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Topological observables and domain wall tension from finite temperature chiral perturbation theory

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第十一届手征有效场论研讨会

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Outline

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- ☼ QCD θ-vacuum
- ☼ Topological observables and domain wall
- ☼ Summary

Instanton and the theta term

Due to the existence of instantons, $G_{\mu\nu} = \pm\tilde{G}_{\mu\nu}$, their contribution to the action:

$$\frac{1}{4}\int d^4x G^c_{\mu\nu}\tilde{G}^{c,\mu\nu} = \pm 8\pi^2 n$$

with n is the winding number.

- True vacuum i,e, theta vacuum is linear superpositions of infinitely $|n>$ vacua:

$$|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$$

The complete massless QCD Lagrangian

$$\mathcal{L}_{QCD} = \mathcal{L}^0_{QCD} - \theta\frac{g^2}{32\pi^2}G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}$$

Theta term and neutron EDM

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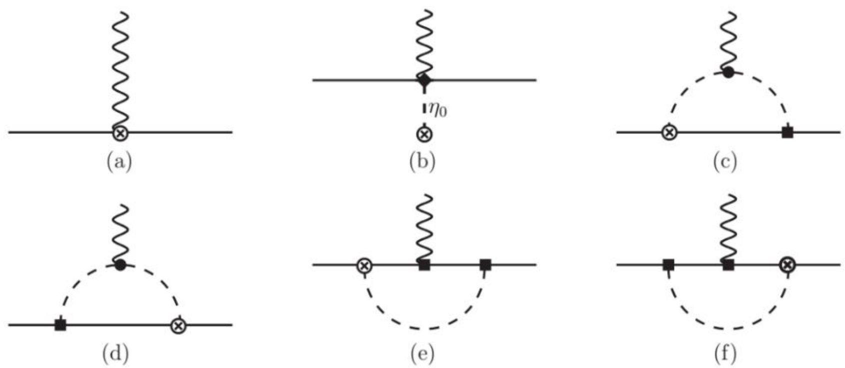
The topological theta term

$$\mathcal{L}_\theta = -\theta \frac{g^2}{32\pi^2} G^c_{\mu\nu} \tilde{G}^{c,\mu\nu} \qquad (\tilde{G}^c_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{c,\rho\sigma})$$

破坏CP对称性

$$\theta = \theta_0 + \arg \det M$$ (when the quarks are massive)



F. K. Guo, U. G. Meißner,
J. High Energy Phys. 12, 97 (2012)

Feynman diagrams contributing to the baryon electric dipole form factor up to next-to-leading order

$$d_n = d_n(\theta)$$

Experiment + Lattice

$|d_n| < 2.9 \times 10^{-26} \mathrm{e} \cdot \mathrm{cm} (90\% \text{ C.L.}) \Rightarrow \theta \lesssim 10^{-10}$

L. Di Luzio, M. Giannotti, E. Nardi, L Visinelli, Phys. Rep. 870, 1–117 (2020).

Cumulants of the QCD topological charge distribution

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- **The QCD partition function in the *θ*-vacuum**

$$Z(\theta) = \int [DG][Dq][D\bar{q}]\, e^{-S_{\rm QCD}[G,q,\bar{q}]-i\theta Q}$$

with the topological charge *Q* given by

F. K. Guo, U. G. Meißner, Phys. Lett. B 749, 278 (2015)

$$Q = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\int d^4x\, G^{\mu\nu}(x)G^{\rho\sigma}(x)$$

- **The partition function *Z*(*θ*) is dominated by the ground state, which defines the vacuum energy density**

$$\boxed{Z(\theta) = e^{-Ve_{\rm vac}(\theta)}, \qquad \text{or} \qquad e_{\rm vac}(\theta) = -\frac{1}{V}\ln Z(\theta)}$$

$$\Rightarrow \quad c_{2n} = \left.\frac{d^{2n}e_{\rm vec}(\theta)}{d\theta^{2n}}\right|_{\theta=0}$$

$$\boxed{\chi_t = \frac{1}{V}\langle Q^2\rangle_{\theta=0} \qquad c_4 = -\frac{1}{V}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)_{\theta=0}}$$

