Unveiling the structure of $T_{c\bar{s}}$ state in the $D_s\pi\pi$ final state

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Based on arXiv:2510.01564

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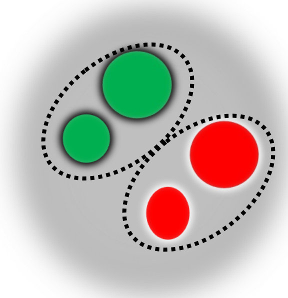
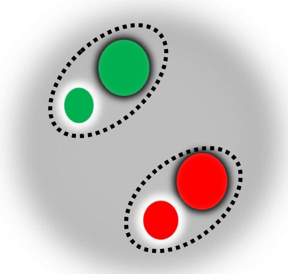
第十一届手征有效场论研讨会，兰州，2026/8/15

Hadron spectra and hadron structures： Exotic states



Compact multiquark



Hadronic molecule

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

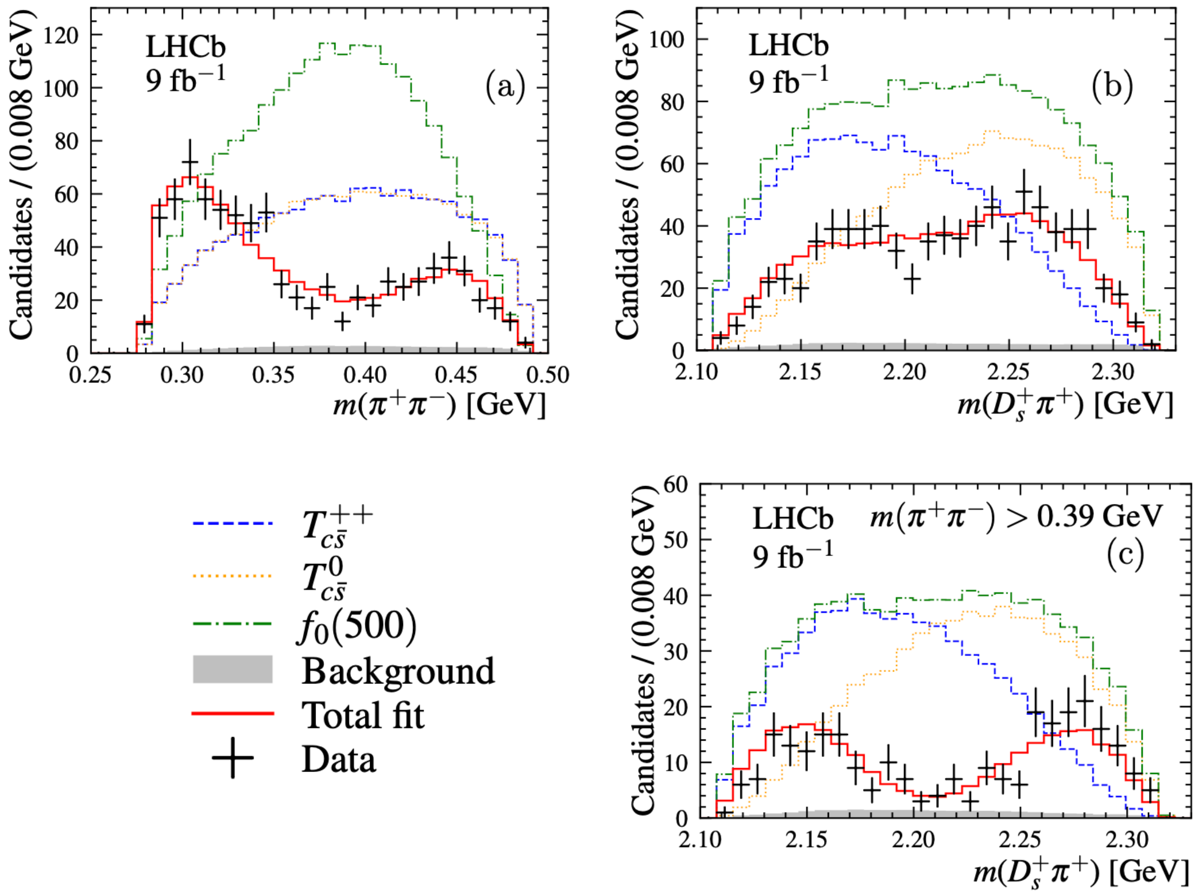
Exotic structure in $B \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays

LHCb, arXiv:2411.03399

$B \to D^{(*)} D_{s1}(2460)$

$\downarrow$

$D_s^+ \pi^+ \pi^-$



Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500)$ + RBW $T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500)$ + K-matrix $T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

Theoretical prediction on the $\pi\pi$ distribution

- Double peak in $\pi\pi$ distribution

Tang, Lin, Guo, Hanhart and Meissner, arXiv:2303.18225

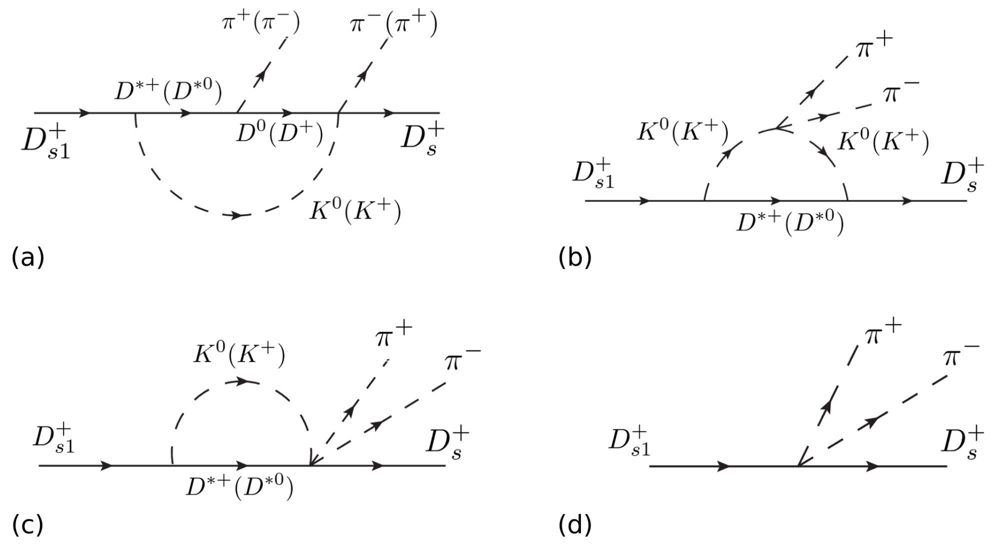
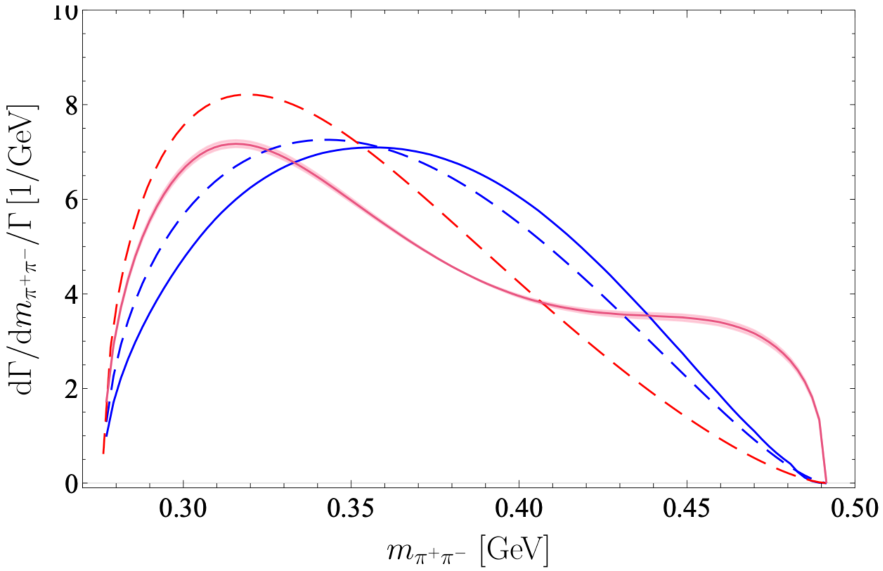


