第十一届手征有效场论研讨会

Efimov physics in three hadron systems

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参考文献：JHEP(2025), J.Phys.G (2025)

2026年8月，兰州

❖ Outlines

1. Backgrounds--What is the Efimov effect?

2. Efimov study in three B/D mesons

3. Summary

Outlines

1. Backgrounds--What is the Efimov effect?
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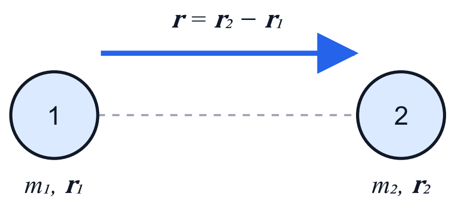
3. Summary

❖ What is the Efimov effect?

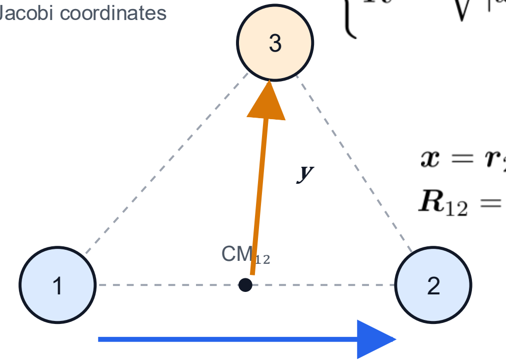
- Solving bound states in three body system with only two-body potentials

Two-body system $\vec{r} \to (r, \theta, \phi)$



Three-body system
Jacobi coordinates

$$\left\{ R = \sqrt{|\vec{x}|^2 + |\vec{y}|^2}, \Omega = (\alpha, \hat{x}, \hat{y}) \right\}$$



$$\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1,\ \boldsymbol{y} = \boldsymbol{r}_3 - \boldsymbol{R}_{12}$$
$$\boldsymbol{R}_{12} = (m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2)/(m_1 + m_2)$$

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{\hat{L}^2}{\hbar^2 r^2}\right) + V(r,\Omega)\right]\psi(r,\Omega) = E\psi(r,\Omega)$$

$$\psi(r,\Omega) = \sum_{lm}\frac{u_{lm}(r)}{r}Y_{lm}(\Omega)$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E\right]u_{lm}(r) + \sum_{l'm'}V_{lm,l'm'}(r)u_{l'm'}(r) = 0$$

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R} - \frac{\hat{\Lambda}^2}{R^2}\right) + V(R,\Omega)\right]\Psi(R,\Omega) = E\Psi(R,\Omega)$$

$$\Psi(R,\Omega) = R^{-5/2}\sum_n f_n(R)\Phi_n(R;\Omega)$$

$$\begin{cases} \hat{H}_\Omega(R)\Phi_n(R;\Omega) = U_n(R)\Phi_n(R;\Omega) \\ \left[-\frac{\hbar^2}{2\mu_R}\frac{d^2}{dR^2} + U_n(R) - E\right]f_n(R) \\ \quad -\frac{\hbar^2}{2\mu_R}\sum_m\left[2P_{nm}(R)\frac{d}{dR} + Q_{nm}(R)\right]f_m(R) = 0 \end{cases}$$

❖ What is the Efimov effect?

☐ when $r_0 \ll R \ll |a|$

$$U_n(R) \simeq \frac{\hbar^2}{2\mu R^2}(s_n^2 - \frac{1}{4}) \rightarrow \left[-\frac{d^2}{dR^2} + \frac{s_n^2 - 1/4}{R^2}\right] f_n(R) = \frac{2\mu E}{\hbar^2} f_n(R)$$

☞ s_n is real: $\quad f_n(R) = AR^{1/2+s_n} + BR^{1/2-s_n}$

☞ $s_n = \pm i s_0$ is pure imaginary :

$$f_n(R) = AR^{1/2+is_0} + BR^{1/2-is_0} \propto \sqrt{R}\sin\left[s_0 \ln\left(\frac{R}{R_0}\right)\right]$$

☐ Discrete scaling symmetry

$$R \rightarrow \lambda R, \quad f(\lambda R) \propto \sin\left[s_0 \ln\left(\frac{R}{R_0}\right) + s_0 \ln\lambda\right]$$

- when taking $\quad s_0 \ln \lambda_0 = \pi \rightarrow \lambda_0 = e^{\pi/s_0} (\approx 22.7 \text{ for three identical scalar bosons})$

$$f(\lambda_0 R) = \pm f(R), \quad E_{n+1} = \lambda_0^2 E_n \qquad N_{\text{node}} \sim \frac{s_0}{\pi} \ln\frac{|a|}{r_0}$$

Efimov effect: discrete scale invariance in a resonant three-body system

❖ What is the Efimov effect?

