



Accessing the Infrared Structure of the Pion TMD with Continuum Schwinger Function Methods

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第十 一 届

手征有效场论研讨会

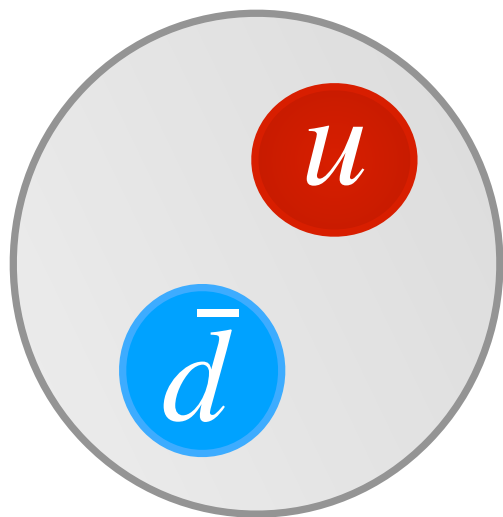
The 11th Workshop on Chiral Effective Field Theory (ChEFT 2026)

2026年8月11日-16日, 兰州

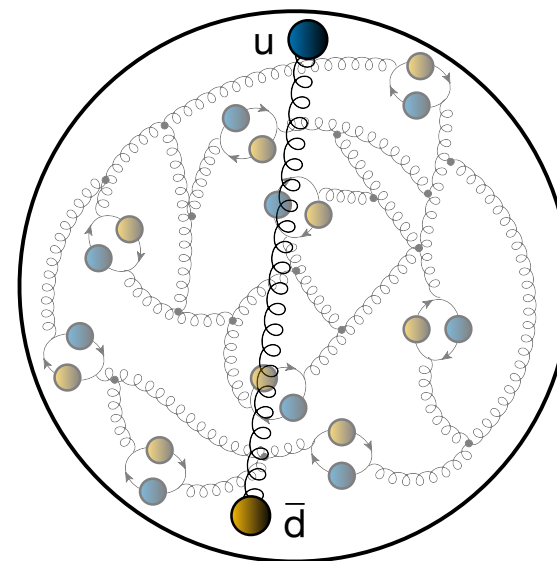
$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$

The pion

- Naive picture:
up quark + down antiquark:



- Realistic picture:
up quark + down antiquark
+ sea quarks + gluons



- Mass: ~ 140 MeV; the lightest hadron; spin 0; P parity $-$; pseudoscalar meson.
- Nambu-Goldstone bosons of spontaneously broken chiral symmetry.



Nambu-Goldstone boson

- Pion does not fit naturally into the mass pattern of constituent quark model:

Maris, Roberts, Tandy, Phys. Lett. B 420 (1998) 267-273

$$f_{\pi} m_{\pi}^2 = 2 m_u \rho_{\pi}$$

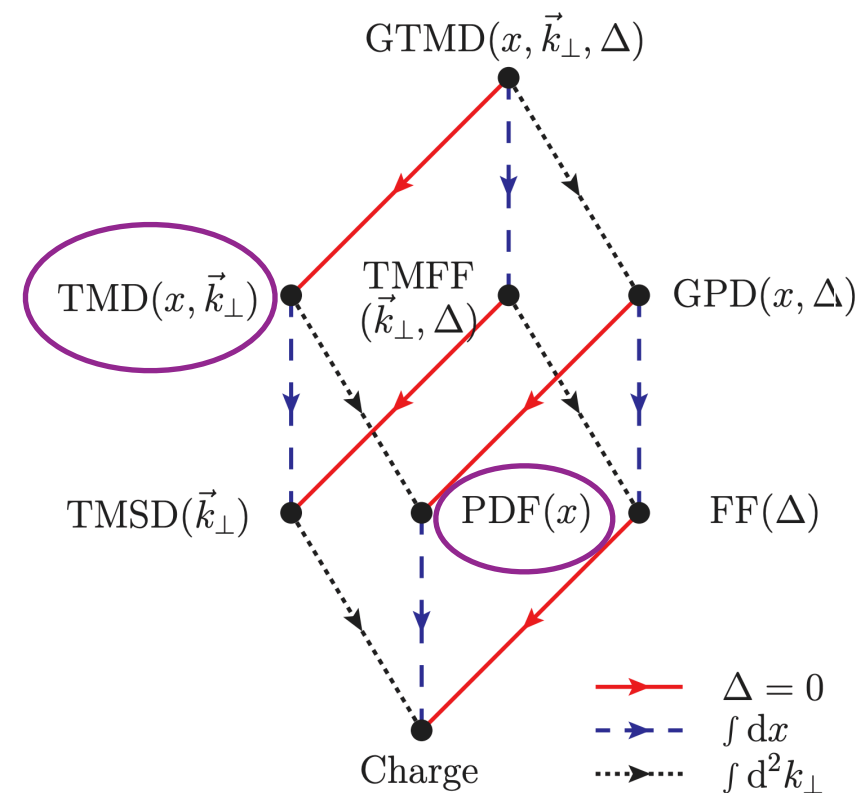
with f_{π} , leptonic decay constant; m_{π} , pion mass; m_u , current quark mass; ρ_{π} , pseudoscalar projection of the pion wave function onto the origin in configuration space.

- ✓ Gell-Mann-Oakes-Renner (GMOR) relation
- ✓ Pion mass m_{π} vanishes in the absence of current quark mass m_u - Nambu-Goldstone boson of chiral symmetry breaking (when current quark mass is zero, QCD Lagrangian possesses a chiral symmetry)
- ✓ Mass square of pion rises linearly with the current quark mass, $m_{\pi}^2 \propto m_u$, whereas in constituent quark model $m_{\text{meson}} \propto m_{\text{quark}}$.



Pion structure

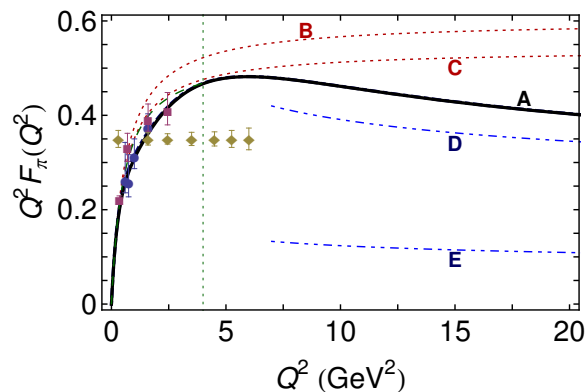
- Form factor: the closest thing we have to a snapshot, the size, shape and makeup of the hadron
 - ✓ Elastic electromagnetic form factor
 - ✓ Two-photon transition form factor
 - ✓ Gravitational form factor
- 1D picture of how quarks move within the hadron
 - ✓ Parton distribution amplitude (PDA)
 - ✓ Parton distribution function (PDF)
- A multidimensional view of the hadron structure
 - ✓ **Transverse momentum dependent distribution function (TMD)**
 - ✓ Generalized parton distribution (GPD)



Lorce, Pasquini, Vanderhaeghen, JHEP 05 (2011) 041.

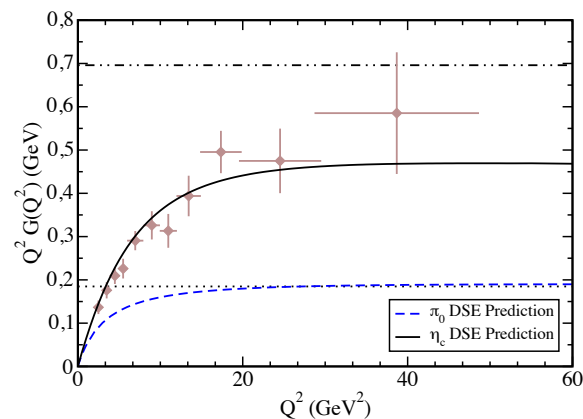


Pion structure



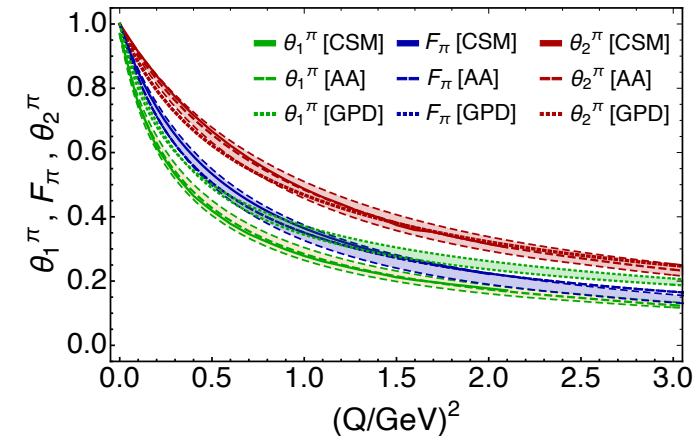
✓ Elastic electromagnetic form factor

Phys. Rev. Lett. 111 (2013) 14, 141802



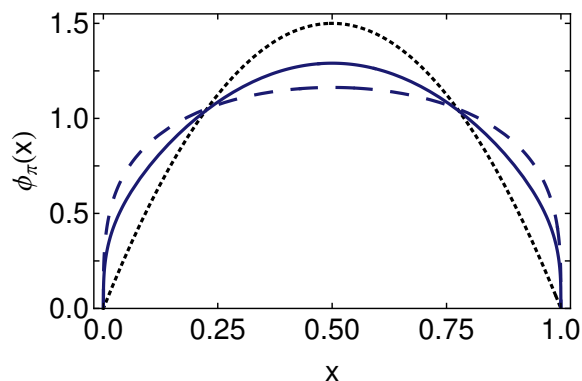
✓ Two-photon transition form factor

Phys. Rev. D 95 (2017) 7, 074014



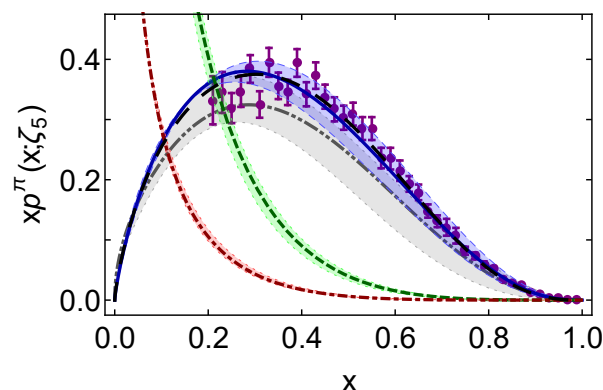
✓ Gravitational form factor

Eur. Phys. J. C 84 (2024) 2, 191



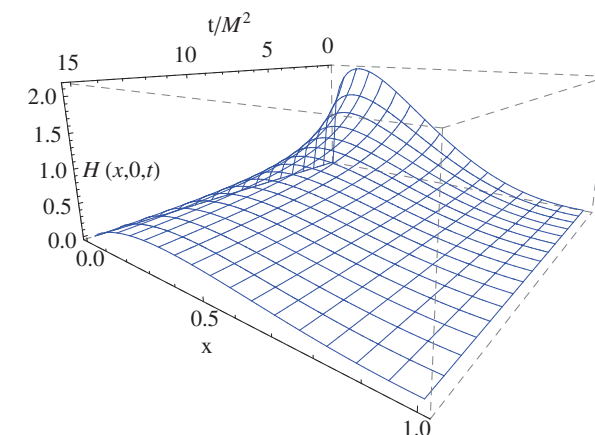
✓ Parton distribution amplitude (PDA)

