The 11th Workshop on Chiral Effective Field Theory
(ChEFT 2026)

A Study on the $K \rightarrow \pi a$ decay within the large-N_C chiral perturbation theory

Based on work to appear on arXiv

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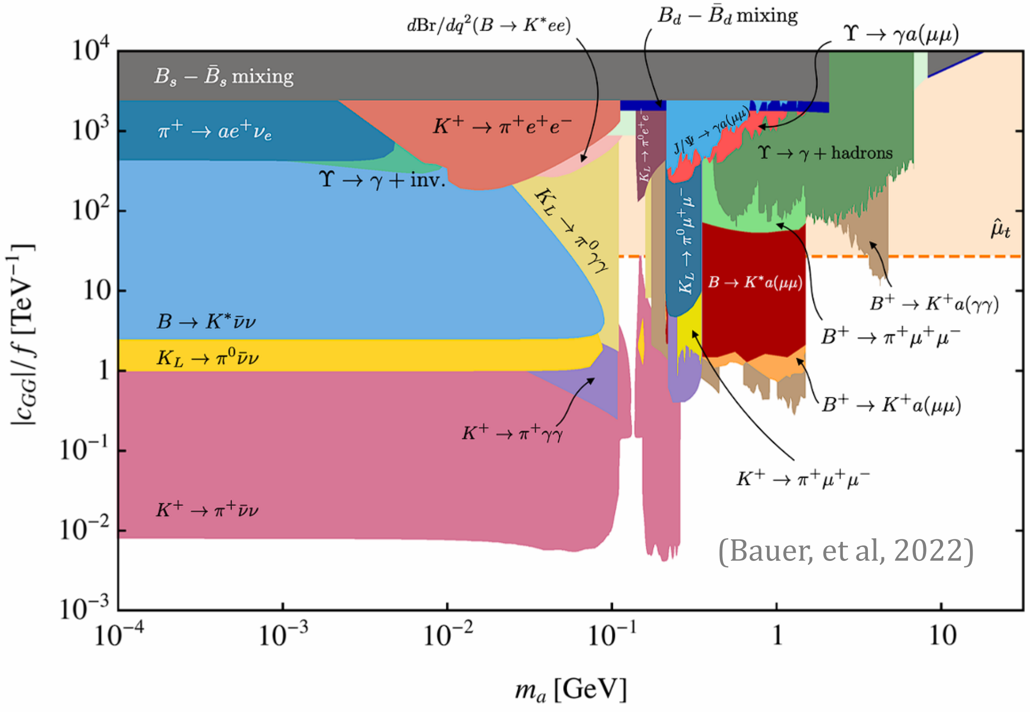
☐ Axion-like particles (ALPs)

- Motivated by the QCD axion and many BSM scenarios.
- Pseudo-Nambu-Goldstone boson (pNGB) from spontaneous $\mathrm{U(1)_{PQ}}$ breaking at $\Lambda \equiv 4\pi f \gg v_{EW}$.
- Effective interactions: dim-5 (or higher) operators suppressed by Λ^{-1}.
- Precision probes: intensity-frontier experiments (BESIII, Belle II, JLab, STCF, ...).
- Flavor physics: rare meson decays strongly constrain ALPs with $m_a \sim$ MeV--GeV.

☐ $K^+ \to \pi^+ a$ decay provides most stringent particle-physics constraint.

- SM: $K^+ \to \pi^+ \nu\bar{\nu}$ is theoretically very clean.
- Experiment: Precisely measured by NA62.
- Long-lived ALPs: $K^+ \to \pi^+ a$ has an identical missing-energy signature,

$$f \gtrsim 5 \times 10^8 \text{ TeV assuming } C_{ds}^{v} = O(1).$$



□ Status of $K \to \pi a$ calculations

- Pioneering EFT calculation (Georgi, et al, 1986)
  - ✓ First analysis of $K \to \pi a$ in a general EFT framework.
  - × Incomplete treatment of ALP terms in the weak effective Lagrangian.
  - × Resulting amplitudes depended on the chiral rotation scheme.
- Resolution (Bauer, et al, 2021)
  - ✓ Implemented the correct flavor-changing quark currents in SU(3) χPT.
  - ✓ Obtained scheme-independent decay amplitudes.
  - ✓ Predicted $\mathcal{B}(K^+ \to \pi^+ a)$ nearly 40 times larger than previous estimates.
- NLO calculation (Cornella, et al, 2024)
  - ✓ Complete NLO analysis in SU(3) χPT.
  - ✓ Included novel ALP-induced operators.
  - ✓ NLO corrections can be sizable; uncertainties are dominated by the relevant LECs.

- Our motivation
  - Previous studies
    - ✓ Based on SU(3) χPT, where η_0 is integrated out.
  - Why go beyond SU(3)?
    - ✓ η-η' mixing is numerically important for processes involving η.
    - ✓ The ALP belongs to the same singlet sector as the η_0.
    - ✓ Large-N_C QCD matching requires the explicit inclusion of the η_0.
  - We study $K \to \pi a$ decay at LO in U(3) χPT (large-N_C χPT), including the explicit η_0 and the associated new weak operators.
- Key question: How important are the explicit η_0 contributions to $K \to \pi a$?

Effective Lagrangian at quark level

ALP effective Lagrangian up to dim-5 at $\mu_0 \simeq 1.2$ GeV (kaon decay scale) (Georgi, et al, 1986)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ALP}}^{D\leq 5}(\mu_0) =& \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \mathcal{L}_{\mathrm{ALP-lepton}} \\
&+ \frac{\partial_\mu a}{f}\left(\bar{q}_L k_Q \gamma^\mu q_L + \bar{q}_R k_q \gamma^\mu q_R\right) \\
&- c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f} G_{\mu\nu}^a \widetilde{G}^{\mu\nu,a} - c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f} F_{\mu\nu}\widetilde{F}^{\mu\nu}
\end{aligned}
$$

$$
k_Q = \begin{pmatrix} [k_U]_{11} & & \\ & [k_D]_{11} & \boxed{[k_D]_{12}} \\ & \boxed{[k_D]_{21}} & [k_D]_{22} \end{pmatrix}, \quad k_q = \begin{pmatrix} [k_u]_{11} & & \\ & [k_d]_{11} & \boxed{[k_d]_{12}} \\ & \boxed{[k_d]_{21}} & [k_d]_{22} \end{pmatrix}
$$

$|\Delta S| = 1$ FCNC ALP-quark couplings induce $K \to \pi a$ directly

$|\Delta S| = 1$ transition can also be induced by the SM nonleptonic weak interaction

$$
\mathcal{L}_{\mathrm{eff}}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{6} c_i(\mu_0)\hat{Q}_i + \mathrm{h.c.}\,,
$$

ignoring higher-order electroweak corrections

$$
\hat{Q}_2 = 4\left(\bar{d}_L \gamma^\mu u_L\right)\left(\bar{u}_L \gamma_\mu s_L\right)
$$

additional operators induced by non-factorizable gluon exchanges

- We match quark and gluon d.o.f. onto mesonic fields using Chiral Perturbation Theory