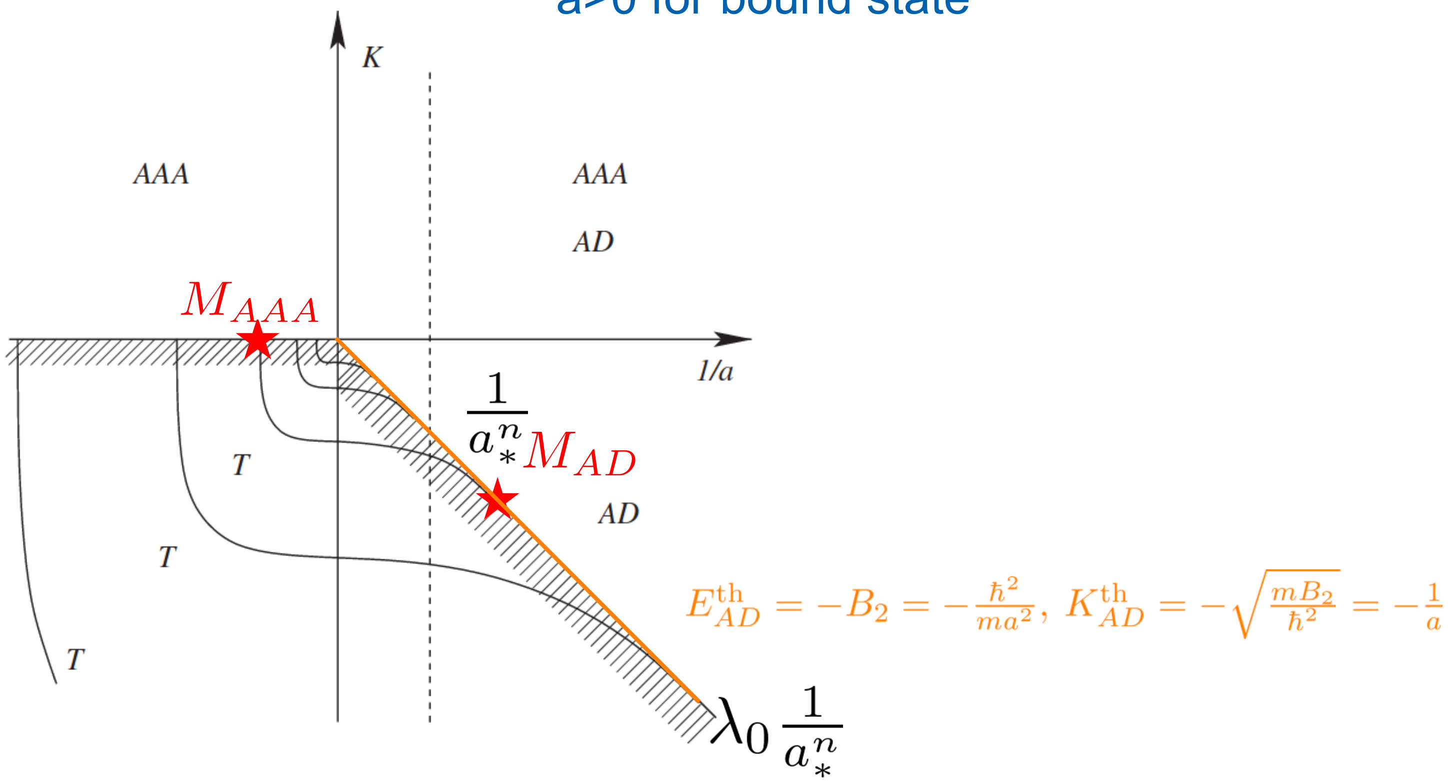
☐ Efimov plot

E. Braaten, HWH, Phys. Rept. 428 259 (2006)

a>0 for bound state

T:

$$K_n(a) = -\sqrt{\frac{mB_3^{(n)}(a)}{\hbar^2}}$$



例如：氚核(2n+1p)，锂11（锂9+2n）

✓ 冷原子系统实现 Nature 440, 315 (2016); Nat. Phys. 5, 227 (2009)