Figure 1. Diagrams for the decay $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ with (a+b+c) and without (d) the D^*K contribution.



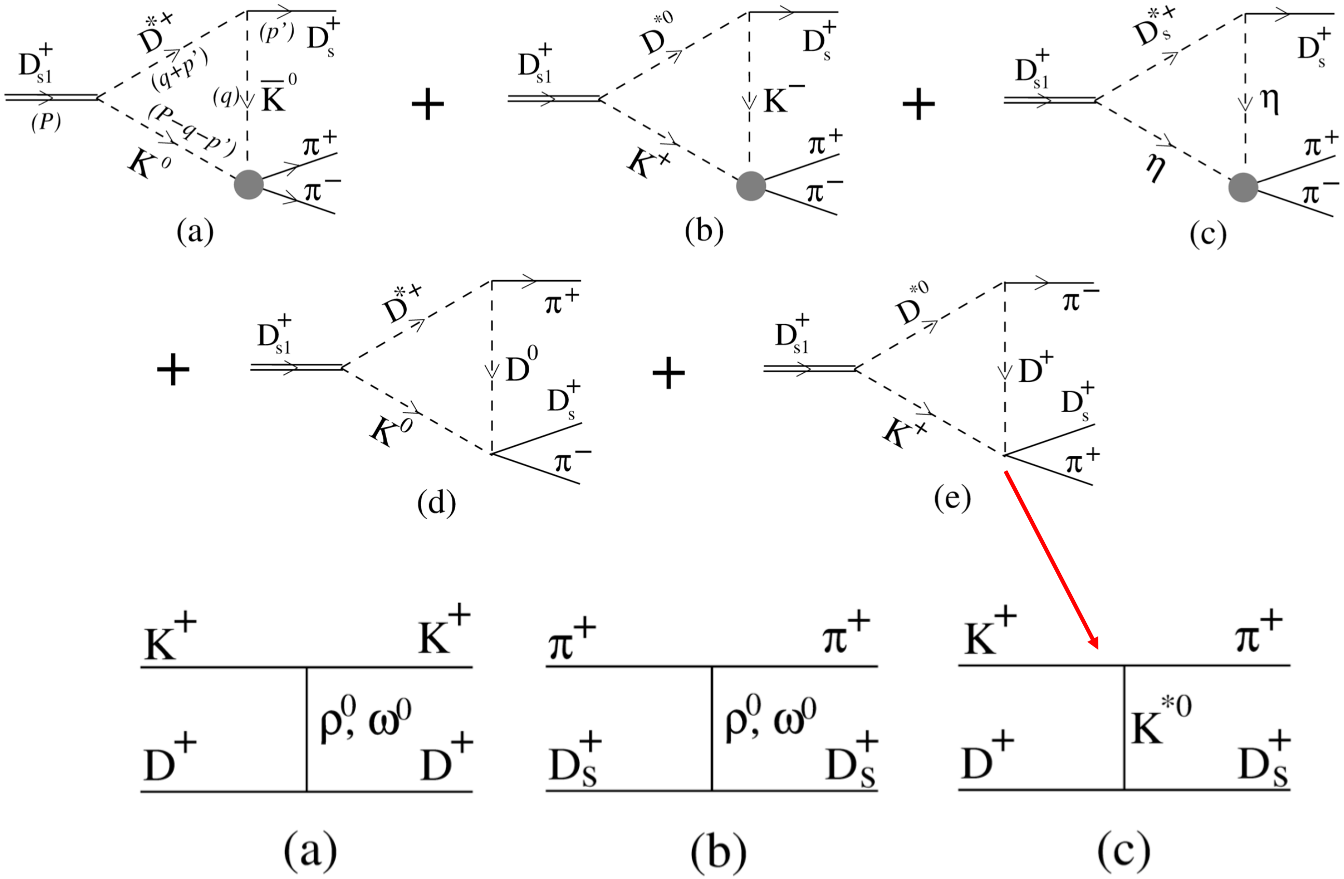
Red: with (a) (b) (c) (d), hadronic molecule
Blue: only with (d), compact state

Solid: with FSI
Dashed: without FSI

Theoretical interpretation

- Molecular $D_{s1}(2460)[D^*K, D_s^*\eta]$ state decay

Roca, Dias and Oset, arXiv:2502.18401



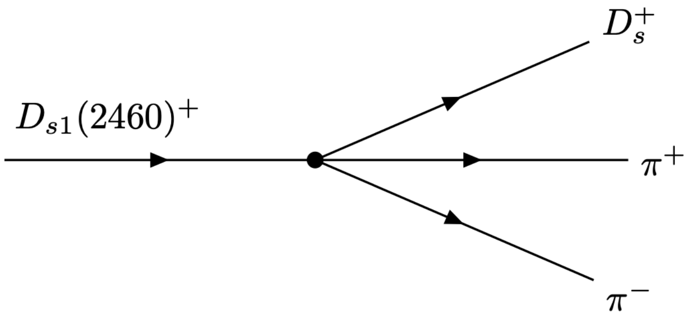
$$V_{K^+D^+,K^+D^+} = 0; \quad V_{\pi^+D_s^+,\pi^+D_s^+} = 0;$$

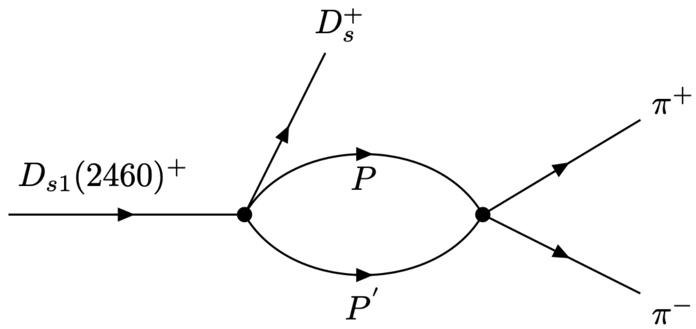
$$V_{K^+D^+,\pi^+D_s^+} = \frac{g^2}{m_{K^*}^2}(p_1 + p_3)_\mu (p_2 + p_4)^\mu.$$

