# A mechanism of power-counting breaking in chiral perturbation theory

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# Outline

- Background & motivations
- Analysis & conclusion
- Summary

# Power-counting rule

- Power-counting scheme of the elements

$$\mathscr{L} = \mathscr{L}^0_{\mathrm{QCD}} + \bar{q}(\not{v}(x) + \not{a}(x)\gamma_5 - s(x) + ip(x)\gamma_5)q$$
$$U = \mathcal{O}(p^0), D_\mu U = \mathcal{O}(p), v_\mu, a_\mu = \mathcal{O}(p), s, p = \mathcal{O}(p^2)$$

- One loop gives $\mathcal{O}(p^2)$

- Chiral dimension

$$D = 2 + \sum_{k=1}^{\infty} 2(k-1)N_{2k} + (n-2)N_L$$

$n$: space-time dimensions; $N_L$: number of loops;

$2k$: the number of vertices from $\mathcal{O}(p^{2k})$ Lagrangian

- Conditions for the validity? Scope of application?

# Contributions at each order

$\mathcal{O}(p^2) + \mathcal{O}(p^4) + \mathcal{O}(p^6)$ Bijnens et.al., Ann. Rev. Nucl. Part. Sci., **64**, 149, (2014)

$$\frac{F_K}{F_\pi} = 1 + 0.176(0.121) + 0.023(0.077)$$

$$\frac{F_\pi}{F_0} = 1 + 0.208(0.313) + 0.088(0.127)$$

$$\frac{M_\pi^2}{M_{\pi\,\mathrm{phys}}^2} = 1.055(0.937) - 0.005(+0.107) - 0.050(-0.044)$$

$$\frac{M_K^2}{M_{K\,\mathrm{phys}}^2} = 1.112(0.994) - 0.069(+0.022) - 0.043(-0.016)$$

$$\frac{M_\eta^2}{M_{\eta\,\mathrm{phys}}^2} = 1.197(0.938) - 0.214(-0.076) + 0.017(0.014)$$

$$a_0^0 = 0.160 + 0.044(0.046) + 0.012(0.012) \quad \pi\pi \text{ scattering lengths}$$

$$a_0^2 = -0.0456 + 0.0016(0.0017) - 0.0001(-0.0003)$$

$$a_0^{1/2} = 0.142 + 0.031(0.027) + 0.051(0.057) \quad \pi K \text{ scattering lengths}$$

$$a_0^{3/2} = -0.071 + 0.007(0.005) + 0.016(0.019) \quad \text{worse convergence}$$

