



南京大學
NANJING UNIVERSITY



NNStar: **An end-to-end AI agent** for nuclear matter and neutron star physics

Y. Ma, Y.-L. Ma, and J.-Y. Xiong, [arXiv:2607.13930 \[nucl-th\] \(2026\)](#).

Presenter: Yao Ma (马垚)

In collaboration with Jia-Ying Xiong (熊佳颖) and Prof. Yong-Liang Ma (马永亮)

Lanzhou University, Lanzhou, Aug. 13th



嚴謹求實
勤奮創新

Motivation

- I. Massive pulsars require **a stiff EoS**, while GW170817 and NICER favor softer matter at intermediate densities;
[P. Demorest et al., Nature 467, 1081 \(2010\).](#)
[B. P. Abbott et al., Phys. Rev. Lett. 121, 161101 \(2018\).](#)
[D. Choudhury et al., Astrophys. J. Lett. 971, L20 \(2024\).](#)
- II. A realistic EoS may need **rapid density dependence**, often seen as a sound-speed peak at a few times saturation density;
[P. Bedaque and A. W. Steiner, Phys. Rev. Lett. 114, 031103 \(2015\).](#)
[I. Tews et al., Astrophys. J. 860, 149 \(2018\).](#)
[E. Annala et al., Nature Phys. 16, 907 \(2020\).](#)
- III. Hadronic models are physically transparent but contain many nonlinear couplings that are **hard to tune globally**;
[J. D. Walecka, Annals Phys. 83, 491 \(1974\).](#)
[B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 \(1986\).](#)
[M. Dutra et al., Phys. Rev. C 90, 055203 \(2014\).](#)
[L.-J. Guo et al., Astrophys. J. 965, 47 \(2024\).](#)
[Y. Fujimoto et al., Phys. Rev. Lett. 129, 252702 \(2022\).](#)
[M. Omana Kuttan et al., Phys. Rev. Lett. 131, 202303 \(2023\).](#)
- IV. ML methods can **accelerate** EoS inference from nuclear and astrophysical data;
- V. LLM agents become reliable when their reasoning is grounded in **verifiable tools**.
[S. Yao et al., arXiv:2210.03629 \(2022\).](#)
[T. Schick et al., arXiv:2302.04761 \(2023\).](#)
[A. M. Bran et al., Nat. Mach. Intell. 6, 525 \(2024\).](#)

Nuclei structures (**low/intermediate densities**)



- Hadron interactions around saturation density $n_0 = 0.16\text{fm}^{-3}$ are crucial to nuclei structures, e.g. ^{24}Mg , ^{90}Zr , ^{116}Sn and ^{208}Pb

$$E(\mathbf{n}, \alpha) = E_0(n) + E_{\text{sym}}(n)\alpha^2 + O(\alpha^4)$$

$$\alpha = (n_n - n_p) / (n_n + n_p)$$

$$E_0(n) = \boxed{E_0(n_0)} + \frac{\boxed{K_0}}{2!}\chi^2 + \frac{\boxed{J_0}}{3!}\chi^3 + O(\chi^4)$$

$$E_{\text{sym}}(n) = \boxed{E_{\text{sym}}(n_r)} + \boxed{L(n_r)}\chi_r + O(\chi_r^2)$$

$$\chi \equiv (n - n_0) / 3n_0$$

$$\mathcal{L} = \bar{\psi} \left[i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho^{a\mu} \right] \psi$$

B. A. Li, C. M. Ko, Nucl. Phys. A 618, 498 (1997).
D. H. Youngblood, H. L. Clark, Y. W. Lui, Phys. Rev. Lett. 82, 691 (1999).
B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000).
S. Karataglidis, K. Amos, B. A. Brown, Phys. Rev. C 65, 044306 (2002).
R. J. Furnstahl, Nucl. Phys. A 706, 85 (2002).
A. E. L. Dieperink, Y. Dewulf, D. Van Neck, Phys. Rev. C 68, 064307 (2003).
J. Piekarewicz, Phys. Rev. C 69, 041301 (2004).
G. Colò, N. Van Giai, J. Meyer, Phys. Rev. C 70, 024307 (2004).
A. W. Steiner, B. A. Li, Phys. Rev. C 72, 041601 (2005).
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E. Khan, J. Margueron, Phys. Rev. Lett. 109, 092501 (2012).
M. Dutra, O. Lourenço, J. S. Sa Martins, Phys. Rev. C 85, 035201 (2012).
M. Dutra, O. Lourenço, S. S. Avancini, Phys. Rev. C 90, 055203 (2014).
M. Oertel, M. Hempel, T. Klähn, Rev. Mod. Phys. 89, 015007 (2017).
I. Tews, J. M. Lattimer, A. Ohnishi, Astrophys. J. 848, 105 (2017).
Y. Wang, C. Guo, Q. Li, Phys. Lett. B 778, 207 (2018).
W.-J. Xie, B.-A. Li, Astrophys. J. 883, 174 (2019).
S. Choi, T. Miyatsu, M.-K. Cheoun, Astrophys. J. 909, 156 (2021).
G. Grams, R. Somasundaram, J. Margueron, Phys. Rev. C 106, 044305 (2022).
A. Sedrakian, J.-J. Li, F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023).

$$n = n_n + n_p$$

$$\begin{aligned} n_0 &\rightarrow 0.155 \pm 0.050 \text{ (fm}^{-3}\text{)} \\ E_0(n_0) &\rightarrow -16.0 \pm 1.0 \text{ (MeV)} \\ E_{\text{sym}}(n_0) &\rightarrow 31.7 \pm 3.2 \text{ (MeV)} \\ K_0 &\rightarrow 250 \pm 50 \text{ (MeV)} \\ L_0 &\rightarrow 58.7 \pm 28.1 \text{ (MeV)} \\ J_0 &\rightarrow -200 \pm 600 \text{ (MeV)} \end{aligned}$$

Neutron star (high densities)

The density in the cores of NSs always reach nearly $8n_0$
M-R relations/ tidal deformations are sensitive to the EOS behavior
throughout the whole density regions

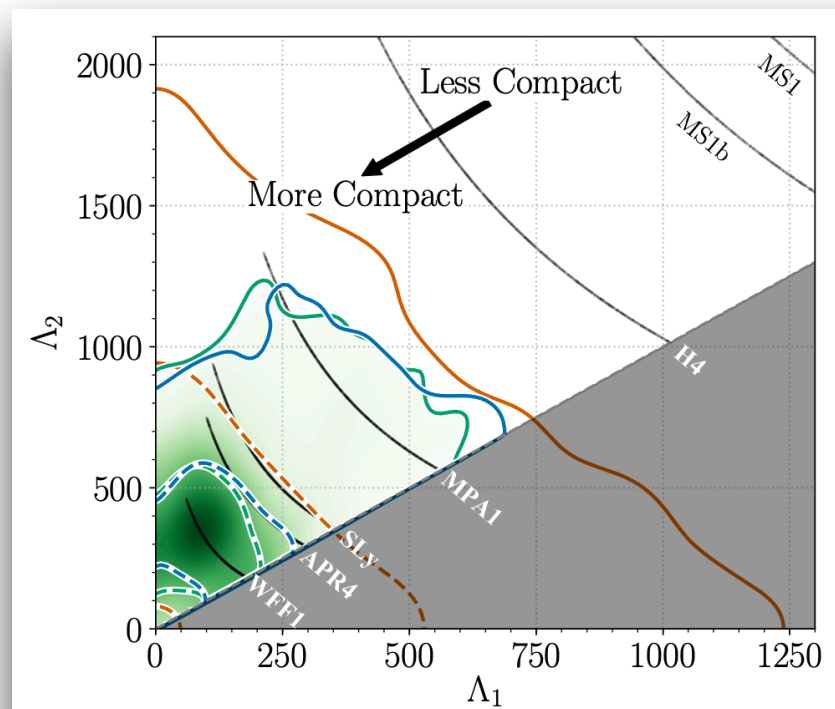


FIG. 1. Marginalized posterior for the tidal deformabilities of the two binary components of GW170817. The green shading shows the posterior obtained using the $\Lambda_a(\Lambda_s, q)$ EOS-insensitive relation to impose a common EOS for the two bodies, while the green, blue, and orange lines denote 50% (dashed) and 90% (solid) credible levels for the posteriors obtained using EOS-insensitive relations, a parametrized EOS without a maximum mass requirement, and independent EOSs (taken from [52]), respectively. The gray shading corresponds to the unphysical region $\Lambda_2 < \Lambda_1$ while the seven black scatter regions give the tidal parameters predicted by characteristic EOS models for this event [113, 115, 121–125].

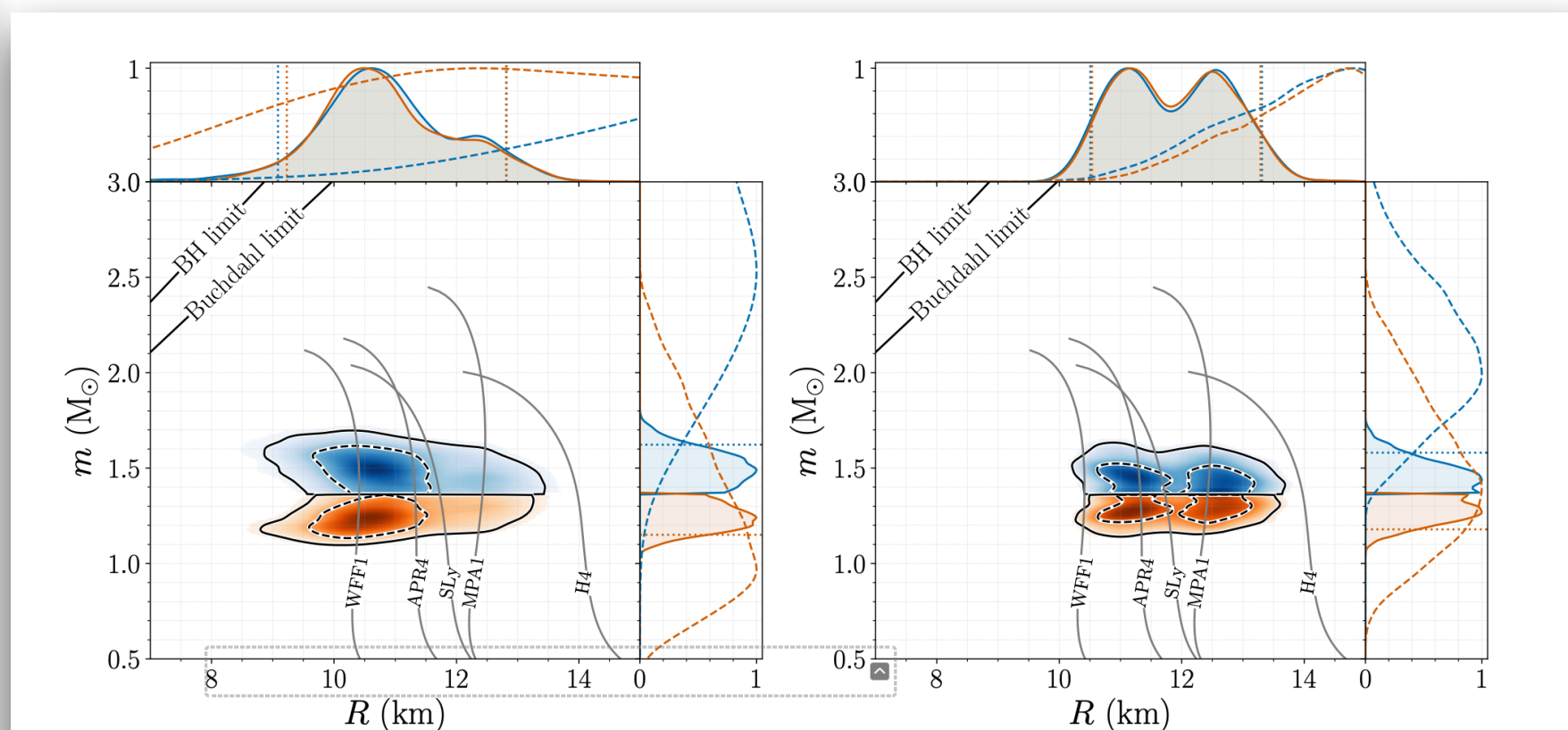


FIG. 3. Marginalized posterior for the mass m and areal radius R of each binary component using EOS-insensitive relations (left panel) and a parametrized EOS where we impose a lower limit on the maximum mass of $1.97 M_\odot$ (right panel). The top blue (bottom orange) posterior corresponds to the heavier (lighter) NS. Example mass-radius curves for selected EOSs are overplotted in gray. The lines in the top left denote the Schwarzschild BH ($R = 2m$) and Buchdahl ($R = 9m/4$) limits. In the one-dimensional plots, solid lines are used for the posteriors, while dashed lines are used for the corresponding parameter priors. Dotted vertical lines are used for the bounds of the 90% credible intervals.

TOV equation

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($ \chi \leq 0.05$)	High-spin priors ($ \chi \leq 0.89$)
Primary mass m_1	$1.36\text{--}1.60 M_\odot$	$1.36\text{--}2.26 M_\odot$
Secondary mass m_2	$1.17\text{--}1.36 M_\odot$	$0.86\text{--}1.36 M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	$0.7\text{--}1.0$	$0.4\text{--}1.0$
Total mass m_{tot}	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy E_{rad}	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance D_L	$40^{+8}_{-14} \text{ Mpc}$	$40^{+8}_{-14} \text{ Mpc}$
Viewing angle Θ	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 119, 161101 (2017)
B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018)

Polytropic process + estimations of strong interaction + TOV equation

Mean-field approach



1. Relativistic mean field approximation

J. D. Walecka, Ann. Phys. 83, 491 (1974).

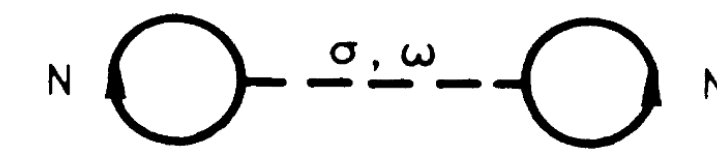
$$\begin{aligned}\mathcal{L} &= \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_I \\ \mathcal{L}_N &= \bar{\psi} \left(i\gamma_\mu \partial^\mu - m_N \right) \psi \\ \mathcal{L}_\sigma &= \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - m_\sigma^2 \phi^2 \right) \\ \mathcal{L}_\omega &= -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ \mathcal{L}_I &= \mathcal{L}_{\sigma N} + \mathcal{L}_{\omega N} = g_\sigma \phi \bar{\psi} \psi - g_\omega \omega^\mu \bar{\psi} \gamma_\mu \psi\end{aligned}$$

$$\begin{aligned}\left(i\gamma_\mu \partial^\mu - g_\omega \gamma_0 \omega^0 - m_N \right) \psi &= 0, \quad m_N^* = m_N - g_\sigma \phi, \\ \phi &= \frac{g_\sigma}{m_\sigma^2} \rho_s, \quad \rho_s = \langle \bar{\psi} \psi \rangle, \\ \omega^0 &= \frac{g_\omega}{m_\omega^2} \rho_B, \quad \rho_B = \langle \psi^\dagger \psi \rangle,\end{aligned}$$

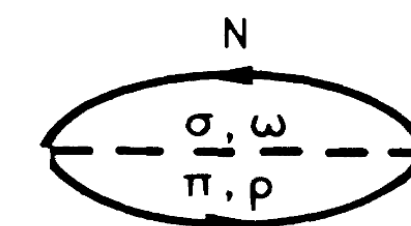
2. Relativistic Hartree-Fock method

$$\begin{aligned}\mathcal{L}_I &= -g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi + \frac{f_\omega}{2M} \bar{\psi} \sigma_{\mu\nu} \partial^\nu \omega^\mu \psi \\ &\quad - g_\rho \bar{\psi} \gamma_\mu \rho^\mu \cdot \tau \psi + \frac{f_\rho}{2M} \bar{\psi} \sigma_{\mu\nu} \partial^\nu \rho^\mu \cdot \tau \psi \\ &\quad - e \bar{\psi} \gamma_\mu \frac{1}{2} (1 + \tau_3) A^\mu \psi + \mathcal{L}_{\pi NN}\end{aligned}$$

$$\Sigma(\mathbf{p}) = \Sigma_S(p) + \gamma_0 \Sigma_0(p) + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}} \Sigma_V(p)$$



(a)



(b)

A. Bouyssy, J.-F. Mathiot, N. V. Giai, and S. Marcos, Phys Rev C 36, 380 (1987).

Static NS: TOV equation

R. C. Tolman, Phys. Rev. 55, 364 (1939).

J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).



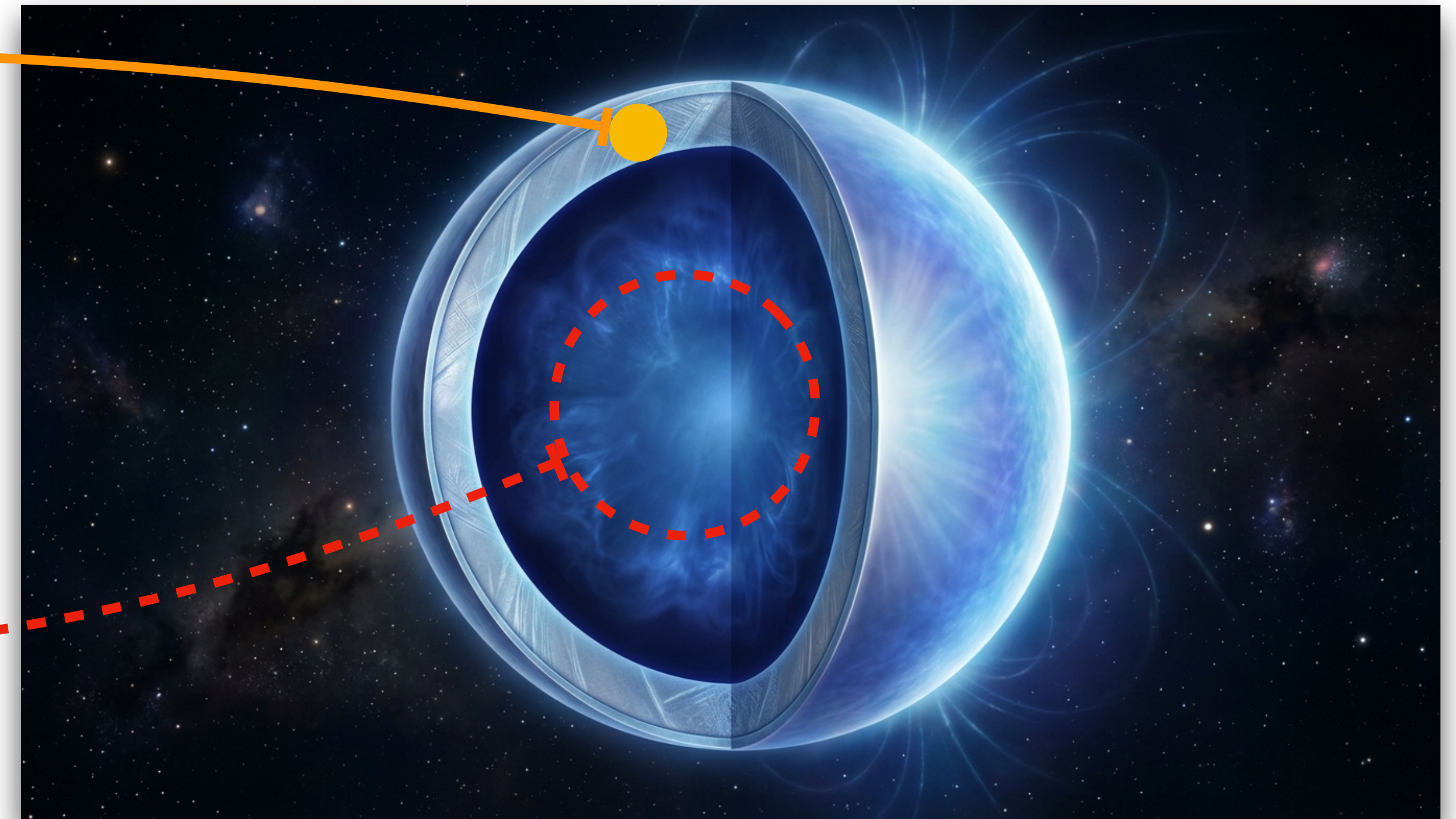
$$\frac{dP}{dr} = -\frac{G\mathcal{M}(r)\varepsilon}{r^2} \left(1 + \frac{P}{\varepsilon}\right) \left(1 + \frac{4\pi r^3 P}{\mathcal{M}(r)}\right) \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}$$

BPS crust

G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170, 299 (1971).

W. D. Arnett and R. L. Bowers, Astrophys. J. Suppl. 33, 415 (1977).

Pure hadronic matter inner core





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Can we extend an EFT/model taking care of QCD symmetry patterns into dense nucleon systems?

An extended linear sigma model in nuclear matter

Phys. Rev. D 109 (2024) 7, 7, YM and Y. L. Ma

Phys. Rev. D 112 (2025) 5, 054026, YM and Y. L. Ma

P-wave problems in light scalar meson sectors below 1 GeV



F. E. Close and N. A. Tornqvist, J. Phys. G 28, R249 (2002)

A tetra-quark picture to include δ ($a_0(980)$) meson and hyperon



Freedoms to be considered

Highlights of the parametrization:

- I. Include **meson exchanges**, e.g. $f_0(500)(\sigma)$ and $a_0(980)(\delta)$
- II. Include **baryon freedoms**, e.g. nucleon and hyperon

$$\begin{aligned}\sigma &= \cos \theta_0 \sigma' + \sin \theta_0 \hat{\sigma}' \\ a_0 &= \cos \theta_8 a'_0 + \sin \theta_8 \hat{a}'_0 \\ f_0 &= \cos \theta_8 f'_0 + \sin \theta_8 \hat{f}'_0\end{aligned}$$

$$\begin{aligned}R^\mu &= V^\mu - A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} - a_1^{\mu +} & K^{*\mu +} - K_1^{\mu +} \\ \rho^{\mu -} - a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} - K_1^{\mu 0} \\ K^{*\mu -} - K_1^{\mu -} & \bar{K}^{*\mu 0} - \bar{K}_1^{\mu 0} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix} \\ L^\mu &= V^\mu + A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} + a_1^{\mu +} & K^{*\mu +} + K_1^{\mu +} \\ \rho^{\mu -} + a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} + K_1^{\mu 0} \\ K^{*\mu -} + K_1^{\mu -} & \bar{K}^{*\mu 0} + \bar{K}_1^{\mu 0} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}\end{aligned}$$

$$\Phi = S + iP = \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

a_0 mesons may be crucial to NS tidal deformations and neutron skin of nucleus

$$B_N \equiv \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

High densities may lead to hyperon cores of NSs

D. Adhikari et al. (PREX), Phys. Rev. Lett. 126, 172502 (2021)
 B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)
 F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022)

N. K. Glendenning, Astrophys. J. 293, 470 (1985)
 N. K. Glendenning and S. A. Moszkowski, Phys. Rev. Lett. 67, 2414 (1991).
 S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Phys. Rev. C 85, 065802 (2012), [Erratum: Phys. Rev. C 90, 019904 (2014)]

Hadron masses and NM properties

The units are in MeV, except n_0 is in fm^{-3}

Y. Sugahara and H. Toki, Nucl. Phys. A579,557 (1994).
F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022).

	$g_{\sigma NN}$	$g_{a_0 NN}$	$g_{f NN}$	$g_{\omega NN}$	$g_{\rho NN}$
TM1	-10.0	—	—	-12.6	-4.63
FSU- δ 6.7	10.2	6.7	—	-13.4	-7.27
el-g30g	-6.17	5.13	2.95	-6.09	5.30
el-g30e	-6.20	-5.03	3.00	6.09	5.30
el-g30eg	-5.97	-0.671	2.68	6.06	3.45
el-g350eg	-6.12	-0.852	2.85	6.37	3.71
el-g3100eg	-6.36	-0.442	3.19	6.73	3.85
el-g3150eg	-6.38	-0.413	3.20	7.09	4.04

Increasing the
four-vector
couplings

	n_0	E_0	$E_{\text{sym}}(n_0)$	$J(n_0)$	$L(n_0)$	$K(n_0)$
Empirical	0.155 ± 0.050	-16.0 ± 1.0	31.7 ± 3.2	-200 ± 600	58.7 ± 28.1	250 ± 50
el-g30g	0.155	-14.6	30.9	451	83.6	418
el-g30e	0.155	-14.6	31.6	479	85.8	419
el-g30eg	0.155	-14.6	30.1	421	82.2	415
el-g350eg	0.155	-15.2	30.9	-392	80.7	370
el-g3100eg	0.155	-15.4	31.4	-1020	71.7	317
el-g3150eg	0.155	-15.6	31.6	-1470	63.7	253

	m_N	m_σ	$m_{f_0(a_0)}$	$m_{f'_0(a'_0)}$	$m_{\sigma'}$	m_ρ	m_ρ
Empirical	938-940	400-800	960-1010	1250-1500	1430-1530	761-765	782-783
el-g30g	939	522	989	1480	1510	763	783
el-g30e	939	525	991	1480	1510	763	783
el-g30eg	939	498	983	1500	1520	763	783
el-g350eg	939	485	991	1470	1510	763	783
el-g3100eg	939	502	994	1470	1510	763	783
el-g3150eg	939	485	991	1470	1510	763	783