Phys. Rev. Lett. 110 (2013) 13, 132001



✓ Parton distribution function (PDF)

Phys. Rev. D 101 (2020) 5, 054014

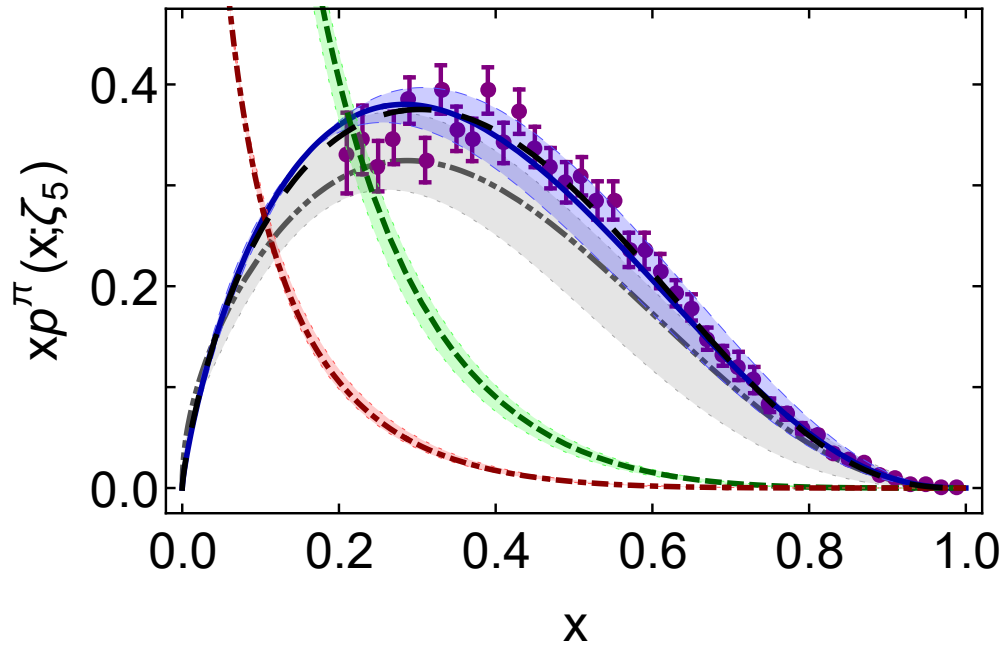


✓ Generalized parton distribution (GPD)

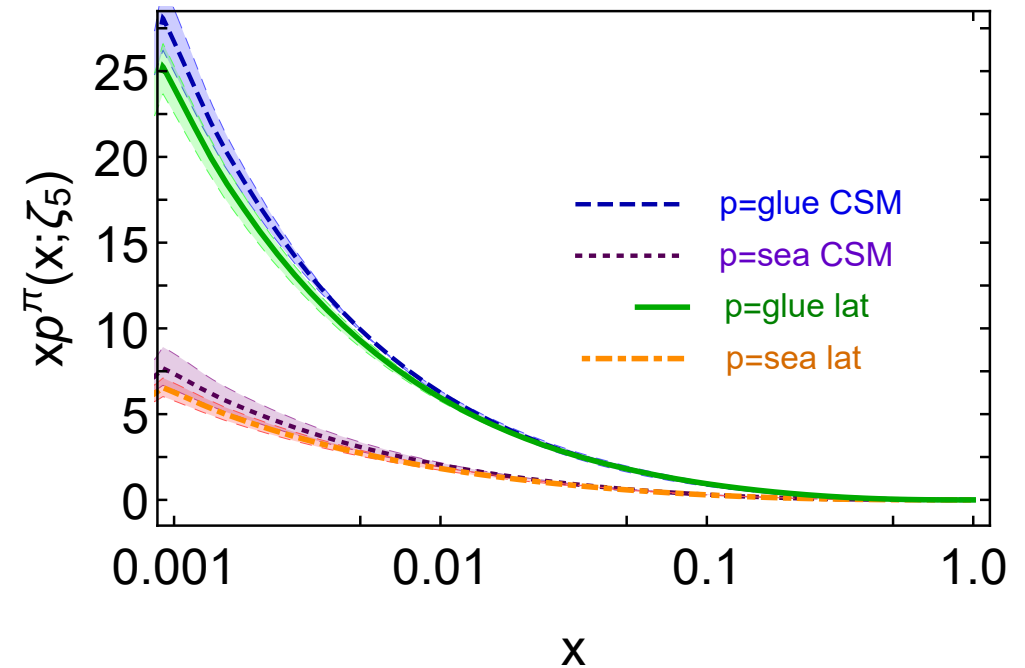
Phys. Lett. B 741 (2015) 190-196



Pion parton distribution function (PDF)



Ding, Raya, Binosi, Chang, Roberts, Schmidt,
Phys. Rev. D 101 (2020) 5, 054014



Cui, Ding, Morgado, Raya, Binosi, Chang, De Soto, Roberts, Rodríguez-Quintero, Schmidt, Phys. Rev. D 105 (2022) 9, L091502.



Future Facilities and experiments on pion PDF

➤ High Intensity, High Luminosity Facilities

✓ Jefferson Lab



✓ CERN: COMPASS++/AMBER



✓ Electron Ion Collider (EIC)

in USA



✓ Electron-ion collider in

China (EicC)

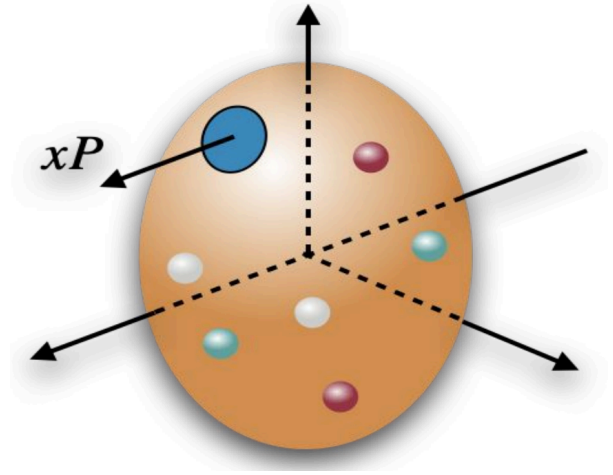
Electron **I**on **C**ollider in **C**hina, **EicC**



Intrinsic longitudinal/transverse momentum of quarks/gluons in hadrons

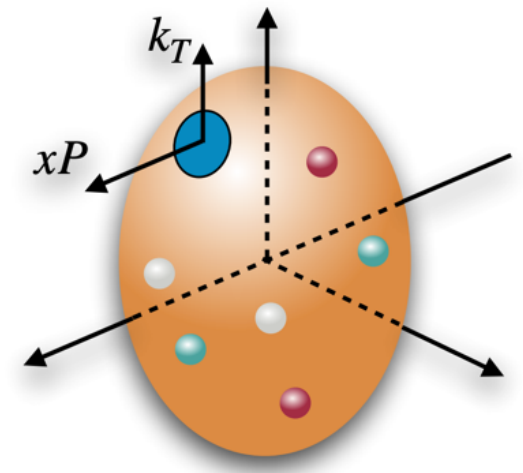
➤ Parton Distribution functions (PDFs) $q(x)$:

- The probability density for finding a particle with a certain longitudinal momentum fraction $x = k^+/P^+$ at resolution scale.



➤ Transverse momentum dependent distribution functions (TMDs) $f(x, k_\perp)$:

- It gives the joint probability density to find a parton inside a fast-moving hadron carrying longitudinal momentum fraction x and transverse momentum $k_\perp$ (defined perpendicular to the hadron's light-cone direction).

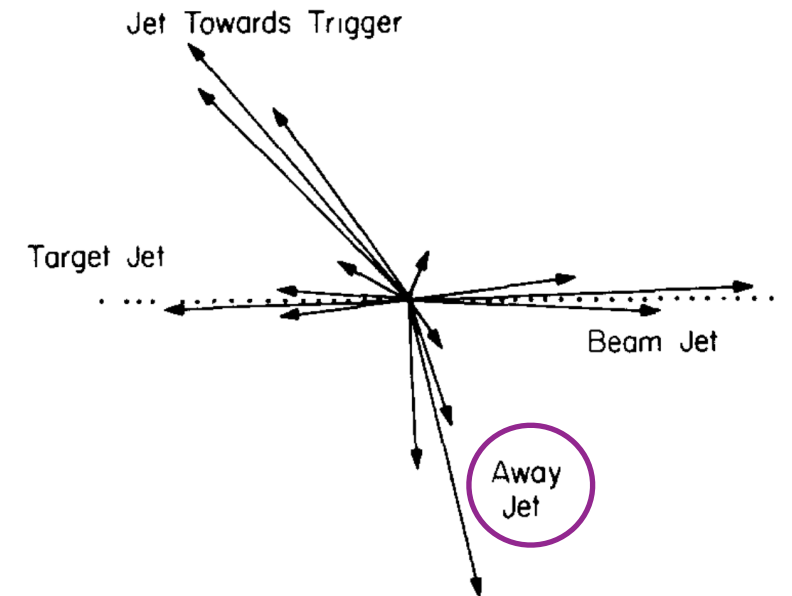


Intrinsic transverse momentum of quarks/gluons in hadrons

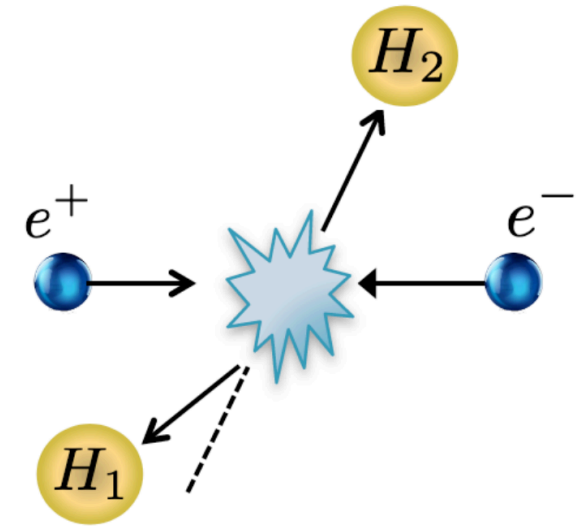
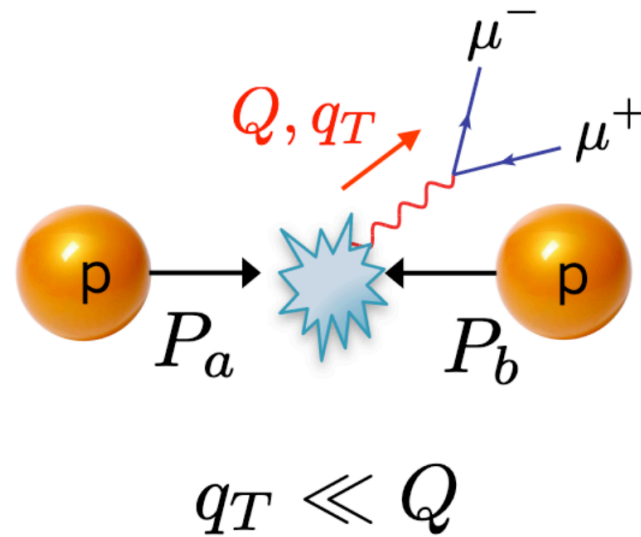
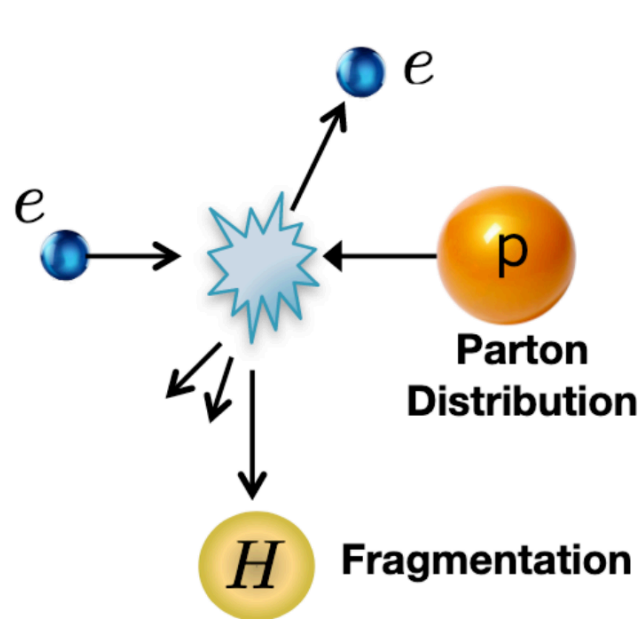
➤ 49 years ago

