Interactions of Heavy Flavored Hadrons

Zhan-Wei Liu

School of Physical Science and Technology, Lanzhou University, China

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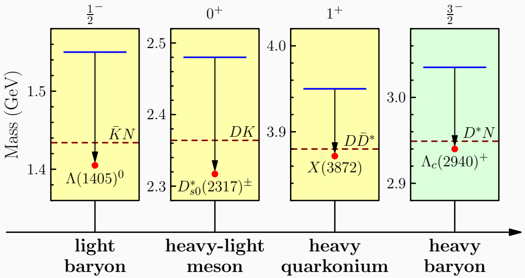
$\Lambda_c(2940)$ **and** $\Lambda_c(2910)$

Hadron interactions are important for hadron spectrum



S.-Q. Luo et. al. Eur. Phys. J. C 80 (2020) 4, 301

- Naive quark model is
  - successful for the ground hadrons;
  - not good when a quark is in P wave.
- S-Wave Hadron-hadron interactions are essential for the mass spectrum.

Experimental discoveries of $\Lambda_c(2940)$ and $\Lambda_c(2910)$

$\Lambda_c(2940)$: $M = 2939.6^{+1.3}_{-1.5}$ MeV and $\Gamma = 20^{+6}_{-5}$ MeV.

- [BaBar:2006itc]: *BABAR* released observation of $\Lambda_c(2940)^+$ in $D^0 p$ invariant mass spectrum.
- [Belle:2006xni]: Later, Belle confirmed $\Lambda_c(2940)^+$ in $\Lambda_c(2940)^+ \to \Sigma_c(2455)^{0,++}\pi^{+,-}$.
- [LHCb:2017jym]: LHCb also observed it, and preferred its possible assignment with $J^P = 3/2^-$.

$\Lambda_c(2910)$: $M = 2913.8 \pm 5.6 \pm 3.8$ MeV and $\Gamma = 51.8 \pm 20.0 \pm 18.8$ MeV.

- [Belle:2022hnm]: the Belle Collaboration reported a new structure called $\Lambda_c(2910)^+$, via the $\bar{B}^0 \to \Sigma_c(2455)\pi\bar{p}$ decay process

Contradiction between experimental measurement and quark model

传统势模型下Λ_c^+重子谱学预测

$J^P(nL)$	Exp. [1]	This work	Ref. [9]	Ref. [50]	Ref. [51]
$\frac{1}{2}^+(1S)$	2286.86	2286	2286	2286	2265
$\frac{1}{2}^+(2S)$	2766.6	2766	2769	2791	2775
$\frac{1}{2}^+(3S)$		3112	3130	3154	3170
$\frac{1}{2}^+(4S)$		3397	3437		
$\frac{1}{2}^-(1P)$	2592.3	2591	2598	2625	2630
$\frac{3}{2}^-(1P)$	2628.1	2629	2627	2636	2640
$\frac{1}{2}^-(2P)$	2939.3	2989	2983		[2780]
$\frac{3}{2}^-(2P)$		3000	3005		[2840]
$\frac{1}{2}^-(3P)$		3296	3303		[2830]
$\frac{3}{2}^-(3P)$		3301	3322		[2885]
$\frac{3}{2}^+(1D)$		2857	2874	2887	2910
$\frac{5}{2}^+(1D)$	2881.53	2879	2880	2887	2910
$\frac{3}{2}^+(2D)$		3188	3189	3120	3035
$\frac{5}{2}^+(2D)$		3198	3209	3125	3140
$\frac{5}{2}^-(1F)$		3075	3097	[2872]	[2900]
$\frac{7}{2}^-(1F)$		3092	3078		3125
$\frac{7}{2}^+(1G)$		3267	3270		3175
$\frac{9}{2}^+(1G)$		3280	3284		

Eur. Phys. J. A 51, 82 (2015)

- Measured $\Lambda_c(2940)^+$ masses are more than 50 MeV smaller than quark model predictions for $\Lambda_c(2P)$.
- $\Lambda_c(2940)^+$ locates below the D^*N threshold about 6 MeV.
  - S wave D^*N should be important.
  - $[D^*N]^{I=0}_{J=1/2}$ or $[D^*N]^{I=0}_{J=3/2}$.