QCD chiral effective Lagrangian in the U(3) framework

SU(3) framework

- In the χ-limit ($m_{u,d,s} \to 0$), $\mathcal{L}_{\mathrm{QCD}}^{n_f=3}$ exhibits a global chiral symmetry $\mathrm{U}(1)_V \otimes \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ in flavor space
- $\mathrm{U}(1)_A$ is broken by the QCD anomaly $\partial_\mu \left(\bar{q}\gamma^\mu\gamma_5 q\right) = 6\omega(x) = \frac{3\alpha_s}{4\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$

U(3) framework

- In the χ-limit and the large-N_C limit (QCD anomaly disappears), chiral symmetry group becomes $G_\chi = \mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$
- It breaks down to $H_\chi = \mathrm{U}(3)_V$ by vacuum, giving rise to 9 massless Goldstone bosons

$$U(x) = \exp\left(\frac{i\phi(x)}{F}\right), \qquad \phi(x) = \sum_{i=0}^{8} \lambda_i \phi_i(x) = \begin{pmatrix} \pi^3 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^3 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

- Under a chiral transformation $(V_L, V_R) \in G_\chi$, $U(x)$ transforms as $U \to V_R U V_L^\dagger$

To reproduce the correct large-N_C behavior of QCD, the U(3) framework is more suitable:

a) In the large-N_C limit, there are nine pNGBs, not eight.

b) SU(3) χPT can be obtained by integrating out the η_0. However, $m_{\eta_0}^2 \sim m_\pi^2$ in the large-N_C limit.

- G_χ is explicitly broken by the light-quark masses, $\mathrm{U}(1)_A$ anomaly, and non-QCD couplings

External source method $\mathcal{L}_{\mathrm{QCD}}^{\mathrm{ext}} = \mathcal{L}_{\mathrm{QCD}}^{0} + \bar{q}_R \gamma^\mu r_\mu q_R + \bar{q}_L \gamma^\mu \ell_\mu q_L - \bar{q}_R(s+ip)q_L - \bar{q}_L(s-ip)q_R - \boxed{\theta\omega},$

QCD chiral Lagrangian is constructed guided by local G_χ symmetry, discrete P and C symmetries, and Lorentz invariance

- Power counting rule: δ expansion $\partial_\mu \sim p^\mu = O(\delta^{1/2})\,, \quad m_{u,d,s} = O(\delta)\,, \quad N_C^{-1} = O(\delta)\,.$

Symmetry breaking effects from $m_{u,d,s}$ and $\mathrm{U}(1)_A$ anomaly are introduced order by order

- Leading-order QCD chiral Lagrangian

$$\mathcal{L}_{\mathrm{eff}}^{(0)} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\rangle - \frac{F^2 M_0^2}{12} X^2$$

M_0^2 can be identified as the $\mathrm{U}(1)_A$ anomaly contribution to the η_0 mass

$$D_\mu U = \partial_\mu U - i r_\mu U + i U \ell_\mu\,, \quad \chi = 2B_0(s+ip)\,, \quad X = \psi + \boxed{\theta}, \quad \psi = \frac{\sqrt{6}\eta_0}{F}\,.$$

Fix external fields to their "physical values", one introduce symmetry breaking and non-QCD couplings

Chiral low-energy constants have their N_C scalings: $F = O\left(N_C^{1/2}\right), B_0 = O(N_C^0), M_0^2 = O(N_C^{-1})$

✓ With the external field $\theta(x)$ coupled to ω, the $aG\tilde{G}$ term does not need to be removed before matching onto χPT

Chiral rotation on quark field

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{\partial_\mu a}{f}\left(\bar{q}_L k_Q \gamma^\mu q_L + \bar{q}_R k_q \gamma^\mu q_R\right) - c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a} - c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

$$q(x) \to \exp\left(-i\kappa_q\gamma_5 c_{GG}\frac{a(x)}{f}\right)q(x)\,, \quad \kappa_q = \mathrm{diag}\left(\kappa_u, \kappa_d, \kappa_s\right)$$

$$\mathcal{L}_{\rm eff}(\kappa_q) = \mathcal{L}^0_{\rm QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{\partial_\mu a}{f}\bar{q}_R\gamma^\mu \hat{k}_q(a) q_R + \frac{\partial_\mu a}{f}\bar{q}_L\gamma^\mu \hat{k}_Q(a) q_L$$
$$- \bar{q}_R \hat{M}(a) q_L - \bar{q}_L \hat{M}^\dagger(a) q_R - 2\hat{c}_{GG}\frac{a}{f}\omega - \hat{c}_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

$$\hat{k}_q(a) = k_q + c_{GG}\kappa_q + \mathcal{O}(a/f)\,, \quad \hat{k}_Q(a) = k_Q - c_{GG}\kappa_q + \mathcal{O}(a/f)\,,$$
$$\hat{M}(a) = M e^{2i\kappa_q c_{GG}\frac{a}{f}}\,, \quad \boxed{\hat{c}_{GG} = \left[1 - \mathrm{Tr}(\kappa_q)\right]c_{GG}},$$
$$\hat{c}_{\gamma\gamma} = c_{\gamma\gamma} - 2N_C \mathrm{Tr}\left(\kappa_q Q_q^2\right)c_{GG}\,.$$

$\mathrm{Tr}(\kappa_q)$ is not necessary to be unity!

$$r_\mu = \frac{\partial_\mu a}{f}\hat{k}_q(a)\,,$$
$$\ell_\mu = \frac{\partial_\mu a}{f}\hat{k}_Q(a)\,,$$
$$s + ip = \hat{M}(a)\,,$$
$$\theta = 2\hat{c}_{GG}\frac{a}{f}\,.$$

Match onto chiral EFT

✓ Physical amplitudes must be independent of κ_q

$$\mathcal{L}^\chi_{\rm eff}(\kappa_q) = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 - \hat{c}_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu} + \mathcal{L}^{(0)}_{\rm eff}(\kappa_q)$$

U(3) chiral Lagrangian for nonleptonic weak interaction

- Four-fermion operators $\hat{Q}_{i=1-6}$ are decomposed into $(8_L, 1_R)$ and $(27_L, 1_R)$ representations under G_χ
- Construct $|\Delta S| = 1$ chiral operators that have the same chiral structures

$$\mathcal{L}^{(-1)}_{\text{weak}}(\kappa_q) = \boxed{e^{i(\kappa_s-\kappa_d)c_{GG}\frac{a}{f}}}\left(\boxed{G_8 O_8 + G_8^\psi O_8^\psi} + \boxed{G_{27}^{1/2} O_{27}^{1/2} + G_{27}^{3/2} O_{27}^{3/2}}\right) + \text{h.c.}\,,$$

induced by chiral rotation

$$L_\mu \equiv i U^\dagger D_\mu U$$

$$D_\mu \psi \equiv -\langle L_\mu \rangle$$

$$O_8 = (L_\mu L^\mu)_{32}\,, \quad O_8^\psi = D_\mu\psi\,(L^\mu)_{32}\,,$$

$$O_{27}^{1/2} = (L_\mu)_{32}(L^\mu)_{11} + (L_\mu)_{31}(L^\mu)_{12} + 2(L_\mu)_{32}(L^\mu)_{22} - 3(L_\mu)_{32}(L^\mu)_{33}\,,$$

$$O_{27}^{3/2} = (L_\mu)_{32}(L^\mu)_{11} + (L_\mu)_{31}(L^\mu)_{12} - (L_\mu)_{32}(L^\mu)_{22}\,.$$