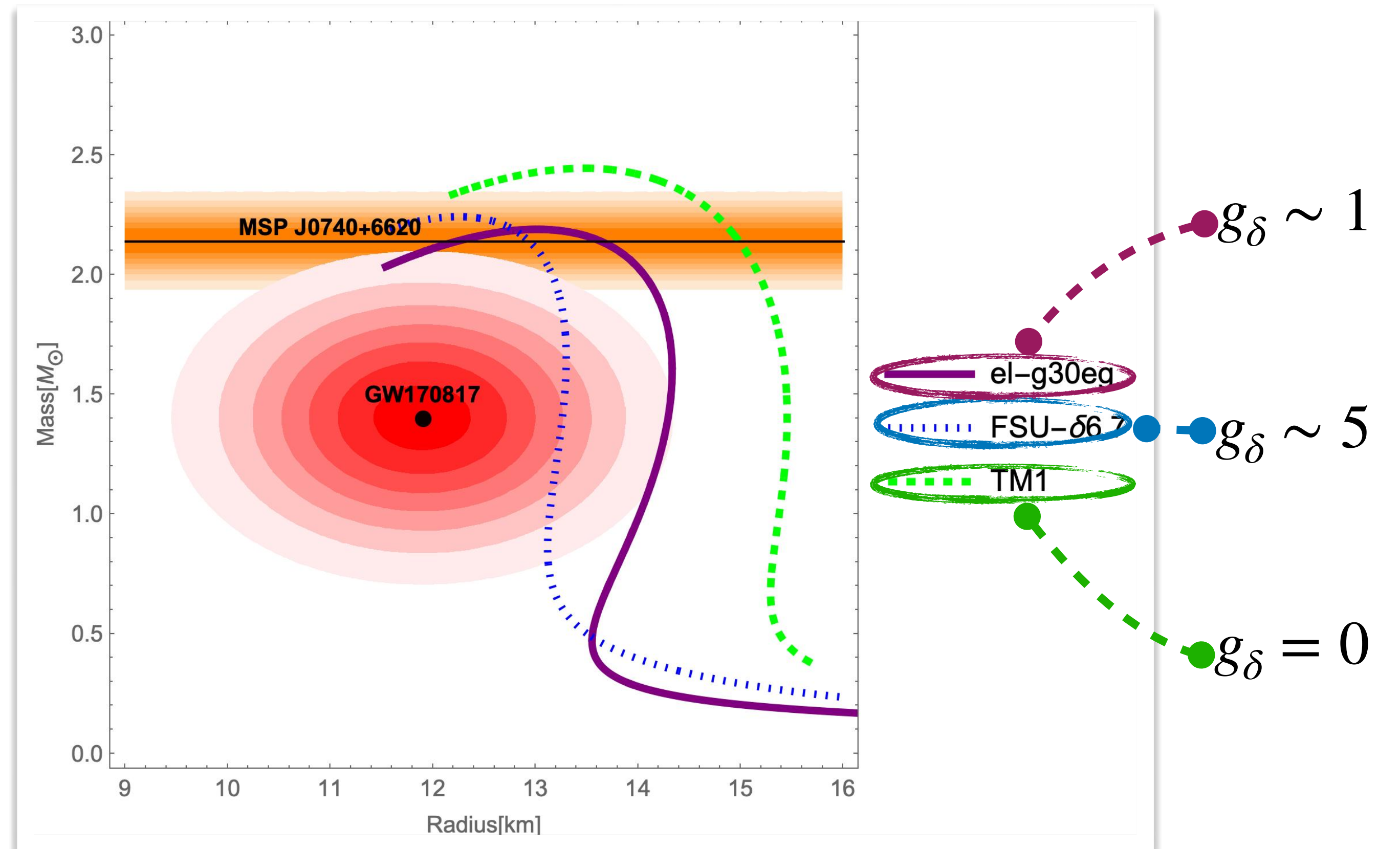
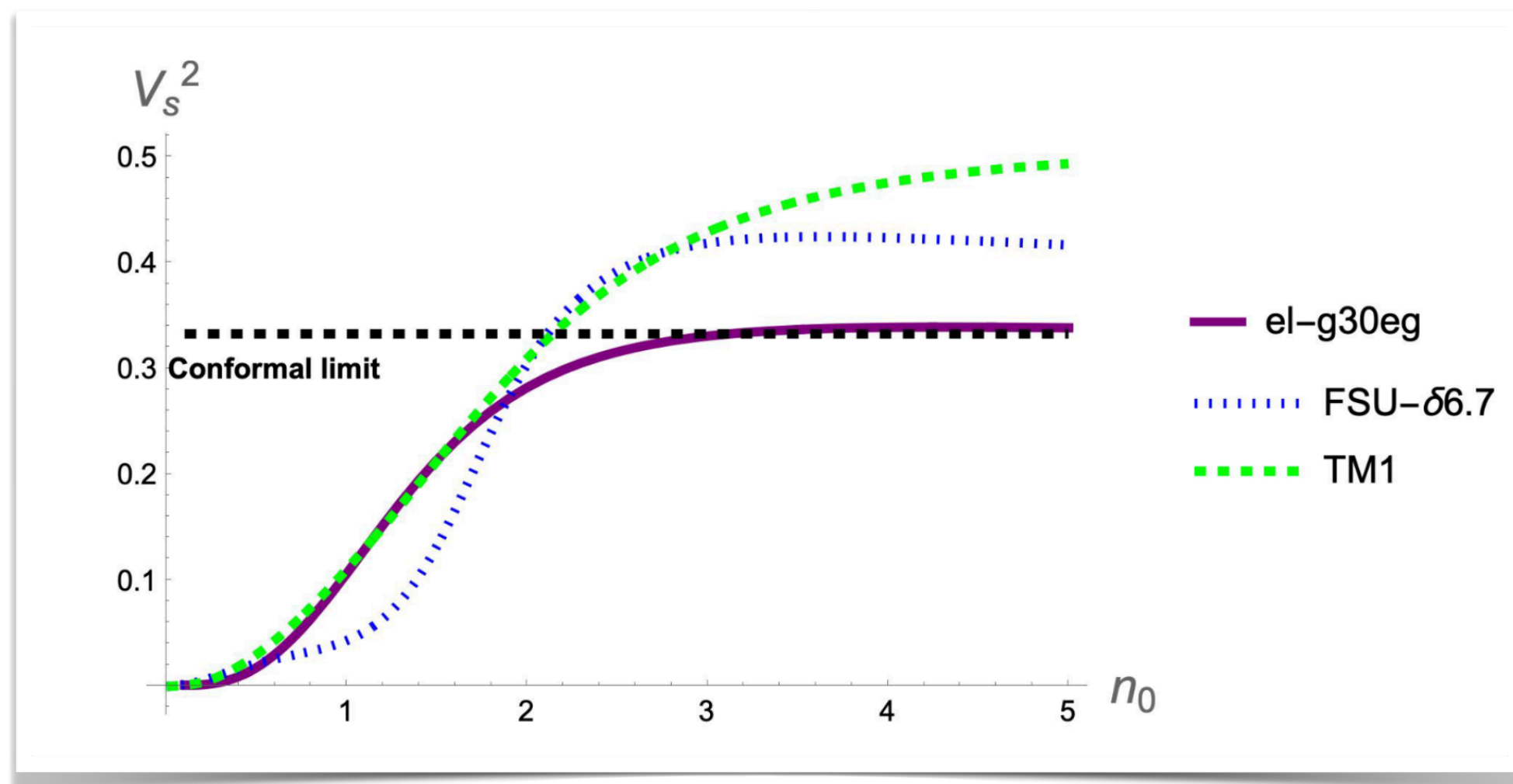
A. Sedrakian, J. J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023)
J. Piekarewicz, Phys. Rev. C 69, 041301 (2004).
M.I. Tews, J. M. Lattimer, A. Ohnishi, and E. E. Kolomeitsev, Astrophys. J. 848, 105 (2017).
M. Dutra, et al., Phys. Rev. C 85, 035201 (2012).
R. L. Workman, et al. (Particle Data Group), PTEP 2022, 083C01 (2022).

Sound speed and M-R relations

σ is considered as the mixing between 4-quark and 2-quark configurations

Phys. Rev. D 109 (2024) 7, 7, YM and Y. L. Ma.
Phys. Rev. D 112 (2025) 5, 054026, YM and Y. L. Ma.

Pure neutron matter



Quite different parameter space from EFT view



MeV

	$C_{\sigma\delta^2}$	$C_{\sigma^2\delta^2}$
Zabari-19	± 1.77	± 0.004
FSU- $\delta 6.7$	—	2.63
el-g30g	-1860	-9.40
el-g30e	-1940	-9.71
el-g30eg	-1480	-7.70
el-g350eg	-1690	-8.75
el-g3100eg	-2190	-11.0
el-g3150eg	-2160	-11.0

	σ		δ	
	p_4	p_2	p_4	p_2
el-g30g	51.7%	48.3%	77.9%	22.1%
el-g30e	51.2%	48.7%	77.2%	22.8%
el-g30eg	54.2%	45.8%	81.5%	18.5%
el-g350eg	52.4%	47.6%	77.3%	20.7%
el-g3100eg	49.2%	50.8%	74.4%	25.6%
el-g3150eg	48.9%	51.1%	74.3%	25.7%

Quite different parameter space!

Slight difference of configurations
but large difference at macroscopic level

N. Zabari, S. Kubis, and W. Wójcik, Phys. Rev. C 99, 035209 (2019)
 F. Li, B.-J. Cai, Y. Zhou, W.-Z. Jiang, and L.-W. Chen, Astrophys. J. 929, 183 (2022).



誠樸雄偉 勵學敦行

A chiral-scale effective field theory

L.-Q. Zhang, YM, and Y.-L. Ma, Phys. Rev. C 112, L031001 (2025).

L.-Q. Zhang, YM, and Y.-L. Ma, Chin. Phys. C 50, 024108 (2026).

A low energy NG boson set: π , κ , σ for SU(3), with HLS: ρ , ω

S. Weinberg, Physica A 96, 327 (1979).

S. Weinberg, Nucl. Phys. B 363, 3 (1991).

R. J. Crewther and L. C. Tunstall, Phys. Rev. D 91, 034016 (2015).

O. Catà, et.al., Phys. Rev. D 100, 095007 (2019).

M. Bando, et.al., Phys. Rev. Lett. 54, 1215 (1985).

$$\theta_\mu^\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu}^a G^{a\mu\nu} + \left(1 + \gamma_m(\alpha_s)\right) \sum_{q=u,d,s} m_q \bar{q}q$$

Scale symmetry considerations

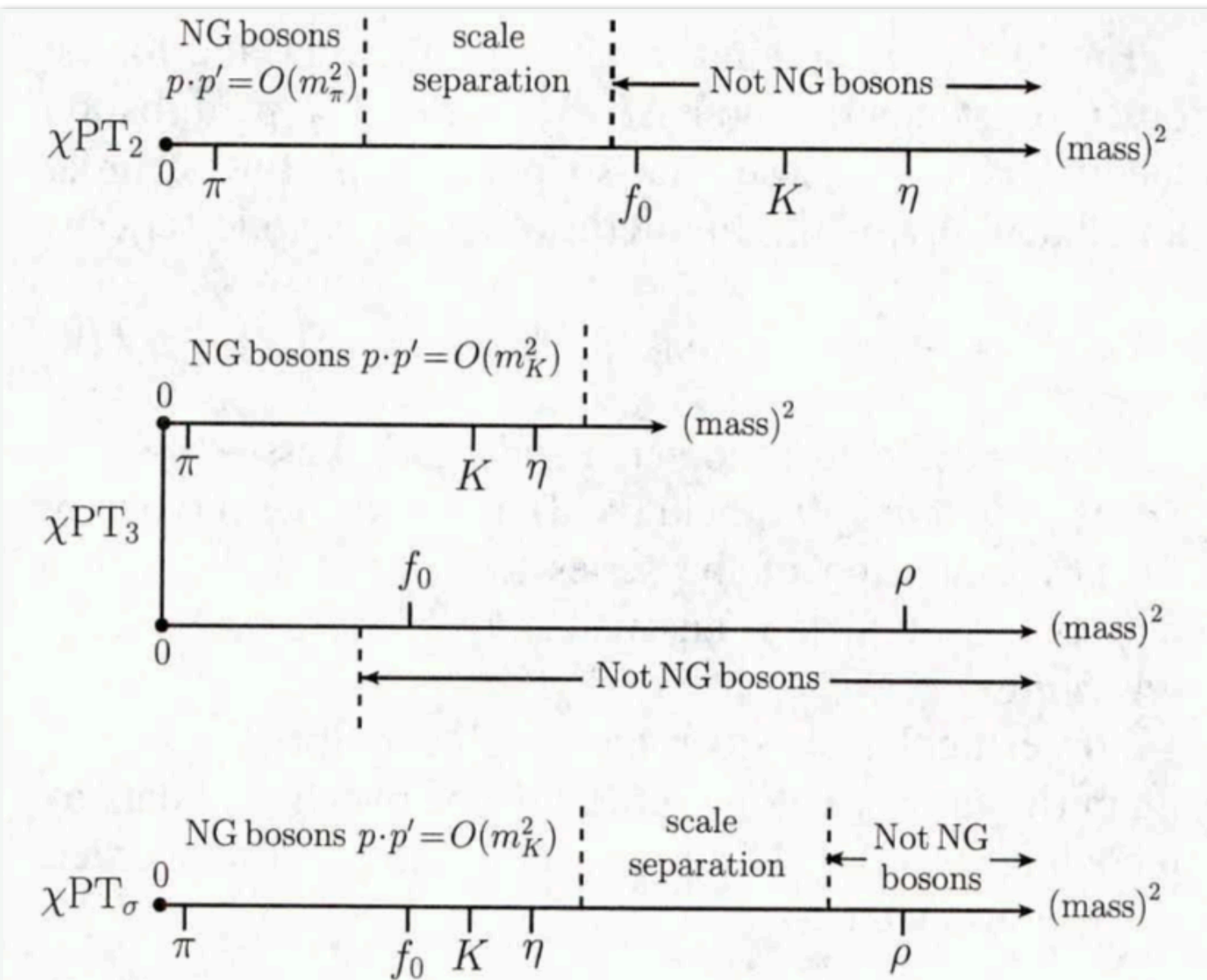


FIG. 4. Scale separations between NG sectors and other hadrons for each type of chiral perturbation theory χ PT discussed in this paper. Note that scale separation in χ PT₂ [chiral $SU(2)_L \times SU(2)_R$, top diagram] is ensured by limiting extrapolations in momenta p, p' to $O(m_\pi)$ [not $O(m_K)$]. In conventional three-flavor theory χ PT₃ (middle diagram), there is no scale separation: the non-NG boson $f_0(500)$ sits in the middle of the NG sector $\{\pi, K, \eta\}$. Our three-flavor proposal χ PT _{σ} (bottom diagram) for $O(m_K)$ extrapolations in momenta implies a clear scale separation between the NG sector $\{\pi, K, \eta, \sigma = f_0\}$ and the non-NG sector $\{\rho, \omega, K^*, N, \eta', \dots\}$.

- Possible infrared fixed point of QCD at low energies

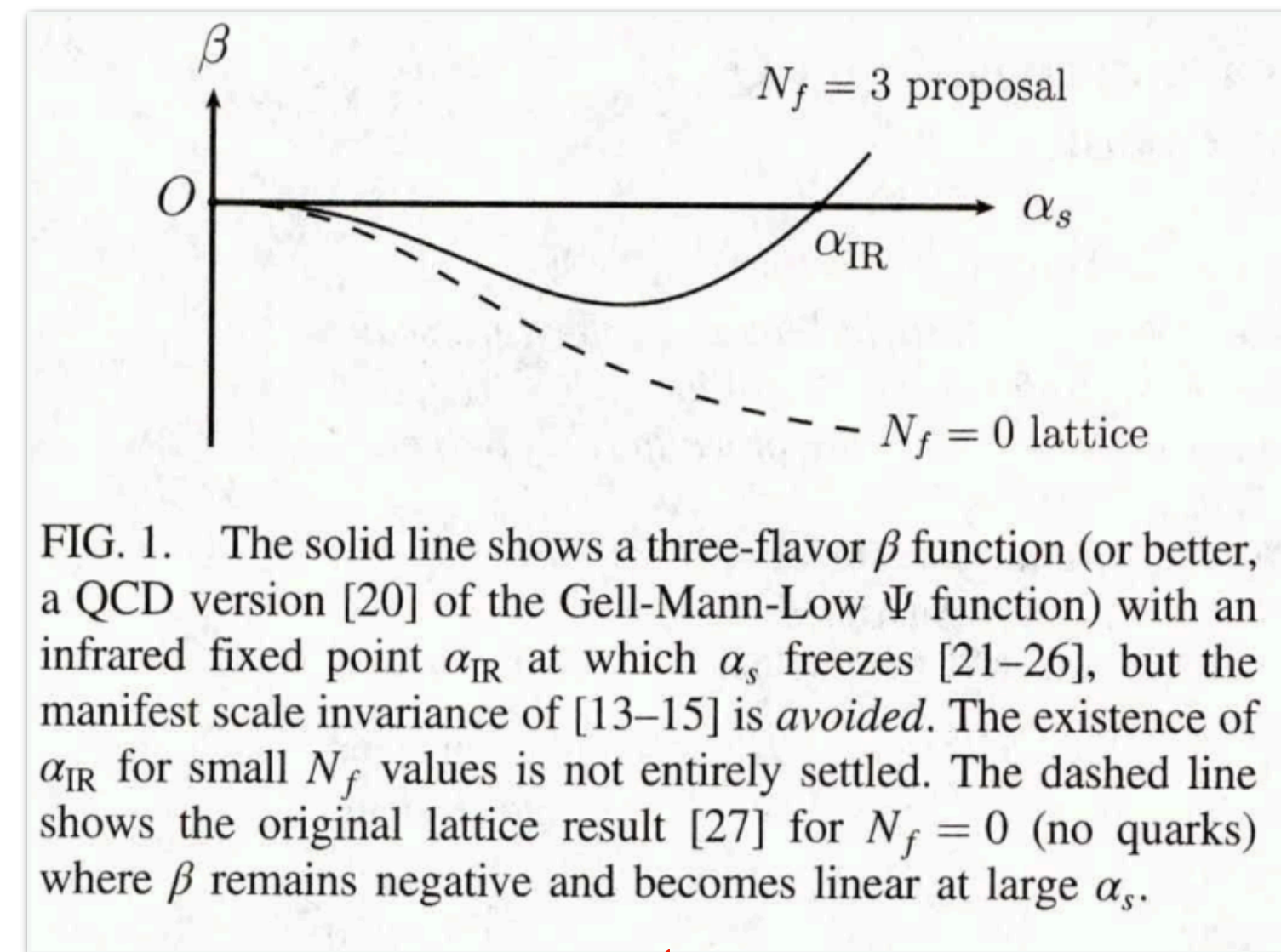


FIG. 1. The solid line shows a three-flavor β function (or better, a QCD version [20] of the Gell-Mann-Low Ψ function) with an infrared fixed point α_{IR} at which α_s freezes [21–26], but the manifest scale invariance of [13–15] is *avoided*. The existence of α_{IR} for small N_f values is not entirely settled. The dashed line shows the original lattice result [27] for $N_f = 0$ (no quarks) where β remains negative and becomes linear at large α_s .

$$\theta_\mu^\mu \Big|_{\alpha_s = \alpha_{\text{IR}}} = \left(1 + \gamma_m(\alpha_{\text{IR}})\right) (m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s) \rightarrow 0, \quad \text{SU}(3)_L \times \text{SU}(3)_R \text{ limit}$$

$K_L \rightarrow \pi^0 \gamma \gamma$ power counting

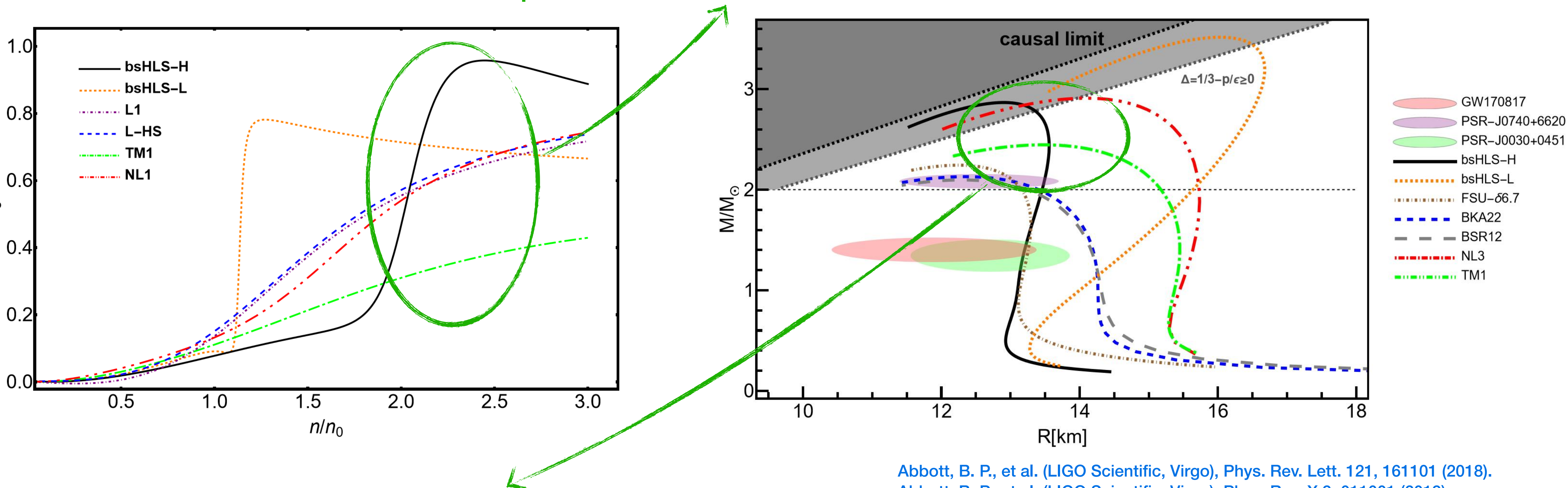
To keep chiral expansion consistent: $f_\pi \sim f_\chi$

$$\chi = f_\chi e^{\sigma/f_\chi} = f_\chi \Phi$$

Sound speed and M-R relations

σ is considered as the dilation from scale symmetry anomaly at low energies

A peak at intermediate densities



A larger mass than past studies

Abbott, B. P., et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018).
 Abbott, B. P., et al. (LIGO Scientific, Virgo), Phys. Rev. X 9, 011001 (2019).
 Riley, T. E., et al., Astrophys. J. Lett. 918, L27 (2021).
 Fujimoto, Y., Fukushima, K., McLerran, L. D., et al., Phys. Rev. Lett. 129, 252702 (2022).
 Riley, T. E., et al., Astrophys. J. Lett. 887, L21 (2019).