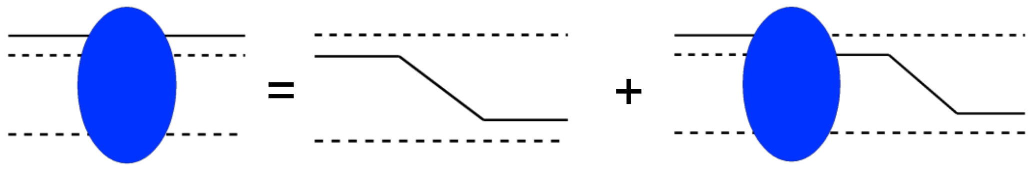
(通过 Feshbach 共振调节a时)

❖ Efimov effect in momentum space

- ☐ Scale separation

$$\gamma = \frac{1}{|a|} \ll p, q \ll \Lambda \simeq \frac{1}{r_0} \simeq M_\pi \longrightarrow$$ Pionless EFT

- ☐ Auxiliary field: dimer-particle scattering



$$A(p) = \frac{4}{\sqrt{3}\pi} \int_0^\infty dx \, \ln\left(\frac{1+x+x^2}{1-x+x^2}\right) A(px), \qquad \text{with } q = px$$

considering $A(p) = p^{s-1}$ $\longrightarrow 1 = \frac{4}{\sqrt{3}\pi} \int_0^\infty dx \, x^{s-1} \ln\left(\frac{1+x+x^2}{1-x+x^2}\right) \longrightarrow 1 = \frac{8}{\sqrt{3}} \frac{\sin(\pi s/6)}{s\cos(\pi s/2)}$ $s = \pm i 1.00624$

- ☐ RGI requires an oscillating three-body force at LO

$$A(p) \propto \frac{1}{p} \sin\left[s_0 \ln\left(\frac{p}{\kappa_0}\right)\right] \longrightarrow g_3(\Lambda) = -\frac{9g^2}{\Lambda^2} H(\Lambda), \quad \text{with } H = \frac{\cos[s_0 \ln \Lambda/\Lambda_0 + \arctan s_0]}{\cos[s_0 \ln \Lambda/\Lambda_0 - \arctan s_0]}$$

$$H(\lambda_0 \Lambda) = H(\Lambda)$$

❖ Hadronic Molecule: r/a $\to$ 0

☐ Successful in deuteron description: the Weinberg compositeness criterion

Deuteron--np bound state: $\boxed{|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\boldsymbol{k})|h_1 h_2\rangle \end{pmatrix},}$ $\begin{pmatrix} H_c & V \\ V & H^0_{hh} \end{pmatrix} \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi|hh\rangle \end{pmatrix} = E \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi|hh\rangle \end{pmatrix}.$

a<0 for bound state

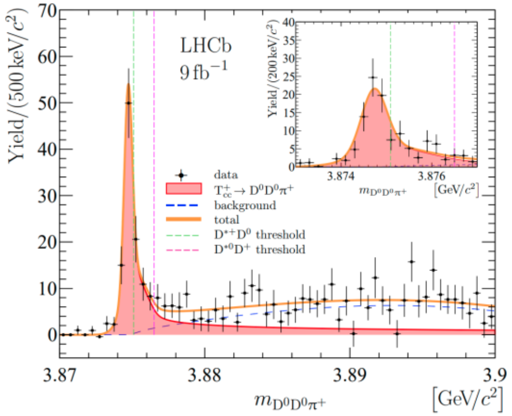
np scattering $T_{\rm NR}(E) = \dfrac{g_0^2}{E - E_0 + \Sigma(E)} + \text{non-pole terms}\,, \longleftrightarrow T_{\rm NR}(E) = -\dfrac{2\pi}{\mu}\dfrac{1}{1/a + (r/2)k^2 - ik},$

$$a = -2\frac{1-\lambda^2}{2-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right), \quad r = -\frac{\lambda^2}{1-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right).$$

$$\boxed{a = -4.3(14)\ \text{fm}[-5.419(7)] \qquad r = 0.0(14)\ \text{fm}[1.764(8)]}$$

