On-Shell Construction of Goldstone–Gauge Amplitudes

郑煜辉

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Based on *https://arxiv.org/abs/2511.09403*

ChPT input meets the on-shell bootstrap

Chiral perturbation theory

- Low-energy EFT of Nambu–Goldstone bosons.
- Organized by a derivative and loop expansion.
- Chiral symmetry implies Adler's zero:

$$\lim_{p_i \to 0} \mathcal{M}_n = 0$$

On-shell methods

- Work directly with physical external states.
- Analyticity locates poles; unitarity fixes residues.
- Recursion builds high-point amplitudes from lower-point data.
- Local contact terms appear as boundary or seed data.

Symmetry and soft input can replace a diagram-by-diagram construction.

On-shell methods organize ChPT amplitudes and operators

- Cheung et al.: soft limits identify special scalar EFTs.
- Low and Yin: soft blocks provide local seed amplitudes.
- Dai et al.: systematic operator counting in mesonic ChPT.
- Low et al.: ChPT amplitude/operator bases with flavor and kinematic relations.

Pure-Goldstone soft blocks are the seed data; the new step is to gauge them.

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Outline

1. On-shell language and recursion
2. Goldstone soft behavior
3. What gauging breaks
4. How the paper repairs recursion
5. Examples, scope, and outlook

On-shell language and spinor variables

$$p_i^2 = 0, \qquad p_{i\,\alpha\dot{\alpha}} = \lambda_{i\alpha}\widetilde{\lambda}_{i\dot{\alpha}} \equiv |i\rangle[i|$$

$$\langle ij\rangle = \epsilon^{\alpha\beta}\lambda_{i\alpha}\lambda_{j\beta}, \qquad [ij] = \epsilon^{\dot{\alpha}\dot{\beta}}\widetilde{\lambda}_{i\dot{\alpha}}\widetilde{\lambda}_{j\dot{\beta}}$$

$$s_{ij} = (p_i + p_j)^2 = \langle ij\rangle[ji]$$

Little-group scaling

$$|i\rangle \to t_i|i\rangle, \qquad |i] \to t_i^{-1}|i]$$

$$\mathcal{M} \to t_i^{-2h_i}\mathcal{M}$$

- On-shellness is automatic
- Helicity is explicit
- Soft limits and poles are compact

BCFW: complex momentum plus Cauchy's theorem

$$p_i \longrightarrow \widehat{p}_i(z), \qquad \widehat{p}_i(0) = p_i, \quad \widehat{p}_i^{\,2}(z) = 0, \quad \sum_i \widehat{p}_i(z) = 0$$

$$\boxed{\mathcal{M}(0) = -\sum_{z_\alpha \text{ finite}} \operatorname*{Res}_{z=z_\alpha} \frac{\widehat{\mathcal{M}}(z)}{z} - \operatorname*{Res}_{z=\infty} \frac{\widehat{\mathcal{M}}(z)}{z}}$$

Finite physical poles

$$\widehat{P}_I^{\,2}(z_I) = 0 \quad \Longrightarrow \quad \mathrm{Res}_{z_I} \widehat{\mathcal{M}} \sim \widehat{\mathcal{M}}_L \widehat{\mathcal{M}}_R$$

Unitarity fixes them from lower-point amplitudes.

Boundary at infinity

$$\operatorname*{Res}_{z=\infty} \frac{\widehat{\mathcal{M}}(z)}{z} = 0 \quad \Longleftrightarrow \quad \text{constructible}$$

If it is nonzero, extra boundary or contact data are needed.

The soft factor removes the boundary at infinity

$$\widehat{p}_i(z) = (1 - a_i z) p_i, \qquad \sum_i a_i p_i = 0, \qquad F_n(z) = \prod_{i=1}^{n} (1 - a_i z)^{\sigma_i}$$

BCFW contour

$$\frac{\widehat{\mathcal{M}}_n(z)}{z}$$

EFT contact terms can leave a residue at infinity.

Soft-recursion contour

$$\frac{\widehat{\mathcal{M}}_n(z)}{z F_n(z)}$$

Extra powers in F_n improve the large-z behavior.

$$\boxed{\mathcal{M}_n(0) = - \sum_{\widehat{P}_I^2(z_I)=0} \operatorname*{Res}_{z=z_I} \frac{\widehat{\mathcal{M}}_n(z)}{z F_n(z)}}$$

NLSM: $\sigma_i = 1$, so Adler's zero cancels the soft poles.

Gauging removes Adler's zero from charged legs

$$\mathcal{M}_{4+1}(\pi^+, \pi^-, \pi^0, \pi^0, \gamma) = e\,(p_5^\mu \epsilon^\nu - p_5^\nu \epsilon^\mu)\,\frac{p_{1\mu} p_{2\nu}}{s_{15} s_{25}}\,\frac{s_{34}}{f^2}$$

Neutral legs

$$p_3 \to 0 \text{ or } p_4 \to 0 \quad \Longrightarrow \quad \mathcal{M} \to 0$$

Charged legs

$$p_1 \to 0 \text{ or } p_2 \to 0 \quad \Longrightarrow \quad \mathcal{M} \not\to 0$$

Fewer Adler-zero factors $\Rightarrow$ worse large-z behavior.