- The weak LECs start at $O(N_C^2)$ in the large-N_C expansion $\left\{G_8, G_8^\psi, G_{27}^{1/2}, G_{27}^{3/2}\right\} = \mathcal{O}(N_C^2)$

$$G_i = G_W g_i\,, \quad G_W \equiv -\frac{F^4 G_F}{\sqrt{2}} V_{ud}^* V_{us}\,,$$

✓ Values of g_i are determined from kaon weak decays (Cirigliano, el al, 2012)(Gerard, et al, 2005)

	p^2	p^4	p^6	$\cdots$
N_C^2	δ^{-1}	δ^0	δ	$\cdots$
N_C	δ^0	δ	δ^2	$\cdots$
N_C^0	δ	δ^2	δ^3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

In $N_C \to \infty$, non-factorizable contributions vanish, and the two color-singlet currents can be matched independently onto mesonic currents

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1}\Big|_{N_C\to\infty} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* V_{us} (\bar{d}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu s_L) + \text{h.c.} \to G_W (L_\mu)_{31}(L^\mu)_{12} + \text{h.c.} + \mathcal{O}(p^4)$$

$$\boxed{g_8\Big|_{N_C\to\infty} = \frac{3}{5}\,, \quad g_{27}^{1/2}\Big|_{N_C\to\infty} = \frac{1}{15}\,, \quad g_{27}^{3/2}\Big|_{N_C\to\infty} = \frac{1}{3}\,,} \quad \boxed{g_8^\psi\Big|_{N_C\to\infty} = \frac{2}{5}\,.}$$

✓ consistent with the SU(3) values (Pich & de Rafael, 1991)

Mixings between fields $A=\left(a,\,\pi^3,\,\eta_8,\,\eta_0\right)^T$

$$\mathcal{L}^{\chi}_{\rm eff}\Big|_{\rm quadratic}=\frac{1}{2}\partial_\mu A^T\begin{pmatrix}1 & -\frac{F}{f}k_{a\pi^3} & -\frac{F}{f}k_{a\eta_8} & -\frac{F}{f}k_{a\eta_0}\\ -\frac{F}{f}k_{a\pi^3} & 1 & 0 & 0\\ -\frac{F}{f}k_{a\eta_8} & 0 & 1 & 0\\ -\frac{F}{f}k_{a\eta_0} & 0 & 0 & 1\end{pmatrix}\partial^\mu A-\frac{1}{2}A^T\begin{pmatrix}m_a^2 & -\frac{F}{f}m^2_{a\pi^3} & -\frac{F}{f}m^2_{a\eta_8} & -\frac{F}{f}m^2_{a\eta_0}\\ -\frac{F}{f}m^2_{a\pi^3} & m^2_{0\pi} & -\frac{1}{\sqrt{3}}\delta_I & -\sqrt{\frac{2}{3}}\delta_I\\ -\frac{F}{f}m^2_{a\eta_8} & -\frac{1}{\sqrt{3}}\delta_I & m^2_{0\eta_8} & -\frac{2\sqrt{2}}{3}\Delta_8\\ -\frac{F}{f}m^2_{a\eta_0} & -\sqrt{\frac{2}{3}}\delta_I & -\frac{2\sqrt{2}}{3}\Delta_8 & m^2_{0\eta_0}\end{pmatrix}A$$

$$+\partial_\mu\pi^+\partial^\mu\pi^- - m^2_{0\pi}\pi^+\pi^- + \partial_\mu K^+\partial^\mu K^- - m^2_{0K^\pm}K^+K^- + \partial_\mu K^0\partial^\mu\bar{K}^0 - m^2_{0K^0}K^0\bar{K}^0 - \frac{F(k_d-k_D)_{21}}{\sqrt{2}f}\partial_\mu a\partial^\mu K^0 - \frac{F(k_d-k_D)_{12}}{\sqrt{2}f}\partial_\mu a\partial^\mu\bar{K}^0$$

- ALP-meson mixing coefficients depend on κ_q

$$k_{a\pi^3}=\tfrac{1}{2}\left[(c^a_{uu}-c^a_{dd})+2c_{GG}\left(\kappa_u-\kappa_d\right)\right], \qquad m^2_{a\pi^3}=2c_{GG}B_0(\kappa_u m_u-\kappa_d m_d),$$

$$k_{a\eta_8}=\tfrac{1}{2\sqrt{3}}\left[(c^a_{uu}+c^a_{dd}-2c^a_{ss})+2c_{GG}\left(\kappa_u+\kappa_d-2\kappa_s\right)\right], \qquad m^2_{a\eta_8}=\tfrac{2}{\sqrt{3}}c_{GG}B_0(\kappa_u m_u+\kappa_d m_d-2\kappa_s m_s),$$

$$k_{a\eta_0}=\tfrac{1}{\sqrt{6}}\left[(c^a_{uu}+c^a_{dd}+c^a_{ss})+2c_{GG}\left(\kappa_u+\kappa_d+\kappa_s\right)\right], \qquad m^2_{a\eta_0}=2\sqrt{\tfrac{2}{3}}c_{GG}B_0(\kappa_u m_u+\kappa_d m_d+\kappa_s m_s)-\tfrac{\sqrt{6}}{3}\hat{c}_{GG}M_0^2,$$

- η-η' mixing at leading order

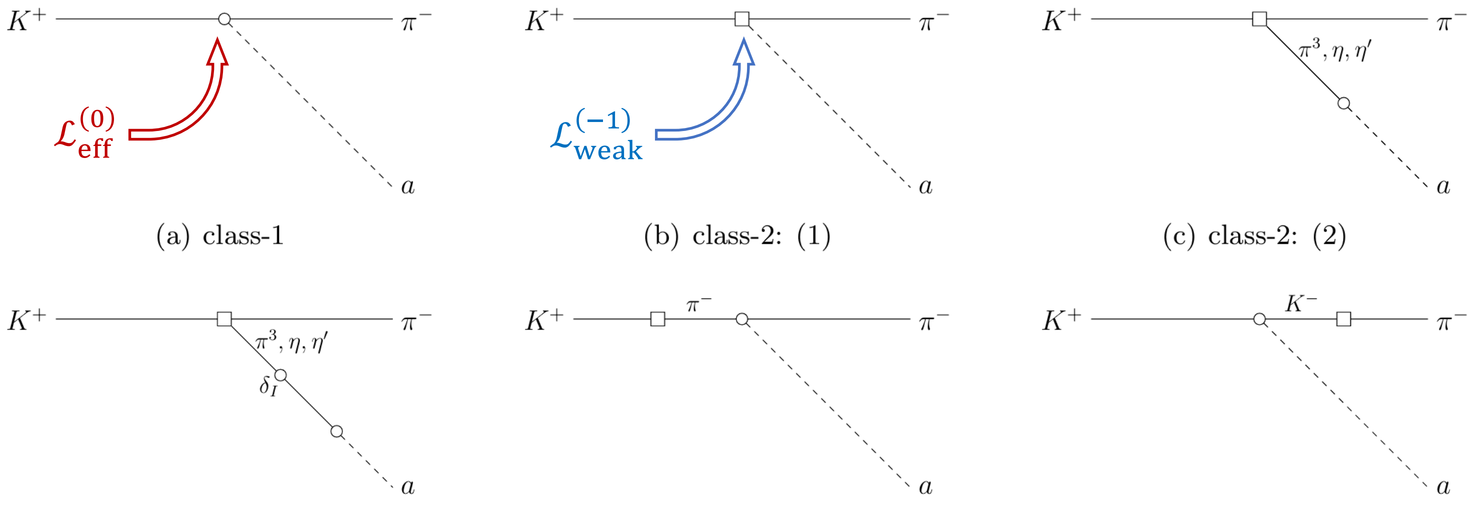
$$\begin{pmatrix}\eta_8 & \eta_0\end{pmatrix}\begin{pmatrix}m^2_{0\eta_8} & -\frac{2\sqrt{2}}{3}\Delta_8\\ -\frac{2\sqrt{2}}{3}\Delta_8 & m^2_{0\eta_0}\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}=\begin{pmatrix}\eta & \eta'\end{pmatrix}\begin{pmatrix}m^2_\eta & \\ & m^2_{\eta'}\end{pmatrix}\begin{pmatrix}\eta\\ \eta'\end{pmatrix}$$

