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$D_{s1}(2460)$ and other open-charm 1^+ states in relativistic chiral effective field theory

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Based on **ZRL**, X.-Q. Xiao, Z.-H. Guo and D.-L. Yao[#], arXiv: 2607.02452 [hep-ph]

第十一届手征有效场论研讨会

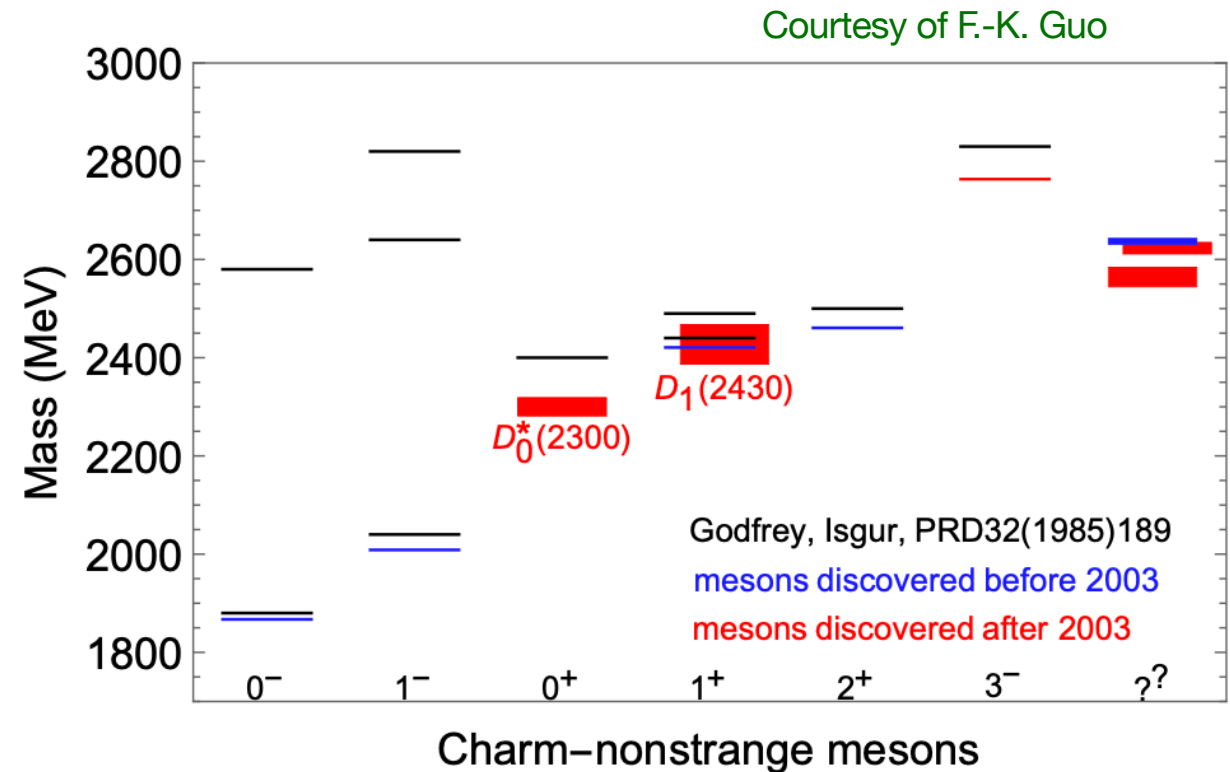
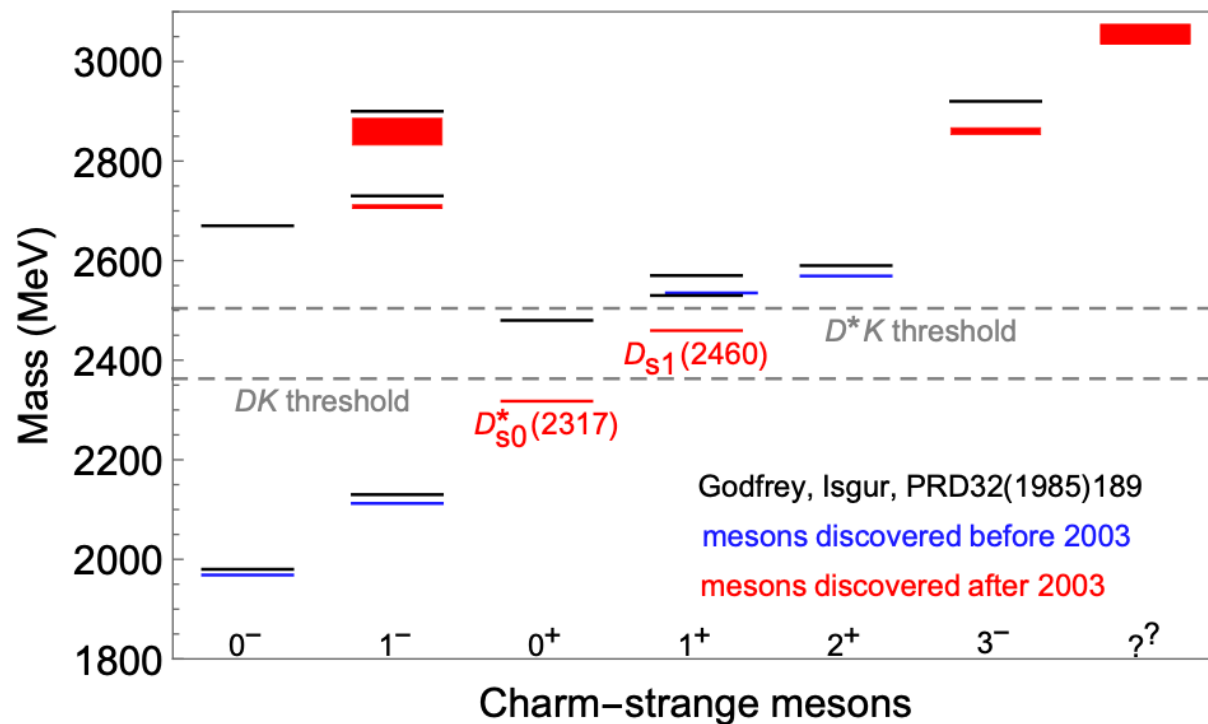
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- Introduction
- Chiral potentials and unitarization
- Scattering length and S-matrix parameters
- Poles and their trajectories
- Summary and outlook

Introduction

Puzzles of charmed mesons

- Spectrum of charm-(non)strange mesons



Courtesy of F.-K. Guo

- ➔ **Puzzle 1:** why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- ➔ **Puzzle 2:** why $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^{*\pm}} - M_{D^\pm}$?
- ➔ **Puzzle 3:** why $M_{D_0(2300)} \gtrsim M_{D_{s0}^*(2317)}$?

A way out of the puzzles lies in the use of ChEFT!