Correlations among particles and jets produced with large transverse momenta,
R.P. Feynman, R.D. Field, G.C. Fox, Nucl. Phys. B128, 1, 1977.

- Drell-Yan process: $A + B \rightarrow h_1 + h_2 + X$
- Abstract: “...If we allow the quarks within the initial hadrons to have a large mean transverse momentum, $\langle k_{\perp} \rangle_{h \rightarrow q}$, then good agreement with data is found...”.
- An assumption: the transverse momentum distribution is independent of x , $f(x, k_{\perp}) = q(x) \mathcal{F}(k_{\perp})$.
- An exponential form was chosen $\mathcal{F}(k_{\perp}) = b^2 \exp(-b |k_{\perp}|) / 2\pi$.
- The mean momentum $\langle k_{\perp} \rangle = \int dk_{\perp} k_{\perp} \mathcal{F}(x, k_{\perp}) = 2/b$.
- Typical choices are $\langle k_{\perp} \rangle = 300 \text{ MeV}$ and $\langle k_{\perp} \rangle = 500 \text{ MeV}$.



Intrinsic transverse momentum of quarks/gluons in hadrons



➤ Semi-Inclusive Deep Inelastic Scattering (SIDIS)

➤ Drell-Yan Process

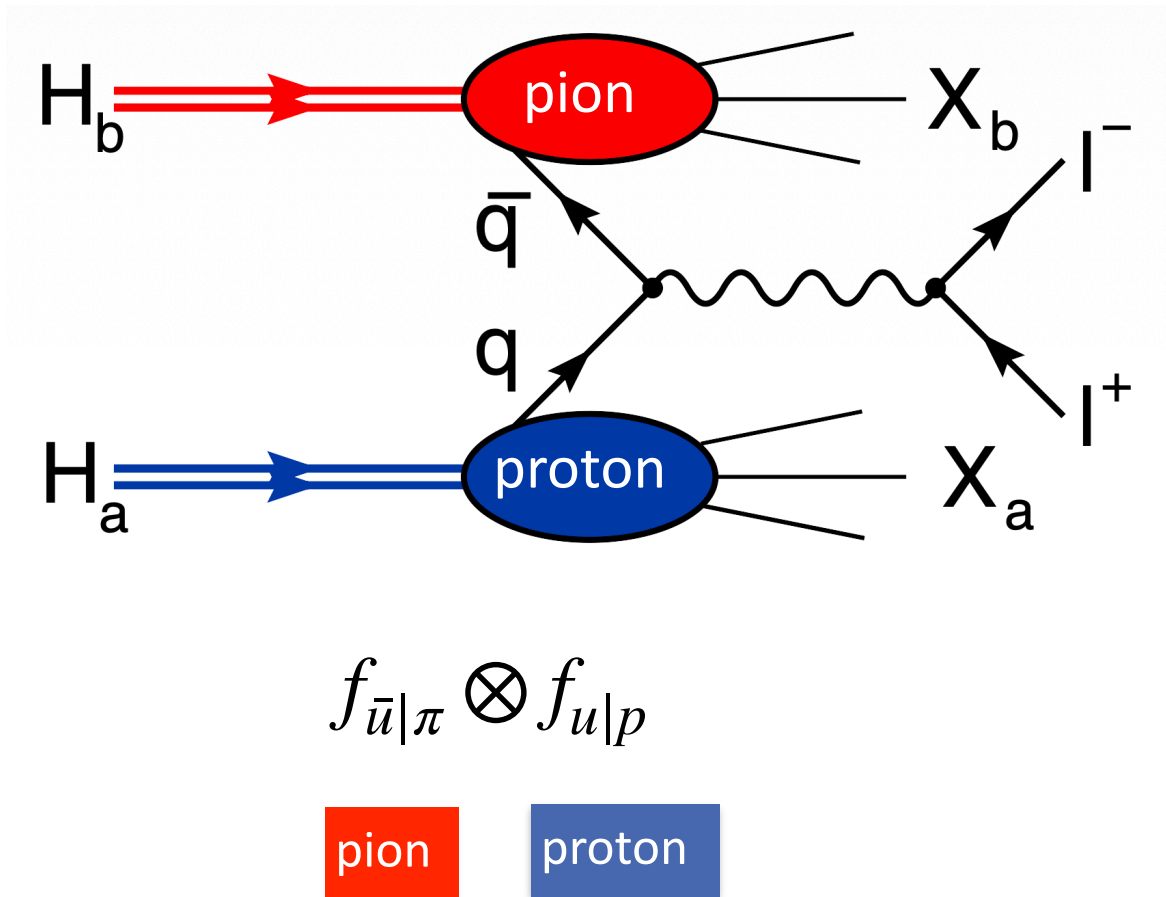
➤ Dihadron production in e^+e^-

- The transverse momentum of the observed hadron or lepton pair is measured in the $q_T \ll Q$ region, where it cannot arise from hard scattering alone and is instead dominated by intrinsic parton transverse momentum and soft radiation.



Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \rightarrow \mu^+ \mu^- + X$

- DY cross section with pions and **polarized proton** is described in terms of six structure functions Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.



$$F_{UU}^1 \sim f_{1,\pi} \otimes f_{1,p}$$

$$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^\perp \otimes h_{1,p}^\perp$$

$$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^\perp \otimes h_{1L,p}^\perp$$

$$F_{UT}^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^\perp$$

$$F_{UT}^{\sin(2\phi-\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1,p}$$

$$F_{UT}^{\sin(2\phi+\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1T,p}^\perp$$

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \rightarrow \mu^+ \mu^- + X$

➤ DY cross section with pions and **polarized proton** is described in terms of six structure functions

	pion	proton
unpolarized TMD	⊗	unpolarized TMD
Boer–Mulders function	⊗	Boer–Mulders function
Boer–Mulders function	⊗	worm-gear L

unpolarized TMD	⊗	Sivers function
Boer–Mulders function	⊗	transversity
Boer–Mulders function	⊗	pretzelosity

$$F_{UU}^1 \sim f_{1,\pi} \otimes f_{1,p}$$

$$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^\perp \otimes h_{1,p}^\perp$$

$$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^\perp \otimes h_{1L,p}^\perp$$

$$F_{UT}^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^\perp$$

$$F_{UT}^{\sin(2\phi-\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1,p}$$

$$F_{UT}^{\sin(2\phi+\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1T,p}^\perp$$

transversely polarized proton target



COMPASS Drell-Yan experiment for single-spin asymmetries in 2015



$$A^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^\perp$$

pion unpolarized TMD

$\otimes$ proton Sivers function

$$A^{\sin(2\phi_{CS}-\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1,p}$$