$$\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}=\begin{pmatrix}c_\theta & s_\theta\\ -s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}\eta\\ \eta'\end{pmatrix} \longrightarrow \begin{pmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{pmatrix}\begin{pmatrix}\eta\\ \eta'\end{pmatrix}$$

Introduce higher-order corrections in η-η' mixing

✓ General η-η' mixing matrix

Leading order $\bar{K} \to \pi a$ decay amplitude in U(3) χPT

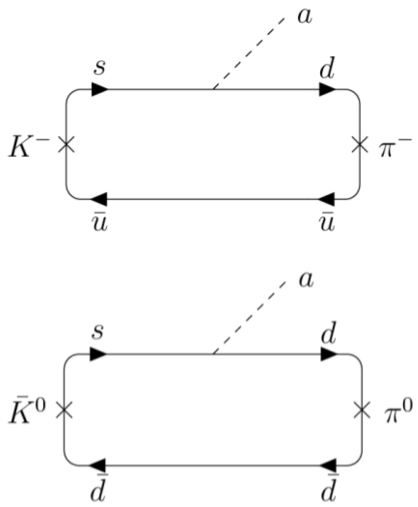


(a) class-1 (b) class-2: (1) (c) class-2: (2)

(d) class-2: (3) (e) class-2: (4) (f) class-2: (5)

Three self-consistency checks:

- ✓ Complete cancellation of the κ_q dependence
- ✓ Recovery of the SU(3) results in the $M_0 \to \infty$ limit
- ✓ Correct large-N_C behavior of the decay amplitudes



$$A_{\bar{K}\to\pi a} = A^{\text{AFC}}_{\bar{K}\to\pi a} + A^{\text{WFC}}_{\bar{K}\to\pi a}$$

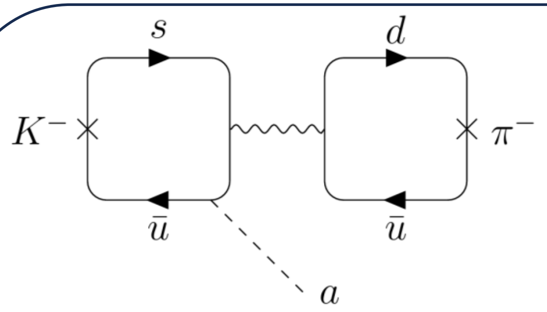
$$A^{\text{AFC}}_{K^-\to\pi^- a} = -\frac{(k_d + k_D)_{12}}{2f}\left(m^2_{K^\pm} - m^2_\pi\right)$$

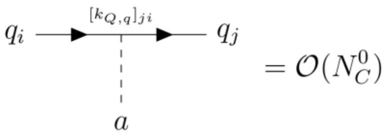
$$A^{\text{AFC}}_{\bar{K}^0\to\pi^0 a} = \frac{(k_d + k_D)_{12}}{2\sqrt{2}f}\left(m^2_{K^0} - m^2_\pi\right)\left\{1 + \delta_I\left[\frac{\beta_{11}(\beta_{11} + \sqrt{2}\beta_{21})}{m^2_\pi - m^2_\eta} + \frac{\beta_{12}(\beta_{12} + \sqrt{2}\beta_{22})}{m^2_\pi - m^2_{\eta'}}\right]\right\}$$

$$A^{\text{WFC}}_{\bar{K}\to\pi a} = G_W \sum_{c_{\text{ALP}}} \frac{c_{\text{ALP}}}{f} F^{c_{\text{ALP}}}_{\bar{K}\to\pi a} \qquad c_{\text{ALP}} \in \{c_{GG}, c^a_{uu}, c^a_{dd}, c^a_{ss}, c^v_{dd} - c^v_{ss}\}$$

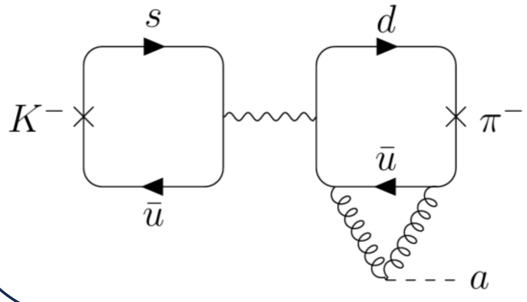
$$c^{v/a}_{uu} = [k_u]_{11} \pm [k_U]_{11}\,, \quad c^{v/a}_{dd} = [k_d]_{11} \pm [k_D]_{11}\,, \quad c^{v/a}_{ss} = [k_d]_{22} \pm [k_D]_{22}$$

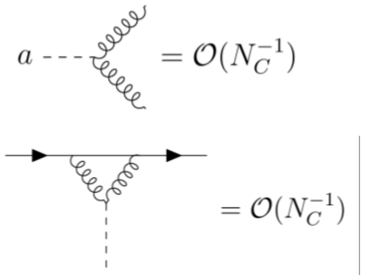
☐ Large-N_C behavior of the $K \to \pi a$ decay amplitudes





$$F^{c_{\rm ALP}}_{K^-\to\pi^- a}\Big|_{c_{\rm ALP}\in\{c^a_{uu},c^a_{dd},c^a_{ss},c^v_{dd}-c^v_{ss}\}} = \mathcal{O}(N_C^{-1})$$





$$F^{c_{GG}}_{K^-\to\pi^- a} = \mathcal{O}(N_C^{-2}) \qquad F^{c_{GG}}_{K^-\to\pi^- a}\Big|_{M_0\to\infty} = \mathcal{O}(N_C^{-1})$$

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m^2_{a,0}a^2 - c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$