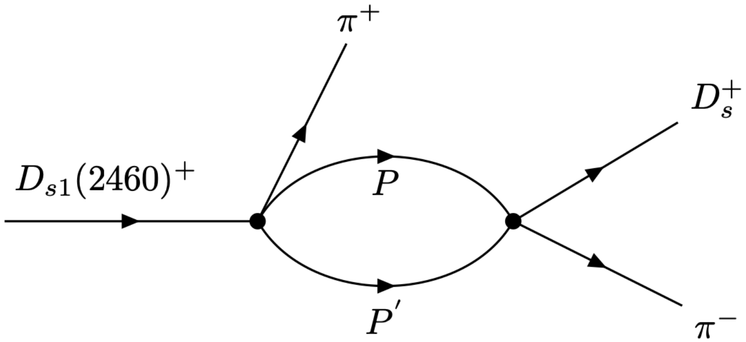
Theoretical interpretation

- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009







$$V^{I=1}_{D_s\pi\to D_s\pi} = 0,$$

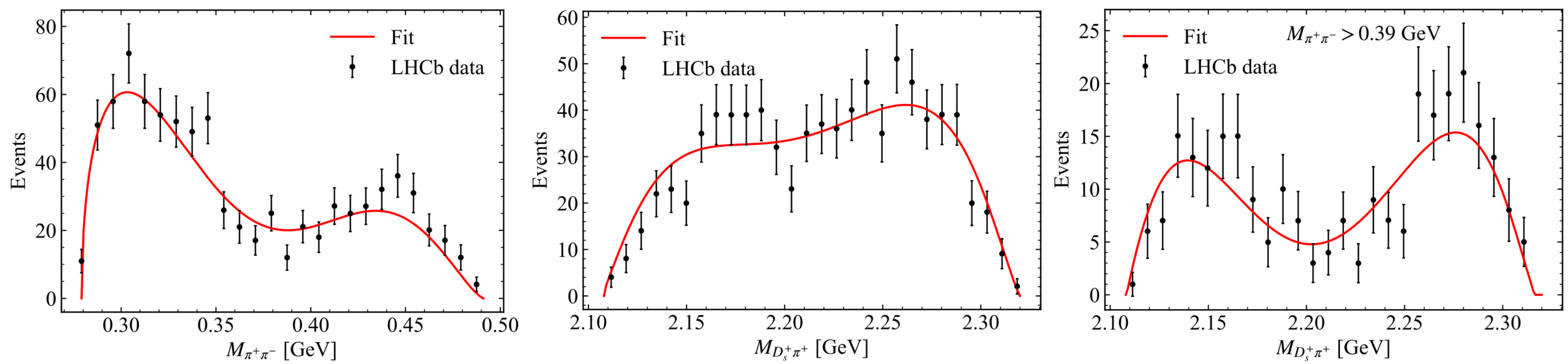
$$V^{I=1}_{D_s\pi\to DK} = -\frac{1}{8f^2}\left(3s - \left(M_1^2 + M_2^2 + m_1^2 + m_2^2\right) - \frac{\Delta_1\Delta_2}{s}\right),$$

$$V^{I=1}_{DK\to DK} = 0,$$

Theoretical interpretation

- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009



- A pole at $\sqrt{s_p} = (2.394 - 0.057i)$GeV on the second Riemann sheet; a pole at $\sqrt{s_p} = (2.246 - 0.093i)$GeV on the third Riemann sheet.

(S, I)	$h_5' = +1$			$h_5' = -1$		
	Re	Im	RS	Re	Im	RS
$(0, \frac{1}{2})$	2107	± 123	II	2107	± 105	II
	2452	± 17	III	2519	± 69	III
$(0, \frac{1}{2})$ $(V_{ii} = 0)$	2466	± 24	III	2388	± 49	III
$(1, 0)$	2318	0	I	2318	0	I
$(1, 1)$	2309	± 111	III	2283	± 196	III

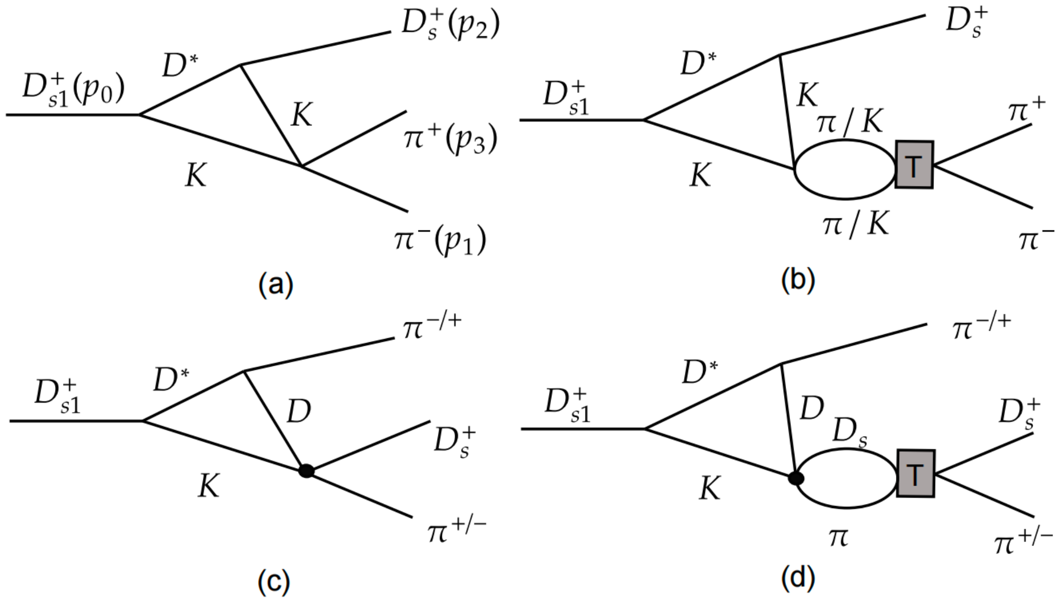
(S, I)	RS	$\sqrt{s_{pole}}$ [MeV]
$(1, 1)$	II	$2466^{+32}_{-27} - i\,271^{+4}_{-5}$
	III	$2225^{+12}_{-9} - i\,178^{+19}_{-17}$

Guo, Hanhart and Meissner,
Eur. Phys. J. A40, 171-179(2009)

Guo, Meissner and Yao,
Phys.Rev.D 92 (2015) 9, 094008

Our model

- The feynman diagrams



- The amplitude

$$
\begin{aligned}
i\mathcal{M}_a &= r_1 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_b &= r_1 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu G_{\pi\pi/KK}(k, m_{23}) T_{\pi\pi/KK\to\pi\pi}(m_{13})}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_c &= r_2 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_d &= r_2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(\vec{k}, \vec{p}_{\rm on}, m_{23})}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}
$$

$\pi\pi - KK$ scattering

- Coupled channel Lippmann Schwinger equations with meson-meson potentials provided by the lowest order chiral Lagrangian