Status of ChEFT study of charmed mesons

- **$P\phi$ scatterings & 0^+ states:** $\phi \in \{\pi, K, \bar{K}, \eta\}$

⇒ Up to NLO: [Kolomeitsev & Lutz, PLB 2004], [Guo, et al., PLB 2006, EPJA 2009], [Gamermann, et al., PRD 2007]

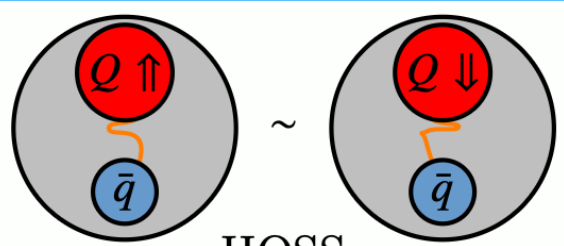
[Liu, et al. PRD 2012], [Altenbuchinger, et al. PRD 2014], [Guo, et al., PRD 2018],

⇒ NNLO: [Liu, et al., PRD 2009], [Huang, et al., PRD 2022] (Heavy Baryon formalism)

[Geng, et al., PRD 2010], [Yao, et al., JHEP 2015], [Du, et al., EPJC 2017] (EOMS scheme)

- **$P^*\phi$ scatterings & 1^+ states:** via heavy quark spin symmetry (HQSS)

$P\phi$ scattering



~

$P^*\phi$ scattering
 $\epsilon^\dagger \cdot \epsilon = -1$

↔

$J_l = \frac{1}{2}$	Exp. mass
$\begin{pmatrix} D^+(0^-) \\ D^{+*}(1^-) \end{pmatrix}$	$\begin{pmatrix} 1869.59 \\ 2010.26 \end{pmatrix}$
$\begin{pmatrix} D_s^+(0^-) \\ D_s^{+*}(1^-) \end{pmatrix}$	$\begin{pmatrix} 1968.28 \\ 2112.10 \end{pmatrix}$
$\begin{pmatrix} D_{s0}^*(0^+) \\ D_{s1}^*(1^+) \end{pmatrix}$	$\begin{pmatrix} 2317.70 \\ 2459.50 \end{pmatrix}$

- **Our work:** [ZRL, X.-Q. Xiao, Z.-H. Guo and D.-L. Yao[#], arXiv: 2607.02452 [hep-ph]]

⇒ Relativistic kinematics: keeping full form of polarization vectors

⇒ Both contact and meson-exchange contributions

⇒ U(3) framework: inclusion of η'

Chiral potentials and unitarization

Chiral Effective Lagrangians

- The Lagrangians up to NLO

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{P^*\phi}^{(1)} + \mathcal{L}_{P\phi}^{(1)} + \mathcal{L}_{P^*P\phi}^{(1)} + \mathcal{L}_{P^*P^*\phi}^{(1)} + \mathcal{L}_{P^*\phi}^{(2)}$$

► LO contact interactions

$$\mathcal{L}_{P^*\phi}^{(1)} = -D_\mu P_\alpha^* D^\mu P^{*\alpha\dagger} + \bar{M}_{P^*}^2 P_\alpha^* P^{*\alpha\dagger} ,$$

$$\mathcal{L}_{P\phi}^{(1)} = D_\mu P D^\mu P^\dagger - \bar{M}_P^2 P P^\dagger$$

✓ Charmed vector mesons

$$P^* = (D^{*0}, D^{*+}, D_s^{*+})^T = (c\bar{u}, c\bar{d}, c\bar{s})^T$$

✓ Pseudo-scalar charmed mesons

$$P = (D^0, D^+, D_s^+)^T = (c\bar{u}, c\bar{d}, c\bar{s})^T$$

✓ Pseudo-scalar Goldstone bosons

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} + \frac{1}{\sqrt{3}}\eta_0$$

Chiral Effective Lagrangians

- The Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{P^*\phi}^{(1)} + \mathcal{L}_{P\phi}^{(1)} + \mathcal{L}_{P^*P\phi}^{(1)} + \mathcal{L}_{P^*P^*\phi}^{(1)} + \mathcal{L}_{P^*\phi}^{(2)}$$

- ▶ LO meson-exchange pieces

$$\mathcal{L}_{P^*P\phi}^{(1)} = ig_0 \left(P_\mu^* u^\mu P^\dagger - P u^\mu P_\mu^{*\dagger} \right), \quad \mathcal{L}_{P^*P^*\phi}^{(1)} = \frac{g_1}{2} \left(P_\alpha^* u_\mu D_\nu P_\beta^{*\dagger} - D_\nu P_\alpha^* u_\mu P_\beta^{*\dagger} \right) \epsilon^{\alpha\beta\mu\nu},$$

- ▶ NLO contact interactions [Eur. Phys. J. A 40, 171 (2009)]

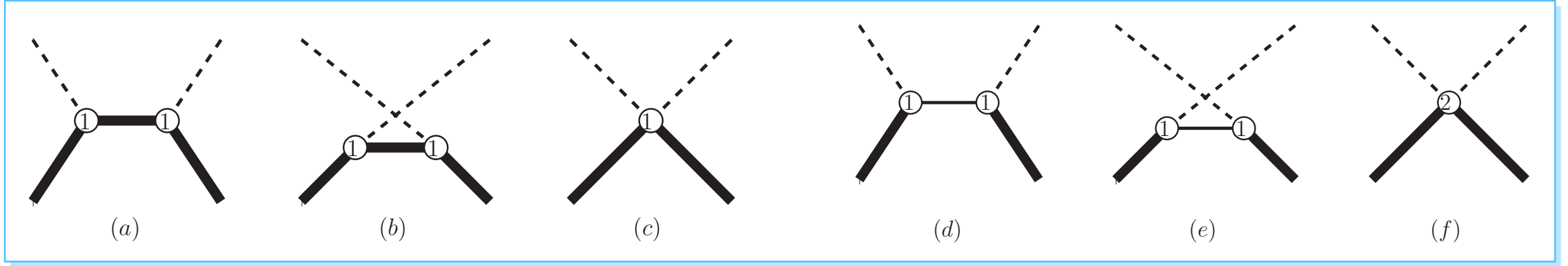
$$\begin{aligned} \mathcal{L}_{P^*\phi}^{(2)} = & -P_\alpha^* \left(-h_0^* \langle \chi_+ \rangle - h_1^* \chi_+ \right) P^{*\alpha\dagger} - P_\alpha^* \left(h_2^* \langle u_\mu u^\mu \rangle - h_3^* u_\mu u^\mu \right) P^{*\alpha\dagger} \\ & - D_\mu P_\alpha^* \left(h_4^* \langle u_\mu u^\nu \rangle - h_5^* \{ u^\mu, u^\nu \} \right) D_\nu P^{*\alpha\dagger}, \end{aligned}$$

✓ $h_{1,2,3,4,5}^*$ and $g_{0,1}$ are LECs.

✓ Heavy quark spin symmetry is applied.