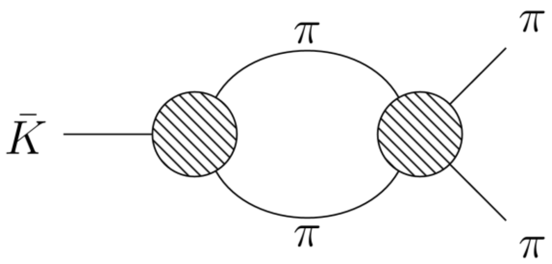
$$q \to e^{-i\gamma_5 c_{GG}\frac{a}{3f}} q$$

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m^2_{a,0}a^2 - \frac{a}{3f}c_{GG}\left[\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right) - 2i\bar{q}M\gamma_5 q\right] + \mathcal{O}\left(\frac{a^2}{f^2}\right)$$

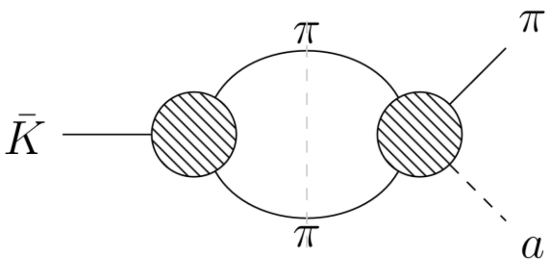
- The following PCAC relation can not be fully reproduced in SU(3) theory

$$\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right) - 2i\bar{q}M\gamma_5 q = \frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a} = \mathcal{O}(N_C^{-1})$$

☐ A first look at higher-order effects: $\pi\pi$ FSI



- Strong phases
- $|\Delta I| = \frac{1}{2}$ rule (long-range enhancement)



- How important is $\pi\pi$ FSI?

$$\mathrm{Im}A^{\mathrm{WFC}}_{\bar{K}\to\pi a} = \sum_I \rho_{\pi\pi}(m_K^2) T^{J=0}_{(\pi\pi,I)\to\pi a}(m_K^2) a^*_{\bar{K}\to(\pi\pi,I)}$$

$$A^{\mathrm{WFC,uni}}_{\bar{K}\to\pi a} = A^{\mathrm{WFC,tree}}_{\bar{K}\to\pi a} + \sum_I \frac{a^{\mathrm{tree}}_{\bar{K}\to(\pi\pi,I)} G_{\pi\pi}(m_K^2) T^{J=0,\mathrm{LO}}_{(\pi\pi,I)\to\pi a}(m_K^2)}{1 - T^{I0,\mathrm{LO}}_{\pi\pi\to\pi\pi}(m_K^2) G_{\pi\pi}(m_K^2)}$$

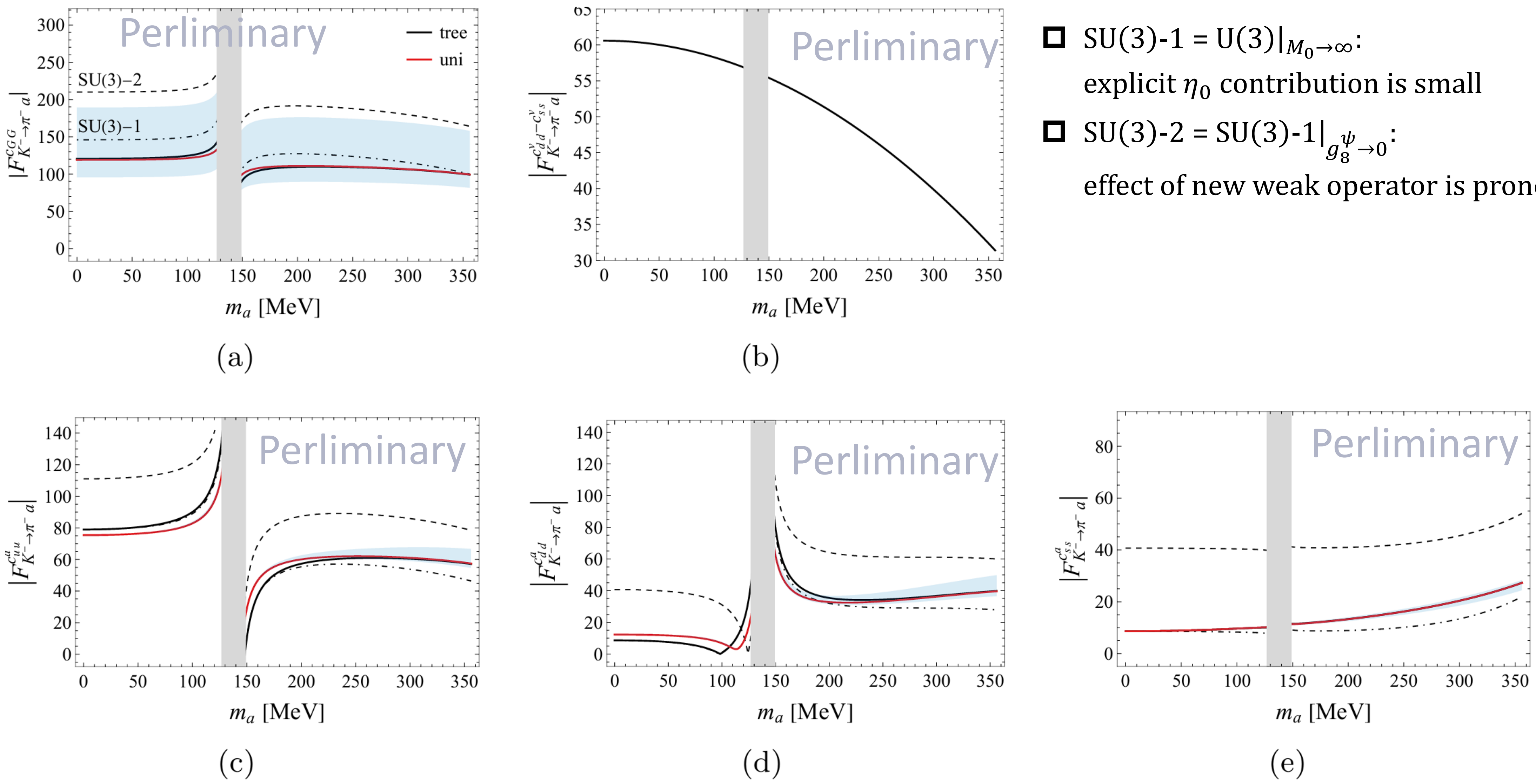
- ☐ K^-: $I = 2$ only, definitely small by the $|\Delta I| = 1/2$ rule
- ☐ $\bar{K}^0$: both $I = 0$ and 2, potentially significant

$$G_{\pi\pi}(s) = -i \int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{1}{((p-q)^2 - m_\pi^2)(q^2 - m_\pi^2)} \Bigg|_{\left(\frac{2}{d-4}+\cdots\right)\to a_{\mathrm{sc}}}$$

$$\mathrm{Im}G_{\pi\pi}(s + i\varepsilon) = \theta(s - 4m_\pi^2)\rho_{\pi\pi}(s)$$