pion Boer-Mulders function

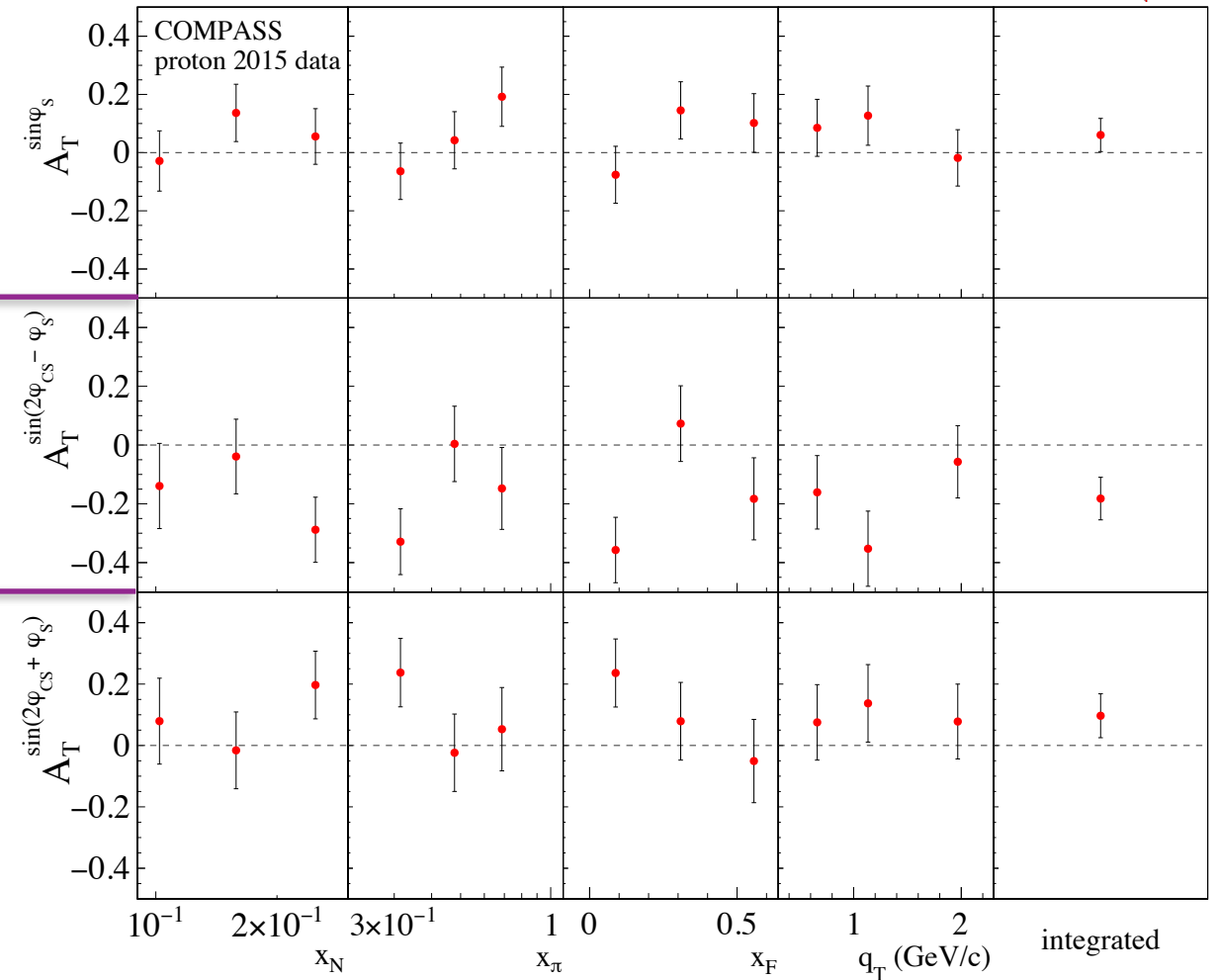
$\otimes$ proton transversity

$$A^{\sin(2\phi_{CS}+\phi_S)} \sim h_{1,\pi}^\perp \otimes h_{1T,p}^\perp$$

pion Boer-Mulders function

$\otimes$ proton pretzelosity

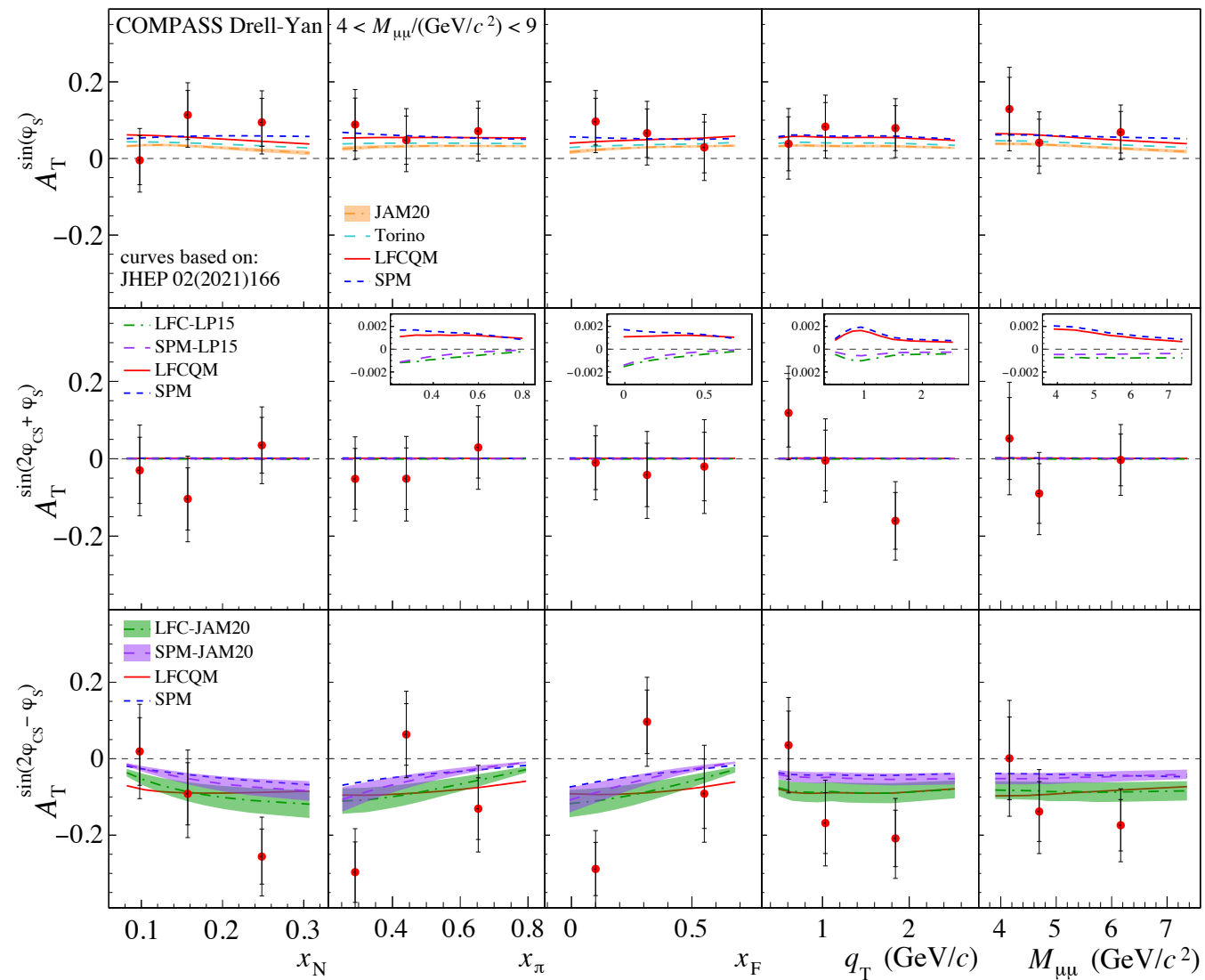
$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}^1}$$



First measurement of transverse-spin-dependent azimuthal asymmetries in the Drell-Yan process, COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)



COMPASS Drell-Yan experiment for single-spin asymmetries in 2018

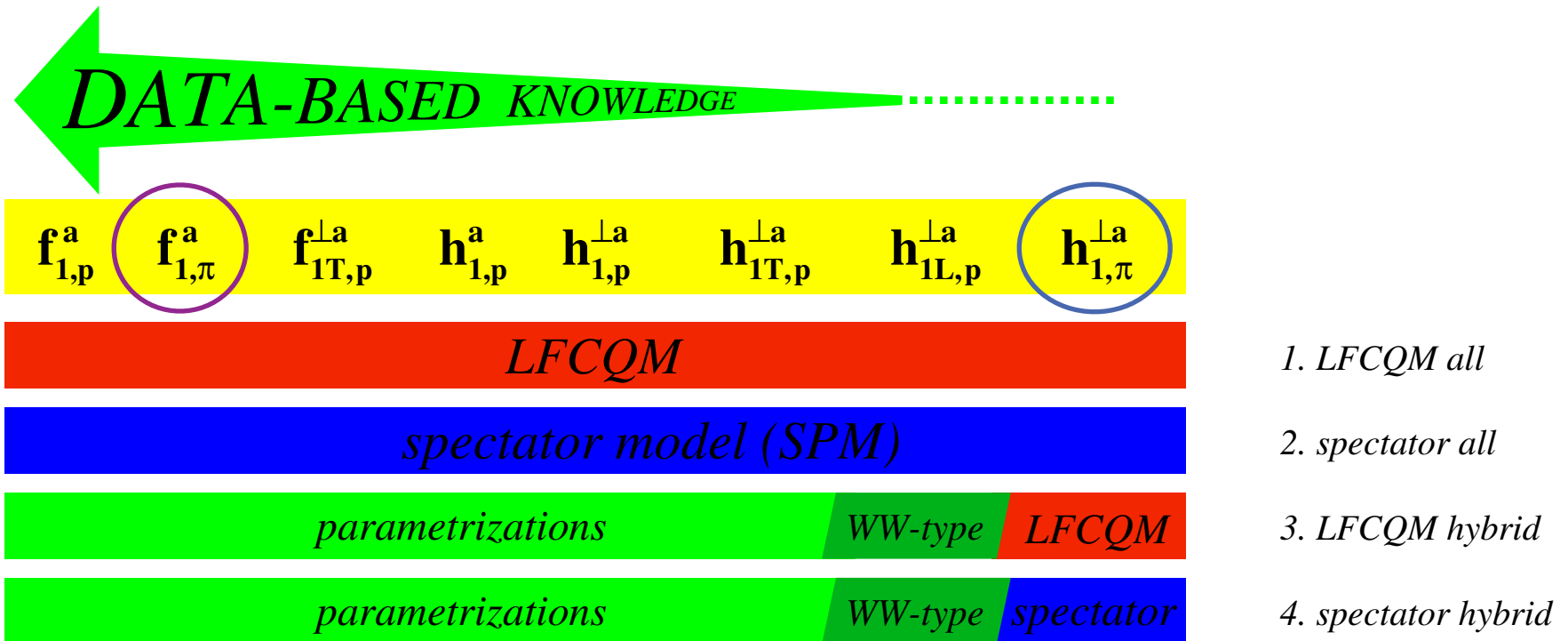


Final COMPASS Results on the Transverse-Spin-Dependent Azimuthal Asymmetries in the Pion-Induced Drell-Yan Process, COMPASS Collaboration, Phys. Rev. Lett. 133 (2024) 071902

Minghui Ding (丁明慧), mhdng@nju.edu.cn, Accessing the Infrared Structure of the Pion TMD with Continuum Schwinger Function Methods



Pion and Proton TMDs: From Best to Least Constrained



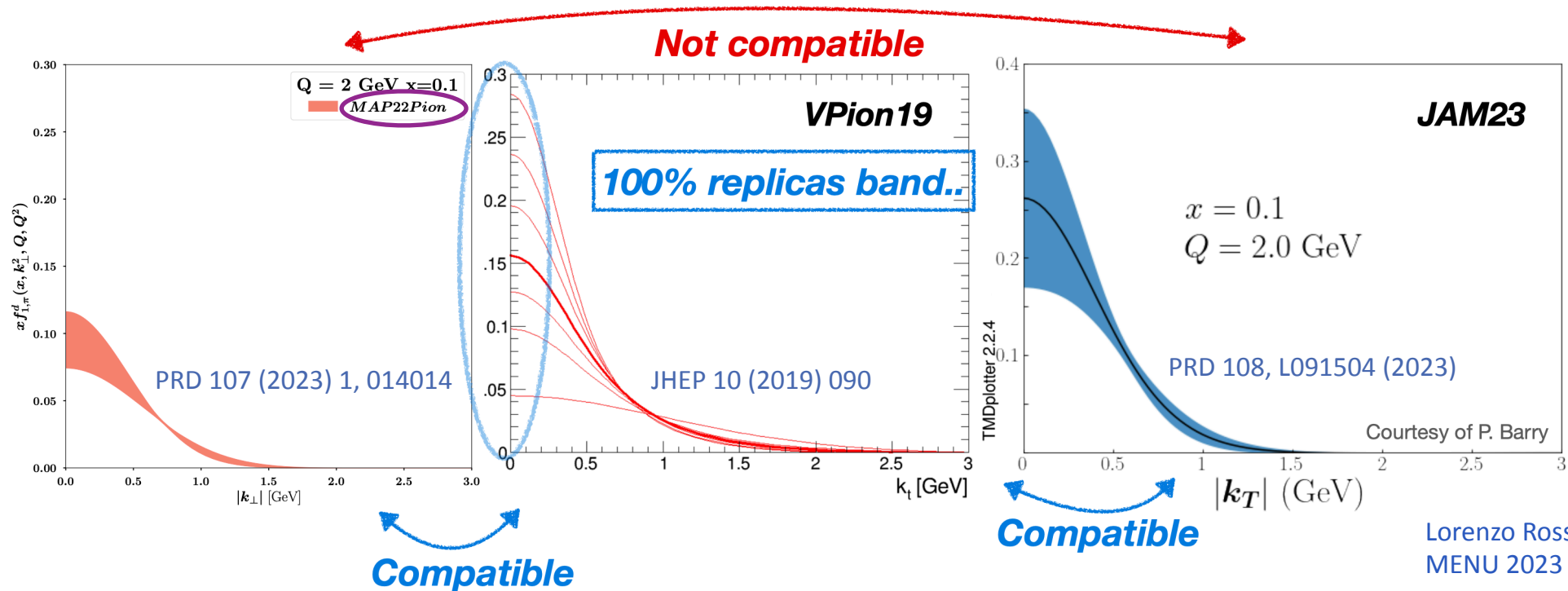
Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.