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Framework

- Physical Λ_c includes two parts:

$$|\Lambda_c, \text{phys}\rangle = c_0|\Lambda_c, \text{bare}\rangle + \int d^3\vec{p}\chi(\vec{p})|D^*N, \vec{p}\rangle$$

- Hamiltonian contains three parts:

$$H = H_0 + H_I + H_{D^*N}$$

  - H_0: to generate the states $|\Lambda_c, \text{bare}\rangle$ in quark model.
  - H_I: coupling between $|\Lambda_c, \text{bare}\rangle$ and $|D^*N, \vec{p}\rangle$.
  - H_{D^*N}: interaction between $|D^*N, \vec{p}\rangle$ and $|D^*N, \vec{p}'\rangle$ and relevant kinetic energies.

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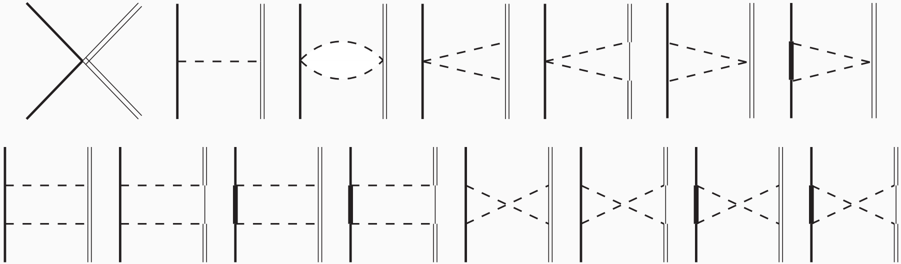
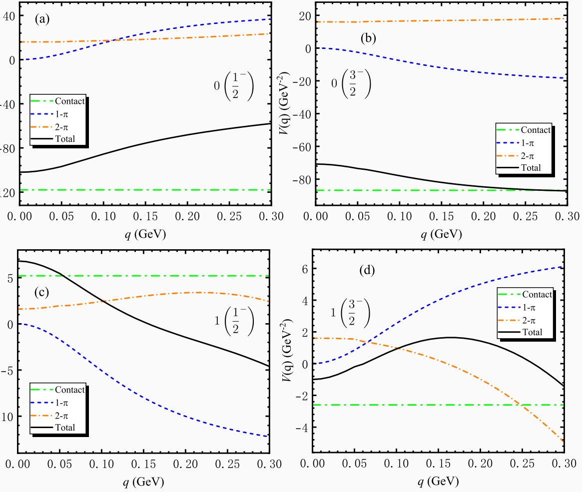
Chiral interactions between $|D^*N\rangle$ and $|D^*N\rangle$



- contact term
- one Boson exchange
- two Boson exchange

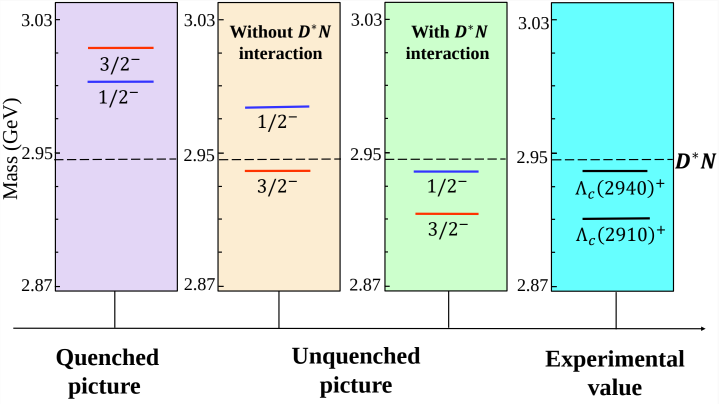
- dimensional regularization is used.
- $\Delta(1232)$ and D mesons are also considered.

Chiral potentials between $|D^* N\rangle$ and $|D^* N\rangle$



- attraction in two $I = 0$ channels is stronger:
  - 2π-exchange potential are repulsive
  - their repulsive interaction is rather weak.
  - The attractive interaction is dominantly provided by the $\mathcal{O}(\epsilon^0)$ contact interaction.

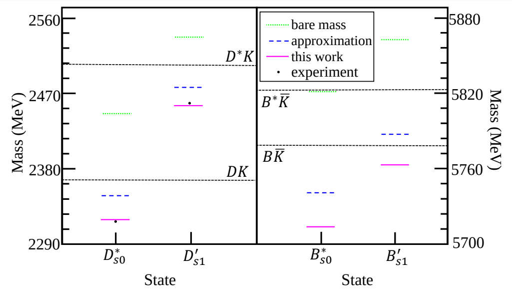
- attraction in two $I = 1$ channels is weak.



