An Infrared Topological Approach to Baryon Structure

第十一届手征有效场论研讨会 • 兰州大学 • 2026年8月13日

林 凡, 王昕杨, arXiv:2601.20274v3 [hep-th]

安徽理工大学, 力学与光电物理学院, 基础物理研究中心

Introduction

Quantum Chromodynamics

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f \left(\mathrm{i}\not{D} - m_f \right) \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

Perturbative UV: Asymptotic Freedom

$$\beta(g) < 0, \qquad \alpha_s(Q^2) \propto \ln^{-1}(Q^2/\Lambda^2_{\text{QCD}})$$

Non-Perturbative IR Dynamics: Core Triad

Yang-Mills Mass Gap   Color Confinement   Chiral Symmetry Breaking (χSB)

Center Vortices & Area Law

Center Topological Defects

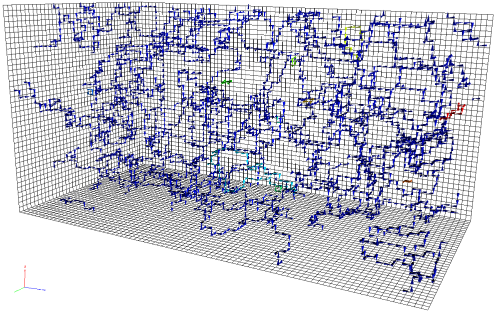
$$z_k = e^{i\frac{2\pi k}{N}} \cdot \mathbb{I} \in \mathbb{Z}_N \subset SU(N)$$

Wilson Loop Phase Shift

$$W(C) \longrightarrow z_k W(C)$$

Random center vortex piercings induce complete statistical dephasing (Area Law):

$$\langle W(C) \rangle \sim \exp\left[-\sigma \cdot \mathrm{Area}(C)\right]$$



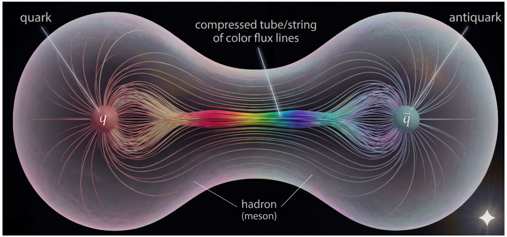
’t Hooft, Nucl. Phys. B 190, 455 (1981); Greensite, Prog. Part. Nucl. Phys. 51, 1 (2003).

Monopole Condensation & Color Flux Tube

Dual Superconducting Vacuum

Color monopole condensation expels chromo-electric fields via the dual Meissner effect, squeezing them into flux tubes:

$$V(R) \simeq \sigma R$$



Color Magnetic Dominance QCD Vacuum: Color monopole condensates account for local flux tube formation; center vortices govern global phase dephasing. A complete framework requires their dynamical unification.

Nambu, Phys. Rev. D 10, 4262 (1974); Mandelstam, Phys. Rep. 23, 245 (1976).

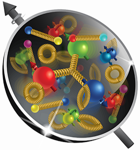
Baryon Structure: Condensates & Topological Defects

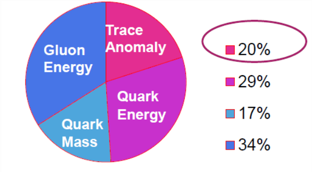
Infrared Condensate Phase

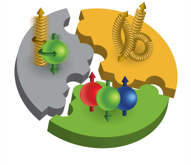
As energy flows into the deep infrared (IR), the quantum state transitions into a highly non-trivial condensate state.

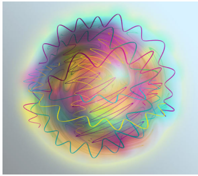
Emergent Topological Defects

Topological defects universally dictate long-range dynamics: Kinks (1+1D), KT vortices (2+1D).





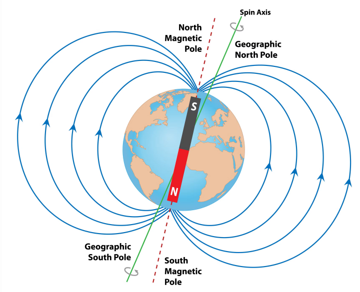


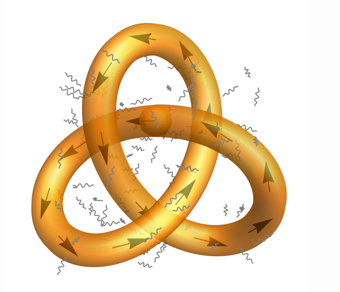


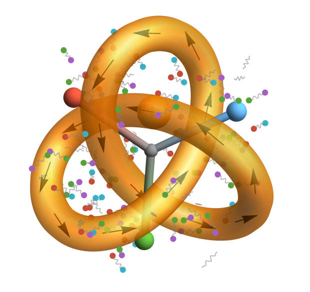
Kosterlitz, Thouless, J. Phys. C 6, 1181 (1973)