Bayesian Analysis

T. Bayes, Philos. Trans. R. Soc. London 53, 370 (1763).



$$p(\boldsymbol{\theta} \mid \mathbf{D}) = \frac{p(\mathbf{D} \mid \boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{D})}$$

prior distribution of parameter space: uniform assumption

$$p(\mathbf{D} \mid \boldsymbol{\theta}) = \prod_{i=1}^n p(D_i \mid \boldsymbol{\theta})$$

BJA benchmark

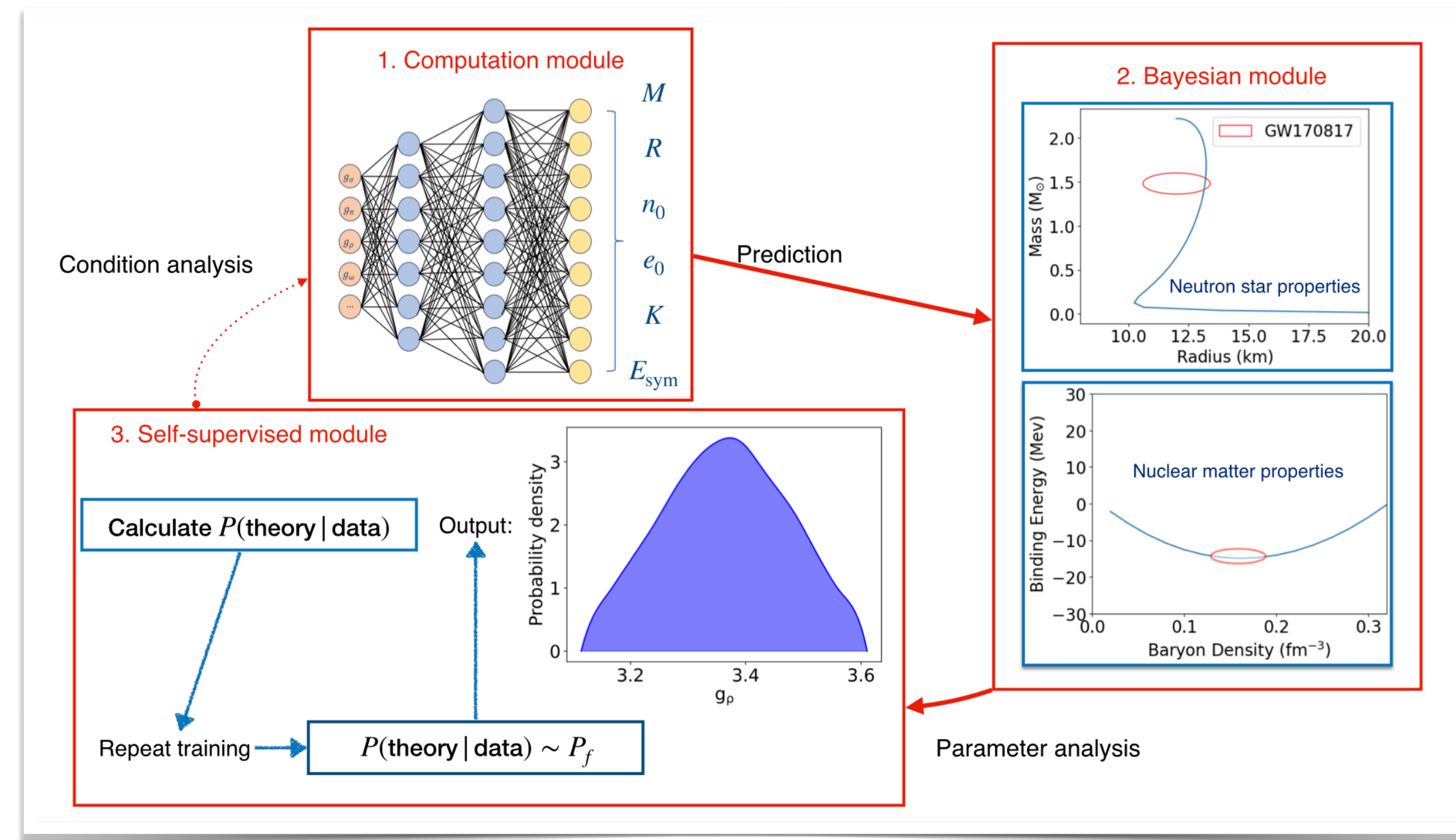
$$p(\mathbf{D}) = \int p(\mathbf{D} \mid \boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}$$

Hard to compute due to high D

A computing platform of studies on nuclear matter

L.-J. Guo, J.-Y. Xiong, YM, and Y.-L. Ma, *Astrophys. J.* 965, 47 (2024)

- The parameter space searching of an EFT of QCD with data across broad scale is a rather labor- and time-consuming work
- AI techs based on machine learning have advantage to analysis massive data and statistical correlations



An application

$$\begin{aligned}\mathcal{L}_{\text{RMF}} = & \bar{\psi} \left[i\gamma_{\mu} \partial^{\mu} - M - g_{\sigma} \sigma - g_{\omega} \gamma_{\mu} \omega^{\mu} - g_{\rho} \gamma_{\mu} \tau_a \rho^{a\mu} \right] \psi \\ & + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} m_{\sigma}^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\ & - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^{\mu} + \frac{1}{4} c_3 \left(\omega_{\mu} \omega^{\mu} \right)^2 \\ & - \frac{1}{4} R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2} m_{\rho}^2 \rho_{\mu}^a \rho^{a\mu}\end{aligned}$$

Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994)

H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nucl. Phys. A 637, 435 (1998)

TABLE II. Optimal values of the parameters calculated from NN. The empirical values $M_N = 938$ MeV, $m_{\omega} = 782$ MeV and $m_{\rho} = 765$ are physical values at vacuum taken as inputs from Ref. [68] for simplicity.

g_{σ}	g_{ω}	g_{ρ}	g_3	c_3	g_2	m_{σ}
9.82	11.8	3.42	1.26	72.6	-1550 MeV	531 MeV

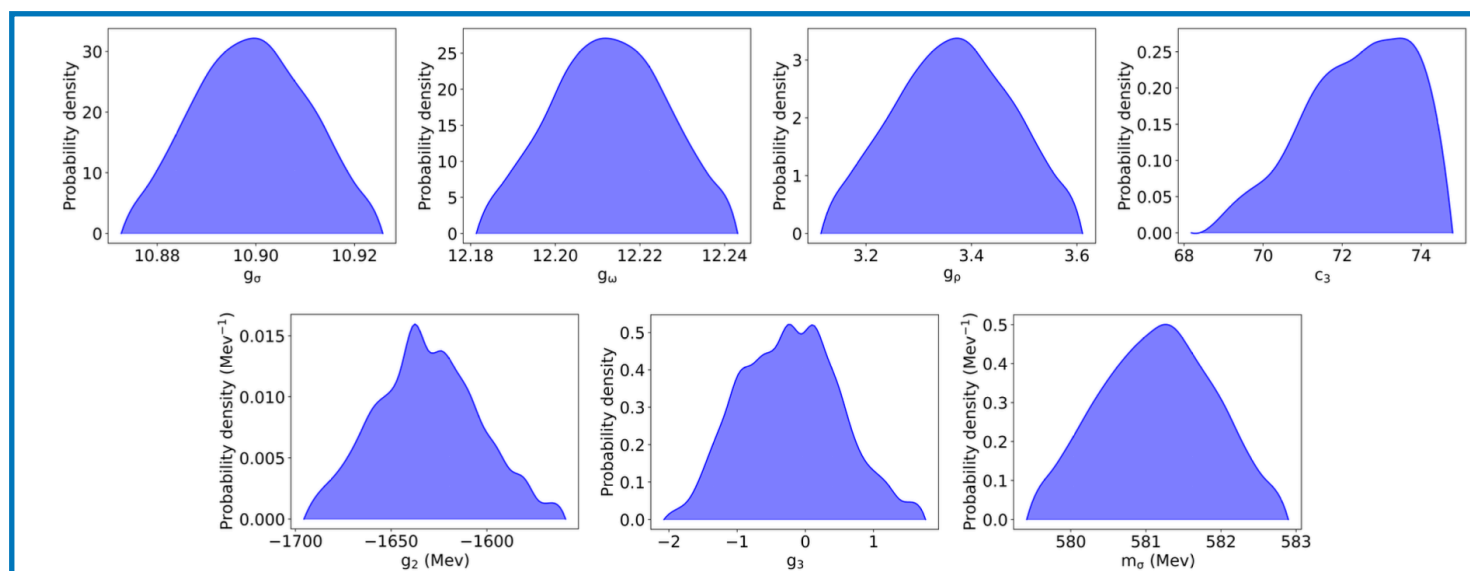


FIG. 5. Distributions of the parameters estimated by NN.

Bayesian analysis

TABLE I. M and R is the mass and radius of the primary star in GW170817. e_0 is the binding energy of nucleon, $E_{\text{sym}}(n) = \frac{1}{2} \frac{\partial^2 E(n, \alpha)}{\partial \alpha^2} \Big|_{\alpha=0}$ is the symmetry energy and $K_0 = 9n_0^2 \frac{\partial^2 E(n, 0)}{\partial n^2} \Big|_{n=n_0}$ is the incompressibility coefficient at n_0 , respectively. Normal distribution is assumed for data error bars with 95% confidence level.

	$M (M_{\odot})$	R (km)	n_0 (fm $^{-3}$)	e_0 (MeV)	E_{sym} (MeV)	K_0 (MeV)
Constraints	1.48 ± 0.12	11.9 ± 1.4	0.155 ± 0.05	-16 ± 1.0	32.0 ± 2.0	250 ± 50
Optimal	—	—	0.161	-15.7	28.6	266

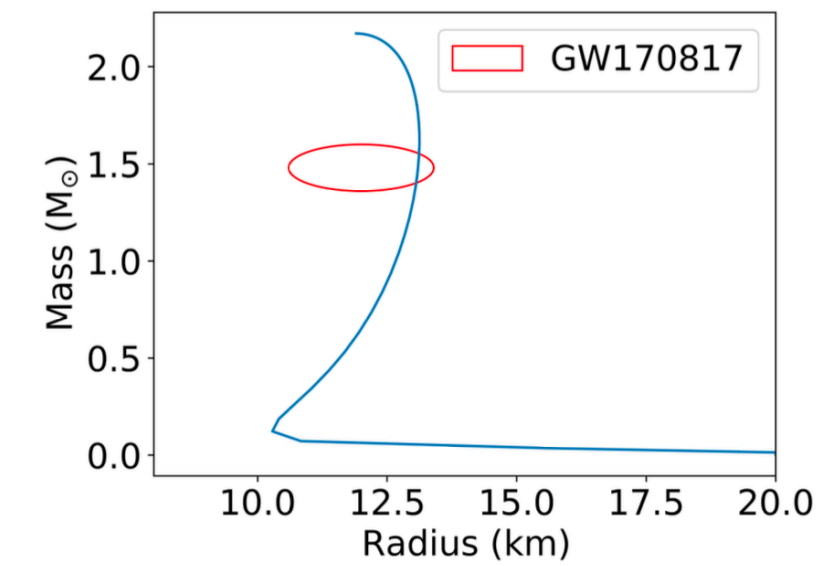


FIG. 3. MR relation obtained by the optimal solution.

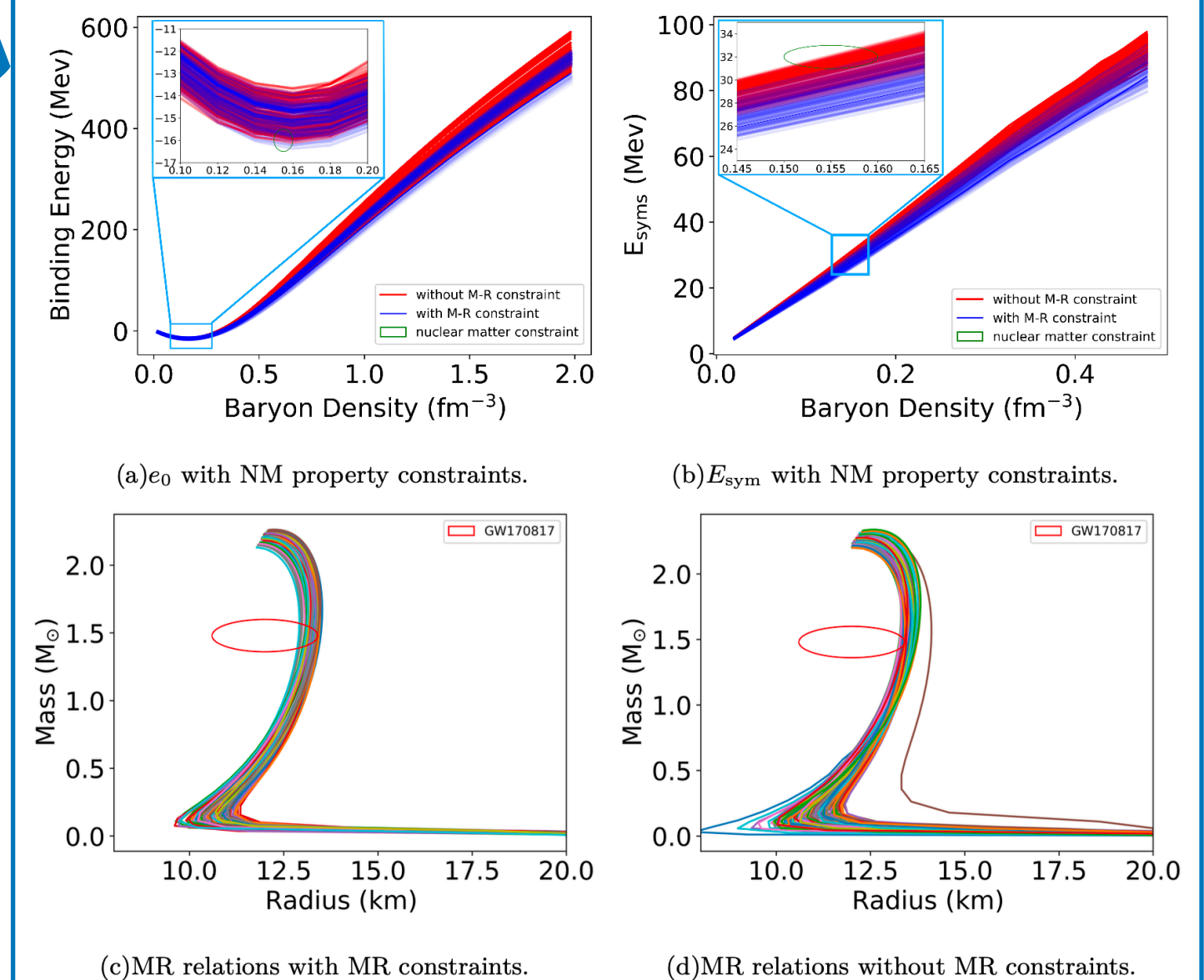


FIG. 4. Regions of the NM properties and MR relations predicted by NN.

δ meson effects

T. Miyatsu, M.-K. Cheoun, and K. Saito, *Astrophys. J.* 929, 82 (2022).

$$\tilde{g}_{\sigma^2\delta^2}\sigma^2\vec{\delta}^2 \quad \tilde{g}_{\sigma\delta^2}\sigma\vec{\delta}^2 \quad g_{\delta}\vec{\delta}\vec{\psi}\vec{\tau}\vec{\psi}$$

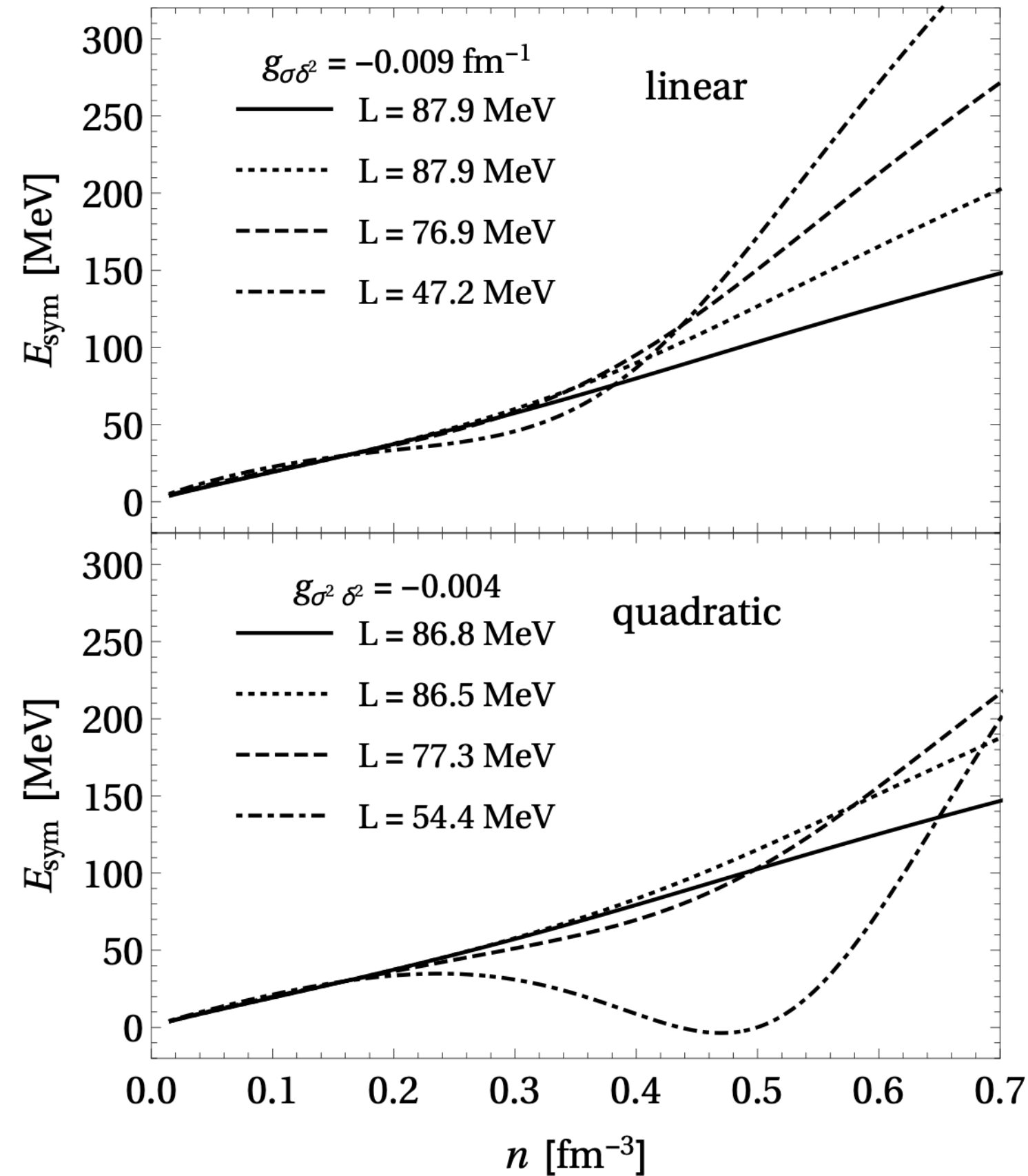


FIG. 2. Symmetry energy E_{sym} for negative value of g_{α} with respect to various values of C_{δ}^2 for both types of coupling.

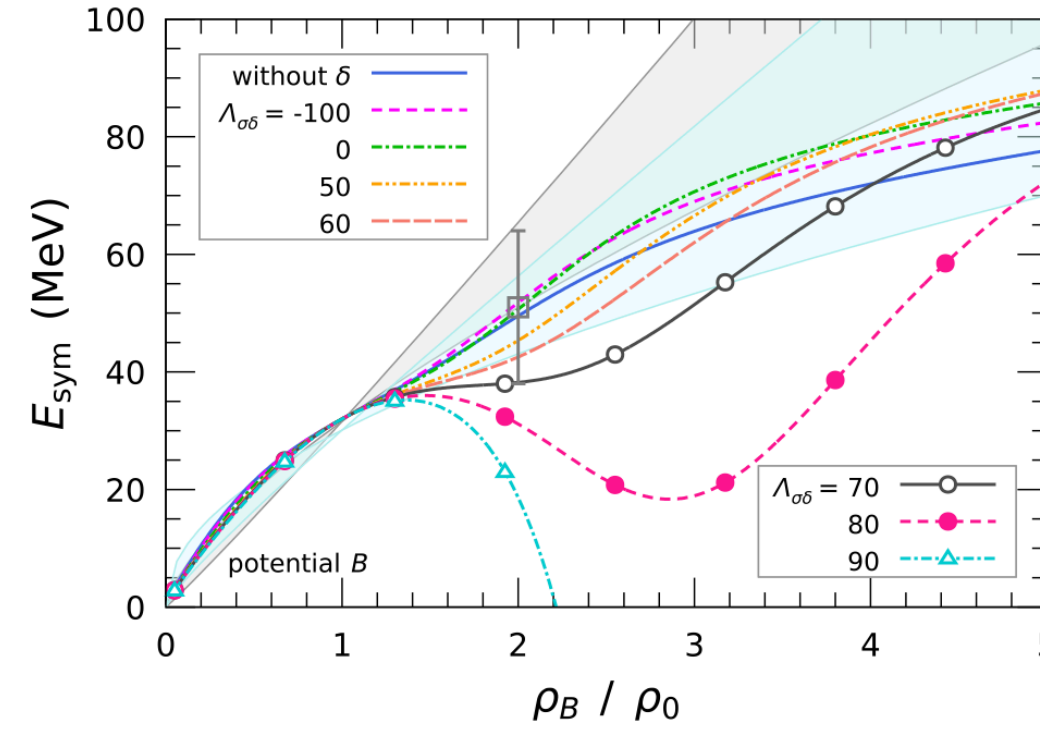


Figure 4. Same as Figure 3 but with the σ - δ mixing. The coupling constants in potential B are used.

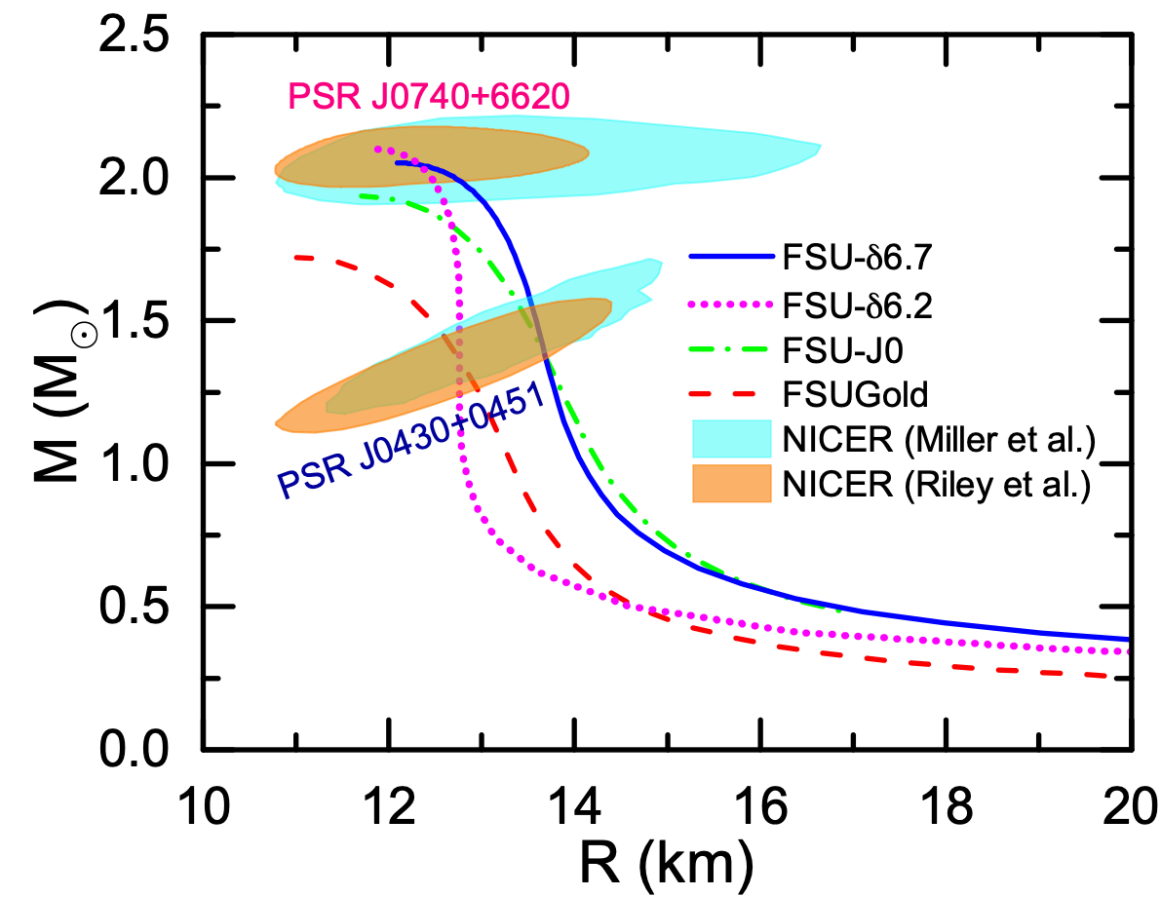


Figure 2. M - R relation for static NSs from FSUGold, FSU-J0, FSU- $\delta 6.7$ and FSU- $\delta 6.2$. The NICER (XMM-Newton) constraints with 68% C.L for PSR J0030+0451 (Riley et al. 2019; Miller et al. 2019) and PSR J0740+6620 (Riley et al. 2021; Miller et al. 2021) are also included for comparison.

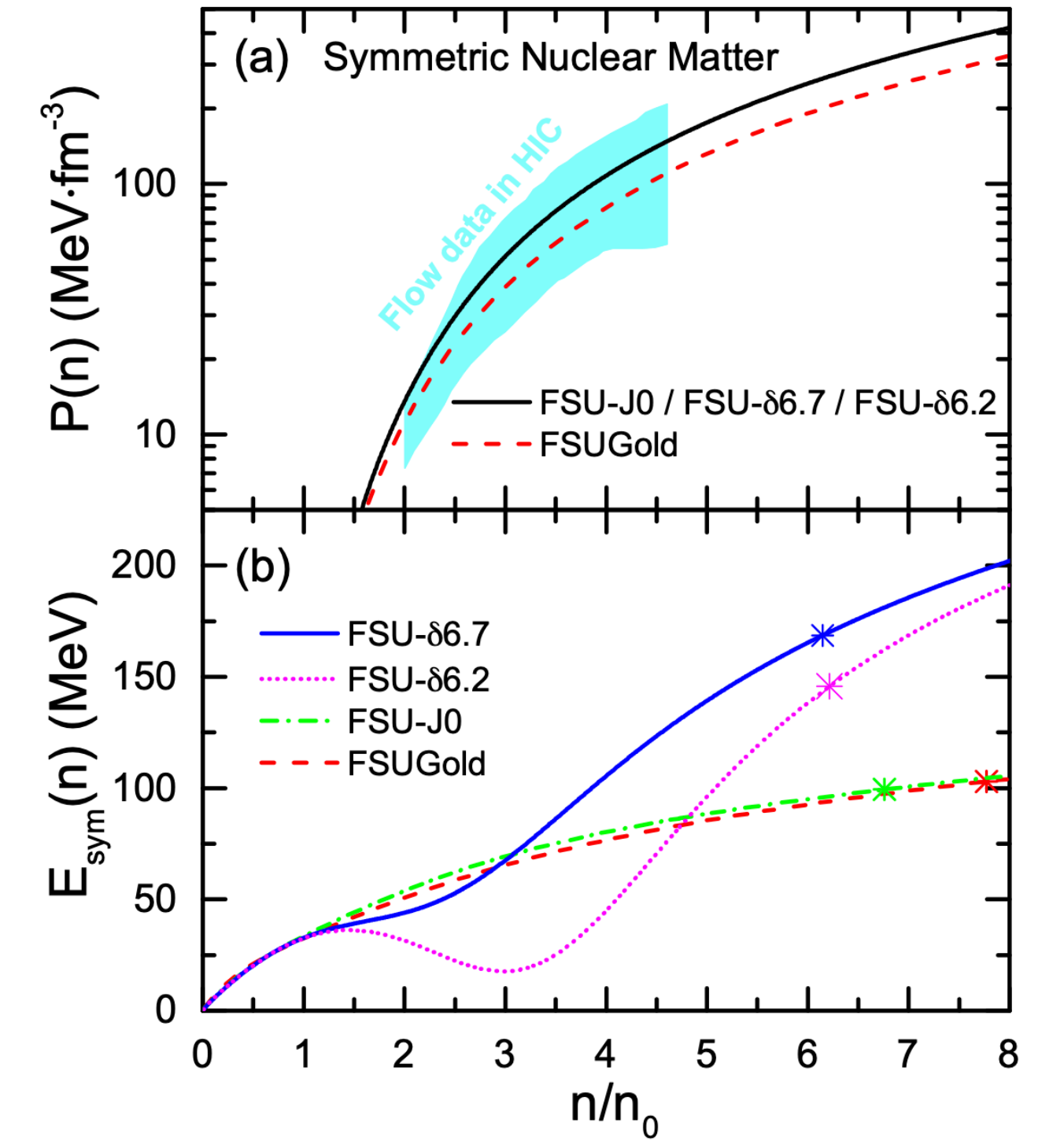


Figure 1. Pressure vs density (a) and density dependence of the symmetry energy (b) predicted by FSUGold, FSU-J0, FSU- $\delta 6.7$ and FSU- $\delta 6.2$. The shaded area in panel (a) represents the constraints from flow data in heavy-ion collisions (Danielewicz et al. 2002). The central density of maximum mass NS configuration is indicated by '*' in panel (b).