Oller, Oset, Nucl.Phys.A 620 (1997) 438-456

$$V_{11} = - < K\bar{K}|\mathcal{L}_2|K\bar{K} >= -\frac{1}{4f^2}(3s + 4m_K^2 - \sum_i p_i^2)$$

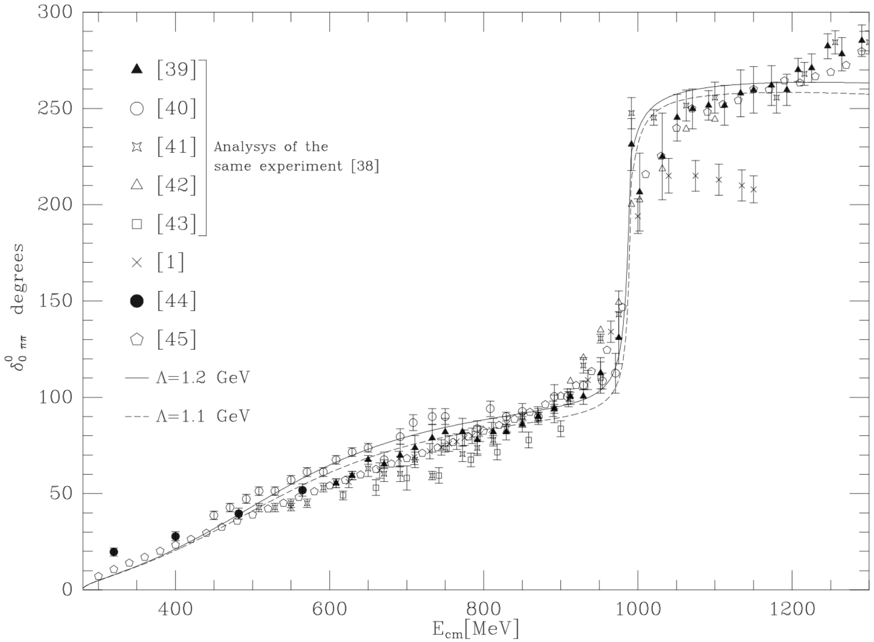
$$V_{21} = - < \pi\pi|\mathcal{L}_2|K\bar{K} >= -\frac{1}{3\sqrt{12}f^2}(\frac{9}{2}s + 3m_K^2 + 3m_\pi^2 - \frac{3}{2}\sum_i p_i^2)$$

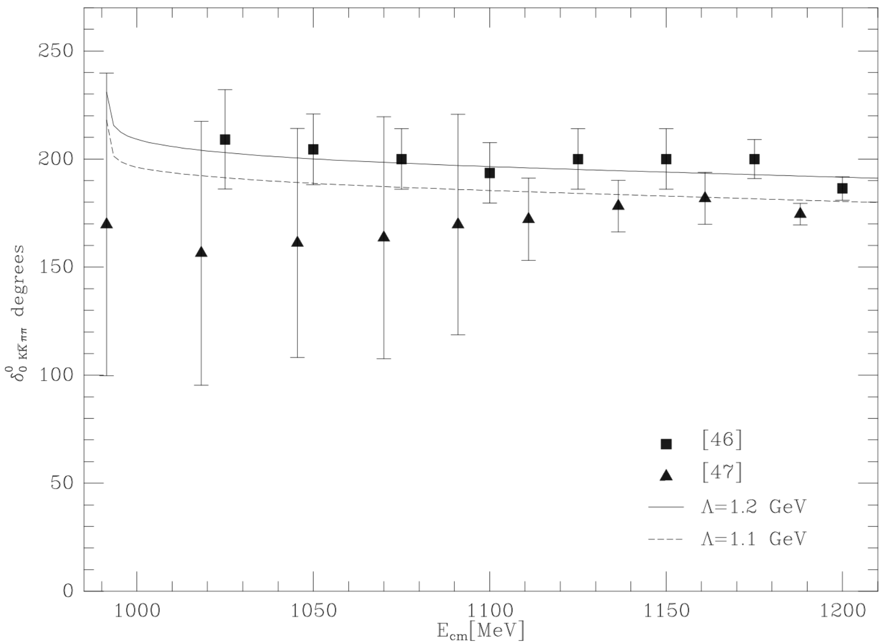
$$V_{22} = - < \pi\pi|\mathcal{L}_2|\pi\pi >= -\frac{1}{9f^2}(9s + \frac{15m_\pi^2}{2} - 3\sum_i p_i^2)$$

$$t_{11} = V_{11} + V_{11}G_{11}t_{11} + V_{12}G_{22}t_{21}$$

$$t_{21} = V_{21} + V_{21}G_{11}t_{11} + V_{22}G_{22}t_{21}$$

$$t_{22} = V_{22} + V_{21}G_{11}t_{12} + V_{22}G_{22}t_{22}$$



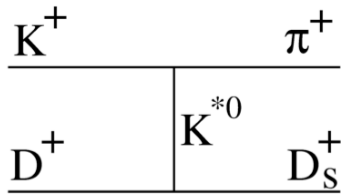


$DK - D_s\pi$ scattering

- For the $DK - D_s\pi$ T-matrix,

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\,\vec{q}\, \frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E)\, T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

$$\mathcal{V}^{K^*}_{DK\to D_s\pi} = \frac{g_{K^*}}{m_{K^*}^2}(p_1 + p_3)_\mu (p_2 + p_4)^\mu$$



Similar for $V^{D^*}_{DK\to D_s\pi}$. The resulting effective potential reads

$$\mathcal{V}_{\alpha\beta} = \left(V^{K^*}_{\alpha\beta} + V^{D^*}_{\alpha\beta}\right)\left(\frac{\Lambda_\alpha^2}{\Lambda_\alpha^2 + p_\alpha^2}\right)^2 \left(\frac{\Lambda_\beta^2}{\Lambda_\beta^2 + p_\beta'^2}\right)^2$$

$\alpha, \beta = 1{,}2$ for $D_s\pi$ and DK channel, respectively. The cutoff for these two channels can be different.