NLO potential

- Feynman diagrams**



- Chiral potentials:** $V(s, t) = V_{\text{ct}}(s, t) + V_{\text{ex}}(s, t)$

contact (c+f):
$$V_{\text{ct}}(s, t) = -\epsilon_1 \cdot \epsilon_3^* \left\{ \mathcal{C}_{\text{LO}} \frac{s-u}{4F_0^2} - \frac{1}{F_0^2} \left[4h_0^* \mathcal{C}_0 - 2h_1^* \mathcal{C}_1 + 2\mathcal{C}_{24} \mathcal{H}_{24} - 2\mathcal{C}_{35} \mathcal{H}_{35} \right] \right\}$$

exchange (a+b+d+e):
$$V_{\text{ex}}(s, t) = \frac{\mathcal{C}_S}{3F_0^2} \left[g_0^2 \mathcal{G}_P(s, t) - g_1^2 \mathcal{G}_{P^*}(s, t) \right] + \frac{\mathcal{C}_U}{3F_0^2} \left[g_0^2 \mathcal{G}_P(u, t) - g_1^2 \mathcal{G}_{P^*}(u, t) \right]$$

NLO potential

- The coefficients:**

- Classified by strangeness and isospin (S,I): 7 channels in total

- $\eta - \eta'$ mixing:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

- LO masses and mixing angle:

$$M_\pi^2 = 2B\hat{m} \ , \quad M_K^2 = B(\hat{m} + m_s)$$

$$M_{\eta'/\eta}^2 = M_K^2 + \frac{M_0^2}{2} \pm \frac{R}{2}$$

$$\sin \theta = - \left(\sqrt{1 + \frac{S^2}{32\Delta^4}} \right)^{-1}$$

(S, I)	Channels	$\mathcal{C}_{\text{LO}}$	$\mathcal{C}_S$	$\mathcal{C}_U$	$\mathcal{C}_0$	$\mathcal{C}_1$	$\mathcal{C}_{24}$	$\mathcal{C}_{35}$
$(-1, 0)$	$D^* \bar{K} \rightarrow D^* \bar{K}$	-1	0	-6	M_K^2	M_K^2	1	-1
$(-1, 1)$	$D^* \bar{K} \rightarrow D^* \bar{K}$	1	0	6	M_K^2	$-M_K^2$	1	1
$(2, \frac{1}{2})$	$D_s^* K \rightarrow D_s^* K$	1	0	6	M_K^2	$-M_K^2$	1	1
$(0, \frac{3}{2})$	$D^* \pi \rightarrow D^* \pi$	1	0	6	M_π^2	$-M_\pi^2$	1	1
$(1, 1)$	$D_s^* \pi \rightarrow D_s^* \pi$	0	0	0	M_π^2	0	1	0
	$D^* K \rightarrow D^* K$	0	0	0	M_K^2	0	1	0
	$D^* K \rightarrow D_s^* \pi$	1	0	6	0	$-\frac{M_K^2 + M_\pi^2}{2}$	0	1
$(1, 0)$	$D^* K \rightarrow D^* K$	-2	12	0	M_K^2	$-2M_K^2$	1	2
	$D^* K \rightarrow D_s^* \eta$	$-\sqrt{3}c_\theta$	$2\sqrt{6}(\sqrt{2}c_\theta + s_\theta)$	$-2\sqrt{3}(c_\theta - \sqrt{2}s_\theta)$	0	$\mathcal{C}_1^{(1,0) K\eta}$	0	$\frac{c_\theta + 2\sqrt{2}s_\theta}{\sqrt{3}}$
	$D_s^* \eta \rightarrow D_s^* \eta$	0	$2(\sqrt{2}c_\theta + s_\theta)^2$	$2(\sqrt{2}c_\theta + s_\theta)^2$	$\mathcal{C}_0^{(1,0) \eta\eta}$	$\mathcal{C}_1^{(1,0) \eta\eta}$	1	$\frac{2(\sqrt{2}c_\theta + s_\theta)^2}{3}$
	$D^* K \rightarrow D_s^* \eta'$	$-\sqrt{3}s_\theta$	$-2\sqrt{6}(c_\theta - \sqrt{2}s_\theta)$	$-2\sqrt{3}(\sqrt{2}c_\theta + s_\theta)$	0	$\mathcal{C}_1^{(1,0) K\eta'}$	0	$\mathcal{C}_{35}^{(1,0) K\eta'}$
	$D_s^* \eta \rightarrow D_s^* \eta'$	0	$\mathcal{C}_S^{(1,0) \eta\eta'}$	$\mathcal{C}_U^{(1,0) \eta\eta'}$	$\mathcal{C}_0^{(1,0) \eta\eta'}$	$\mathcal{C}_1^{(1,0) \eta\eta'}$	0	$\mathcal{C}_{35}^{(1,0) \eta\eta'}$
	$D_s^* \eta' \rightarrow D_s^* \eta'$	0	$2(c_\theta - \sqrt{2}s_\theta)^2$	$2(c_\theta - \sqrt{2}s_\theta)^2$	$\mathcal{C}_0^{(1,0) \eta'\eta'}$	$\mathcal{C}_1^{(1,0) \eta'\eta'}$	1	$\mathcal{C}_{35}^{(1,0) \eta'\eta'}$
$(0, \frac{1}{2})$	$D^* \pi \rightarrow D^* \pi$	-2	9	-3	M_π^2	$-M_\pi^2$	1	1
	$D^* \eta \rightarrow D^* \eta$	0	$(c_\theta - \sqrt{2}s_\theta)^2$	$(c_\theta - \sqrt{2}s_\theta)^2$	$\mathcal{C}_0^{(0, \frac{1}{2}) \eta\eta}$	$\mathcal{C}_1^{(0, \frac{1}{2}) \eta\eta}$	1	$\mathcal{C}_{35}^{(0, \frac{1}{2}) \eta\eta}$
	$D_s^* \bar{K} \rightarrow D_s^* \bar{K}$	-1	6	0	M_K^2	$-M_K^2$	1	1
	$D^* \eta \rightarrow D^* \pi$	0	$3(c_\theta - \sqrt{2}s_\theta)$	$3(c_\theta - \sqrt{2}s_\theta)$	0	$M_\pi^2(\sqrt{2}s_\theta - c_\theta)$	0	$c_\theta - \sqrt{2}s_\theta$
	$D_s^* \bar{K} \rightarrow D^* \pi$	$-\frac{\sqrt{6}}{2}$	$3\sqrt{6}$	0	0	$-\frac{\sqrt{6}(M_K^2 + M_\pi^2)}{4}$	0	$\frac{\sqrt{6}}{2}$
	$D_s^* \bar{K} \rightarrow D^* \eta$	$-\frac{\sqrt{6}}{2}c_\theta$	$\sqrt{6}(c_\theta - \sqrt{2}s_\theta)$	$-2\sqrt{3}(\sqrt{2}c_\theta + s_\theta)$	0	$\mathcal{C}_1^{(0, \frac{1}{2}) \bar{K}\eta}$	0	$\mathcal{C}_{35}^{(0, \frac{1}{2}) \bar{K}\eta}$
	$D^* \eta' \rightarrow D^* \pi$	0	$3(\sqrt{2}c_\theta + s_\theta)$	$3(\sqrt{2}c_\theta + s_\theta)$	0	$-M_\pi^2(\sqrt{2}c_\theta + s_\theta)$	0	$s_\theta + \sqrt{2}c_\theta$
	$D^* \eta \rightarrow D^* \eta'$	0	$\mathcal{C}_{(c_\theta, s_\theta)}^{(0, \frac{1}{2})}$	$\mathcal{C}_{(c_\theta, s_\theta)}^{(0, \frac{1}{2})}$	$\mathcal{C}_0^{(0, \frac{1}{2}) \eta\eta'}$	$\mathcal{C}_1^{(0, \frac{1}{2}) \eta\eta'}$	0	$\mathcal{C}_{35}^{(0, \frac{1}{2}) \eta\eta'}$
	$D_s^* \bar{K} \rightarrow D^* \eta'$	$-\frac{\sqrt{6}}{2}s_\theta$	$\sqrt{6}(\sqrt{2}c_\theta + s_\theta)$	$2\sqrt{3}(c_\theta - \sqrt{2}s_\theta)$	0	$\mathcal{C}_1^{(0, \frac{1}{2}) \bar{K}\eta'}$	0	$\mathcal{C}_{35}^{(0, \frac{1}{2}) \bar{K}\eta'}$
	$D^* \eta' \rightarrow D^* \eta'$	0	$(\sqrt{2}c_\theta + s_\theta)^2$	$(\sqrt{2}c_\theta + s_\theta)^2$	$\mathcal{C}_0^{(0, \frac{1}{2}) \eta'\eta'}$	$\mathcal{C}_1^{(0, \frac{1}{2}) \eta'\eta'}$	1	$\mathcal{C}_{35}^{(0, \frac{1}{2}) \eta'\eta'}$