- The pion unpolarized TMDs $f_{1,\pi}(x, k_\perp)$ have been studied and much progress was achieved in incorporating effects of QCD evolution.
- The pion Boer-Mulders function $h_{1,\pi}^\perp(x, k_\perp)$ is the least known of the TMDs needed to describe the pion-proton DY process at leading twist. No extractions are currently available for this TMD.



Unpolarized Pion TMD: Phenomenological Extractions

Comparison of pion TMD PDFs

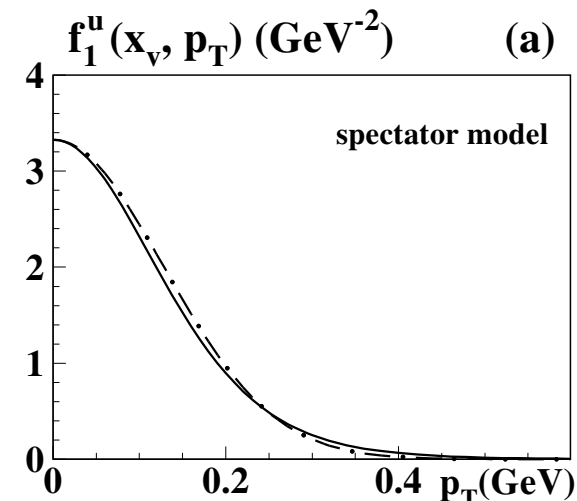
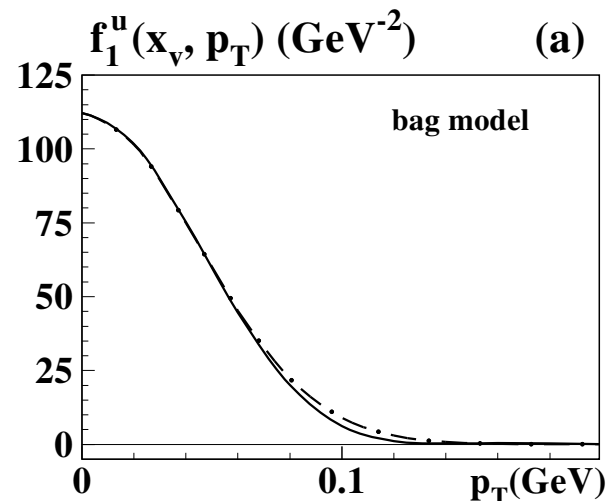
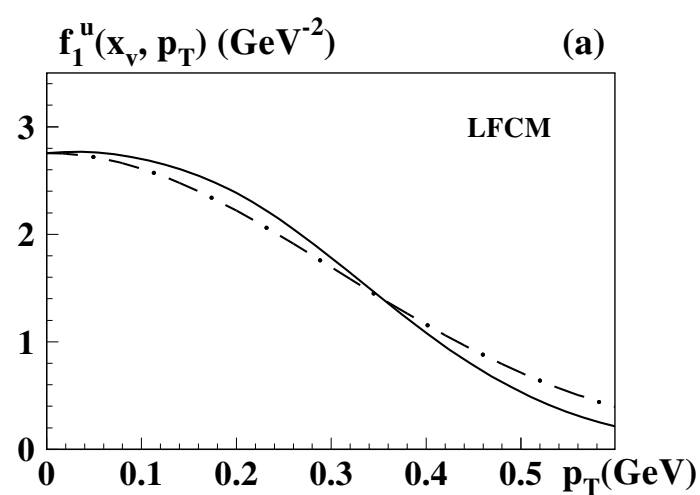


➤ **Unpolarised Drell-Yan lepton-pair production, E615 and E537 experiments data.**

Extraction of pion transverse momentum distributions from Drell-Yan data, MAP Collaboration, Phys. Rev. D 107, 014014 (2023)



Unpolarized Pion TMD: in models



Gaussian approximations

- light-front constituent model (LFCM),
- bag model,
- spectator model.

(a) Pion	LFCM		
TMD	$\langle p_T \rangle$	$\langle p_T^2 \rangle^{1/2}$	R_G
f_1^q	0.28	0.32	0.99
(b) Pion	LFCM	Bag	Spectator
TMD	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$
f_1^q	0.420	0.063	0.180

$$R_G = \frac{2}{\sqrt{\pi}} \frac{\langle p_T \rangle}{\langle p_T^2 \rangle^{1/2}}$$

Lorcé, Pasquini, Schweitzer, Transverse pion structure beyond leading twist in constituent models, Eur. Phys. J. C 76 (2016) 7, 415.

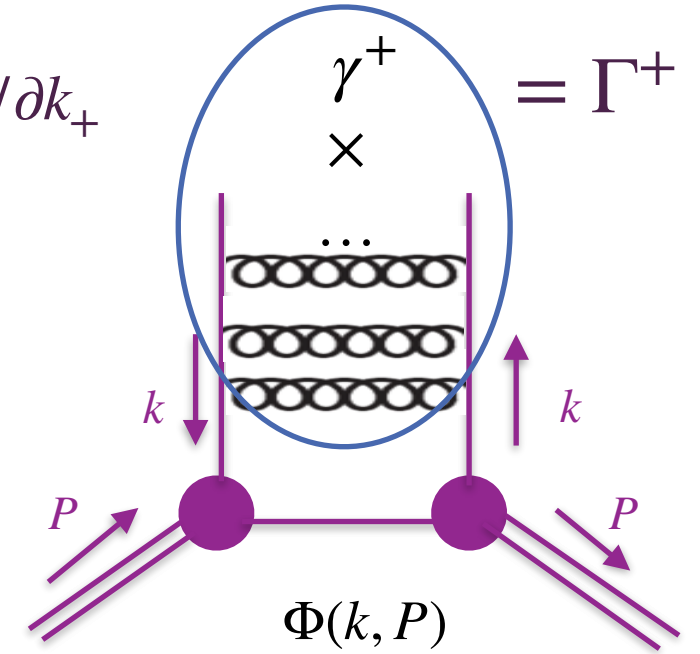


Twist-2 PDF of pion

$$q(x) \sim \int dk^- dk_\perp^2 \text{Tr} [\gamma^+ \Phi(k, P)] \Big|_{k^+ = xP^+} \sim \int d^4k \text{Tr} [\gamma^+ \Phi(k, P)] \delta(k^+ - xP^+)$$

- Because of gluon dressing, the dressed quark-photon vertex is used, $\gamma^+ \rightarrow \Gamma^+$, which is constrained by Ward's identity $\Gamma^+(k) = \partial S^{-1}(k) / \partial k_+$
- The rest is $S(k) \Gamma_\pi(k, -P) S(k+P) \Gamma_\pi(k, P) S(k)$

$$\begin{aligned} q_A(x) &\sim \int dk^- dk_\perp^2 \text{Tr} [\gamma^+ \Phi(k, P)]_{k^+ = xP^+} \\ &\sim \int d^4k \text{Tr} [\Gamma^+(k) S(k) \Gamma_\pi(k, -P) S(k+P) \\ &\quad \times \Gamma_\pi(k, P) S(k)] \delta(k^+ - xP^+) \end{aligned}$$



the hadronic part = quark-quark correlation function



Twist-2 PDF of pion

➤ DF: (term A)
$$q_A(x) = \int d^4k \text{Tr} \left[n^\mu \frac{\partial S(k)}{\partial k_\mu} \Gamma_\pi(k, -P) S(k+P) \Gamma_\pi(k, P) \right] \delta(n \cdot k - x n \cdot P).$$

- A second contribution, referred to as “term BC”, is introduced to restore symmetry: the pion PDF at hadron scale is expected to be a symmetric function if we consider isospin symmetry: $q_{\zeta_H}(x) = q_{\zeta_H}(1-x).$

$$q_{BC}(x) = \int d^4k \text{Tr} \left[n^\mu S(k) \frac{\partial \Gamma_\pi(k, -P)}{\partial k_\mu} S(k+P) \Gamma_\pi(k, P) \right] \delta(n \cdot k - x n \cdot P).$$

➤ DF:

$$q(x) = q_A(x) + q_{BC}(x) = \int d^4k \text{Tr} \left[n^\mu \frac{\partial [S(k) \Gamma_\pi(k, -P)]}{\partial k_\mu} S(k+P) \Gamma_\pi(k, P) \right] \delta(n \cdot k - x n \cdot P).$$

Chang, Mezrag, Moutarde, Roberts, Rodríguez-Quintero, Tandy, Phys. Lett. B737 (2014) 23-29



Twist-2 PDF of pion

- PDF
$$q(x) = \int d^4k \text{Tr} [\gamma^+ \Phi(k, P)] \delta(k^+ - xP^+)$$
- Mellin moments
$$\langle x^n \rangle = \int dx x^n q(x) \sim \int d^4k \text{Tr} [\gamma^+ \Phi(k, P)] (k^+)^n / (P^+)^n$$
- The calculation only requires the quark propagator and meson Bethe-Salpeter amplitude, which are calculated using the conventional method.
- Gauge sector: $g(k^2) = g_{IR}(k^2) + g_{UV}(k^2)$, with $g_{IR}(k^2) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2}$ and $g_{UV}(k^2)$ is from one-loop perturbative calculation.



Continuum Schwinger function Methods (CSMs)

Quark propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Quark-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Gluon propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$T = K + K \circ T$$

$$T \xrightarrow{p^2 \rightarrow -M^2} \Psi = \bar{\Psi}$$

$$\Rightarrow \Psi = K \circ \Psi$$

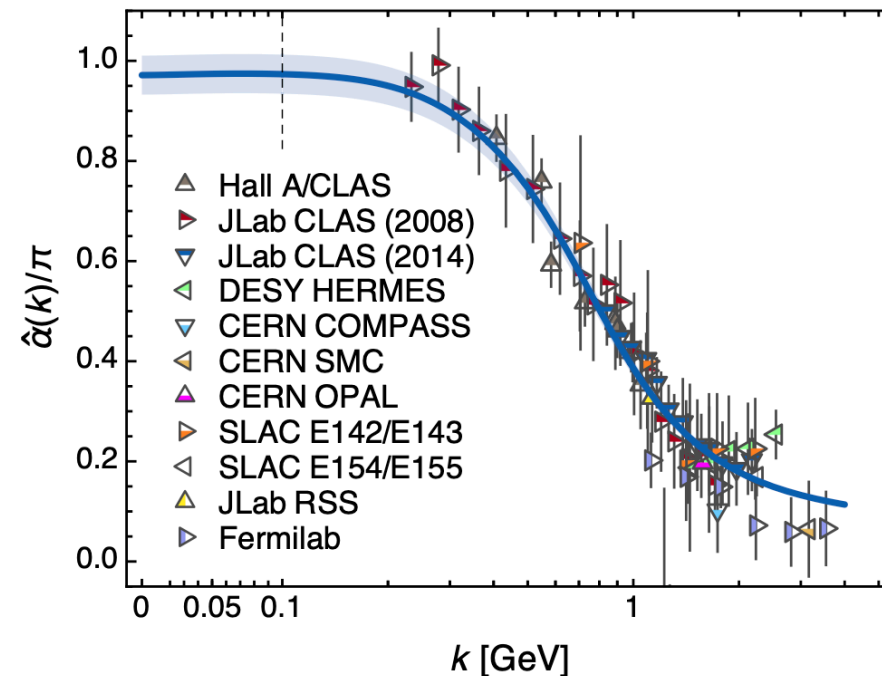
✓ Dyson-Schwinger Equation

✓ Bethe-Salpeter Equation

Process-independent effective charge

➤ Absence of a Landau pole

- ✓ Owing to the appearance of a gluon mass scale
- ✓ $\sqrt{k^2} < \zeta_H$, the running of the coupling slows down, at some point, the running ceases, effectively conformal
- ✓ $\zeta_H \approx 0.331(2)$ GeV
- ✓ Deep infrared, saturates to $\hat{\alpha}(k^2 = 0) = \pi \times 0.97$
- ✓ Gluons are screened, play no dynamical role
- ✓ Valence quasiparticles carry all hadron properties at hadron scale ζ_H .



Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, Phys. Rev. D 96, 054026 (2017)



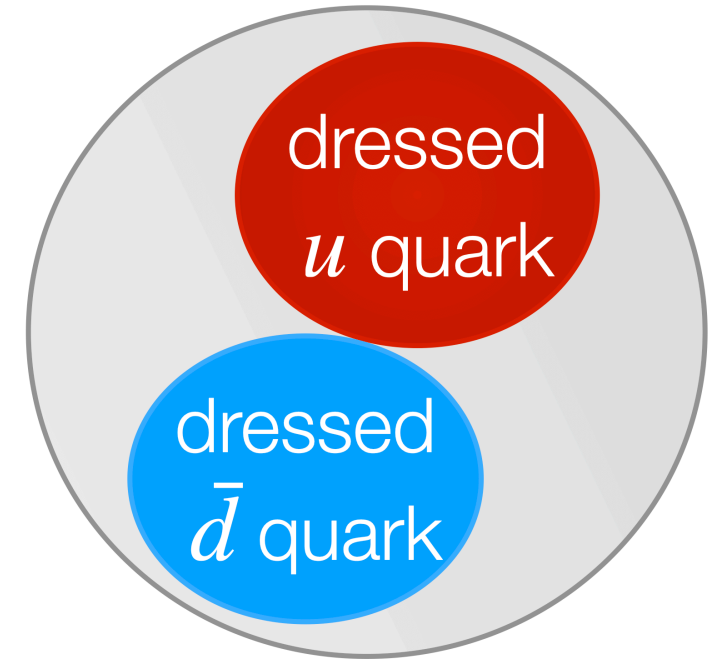
Pion at hadron scale

- At a hadron scale ζ_H , dressed valence-quarks carry all the pion's light-front momentum and the glue and sea distributions vanish.

Ding, Raya, Binosi, Chang, Roberts, Schmidt, Phys. Rev. D 101 (2020) 5, 054014

- In isospin limit $m_u = m_{\bar{d}}$, pion PDF is a symmetric function with respect to $x=1/2$.

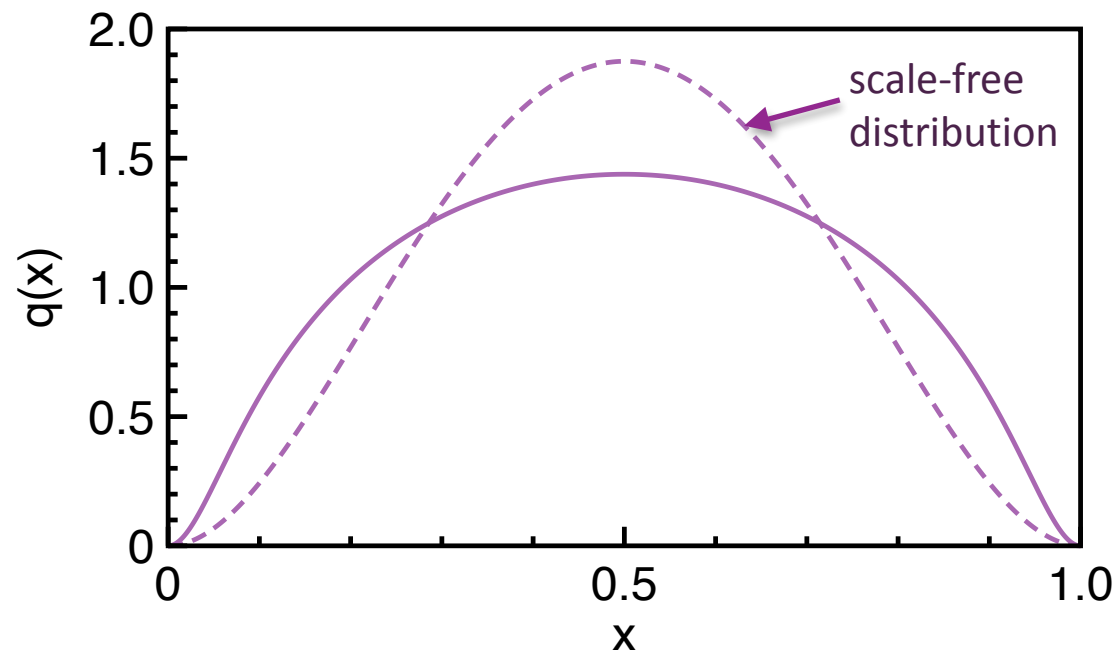
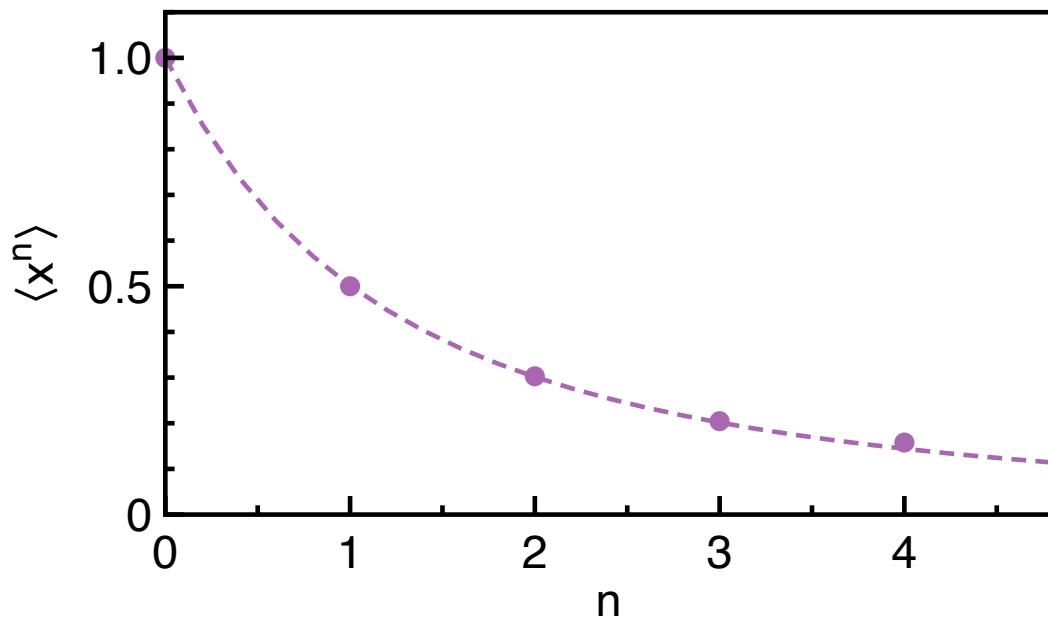
$$u^\pi(1-x, \zeta_H) = u^\pi(x, \zeta_H)$$



- ✓ Pion picture at hadron scale: dressed up quark + dressed down antiquark.



Twist-2 PDF of pion



➤ Lowest five Mellin moments of the pion valence-quark DF.

➤ Scale free result $q_{\text{sf}}(x) = 30x^2(1 - x)^2$

✓ Large x behaviour: $q^\pi(x; \zeta_H) \sim (1 - x)^2$

$$q(x) = \frac{1}{2} \ln \left[1 + \frac{x^2(1 - x)^2}{\rho_{DF}^2} \right], \quad \rho_{DF} = 0.061$$

✓ **Broad function**, induced by dynamical chiral symmetry breaking, and emergence of hadron mass (EHM).

Ding, Raya, Binosi, Chang, Roberts, Schmidt,
Phys. Rev. D 101 (2020) 5, 054014



Twist-2 TMDs of pion

unpolarized TMD Boer–Mulders function

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int dk^- \Phi_{ij}(k, P, S) \Big|_{k^+ = xP^+} = f_1(x, \mathbf{k}_\perp) \gamma^+ + h_1^\perp(x, \mathbf{k}_\perp) \sigma^{+i}$$

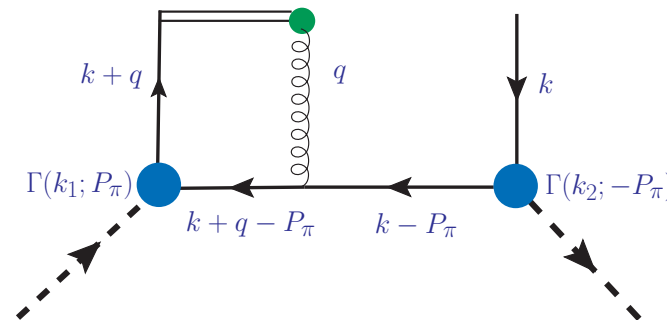
$$\boxed{f_1(x, \mathbf{k}_\perp)} = \int dk^- \text{Tr} [\gamma^+ \Phi(k)] \Big|_{k^+ = xP^+} = \int d^4 q \text{Tr} [\gamma^+ \Phi(q)] \delta(q^+ - xP^+) \delta(\mathbf{k}_\perp - \mathbf{q}_\perp)$$

$$h_1^\perp(x, \mathbf{k}_\perp) = \int dk^- \text{Tr} [\sigma^{+i} \Phi(k)] \Big|_{k^+ = xP^+} = \int d^4 q \text{Tr} [\sigma^{+i} \Phi(q)] \delta(q^+ - xP^+) \delta(\mathbf{k}_\perp - \mathbf{q}_\perp) = 0$$

➤ Wilson line contribution in

$$\Phi_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int d^4 \xi e^{ik \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{U}(0, \xi) \psi_i(\xi) | P, S \rangle$$

➤ Interaction between the eikonalised quark and the spectator $\Rightarrow$ nonzero $h_1^\perp(x, \mathbf{k}_\perp)$



Cheng, Cui, Ding, Roberts, Schmidt, Eur. Phys. J. C 85 (2025) 1, 115

Cheng, Ding, Binosi, Roberts, arXiv: 2603.25941 [hep-ph]