UV divergence in the triangle loop

- S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

- The amplitude

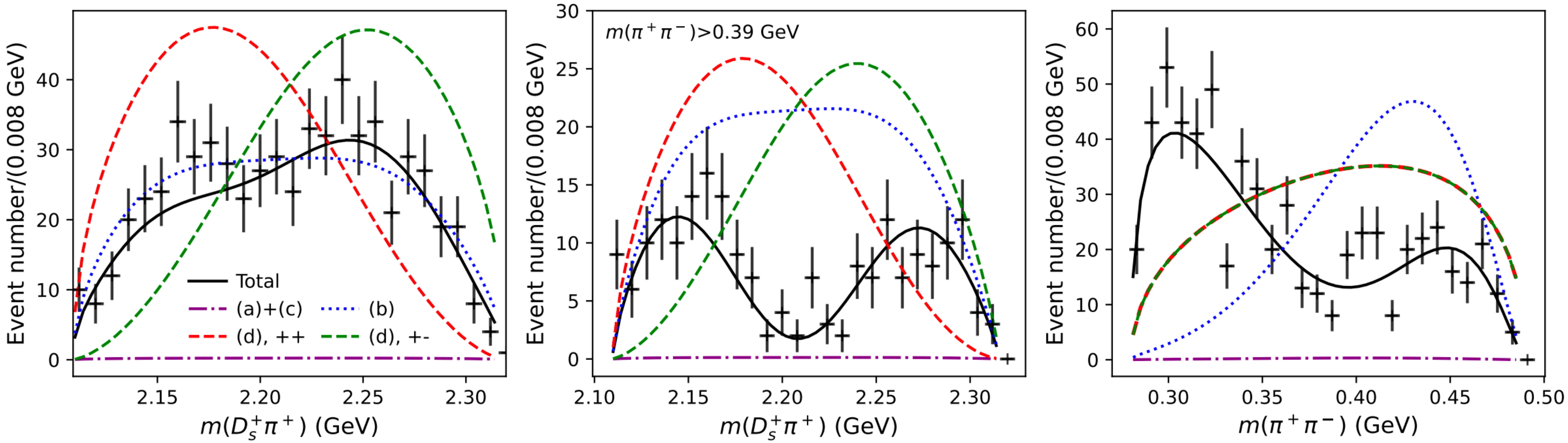
$$\begin{aligned}
i\mathcal{M}_a &= r_1 \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_b &= r_1 \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu G_{\pi\pi/KK}(k, m_{23}) T_{\pi\pi/KK\to\pi\pi}(m_{13})}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_c &= r_2 \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_d &= r_2 \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(\vec{k}, \vec{p}_{\rm on}, m_{23})}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}$$

$$\begin{aligned}
L_\mu &= P_{\mu\nu}(p_2 + q, m_{D^*})(p_2 - q)^\nu \\
N_\mu &= P_{\mu\nu}(p_1 + q, m_{D^*})(p_1 - q)^\nu (p_2 - q + 2p_3)^\alpha \\
&\quad \times P_{\alpha\beta}(p_2 - q, m_{K^*})(p_2 + q)^\beta
\end{aligned}$$

$$P_{\mu\nu}(p, m) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

- Regularize the loop integral using dimensional regularization, and use the Msbar renormalization scheme.

Fit to the experimental lineshapes

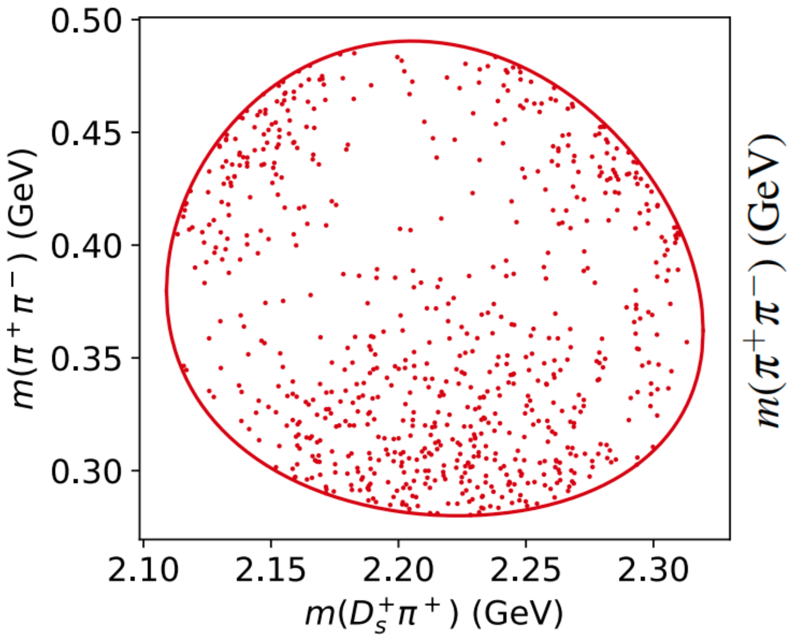


- The parameters of the model determined from the fit：

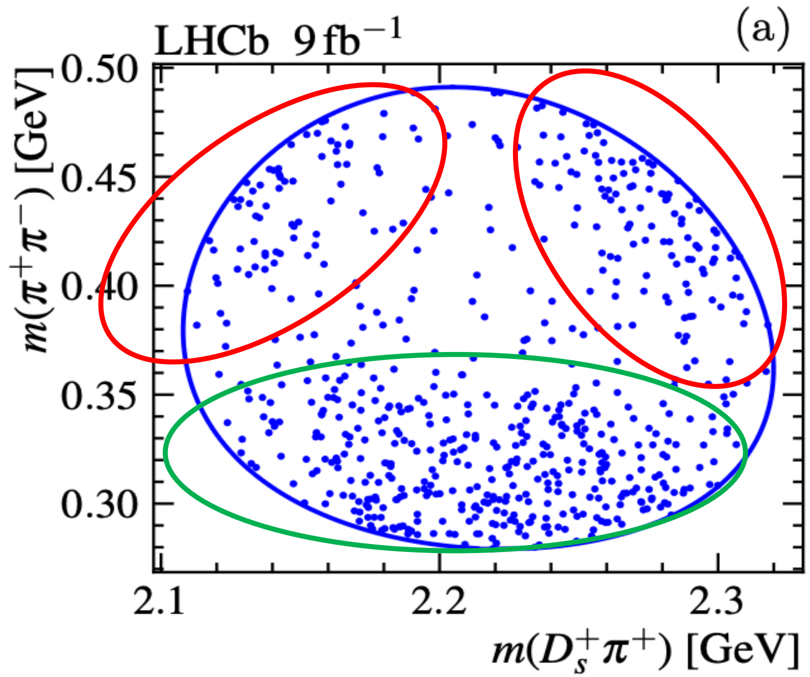
Parameter	
Λ_1 [GeV]	1.60 ± 0.07
Λ_2 [GeV]	0.557 ± 0.024
g_{K^*}	61.4 ± 2.4
r_1	7.1 ± 0.8
r_2	0.121 ± 0.014
χ^2/d.o.f.	1.43
E_p [MeV]	$2211.0^{+31.4}_{-22.6} - 154.5^{+20.7}_{-17.1} i$

- The pole position is in second Riemann sheet.