S. Weinberg, Phys. Rev. 137 B672 (1965);
Guo et al. RMP (2018)

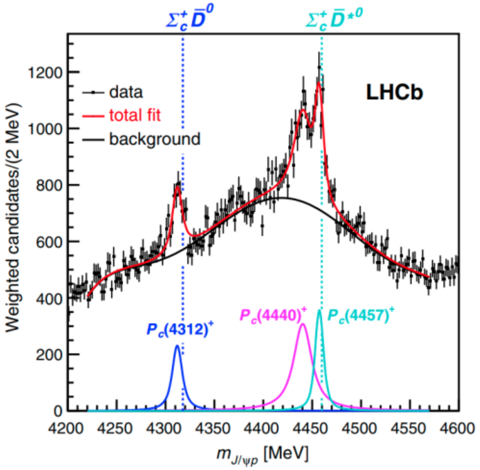
☐ Emergence of near-threshold structures



LHCb Nature Commun. 13 3351 (2022)

T_{cc} @2021

$D^{*+}D^0$-$D^{*0}D^+$



LHCb PRL 122 222001 (2019)

$P_{c\bar{c}}$ @2015

$\bar{D}^{(*)}\Sigma_c^{(*)}$

Outlines

❖ Three-B-Mesons in Pionless EFT

- Zb(10610) and Zb ′ (10650) are weak bound states of two B mesons

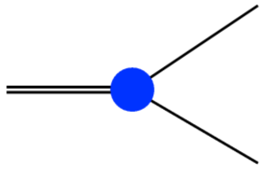
  $(\bar{B}B^* + B\bar{B}^*)/\sqrt{2}$ $\quad B^*\bar{B}^*$

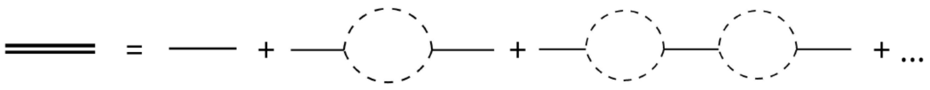
Exp. $\quad E_B^Z = -3(2)\,\mathrm{MeV}, E_B^{Z'} = -2.7(16)\,\mathrm{MeV}$ PDG, https://pdg.lbl.gov

HQEFT $\quad E_B^Z = 5.0(2.5)\,\mathrm{MeV}, E_B^{Z'} = 1.0(0.5)\,\mathrm{MeV}$ M. Cleven et al., EPJA 47 120 (2011)

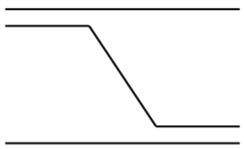
$$\boxed{a_Z = 1.2(3)\,\mathrm{fm}, a_{Z'} = 2.7(0.7)\,\mathrm{fm}, r \approx \frac{1}{M_\pi} = 1.4\,\mathrm{fm}}$$

- The leading order pionless EFT for three-B-meson dynamics

LO:  $\sim g_Z \mathcal{O}_{IS}$

Dressed dimer propagator: 

$$iS_{ij}^D(p_0, \vec{p}) = -i\frac{2\pi}{g^2\mu_D S_1^D c_D}\frac{1}{-\gamma_D + \sqrt{-2\mu_D\left(p_0 - \frac{\vec{p}^2}{2(m_i+m_j)}\right) - i\varepsilon}},$$

Independent transitions:  $\mathcal{M}_{ZB\to ZB}, \mathcal{M}_{ZB^*\to ZB^*}, \quad \mathcal{M}_{ZB^*\to Z'B}, \mathcal{M}_{Z'B^*\to Z'B^*}$