Central Paradigm: The Gluon Knot

- **Fundamental IR Degree of Freedom :** Gluon knots act as the fundamental infrared degrees of freedom in Yang-Mills theory.
- **Baryon Frame Architecture:** The physical baryon embeds a gluon knot as its structural frame.
- **Unified Mechanism:** Unifies flux tube squeezing (confinement) with dynamical chiral symmetry breaking.







Chiral Effective Field Theory & Skyrmion

Chiral Symmetry Breaking & χEFT

$$\mathcal{L}_\chi = \frac{f_\pi^2}{4}\,\mathrm{Tr}(\partial^\mu U^\dagger \partial_\mu U) + \cdots\,, \quad U(x) \in \mathrm{SU}(N_f)$$

Spontaneous symmetry breaking gives rise to pseudo-Nambu-Goldstone bosons (π, K).

Topological Solitons (Skyrmions)

$$\pi_3(\mathrm{SU}(N_f)) = \mathbb{Z} \quad \longleftrightarrow \quad B^\mu \propto \epsilon^{\mu\nu\sigma\rho}\,\mathrm{Tr}(U^\dagger \partial_\nu U U^\dagger \partial_\sigma U U^\dagger \partial_\rho U)$$

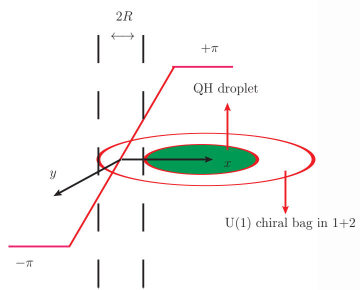
Skyrmions stabilize chiral solitons where topological charge directly maps to baryon number via anomaly matching.

Skyrme, Proc. R. Soc. A 260, 127 (1961); Witten, Nucl. Phys. B 223, 422 (1983).

One-Flavor Baryons on η' Domain Walls

Quantum Hall Droplets, Chiral Bags, and Vortex Solitons

- η' domain walls support $SU(N_c)_{-1} \leftrightarrow U(1)_{N_c}$ Chern-Simons level-rank duality.
- Baryons emerge as Quantum Hall droplets / chiral bags via the Cheshire Cat Principle.
- Chern-Simons-Higgs effective description realizes baryons as quantum vortex excitations.



$$S_A[a,\phi] = \int \mathrm{d}^3x \left[|\partial_\mu\phi - \mathrm{i}a_\mu\phi|^2 - V(\phi^*\phi) + \frac{N_c}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho \right]$$

Zohar Komargodski, arXiv:1812.09253 [hep-th]; **Yong-Liang Ma**, Maciej A. Nowak, Mannque Rho, and Ismail Zahed, Phys. Rev. Lett., 123:172301, 2019; **Fan Lin and Yong-Liang Ma, JHEP, 05:270, 2024; Symmetry, 17(4):477, 2025. Phys. Rev. D, 112(1):014009, 2025.**

Yang-Mills Theory

’t Hooft Projection & Magnetic Monopoles

Yang-Mills Lagrangian

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2g^2}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}), \quad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]$$

Abelian Projection / Dominance & Topological Singularities

$$\mathrm{SU}(N) \xrightarrow{\text{Maximal Abelian Gauge}} \mathrm{U}(1)^{N-1}$$

Partial gauge fixing isolates relatively trivial diagonal components. The second homotopy group dictates color monopole singularities of the off-diagonal gluon in the IR:

$$\pi_2\left(\mathrm{SU}(N)/\mathrm{U}(1)^{N-1}\right) \cong \mathbb{Z}^{N-1} \quad \longleftrightarrow \quad \text{Color-Magnetic Monopoles}$$

’t Hooft, Nucl. Phys. B 190, 455 (1981); Wu, Yang, Phys. Rev. D 14, 437 (1976).