- D^*N interaction is important to obtain the correct mass spectrum.
- D^*N interaction makes the $1/2^-$ and $3/2^-$ masses order reversed.
- We can interpret $\Lambda_c(2940)$ and $\Lambda_c(2910)$ simultaneously.

Z. L. Zhang, Z. W. Liu, S. Q. Luo, F. L. Wang, B. Wang and H. Xu, Phys. Rev. D **107** (2023) no.3, 034036.

Same framework can also successfully be applied into D_s family



- We can interpret $D^*_{s0}(2317)$ and $D'_{s1}(2460)$ simultaneously.

- We also predict the B_s mass spectrum.

Z. L. Zhang, Z. W. Liu, S. Q. Luo, P. Chen and Z. H. Guo, Phys. Rev. D **110** (2024) no.9, 094037

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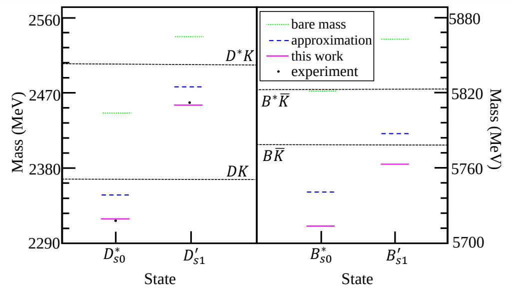

Pure $D^{(*)}K$ molecule assumption without $c\bar{s}$ core

- from experimental measurements: $\rightarrow E_{\text{bind}} \sim 50$ MeV
  much larger than that of deuteron.
- in our framework: E_{bind} around 2 MeV and 1 MeV, respectively
  similar to that of deuteron.

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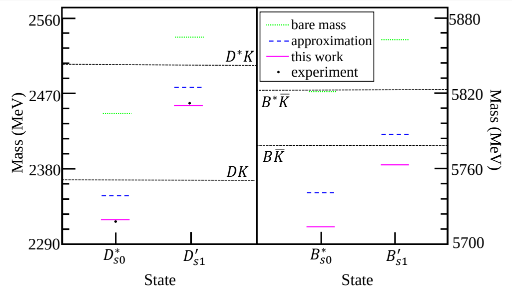

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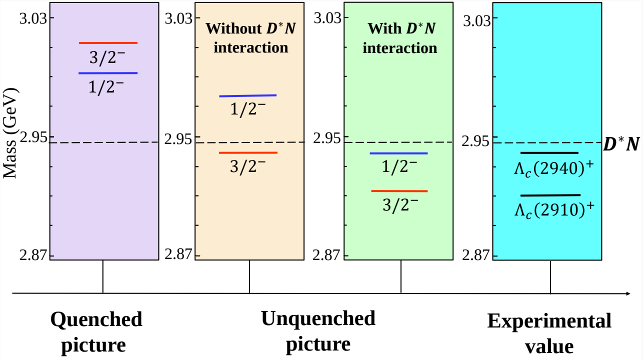
$c\bar{s}$ core strengthens the dressed attraction of $D^{(*)}K$ in our framework!

Looking back at $\Lambda_c(2910)$ starting from a molecule picture



- $\Lambda_c(2910)$ has a binding energy more than 30 MeV below D^*N threshold.
- The physical states have about only about 25% bare $\Lambda_c^{\text{bare}}(2P)$ which nevertheless provide strong attraction for D^*N.
- Without $\Lambda_c^{\text{bare}}(2P)$, binding energy of $(D^*N)_{J=3/2^-}$ is few MeV in our framework.

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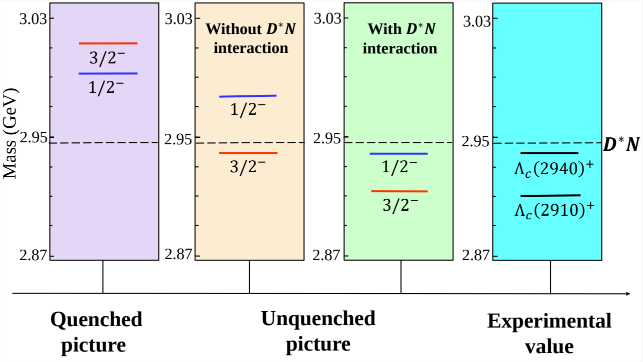

The bare $\Lambda_c^{\text{bare}}(2P)$

- is not dominant in $\Lambda_c(2910)$;
- changes the dressed interaction of D^*N greatly!