A general quantum hadrodynamics (GQHD) model

YM, Y.-L. Ma, and J.-Y. Xiong, arXiv:2603.01933 [nucl-th] (2026)



$$\mathcal{L}_N = \bar{\Psi} \left(i\gamma^\mu \partial_\mu - m_N \right) \Psi,$$

$$\mathcal{L}_\sigma = \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{2} g_4 \sigma^2 \omega^\mu \omega_\mu - \frac{1}{2} g_5 \sigma^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu - g_6 \sigma \vec{\rho}^\mu \cdot \vec{\rho}_\mu - g_7 \sigma \omega^\mu \omega_\mu,$$

$$\mathcal{L}_\omega = -\frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{1}{4} c_3 \left(\omega^\mu \omega_\mu \right)^2 + \frac{1}{2} c_4 \omega^\mu \omega_\mu \vec{\rho}^\mu \cdot \vec{\rho}_\mu,$$

$$\mathcal{L}_\rho = -\frac{1}{4} \vec{P}^{\mu\nu} \cdot \vec{P}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu + \frac{1}{4} d_3 \left(\vec{\rho}^\mu \cdot \vec{\rho}_\mu \right)^2,$$

$$\begin{aligned} \mathcal{L}_{a_0} = & \frac{1}{2} \partial_\mu \vec{a}_0 \cdot \partial^\mu \vec{a}_0 - m_{a_0}^2 \vec{a}_0 \cdot \vec{a}_0 + \frac{1}{4} b_3 \left(\vec{a}_0 \cdot \vec{a}_0 \right)^2 + \frac{1}{2} b_4 \left(\vec{a}_0 \cdot \vec{a}_0 \right) \left(\vec{\rho}^\mu \cdot \vec{\rho}_\mu \right) + \frac{1}{2} b_5 \sigma \left(\vec{a}_0 \cdot \vec{a}_0 \right) \\ & + \frac{1}{2} b_6 \vec{a}_0 \cdot \vec{a}_0 \sigma^2 + \frac{1}{2} b_7 \vec{a}_0 \cdot \vec{a}_0 \omega^\mu \omega_\mu + b_8 \vec{a}_0 \cdot \vec{\rho}^\mu \omega_\mu \sigma + b_9 \vec{a}_0 \cdot \vec{\rho}_\mu \omega^\mu, \end{aligned}$$

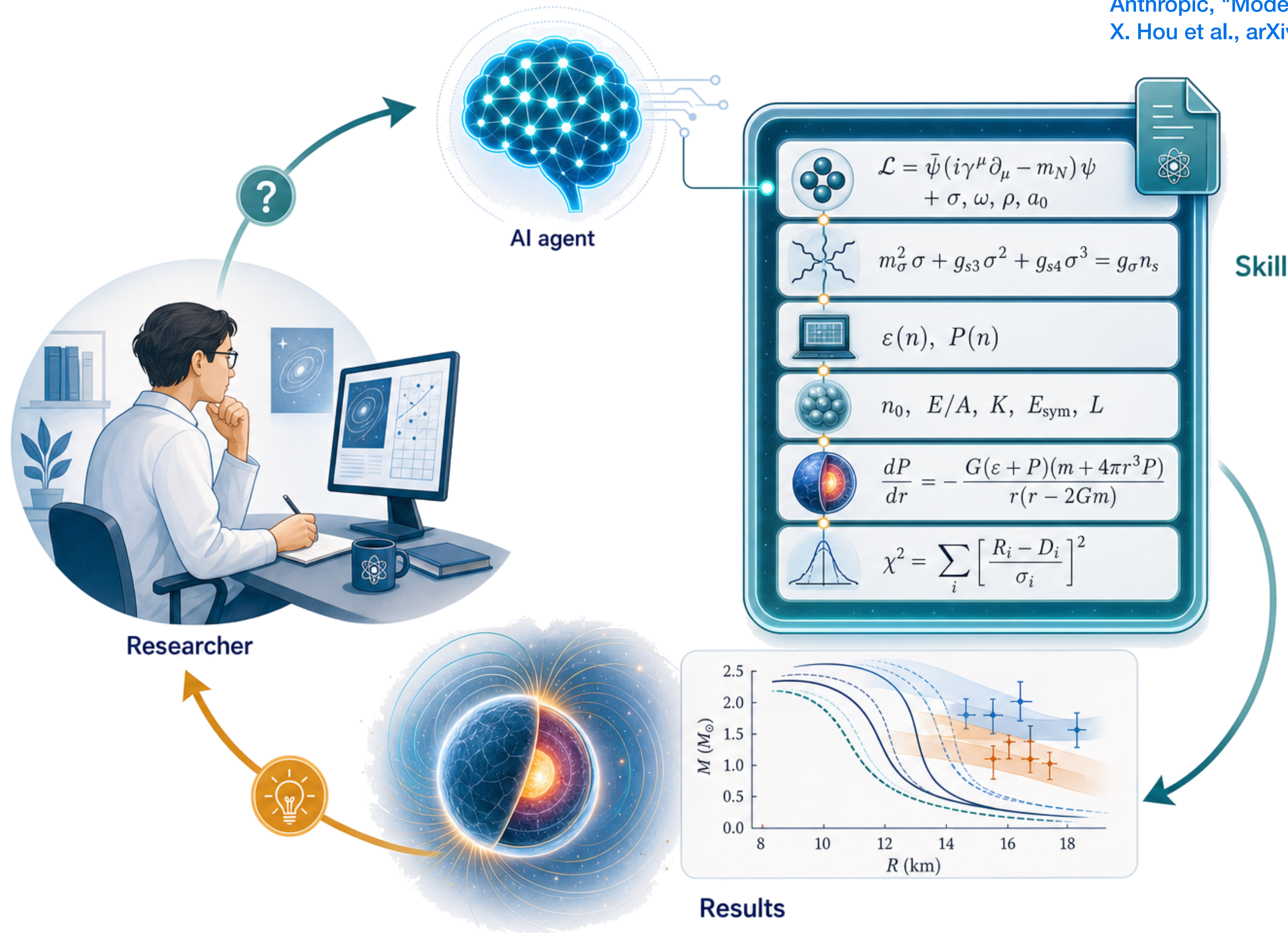
$$\mathcal{L}_I = \bar{\Psi} \left(g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\rho}_\mu + g_{a_0} \vec{a}_0 \right) \Psi,$$

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{a_0} + \mathcal{L}_I.$$

Including all possible operators with (iso-)scalars and vectors within dimension 4, but lead to complexities in solving EOMs based on neural network so that the need for GPU resources grow out of control

The agentic working flow

Anthropic, "Model Context Protocol", <https://modelcontextprotocol.io> (2024).
X. Hou et al., arXiv:2503.23278 (2025).

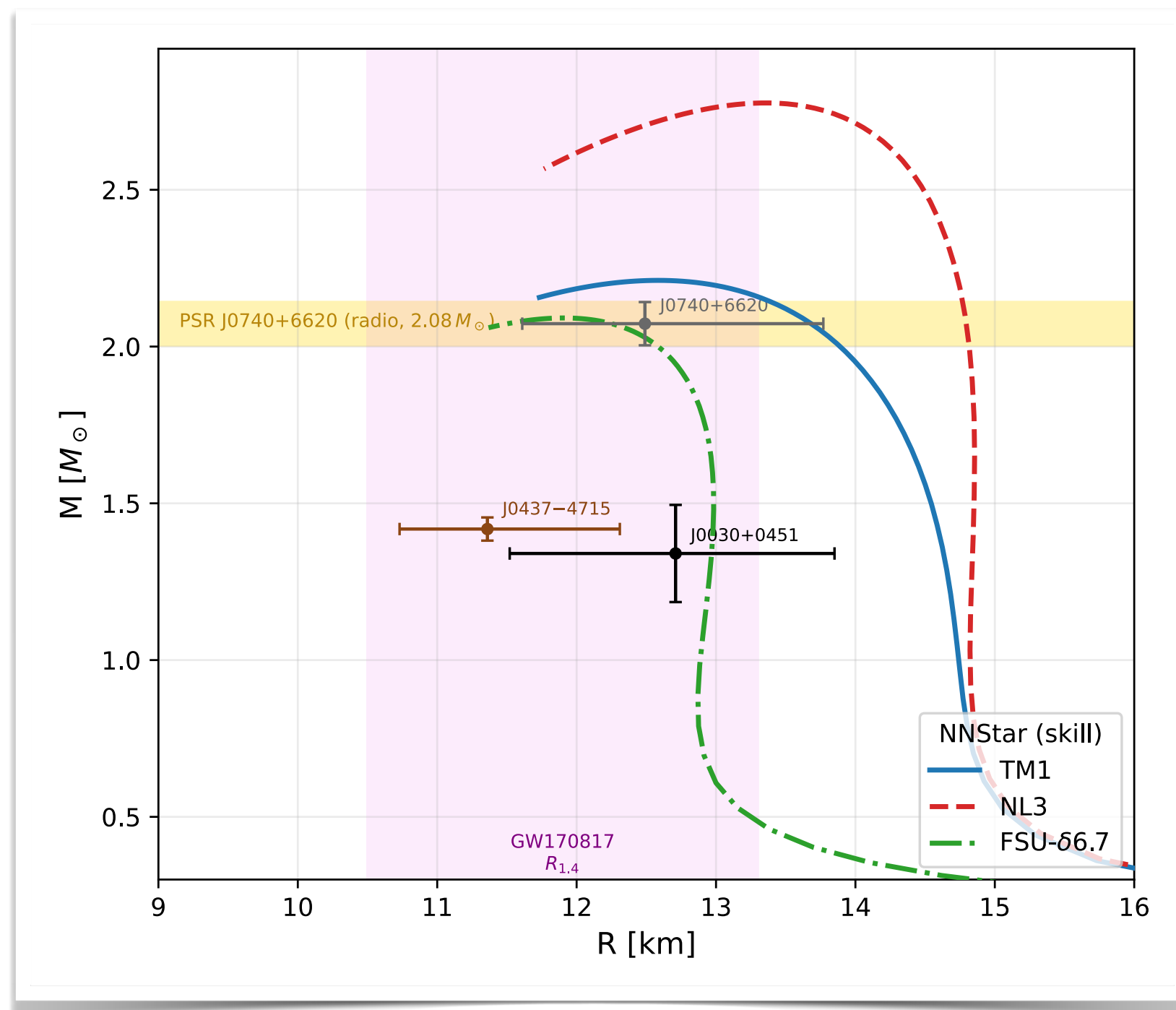


Thanks to MCP

Just knowledge
of physics

Validations in Walecka models

Model	Source	n_0 [fm ⁻³]	E [MeV]	E_{sym} [MeV]	L [MeV]	K [MeV]	M_{max} [$M_{\odot}$]	$R_{1.4}$ [km]
TM1	NNStar	0.145	-15.9	36.6	109.9	275	2.21	14.60
	Ref.	0.142	-16.2	36.0	108	257	—	—
NL3	NNStar	0.151	-16.6	38.7	122.9	264	2.78	14.85
	Ref.	0.148	-16.3	38.2	121	246	—	—
FSU- $\delta 6.7$	NNStar	0.149	-16.4	32.2	52.5	231	2.09	12.98
	Ref.	0.148	-16.3	32.7	53.5	229	—	—



Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994).

G. A. Lalazissis et al., Phys. Rev. C 55, 540 (1997).

B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. 95, 122501 (2005).

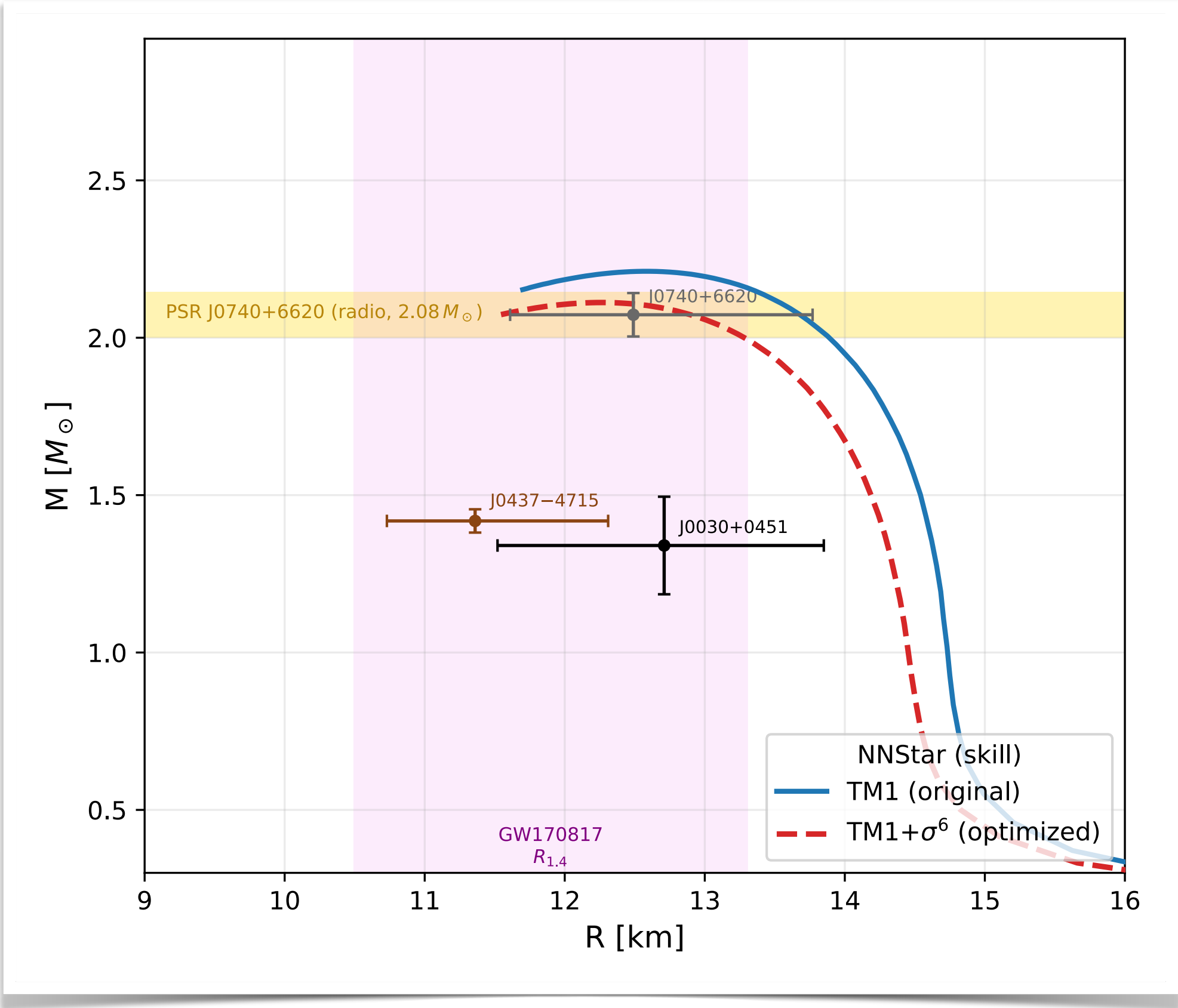
B. P. Abbott et al., Phys. Rev. Lett. 121, 161101 (2018).

T. Salmi et al., Astrophys. J. 974, 294 (2024).

D. Choudhury et al., Astrophys. J. Lett. 971, L20 (2024).

Model extension (σ^6 added to TM1)

J. D. Walecka, *Annals Phys.* 83, 491 (1974).
J. Boguta and A. R. Bodmer, *Nucl. Phys. A* 292, 413 (1977).
Y. Sugahara and H. Toki, *Nucl. Phys. A* 579, 557 (1994).



Observable	Empirical	TM1	TM1+ σ^6
$n_0 [\text{fm}^{-3}]$	0.16 ± 0.05	0.145	0.141
$E(n_0) [\text{MeV}]$	-16.0 ± 1.0	-15.9	-16.1
$K(n_0) [\text{MeV}]$	230 ± 30	275	231
$E_{\text{sym}}(n_0) [\text{MeV}]$	30.9 ± 1.9	36.6	30.1
$L(n_0) [\text{MeV}]$	52.5 ± 17.5	110	90
$m_\sigma [\text{MeV}]$	600 ± 200	511	492
$M_{\text{max}} [M_\odot]$	≥ 2.08	2.21	2.11
$R_{1.4} [\text{km}]$	12.7 ± 1.1	14.6	14.3
χ^2	—	25.5	7.2

M. Dutra et al., *Phys. Rev. C* 90, 055203 (2014).
J. M. Lattimer and Y. Lim, *Astrophys. J.* 771, 51 (2013).
B. T. Reed et al., *Phys. Rev. Lett.* 126, 172503 (2021).

LLM benchmark



<https://github.com/AaronMahn/NM-rmf-benchmark-scheme>

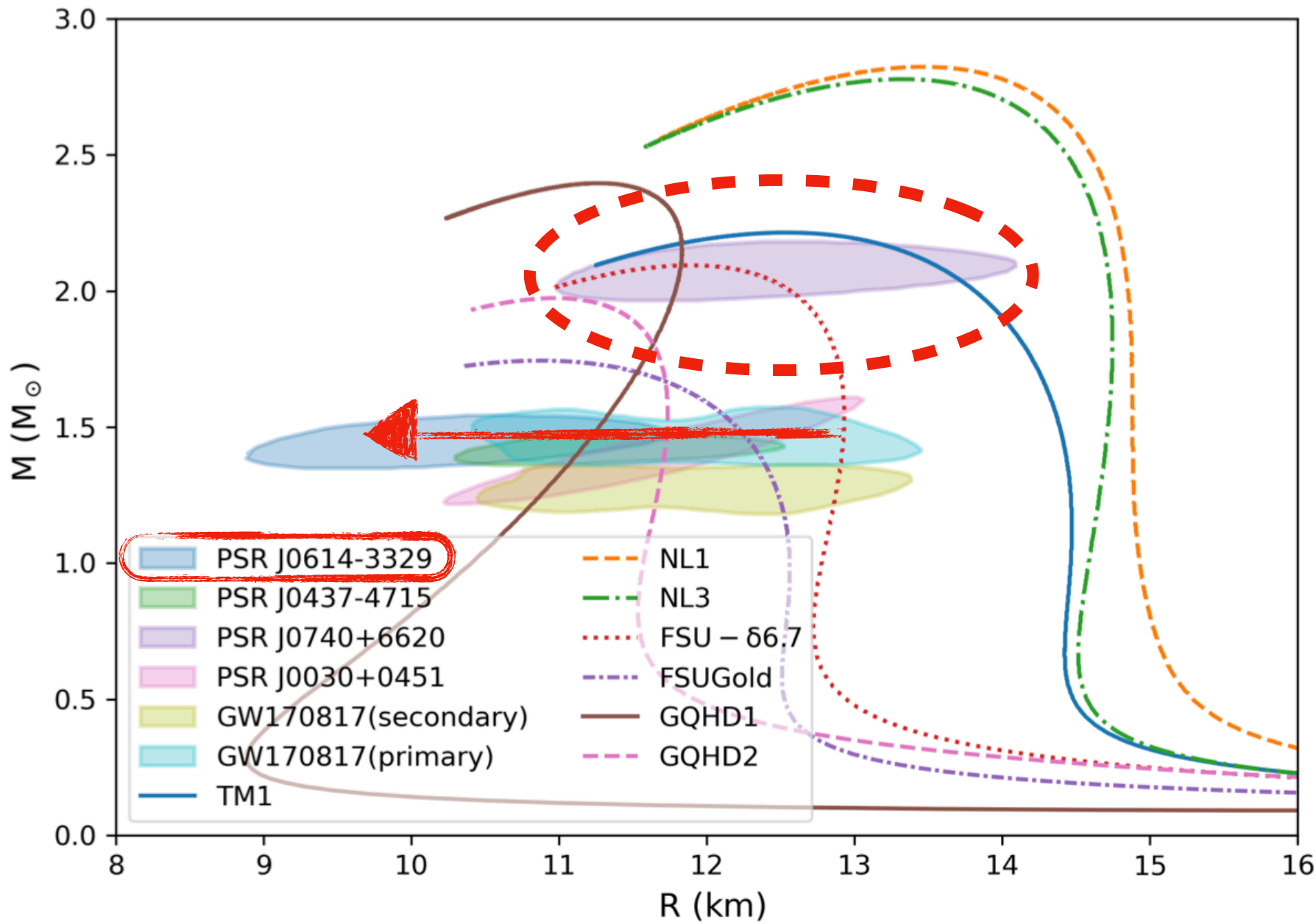
Model	Mode	Valid	matter (%)	M (%)	R (%)	Tokens (k)	Time (s)
GPT-5.5	+skill	3/3	0.0	0.0	0.0	65	109
	−skill	3/3	0.0	7.7	16.0	52	204
Claude-Opus-4.8	+skill	3/3	0.0	0.3	0.8	77	103
	−skill	3/3	186.5	34.8	17.0	116	352
Gemini-3.1-Pro	+skill	3/3	0.0	0.1	0.3	1218	386
	−skill	2/3	52.4	1.0	1.0	1341	364
Qwen3.7-Max	+skill	2/3	0.0	0.0	0.0	112	161
	−skill	0/3	–	–	–	–	154
DeepSeek-V4-Pro	+skill	3/3	0.0	2.4	1.7	63	394
	−skill	2/3	2214.3	100.0	$\sim 10^{10}$	55	606
Kimi-K2.6	+skill	2/3	101.1	79.9	75.0	75	605
	−skill	2/3	1334.4	100.0	85.7	56	609

Tested with openclaw

Phenomenological Constraints

(1 σ)

[M. Dutra et al., Phys. Rev. C 90, 055203 \(2014\)](#)
[J. M. Lattimer and Y. Lim, Astrophys. J. 771, 51 \(2013\)](#)
[P. Danielewicz and J. Lee, Nucl. Phys. A 922, 1 \(2014\)](#)
[M. Dutra et al., Phys. Rev. C 85, 035201 \(2012\)](#)
[A. Le Fèvre et al., Nucl. Phys. A 945, 112 \(2016\)](#)
[B. A. Brown, Phys. Rev. Lett. 111, 232502 \(2013\)](#)
[B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 \(2021\)](#)
[J. Antoniadis et al., Science 340, 6131 \(2013\)](#)
[B. P. Abbott et al. \(LIGO Scientific, Virgo\), Phys. Rev. Lett. 119, 161101 \(2017\)](#)
[S. Vinciguerra et al., Astrophys. J. 961, 62 \(2024\)](#)
[D. Choudhury et al., Astrophys. J. Lett. 971, L20 \(2024\)](#)
[L. Mauviard et al., arXiv:2506.14883 \(2025\)](#)
[B. P. Abbott et al. \(LIGO Scientific, Virgo\), Phys. Rev. Lett. 121, 161101 \(2018\)](#)
[T. Salmi et al., Astrophys. J. 974, 294 \(2024\)](#)