CFN Decomposition & Local Cartan Basis

Local Cartan Basis & Cho–Faddeev–Niemi (CFN) Decomposition

$$m_i(x) = g(x)H_i g^\dagger(x) = m_i^a T^a \in \mathrm{SU}(N)/\mathrm{U}(1)^{N-1}, \quad i = 1, \dots, N-1$$

gauge transformation $g(x)$ carry color monopole topological charges:

$$A_\mu^a = C_{i,\mu} m_i^a + H_\mu^a + X_\mu^a,$$
$$H_\mu^a = f^{abc}\partial_\mu m_i^b m_i^c, \qquad X_\mu^a = \left(\rho_{ij} f^{abc} + \sigma_{ij} d^{abc}\right)\partial_\mu m_i^b m_j^c$$

Physical Degrees of Freedom

- **Monopole Field (H_μ^a):** Covariant connection carrying color-magnetic charges.
- **Abelian-Higgs Multiplets:** $C_{i,\mu}$ (diagonal $[\mathrm{U}(1)]^{N-1}$ gluons) and $\phi_{ij} = \rho_{ij} + \mathrm{i}\sigma_{ij}$ (off-diagonal complex scalars).

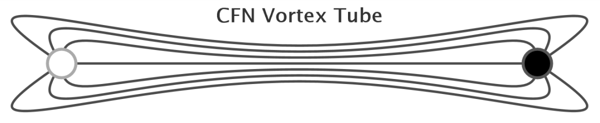
Cho, Phys. Rev. D 21, 1080 (1980); Faddeev, Niemi, Phys. Rev. Lett. 82, 1624 (1999), Phys. Lett. B, 449:214–218, 1999.

Color Monopole Charges & CFN Vortex Tubes

Topological Charge Matching

$$\text{Color Monopole Charge:} \quad \boldsymbol{g}_m = \oint_{\Sigma} \boldsymbol{F}^H \cdot \mathrm{d}\boldsymbol{\sigma} = 2\pi\xi_\alpha \boldsymbol{\alpha} \quad (\xi_\alpha \in \mathbb{Z})$$

$$\text{CFN Vortex Flux:} \quad \boldsymbol{\alpha}^{(ij)} \cdot \boldsymbol{\Phi}_{\mathrm{CFN}} = 2\pi n \implies \boldsymbol{\Phi}_{\mathrm{CFN}} = 2\pi\zeta_\alpha \boldsymbol{\alpha}^{(ij)} \quad (\zeta_\alpha \in \mathbb{Z})$$



Color monopole charges and CFN flux tubes obey matching co-root lattice topological conditions. Color monopoles act as physical sources that terminate CFN vortex tubes, confining $M\bar{M}$ pairs.

Fractional Vortex Structure & Energy Favorability

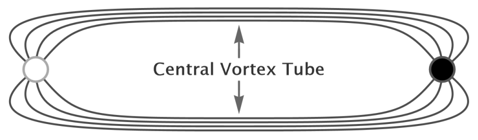
Fractional Flux Quantization

$$Z_N \text{ Vacuum Symmetry} \quad \longrightarrow \quad \omega^i \cdot \mathbf{\Phi}_{\sigma_i} = 2\pi \left(n_i - \frac{n}{N} \right) \quad \xrightarrow{\text{energy min.}} \quad \mathbf{\Phi}_{\sigma_i} = 4\pi\omega^i$$

An intact CFN root vortex can fractionates into a center vortices pair with fundamental weight fluxes.

Energy Minimization in SU(3)

$$\frac{E_\sigma}{E_{\text{CFN}}} \approx \frac{|2\omega^i|^2 + |2\omega^j|^2}{2|\alpha^{ij}|^2} = \frac{N-1}{N} = \frac{2}{3} < 1$$



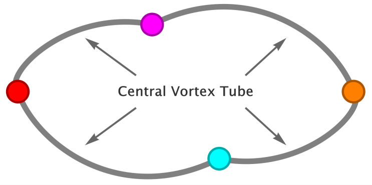
Fractionated center vortices pair carries lower energy than an intact CFN vortex, driving spontaneous dissociation.

Center Vortex Network & Gluon Knots

Center Vortex Percolation & Confinement

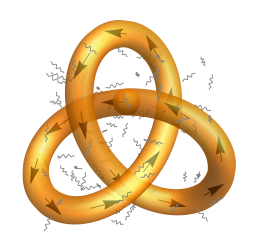
Random $\mathbb{Z}_N$ center vortex network crossings induce full statistical dephasing, driving the area law of Wilson loops:

$$\langle W(C)\rangle \sim \prod_i z_i \sim \exp\left[-\sigma \cdot \mathrm{Area}(\mathcal{S})\right], \quad z_i \in \mathbb{Z}_N$$



Topological Stabilization

Non-trivial knot topology provides the essential helicity and string tension required to dynamically stabilize color monopoles and the vortex network.