$$f_s = 3.786 + 1.202(1.231) + 0.717(0.688) \quad K_{l4}\text{form factors}$$

$$g_p = 3.786 + 0.952(0.857) + 0.212(0.309)$$

# Contributions at each order

## Geometric sequence fit

| physical quantities | LO\|Pct$_{\rm LO}$(%) | NLO\|Pct$_{\rm NLO}$(%) | NNLO\|Pct$_{\rm NNLO}$(%) | HO\|Pct$_{\rm HO}$(%) | Theory | Inputs |
|---|---|---|---|---|---|---|
| $m_s/\hat{m}\|_1$ | 25.8(94.8) | 2.0(7.2) | $-1.1$(4.0) | 0.6(2.0) | 27.3 | $27.3^{+0.7}_{-1.3}$ |
| $m_s/\hat{m}\|_2$ | 24.2(88.7) | 3.3(12.2) | $-0.8$(2.8) | 0.5(1.9) | 27.3 | $27.3^{+0.7}_{-1.3}$ |
| $F_K/F_\pi$ | 1.000(83.4) | 0.169(14.1) | 0.023(1.9) | 0.007(0.6) | 1.199 | $1.199 \pm 0.003$ |
| $f_s$ | 3.782(66.2) | 1.322(23.1) | 0.371(6.5) | 0.235(4.1) | 5.709 | $5.712 \pm 0.032$ |
| $g_p$ | 3.782(76.7) | 0.776(15.7) | 0.366(7.4) | 0.007(0.1) | 4.931 | $4.958 \pm 0.085$ |
| $a_0^0$ | 0.1592(72.5) | 0.0453(20.6) | 0.0098(4.5) | 0.0053(2.4) | 0.2196 | $0.2196 \pm 0.0034$ |
| $10a_0^2$ | $-0.455$(103.8) | 0.022(5.0) | $-0.010$(2.2) | 0.005(1.1) | $-0.438$ | $-0.444 \pm 0.012$ |
| $a_0^{1/2}m_\pi$ | 0.142(62.6) | 0.033(14.6) | 0.049(21.7) | 0.002(1.0) | 0.226 | $0.224 \pm 0.022$ |
| $10a_0^{3/2}m_\pi$ | $-0.709$(150.2) | 0.094(19.8) | 0.142(30.1) | 0.001(0.3) | $-0.472$ | $-0.448 \pm 0.077$ |
| $F_S^\pi(0)/2B_0$ | 1.000(98.1) | 0.019(1.9) | 0.000(0.0) | 0.000(0.0) | 1.019 | – |
| $10^3 l_1^r$ | – | $-3.2$(81.5) | $-0.6$(15.1) | $-0.1$(3.4) | $-4.0$ | $-4.0 \pm 0.6$ |
| $10^3 l_2^r$ | – | 3.0(145.6) | $-1.3$(66.5) | 0.4(20.8) | 2.0 | $1.9 \pm 0.2$ |
| $10^3 l_3^r$ | – | 0.2(102.5) | $-0.0$(2.6) | 0.0(0.1) | 0.2 | $0.3 \pm 1.1$ |
| $10^3 l_4^r$ | – | 6.3(96.8) | 0.2(3.1) | 0.0(0.1) | 6.6 | $6.2 \pm 1.3$ |

Yang et al., Phys. Rev. D, **102**, 094009, (2020)

# Contributions at each order

## Bayesian fit

| Observables | LO$\vert$Pct$_{\mathrm{LO}}$ | NLO$\vert$Pct$_{\mathrm{NLO}}$ | NNLO$\vert$Pct$_{\mathrm{NNLO}}$ | HO$\vert$Pct$_{\mathrm{HO}}$ | Theory | Experiment |
|---|---|---|---|---|---|---|
| $m_s/\hat{m}\vert_1$ | $25.84(95.1\%)$ | $1.45(5.3\%)$ | $-0.12(-0.5\%)$ | $0.003(0.01\%)$ | $27.2 \pm 4.1$ | $27.3^{+0.7}_{-1.3}$ |
| $m_s/\hat{m}\vert_2$ | $24.21(88.4\%)$ | $3.60(13.1\%)$ | $-0.38(-1.4\%)$ | $-0.020(-0.07\%$ | $27.4 \pm 10.7$ | $27.3^{+0.7}_{-1.3}$ |
| $F_K/F_\pi$ | $1.000(83.2\%)$ | $0.197(16.4\%)$ | $0.007(0.6\%)$ | $-0.002(-0.12\%$ | $1.202 \pm 0.050$ | $1.199 \pm 0.003$ |
| $f_s$ | $3.782(66.7\%)$ | $1.342(23.7\%)$ | $0.494(8.7\%)$ | $0.050(0.88\%)$ | $5.668 \pm 0.351$ | $5.712 \pm 0.032$ |
| $g_p$ | $3.782(76.9\%)$ | $0.834(17.0\%)$ | $0.284(5.8\%)$ | $0.018(0.37\%)$ | $4.918 \pm 0.080$ | $4.958 \pm 0.085$ |
| $a_0^0$ | $0.159(72.2\%)$ | $0.045(20.6\%)$ | $0.015(6.8\%)$ | $0.001(0.34\%)$ | $0.2204 \pm 0.004$ | $0.2196 \pm 0.0034$ |
| $10a_0^2$ | $-0.455(104.1\%$ | $0.020(-4.6\%)$ | $0.003(-0.7\%)$ | $-0.005(1.20\%)$ | $-0.437 \pm 0.017$ | $-0.444 \pm 0.012$ |
| $a_0^{1/2}m_\pi$ | $0.142(63.2\%)$ | $0.034(15.2\%)$ | $0.049(21.6\%)$ | $0.000(0.00\%)$ | $0.225 \pm 0.013$ | $0.224 \pm 0.022$ |
| $10a_0^{3/2}m_\pi$ | $-0.709(161.9\%)$ | $0.093(-21.2\%)$ | $0.179(-40.8\%)$ | $0.000(0.04\%)$ | $-0.438 \pm 0.061$ | $-0.448 \pm 0.077$ |
| $10^3 l_1^r$ | $0(0.0\%)$ | $-3.57(90.4\%)$ | $-0.38(9.7\%)$ | $0.004(-0.11\%$ | $-4.0 \pm 0.94$ | $-4.0 \pm 0.6$ |
| $10^3 l_2^r$ | $0(0.0\%)$ | $3.32(174.8\%)$ | $-1.36(-71.8\%$ | $-0.057(-3.01\%$ | $1.9 \pm 0.82$ | $1.9 \pm 0.2$ |
| $10^3 l_3^r$ | $0(0.0\%)$ | $-0.25(-115.1\%)$ | $0.46(214.6\%)$ | $0.001(0.48\%)$ | $0.2 \pm 3.01$ | $0.3 \pm 1.1$ |
| $10^3 l_4^r$ | $0(0.0\%)$ | $6.85(104.8\%)$ | $-0.27(-4.1\%)$ | $-0.044(-0.67\%$ | $6.5 \pm 1.91$ | $6.2 \pm 1.3$ |
| $f_s'$ | | | | | $0.472 \pm 0.461$ | $0.868 \pm 0.049$ |
| $g'$ | | | | | $0.508 \pm 0.029$ | $0.508 \pm 0.122$ |
| $\langle r^2\rangle_S^\pi$ | | | | | $0.59 \pm 0.07$ | $0.61 \pm 0.04$ |
| $c_S^\pi$ | | | | | $11 \pm 1$ | $11 \pm 1$ |

