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Perturbative pion interaction and ab initio calculation

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OUTLINE

1 Background and motivation

2 Perturbative pion interaction (PPI) scheme

3 PPI for Nuclei: From ^{3}H to ^{16}O

4 Summary and outlook

Background and motivation

1935's
Yukawa: Meson theory

1950's
The ''Pion Theories''

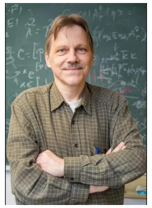
1960's
OBE Model (σ, ρ, ω): success.

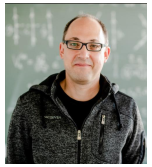
1970's
Diverse two-pion-exchange models
(Paris, Bonn, etc.)

1980's
Discover QCD: Quark Models.

1990
Weinberg: Chiral EFT

U. van Kolck: EFT Power Counting, Renormalization of OPE





Ulf-G. Meißner, E. Epelbaum, and R. Machleidt: Chiral EFT and Nuclear forces

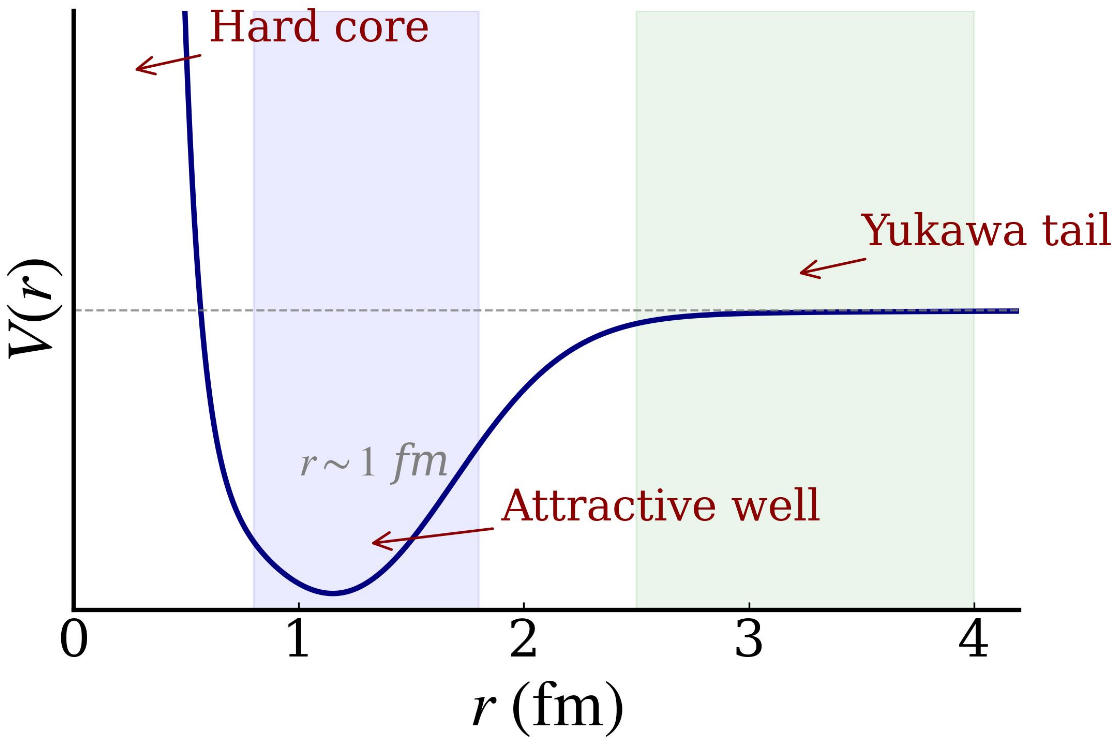
Long and Yang:
MMW power counting

our works:
optimize power counting
perturbative ab initio

Background and motivation

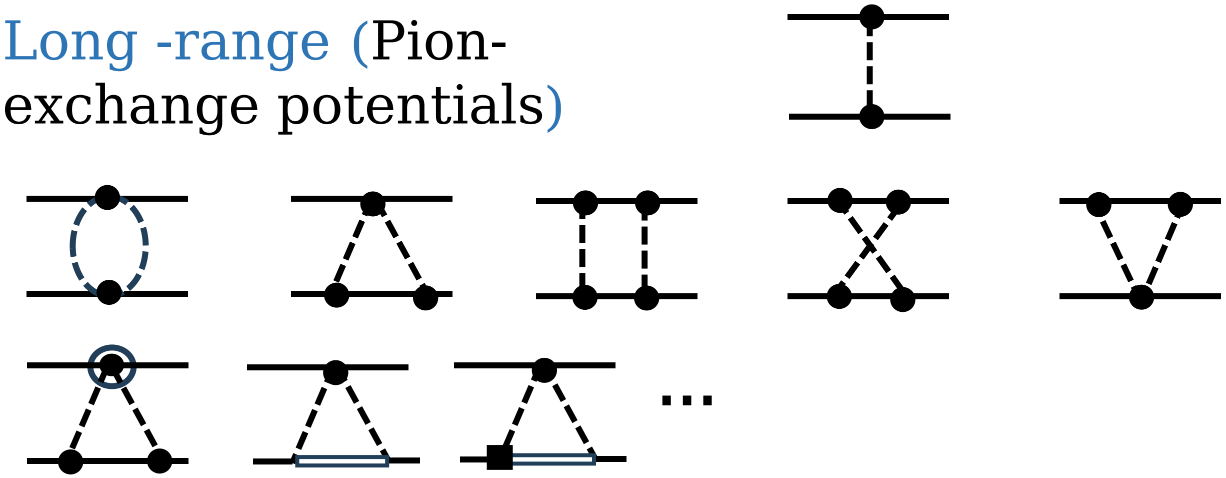
Two-body Nuclear forces (**2NF**)

Phenomenological NN interaction

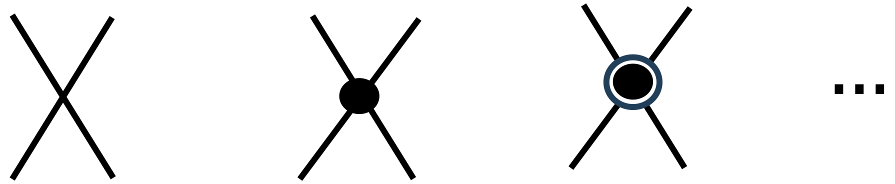


Chiral Nuclear forces

Long -range (Pion-exchange potentials)



Short-range (contact or counter term)