Topological susceptibility χ_t **The fourth cumulant c_4** $b_2 = -c_4/(12\chi_t)$

Chiral Perturbation Theory

At low energy:
- Effective field theories
- Lattice QCD
- Phenomenological approaches

Chiral Perturbation Theory: an effective field theory for QCD in the nonperturbative regime

- Weinberg's theorem: symmetries, first principles (unitarity, causality, cluster decomposition)) S-matrix and correlation functions.
- Power counting scheme

- Degree of freedom: the lightest pseudoscalar Goldstone bosons (SU(2): $\pi^0,\ \pi^\pm$; SU(3): $\pi^0,\ \pi^\pm,\ K^\pm,\ K^0,\ \bar{K}^0,\ \eta$)

One-loop generating functional

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Originally, the one-loop generational functional $Z_{\mathrm{one-loop}}$ is expanded around the functional for free fields at $\theta = 0$

J. Gasser and H. Leutwyler, NPB 250, 465 (1985)

$$Z_{\mathrm{one-loop}} = \frac{i}{2} \ln \det D^0 + \frac{i}{2} \mathrm{Tr}\left[(D^0)^{-1} \delta \right] + \cdots$$

- $\delta = \bar{\sigma}_{PQ} + \ldots = \sigma^{\triangle}_{PQ} + \sigma^{\chi}_{PQ} + \ldots$
- $\sigma^{\triangle}_{PQ} = \frac{1}{8} \langle [\tau^a, U^\dagger D_\mu U][\tau^b, U^\dagger D^\mu U] \rangle$ (in the vacuum, this term vanishes)
- $\sigma^{\chi}_{PQ} = \frac{1}{8} \langle \{\tau^a, \tau^b\} (\chi^+ U + U^\dagger \chi) \rangle - \delta_{PQ} M_P^2(0)$

It was then applied to study the topological susceptibility. And the loops were calculated using the Goldstone boson masses at $\theta = 0$, while the θ-dependence is kept in the first part of σ^{χ}_{PQ}:

Y. Y. Mao and T. W. Chiu, PRD 80, 34502 (2009); V. Bernard, S. Descotes-Genon, and G. Toucas, JHEP 12, 80 (2012)

$$\sigma^{\chi}_{PQ} \quad \Rightarrow \quad \sigma^{\chi}_{PQ} = \frac{1}{8} \langle \{\tau^a, \tau^b\} (\chi_\theta^+ U + U^\dagger \chi_\theta) \rangle - \delta_{PQ} M_P^2(0)$$

One-loop contributions

In fact, the pseudoscalar meson masses are θ-dependent $\Rightarrow M_P^2(\theta)$. If we expand the one-loop generating functional around the one for the free fields in a θ-vacuum, we get F. K. Guo and U.-G. Meißner, PLB 749, 278 (2015)

$$Z(\theta) = \frac{i}{2} \ln \det D^0(\theta) = \frac{i}{2} \mathrm{Tr} \ln D^0(\theta)$$

In this case, σ_{PQ}^{χ} vanishes. And $Z(\theta) = \frac{i}{2} \mathrm{Tr} \ln D^0(\theta)$ is the only term left in the one-loop generating functional. **The discussion is not only valid for $N = 2$ case but also for an arbitrary N case.**

- The differential operator $D_{PQ}^0(\theta)$: $D_{PQ}^0(\theta) = [\partial^\mu \partial_\mu + M_p^2(\theta)]\delta_{PQ}$

$\Rightarrow$ Loop contribution

$$\mathcal{L}_{\text{loop}} = \frac{1}{V} \frac{i}{2} \mathrm{Tr} \ln D^0(\theta) = \sum_P \frac{M_P^4(\theta)}{128\pi^2} \left[1 - 2 \ln \frac{M_P^2(\theta)}{\mu^2} - 64\pi^2 \lambda\right]$$