Partial-wave decomposition

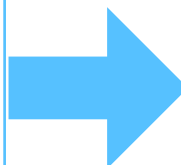
- Helicity amplitude $V_{\lambda_3\lambda_1} \iff$ Covariant amplitude $V_i(s, t)$

$$V_{\lambda_3\lambda_1}(p_2, p_4; \Sigma) = \epsilon_{\lambda_3}^{\mu\dagger}(p_3) V_{\mu\nu}(p_2, p_4; \Sigma) \epsilon_{\lambda_1}^{\nu}(p_1)$$

$$V^{\mu\nu} = V_1 (g^{\mu\nu} - \hat{\Sigma}^{\mu}\hat{\Sigma}^{\nu}) + V_2 \hat{\Sigma}^{\mu}\hat{\Sigma}^{\nu} + V_3 \hat{\Sigma}^{\mu}p_{4T}^{\nu} + V_4 p_{2T}^{\mu}\hat{\Sigma}^{\nu} + V_5 p_{2T}^{\mu}p_{4T}^{\nu}$$

- P and T parity transformation: 5 out of 9 are independent:

$$\begin{aligned} \mathbb{P} : \quad & V_{\lambda_3\lambda_1} = (-1)^{\lambda_3-\lambda_1} V_{-\lambda_3-\lambda_1} \\ \mathbb{T} : \quad & V_{\lambda_3\lambda_1} = (-1)^{\lambda_3-\lambda_1} V_{\lambda_1\lambda_3} \end{aligned}$$



$$\begin{aligned} V_{++} &= V_{--} = -\frac{1}{2} [(z_s + 1)V_1 + p_{\text{cm}}\bar{p}_{\text{cm}}(1 - z_s^2)V_5] \quad , \quad z_s = \cos \theta \\ V_{+-} &= V_{-+} = \frac{1}{2} [(z_s - 1)V_1 + p_{\text{cm}}\bar{p}_{\text{cm}}(1 - z_s^2)V_5] \quad , \\ V_{+0} &= -V_{-0} = -\frac{(1 - z_s^2)^{\frac{1}{2}}}{\sqrt{2}m_1} [\omega_1 V_1 - p_{\text{cm}}^2 V_4 - z_s \omega_1 p_{\text{cm}}\bar{p}_{\text{cm}} V_5] \quad , \\ V_{0+} &= -V_{0-} = \frac{(1 - z_s^2)^{\frac{1}{2}}}{\sqrt{2}m_3} [\omega_3 V_1 - \bar{p}_{\text{cm}}^2 V_3 - z_s \omega_3 p_{\text{cm}}\bar{p}_{\text{cm}} V_5] \quad , \\ V_{00} &= \frac{1}{m_1 m_3} [z_s p_{\text{cm}}^2 \omega_3 V_4 + z_s \omega_1 (\bar{p}_{\text{cm}}^2 V_3 - \omega_3 V_1) + p_{\text{cm}}\bar{p}_{\text{cm}}(V_2 + z_s^2 \omega_1 \omega_3 V_5)] \quad , \end{aligned}$$

Partial-wave decomposition

- Partial-wave amplitudes:

$$V_{\lambda_3\lambda_1}(s, t) = \sum_{J=\max\{|\lambda_3|, |\lambda_1|\}}^{\infty} (2J+1) V_{\lambda_3\lambda_1}^J(s) d_{\lambda_1\lambda_3}^J(z_s) \quad V_{\lambda_3\lambda_1}^J(s) = \frac{1}{2} \int_{-1}^1 dz_s V_{\lambda_3\lambda_1}(s, t(s, z_s)) d_{\lambda_1\lambda_3}^J(z_s)$$

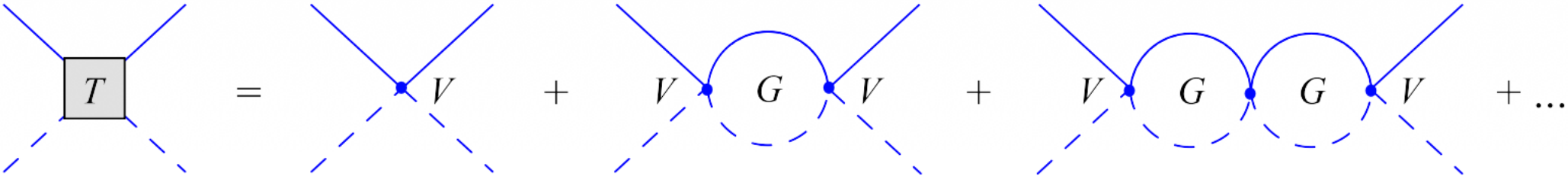
- Helicity basis to **JLS basis** (for definite parity)

$$V_{JLS} = \sum_{\lambda_1\lambda_3} \mathcal{U}_{\lambda_3 0}^{JLS} V_{\lambda_3\lambda_1}^J [\mathcal{U}_{\lambda_1 0}^{JLS}]^\dagger, \quad \mathcal{U}_{\lambda_i 0}^{JLS} = \left(\frac{2L+1}{2S+1} \right)^{\frac{1}{2}} \langle L 0 S \lambda | J \lambda \rangle \langle S_i \lambda_i 0 0 | S \lambda \rangle,$$