Background and motivation

Nuclear Physics Around the Unitarity Limit

the range of Nuclear force: $R \sim m_{\pi}^{-1} \approx 1.4$ fm

NN scattering length in S-waves: $a_s = -23.74$ fm, $a_t = 5.4$ fm

$a_{s,t} \gg R$

$$\frac{1}{a_{s,t}} < Q_A < \frac{1}{R}\,; \quad Q_A = \sqrt{2mN\frac{B_A}{A}}$$

$$\frac{1}{a_{s,t}} \rightarrow 0 \text{ (Unitarity Limit)}$$

$SU(4)_W$ Wigner symmetry

corrections. Our conjecture places nuclei in a sweet spot: bound weakly enough to be insensitive to the details of the interaction but dense enough to be insensitive to the exact values of the large two-particle scattering lengths. One

S. König, Harald W. Grießhammer, H.-W. Hammer, and U. van Kolck, PRL 118, 202501 (2017)

NN system appears close to the unitarity limit

Simplification of chiral nuclear forces

Distorted-Wave Born Approximation (DWBA)

$$T^{(0)} = V^{(0)} + V^{(0)} G T^{(0)}$$

$$T^{(1)} = (1 + T^{(0)} G) V^{(1)} (1 + G T^{(0)})$$

$$T^{(2)} = (1 + T^{(0)} G)(V^{(2)} + V^{(1)} G V^{(1)})(1 + G T^{(0)})$$

$$\vdots$$

Higher-order potentials correct the amplitude **perturbatively.**

From Symmetry to Reality

1. LO: Symmetry preserved

- Leading-order interaction retains SU(4) symmetry.