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$\Lambda(1405)$ **and** $\Lambda(1670)$

$\Lambda(1/2^-)$ resonances

- $\Lambda(1405)$ is close to the $\bar{K}N$ threshold.
- Its molecular interpretation can easily explain why $m_{\Lambda(1405)} > m_{N(1535)}$.

- If so, where can one find the lightest P-wave $|uds\rangle$ baryon expected in the conventional quark model?
- We will show this P-wave triquark core goes in $\Lambda(1670)$.

$\Lambda(1405)$ and $\Lambda(1670)$ with K^-p scattering

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

$$v^I = \sum_{\alpha,\beta} \int d^3\vec{k}\, d^3\vec{k}' \, |\alpha(\vec{k})\rangle \, V^I_{\alpha,\beta}(k,k') \, \langle\beta(\vec{k}')| \, ,$$

$$V_{\alpha,\beta}(k,k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)}\sqrt{\omega_{\beta_M}(k')}}$$

$|\alpha\rangle$=$|\pi\Sigma\rangle$, $|\bar{K}N\rangle$, $|\eta\Lambda\rangle$, $|K\Xi\rangle$, $|\pi\Lambda\rangle$

- two scenarios: with or without a bare baryon

$$g^I = \sum_{\alpha,B_0} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle \, G^{I\dagger}_{\alpha,B_0}(k) \, \langle B_0| + |B_0\rangle \, G^I_{\alpha,B_0}(k) \, \langle\alpha(\vec{k})| \right\} ,$$

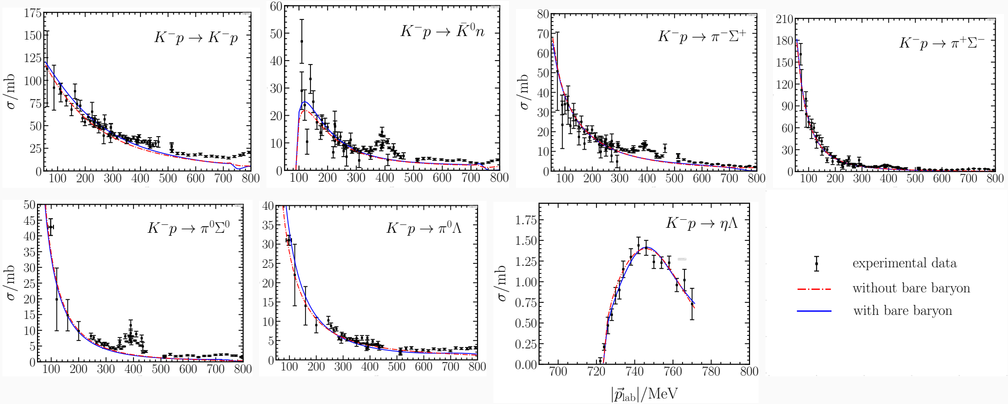
where

$$G^I_{\alpha,B_0}(k) = \frac{\sqrt{3}\, g^I_{\alpha,B_0}}{2\pi f} \sqrt{\omega_\pi(k)}\; u(k).$$

$$H^I_{\text{int}} = g^I + v^I.$$

$\Lambda(1405)$ and $\Lambda(1670)$ with $K^- p$ scattering

We can fit the cross sections of $K^- p$ well both with and without a bare baryon.

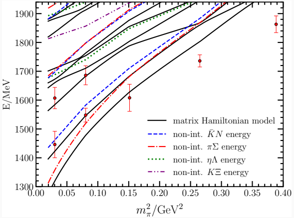


Two-pole structure of $\Lambda(1405)$: $1324 - i67$ MeV, $1428 - i24$ MeV ;

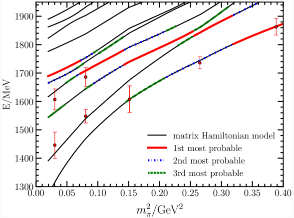
Pole for $\Lambda(1670)$: $1674 - i11$ MeV.

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Spectrum of $\Lambda(1/2^-)$ on the Lattice



without a bare baryon



with a bare baryon

Λ Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting lattice QCD data at large pion masses.
- The bare triquark core is important for $\Lambda(1670)$.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state.

  containing very little of bare baryon at physical pion mass.