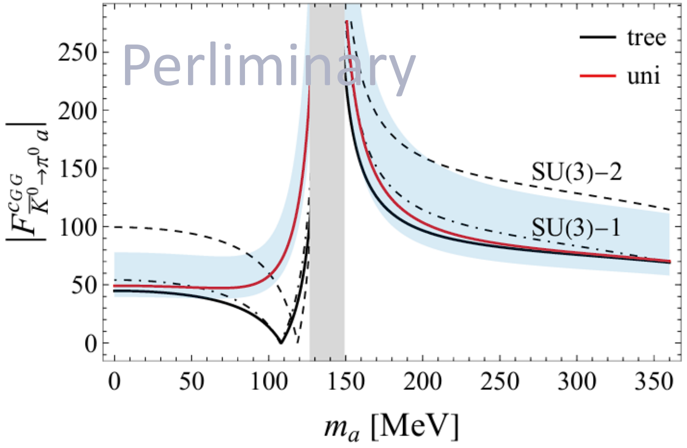
Determined by S-wave $\pi\pi$ phase shifts

Numeric discussion

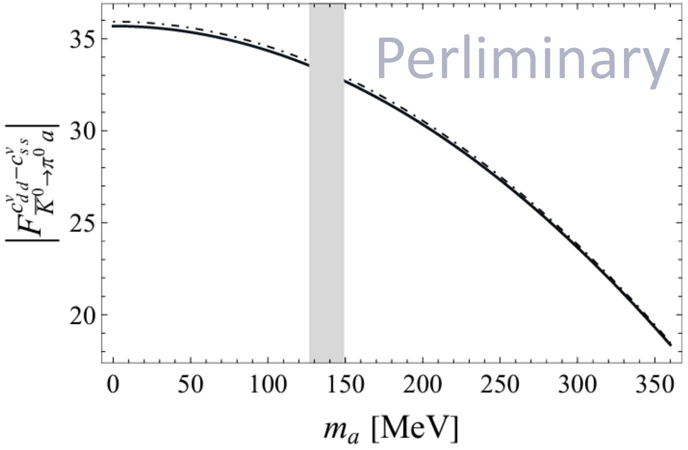


- SU(3)-1 = U(3)$|_{M_0\to\infty}$:
  explicit η_0 contribution is small
- SU(3)-2 = SU(3)-1$|_{g_8^\psi\to 0}$:
  effect of new weak operator is pronounced

Numeric discussion



(a)



(b)

- SU(3)-1 = U(3)$|_{M_0\to\infty}$:
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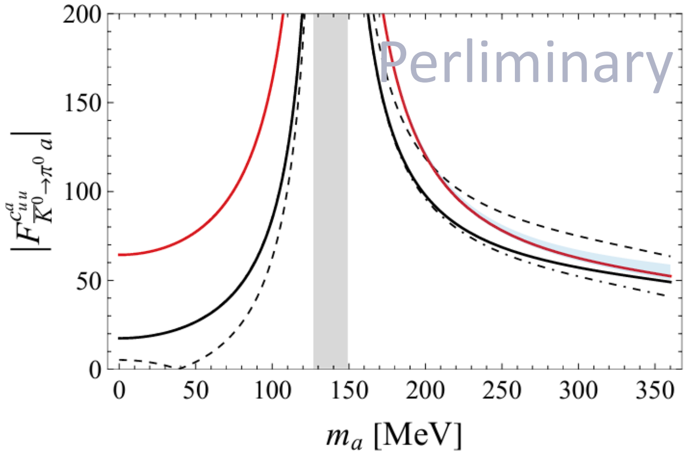
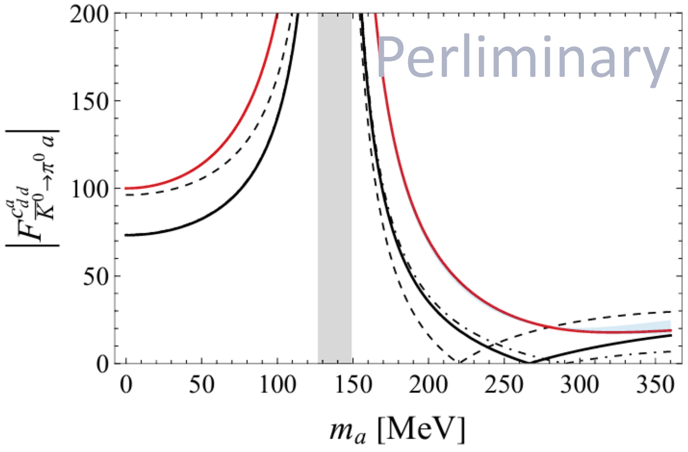
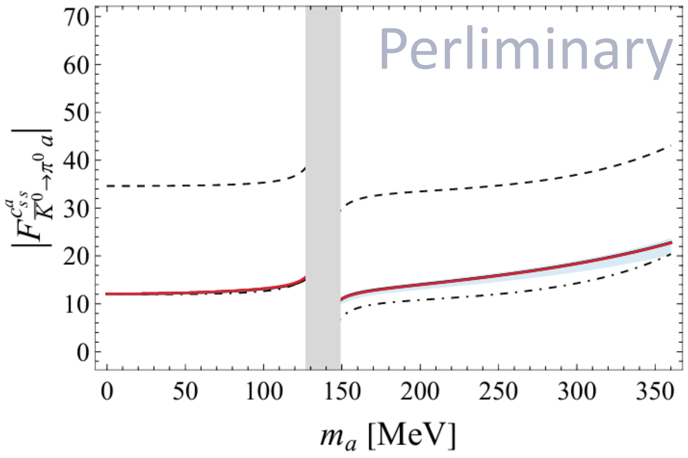


(c)



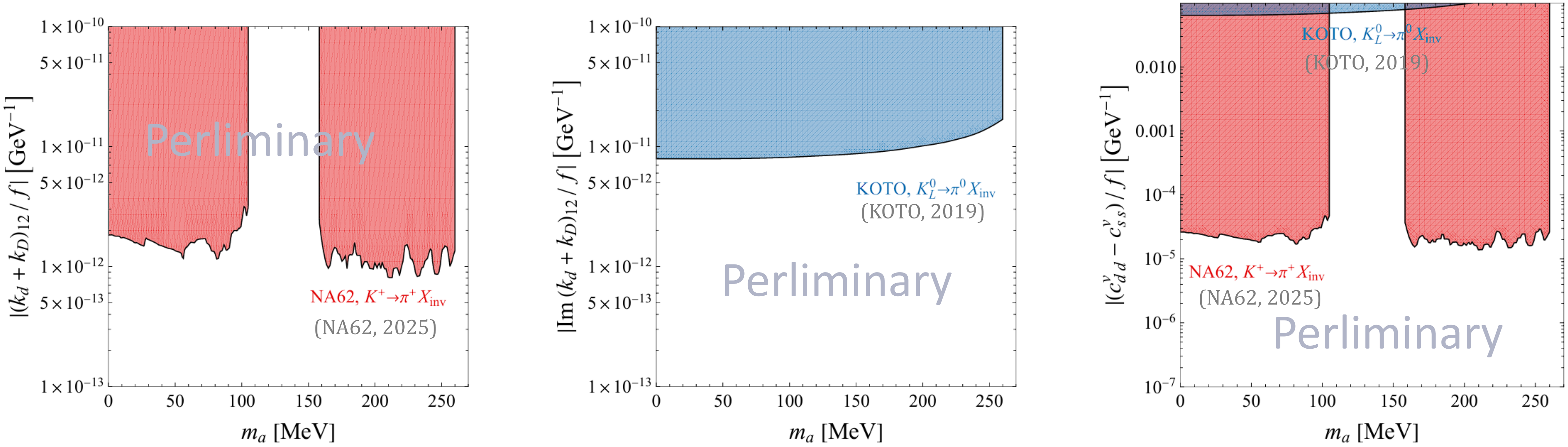
(d)