Dalitz plot



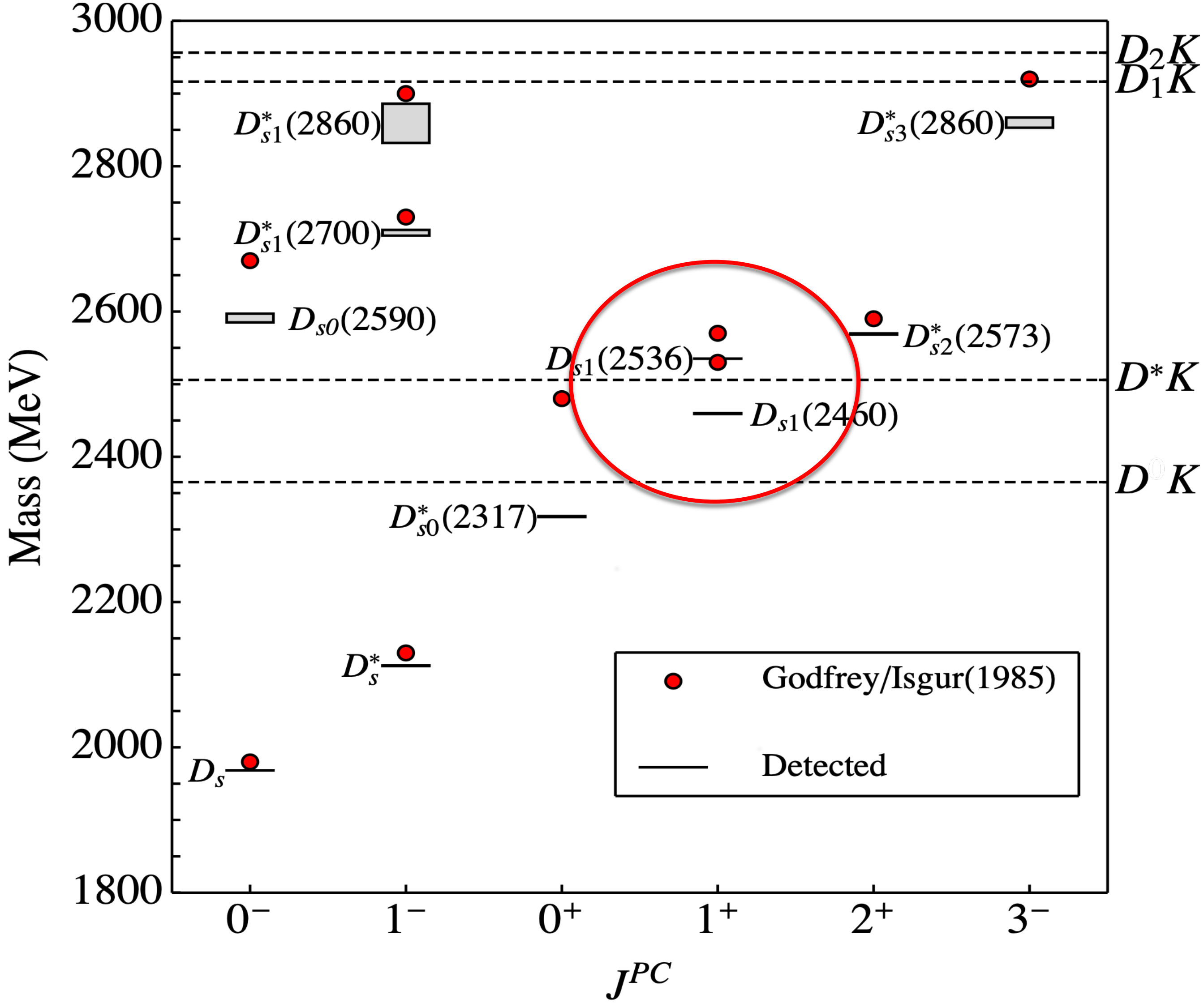
LHCb, arXiv:2411.03399



Is possible to investigate the structure of D_{s1} states?

- $D_{s1}(2460) \to D_s^+ \pi^+ \pi^-$
- $D_{s1}(2536) \to D_s^+ \pi^+ \pi^-$

D_s mesons in quark model



Coupled-channel framework

- Quark model bare state + hadronic molecule

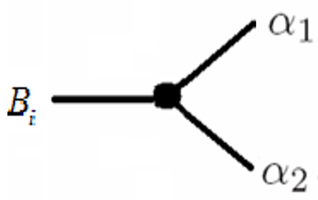
Yang, Wang, Wu, Makoto, Zhu, Phys.Rev.Lett. 128,112001(2022)

The Hamiltonian reads: $$H = H_0 + H_I,$$

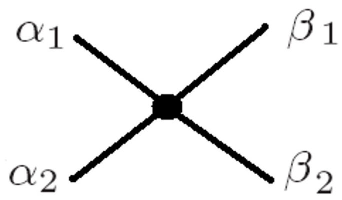
The non-interacting one: $$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

The interacting one: $$H_I = g + v$$

bare state core -> channel：



channel -> channel：



$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

truncate the hard vertices given by usual QPC

P. G. Ortega, et al,
Phys. Rev. D 94, 074037 (2016)

Effective Lagrangian: (exchanging mesons, e.g. ρ/ω)

Form factor: $$\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$$

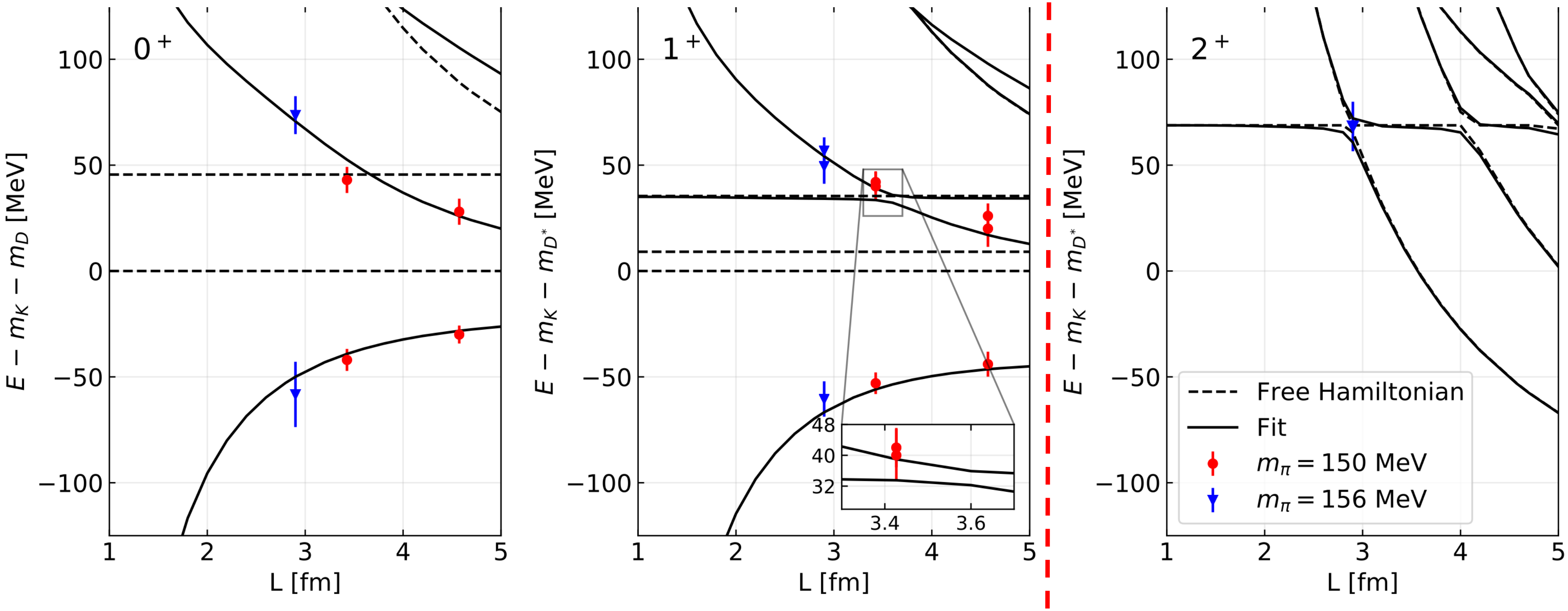
Fit the lattice data of $D_s(2317,2460,2536)$

$$(H_0+H_\mathrm{I})|\Psi\rangle = E|\Psi\rangle$$

Eigenvalues ⟷ Lattice levels

Lattice data from: C. B. Lang et al., **Phys. Rev. D 90, 034510 (2014);**
G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**

Fit | Postpredict



Component and pole mass of D_s states

Yang, Wang, Wu, Makoto, Zhu, Phys.Rev.Lett. 128,112001(2022)

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

G. S. Bali et al., Phys. Rev. D 96, 074501 (2017)

Lattice
2348(4)(+6)

$D_{s1}(2460)$

- Both the bare $c\bar{s}$ core and molecular components are significant and essential.