Divergences

The divergence at $d = 4$ dimension

$$\lambda = \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}\left[1 + \Gamma'(1) + \ln(4\pi)\right]\right\}$$

From the $\mathcal{O}(p^4)$ tree-level Lagrangian

$$\mathcal{L}^{(4,\text{tree})} \quad \Longrightarrow \quad \mathcal{L}^{(4,\infty)}_{\text{tree}} = \tfrac{\lambda}{2}\textstyle\sum_P M_P^4(\theta)$$

- $L_6 = L_6^r + \frac{11}{144}\lambda, \quad L_8 = L_8^r + \frac{5}{48}\lambda$
- $L_7 = L_7^r, \quad H_2 = H_2^r + \frac{5}{24}\lambda$

From the loop contribution

$$\mathcal{L}^{(4,\infty)}_{\text{loop}} = -\tfrac{\lambda}{2}\textstyle\sum_P M_P^4(\theta)$$

$\Longrightarrow$ **The finial result is finite!!!** And the θ vacuum energy density is obtained by summing up all contributions.

SU(2) CHPT: LO

- **The leading-order chiral Lagrangian of SU(2) CHPT**

$$\mathcal{L}_{p^2}^{(\text{tree})} = \frac{F^2}{4}\left\langle \chi_\theta U^\dagger + U\chi_\theta^\dagger \right\rangle$$

$$\chi_\theta = 2B\mathcal{M}_q \exp(i\mathcal{X}\theta)$$

$$U = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix}$$

$$\mathcal{L}_{p^2}^{(\text{tree})} = F^2 B(m_u \cos\phi_u + m_d \cos\phi_d)$$

- **Explicit analytical solution for the vacuum state**

$$\varphi = \arctan\left[\frac{z\sin(\mathcal{X}_u\theta) - \sin(\mathcal{X}_d\theta)}{z\cos(\mathcal{X}_u\theta) + \cos(\mathcal{X}_d\theta)}\right]$$

- **If we choose $\mathcal{X}_u = \mathcal{X}_d = 1/N$**

$$\tan\varphi = -\epsilon\tan\frac{\theta}{2} \qquad \epsilon = (m_d - m_u)/(m_u + m_d)$$

SU(2) CHPT: NLO

- **The next-to-leading-order chiral Lagrangian of SU(2) CHPT**
➔ Tree-level Lagrangian

$$\mathcal{L}_{p^4}^{(\text{tree})} = \frac{l_3}{16}\left\langle \chi_\theta U^\dagger + U\chi_\theta^\dagger \right\rangle^2 - \frac{l_7}{16}\left\langle \chi_\theta U^\dagger - U\chi_\theta^\dagger \right\rangle^2 + \frac{h_1 + h_3}{4}\left\langle \chi_\theta^\dagger \chi_\theta \right\rangle + \frac{h_1 - h_3}{2}\mathrm{Re}(\det \chi_\theta).$$

- **The next-to-leading-order chiral Lagrangian of SU(2) CHPT**
➔ One-loop contribution

$$\mathcal{L}_{p^4}^{(\text{loop})} = \frac{3}{2}\int i\frac{d^d p}{(2\pi)^d}\ln\left[-p^2 + M_\pi^2(\theta)\right] = \frac{3M_\pi^4(\theta)}{128\pi^2}\left[1 - 2\ln\frac{M_\pi^2(\theta)}{\mu^2}\right] + \mathcal{L}_{p^4}^{(\text{div})}$$

Free energy density up to NLO

- **Vacuum free energy density at zero temperature**

$$V(\theta) = -M_\pi^2(\theta)F_\pi^2\left\{1 - 2\frac{M_\pi^2}{F_\pi^2}\left[-\frac{(1-z)^2}{(1+z)^2}l_7^r + l_3^r + l_4^r - \frac{3}{64\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right] + \frac{M_\pi^2(\theta)}{F_\pi^2} \times \left[\frac{4z^2}{(1+z)^4}\frac{M_\pi^8 \sin^2(\theta)}{M_\pi^8(\theta)}l_7^r + (h_1^r - h_3^r + l_3^r) - \frac{3}{64\pi^2}\left(\log\left(\frac{M_\pi^2(\theta)}{\mu^2}\right) - \frac{1}{2}\right)\right]\right\}.$$