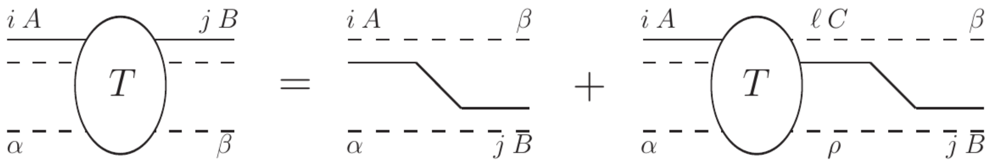
❖ Dimer-particle scattering

☐ The single-channel integral equations

$$S\text{-wave } ZB : (I,S) = (3/2,1), (1/2,1)$$

$$S\text{-wave } ZB^{*} : (I,S) = (1/2(3/2), 0(2))$$

$$S\text{-wave } Z'B^{*} : (I,S) = (3/2,1), (3/2,1), (1/2,0)$$
$$(1/2,1), (1/2,2)$$
$$(3/2,0)$$

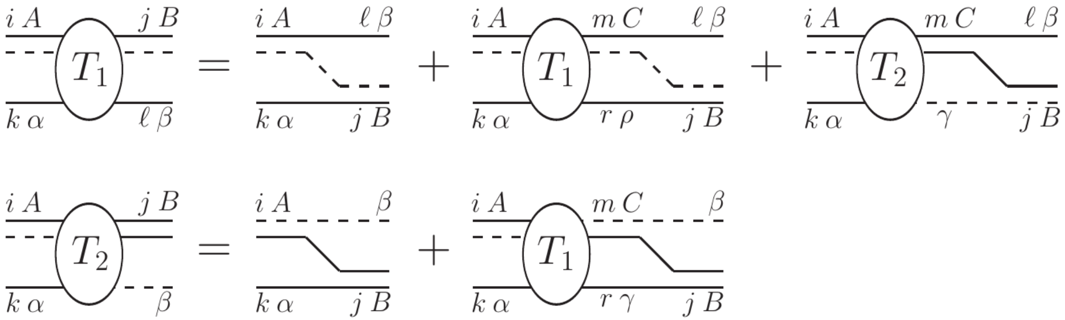


$$T^{IS}(E,k,p) = \langle \mathcal{O}^{IS} \rangle \mathcal{M}_0(k,p) + \int_0^{\Lambda} dq \, \langle \mathcal{O}^{IS} \rangle \mathcal{M}_1(p,q) T^{IS}(E,k,q)$$

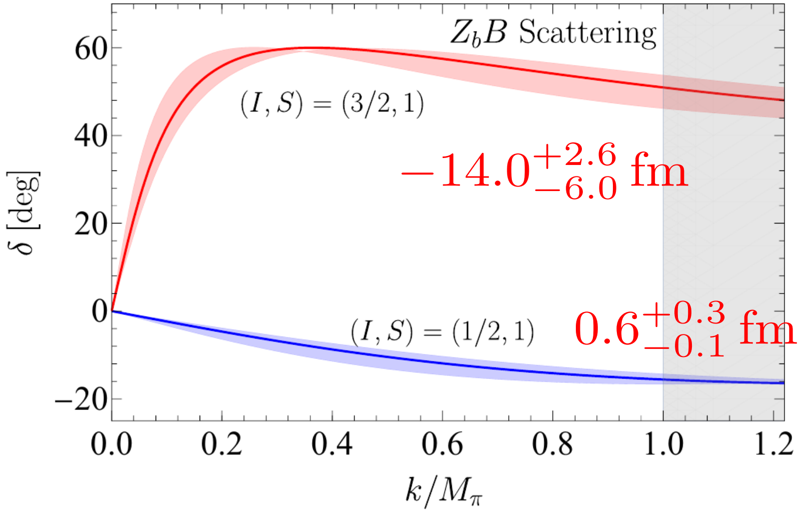
☐ The coupled-channel integral equations