2. Subleading orders: Perturbative symmetry breaking

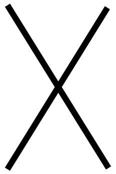
- Finite $a_{s,\mathrm{t}}$, range corrections, isospin-breaking effects.
- Observables shifted to physical values perturbatively.

Perturbative pion interaction (PPI) scheme

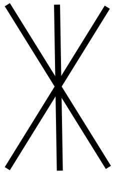
S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

1. The LO potential (same as pionless EFT)

2NF: $V_2^{(0)}(\boldsymbol{p}', \boldsymbol{p}) = \boldsymbol{C_s} + \boldsymbol{C_t}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$



3NF: $V_3^{(0)}(\boldsymbol{p}', \boldsymbol{q}'; \boldsymbol{p}, \boldsymbol{q}) = \boldsymbol{h}$



a. LO amplitude is analytically solvable

$$T^{(0)} = \frac{C_0}{1 - C_0 I(k)} \; ; I(k) = \int \frac{d^3q}{(2\pi)^3} \frac{f_\Lambda^2(q)}{k^2 - q^2 + i\epsilon}$$

b. Symmetry is preserved at LO

- LO potential is spatially isotropic, L and S are separately conserved.
- Higher-order terms are added order by order, making it easy to identify the effects of different symmetry-breaking sources.

PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

2. Tensor force in one-pion exchange (OPE)

$$V_\pi(\mathbf{r}) = \frac{\alpha_\pi}{4m_N}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\widehat{S}_{12} T(r) + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 Y(r)\right]$$

where $\alpha_\pi = g_A^2 m_N / (16\pi f_\pi^2)$, and

$$\widehat{S}_{12} \equiv 3(\boldsymbol{\sigma}_1 \cdot \hat{r})(\boldsymbol{\sigma}_2 \cdot \hat{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$T(r) \equiv \frac{e^{-m_\pi r}}{r}\left(\frac{3}{r^2} + \frac{3m_\pi}{r} + m_\pi^2\right),$$

$$Y(r) \equiv m_\pi^2 \frac{e^{-m_\pi r}}{r}.$$

In ${}^3S_1$ - ${}^3D_1$ channel, when

$$r \to m_\pi^{-1}$$

$$\frac{\langle S|\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \widehat{S}_{12} T(m_\pi^{-1})|D\rangle}{\langle S|\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 Y(m_\pi^{-1})|S\rangle} = 14\sqrt{2}$$

Tensor potential $\gg$ Yukawa potential

BUT

the mixing angle ϵ_1 is notably small in NN scattering !

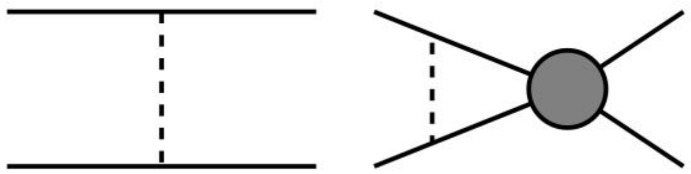
($\epsilon_1 \leq 3^\circ$ for $k \leq 300$ MeV)

PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

3. Double suppression at NLO

$T_\pi^{(1)}$:



The NLO ϵ_1:

$$\epsilon_{1,\pi} = \frac{m_N k}{4\pi} \frac{k}{(a^{-2}+k^2)^{\frac{1}{2}}} \times \left[\frac{1}{ak} \langle k, {}^3S_1 | V_\pi | k, {}^3D_1 \rangle - \frac{g_A^2}{\sqrt{2} f_\pi^2} \Pi\left(\frac{m_\pi}{k}\right) \right],$$