- **Vacuum free energy density at finite temperature**

$$V(\theta, T) = V(\theta) + \frac{3T}{2\pi^2}\int_0^\infty p^2 \times \log\left[1 - \exp\left(-\frac{\sqrt{p^2 + M_\pi^2(\theta)}}{T}\right)\right]dp$$

Topological susceptibility and b2

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- **Topological susceptibility at vanishing temperature**

$$\chi_t = \frac{1}{2}F^2 B\bar{m}\left(1-\epsilon^2\right)\left\{1 - \frac{2B\bar{m}}{F^2}\left(\frac{3}{32\pi^2}\ln\frac{2B\bar{m}}{\mu^2} - 2\left[l_3^r + h_1^r - h_3 - l_7\left(1-\epsilon^2\right)\right]\right)\right\} + \mathcal{O}\left(p^6\right)$$

- **Lattice data**

$$\chi_t^{1/4} = 75.6(2)\ \mathrm{MeV}$$

S. Borsanyi et al, Nature 539, 69-71 (2016)

$$\boxed{\chi_t^{1/4} = 75.5(5)\ \mathrm{MeV}}$$

- **SU(2) CHPT**

- **The normalized fourth cumulant $b_2 = -\frac{c_4}{12\chi_t}$ at vanishing finite temperature**

$$c_4 = -\frac{1}{8}F^2 B\bar{m}\left(1+2\epsilon^2-3\epsilon^4\right) + B^2\bar{m}^2\left(1-\epsilon^2\right)\left\{\frac{9}{128\pi^2}\left(1-\epsilon^2\right) + \frac{3}{32\pi^2}\ln\frac{2B\bar{m}}{\mu^2} - 2\left[l_3^r + h_1^r - h_3 - l_7\left(1+2\epsilon^2-3\epsilon^4\right)\right]\right\} + \mathcal{O}\left(p^6\right).$$

$$\boxed{b_2 = -0.029(2)}$$

- **SU(3) CHPT**

$$\chi_t^{1/4} = \sqrt{m_a f_a} = 75.6(6)\ \mathrm{MeV}$$

$$b_2 = \frac{\lambda_4 f_a^2}{12 m_a^2} = -0.028(3)$$

Z.-Y. Lu, M.-L. Du, F.-K. Guo, U.-G. Meißner, and T. Vonk, JHEP 05, 001 (2020).

Cumulants of the QCD topological charge distribution

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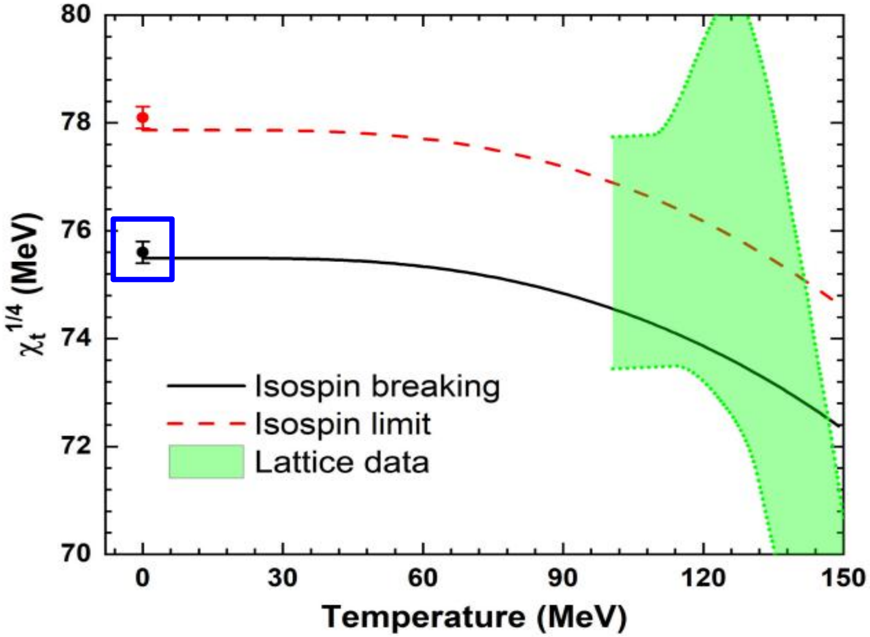