$$S\text{-wave } ZB^{*} - Z'B :$$

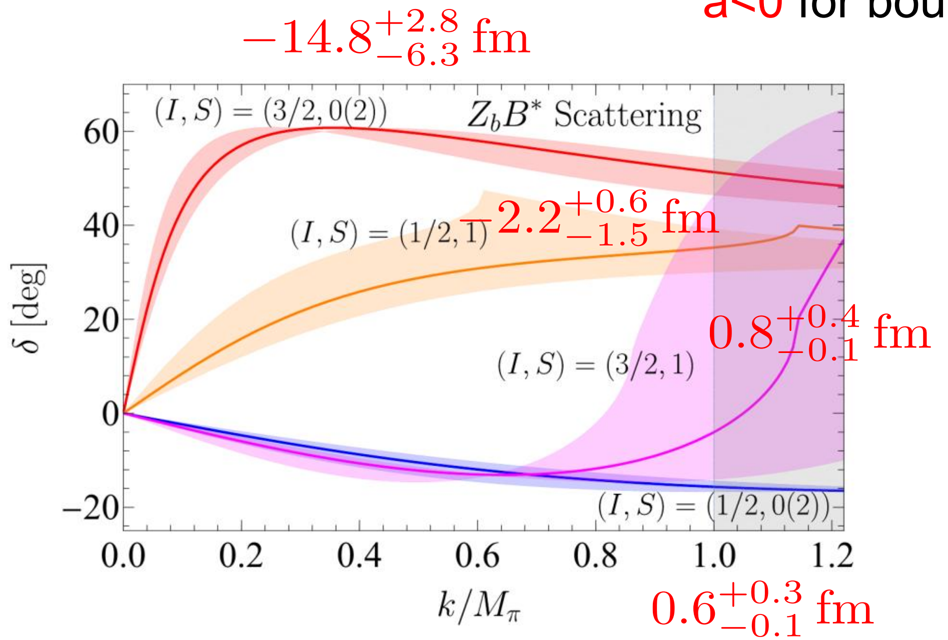
$$(I,S) = (1/2,1), (3/2,1)$$

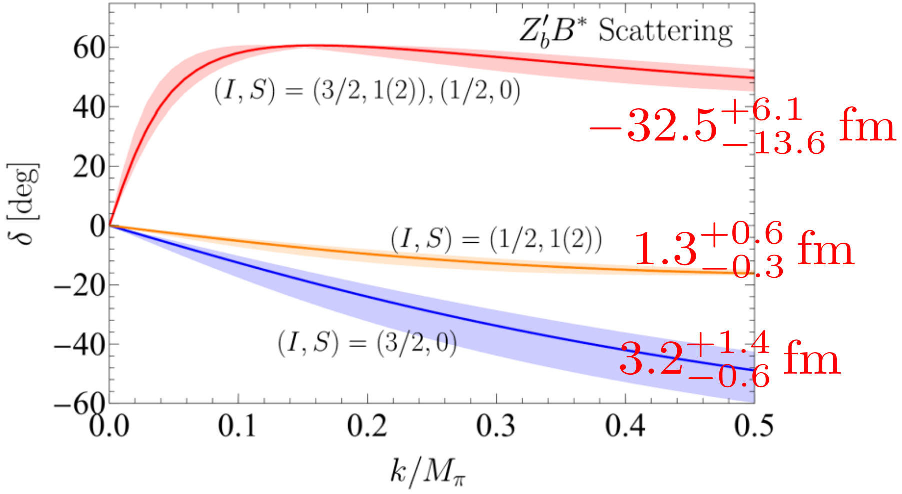


❖ Dimer-particle scattering



a<0 for bound state





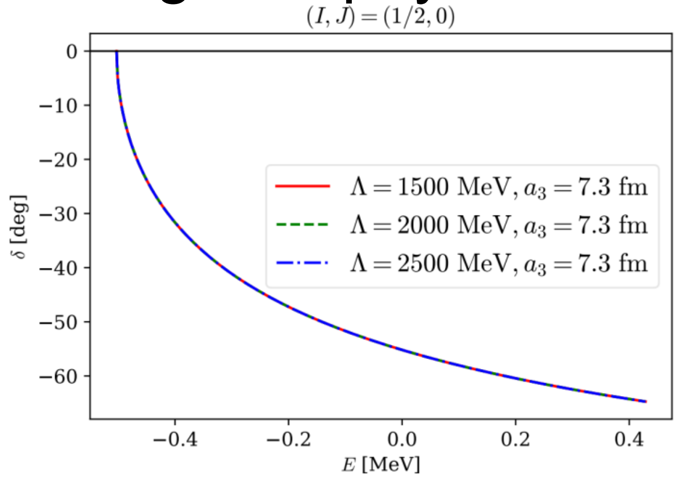
Hints that no Efimov state emerges in these ZB channels:

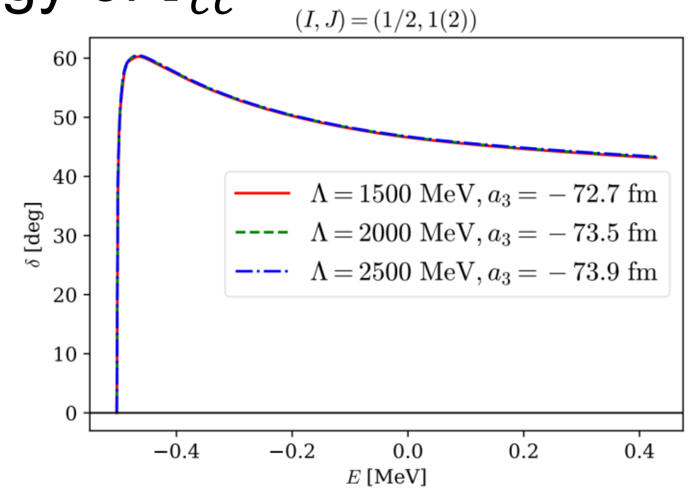
- Convergence achieves as cutoff grows to infinity
- Large positive phase shift but associated with negative scattering length

No three body force in LO pionless EFT

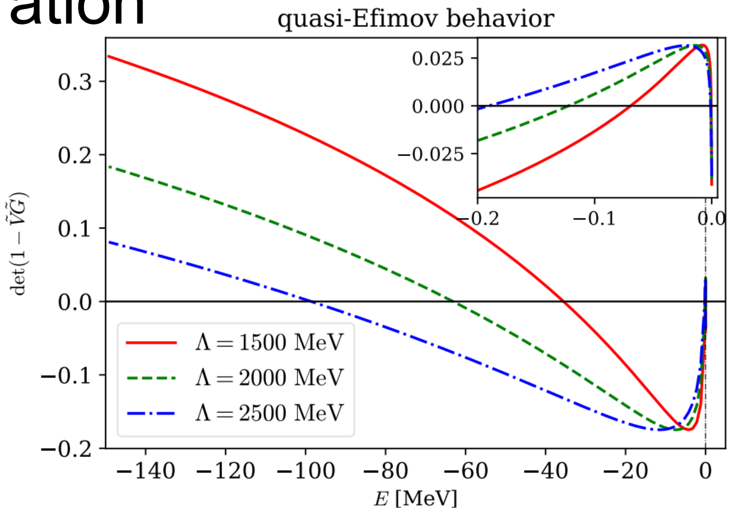
❖ Three-D-Mesons in Pionless EFT

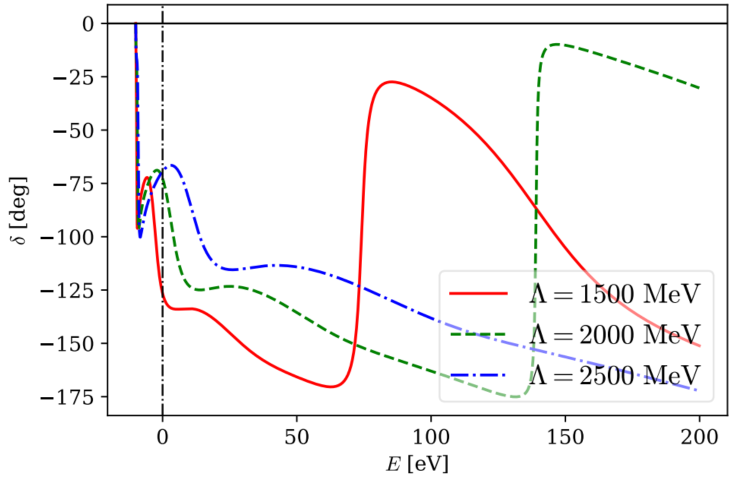
- $T_{cc}^* D^*$ scattering with physical binding energy of T_{cc}^*