Pan et al., Frontiers of Physics, **19**, 64203, (2024)

# Motivations

- Why power-counting rule hold or not?

  $SU(2)$ or $SU(3)$? $s$ mass? Number of diagrams? Loop diagrams? Loop integrals? LECs? And/or ...?

- Only consider all diagrams at a given order

  Not each diagrams. Possible cancellation.

- Only compare $\pi K$ and $\pi\pi$ scattering lengths

  Possible exist other mechanisms. A preliminary study. Other processes

- Qualitative and semi-quantitative estimates

- For truncation error estimation

  Theoretical error, which order is enough? LECs

# Review of $\pi K$ scattering lengths

$$\pi + K \to \pi + K$$

Scattering amplitude

$$T(s,t,u) = T_2(s,t,u) + T_4(s,t,u) + T_6(s,t,u) + \mathcal{O}(p^8)$$

Only $I = 3/2$ channel $\pi^+ K^+ \to \pi^+ K^+$

$$T^{\frac{1}{2}}(s,t,u) = -\frac{1}{2} T^{\frac{3}{2}}(s,t,u) + \frac{3}{2} T^{\frac{3}{2}}(u,t,s)$$

Scattering lengths $q_{\pi K}^2 = \frac{s}{4}\left(1 - \frac{(m_K + m_\pi)^2}{s}\right)\left(1 - \frac{(m_K - m_\pi)^2}{s}\right)$

$$T^I(s,t,u) = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

$$t_\ell^I(s) = \frac{1}{2}\sqrt{s}\, q_{\pi K}^{2\ell}\left(a_\ell^I + b_\ell^I q_{\pi K}^2 + \mathcal{O}(q_{\pi K}^4)\right)$$

At the threshold, $S$ wave, $\ell = 0$

$$a_0^I = \frac{T^I\left((m_\pi + m_K)^2,\, 0,\, (m_\pi - m_K)^2\right)}{8\pi(m_\pi + m_K)}, \qquad I = \frac{1}{2}, \frac{3}{2}$$

Bijnens et al. JHEP05(2004)036

# Review of $\pi\pi$ scattering lengths

$\pi\pi$ scattering amplitude

$$A\left(\pi^a\pi^b \to \pi^c\pi^d\right) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,t,s)$$

$$T^0(s,t) = 3A(s,t,u) + A(t,u,s) + A(u,s,t),$$

$$T^1(s,t) = A(t,u,s) - A(u,s,t),$$

$$T^2(s,t) = A(t,u,s) + A(u,s,t),$$

$\pi\pi$ scattering lengths $q_{\pi\pi}^2 = \frac{1}{4}(s - 4m_\pi^2)$

$$T^I(s,t) = 32\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\, t_\ell^I(s)$$

$$t_\ell^I(s) = q_{\pi\pi}^{2\ell}\left(a_\ell^I + b_\ell^I q_{\pi\pi}^2 + \mathcal{O}(q_{\pi\pi}^4)\right),$$