Twist-2 unpolarised TMD of pion

➤ TMD $f_1(x, \mathbf{k}_\perp) = \int d^4q \text{Tr} [\gamma^+ \Phi(q)] \delta(q^+ - xP^+) \delta(\mathbf{k}_\perp - \mathbf{q}_\perp)$

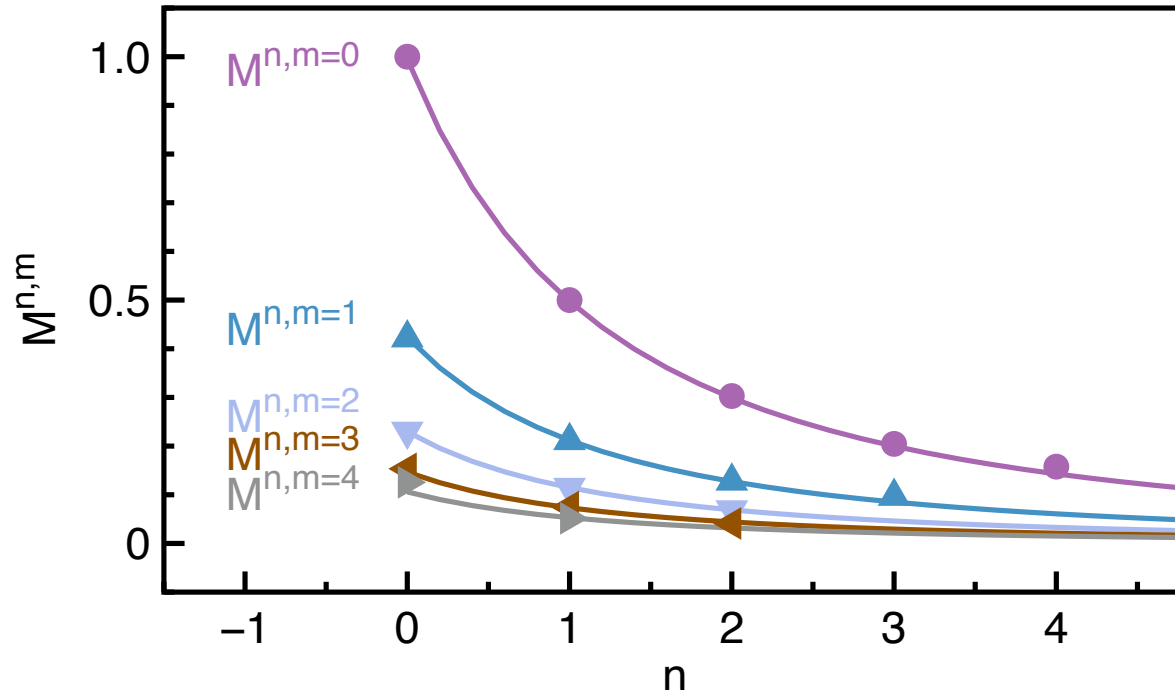
➤ Mellin-transverse moments

$$\begin{aligned} \mathcal{M}^{n,m} &= \int_0^1 dx x^n \int d^2k_T k_T^m f_1(x, k_T^2) \\ &\sim \int d^4k \text{Tr} [\gamma^+ \Phi(k, P)] (k^+)^n / (P^+)^n (q_{Tn})^m \end{aligned}$$

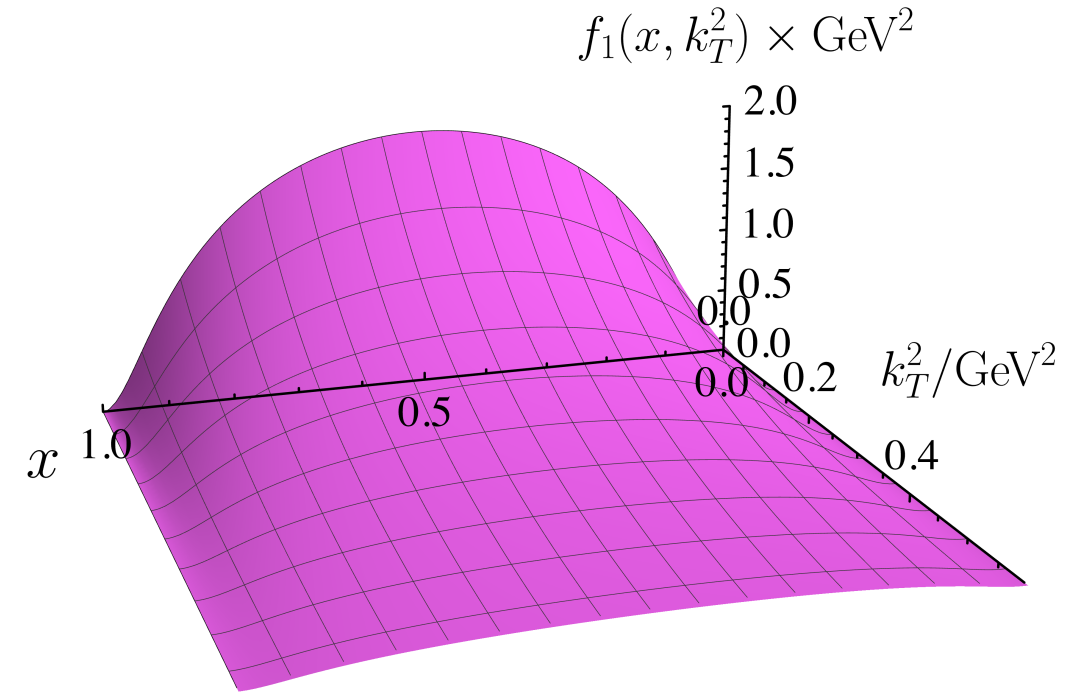
where $q_{Tn} = |\mathbf{q}_T|$ is the magnitude of the transverse momentum, defined using the transverse basis vectors n_{T_1} and n_{T_2} as $q_{Tn} = \sqrt{(n_{T_1} \cdot \mathbf{q})^2 + (n_{T_2} \cdot \mathbf{q})^2}$, $n_{T_1} = (0, 1, 0, 0)$, $n_{T_2} = (0, 0, 1, 0)$.



Twist-2 unpolarised TMD of pion



- Lowest seventeen moments Mellin-transverse moments of the pion unpolarised valence-quark TMD. [Ding, Phys. Lett. B 879 \(2026\) 140677](#)



$$f_1(x, k_T^2) = \frac{1}{2} \ln \left[1 + \frac{x^2(1-x)^2}{\rho_{TMD}^2} \right] \frac{e^{-k_T^2/\langle k_T^2 \rangle}}{\pi \langle k_T^2 \rangle},$$

with $\langle k_T^2 \rangle = 0.231 \text{ GeV}^2$ and $\rho_{TMD} = \rho_{DF}$.

$$\langle k_T \rangle = 0.422 \text{ GeV}$$



Gaussianity of the transverse-momentum distribution

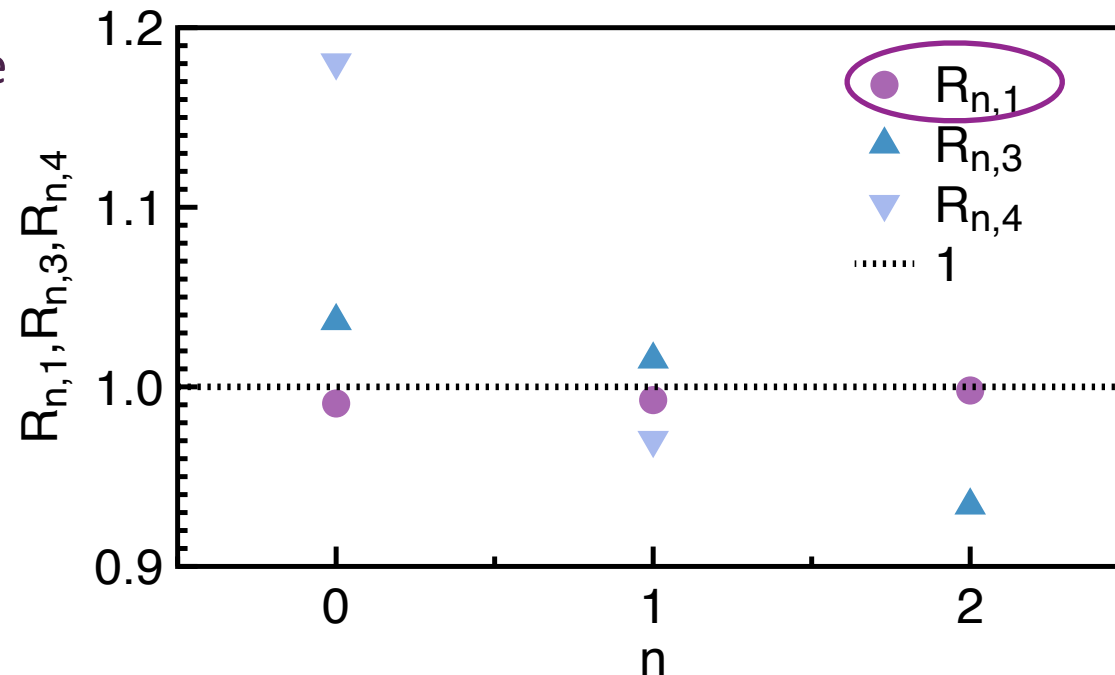
- To quantify deviations from a Gaussian transverse profile, we construct dimensionless ratios

$$R_{n,m} \equiv \frac{1}{\Gamma\left(1 + \frac{m}{2}\right)} \frac{\mathcal{M}^{n,m} / \mathcal{M}^{n,0}}{(\mathcal{M}^{n,2} / \mathcal{M}^{n,0})^{m/2}},$$

- which satisfy $R_{n,m} = 1$ for all m if the transverse-momentum dependence is purely Gaussian.

- $\mathcal{M}^{0,1} = \langle k_T \rangle = 0.422 \text{ GeV}$, $\mathcal{M}^{0,2} = \langle k_T^2 \rangle = 0.231 \text{ GeV}^2$. Therefore $\langle k_T \rangle \simeq \left(\frac{\pi}{4} \langle k_T^2 \rangle \right)^{1/2}$ confirming **near-Gaussian behavior of the transverse-momentum distribution at the hadron scale.**

- The intrinsic transverse width $\langle k_{\perp}^2 \rangle = 0.267 \text{ GeV}^2$ ($\omega = 0.4 \text{ GeV}$), and $\langle k_{\perp}^2 \rangle = 0.193 \text{ GeV}^2$ ($\omega = 0.6 \text{ GeV}$), In all cases, $\langle k_T \rangle \simeq \left(\frac{\pi}{4} \langle k_T^2 \rangle \right)^{1/2}$ is satisfied to high precision.