$$F(T,\theta) - F(T,0) = \frac{1}{2}\chi(T)\theta^2\left[1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right]$$

$$\chi^{1/4} = \begin{cases} 75.5(5)\ \mathrm{MeV}, & \mathrm{SU(2)CHPT} \\ 75.6(6)\ \mathrm{MeV}, & \mathrm{SU(3)CHPT} \\ \boxed{75.6(2)\ \mathrm{MeV}, \ \mathrm{Lattice\ data}} \end{cases}$$

ZYL, Q. Tang, S.-P. Wang, Y. Huang, Z. Zhang, B. Zhang, Phys. Rev. D 113, 014013 (2026)

ZYL, M.-L. Du, F.-K. Guo, U.-G. Meißner, T. Vonk, JHEP 05, 001 (2020)

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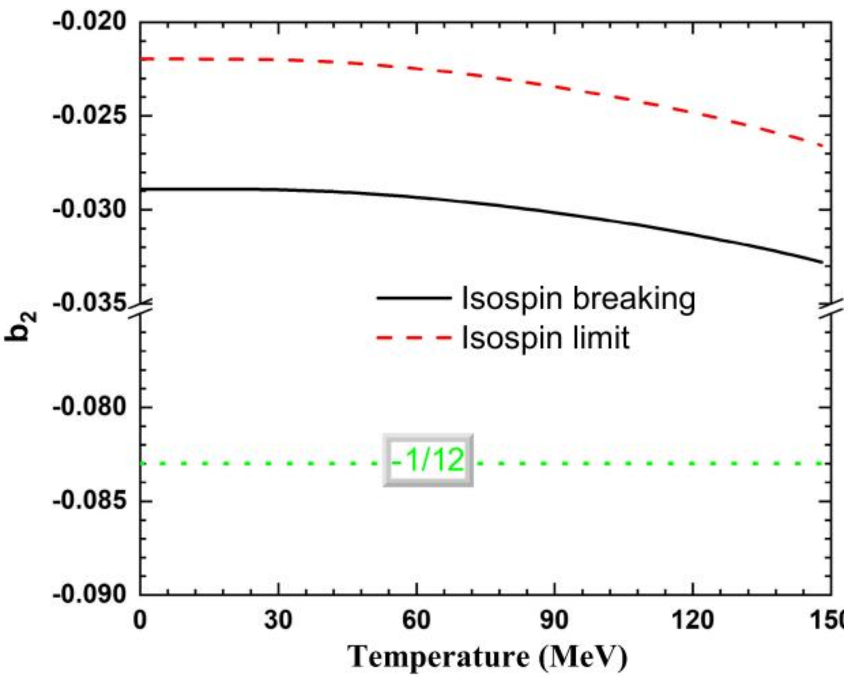
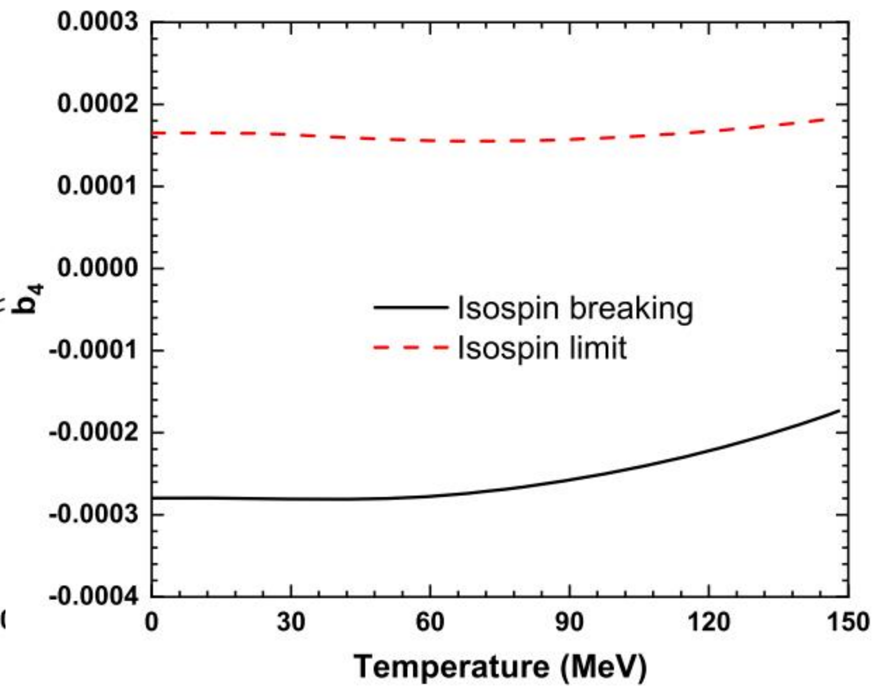
	$\chi_t^{1/4}$ (MeV)	b_2	b_4
Isospin breaking	75.5(5)	−0.029(2)	−0.00028(7)
Isospin limit	77.9(4)	−0.022(1)	−0.00017(6)