At the threshold, $S$ wave, $\ell = 0$

$$a_0^I = \frac{T^I(4m_\pi^2, 0, 0)}{32\pi}, \qquad I = 0,\ 2$$

Bijnens et al. JHEP01(2004)050

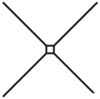
# Lagrangian and LECs

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots \\
&= \frac{F_0^2}{4}\mathrm{Tr}\big(D_\mu U D^\mu U^\dagger\big) + \frac{F_0^2}{4}\mathrm{Tr}\big(\chi U^\dagger + U\chi^\dagger\big) \\
&\quad + \sum_{i=1}^{10} L_i O_i^{(4)} + \sum_{i=1}^{90} C_i O_i^{(6)} + \cdots
\end{aligned}
$$

Precision: $F_0, B_0 > L_i > C_i$

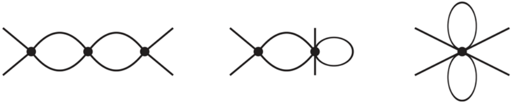
Two-loop diagram $\Rightarrow$ one-loop diagram $\Rightarrow$ tree

# $\pi K$ scattering at $\mathcal{O}(p^6)$

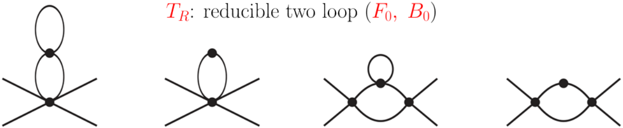


$T_C$: tree ($C_i$)

$T_L$: one loop ($L_i$)



$T_R$: reducible two loop ($F_0$, $B_0$)



$T_H$: sunset ($F_0$, $B_0$) $T_V$: vertex ($F_0$, $B_0$)

$F_0, B_0, L_i$ are more precise than $C_i$

# $\pi K$ scattering at $\mathcal{O}(p^6)$

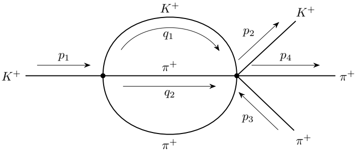
$$T^{3/2} = T_C + T_L + T_R + T_H + T_V,$$

| Process | Observable | $T_C$ | $T_L$ | $T_R$ | $T_H$ | $T_V$ | $T$ |
|---|---|---|---|---|---|---|---|
| $\pi K$ | $\mathcal{T}^{3/2}_{\pi K}$ | -0.074 | 0.588 | 1.763 | 10.815 | -11.311 | 1.782 |
| $\pi K$ | $10 a_0^{3/2} m_\pi$ | -0.00642 | 0.0514 | 0.154 | 0.945 | -0.988 | 0.156 |
| $\pi\pi$ | $\mathcal{A}_{\pi\pi}$ | -0.0395 | 0.000287 | 1.53 | -0.468 | -0.740 | 0.283 |
| $\pi\pi$ | $10 a_0^2$ | -0.0133 | -0.00321 | 0.0636 | 0.0715 | -0.120 | -0.00141 |

# Sunset diagrams

## An example

$$\mathcal{M}_{H1} = \frac{1}{432 F_0^6 M_\pi^4} \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{1}{(q_1^2 - M_\pi^2)(q_2^2 - M_\pi^2)((-p_1 + q_1 + q_2)^2 - M_K^2)}$$
$$\times \Big[ -2(p_1 \cdot q_1)(11 M_K^2 + 7 M_\pi^2 + 2s - 7t - 4u) - 2(p_1 \cdot q_2)(M_K^2 + 21 M_\pi^2 - 10s - 8t + 7u)$$
$$+ M_\pi^2 (2 M_\pi^2 - 14(q_1 \cdot q_2) - 4 q_1^2 - 16 q_2^2 + 2s - 7t - 10u) \Big]$$
$$\times \Big[ M_\pi^2 (-2 M_K^2 + 4(q_1 \cdot q_2) - 2 q_1^2 + 4 q_2^2 + s + t + u)$$
$$+ 2(p_1 \cdot q_1)(M_\pi^2 + s + t - u) + 4(p_1 \cdot q_2)(2 M_\pi^2 - s - t + u) \Big].$$