$D_{s1}(2536)$

- Mainly pure $c\bar{s}$.

Coupling of D_{s1} to D^*K

❖ S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

The coupling of D_{s1} to D^*K can be obtained from the residue of the T-matrix:

$$\begin{aligned} g_i^2 &= \frac{r}{2\pi} \int_0^{2\pi} T_{ii}(z(\theta)) e^{i\theta} d\theta \\ &= \lim_{s \to z_p} (s - z_p) T_{ii}(s) \end{aligned}$$

- $D_{s1}(2460)$: $g_S \gg g_D$
- $D_{s1}(2536)$: $\frac{g_S}{g_D} = 0.08\ e^{2.67i}$

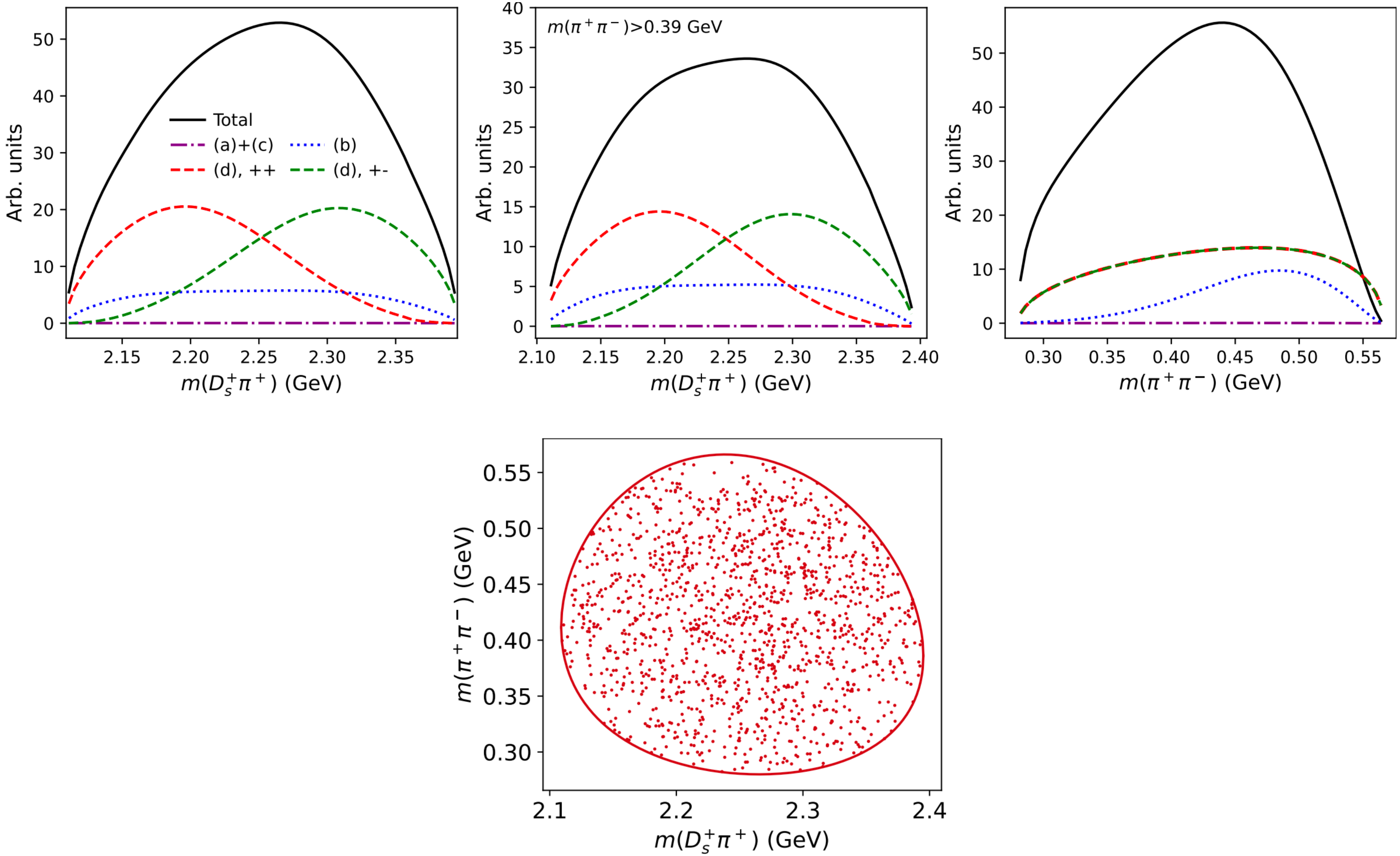
❖ The $D_{s1}(2536)$ coupling can also obtained using the experimental measurment on the S- and D-wave decay:

LHCb，JHEP 10 (2023) 106

$$\frac{g_S}{g_D} = 0.1\ e^{-0.7i}$$

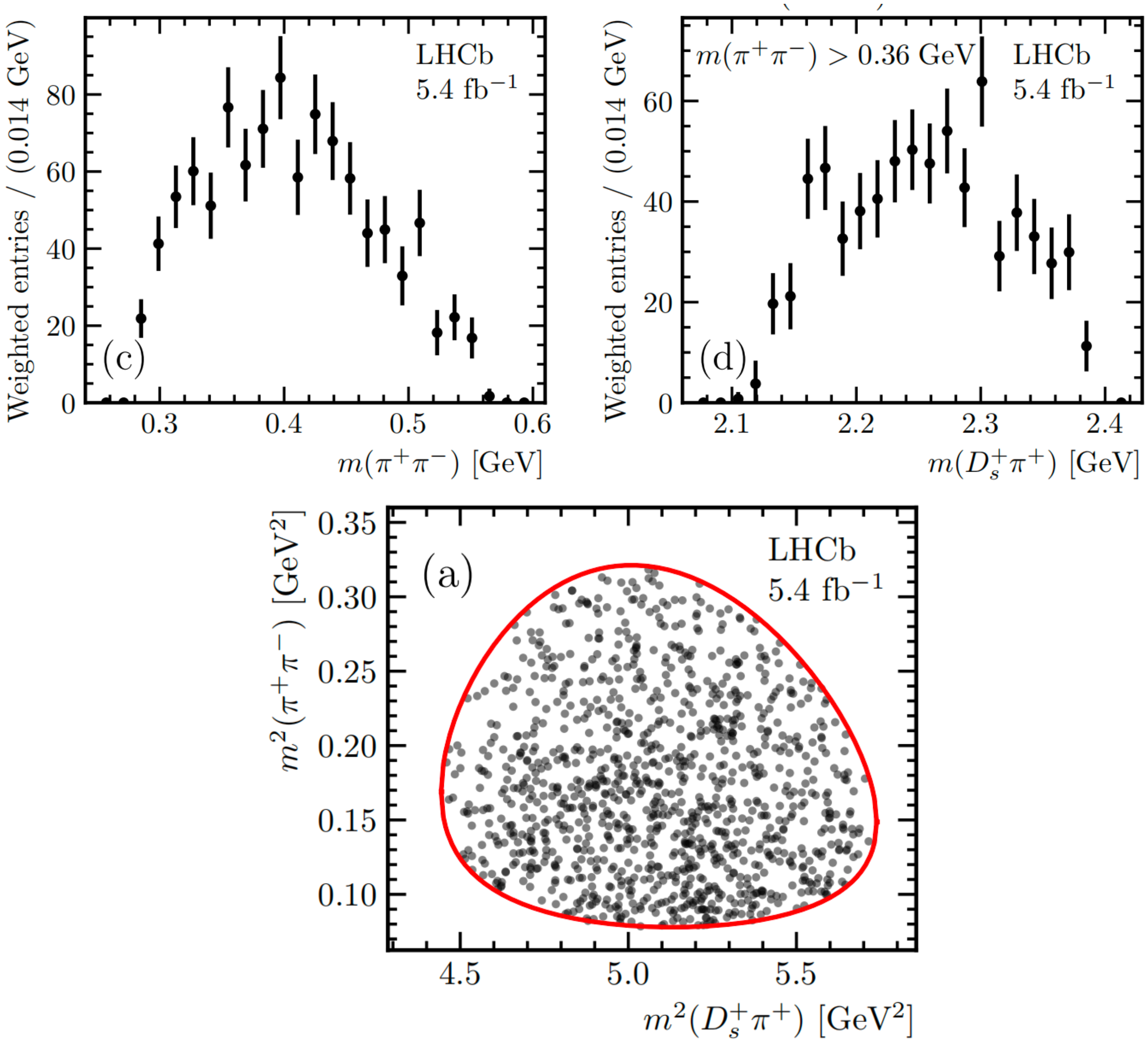
Predicted lineshapes and Dalitz plot for $D_{s1}(2536)$ decay

- No two-peak structure at $m(\pi^+\pi^-) > 0.39$ GeV



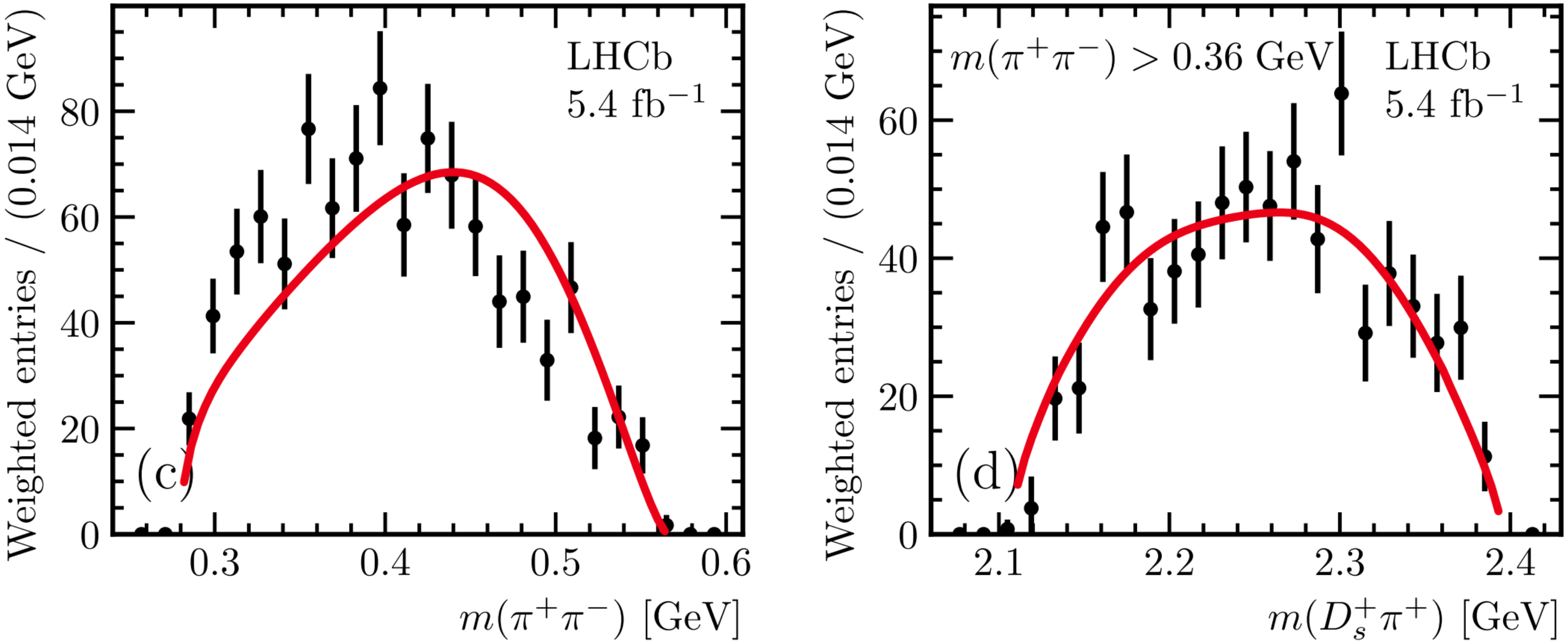
Measured lineshapes and Dalitz plot for $D_{s1}(2536)$ decay

LHCb, arXiv:2608.10826



Comparison for $D_{s1}(2536)$ decay

LHCb, arXiv:2608.10826



Red line: our theoretical prediction

Summary

- The $T_{c\bar{s}}$ lineshape can be discribed through $D_s\pi - DK$ coupled-channel interactions, revealing how off-diagonal potential terms generate the observed resonance pole;

- **Internal structure of D_{s1}:** What can the $D_s\pi\pi$ distributions reveal about the internal structure of $D_{s1}(2460/2536)$? After decades of debate over whether these states are molecules, conventional $c\bar{s}$, or mixtures thereof, their decay kinematics offer a qualitatively new experimental window. The structural contrast between them turns their decay pathways into complementary filters of exotic state formation.

arXiv:2510.01564

integrated luminosity of $5.4\,\mathrm{fb}^{-1}$. Already at the level of the raw Dalitz plots, the two modes exhibit visibly different structures, even though the parent states have the same quantum numbers and very similar masses, implying that their decay dynamics cannot be identical. This points to a difference in the internal structure of the two states, such as a sizable molecular D^*K component in the $D_{s1}(2460)^+$ wave function that is absent or strongly suppressed for the $D_{s1}(2536)^+$ state.

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