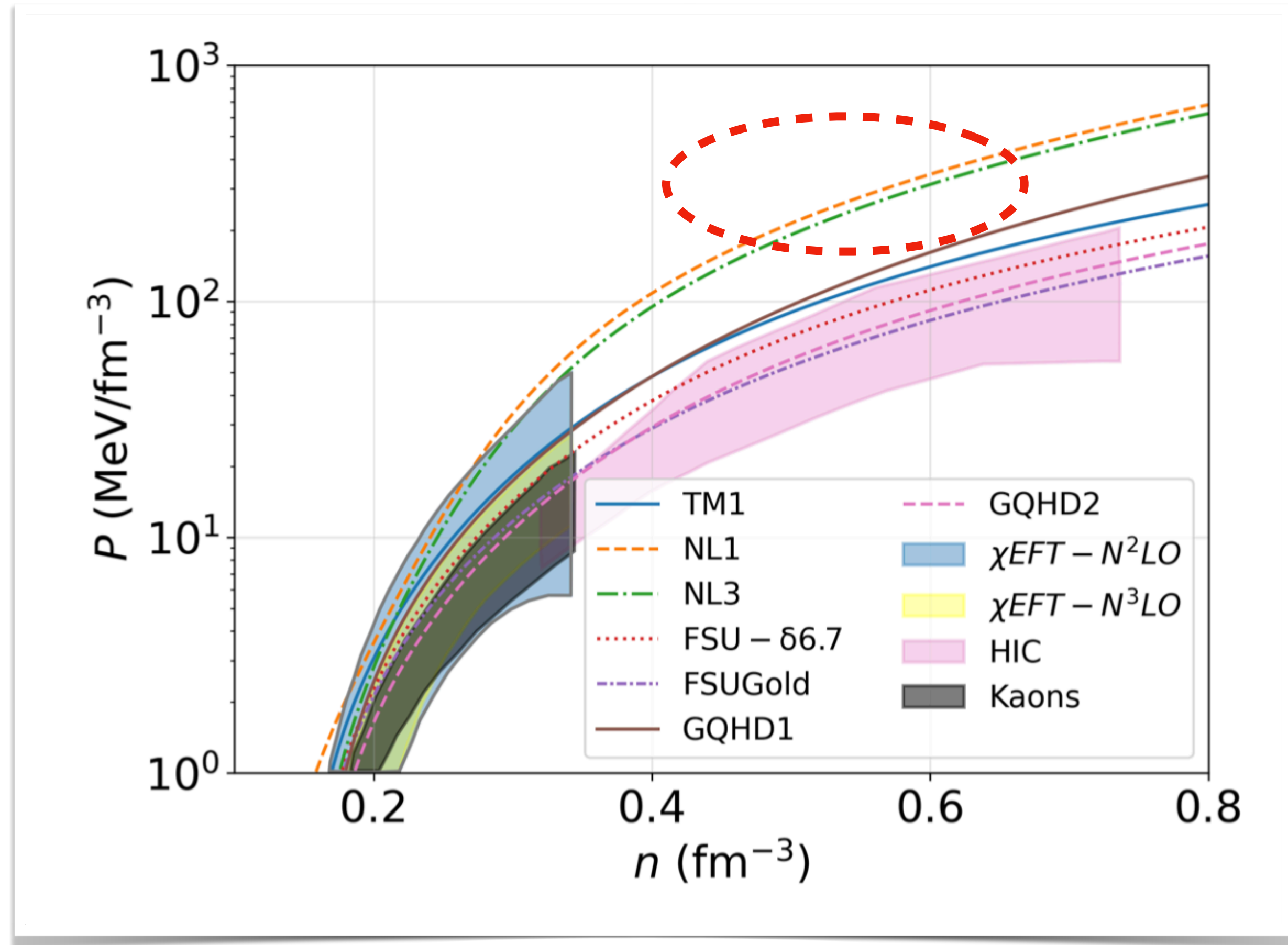


M-R relations

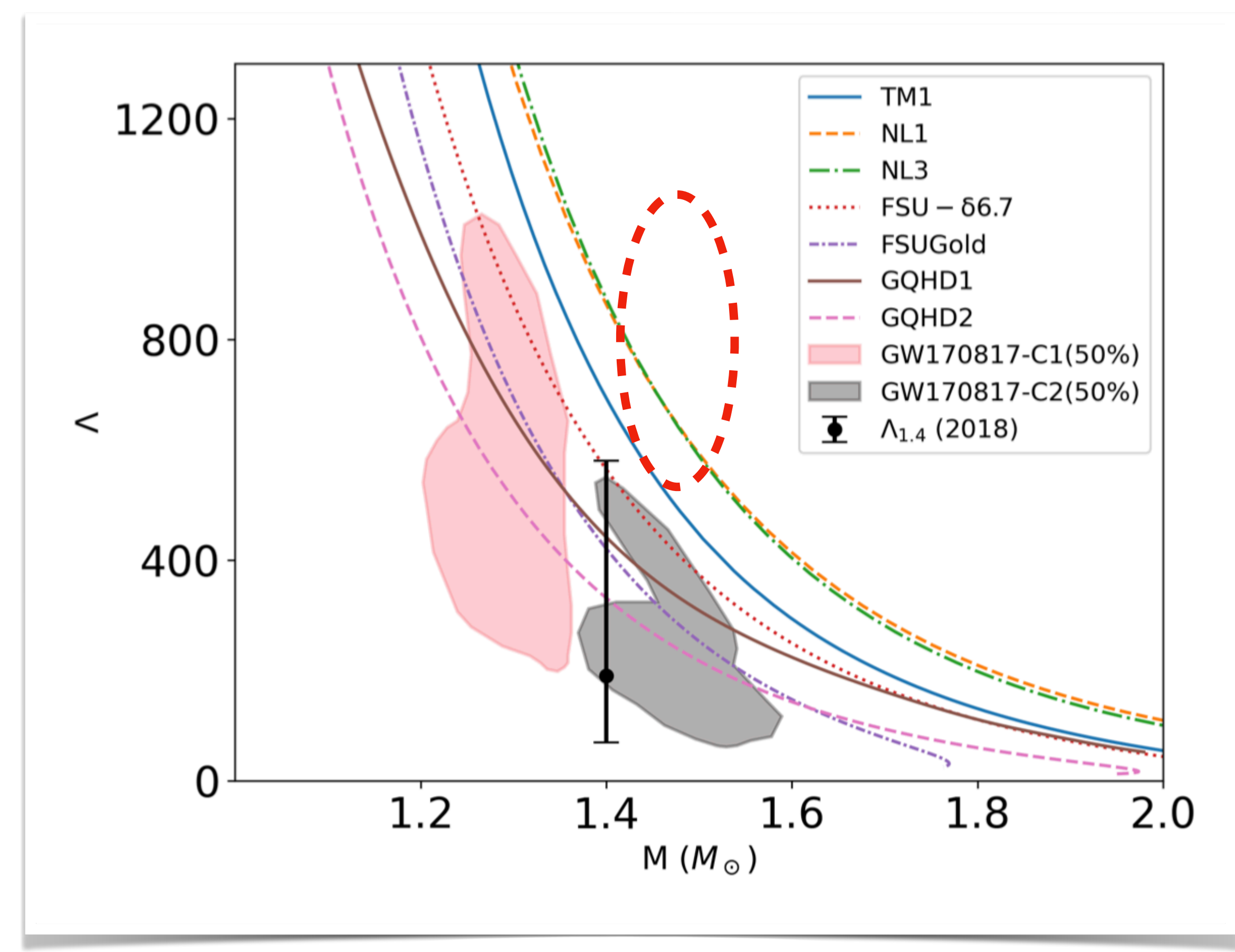
	Empirical	TM1	NL1	NL3	FSU- $\delta 6.7$	FSUGold	GQHD1	GQHD2
$E(n_0)$	-16.0 ± 1.0	-16.2	-16.5	-16.3	-16.3	-16.3	-17.2	-16.1
n_0	0.16 ± 0.05	0.142	0.152	0.148	0.148	0.148	0.159	0.155
$E_{\text{sym}}(n_0)$	30.9 ± 1.9	36.0	43.5	38.2	32.7	29.1	32.3	33.4
$L(n_0)$	52.5 ± 17.5	108	140	121	53.5	53.1	49.6	49.6
$K(n_0)$	230 ± 30	257	207	246	229	228	349	214
$E(1.5n_0)$	-13.3 ± 0.5	-13.0	-13.1	-12.6	-13.3	-13.5	-12.5	-13.3
$P(1.5n_0)$	3.41 ± 1.29	4.26	5.52	5.57	3.97	3.58	6.52	3.67
$L(\frac{2}{3}n_0)$	71.5 ± 22.6	69.6	85.0	74.3	54.5	39.2	69.9	55.2
m_σ	600 ± 200	511	492	503	492	492	445	504
BJA $\uparrow$		-4.51	-15.3	-11	3.8	4.28	1.13	8.03

NM properties

HIC and tidal deformaton

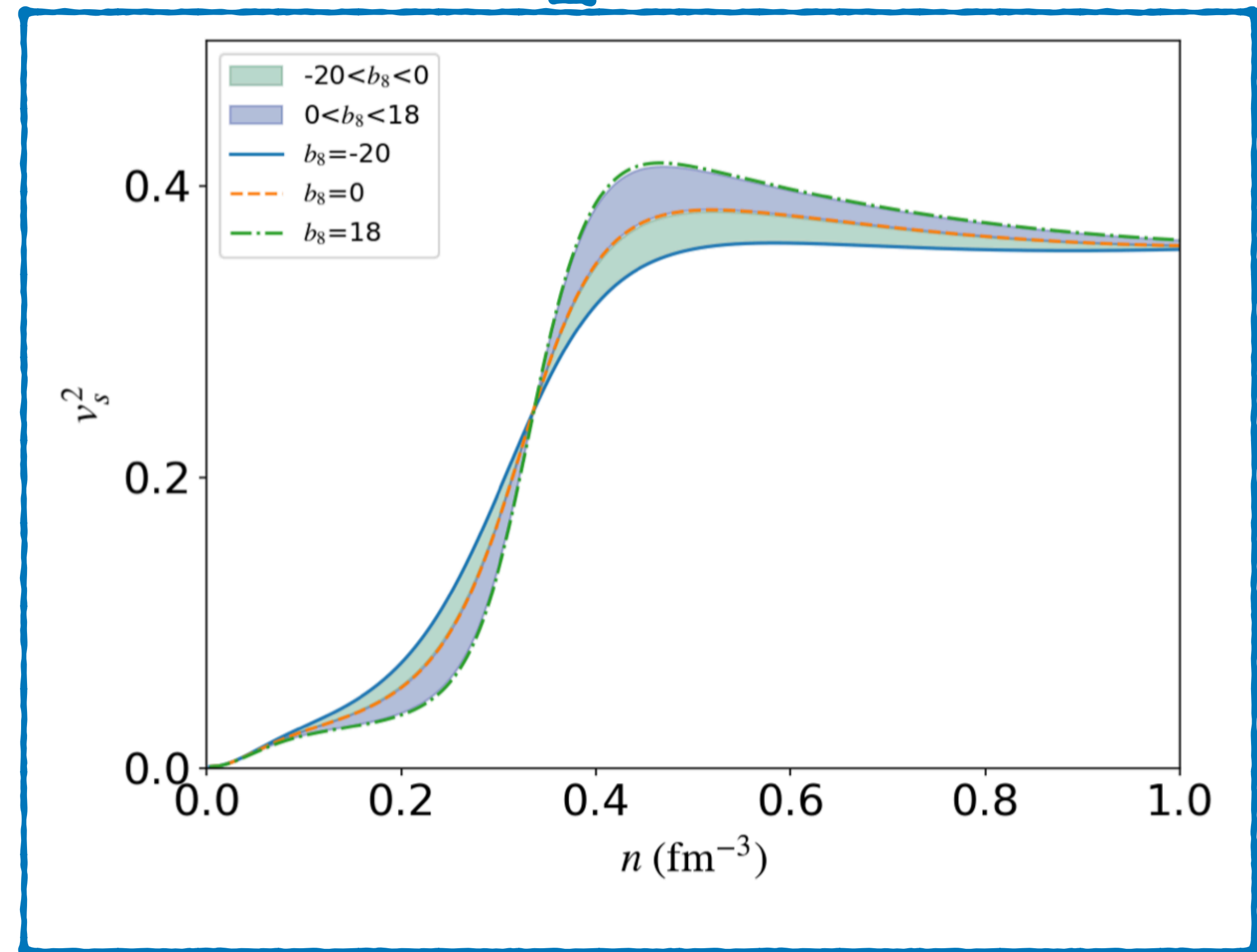
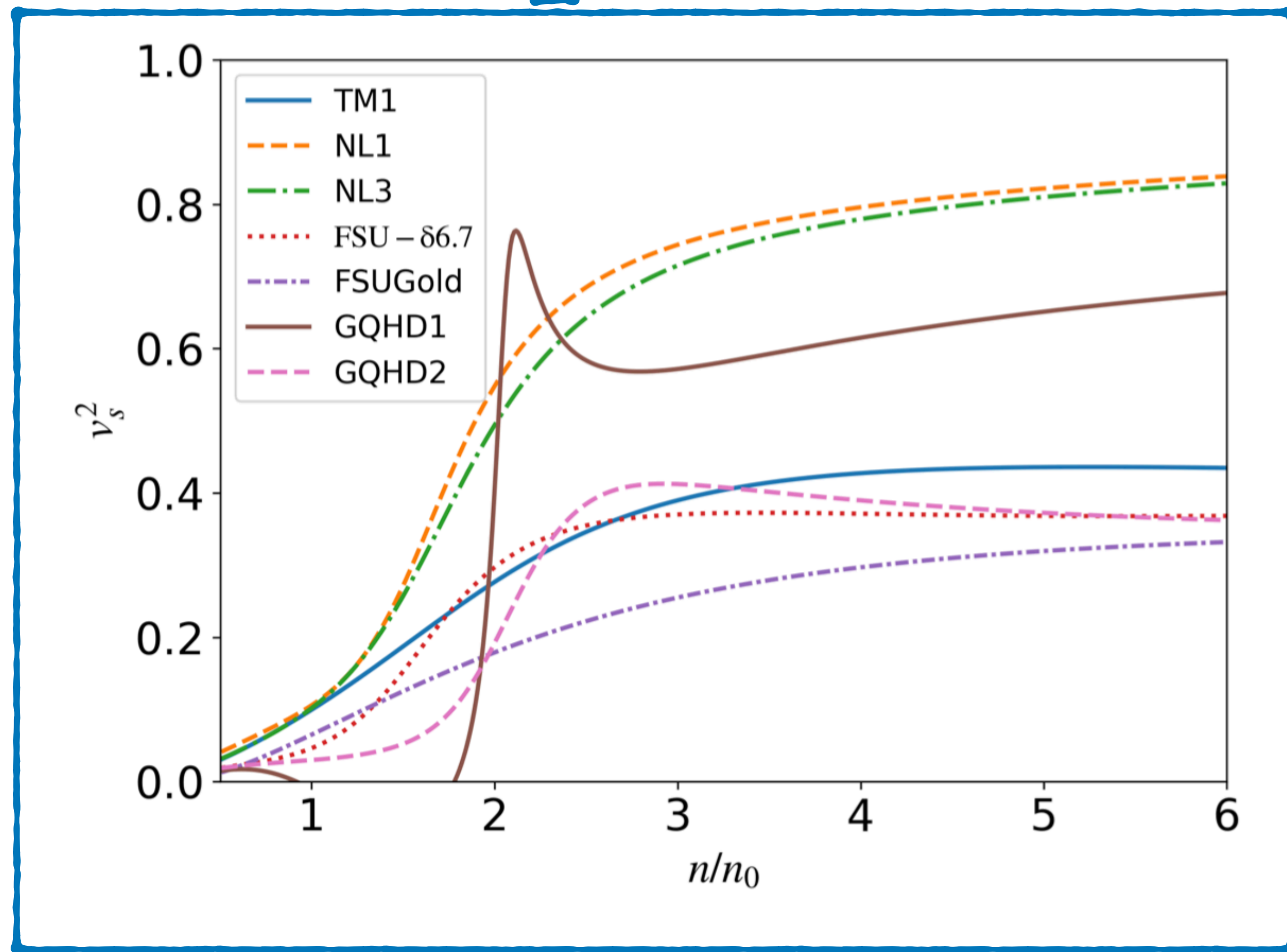


P. Danielewicz, R. Lacey, and W. G. Lynch, *Science* 298, 1592 (2002).
 S. Huth et al., *Nature* 606, 276 (2022).
 W. G. Lynch et al., *Prog. Part. Nucl. Phys.* 62, 427 (2009).
 C. Fuchs, *Prog. Part. Nucl. Phys.* 56, 1 (2006).



B. P. Abbott et al. (LIGO Scientific, Virgo), *Phys. Rev. Lett.* 119, 161101 (2017).
 B. P. Abbott et al. (LIGO Scientific, Virgo), *Phys. Rev. Lett.* 121, 161101 (2018).
 The LIGO Scientific Collaboration and The Virgo Collaboration, LIGO-P1800115-v12, <https://dcc.ligo.org/LIGO-P1800115/>.
 P. J. Kalita et al., arXiv:2510.23405 (2025).

Sound velocity and $\rho\omega a_0\sigma$ mixing



Summary and outlook

- I. More and more data from experiments and observations offer **a good window** for nuclear physics;
- II. It's rather important to link the data and fundamental theory of strong interaction based on **low energy effective theories/ models**;
- III. An agentic research scheme can help to **reduce time- and labor-consuming** in such a kind of studies.



Thank you!

Backup

The chiral transformations



$$SU(3)_R \otimes SU(3)_L$$

$$\Phi' \rightarrow g_L \Phi' g_R^\dagger, \quad \hat{\Phi}' \rightarrow g_L \hat{\Phi}' g_R^\dagger$$

$$L_\mu \rightarrow g_L L_\mu g_L^\dagger, \quad R_\mu \rightarrow g_R R_\mu g_R^\dagger$$

$$N_R^{(RR)} \rightarrow g_R N_R^{(RR)} g_R^\dagger$$

$$N_R^{(LL)} \rightarrow g_R N_R^{(LL)} g_L^\dagger$$

Scalars Vectors Baryons

$$2\text{-quark} \quad U(1)_A \quad 4\text{-quark}$$

$$\Phi' \rightarrow e^{2i\nu} \Phi', \quad \hat{\Phi}' \rightarrow e^{-4i\nu} \hat{\Phi}'$$

$$L_\mu \rightarrow L_\mu, \quad R_\mu \rightarrow R_\mu$$

$$N_R^{(RR)} \rightarrow e^{-3i\nu} N_R^{(RR)}, \quad N_L^{(RR)} \rightarrow e^{-i\nu} N_L^{(RR)}$$

$$N_R^{(LL)} \rightarrow e^{i\nu} N_R^{(LL)}, \quad N_L^{(LL)} \rightarrow e^{3i\nu} N_L^{(LL)}$$

di-quark approximation

Power counting rules



- ① The operators are limited **within dimension-4**, for the higher dimensional operators are suppressed by the cutoff scale;
- ② The **quark number** of an operator is limited within 8 and the number of flavor space traces is limited to only 1, for its suppression by N_c ;
- ③ The **explicit symmetry** breaking caused by quark mass is treated as **perturbation**, which is crucial to strangeness in NS.

A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008).
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005).
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009).
D. Parganlija, F. Giacosa, and D. H. Rischke, Phys. Rev. D 82, 054024 (2010).

The Lagrangian at the leading order for RMF



$$\mathcal{V}_M = c_2 \text{Tr } S'^2 - d_2 \text{Tr } \hat{S}'^2 - c_4 \text{Tr } S'^4 - 2e_3 \epsilon_{abc} \epsilon_{def} S'_{ad} S'_{be} \hat{S}'_{cf}$$

Quark configuration mixing

$$\mathcal{V}_V = \tilde{h}_2 \text{Tr } (S'^2 V^2) + \tilde{g}_3 \text{Tr } V^4 +$$

$$a_1 \epsilon_{abc} \epsilon_{def} V_{ad} V_{be} (S'^2)_{cf}$$

Masses by spontaneous chiral symmetry breaking

$$\mathcal{L}_B^{\text{RMF}} = \text{Tr} \left(\bar{B} i \gamma_\mu \partial^\mu B \right) + c \text{Tr} \left(\bar{B} \gamma^0 V B \right) - g \text{Tr} \left(\bar{B} S' B \right)$$

$$+ h \epsilon_{abc} \epsilon_{def} \bar{B}_{ad} \gamma^0 B_{be} V_{cf}$$

$$- e \epsilon_{abc} \epsilon_{def} \bar{B}_{ad} \gamma^0 B_{be} S'_{cf}$$

Only two quark configuration couples to baryon at the leading order

The lowest order Lagrangians (The system only with nucleons)



The scalar meson masses by spontaneous chiral symmetry breaking

$$\mathcal{L}_M = \frac{1}{2} \text{Tr} \left(\partial_\mu \Phi' \partial^\mu \Phi'^\dagger \right) + \frac{1}{2} \text{Tr} \left(\partial_\mu \hat{\Phi}' \partial^\mu \hat{\Phi}'^\dagger \right) + c_2 \text{Tr} \left(\Phi' \Phi'^\dagger \right) - c_4 \text{Tr} \left(\Phi' \Phi'^\dagger \Phi' \Phi'^\dagger \right) - d_2 \text{Tr} \left(\hat{\Phi}' \hat{\Phi}'^\dagger \right) -$$

$$e_3 \left(\epsilon_{abc} \epsilon^{def} \Phi'_d{}^a \Phi'_e{}^b \hat{\Phi}'_f{}^c + \text{h.c.} \right) - c_3 \left[\gamma_1 \ln \left(\frac{\det \Phi'}{\det \hat{\Phi}'^\dagger} \right) + (1 - \gamma_1) \ln \left(\frac{\text{Tr} \left(\Phi' \hat{\Phi}'^\dagger \right)}{\text{Tr} \left(\hat{\Phi}' \Phi'^\dagger \right)} \right) \right]^2$$

Two quark configurations mixing

A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005)
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008)
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009)



$$\begin{aligned}
\mathcal{L}_V = & -\frac{1}{8} \text{Tr} \left(L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \right) + g_1 \left[\text{Tr} \left(\partial_\nu R_\mu R^\mu R^\nu \right) + \text{Tr} \left(\partial_\nu L_\mu L^\mu L^\nu \right) \right] + \\
& g_2 \left[\text{Tr} \left(\partial_\nu R_\mu R^\nu R^\mu \right) + \text{Tr} \left(\partial_\nu L_\mu L^\nu L^\mu \right) \right] + g_3 \left[\text{Tr} \left(R_\mu R^\mu R_\nu R^\nu \right) + \text{Tr} \left(L_\mu L^\mu L_\nu L^\nu \right) \right] + \\
& g_4 \left[\text{Tr} \left(R_\mu R^\nu R_\mu R^\nu \right) + \text{Tr} \left(L_\mu L^\nu L_\mu L^\nu \right) \right] + h_1 \left[\text{Tr} \left(\Phi'^\dagger \Phi' R_\mu R^\mu \right) + \text{Tr} \left(\Phi' \Phi'^\dagger L_\mu L^\mu \right) \right] + \\
& h_2 \left[\text{Tr} \left(L^\mu \Phi' R_\mu \Phi'^\dagger \right) \right] + h_3 \left[\text{Tr} \left(\Phi'^\dagger \partial_\mu \Phi' R^\mu \right) + \text{Tr} \left(\Phi' \partial_\mu \Phi'^\dagger L^\mu \right) \right] + \\
& \frac{a_1}{2} \epsilon_{abc} \epsilon^{def} \left[\left(R_\mu \right)_{ad} \left(R_\nu \right)_{be} \left(\Phi'^\dagger \Phi \right)_{cf} + \left(L_\mu \right)_{ad} \left(L_\nu \right)_{be} \left(\Phi \Phi'^\dagger \right)_{cf} \right] + \\
& \frac{a_2}{2} \epsilon_{abc} \epsilon^{def} \left[\left(R_\mu \right)_{ad} \left(R_\nu \right)_{be} \left(R^\mu R^\nu \right)_{cf} + \left(L_\mu \right)_{ad} \left(L_\nu \right)_{be} \left(L^\mu L^\nu \right)_{cf} \right] + \\
& \frac{a_3}{2} \epsilon_{abc} \epsilon^{def} \left[\left(R_\mu \right)_{ad} \left(R^\mu \right)_{be} \left(R^\nu R_\nu \right)_{cf} + \left(L_\mu \right)_{ad} \left(L^\mu \right)_{be} \left(L^\nu L_\nu \right)_{cf} \right] + \\
& \frac{a_4}{2} \epsilon_{abc} \epsilon^{def} \left[\left(R_\mu \right)_{ad} \left(R^\nu \right)_{be} \left(\partial^\mu R_\nu \right)_{cf} + \left(L_\mu \right)_{ad} \left(L^\nu \right)_{be} \left(\partial^\mu L_\nu \right)_{cf} \right]
\end{aligned}$$