sunset diagram

simplify to

$$H\left(m_1^2, m_2^2, m_3^2; p^2\right) = \langle\langle 1 \rangle\rangle$$
$$H_\mu\left(m_1^2, m_2^2, m_3^2; p^2\right) = \langle\langle q_\mu \rangle\rangle = p_\mu H_1\left(m_1^2, m_2^2, m_3^2; p^2\right)$$
$$H_{\mu\nu}\left(m_1^2, m_2^2, m_3^2; p^2\right) = \langle\langle q_\mu q_\nu \rangle\rangle = p_\mu p_\nu H_{21}\left(m_1^2, m_2^2, m_3^2; p^2\right) + g_{\mu\nu} H_{22}\left(m_1^2, m_2^2, m_3^2; p^2\right)$$

$$\langle\langle X \rangle\rangle_H \equiv \frac{1}{i^2} \int \frac{d^d q}{(2\pi)^d} \frac{d^d r}{(2\pi)^d} \frac{X}{\left(q^2 - m_1^2\right)\left(r^2 - m_2^2\right)\left((q + r - p)^2 - m_3^2\right)}$$

# Sunset diagrams

Simplify the two-loop integral to $H$ two typical cases

$$\mathcal{M}_{H1}^{(1)} = \frac{11M_K^4}{108F_0^6M_\pi^2}\int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{p_1\cdot q_1}{(q_1^2-M_\pi^2)(q_2^2-M_\pi^2)((-p_1+q_1+q_2)^2-M_K^2)}$$

$$= \frac{11M_K^6}{108F_0^6M_\pi^2}H_1(M_\pi^2,M_\pi^2,M_K^2;p_1^2)$$

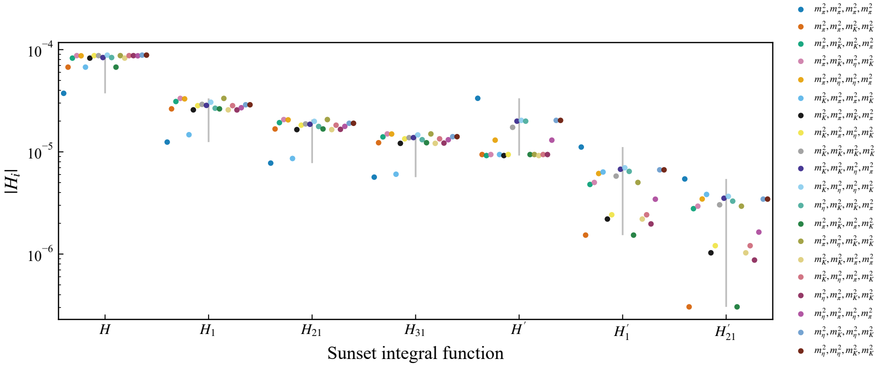
$$\mathcal{M}_{H1}^{(2)} = \frac{2M_K^2}{27F_0^6}\int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{q_2^2}{(q_1^2-M_\pi^2)(q_2^2-M_\pi^2)((-p_1+q_1+q_2)^2-M_K^2)}$$

$$= \frac{2M_K^2}{27F_0^6}\left[A(M_\pi^2)A(M_K^2)+M_\pi^2H(M_\pi^2,M_\pi^2,M_K^2;p_1^2)\right]$$

$$A(m^2) = \frac{1}{i}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m^2}$$