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Comparison with recent BaSc lattice simulations

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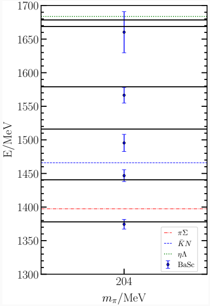
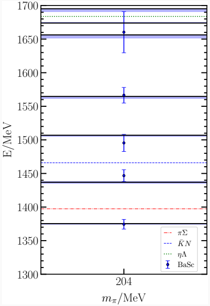


without
a bare baryon



with
a bare baryon

- The BaSc lattice collaboration observed all HEFT states with multiquark interpolating operators;
- The right HEFT results with bare Λ fit the lattice simulations better;
- The left HEFT results without bare triquark core lose the 1σ consistence with the lattice simulations.

Baryon Scattering (BaSc) Collaboration, Phys.Rev.Lett. 132 (2024) 5, 051901; Phys.Rev.D 109 (2024) 1, 014511

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

How to describe the constituents of a resonance like $\Lambda(1405)$?

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Hermit $\hat{H}$ vs complex E

Gamow vector with complex scaling method

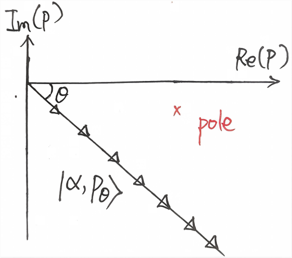
- introduce the two-particle basis with complex-scaling momentum $p \to p_\theta \equiv pe^{-i\theta}$

$$|\alpha, p_\theta\rangle = |\pi\Sigma, p_\theta\rangle, |\bar{K}N, p_\theta\rangle$$

- solve the Hamiltonian equation in the above basis

$$H\left|\psi^{\text{Gamow}}\right\rangle = (H_0 + V)\left|\psi^{\text{Gamow}}\right\rangle = E_{\text{pole}}\left|\psi^{\text{Gamow}}\right\rangle .$$

- then obtain resonance pole E_{pole} and corresponding Gamow wavefunction $\left|\psi^{\text{Gamow}}\right\rangle$.



The Gamow vector is spanned in the complex-momentum space which cannot be measured directly.

Alternatively Gamow wavefunction can be obtained by residues of T matrix

- obtain the T matrix by solving the Lippmann-Schwinger equation, equivalent to the resummation of the above diagrams.
- search for the poles of T matrix on the unphysical Riemann sheet.
- extract the residues $\gamma_\alpha(p_\theta)$ of T matrix

$$T_{\alpha,\beta}(p_\theta, p'_\theta; E) \approx \frac{\gamma_\alpha(p_\theta)\,\gamma_\beta(p'_\theta)}{E - E_{\text{pole}}} + \cdots\,,$$

- finally get the relation between the Gamow wavefunction and residues of T matrix

$$\psi_\alpha^{\text{Gamow}}(p_\theta) \equiv \langle \alpha, p_\theta | \psi^{\text{Gamow}} \rangle = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)}\,,$$

Advantage and Disadvantage of Gamow wavefunction

Disadvantage:

- Gamow wavefunctions diverge at infinite distance.
- Complex probability or negative probability.
- Strong tensions between
  - Gamow vector in Complex momentum space;
  - Experimental measurements in Real momentum space.

Advantage:

constrained by Hamiltonian eigen equation.

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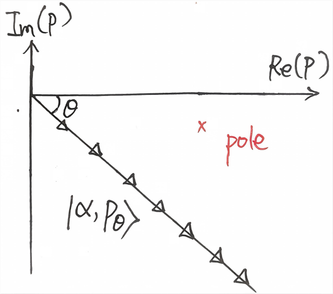
(5) consistent with experimental measurements.

A natural transition to the representation in the real world



$$\psi_\alpha^{\text{Gamow}}(p_\theta) = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)} \,,$$

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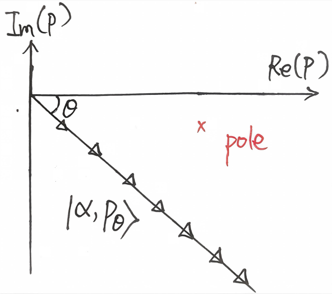

$$\psi_\alpha^{\text{Gamow}}(p_\theta) = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)},$$

By analytical continuation of the residue $\gamma_\alpha(p_\theta) \to \gamma_\alpha(p)$,

$$|\psi^{\text{phys}}\rangle = \sum_\alpha \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle .$$