The vector meson masses by spontaneous chiral symmetry breaking

Di-quark approximation

L. Olbrich, Master's thesis, Goethe U., Frankfurt (main) (2015).

L. Olbrich, M. Zétényi, F. Giacosa, and D. H. Rischke, Phys. Rev. D 93, 034021 (2016).

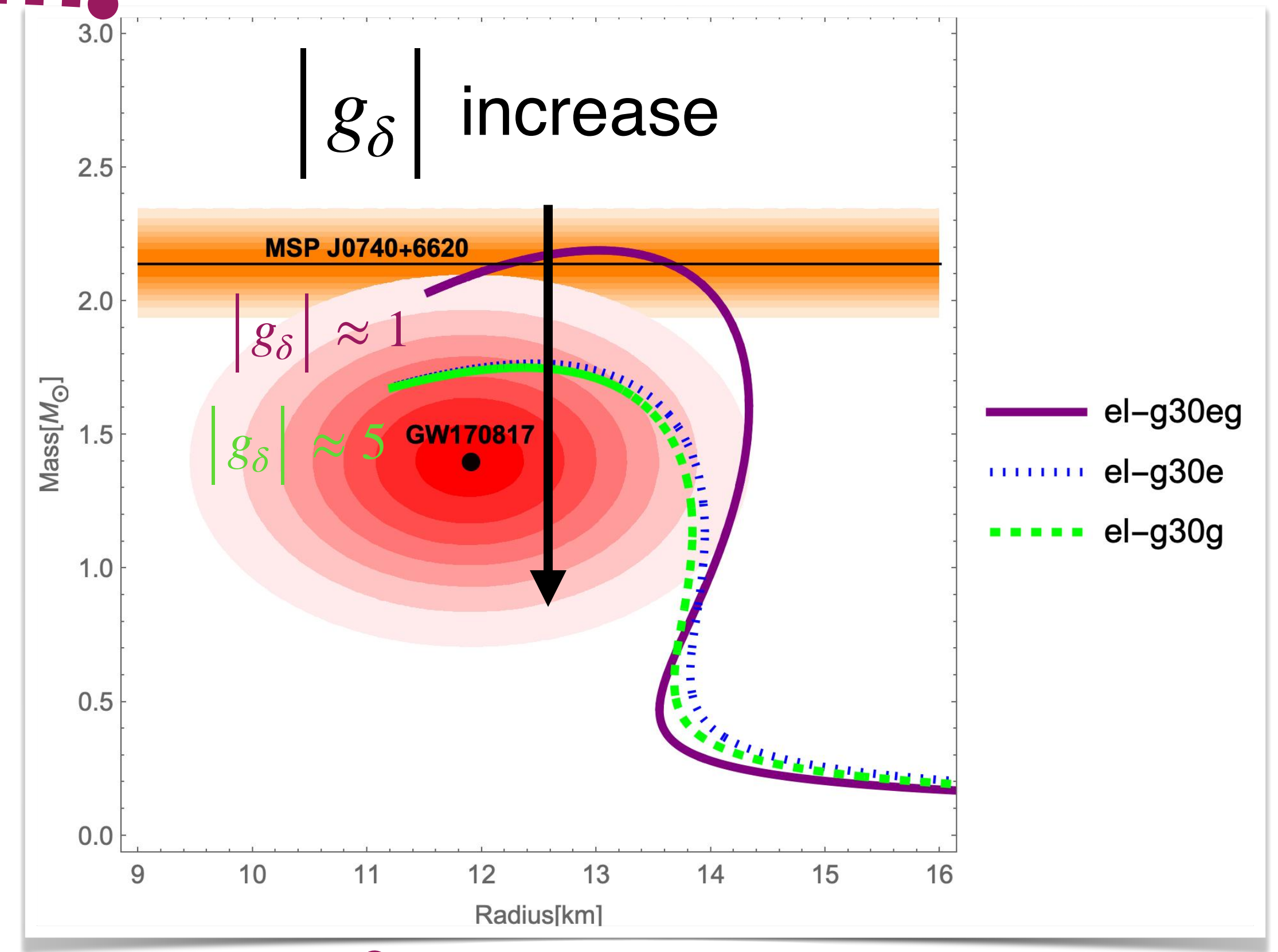
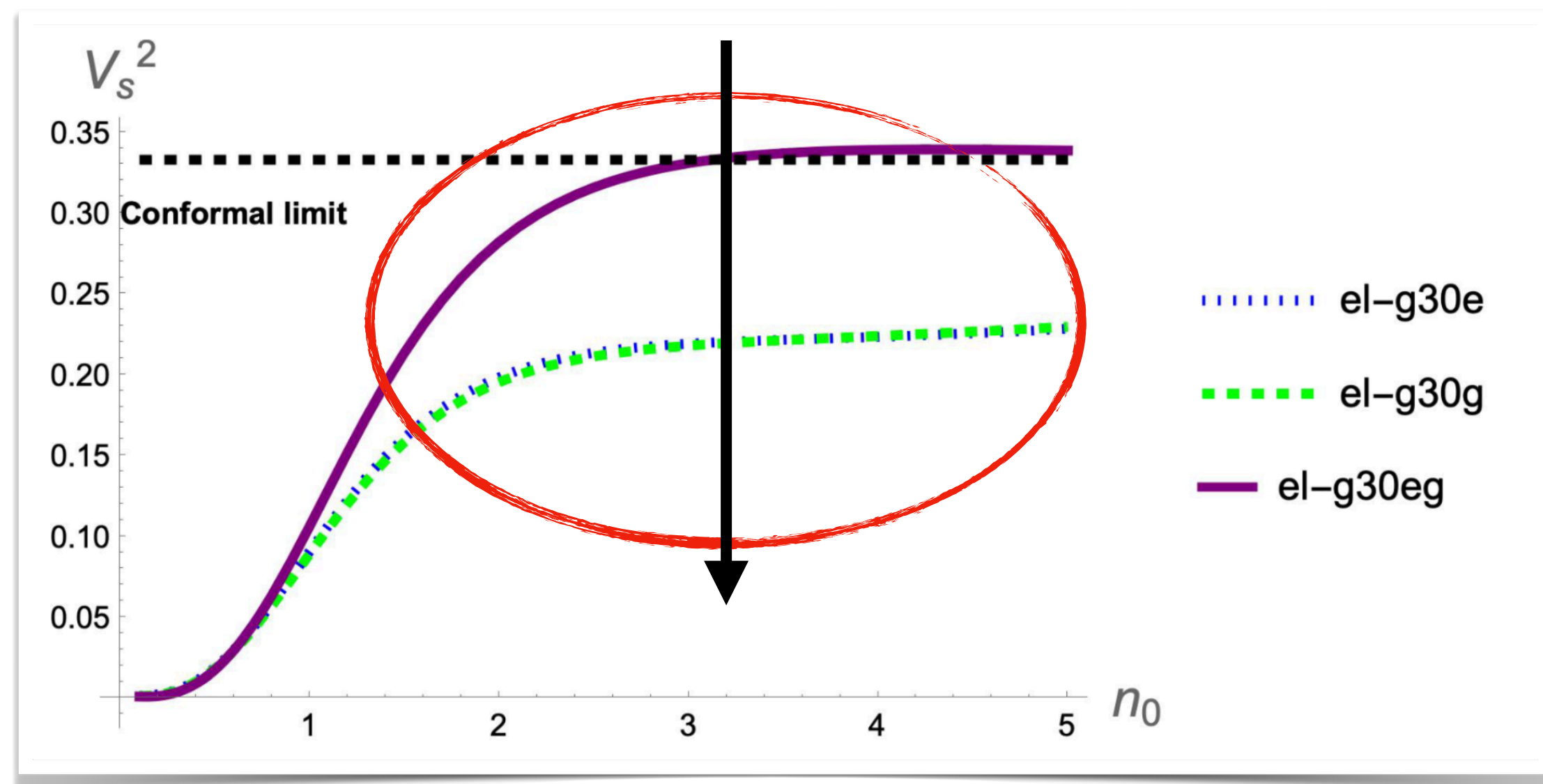
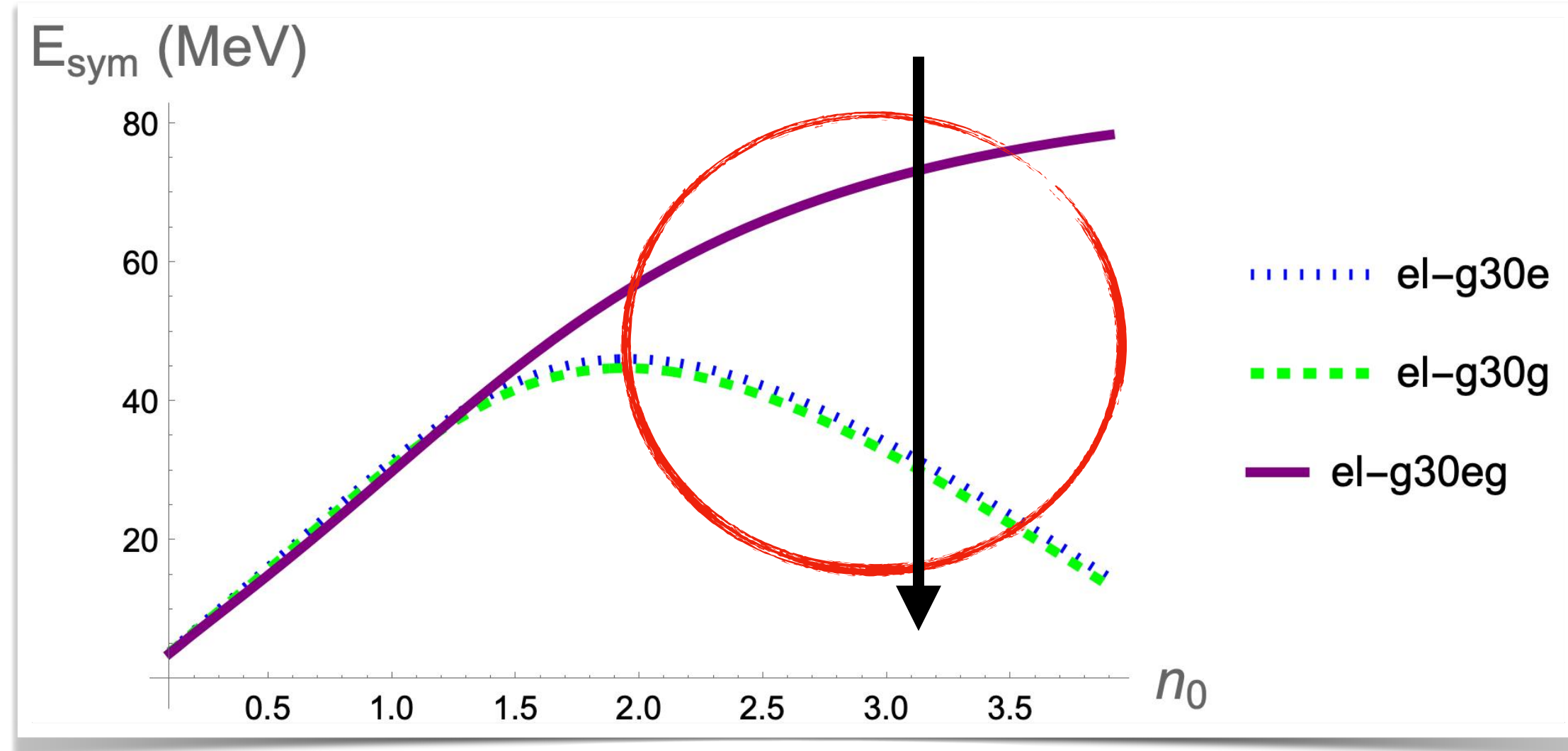


$$N_{R, L}^{(RR)} = \frac{1}{\sqrt{2}} \frac{1 \pm \gamma_5}{2} B, \quad N_{R, L}^{(LL)} = -\frac{1}{\sqrt{2}} \frac{1 \pm \gamma_5}{2} B$$

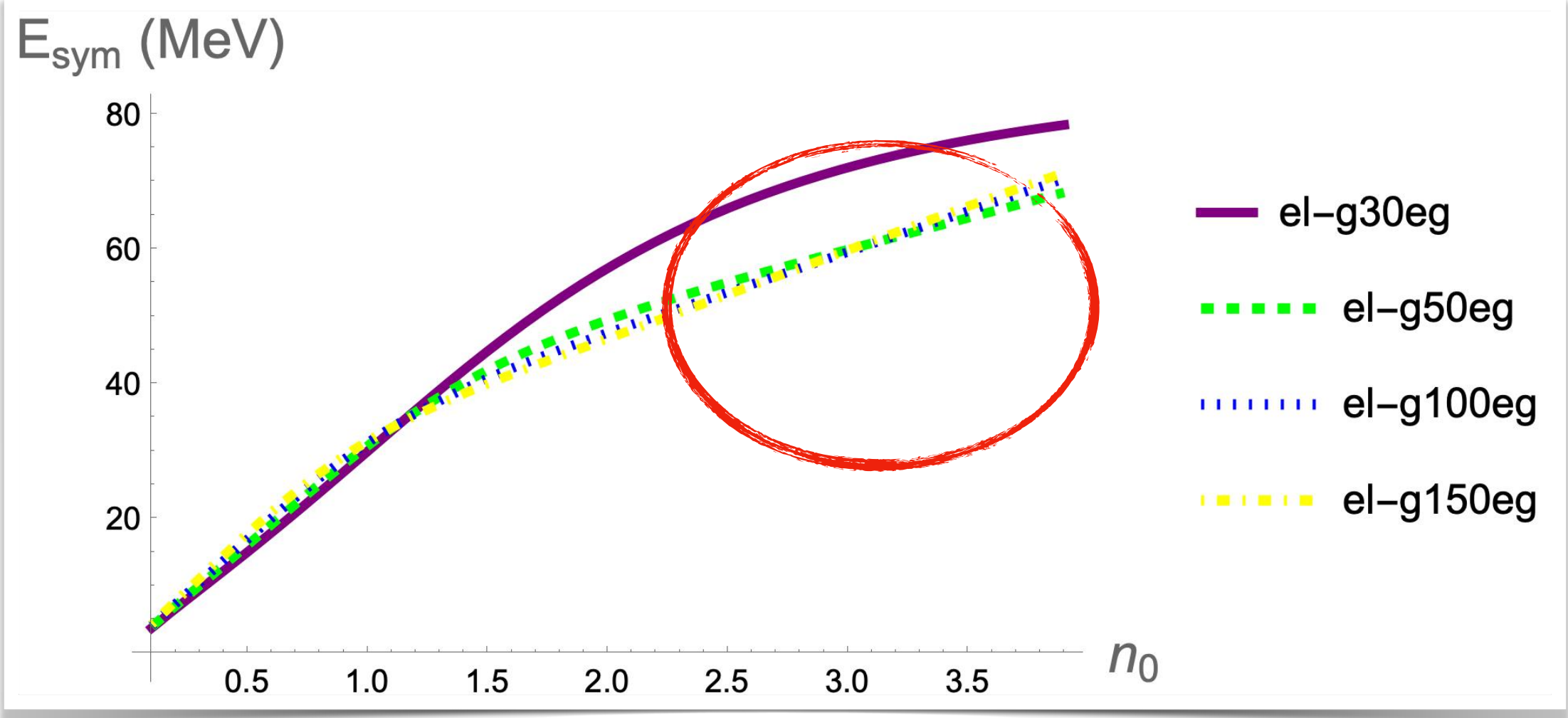
$$\begin{aligned} \mathcal{L}_B = & \text{Tr}(\bar{B} i \not{\partial} B) + c \text{Tr} \left(\bar{B} \gamma_\mu V^\mu B + \bar{B} \gamma_\mu \gamma_5 A^\mu B \right) + c' \text{Tr} \left(\bar{B} \gamma_\mu \bar{B} V^\mu + \bar{B} \gamma_\mu \gamma_5 \bar{B} A^\mu \right) + \\ & h \epsilon_{abc} \epsilon^{def} \left[(\bar{B})_{ad} \gamma_\mu (B)_{be} (V^\mu)_{cf} + (\bar{B})_{ad} \gamma_\mu \gamma_5 (B)_{be} (A^\mu)_{cf} \right] - \\ & \frac{g}{2} \text{Tr} \left[\bar{B} (\Phi' + \Phi'^\dagger) B + \bar{B} \gamma_5 (\Phi' - \Phi'^\dagger) B \right] - \\ & \frac{e}{2} \epsilon_{abc} \epsilon^{def} \left[(\bar{B})_{ad} (\Phi' + \Phi'^\dagger)_{be} (B)_{cf} + (\bar{B})_{ad} \gamma_5 (\Phi' - \Phi'^\dagger)_{be} (B)_{cf} \right] \end{aligned}$$

The baryon masses by spontaneous chiral symmetry breaking

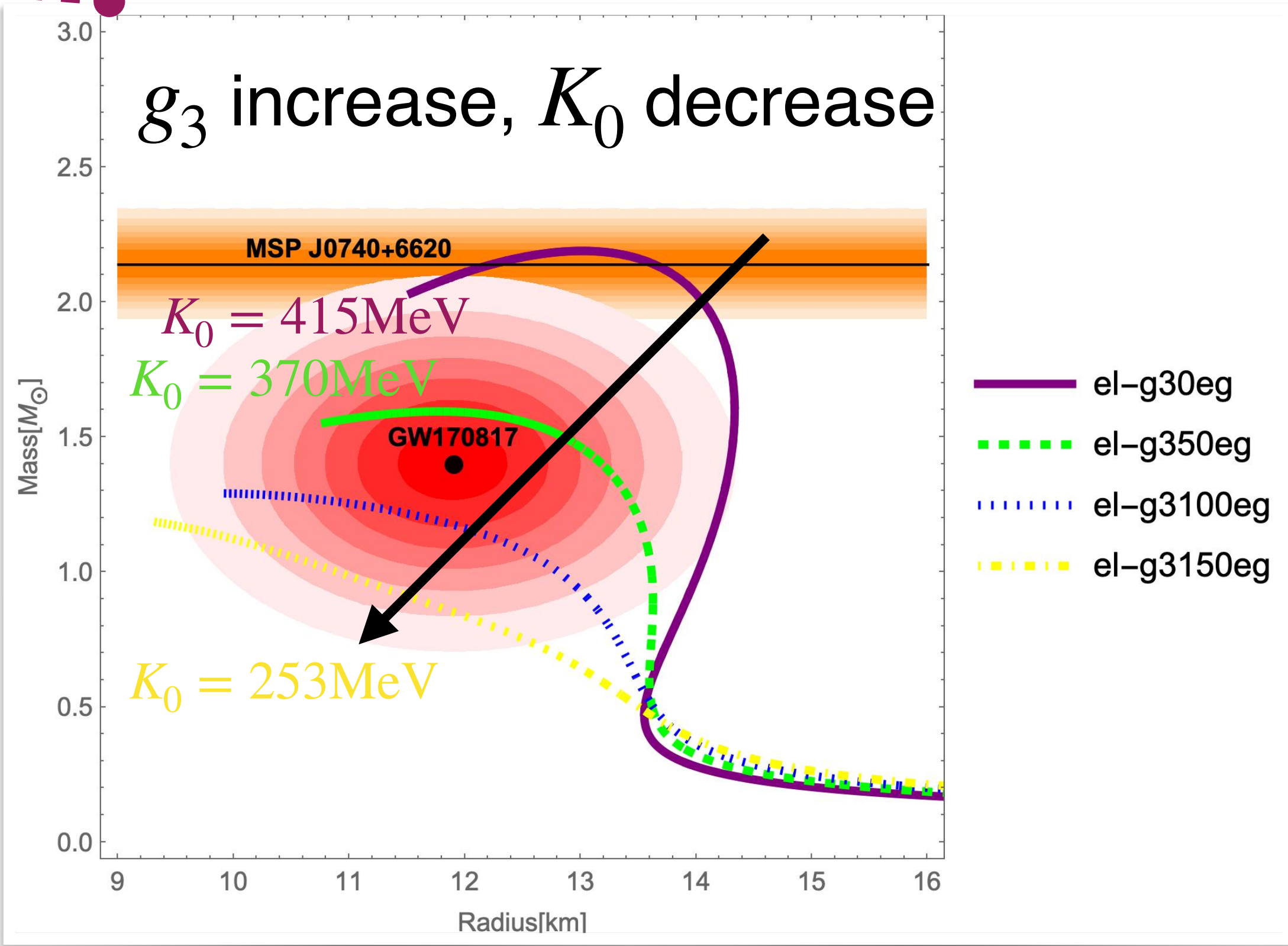
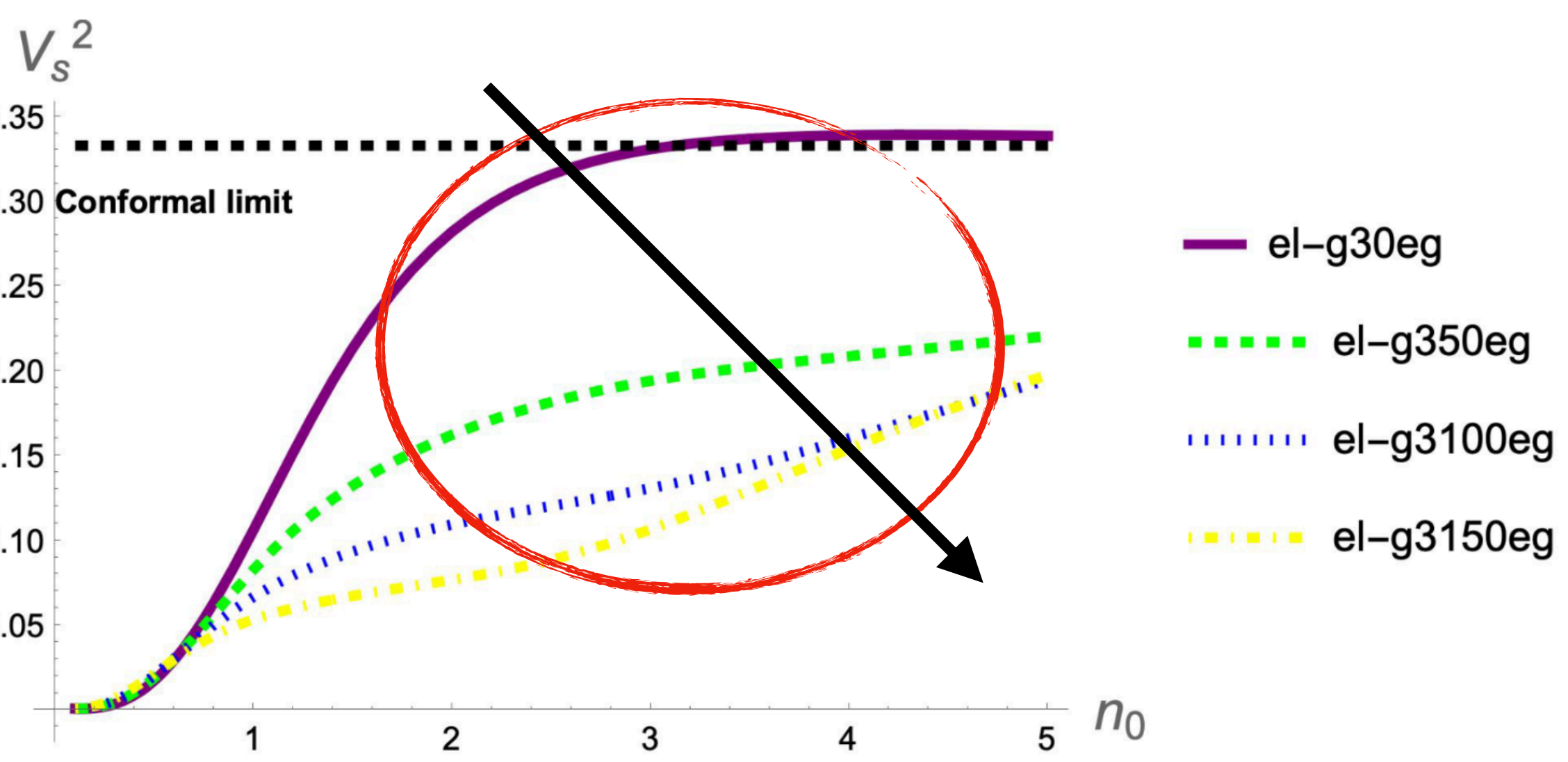
Comparison among different g_{aNN} cases



Comparison among different g_3 cases



H. T. Cromartie et al. (NANOGrav), Nature Astron. 4, 72 (2019).
B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018),



Hyperon effects (Strangeness)

$$\mathcal{L}_b = \sum_b \bar{\psi}_b \left[\gamma^\mu \left(i\partial_\mu - g_{\omega b} \omega_\mu - g_{\rho b} \tau_b \cdot \rho_\mu \right) - (m_b - g_{\sigma b} \sigma) \right] \psi_b$$

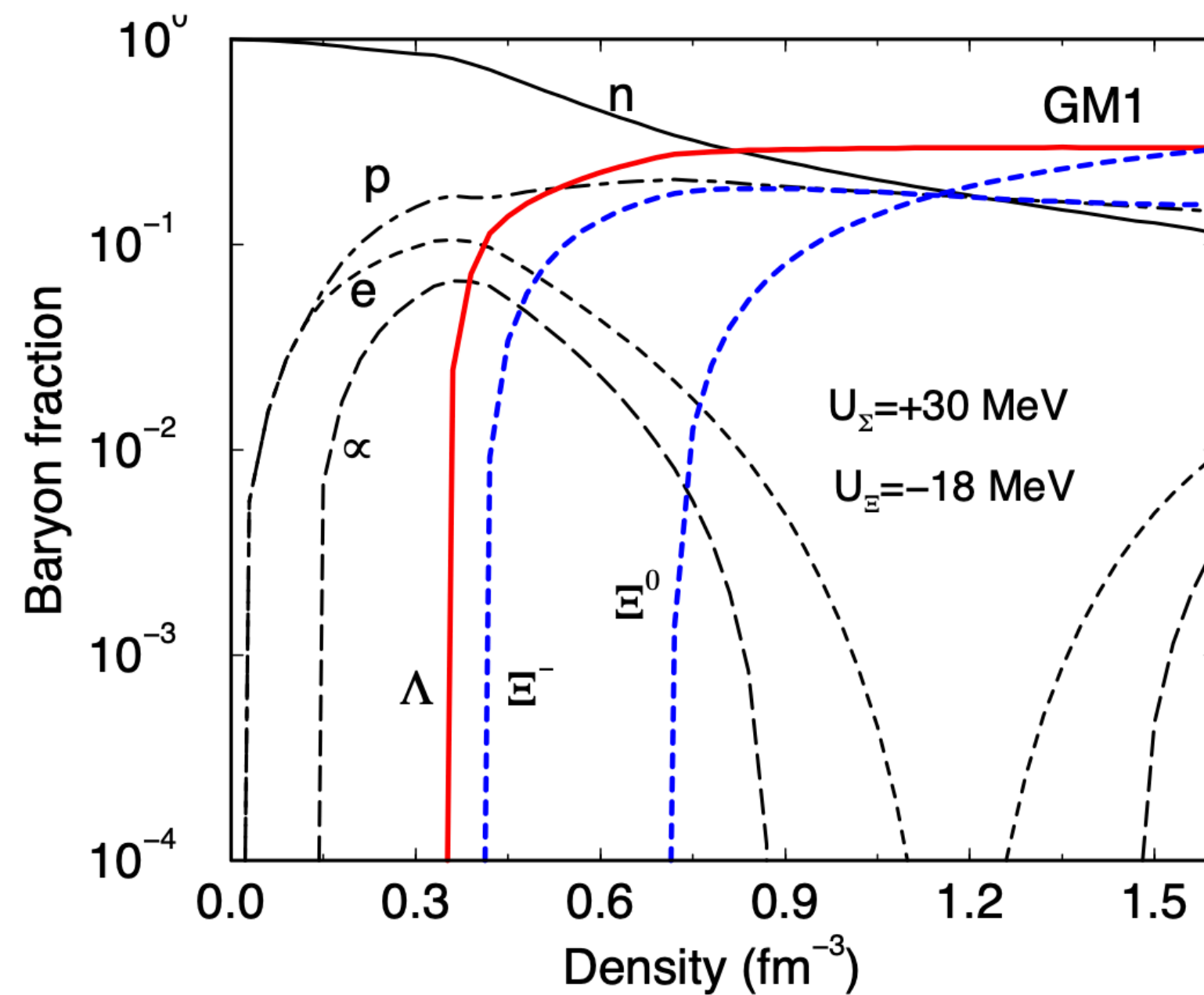


FIG. 31 Neutron star matter fractions of baryons and leptons, calculated as a function of density. From Schaffner-Bielich, 2008.