# Sunset diagrams

## $H$ integrals

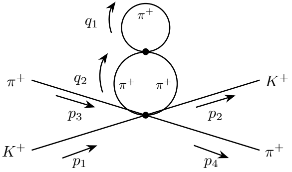



Not all $H$s are at the same order, $H$ is the largest one

# Reducible two-loop diagrams

## An example

$$\mathcal{M}_{R1} = -\frac{1}{108F_0^6 M_\pi^2}\int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{(2M_\pi^2 - 6(q_1\cdot q_2) - q_1^2 - q_2^2)}{(q_1^2 - M_\pi^2)(q_2^2 - M_\pi^2)^2}$$
$$\times\Big[2(p_1\cdot q_2)(10M_K^2 + 4M_\pi^2 + 6s - 5t - 5u)$$
$$+ M_\pi^2(28M_K^2 + 26M_\pi^2 + 6q_2^2 - 16s - 7t - 4u)\Big]$$




reducible two-loop diagram

simplify to

$$A(m^2) = \frac{1}{i}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2 - m^2} \sim -\frac{m^2}{16\pi^2}\ln\left(m^2\right) + \cdots$$

$$B\left(m_1^2, m_2^2, p^2\right) = \frac{1}{i}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2 - m_1^2)((q-p)^2 - m_2^2)}$$
$$\sim -\frac{1}{16\pi^2}\frac{m_1^2\ln\left(m_1^2\right) - m_2^2\ln\left(m_2^2\right)}{m_1^2 - m_2^2} + \cdots$$

$$H\left(m_1^2, m_2^2, m_3^2; 0\right) \sim -\frac{1}{(16\pi^2)^2}\Bigg(m_1^2\Big(\frac{1}{12}\pi^2 + \frac{3}{2} - \ln_1$$
$$+ \frac{1}{2}(-\ln_2\ln_3 + \ln_1\ln_4)\Big)$$
$$+ m_2^2\Big(\frac{1}{12}\pi^2 + \frac{3}{2} - \ln_2 + \frac{1}{2}(-\ln_1\ln_3 + \ln_2\ln_4)\Big)$$
$$+ m_3^2\Big(\frac{1}{12}\pi^2 + \frac{3}{2} - \ln_3 + \frac{1}{2}(-\ln_1\ln_2 + \ln_3\ln_4)\Big)\Bigg) + \cdots$$

# Reducible two-loop diagrams

Simplify the reducible two-loop diagram to $A$, $B$, $H$

$$\mathcal{M}_{R1}^{(1)} = \frac{4s}{27F_0^6} \iint \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{q_1^2}{(q_1^2 - M_\pi^2)(q_2^2 - M_\pi^2)^2}$$
$$= \frac{4sM_\pi^2}{27F_0^6} A(M_\pi^2) B(0, M_\pi^2, M_\pi^2)$$

$$\mathcal{M}_{R1}^{(2)} = \frac{s}{9F_0^6 M_\pi^2} \iint \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{(p_1 \cdot q_2)\, q_2^2}{(q_1^2 - M_\pi^2)(q_2^2 - M_\pi^2)^2} = 0$$

$$A \cdot B \sim \frac{m^2}{(16\pi^2)^2} \ln\!\left(m^2\right) \frac{m_1^2 \ln\!\left(m_1^2\right) - m_2^2 \ln\!\left(m_2^2\right)}{m_1^2 - m_2^2} + \cdots \sim H \quad \ln_i = \ln\!\left(m_i^2\right)$$

power-counting rule

Parity or odd loop momenta $\Rightarrow$ many $A \cdot B = 0$

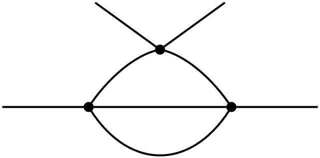
# Vertex diagrams

## An example

$$\langle\langle X\rangle\rangle_V = \frac{1}{i^2}\int \frac{d^dr}{(2\pi)^d}\frac{d^ds}{(2\pi)^d}\frac{X}{(r^2-m_1^2)((r-q)^2-m_2^2)(s^2-m_3^2)((r+s-p)^2-m_4^2)}$$

## Typical integral structure

$$P_{211}^{ij}(m^2,m_3^2,m_4^2,k^2) = \frac{1}{i^2}\int \frac{d^dr}{(2\pi)^d}\frac{d^ds}{(2\pi)^d}\frac{(r\cdot k)^i(s\cdot k)^j}{(r^2-m^2)^2(s^2-m_3^2)((r+s+k)^2-m_4^2)}$$



vertex diagram

An example

$$\begin{aligned}P_{211}^{20}(1,2,3;k^2) \sim \frac{1}{(16\pi^2)^2}k^2\Bigg[&\frac{m_1^2}{2}\left(\frac{1}{8}+\frac{\pi^2}{12}+\ln_1^2-\frac{1}{2}\ln_1\right)\\ &+\frac{m_2^2}{4}\left(\frac{7}{8}+\frac{\pi^2}{12}+\ln_2^2-\frac{3}{2}\ln_2\right)\\ &+\frac{m_3^2}{4}\left(\frac{7}{8}+\frac{\pi^2}{12}+\ln_3^2-\frac{3}{2}\ln_3\right)\\ &+\frac{1}{2}m_1^2h_{1;123}+\frac{1}{4}m_2^2h_{1;213}+\frac{1}{4}m_3^2h_{1;312}\Bigg]+\cdots\end{aligned}$$