Mithat Unsal and Laurence G. Yaffe, Phys. Rev. D, 78:065035, 2008; Yui Hayashi, Yuya Tanizaki, Mithat Unsal, Phys. Rev. Lett. 133, 171902 (2024); JHEP, 02:126, 2026;

Faddeev–Niemi theory & Topological Soliton

Effective Infrared Action

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2g^2}\,\text{Tr}(G_{\mu\nu}G^{\mu\nu}) \quad \xrightarrow{\text{Infrared}} \quad \mathcal{L}_{\text{FN}} = (\partial_\mu m_i^a)^2 + \frac{1}{e_i^2}(f^{bcd} m_i^b \partial_\nu m_j^c \partial_\mu m_j^d)^2$$

Integrating out massive off-diagonal gluons yields an IR effective Lagrangian governed by local Cartan directions $m_i^a \in \text{SU}(N)/\text{U}(1)^{N-1}$ dynamics.

Gluon Knots as Topological Solitons

$$\mathcal{H} = \frac{1}{4\pi^2}\sum_i \int_{\mathbb{S}^3_\infty} A_i^H \wedge F_i^H \quad \longleftrightarrow \quad \pi_3\left(\text{SU}(N)/\text{U}(1)^{N-1}\right) \cong \mathbb{Z}$$

Non-zero Hopf invariant $\mathcal{H}$ guarantees topological stability via the energy bound ($E \geq c\,|\mathcal{H}|^{3/4}$), establishing **gluon knots as stable IR degrees of freedom in Yang-Mills thoery which generates the mass gap**.

Faddeev, Niemi, Phys. Rev. Lett. 82, 1624 (1999), Phys. Lett. B, 449:214–218, 1999; Nature 387, 58 (1997).

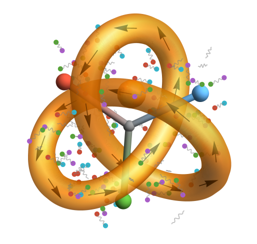
Baryon Structure

Dual Ginzburg–Landau Theory & Baryon Structure

Dual Superconductivity Formulation

$$\mathcal{L}_{\mathrm{GL}} = -\frac{1}{4}(\overrightarrow{M}_{\mu\nu})^2 + \sum_{\alpha}\left[\left|D_\mu \chi_\alpha\right|^2 - V\left(|\chi_\alpha|^2\right)\right]$$

Lattice QCD confirms a weak Type-II dual superconductor. The dual Meissner effect squeezes chromoelectric fields into a Y-shaped flux junction centered at the knot.



Shinji Maedan and Tsuneo Suzuki, Prog. Theor. Phys., 81:229–240, 1989; Progress of theoretical physics, 84(1):130–141, 1990. A. D'Alessandro and Massimo D'Elia, B Proc. Suppl., 164:236–239, 2007; N. Cardoso, M. Cardoso, and P. Bicudo, arXiv:1004.0166 [hep-lat]

Baryon Mass: Trace Anomaly & Magnetic Catalysis

Gluon Condensate & Trace anomaly

$$T^{\mu}{}_{\mu} = \frac{\beta(g)}{2g} G^{a}_{\mu\nu} G^{\mu\nu,a} + \sum_{f} m_f (1+\gamma_m) \bar{\psi}_f \psi_f$$

The gluon knot condensate directly provides the dominant $\sim 40\%$ trace anomaly contribution to nucleon mass (Yang-Mills Mass Gap).

Chiral Condensate via Magnetic Catalysis

$$\langle 0|\bar{\psi}\psi|0\rangle \sim |g\,\boldsymbol{B}|$$

Concentrated color-magnetic fields inside the knot core catalyze chiral symmetry breaking, generating dynamical constituent quark masses.

R. J. Crewther, Phys. Rev. Lett. 281421 (1972); Yi-Bo Yang, Jian Liang, Yu-Jiang Bi, Ying Chen, Terrence Draper, Keh-Fei Liu, and Zhaofeng Liu, Phys. Rev. Lett. 121(21):212001, 2018; V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. Lett., 73:3499–3502, 1994; Phys. Lett. B, 349:477–483,1995; Phys. Rev. Lett., 83:1291–1294, 1999.

Vacuum Topology & Axial Anomaly

Homotopy Isomorphism

$$\pi_3\left(\mathrm{SU}(N)/\mathrm{U}(1)^{N-1}\right) \cong \pi_3\left(\mathrm{SU}(N)\right) \cong \mathbb{Z}$$

Hopf-Winding Relation in $\mathrm{SU}(3)$

$$N(g) = \frac{1}{24\pi^2}\int_{\mathbb{S}^3_\infty} d^3S\,\epsilon^{ijk}\mathrm{tr}\left(g\partial_i g^\dagger\, g\partial_j g^\dagger\, g\partial_k g^\dagger\right) = \frac{\mathcal{H}}{4} = \frac{1}{16\pi^2}\int A_i^H \wedge F_i^H$$

Opposite component topological charges yield $\mathcal{H} = \mathcal{H}_{(11)} + \mathcal{H}_{(22)} = 0$, possibly selecting as the topologically trivial ground state for the nucleons.