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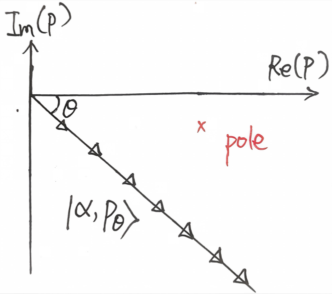

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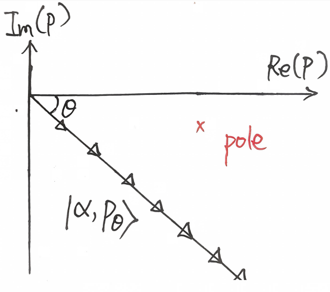
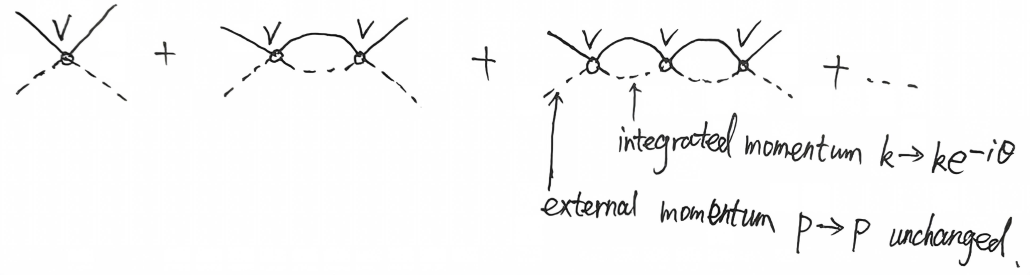
A natural transition to the representation in the real world



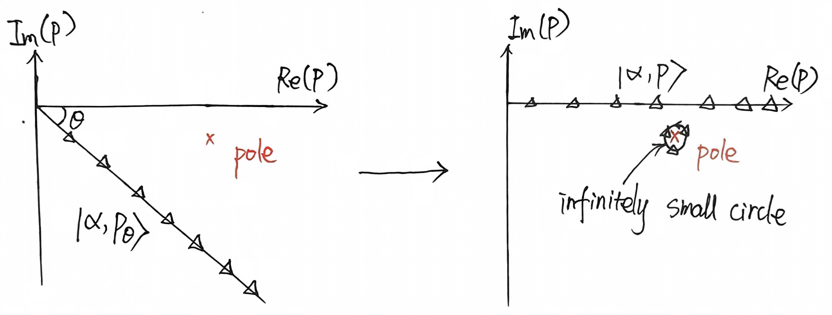
$$\psi_\alpha^{\text{Gamow}}(p_\theta) = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)},$$

By analytical continuation of the residue $\gamma_\alpha(p_\theta) \to \gamma_\alpha(p)$,

$$|\psi^{\text{phys}}\rangle = \sum_\alpha \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle .$$



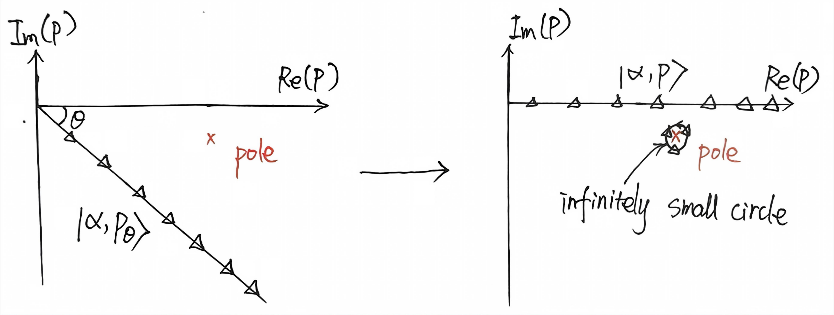
A New Relations between Gamow and Physical Representations



- By introducing a auxiliary vector state spanned around the basis states with the kinetic energy approaching the pole

$$|\psi^{\text{auxiliary}}\rangle = \sum_{\{\alpha \,|\, \theta_\alpha \neq 0\}} \oint_{\mathcal{C}_\epsilon^\alpha \to p_\alpha^{\text{on}}} dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle,$$