# Sunset vs vertex diagrams

Compare sunset and vertex integrals

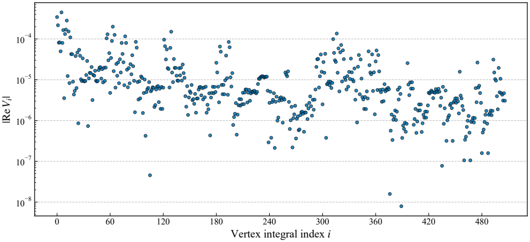
$$P_{211}^{20}(1,2,3;k^2) \sim \frac{k^2 m^2}{(16\pi^2)^2}\mathcal{O}(1-10)$$ $k$: external momentum

$$H \sim \frac{m^2}{(16\pi^2)^2}[\mathcal{O}(1) + \mathcal{O}(\ln_i) + \mathcal{O}(\ln_i \ln_j)] \sim \frac{m^2}{(16\pi^2)^2}\mathcal{O}(1-10)$$

$$|H_{\pi\pi}| < |H_{\pi K}|, \quad |P_{\pi\pi}| < |P_{\pi K}|$$

It seems

$$k^2 \lesssim 1 \Rightarrow |P| \lesssim |H|$$ most cases, but ...



# Sunset vs vertex diagrams

$$\mathcal{M}_{H1} = \frac{1}{432F_0^6 M_\pi^4} \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \frac{1}{(q_1^2 - M_\pi^2)(q_2^2 - M_\pi^2)((-p_1+q_1+q_2)^2 - M_K^2)}$$
$$\times \Big[ -2(p_1\cdot q_1)(11M_K^2 + 7M_\pi^2 + 2s - 7t - 4u) - 2(p_1\cdot q_2)(M_K^2 + 21M_\pi^2 - 10s - 8t + 7u)$$
$$+ M_\pi^2(2M_\pi^2 - 14(q_1\cdot q_2) - 4q_1^2 - 16q_2^2 + 2s - 7t - 10u)\Big]$$
$$\times \Big[M_\pi^2(-2M_K^2 + 4(q_1\cdot q_2) - 2q_1^2 + 4q_2^2 + s + t + u)$$
$$+ 2(p_1\cdot q_1)(M_\pi^2 + s + t - u) + 4(p_1\cdot q_2)(2M_\pi^2 - s - t + u)\Big]$$

$$\langle\langle X\rangle\rangle_V = \frac{1}{i^2}\int \frac{d^dr}{(2\pi)^d}\frac{d^ds}{(2\pi)^d}\frac{X}{(r^2-m_1^2)((r-q)^2-m_2^2)(s^2-m_3^2)((r+s-p)^2-m_4^2)}$$

## Wick rotation

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{[\ell^2-\Delta]^m} = (-1)^m \frac{i}{(2\pi)^d}\int d^d\ell_E \frac{1}{[\ell_E^2+\Delta]^m}$$

$\Rightarrow \mathrm{sign}H = -\mathrm{sign}P$

# Sunset vs vertex diagrams

Consider the numbers

$$\mathrm{sign}H = -\mathrm{sign}P$$

$$\frac{\text{number of } V}{\text{number of } H} \sim 4 \Rightarrow |\sum H| \lesssim |\sum P| \quad \text{cancellation}$$

$$|\sum H_{\pi\pi}| < |\sum H_{\pi K}|, \quad |\sum P_{\pi\pi}| < |\sum P_{\pi K}|$$