► S wave:	$V_{J=1, L=0}(s) = \frac{1}{3} [2V_{++}^1(s) + 2V_{+0}^1(s) + 2V_{+-}^1(s) + 2V_{0+}^1(s) + V_{00}^1(s)]$	1 ⁺
► P wave:	$V_{J=0, L=1}(s) = V_{00}^0(s),$	0 ⁻
	$V_{J=1, L=1}(s) = V_{++}^1(s) - V_{+-}^1(s),$	1 ⁻
	$V_{J=2, L=1}(s) = \frac{1}{5} [2V_{00}^2(s) + 3(V_{++}^2(s) + V_{+-}^2(s)) + 2\sqrt{3}(V_{0+}^2(s) + V_{+0}^2(s))]$	2 ⁻

The on-shell unitarization approach

- **Bethe-Salpeter equation (BSE) with on-shell approximation**



$$T_{JL}^{(S,I)}(s) = V_{JL}^{(S,I)}(s) \cdot \left[1 - G(s) \cdot V_{JL}^{(S,I)}(s) \right]^{-1} \quad V_{JL}^{(S,I)}(s) = \begin{pmatrix} [V_{JL}^{(S,I)}]_{1 \rightarrow 1} & \cdots [V_{JL}^{(S,I)}]_{1 \rightarrow j} & \cdots \\ \vdots & \ddots & \vdots & \ddots \\ [V_{JL}^{(S,I)}]_{i \rightarrow 1} & \cdots [V_{JL}^{(S,I)}]_{i \rightarrow j} & \cdots \\ \vdots & \ddots & \vdots & \ddots \end{pmatrix}$$

► **Two-point loop functions:** $G = \{ \cdots, g_i(s), \cdots \}$, $\sigma_i(s) \equiv \lambda^{1/2}(s, M_{P_i^*}^2, M_{\phi_i}^2)$

$$g_i(s) = \frac{1}{16\pi^2} \left\{ a_i(\mu) + \ln \frac{M_{P_i^*}^2}{\mu^2} + \frac{s - M_{P_i^*}^2 + M_{\phi_i}^2}{2s} \ln \frac{M_{\phi_i}^2}{M_{P_i^*}^2} + \frac{\sigma_i(s)}{2s} \left[\ln(s - M_{\phi_i}^2 + M_{P_i^*}^2 + \sigma_i(s)) \right. \right. \\ \left. \left. - \ln(-s + M_{\phi_i}^2 - M_{P_i^*}^2 + \sigma_i(s)) + \ln(s + M_{\phi_i}^2 - M_{P_i^*}^2 + \sigma_i(s)) - \ln(-s - M_{\phi_i}^2 + M_{P_i^*}^2 + \sigma_i(s)) \right] \right\}$$

Scattering length and S-matrix parameters

Parameter setup

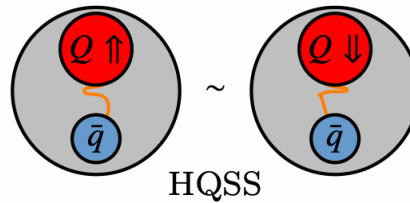
- Values of LECs in contact term:

- by mass difference:

$$h_1^* = \frac{M_{D_s^*}^2 - M_{D^*}^2}{4(M_K^2 - M_\pi^2)} = 0.47$$

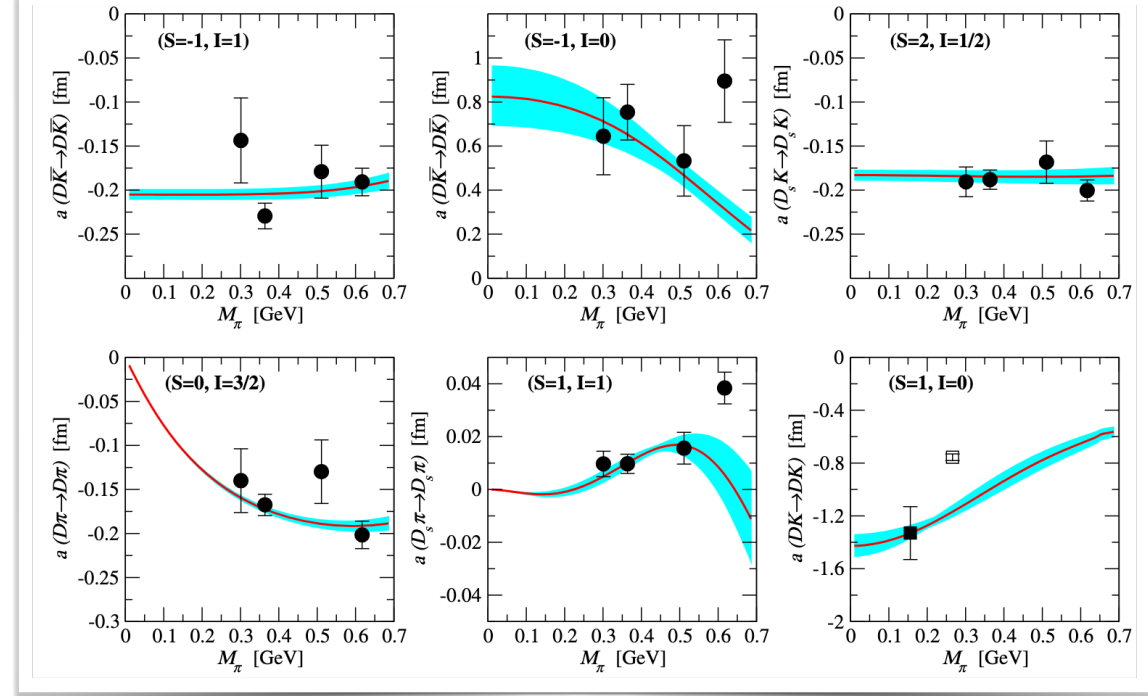
- via HQSS:

$$h_i^* = h_i, \quad i = 0, 2, 3, 4, 5$$



LECs	h_0^*	h_1^*	h_2^*	h_3^*	h_4^*	h_5^*
Value	0.033	0.47	$0.08^{+0.31}_{-0.34}$	$3.79^{+0.41}_{-0.41}$	$-0.06^{+0.08}_{-0.07}$	$-0.48^{+0.05}_{-0.05}$

[Z.-H. Guo, Ulf-G. Meissner and D.-L. Yao#, PRD 2015]



- Values of LO coupling in meson-exchange contributions

- by decay width:

$$\Gamma(D^{*+}(2010) \rightarrow D^0 \pi^+) = \frac{g_0^2}{12\pi F_\pi^2} \frac{|\vec{q}_\pi|^3}{M_{D^{*+}}^2} \longrightarrow g_0 = 1095.0 \pm 15.8 \text{ MeV}$$