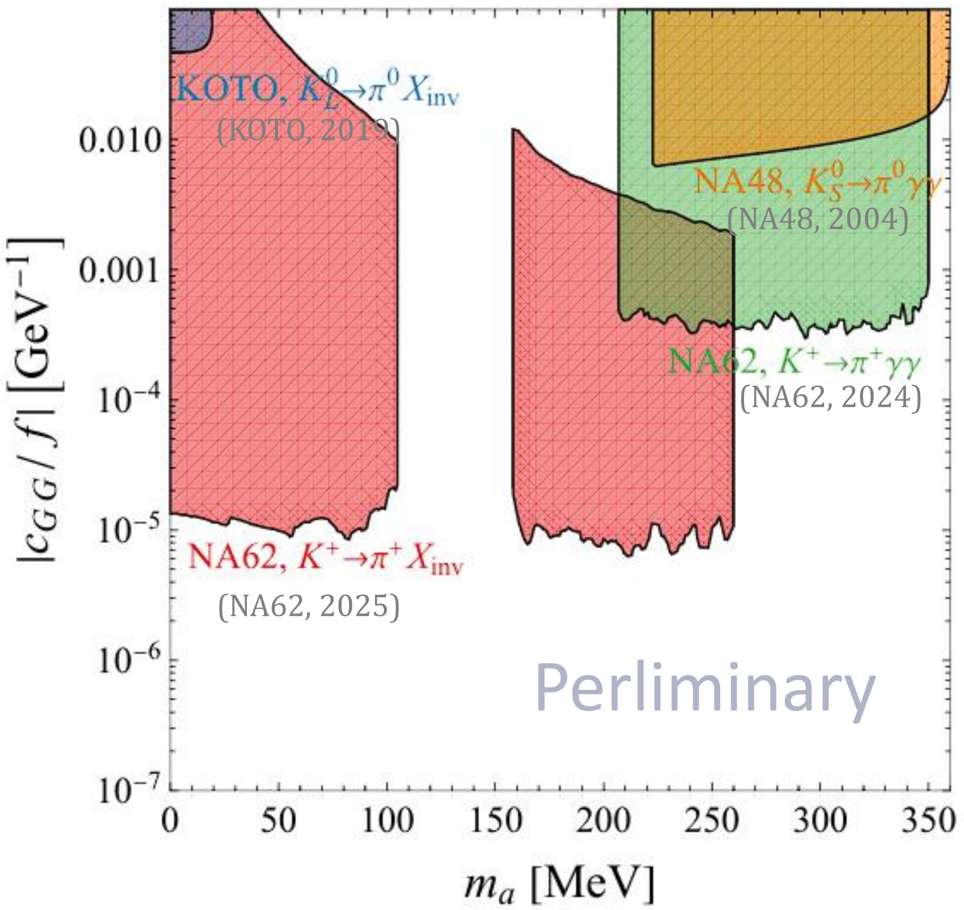
(e)

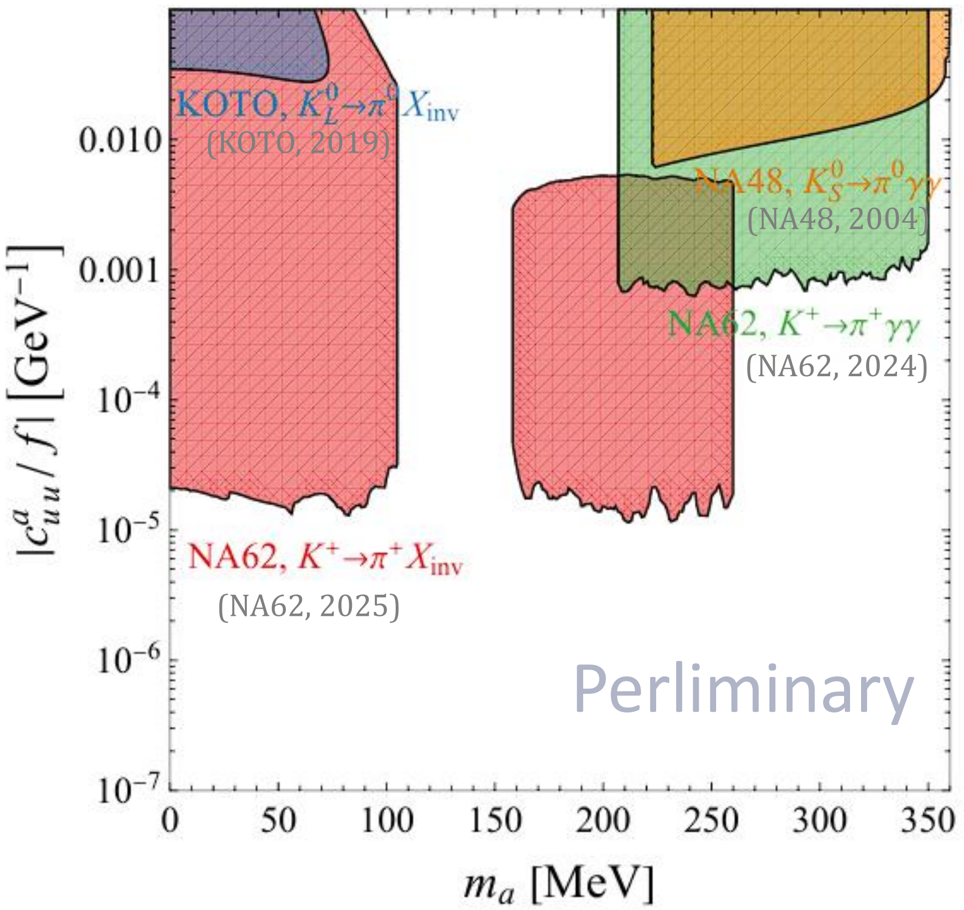
☐ ALP constraints from kaon decays

- Assume ALPs decay only into $\gamma\gamma$
- Switch on one ALP coupling (kaon scale) at a time