- Three identical scalar scattering with 2-body binding energy artificially set for illustration





Three-D-Mesons in Pionless EFT

Coupled channel effects

$$T_i(p) = \mathcal{A}_{ij}\frac{4}{\sqrt{3}\pi p}\int_0^\infty dq\, T_j(q)\frac{1}{2}\ln\frac{p^2+pq+q^2}{p^2-pq+q^2}.$$

$$\xrightarrow{T_i \sim p^{s_i-1}} \quad 1 = \frac{4\lambda_i}{\sqrt{3}}\frac{1}{s_i}\frac{\sin\frac{\pi}{6}s_i}{\cos\frac{\pi}{2}s_i}$$

imaginary s_i solution when $\lambda_i > \lambda_c = \frac{3\sqrt{3}}{2\pi} \approx 0.827$

S-wave D^*D^* dimer: $(I,J) = T_0(1{,}0),\ T_1(0{,}1),\ T_2(1{,}2)$

	Only T_1	Multi-dimer coupled	
$(I,S) = (1/2,0)\ D^*D^*D^*$	$\lambda = -1$		
$(I,S) = (1/2,1)\ D^*D^*D^*$	$\lambda = 1/2$	$\begin{array}{c} T_0 \\ T_1 \\ T_2 \end{array}\begin{pmatrix} -\frac{1}{3} & 1 & -\frac{\sqrt{5}}{3} \\ 1 & \frac{1}{2} & -\frac{\sqrt{5}}{2} \\ -\frac{\sqrt{5}}{3} & -\frac{\sqrt{5}}{2} & -\frac{1}{6} \end{pmatrix}$	$\lambda = 2, -1, -1$
$(I,S) = (1/2,2)\ D^*D^*D^*$	$\lambda = 1/2$	$\begin{array}{c} T_1 \\ T_2 \end{array}\begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$	$\lambda = 2, -1$

❖ Three-D-Mesons in Pionless EFT

□ Coupled channel effects

$$(I, S) = (1/2, 1)\, D^* D^* D^*,\ \text{with}\ T_0, T_1, T_2\ \text{coupled}$$

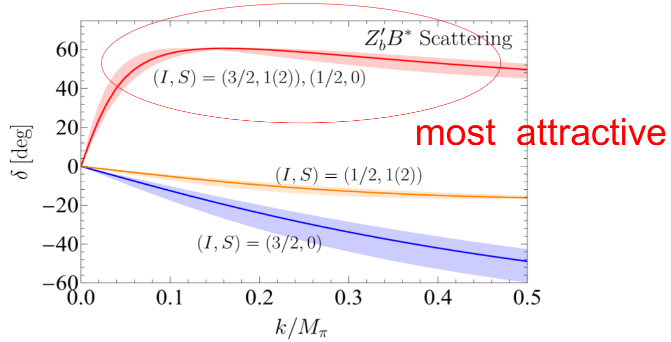
$\mathcal{B}_2$[MeV]	a_{sc}[fm]	$\mathcal{B}_3^{(1)}$	$\mathcal{B}_3^{(2)}$	$\mathcal{B}_3^{(3)}$	$\mathcal{B}_3^{(1)}/\mathcal{B}_3^{(2)}$
0.01	44.03	54.592	0.185	0.011	295
0.5	6.23	64.158	0.980	0.620†	66
1.0	4.40	69.099	1.557	—	44
5.0	1.97	91.365	5.521	—	17

† virtual state.

Summary

- Two-body hadronic molecule leaves imprints in corresponding 3body observables
  - Direct evidence by confirmation of Efimov effects

    No Efimov effects in $Z^{(\prime)}B^{(*)}/T_{cc}^{*}D^{(*)}$ scattering
  - Coupled-channel effects can promote the emergence of Efimov effects



In future

- Finite range corrections to mimic real hadron systems

Thank you for your attention!