ZYL, Q. Tang, S.-P. Wang, Y. Huang, Z. Zhang, B. Zhang, Phys. Rev. D 113, 014013 (2026)





$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} = -\frac{c_4}{12\chi_t}$$

$$b_4 = \frac{\left[\langle Q^6\rangle - 15\langle Q^2\rangle\langle Q^4\rangle + 30\langle Q^2\rangle^3\right]_{\theta=0}}{360\langle Q^2\rangle_{\theta=0}} = \frac{c_6}{360\chi_t}$$

CHPT at finite isospin density

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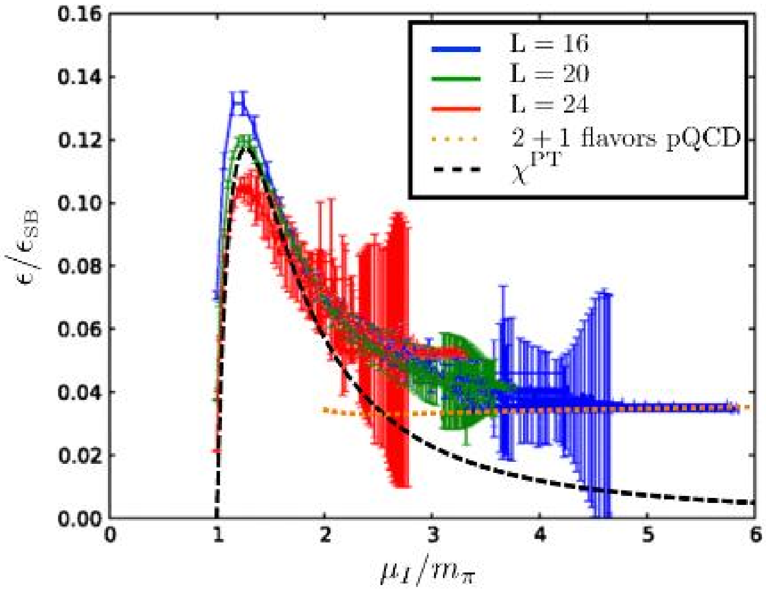