- via HQSS: $g_1 M_D^* = g_0$

Scattering length

► Prediction of S-wave scattering lengths

$$a_{JL}^{(S,I)}(P_i^* \phi_i \rightarrow P_i^* \phi_i) = - \frac{1}{8\pi \sqrt{s_{\text{th}}^i}} \lim_{s \rightarrow s_{\text{th}}^i} \left[\frac{\mathcal{T}_{JL, P_i^* \phi_i \rightarrow P_i^* \phi_i}^{(S,I)}(s)}{p_{i,\text{cm}}^{2L}} \right]$$

(S, I)	Process	CT	CT + Ex	HMChPT [75]
$(-1, 0)$	$D^* \bar{K} \rightarrow D^* \bar{K}$	0.41(2)	0.41(2)	$0.29 + i 5.2 \times 10^{-6}$
$(-1, 1)$	$D^* \bar{K} \rightarrow D^* \bar{K}$	-0.18(1)	-0.18(1)	$-0.19 - i 1.7 \times 10^{-6}$
$(2, \frac{1}{2})$	$D_s^* K \rightarrow D_s^* K$	-0.17(1)	-0.17(1)	-0.14
$(0, \frac{3}{2})$	$D^* \pi \rightarrow D^* \pi$	-0.10(1)	-0.10(1)	$-0.13 - i 3.6 \times 10^{-4}$
$(1, 1)$	$D_s^* \pi \rightarrow D_s^* \pi$	-0.001(1)	-0.004(1)	-0.039
	$D^* K \rightarrow D^* K$	$0.01(1) + i 0.07(1)$	$0.01(1) + i 0.05(1)$	$-0.022 + i 0.10$
$(1, 0)$	$D^* K \rightarrow D^* K$	$-0.86^{+0.09}_{-0.14}$	$-0.85^{+0.09}_{-0.14}$	$0.76 - i 5.2 \times 10^{-6}$
	$D_s^* \eta \rightarrow D_s^* \eta$	$-0.22(1) + i 0.04(1)$	$-0.22(1) + i 0.04(1)$	$0.18 + i 0.19$
	$D_s^* \eta' \rightarrow D_s^* \eta'$	$-0.24(2) + i 0.03(1)$	$-0.26(2) + i 0.04(1)$	-
$(0, \frac{1}{2})$	$D^* \pi \rightarrow D^* \pi$	0.35(1)	$0.36^{+0.01}_{-0.02}$	$0.27 - i 3.6 \times 10^{-4}$
	$D^* \eta \rightarrow D^* \eta$	$-0.03(2) + i 0.04(1)$	$-0.05(2) + i 0.04(1)$	$0.051 + i 0.094$
	$D_s^* \bar{K} \rightarrow D_s^* \bar{K}$	$-0.12(3) + i 0.18^{+0.04}_{-0.03}$	$-0.14^{+0.03}_{-0.02} + i 0.16^{+0.04}_{-0.03}$	$0.35 + i 0.27$
	$D^* \eta' \rightarrow D^* \eta'$	$-0.16(2) + i 0.01(1)$	$-0.16(2) + i 0.01(1)$	-

The meson-exchange diagrams contribute negligibly in all channels!

Scattering length

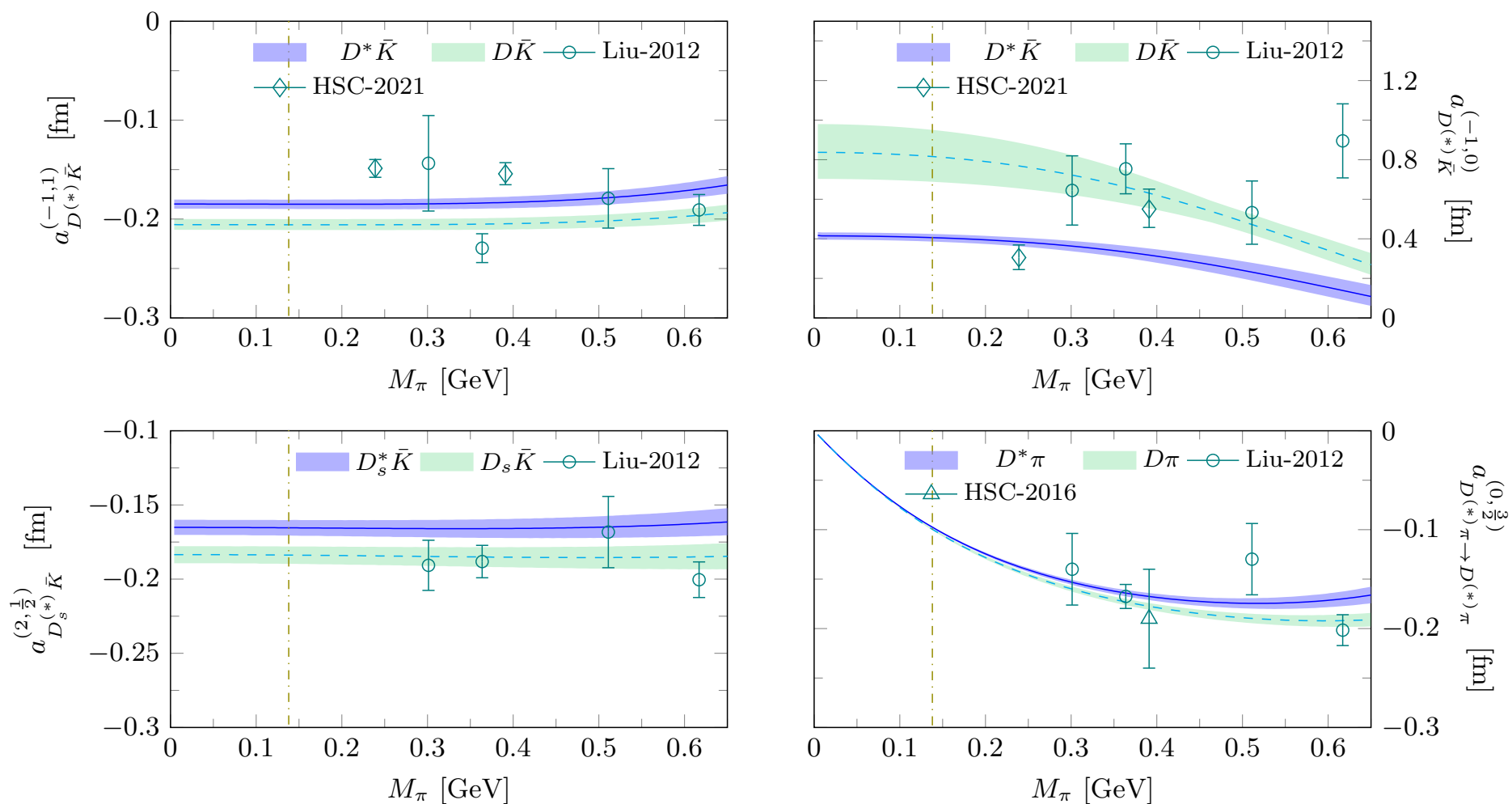
► Prediction of P-wave scattering length