Axial Anomaly & Quark Zero Modes

$$\partial_\mu j^{5,\mu} = 2im\bar{\psi}\gamma^5\psi + \frac{g^2 N_f}{8\pi^2}\mathrm{Tr}(F_{\mu\nu}\star F^{\mu\nu}) \quad\Longrightarrow\quad \langle\bar{\psi}\psi\rangle = -\frac{\pi}{V}\mathrm{sign}(m)\overline{\nu(0)},$$

Fluctuations in $\mathcal{H}$ drive topological tunneling, inducing localized Dirac zero modes and further chiral symmetry breaking (Banks–Casher relation).

Adler, Phys. Rev. 177, 2426 (1969); Bell, Jackiw, Nuovo Cimento A 60, 47 (1969).

Topological Current of Gluon Spin

Jaffe–Manohar Spin Decomposition

$$\boldsymbol{J}_{\text{QCD}} = \int \mathrm{d}^3x \left[\boldsymbol{\psi}_f^\dagger \frac{\boldsymbol{\Sigma}}{2} \boldsymbol{\psi}_f + \boldsymbol{\psi}_f^\dagger (\boldsymbol{x} \times -\mathrm{i}\boldsymbol{\partial}) \boldsymbol{\psi}_f + \boldsymbol{E}_a \times \boldsymbol{A}_a + E_a^i (\boldsymbol{x} \times \boldsymbol{\partial}) A_a^i \right]$$

Topological charges of the gluon knot drive a spatial current $\boldsymbol{H} \propto \boldsymbol{E}_i^H \times \boldsymbol{A}_i^H$, mapping directly into the gluon spin operator.

Ji Decomposition & Poynting Vector

$$\boldsymbol{J}_{\text{QCD}} = \int \mathrm{d}^3x \left[\boldsymbol{\psi}_f^\dagger \frac{\boldsymbol{\Sigma}}{2} \boldsymbol{\psi}_f + \boldsymbol{\psi}_f^\dagger (\boldsymbol{x} \times -\mathrm{i}\boldsymbol{D}) \boldsymbol{\psi}_f + \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) \right]$$

Time-varying color-magnetic fields inside the knot induce orthogonal color-electric fields, generating Poynting vector angular momentum.

Jaffe, Manohar, Nucl. Phys. B 337, 509 (1990); Ji, Phys. Rev. Lett. 78, 610 (1997).

Gluon Knots in Mesonic Spectra

Light Scalar Mesons: The $f_0(500)$ Resonance

The $f_0(500)$ (σ-meson) is identified as the most basic gluon knot, setting the baseline energy scale for chiral symmetry breaking and the trace anomaly.

Glueball Classification

Glueball states may be systematically described as topological gluon knots governed by distinct linking structures.

Heavy Quarkonia

Heavy $Q\bar{Q}$ bound states ($J/\psi, \Upsilon$) can form color flux tubes wrapped tightly by gluon knots.

Vector and Pseudoscalar Mesons

The empirical relation $m_\sigma + m_\pi \approx m_\rho$ allows transient knot components in vector mesons (ρ, ω), whereas light pseudoscalars (π, K) lack internal knots.

G. E. Brown, Mannque Rho, Phys. Rev. Lett. 66, 2720, 1991; Yong-Liang Ma, Mannque Rho, Prog.Part.Nucl.Phys. 113 (2020) 103791; Yan-Ling Li, Yong-Liang Ma, Mannque Rho, Phys.Rev.D 95 (2017) 11, 114011.

Conclusions

A Unified Infrared Degree of Freedom

Gluon knots act as the fundamental dynamical degrees of freedom for $SU(N)$ Yang-Mills theory in IR, unifying the confinement mechanism, satisfying abelian dominance, giving Yang-Mills mass gap.

Simultaneous Explanation of Baryon Structure

- **Quark Confinement:** Dual Meissner effect squeezes fields into Y-junction flux tubes.
- **Chiral Symmetry Breaking:** Magnetic catalysis and Dirac zero modes drive χSB.
- **Mass and Spin:** Natural alignment with trace anomaly and Poynting angular momentum of gluon field.

Offers a unified non-perturbative framework consistent with empirical baryon properties.

Thanks for Your Attention