Consist with the results

| Process | Observable | $T_C$ | $T_L$ | $T_R$ | $T_H$ | $T_V$ | $T$ |
|---|---|---|---|---|---|---|---|
| $\pi K$ | $\mathcal{T}_{\pi K}^{3/2}$ | -0.074 | 0.588 | 1.763 | 10.815 | -11.311 | 1.782 |
| $\pi K$ | $10a_0^{3/2}m_\pi$ | -0.00642 | 0.0514 | 0.154 | 0.945 | -0.988 | 0.156 |
| $\pi\pi$ | $\mathcal{A}_{\pi\pi}$ | -0.0395 | 0.000287 | 1.53 | -0.468 | -0.740 | 0.283 |
| $\pi\pi$ | $10a_0^2$ | -0.0133 | -0.00321 | 0.0636 | 0.0715 | -0.120 | -0.00141 |

# One-loop diagrams

Amplitude

$$T_{\text{one-loop}} = \sum L_i \cdot B\left(m_1^2, m_2^2, p^2\right) \cdot \mathcal{D}^L_{m_j} \quad \mathcal{D}^L_{m_j} \text{ : mass factor}$$

Estimation

$$B\left(m_i^2, m_i^2, p^2\right) \sim -\frac{1}{16\pi^2}(\ln m_i^2 + 1) + \cdots$$

$$B(m_\pi^2, m_\pi^2, p^2) \sim 10^{-2} > B(m_K^2, m_K^2, p^2) \sim 10^{-4}$$

$$\mathcal{D}^L_{m_\pi} \sim m_\pi^6, \mathcal{D}^L_{m_K} \sim m_K^6 \Rightarrow \mathcal{D}^L_{m_\pi}/\mathcal{D}^L_{m_K} \sim 10^{-3}$$

$$L_i \sim 10^{-3}$$

$$\Rightarrow \frac{T_L^{(\pi)}}{T_L^{(K)}} \sim \frac{1}{10}$$

Keep power-counting rule

# One-loop vs two-loop diagrams

number of one-loop diagrams $<$ number of two-loop diagrams

$$\Rightarrow |T_L| < |T_R|,\ |T_H|,\ |T_V|$$

Consist with the results

| Process | Observable | $T_C$ | $T_L$ | $T_R$ | $T_H$ | $T_V$ | $T$ |
|---|---|---|---|---|---|---|---|
| $\pi K$ | $\mathcal{T}_{\pi K}^{3/2}$ | -0.074 | 0.588 | 1.763 | 10.815 | -11.311 | 1.782 |
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| $\pi\pi$ | $10a_0^2$ | -0.0133 | -0.00321 | 0.0636 | 0.0715 | -0.120 | -0.00141 |

# Tree diagrams

The amplitude

$$T_C = \sum C_i^r \cdot \mathcal{D}^C_{m_j}, \quad \frac{\mathcal{D}^C_{m_K}}{\mathcal{D}^C_{m_\pi}} \sim \frac{m_K^6}{m_\pi^6} \sim 10^3$$

$C_i^r$ is fitted, imprecision and adjustable

But the adjustable space is not very large

$\Rightarrow \pi K$ scattering length bad convergence

| Process | Observable | $T_C$ | $T_L$ | $T_R$ | $T_H$ | $T_V$ | $T$ |
|---|---|---|---|---|---|---|---|
| $\pi K$ | $\mathcal{T}^{3/2}_{\pi K}$ | -0.074 | 0.588 | 1.763 | 10.815 | -11.311 | 1.782 |
| $\pi K$ | $10a_0^{3/2}m_\pi$ | -0.00642 | 0.0514 | 0.154 | 0.945 | -0.988 | 0.156 |
| $\pi\pi$ | $\mathcal{A}_{\pi\pi}$ | -0.0395 | 0.000287 | 1.53 | -0.468 | -0.740 | 0.283 |
| $\pi\pi$ | $10a_0^2$ | -0.0133 | -0.00321 | 0.0636 | 0.0715 | -0.120 | -0.00141 |

# Summary

- Total sunset and vertex diagrams break the power-counting rule
- Why $\pi\pi$ scattering lengths seems keeping the power-counting rule?

  Small $m_\pi$, symmetry, adjustable $C_i$

- Why $\pi K$ scattering lengths seems breaking the power-counting rule?

  Large difference of $m_K$ and $m_\pi$, $C_i$ adjustable range

# Thank you for your attention!