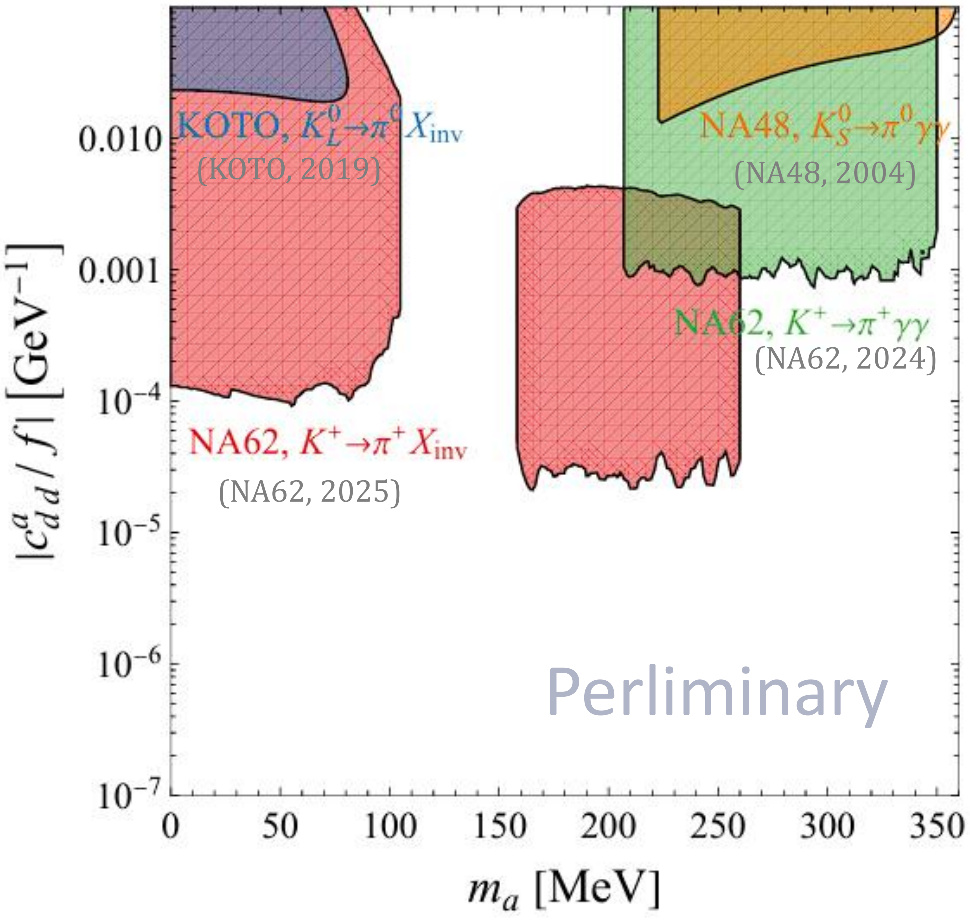


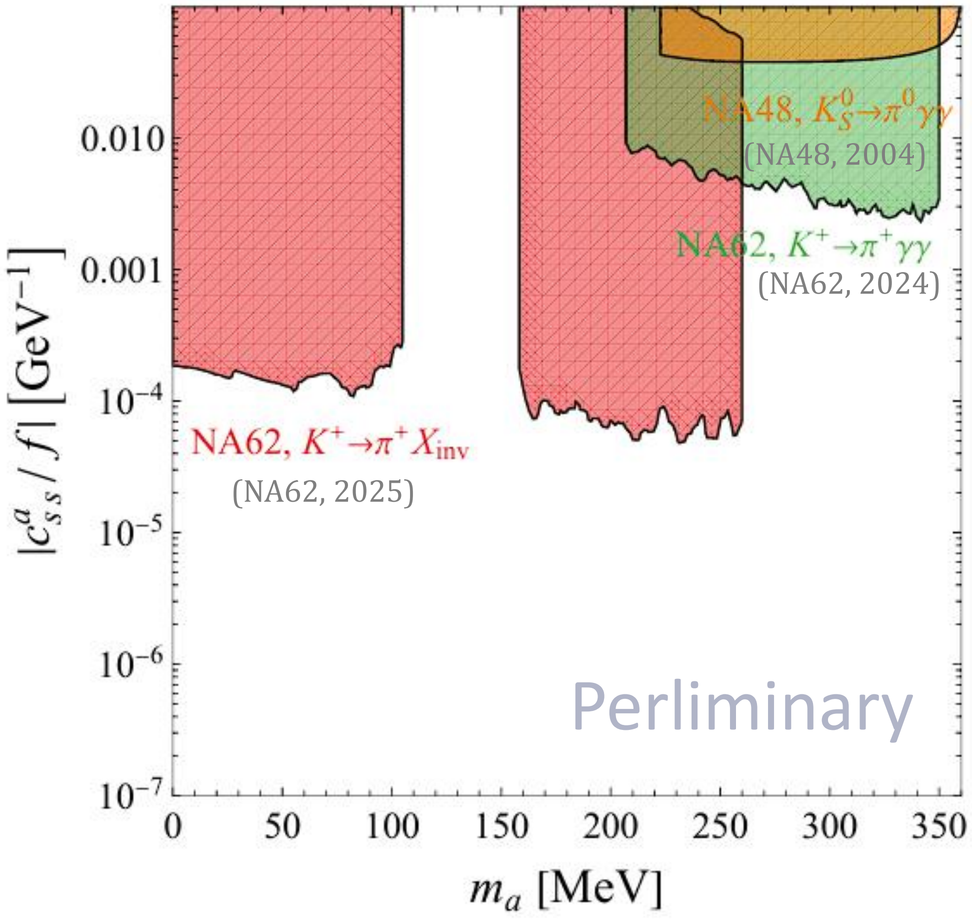
❑ ALP constraints from kaon decays





ALP constraints from kaon decays





Summary

- In this talk, we have studied the $K \to \pi a$ decay at leading order in U(3) χPT.
- We verified that our treatment of ALP interactions in U(3) χPT is self-consistent, as the resulting decay amplitudes are independent of the reparameterization parameters.
- U(3) χPT fully reproduces the large-N_C behavior of QCD. We show that our amplitudes exhibit the correct large-N_C scaling, especially for the c_{GG} contribution.
- Numerically, we found that the explicit η_0 contribution is small in the $K \to \pi a$ decay, whereas the new weak operator unique to U(3) χPT gives a sizable contribution. This demonstrates the importance of studying the $K \to \pi a$ decay within the U(3) framework.

Outlook

- Extending the present work to the complete one-loop calculation in U(3) χPT would be an important next step toward a precision prediction of the $K \to \pi a$ decay. The main challenge is our limited knowledge of the many weak low-energy constants that enter at higher orders. We hope that this difficulty can be overcome in the future with the help of nonperturbative approaches, such as the large-N_C expansion and lattice QCD.

Thank you for your attention

- Backup:

  - The fraction of ALPs escape the detector:

$$F_{esc}^{exp} = F\left(m_a, \tau_0, \beta_K^{exp}, L_{max}^{exp}\right) = \int_0^{\frac{\pi}{2}} d\theta \sin\theta \exp\left(-\frac{L_{max}^{exp}}{\Delta x_L}\right), \qquad \Delta x_L = \frac{p_{a,L}\tau_0}{m_a}, \qquad \tau_0 = \Gamma_{a\to\gamma\gamma}^{-1}$$

$$p_{a,L} = \gamma_K^{exp}\left(\beta_K^{exp} E_a^* + p_a^* \cos\theta\right)$$

- Long-lived: $F_{esc}^{exp}\mathcal{B}(K \to \pi a) \leq \mathcal{B}(K \to \pi X_{inv})_{UL}^{exp}$
- $\gamma\gamma$: $\left(1 - F_{esc}^{exp}\right)\mathcal{B}(K \to \pi a)\mathcal{B}(a \to \gamma\gamma) \leq [\mathcal{B}(K \to \pi a)\mathcal{B}(a \to \gamma\gamma)]_{UL}^{exp}$