unitarity limits $(1/a \to 0)$

chiral limits $(m_\pi \to 0)$

where

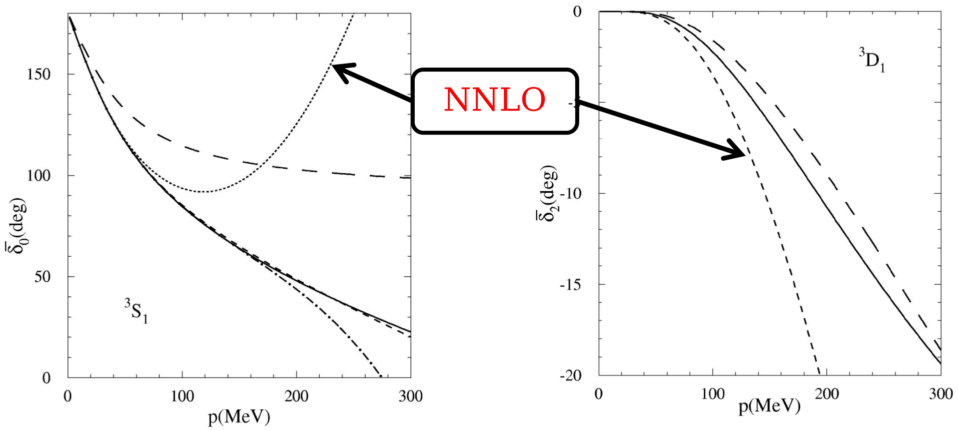
$$\Pi\left(\frac{m_\pi}{k}\right) \equiv \int_0^\infty \mathrm{d}r' r'^2 n_0(kr') T(r') j_2(kr'),$$

PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

3. N^2LO

KSW at NNLO



D.B.Kaplan, M.J.Savage, M.B.Wise, (KSW) NPB 534 329-355 (1998); PLB 424 390–396 (1998)
S.Fleming, T.Mehen, I.W.Stewart, NPA 677 313–366 (2000)

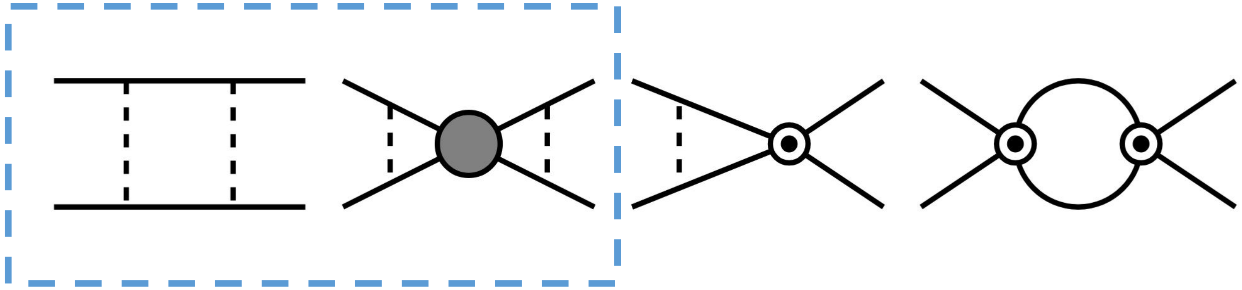
Why KSW Fails?

- Tensor force suppression is only effective near on-shell ($p' \approx p \approx k$)
- Off-shell iteration of OPE to N^2LO

PPI Solution

- Promote $\boldsymbol{C_2^{SD}}$ at NLO
- Counterterm suppresses tensor OPE in S-wave intermediate states

$T^{(2)}$:



PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

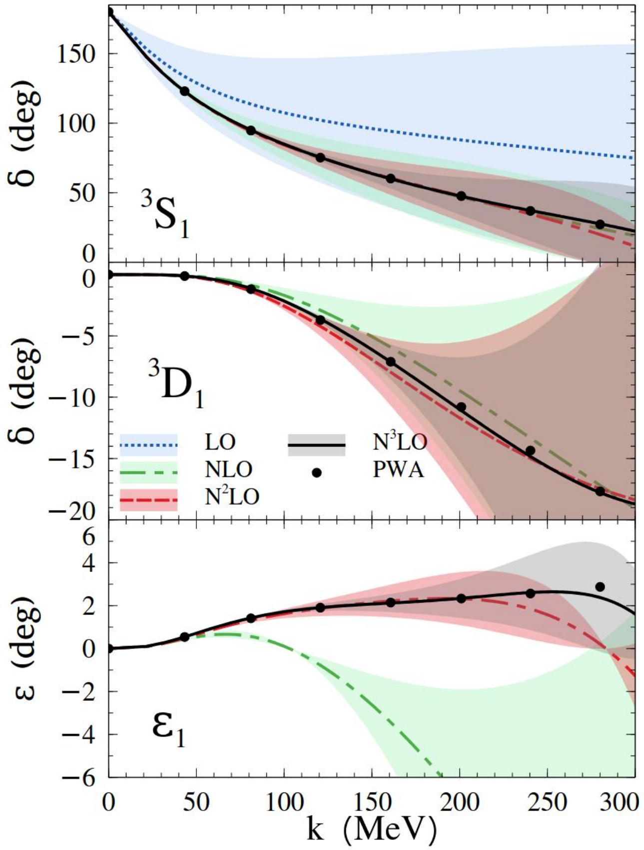
4. PPI Power Counting & NN Phase Shifts

PPI Power Counting

	LO	NLO	N^2LO	N^3LO
π		OPE		TPE
$^3S_1$ $^1S_0$	C_0	C_2	C_4	C_6
SD		C_2	C_4	C_6
$^3D_1$				C_4
$^3P_0$		C_2	C_4	C_6
$^3P_{1,2}$ $^1P_1$				C_2

TABLE I. The PPI power counting of NN forces. Rows labeled by a partial wave describes contact interactions acting in that partial wave. Pion exchanges are in the row led by "π".

NN Phase Shifts upto N^3LO



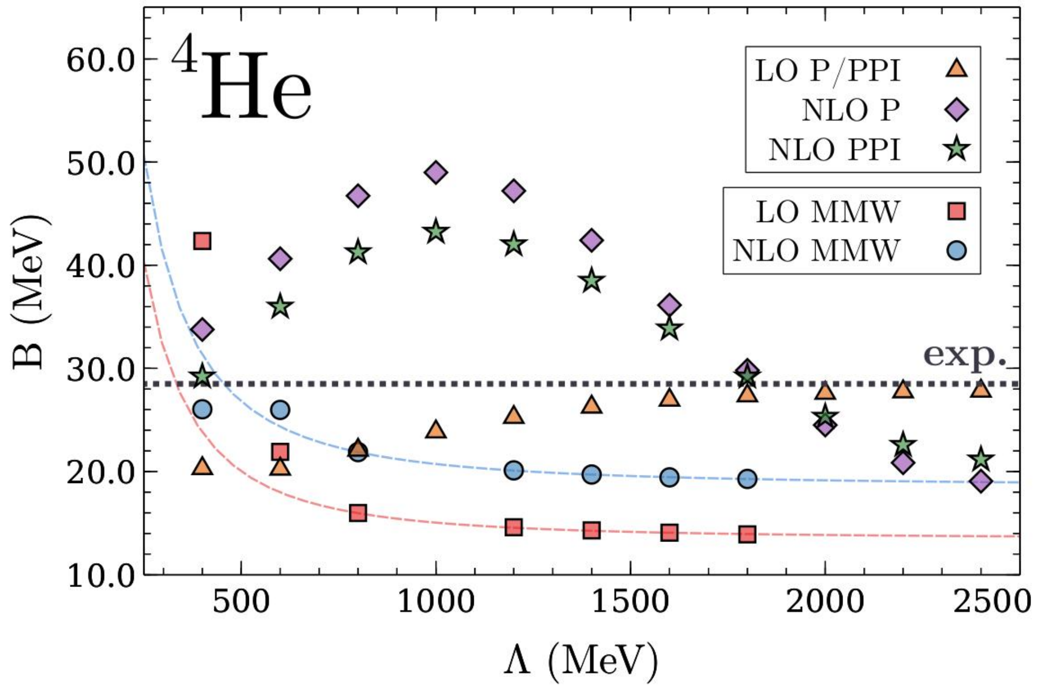
- Discrepancy $\lesssim 1°$ up to k $\lesssim$ 300 MeV
- Breakdown scale $\alpha_\pi^{-1} \simeq 270$ MeV

PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

4. Results of ^{3}H and ^{4}He upto NLO

cutoff dependence of ^{4}He Binding energy



3NF at LO

- Fixed by ^{3}H binding energy

4NF at NLO

- Large cutoff variations in both Pionless EFT and PPI

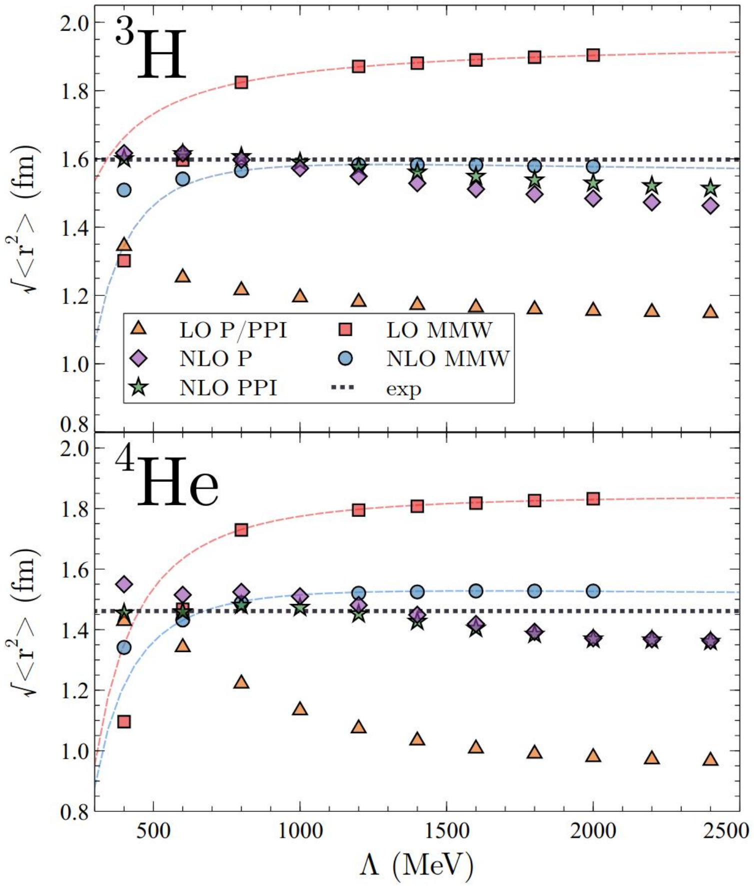
- → A contact 4NF is required at NLO

PPI scheme

S. Lyu, L. Zuo, R. Peng, S. König, B. Long, arXiv:2511.12522 [nucl-th]

4. Results of ^{3}H and ^{4}He upto NLO

point charge radii of ^{3}H and ^{4}He



4NF at NLO added

- Fixed by ^{4}He binding energy

NLO radius predictions:

$$r_{^3\mathrm{H}} = 1.51 \pm 0.11 \text{ fm}$$

$$r_{^4\mathrm{He}} = 1.36 \pm 0.25 \text{ fm}$$

EFT uncertainty at NLO

$$\Delta_r = r \times (Q_A \alpha_\pi)^2$$

$Q_3\alpha_\pi \simeq 0.27$; $Q_4\alpha_\pi \simeq 0.43$

PPI for Nuclei: From ^3H to ^{16}O （preliminary）

1. limit cycle of 3NF

$$H(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_\star) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_\star) + \arctan(1/s_0)]}$$

Difficulties in many-body calculations:

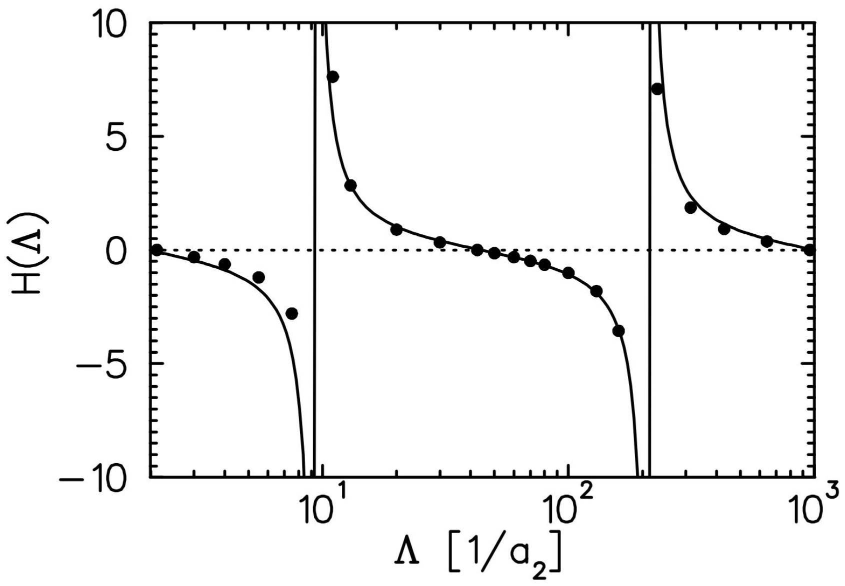


FIG. 3. Three-body force as a function of the cutoff Λ: numerical solution (dots) and Eq. (12).

• Large cutoff: convergence becomes prohibitively slow

• Few-body forces: complex structure, hard to implement

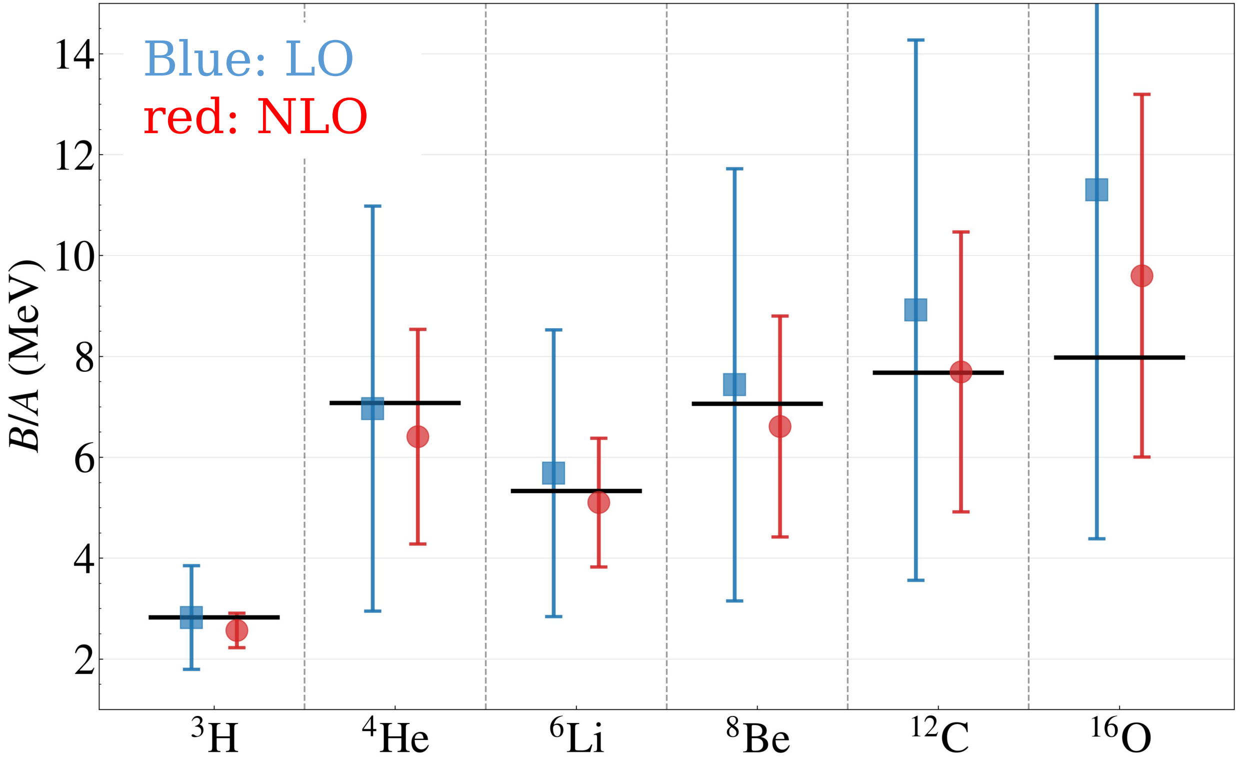
Λ : 3NF vanishes

P. F. Bedaque, H.-W. Hammer,and U. van Kolck, PRL. 82, 463 (1999).

PPI for Nuclei: From ^{3}H to ^{16}O (preliminary)