(S, I) Process	$0^- [^3P_0]$		$1^- [^3P_1]$		$2^- [^3P_2]$	
	CT	CT + Ex	CT	CT+Ex	CT	CT+Ex
$(-1, 0) D^* \bar{K} \rightarrow D^* \bar{K}$	$-4.9^{+1.1}_{-1.0}$	$-4.7^{+1.1}_{-1.0}$	$-4.6^{+1.1}_{-1.0}$	$-4.4^{+1.1}_{-1.0}$	$-4.6^{+1.1}_{-1.0}$	$-5.8^{+1.1}_{-1.0}$
$(-1, 1) D^* \bar{K} \rightarrow D^* \bar{K}$	$4.4^{+0.8}_{-0.7}$	$4.2^{+0.8}_{-0.7}$	$4.1(7)$	$3.9(7)$	$4.1(7)$	$5.3(7)$
$(2, 1/2) D_s^* K \rightarrow D_s^* K$	$4.2(7)$	$4.0(7)$	$3.9(7)$	$3.6(7)$	$3.9(7)$	$6.1(7)$
$(0, 3/2) D^* \pi \rightarrow D^* \pi$	$4.2^{+0.8}_{-0.7}$	$-101.8^{+0.8}_{-0.7}$	$4.0(7)$	$106.1(7)$	$4.0(7)$	$-93.0(7)$
$(1, 1) D_s^* \pi \rightarrow D_s^* \pi$	$-0.4^{+0.8}_{-0.7}$	$-0.5(7)$	$-0.4^{+0.8}_{-0.7}$	$-32.9^{+0.9}_{-0.8}$	$-0.4^{+0.8}_{-0.7}$	$-6.4^{+0.7}_{-0.6}$
$D^* K \rightarrow D^* K$	$0.6(7) + i 1.0(2)$	$0.5(7) + i 0.9(2)$	$0.5(7) + i 0.9(2)$	$0.4(7) + i 0.7(2)$	$0.5(7) + i 0.9(2)$	$1.4^{+0.7}_{-0.6} + i 2.0(3)$
$(1, 0) D^* K \rightarrow D^* K$	$13.3(1.6)$	$5.4^{+1.3}_{-1.2}$	$14.1^{+1.6}_{-1.5}$	$10.7(1.9)$	$13.9^{+1.6}_{-1.5}$	$13.2(1.4)$
$D_s^* \eta \rightarrow D_s^* \eta$	$3.5^{+0.7}_{-0.6} + i 0.2(1)$	$2.6^{+0.7}_{-0.6} + i 0.04(0)$	$3.4^{+0.7}_{-0.6} + i 0.4(1)$	$2.8^{+0.7}_{-0.6} + i 0.02(0)$	$3.4^{+0.7}_{-0.6} + i 0.4(1)$	$4.3^{+0.7}_{-0.6} + i 0.2(1)$
$D_s^* \eta' \rightarrow D_s^* \eta'$	$-5.5^{+1.7}_{-1.5} + i 7.9^{+2.4}_{-2.1}$	$-2.9^{+2.6}_{-2.2} + i 12.6^{+2.8}_{-3.2}$	$-4.7^{+1.7}_{-1.6} + i 5.3^{+1.7}_{-1.5}$	$-4.8^{+1.7}_{-1.5} + i 7.1^{+2.3}_{-2.0}$	$-4.9^{+1.8}_{-1.6} + i 5.2^{+1.8}_{-1.5}$	$-6.4^{+2.0}_{-1.9} + i 6.6^{+2.3}_{-1.9}$
$(0, 1/2) D^* \pi \rightarrow D^* \pi$	$7.9(1.0)$	$57.9^{+1.6}_{-1.4}$	$8.4^{+1.1}_{-1.0}$	$-16.1^{+4.5}_{-3.9}$	$8.2(1.0)$	$55.4^{+0.9}_{-0.8}$
$D^* \eta \rightarrow D^* \eta$	$0.6^{+0.8}_{-1.1} + i 8.2^{+1.7}_{-1.6}$	$3.4^{+0.1}_{-0.3} + i 5.0^{+1.7}_{-1.4}$	$-0.2^{+0.9}_{-1.1} + i 7.5^{+1.6}_{-1.5}$	$2.4^{+0.2}_{-0.5} + i 5.9^{+1.7}_{-1.5}$	$-0.4^{+0.9}_{-1.2} + i 7.6^{+1.6}_{-1.5}$	$-0.6^{+1.2}_{-1.5} + i 10.8(1.9)$
$D_s^* \bar{K} \rightarrow D_s^* \bar{K}$	$1.5(7) + i 5.1^{+1.0}_{-0.9}$	$3.1(3) + i 3.0^{+1.1}_{-1.0}$	$0.9^{+0.9}_{-0.8} + i 5.1(9)$	$2.4^{+0.4}_{-0.5} + i 4.1(1.0)$	$0.8^{+0.9}_{-0.8} + i 5.1(9)$	$1.3(8) + i 5.4(9)$
$D^* \eta' \rightarrow D^* \eta'$	$-4.2^{+1.3}_{-1.2} + i 2.6^{+0.6}_{-0.5}$	$-4.6^{+1.4}_{-1.3} + i 3.8^{+0.9}_{-0.8}$	$-3.5^{+1.2}_{-1.1} + i 2.0^{+0.5}_{-0.4}$	$-3.9^{+1.3}_{-1.2} + i 2.5^{+0.6}_{-0.5}$	$-3.6^{+1.3}_{-1.1} + i 1.9^{+0.5}_{-0.4}$	$-4.5^{+1.4}_{-1.2} + i 2.3^{+0.6}_{-0.5}$

The discrepancies between “CT” and “CT+Ex” results (the first five channels) are mainly attributable to the u-channel exchanges.