A New Relations between Gamow and Physical Representations

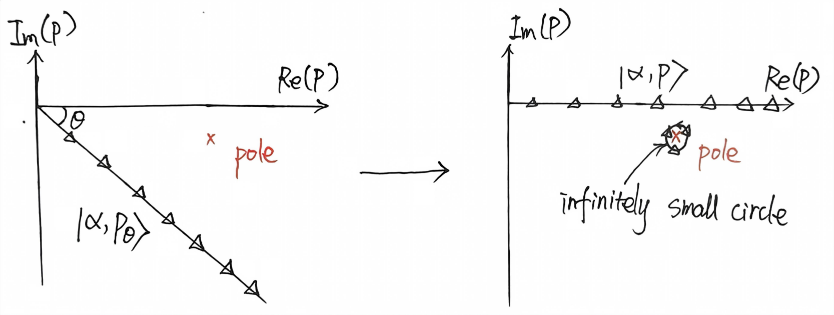


- we can prove

$$H\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle\right) = E_{\text{pole}}\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle\right)$$

  - $|\psi^{\text{phys}}\rangle$ in the ordinary Hilbert space
  - $|\psi^{\text{auxiliary}}\rangle$ in the special complex space closely around the pole

A New Relations between Gamow and Physical Representations



- With respect to the Hamiltonian equation,

$$|\psi^{\text{Gamow}}\rangle \cong |\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle\,,$$

which is a new essential relation between $|\psi^{\text{Gamow}}\rangle$ and $|\psi^{\text{phys}}\rangle$ in addition to their analytic connection.

Our framework for the dynamics of $\Lambda(1405)$

- We noticed
  - $\Lambda(1405)$ is mainly a $\bar{K}N$-$\pi\Sigma$ dynamically generated state.
  - The bare triquark core is important for $\Lambda(1670)$.

- In this work we use a simple separable interaction in the $\pi\Sigma$ and $\bar{K}N$ channels

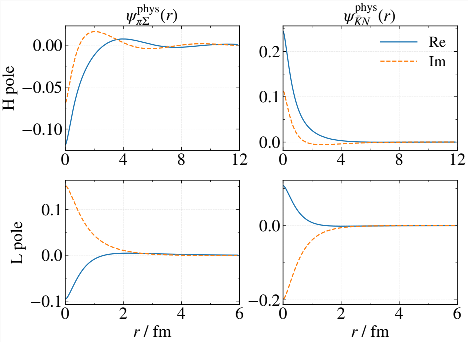
$$V_{\alpha\beta}(p,p') = \frac{3v_{\alpha\beta}}{4\pi^2 f_\pi^2}\, u_\alpha(p)\, u_\beta(p')\,, \quad u_\alpha(p) = \left(1+\frac{p^2}{\Lambda_\alpha^2}\right)^{-2}.$$

By adjusting the coupling parameters, we obtain

- the higher-mass (H) pole $E^{\mathrm{H}}_{\mathrm{pole}} = 1430 - 22\,i$ MeV,
- the lower-mass (L) pole $E^{\mathrm{L}}_{\mathrm{pole}} = 1338 - 89\,i$ MeV.

Coordinate Wavefunctions of $\Lambda(1405)$

The wavefunctions $\psi_\alpha^{\text{phys}}(r) \equiv \langle \alpha, r | \psi^{\text{phys}} \rangle$ in coordinate space can be obtained by Fourier transformation.



In $\Lambda(1405)$, $\pi\Sigma$ and $\bar{K}N$ are confined within finite ranges.

Temporal evolution for the two $\Lambda(1405)$ poles

With

$$|\psi^{\text{phys}}\rangle = \sum_\alpha \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle ,$$

we can calculate the evolution of the states

$$|\psi^{\text{phys}}, t\rangle = e^{-iHt} |\psi^{\text{phys}}\rangle .$$

In detail

- insert the completeness relation with $|\alpha, p\rangle$
- discretize the resulting momentum integrals using Gaussian quadrature
- thus reduce time evolution to a finite-dimensional matrix-vector operation for numerical solutions.