2. The binding energy

Binding energy per nucleon at
Λ = 204 MeV from NCSM



EFT uncertainty

$$\Delta^{B/A}_{\mathrm{EFT}} = (B^{(0)}/A) \times (Q_A/M_{\mathrm{hi}})^i$$

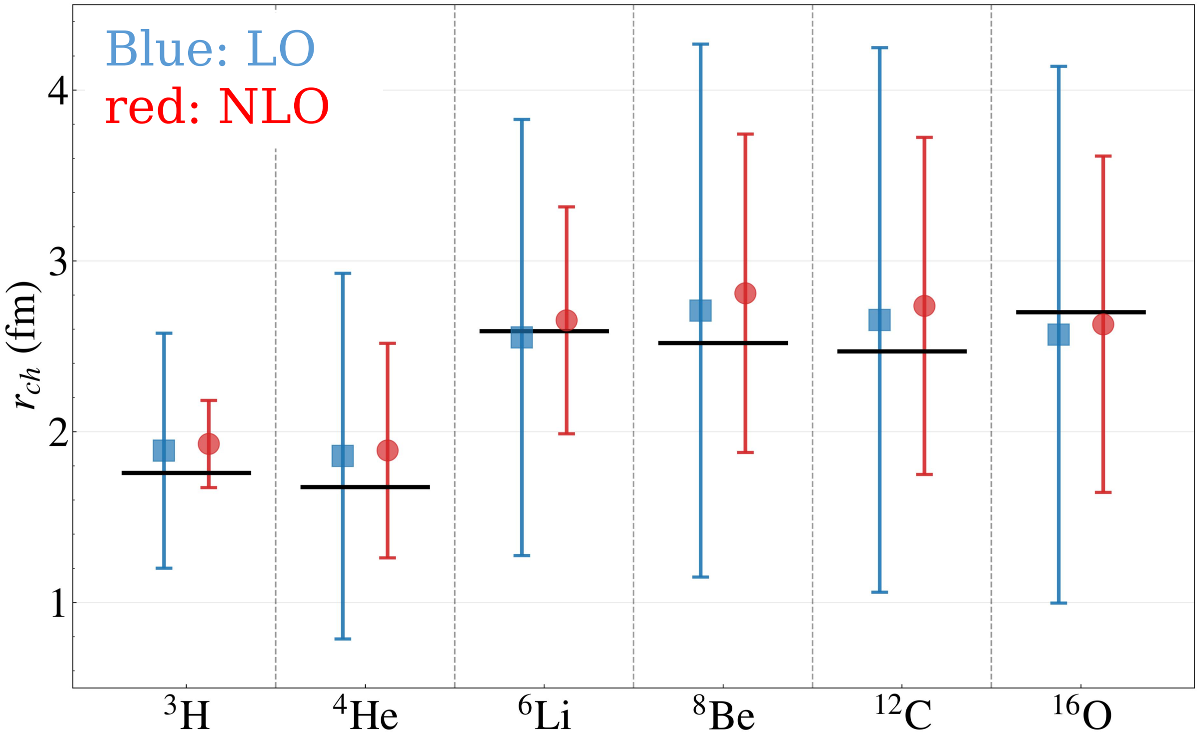
$$\alpha_\pi^{-1} \rightarrow M_{\mathrm{hi}} = 200\,\mathrm{MeV}$$

- LO: ^{3}H reproduced
- NLO corrections → all nuclei less bound
- NLO lacks: 4NF & 3NF corrections

PPI for Nuclei: From ^{3}H to ^{16}O (preliminary)

2. The charge radius

charge radius from NCSM



NLO corrections:

- Radii → larger at NLO
- Both B and r_{ch} within EFT error estimates

Summary and outlook

Summary :

- PPI: A chiral force scheme with OPE treated perturbatively and Approximate SU(4) symmetry preserved at LO
- Tested: NN scattering + ^{3}H,^{4}He → predictive power confirmed
- Extended to heavier nuclei: ^{3}H to ^{16}O (NCSM, up to NLO) → remarkable performance

Outlook:

- Include few-body forces systematically
- Push to heavier nuclei
- Include more symmetry-breaking effects (e.g., Coulomb)

Thanks for your attention !