Large k_T behavior of $f_1^\pi(x, k_\perp)$

- It is possible to determine the power behavior at large $k_\perp$ for the different distributions, for example Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008) 023

$$f_1(x, k_\perp^2) \sim \frac{1}{k_\perp^2} \alpha_s \sum_{i=q, \bar{q}, g} q^i(x) \otimes K_1^i$$
$$h_1^\perp(x, k_\perp^2) \sim \frac{M^2}{k_\perp^4} \alpha_s \sum_{i=q, \bar{q}, g} [\text{collinear twist-three distributions}] \otimes K_3^i$$

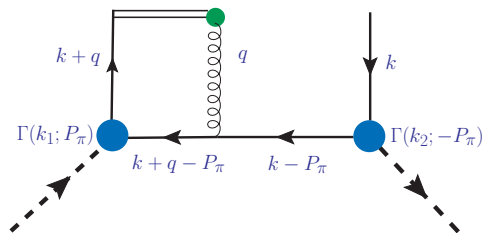
- If this is true, then $\langle k_\perp^m \rangle = \int dk_\perp^2 (|k_\perp|)^m f_1(x, k_\perp)$ (power divergence for $m \geq 1$)
- The calculated TMD primarily describes the soft ***nonperturbative component of the transverse-momentum distribution*** and does not reproduce the perturbative large- k_T tail expected in full QCD.



Small k_T behavior of $f_1^\pi(x, k_\perp)$

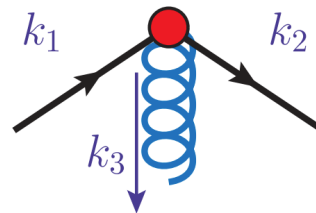
- Within the present impulse approximation framework, the TMD is determined solely by the dressed-quark propagator and pion Bethe–Salpeter amplitude, whose ultraviolet behavior leads to a substantially faster falloff.
- Lower transverse moments remain numerically stable and provide reliable information on the infrared structure, whereas higher moments become increasingly sensitive to ultraviolet details and should therefore be interpreted with appropriate caution.
- A more complete description of the ultraviolet region will require

✓ explicit incorporation of gauge-link contributions,



Cheng, Cui, Ding, Roberts, Schmidt,
Eur. Phys. J. C 85 (2025) 1, 115

✓ exchange-current effects beyond impulse approximation,



Costa et al, Phys. Rev. C 104 (2021) 4, 045201.

✓ transverse components of the dressed quark–photon vertex.

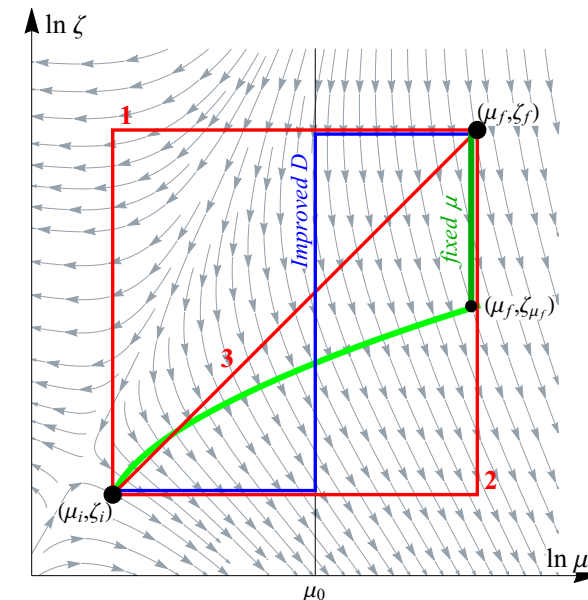
$$\begin{aligned}\Gamma_\mu(k, p) &= \Gamma_\mu^L(k, p) + \Gamma_\mu^T(k, p), \\ \Gamma_\mu^L(k, p) &= \sum_{j=1}^4 \lambda_j(k, p) L_\mu^j(k, p), \\ \Gamma_\mu^T(k, p) &= \sum_{j=1}^8 \tau_j(k, p) T_\mu^j(k, p),\end{aligned}$$

Qin, Chang, Liu, Roberts, Schmidt,
Phys. Lett. B 722 (2013) 384-388.

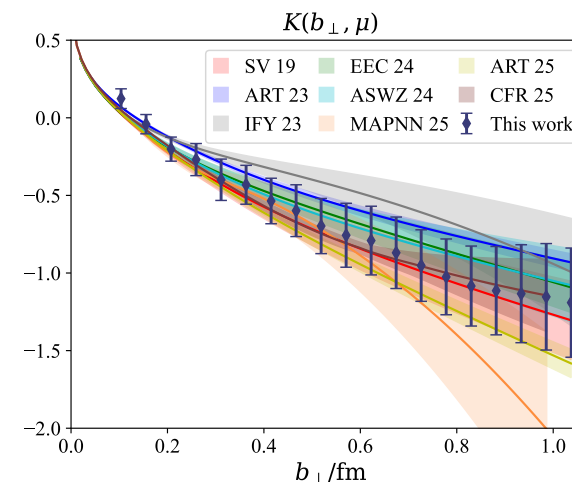


QCD evolution

- TMDs generally depend on a renormalization scale μ and rapidity scale ζ , the latter governing Collins–Soper evolution.
- The present calculation is performed at the hadron scale μ_H , where the pion is described solely in terms of dressed valence quark and antiquark. The x -dependent functions correspond to collinear distributions and the exponential factors are “primordial shapes” of TMDs at the initial scale. This particular dependence is often used in phenomenology, corresponds to the Gaussian Ansatz and is supported in models.
- Consequently, the resulting TMD may be defined at a fixed hadron-scale rapidity parameter ζ_H , naturally of order μ_H^2 .



Scimemi and Vladimirov, J. High Energ. Phys. 2018, 3 (2018)

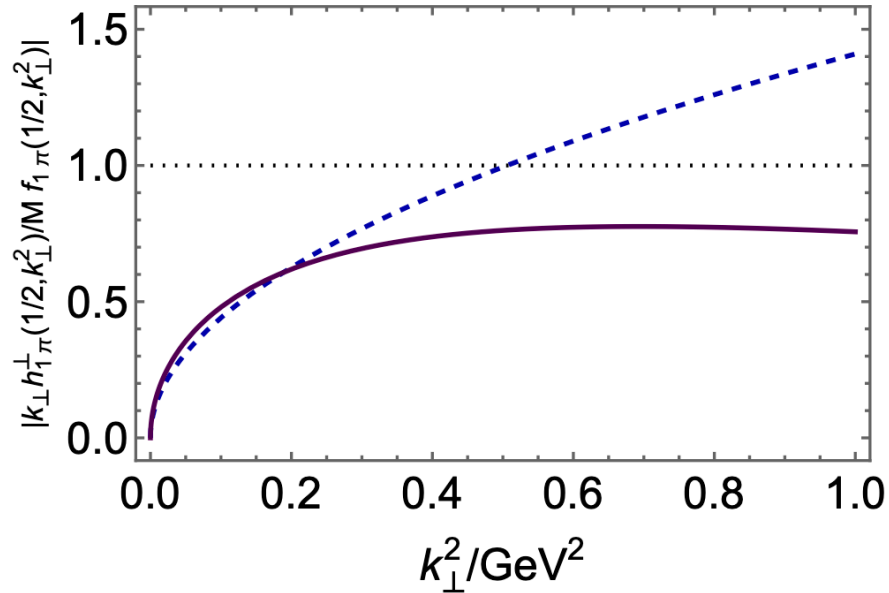


Jin-Xin Tan et al., Phys.Rev.D 113 (2026) 5, 054505



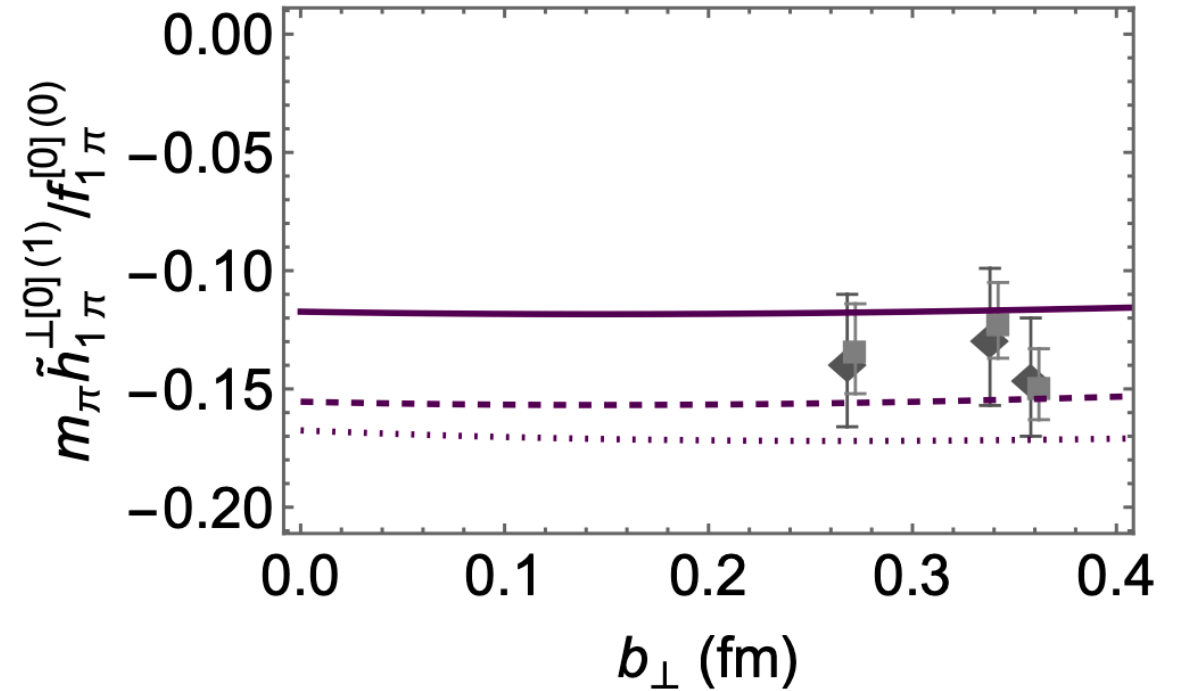
TMDs of pion: Boer-Mulders function

Cheng et al, *Eur. Phys. J. C* 85 (2025) 1, 115



- **Positivity constraint:** the bound is violated if the curve crosses the horizontal dotted line.

$|k_\perp h_{1\pi}^{\perp \text{MI}}(x = 1/2, k_\perp^2) / M f_{1\pi}(x = 1/2, k_\perp^2)|$ -- dashed blue
cf. $|k_\perp h_{1\pi}^\perp(x = 1/2, k_\perp^2) / M f_{1\pi}(x = 1/2, k_\perp^2)|$ solid purple.



- **Generalized Boer-Mulders shift.** Dotted purple curve -- hadron scale physical pion mass result; dashed purple curve -- hadron scale heavy pion ($m_\pi = 0.518$ GeV); and solid purple curve -- heavy pion result evolved $\zeta_{\mathcal{H}} \rightarrow \zeta_2$. LQCD results, obtained with $m_\pi = 0.518$ GeV at a resolving scale $\zeta = \zeta_2 = 2$ GeV.



Summary

- Determination of the infrared structure of the pion twist-2 unpolarized valence-quark TMD.
- A transverse-projection method was proposed that enables the computation of transverse-momentum moments of the TMD.
- The method is general and readily extendable to other hadrons and transverse observables, establishing a pathway for studying three-dimensional hadron structure.

Outlook

- Boer-Mulders function with realistic interaction, Kaon, proton.
- Collins-Soper kernel, QCD evolution.
- Comparison with COMPASS 2018 Pion-Induced Polarized Drell–Yan Data.



Thank you!