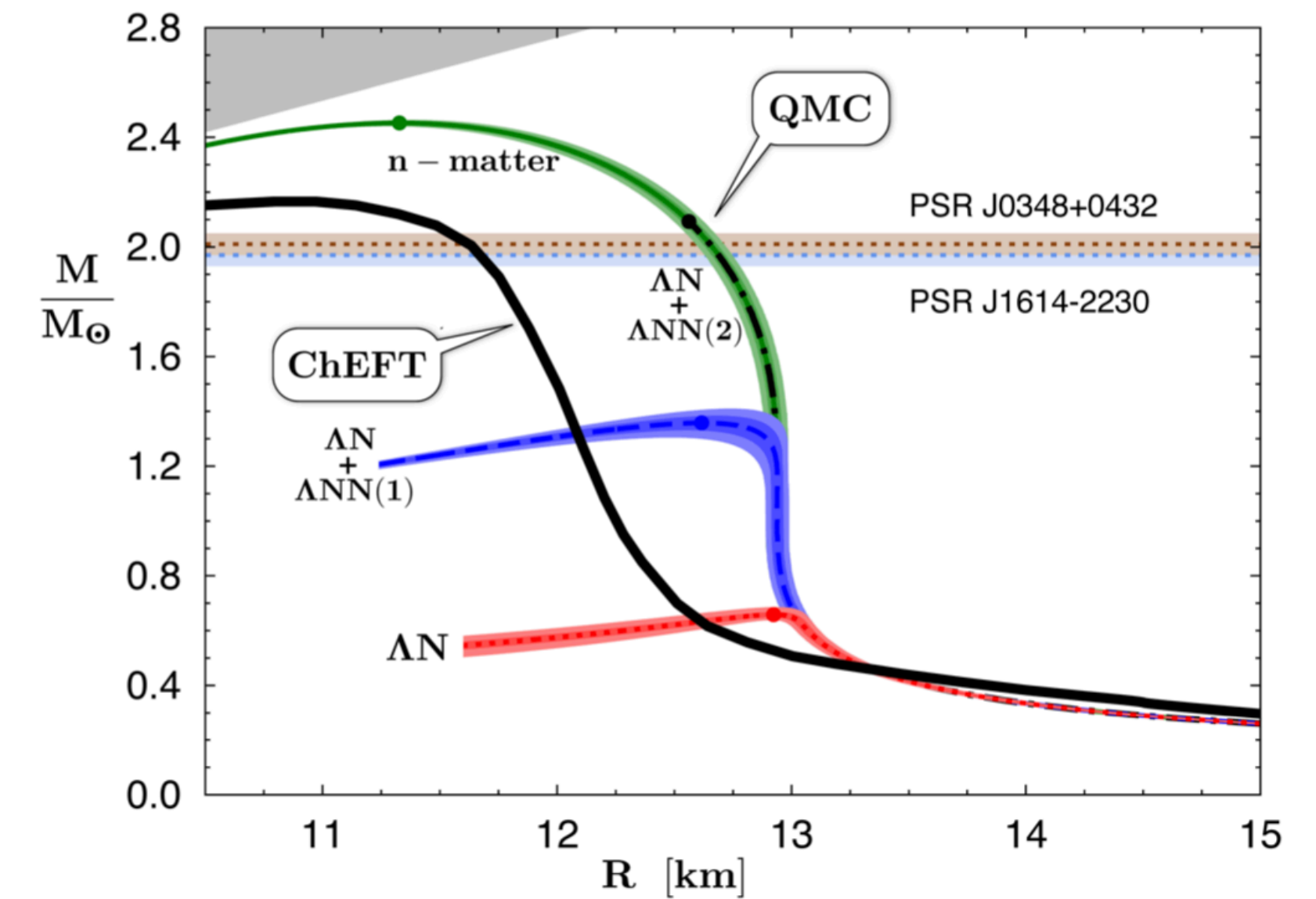


FIG. 33 Mass-radius relationship for various EoS scenarios of neutron stars, including nucleons and leptons only (Hell and Weise, 2014) as well as upon including Λ hyperons (Lonardoni *et al.*, 2015). From Weise, 2015.

Chemical potential equilibrium, e.g. $\mu_\Lambda = \mu_p + \mu_e$

A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016).

Strangeness in this framework under RMF (Neglecting the 4-quark configurations but with **explicit** symmetry breaking terms at leading order)



In collaboration with Lu-Qi Zhang (张璐琦)

$$\mathcal{V}_M = c_2 \text{Tr} S'^2 - c_4 \text{Tr} S'^4 - b_1 \text{Tr} (\xi S'^3) - 2G \text{Tr} (\xi S')$$

$$\begin{aligned} \mathcal{V}_V = & \tilde{h}_2 \text{Tr} (V^2 S'^2) + a_1 \epsilon^{ijk} \epsilon^{lmn} (V)_{il} (V)_{jm} (S'^2)_{kn} + b_2 \text{Tr} (V^2 \xi S') \\ & + b_3 \epsilon^{ijk} \epsilon^{lmn} (V)_{il} (V)_{jm} (\xi S')_{kn} + \tilde{g}_3 \text{Tr} V^4 + \tilde{a}_2 \epsilon^{ijk} \epsilon^{lmn} (V)_{il} (V)_{jm} (V^2)_{kn} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{RMF}}^B = & \text{Tr} (\bar{B} i \gamma_\mu \partial^\mu B) + c \text{Tr} [\bar{B} \gamma_0 V B] + c' \text{Tr} [\bar{B} \gamma_0 B V] \\ & + h \epsilon^{ijk} \epsilon^{lmn} (\bar{B})_{il} (V \gamma_0)_{jm} (B)_{kn} - g \text{Tr} [\bar{B} S' B] \\ & - e \epsilon^{ijk} \epsilon^{lmn} (\bar{B})_{il} (S')_{jm} (B)_{kn} - b_4 \text{Tr} [\bar{B} \xi B] - b_5 \epsilon^{ijk} \epsilon^{lmn} (\bar{B})_{il} (\xi)_{jm} (B)_{kn} \end{aligned}$$

Mass splittings (beta equilibrium)

Strangeness in this framework under RMF (Neglecting the 4-quark configurations)



In collaboration with Lu-Qi Zhang (张璐琦)

$$S = \frac{1}{\sqrt{2}} (\lambda_8 f_0 + \lambda_3 a_0) + \frac{1}{\sqrt{3}} \mathbf{I} \cdot \boldsymbol{\sigma}$$

$$V = \frac{1}{2} \begin{pmatrix} \rho + \omega & 0 & 0 \\ 0 & -\rho + \omega & 0 \\ 0 & 0 & \sqrt{2}\phi \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Neglected in EOS after the determination of mass spectrum

Physical couplings and mass formula



$$g_{\rho NN} = -\frac{1}{2}(c + h)$$

$$g_{\omega NN} = -\frac{1}{2}(c - h)$$

$$g_{\omega\Lambda\Lambda} = -\frac{1}{6}(c + c' - 4h)$$

$$g_{\sigma NN} = g_{\sigma\Lambda\Lambda} = -\frac{1}{\sqrt{3}}(g - e)$$

$$\xi = \xi_1\lambda_1 + \xi_3\lambda_3 + \xi_8\lambda_8$$



Non-zero $\langle a_0 \rangle$ and $\langle f_0 \rangle$

Gell-Mann Okubo Formula

$$m_{\Sigma^+} = \frac{1}{2}(-2m_n + 3m_\Lambda - 2m_{\Xi^-} + 3m_{\Sigma^0})$$

$$m_{\Sigma^-} = \frac{1}{2}(2m_n - 3m_\Lambda + 2m_{\Xi^-} + m_{\Sigma^0})$$

$$m_{\Xi^0} = -m_n - m_p + 3m_\Lambda - m_{\Xi^-} + m_{\Sigma^0}.$$

$$m_{a_0}^2 + m_{f_0}^2 = 2m_\sigma^2$$

Hadron spectra and nuclear matter properties

	Empirical	$\sigma_{\pi N-75}^{+}$	$\sigma_{\pi N-100}^{-}$	$\sigma_{\pi N-400}^{-}$	$\sigma_{\pi N-600}^{-}$
$n_0(\text{fm}^{-3})$	0.1550 ± 0.0500	0.1592	0.1592	0.1592	0.1592
$e_0(\text{MeV})$	-15.00 ± 1.00	-16.00	-16.00	-16.02	-16.00
$K(n_0)(\text{MeV})$	230.0 ± 30.0	240.8	241.3	241.6	239.2
$E_{\text{sym}}(n_0)(\text{MeV})$	30.90 ± 1.90	30.00	30.00	30.01	30.00
$L(n_0)(\text{MeV})$	52.50 ± 17.50	54.91	52.18	70.94	76.74

→ Align with the empirical values

$$U_{\Lambda} \approx -28 \text{ MeV}$$

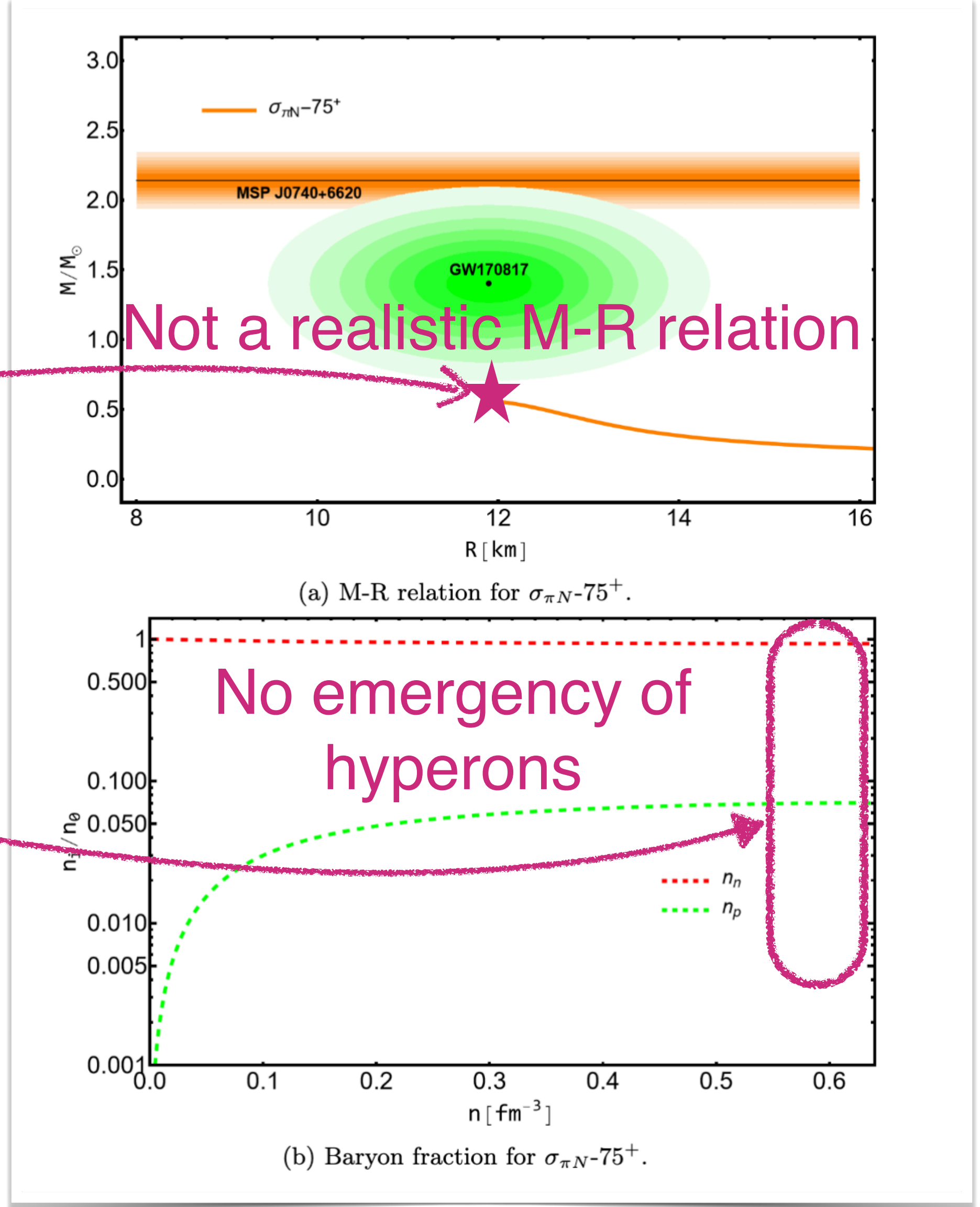
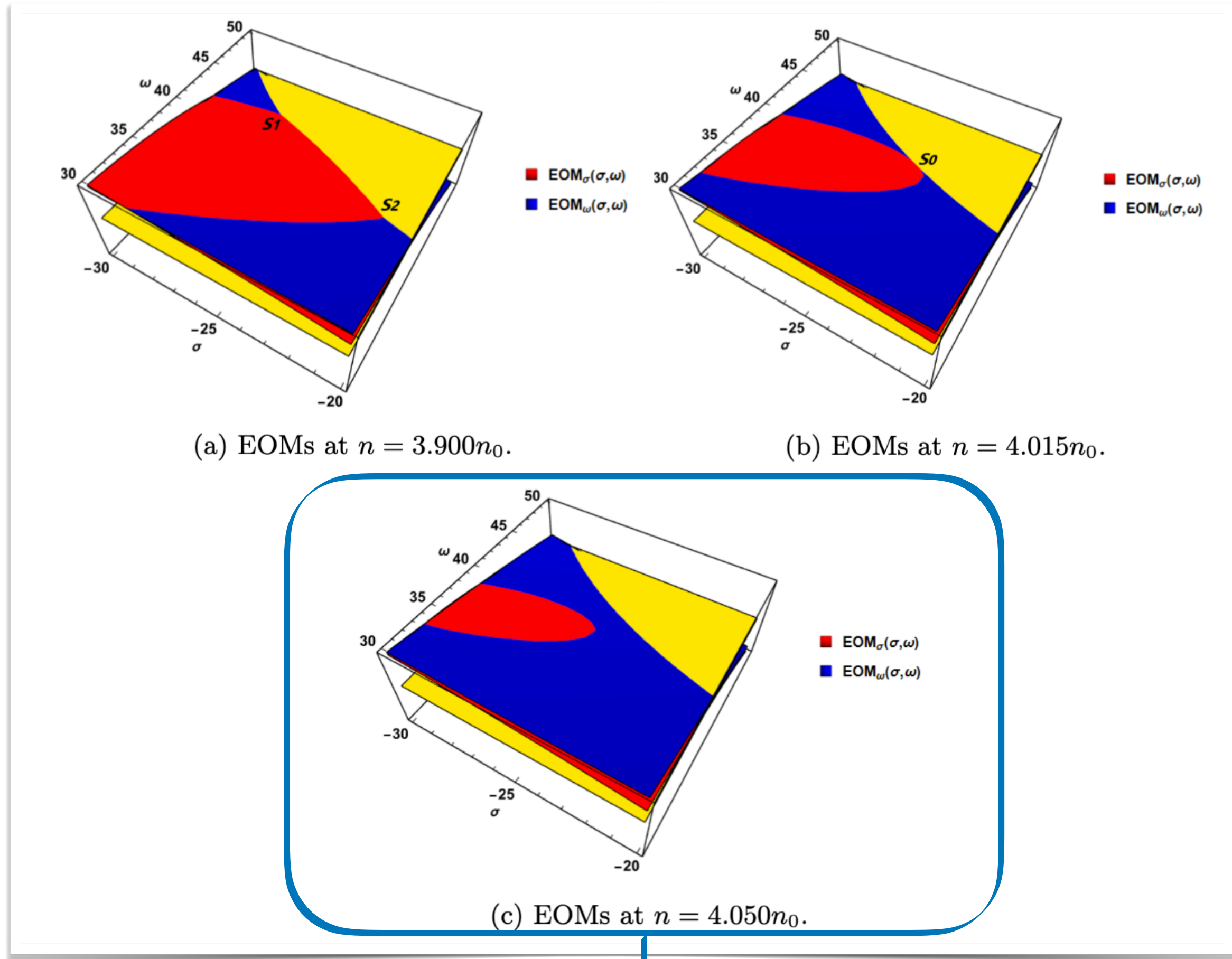
$$m_N = m + \sigma_{\pi N}(0)$$

Account for the contribution of spontaneous and explicit breaking

	Empirical	$\sigma_{\pi N-75}^{+}$	$\sigma_{\pi N-100}^{-}$	$\sigma_{\pi N-400}^{-}$	$\sigma_{\pi N-600}^{-}$
m_n	$939.565 \pm (5 \times 10^{-7})$	939.565	939.565	939.565	939.565
m_p	$938.272 \pm (2.9 \times 10^{-7})$	938.272	938.272	938.272	938.272
m_{Λ}	$1115.68 \pm (0.006)$	1115.59	1115.54	1115.57	1115.58
m_{Σ^+}	1189.37 ± 0.06	1192.30	1191.21	1191.74	1191.79
m_{Σ^0}	1192.64 ± 0.02	1192.17	1192.18	1192.84	1191.98
m_{Σ^-}	1197.45 ± 0.03	1192.03	1191.15	1191.95	1192.16
m_{Ξ^0}	1314.82 ± 0.21	1331.34	1330.65	1330.90	1330.90
m_{Ξ^-}	1321.70 ± 0.09	1329.77	1329.30	1329.82	1329.98
m_{a_0}	995.000 ± 25.000	980.132	979.998	980.024	980.040
m_{f_0}	995.000 ± 15.000	990.119	989.962	989.971	989.974
m_{σ}	475.000 ± 75.000	985.183	984.993	986.010	985.019
m_{ρ}	773.000 ± 2.000	775.261	775.260	775.260	775.260
m_{ω}	782.660 ± 0.13	782.659	782.660	782.660	782.660
m_{ϕ}	1019.46 ± 0.02	1019.46	1019.46	1019.46	1019.46
$\sigma_{\pi N}(0)$	60.5000 ± 28.5000	75.0541	-104.048	-402.087	-625.663
m	878.419 ± 29.1465	863.946	1043.05	1341.09	1564.66

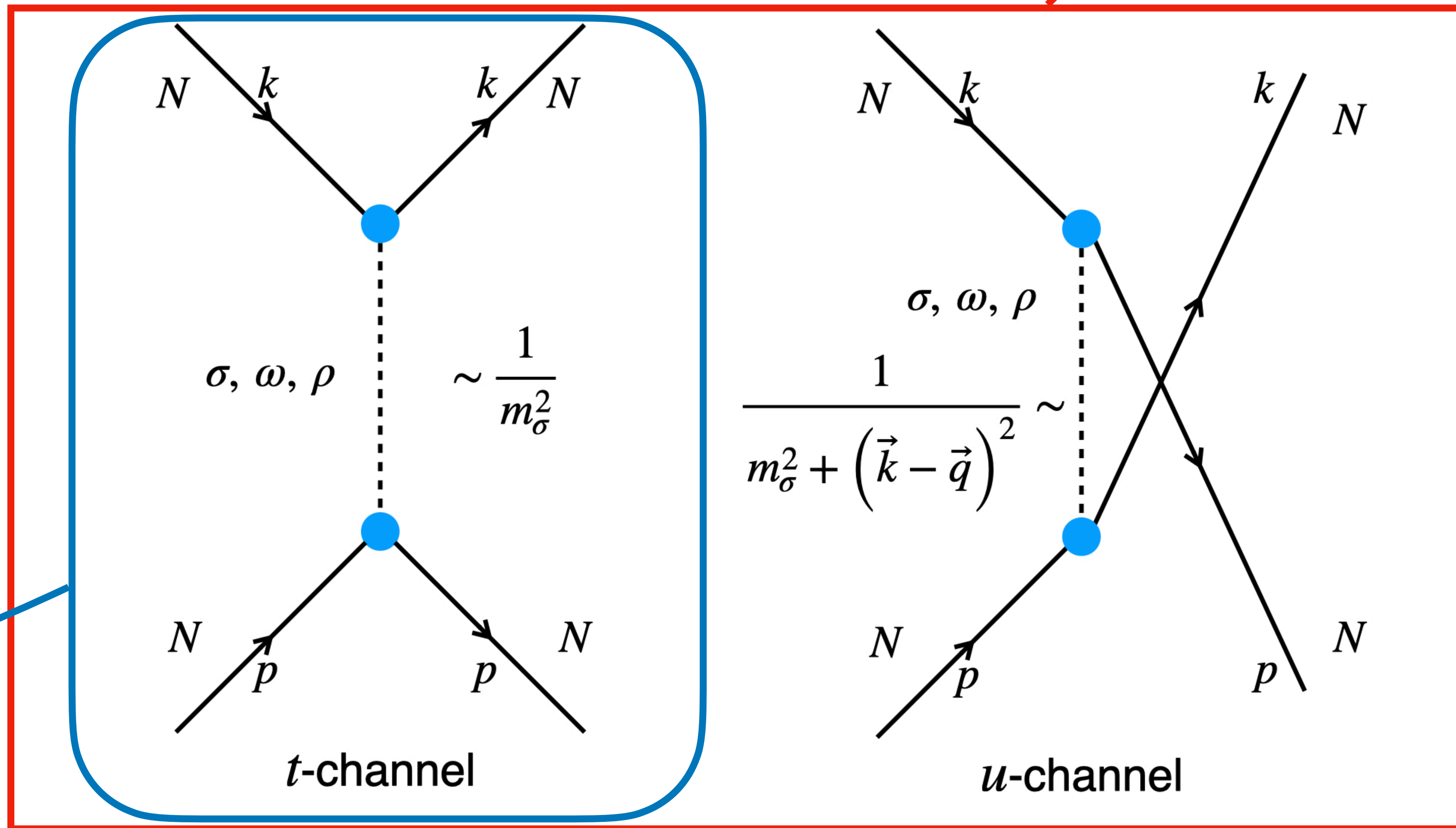
Problems in M-R relation and RMF approach for $\sigma_{\pi N-75}^+$