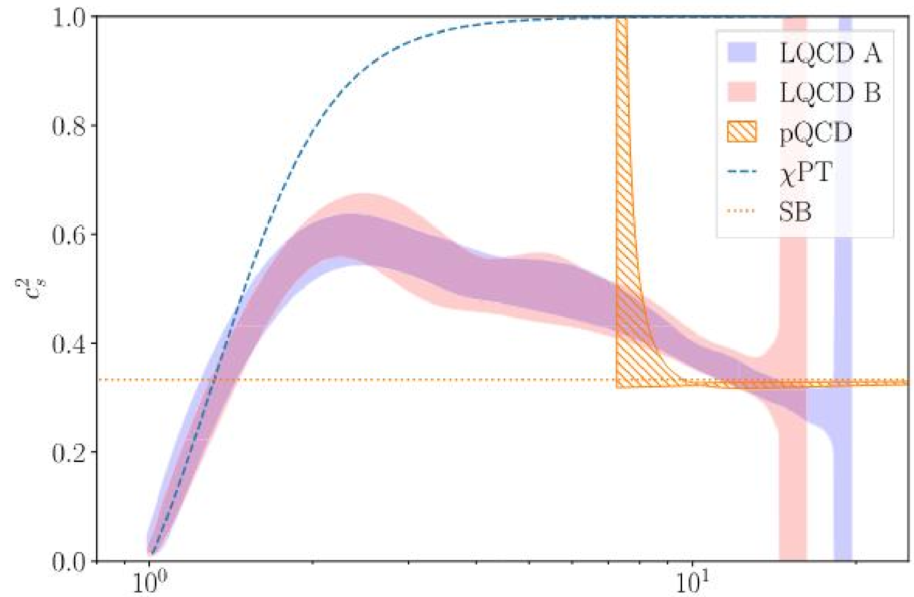

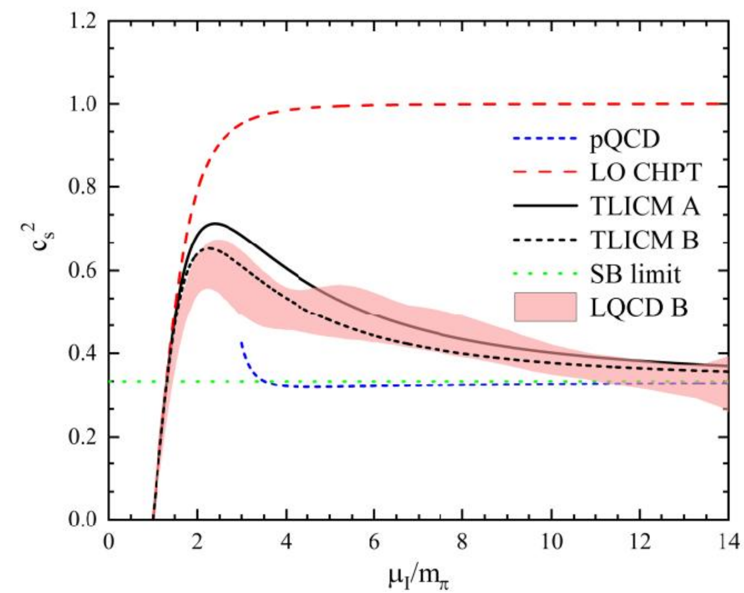

S. Carignano, A. Mammarella, M. Mannarelli, Phys. Rev. D 93, 051503 (2016).

Low density ➔ High density

Summary

- We calculate the QCD θ-vacuum energy density within SU(2) chiral perturbation theory up to next-to-leading order, **including both tree-level and one-loop contributions at finite temperature**.

- The **topological susceptibility and higher-order cumulants (b2, b4)** are systematically studied, showing good agreement with lattice QCD results.

- The **temperature and density dependence of topological observables** reveals the modification of axion properties in hot and dense QCD matter.

- The reduction of the axion potential barrier in the chirally restored phase **leads to thicker domain walls with reduced tension**, which may have implications for axion dynamics in compact astrophysical environments.