Three ingredients repair three different obstructions

Obstruction	New input
Charged legs have no Adler zero	Charge-flow decomposition
A soft photon gives a pole	Leading and subleading soft theorem
Internal photon exchange is ambiguous up to a contact term	$J = 1$ partial-wave projection

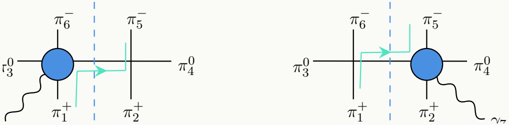
Decompose first, recurse second, project internal vector exchange last.

Charge flow restores most Adler zeros

$$\mathcal{M}_{n+1}(\phi_1^{q_1},\dots,\phi_n^{q_n};\gamma)=\sum_{i=1}^{n} q_i m_i,\qquad \sum_i q_i = 0,\qquad \boxed{\mathcal{M}_{n+1}=\sum_{i=1}^{n-1} Q_i A_i}$$

$$Q_i=\sum_{j=1}^{i} q_j,\qquad A_i = m_i - m_{i+1}$$

$$A_i \equiv \mathcal{M}(\phi_1^0,\dots,\phi_i^{+1},\phi_{i+1}^{-1},\dots,\phi_n^0;\gamma)$$



The green arrow tracks one effective current.

Each A_i is gauge invariant, and $\lim_{p_j\to 0} A_i = 0$ for $j \neq i, i+1$.

The soft photon theorem becomes recursive data

$$\mathcal{M}_{n+1}(\epsilon p_s) = \left(\frac{1}{\epsilon} S^{(0)} + S^{(1)} + O(\epsilon)\right) \mathcal{M}_n$$

$$S^{(0)} = e \sum_i \frac{\langle ri \rangle}{\langle rs \rangle \langle si \rangle} q_i, \qquad S^{(1)} = e \sum_i \frac{q_i}{\langle si \rangle} \tilde{\lambda}_s^{\dot{\alpha}} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}}$$

At $z = 1/a_s$, the photon soft pole is fixed by $S^{(0)}$ and $S^{(1)}$.

Gauged soft recursion closes for the NLSM

$$F(z) = \prod_{\text{neutral Goldstones } j} (1 - a_j z)^{\sigma} \prod_{\text{gauge bosons } s} (1 - a_s z)$$

$$\boxed{A_{i_1 \ldots i_l} = -\sum_I \operatorname*{Res}_{z=z_I^{\pm}} \frac{\widehat{A}_{I_L}(z)\widehat{A}_{I_R}(z)}{zF(z)\widehat{P}_I^2(z)} - \sum_s \operatorname*{Res}_{z=1/a_s} \frac{\widehat{A}_{i_1 \ldots i_l}(z)}{zF(z)}}$$

Hard poles: lower-point factorization.

Gauge-soft poles: remove one gauge boson.

Boundary: $\rho < \sigma$ for one gauge boson.

NLSM: $(\rho, \sigma) = (0, 1)$, so the boundary at infinity vanishes.

$J = 1$ identifies internal photon exchange

$$A_{e^2}(1^{+q}, 2^{-q}, 3^{+q'}, 4^{-q'}) = qq'e^2\frac{s_{13} - s_{14}}{s_{12}}$$

Vector exchange

$$\frac{\langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle}{\langle 12\rangle\langle 34\rangle} = P_1(\cos\theta) = \cos\theta$$

$J = 1$ symmetric contraction

Local scalar contact term

$$\frac{\langle 13\rangle\langle 24\rangle - \langle 14\rangle\langle 23\rangle}{\langle 12\rangle\langle 34\rangle} = P_0(\cos\theta) = 1$$

$J = 0$ antisymmetric contraction

The $J = 1$ rule separates photon exchange from a contact-term ambiguity.

Worked example: $\pi^+\pi^-\pi^0\pi^0\gamma$

$$\mathcal{M}_e = Q_1 A_1 + Q_2 A_2 + Q_3 A_3, \qquad A_1 : \ \phi_1^{+1}, \phi_2^{-1}, \phi_3^{0}, \phi_4^{0}$$

$$A_1 = (S^{(0)} + S^{(1)})\mathcal{M}_4, \qquad \mathcal{M}_4 = \frac{s_{12}}{f^2}$$

$$\boxed{A_1 = \frac{e}{f^2} \frac{\langle 12 \rangle}{\langle s1 \rangle \langle s2 \rangle} s_{34}}$$

- Five particles: use a Risager-type shift.
- Only the photon soft pole contributes.
- s_{34} makes the two neutral-pion Adler zeros manifest.

The same logic extends to more photons and to gluons

Higher multiplicity

- Hard and soft poles
- $6\phi + 1\gamma$ checked explicitly
- Contact terms recovered

Several photons

- Same helicity: commuting soft operators
- Opposite helicity: Compton poles

Non-Abelian gauging

- Charge flow $\rightarrow$ color flow
- Gauge one current at a time
- Recombine the couplings

The output is a recursive algorithm, not only a set of low-point examples.

Take-home message — questions welcome

$$\text{pure-Goldstone soft blocks} + \text{gauge soft theorems} + \text{unitarity} + \text{spin}$$
$$\Longrightarrow \boxed{\text{gauged NLSM tree amplitudes}}$$

Main idea

- Decompose into gauge-invariant currents.
- Recover Adler zeros component by component.
- Use gauge-soft poles as recursive data.

Scope

- Tree level
- Massless Goldstones
- Minimal gauge coupling
- Loop extension remains open