Solution plane for EOMs of mesons

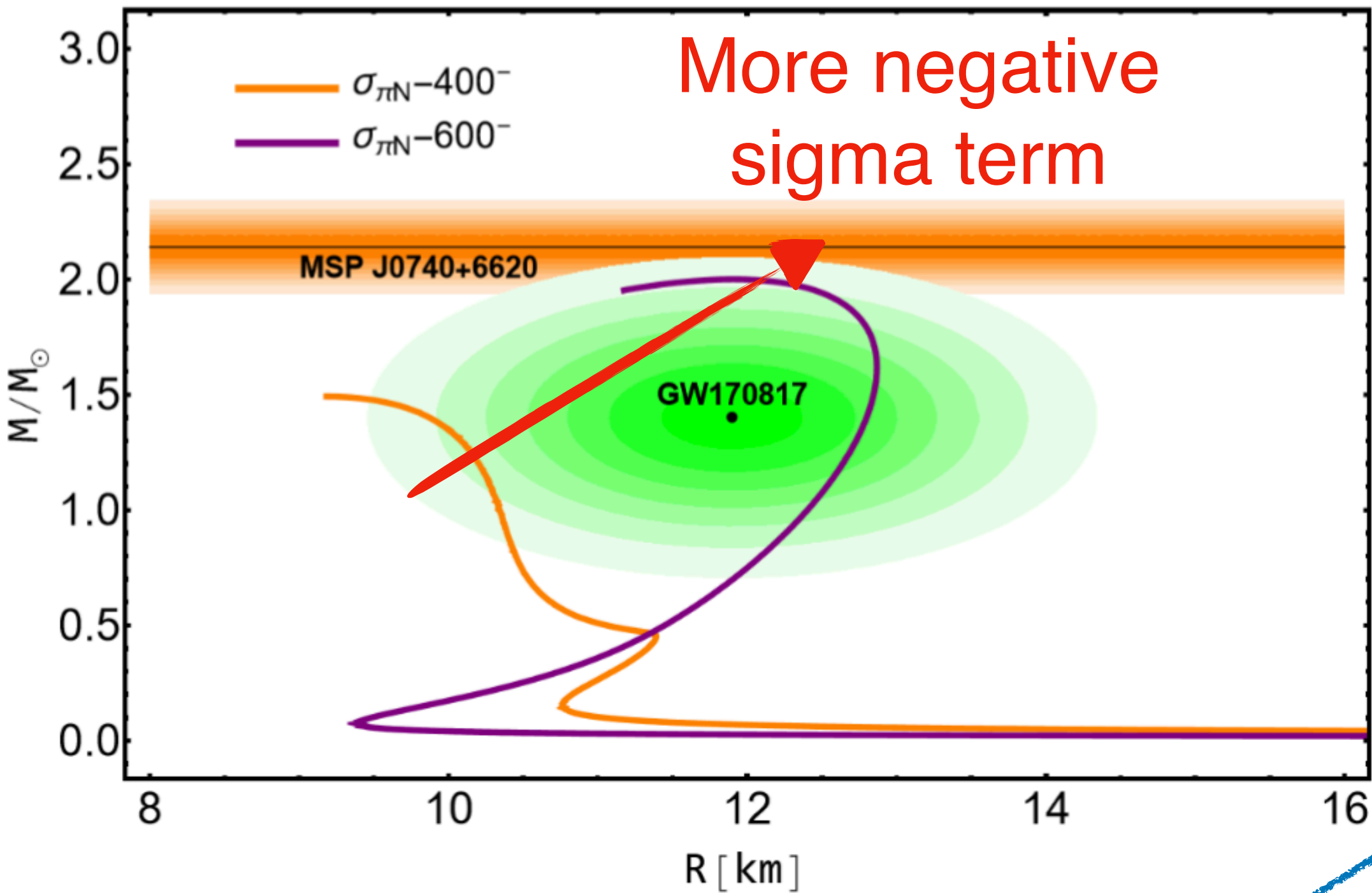


Ignorance of exchange terms of nucleon interaction

Relativistic
mean field

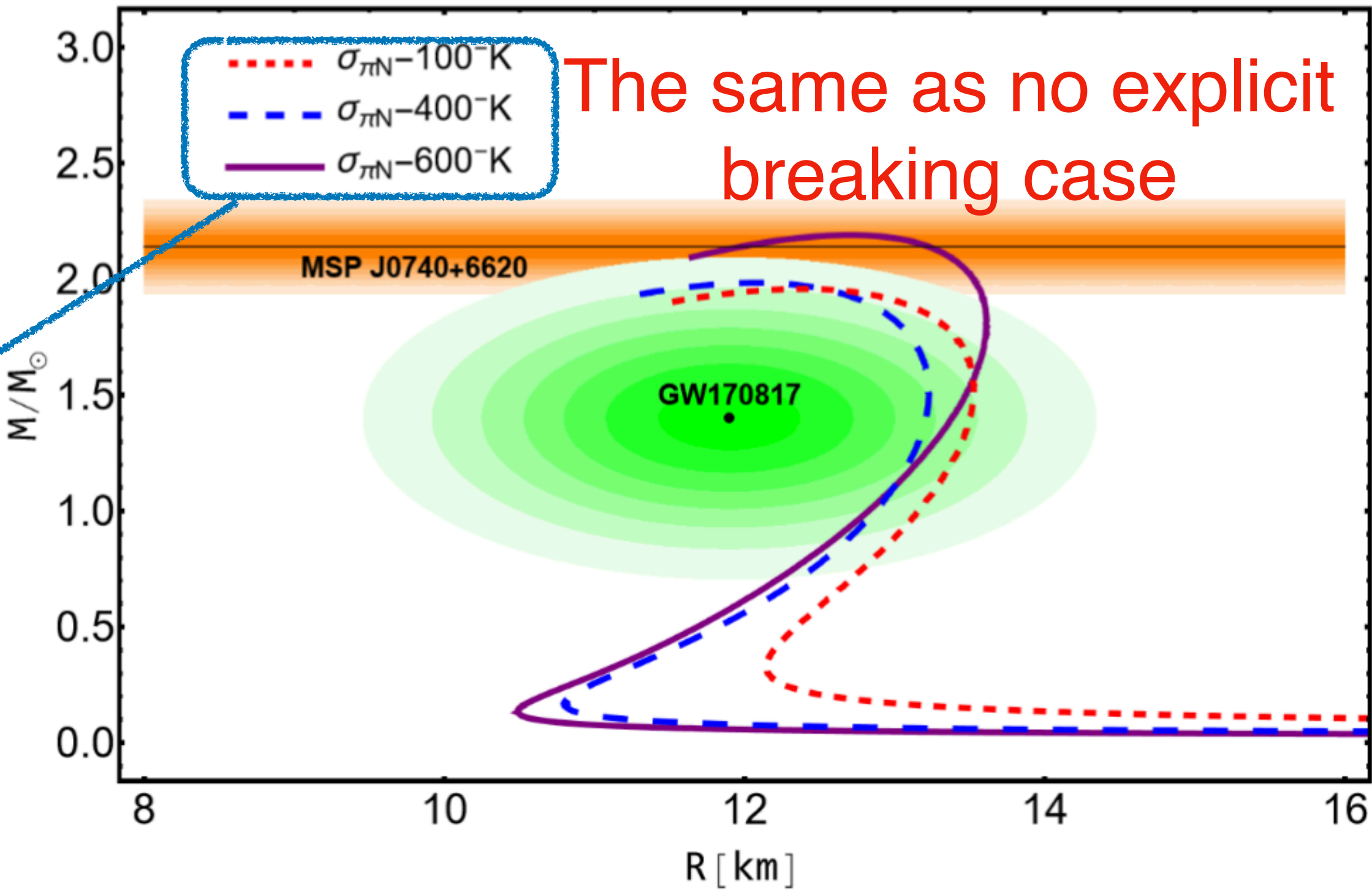


Effects of sigma term and incompressibility



Still negative sigma term

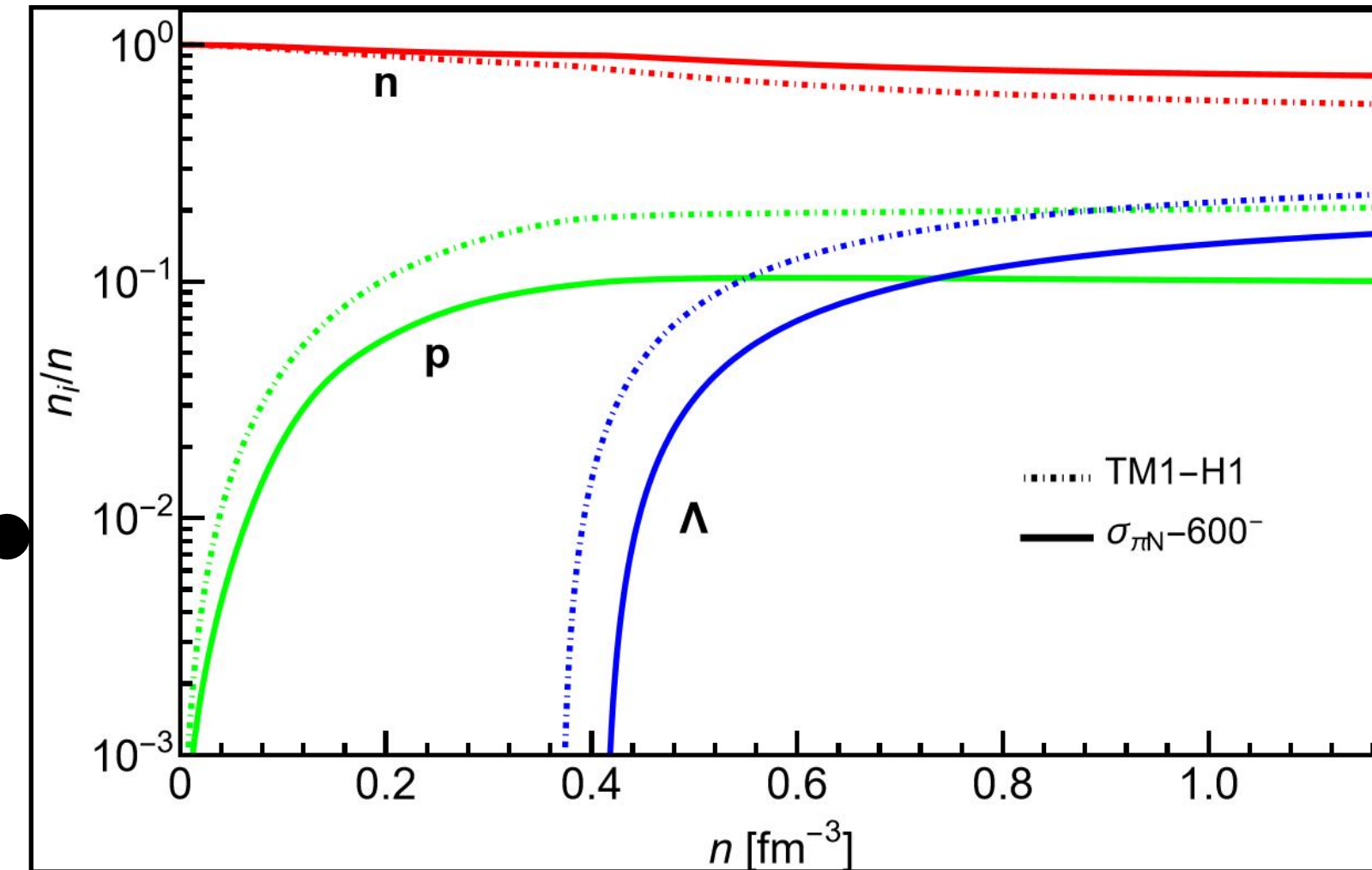
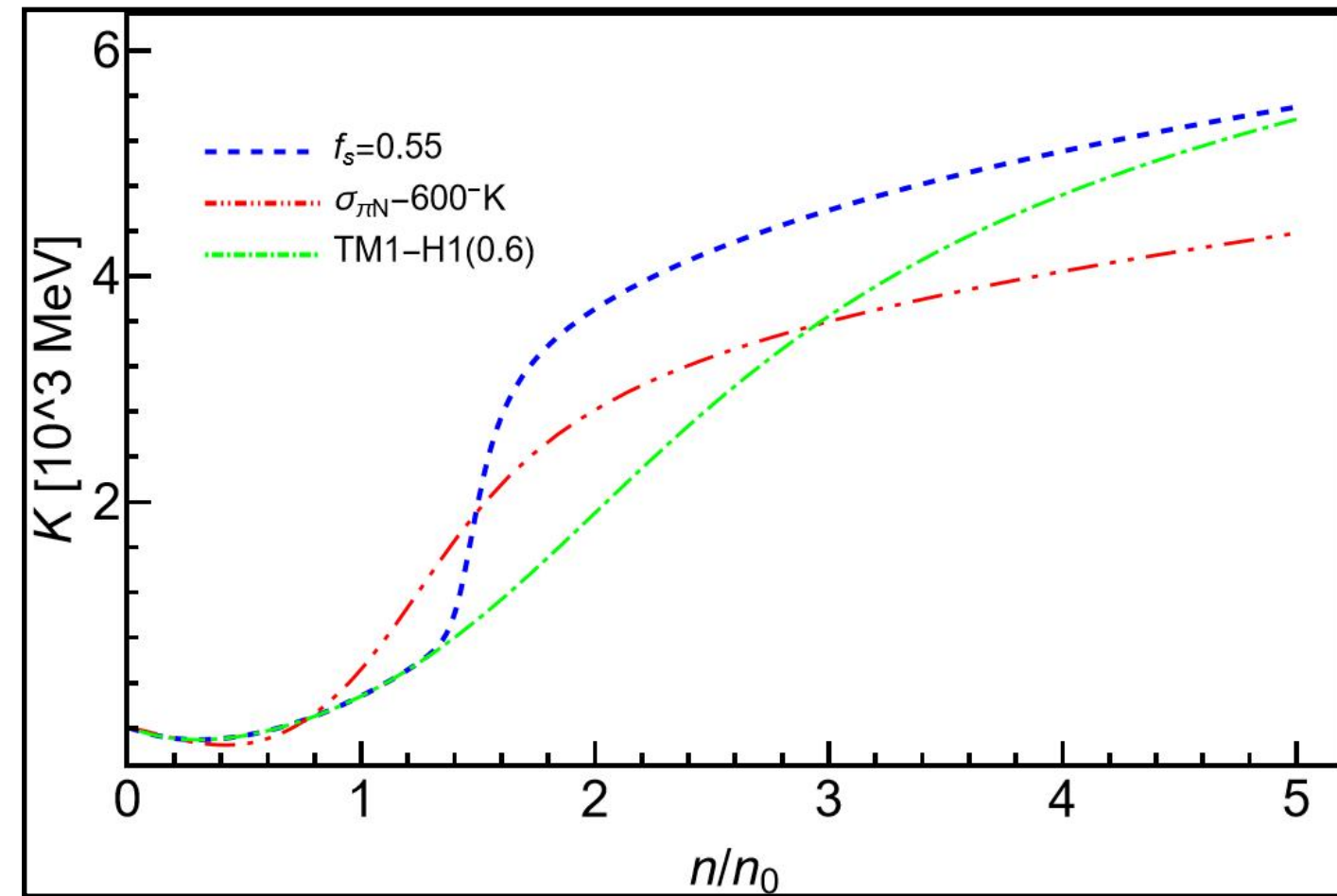
	Empirical	$\sigma_{\pi N} = -100$ MeV	$\sigma_{\pi N} = -400$ MeV	$\sigma_{\pi N} = -600$ MeV
n_0 (fm ⁻³)	0.1550 ± 0.0500	0.1592	0.1592	0.1592
e_0 (MeV)	-15.00 ± 1.00	-16.00	-15.99	-16.00
$K(n_0)$ (MeV)	—	521.9	520.3	518.8
$E_{\text{sym}}(n_0)$ (MeV)	30.90 ± 1.90	30.00	30.00	30.01
$L(n_0)$ (MeV)	52.50 ± 17.50	80.15	93.28	76.64



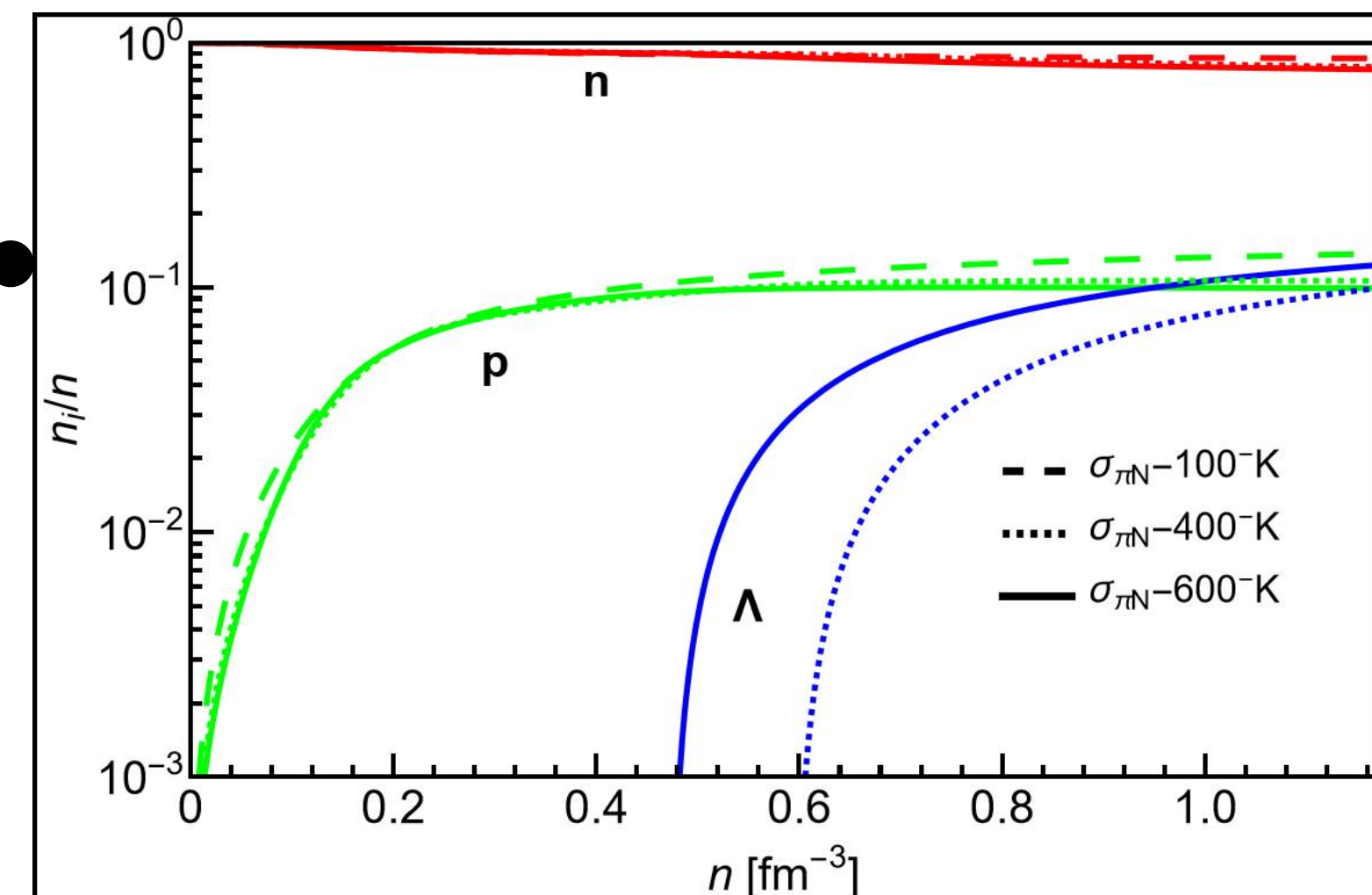
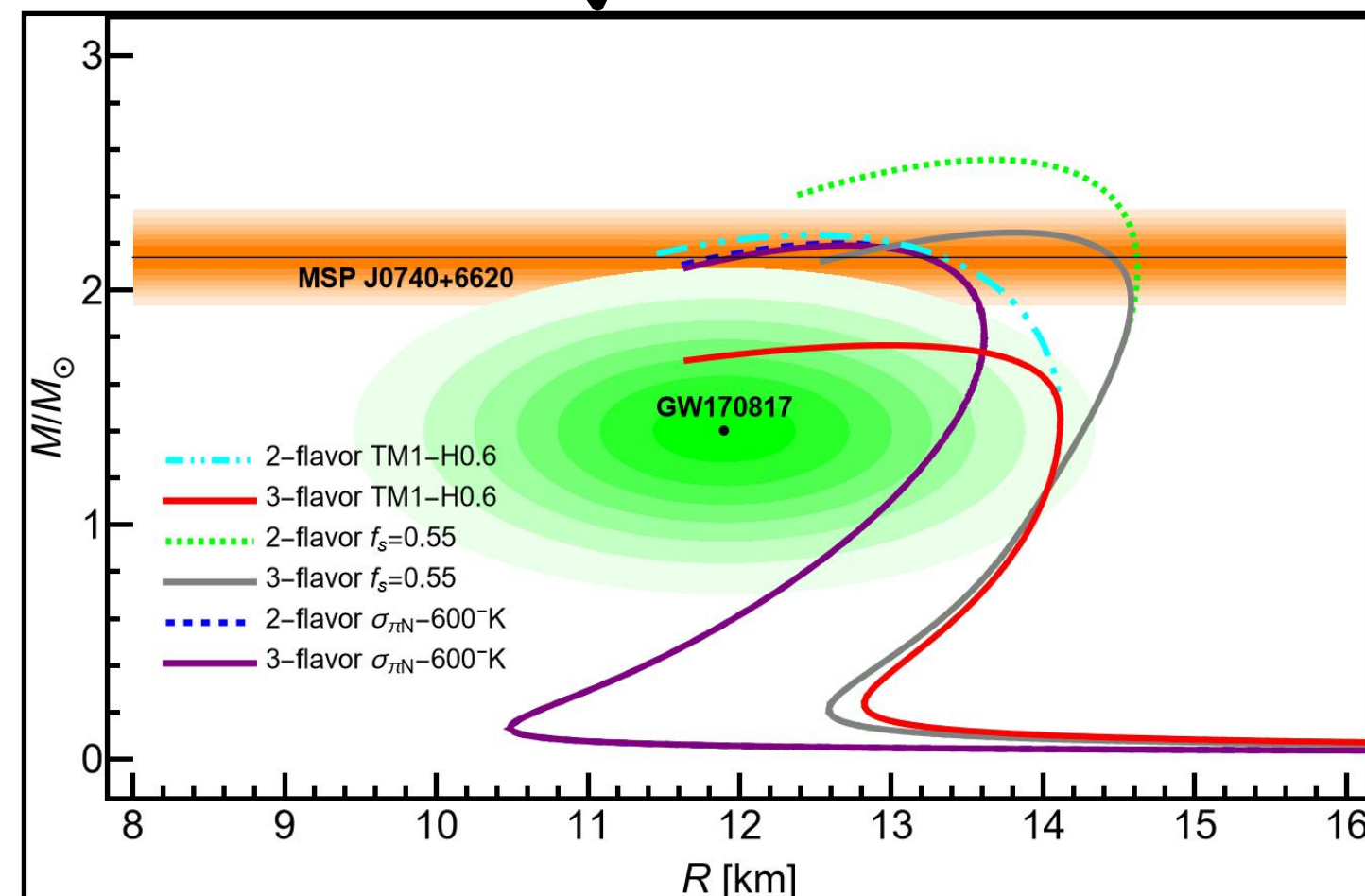
Comparison with Walecka models

Y. Zhang, J. Hu, and P. Liu, Phys. Rev. C 97, 015805 (2018).

K. A. Maslov, E. E. Kolomeitsev, and D. N. Voskresensky, Phys. Rev. C 92, 052801 (2015).



Hyperon
Ratio



LO chiral-scale Lagrangian

$$\begin{aligned}\mathcal{L}_M = & f_\pi^2 \Phi^2 \text{Tr} \left(\hat{\alpha}_\perp^\mu \hat{\alpha}_{\mu\perp} \right) + f_{\sigma\rho}^2 \Phi^2 \text{Tr} \left(\hat{\alpha}_\parallel^\mu \hat{\alpha}_{\mu\parallel} \right) + f_0^2 \Phi^2 \text{Tr} \left(\hat{\alpha}_\parallel^\mu \right) \text{Tr} \left(\hat{\alpha}_{\mu\parallel} \right) - \\ & \frac{1}{2g_\rho^2} \text{Tr} \left(V_{\mu\nu} V^{\mu\nu} \right) - \frac{1}{2g_0^2} \text{Tr} \left(V_{\mu\nu} \right) \text{Tr} \left(V^{\mu\nu} \right) + \\ & \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{f_\pi^2}{4} \Phi^{3-\gamma_m} \text{Tr} \left(\mathcal{M} U^\dagger + U \mathcal{M}^\dagger \right) + h_5 \Phi^4 + h_6 \Phi^{4+\beta'}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_B = & \bar{N} i \gamma_\mu D^\mu N - m_N \Phi \bar{N} N \\ & + \left[g_A C_A + g_A (1 - C_A) \Phi^{\beta'} \right] \bar{N} \hat{\alpha}_\perp^\mu \gamma_\mu \gamma_5 N + \left[g_{V_\rho} C_{V_\rho} + g_{V_\rho} (1 - C_{V_\rho}) \Phi^{\beta'} \right] \bar{N} \hat{\alpha}_\parallel^\mu \gamma_\mu N \\ & + \frac{1}{2} \left[g_{V_0} C_{V_0} + g_{V_0} (1 - C_{V_0}) \Phi^{\beta'} \right] \text{Tr} \left[\hat{\alpha}_\parallel^\mu \right] \bar{N} \gamma_\mu N\end{aligned}$$

H. K. Lee, W.-G. Paeng, and M. Rho, Phys. Rev. D 92, 125033 (2015)
Y. L. Li, Y. L. Ma, and M. Rho, Phys. Rev. D 95, 114011 (2017)