Resonances decay with the scattering stated emitted

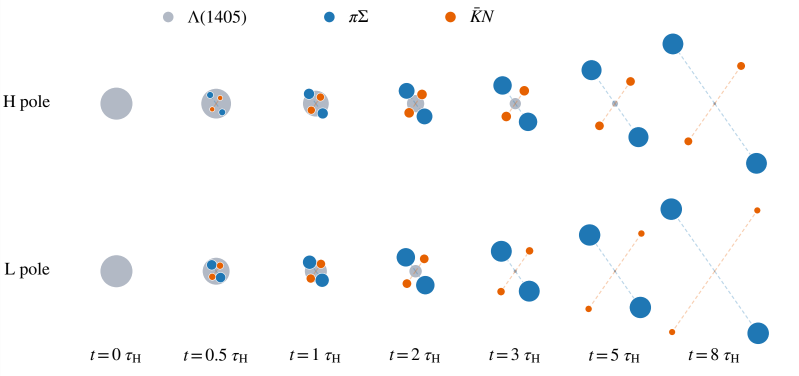


Illustration of the time evolution $|\psi^{\text{phys}}, t\rangle = e^{-iHt}\,|\psi^{\text{phys}}\rangle$ of the resonance components

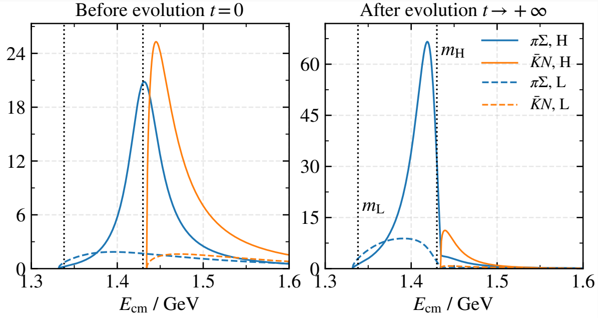
- The gray area is proportional to the survival probability
  and decreases with time in units of $\tau_H = 1\,/\,|\Gamma^{H}_{\text{pole}}|$
- The emitted $\pi\Sigma$ and $\bar{K}N$ move away from the interaction region.
  Their distances from the center are proportional to their root-mean-square radii.

The asymptotic state at the infinite time

The asymptotic state $|\psi^{\text{phys}}, t \to +\infty\rangle$ in the Schrödinger picture

$$|\psi^{\text{phys}}, t \to +\infty\rangle = \sum_\alpha \int dp\, p^2 \psi_\alpha^{\text{phys}}(p) \times \left(e^{-i\omega_\alpha(p)t} |\alpha, p\rangle + \sum_\beta \int dq\, q^2 \frac{T_{\beta,\alpha}(q,p;E)}{\omega_\beta(q) - \omega_\alpha(p) + i\epsilon} e^{-i\omega_\beta(q)t} |\beta, q\rangle \right) .$$

The resulting distribution $|f_\alpha(E)|^2 = |\langle \alpha, E \,|\, \psi^{\text{phys}}(t = +\infty)\rangle|^2$

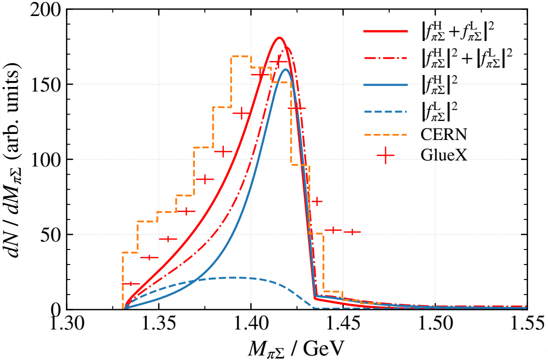


For the high-mass pole

- the $\bar{K}N$ component is significant when initially formed
- most of it turns into the $\pi\Sigma$ rather than $\bar{K}N$ scattering states because of
  - energy conservation when $t \to \infty$
  - the larger mass of $\bar{K}N$ than $m^{\rm H}_{\rm pole}$

Y. Zhuge and Z. W. Liu, arXiv:2606.07411.

Comparison with the experimental measurements



Y. Zhuge and Z. W. Liu, arXiv:2606.07411.

We give the full results with both

- the strongest interference scenario $|f^{\rm H}_{\pi\Sigma}(E)+f^{\rm L}_{\pi\Sigma}(E)|^2$
- no interference assumption $|f^{\rm H}_{\pi\Sigma}(E)|^2+|f^{\rm L}_{\pi\Sigma}(E)|^2$

The currently observed $\pi\Sigma$ spectrum prefers the strongest interference scenario.

Summary

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- We have studied the mass spectrum of $\Lambda_c(2940)$, $\Lambda_c(2910)$, $\Lambda(1405)$, $\Lambda(1670)$, ...

- Both bare triquark states in quark model and meson-baryon channels are important.

- We have developed a physical description $|\psi^{\text{phys}}_{\text{H/L}}\rangle$ of the two-pole $\Lambda(1405)$ in the real momentum basis.

Thank you for your attention!