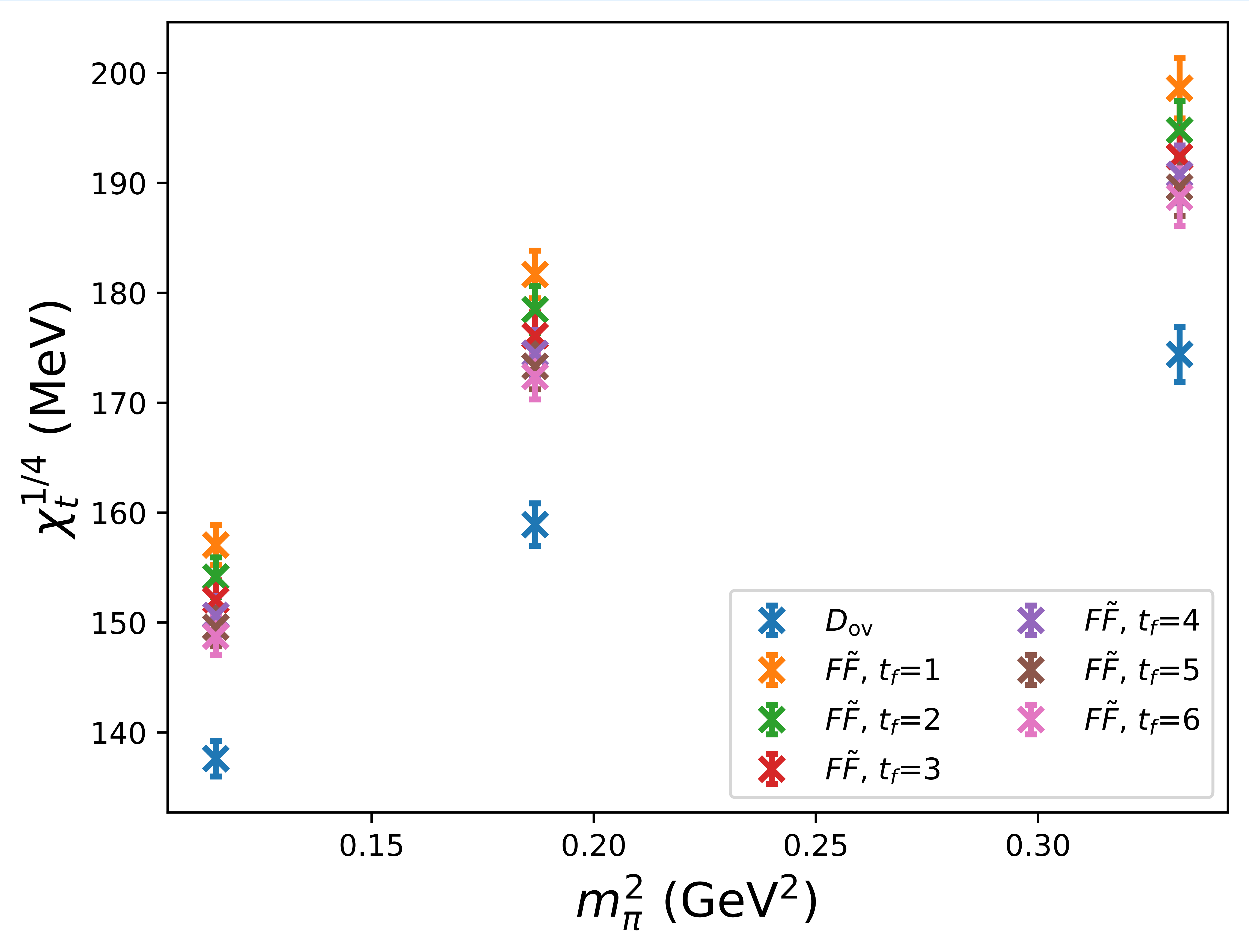




✦ The topological charges of individual configurations with different definitions (smearings) can be different, while the distributions are similar

Topological Charges on the Lattice

$$\chi_t = \frac{1}{V}\langle Q^2 \rangle$$



✦ For physical observables such as the topological susceptibility, the $D_{\rm ov}$ definition has smaller discrete effects than the $G\tilde{G}$ definition with Wilson flow

Chiral Fermions

$$\partial_\mu A_\mu^0 = \sum_{f=u,d,s} 2m_f P_f - 2iN_f q$$

For overlap fermions, the anomalous Ward identity (AMI) has been proven (with chiral axial vector current) and numerically checked at finite lattice spacings

P. Hasenfratz, et. al., NPB643:280 (2002)

***J. Liang** et. al., PRD98:074505 (2018)*

With the AWI respected, one has $d_n \to 0$ when $m_q \to 0$ even at finite lattice spacings, reliable chiral limit

D. Guadagnoli, et. al., JHEP 0304, 019 (2003)