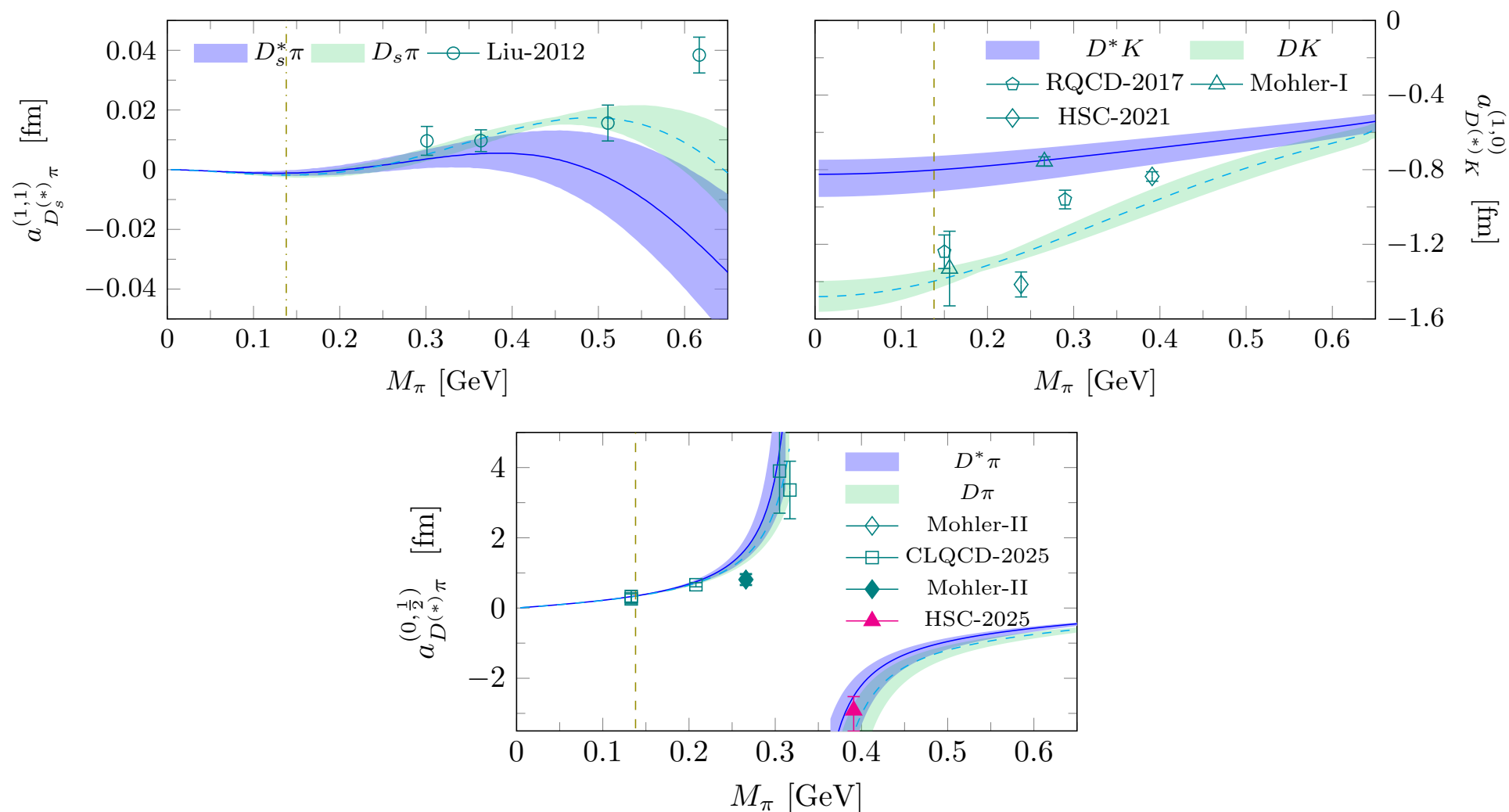
Pion-mass dependence of S-wave scattering lengths

- The four single channels: solid blue ($P^*\phi$) vs dashed green ($P\phi$)



Pion-mass dependence of S-wave scattering lengths

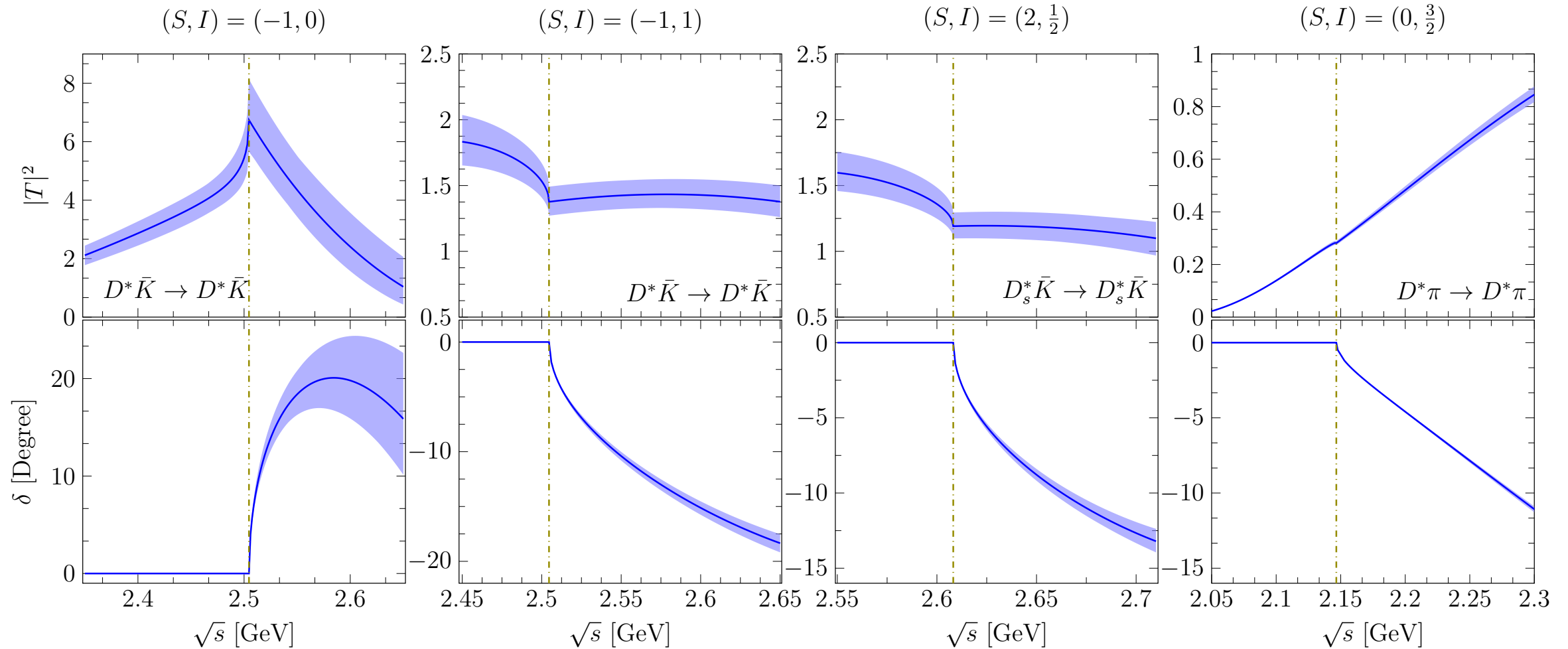
- The three coupled channels: solid blue ($P^*\phi$) vs dashed green ($P\phi$)



Goodness of HQSS: The most recent $P\phi$ and $P^*\phi$ data in $(0,1/2)$ channel are well described!

S-matrix: phase shifts and inelasticity

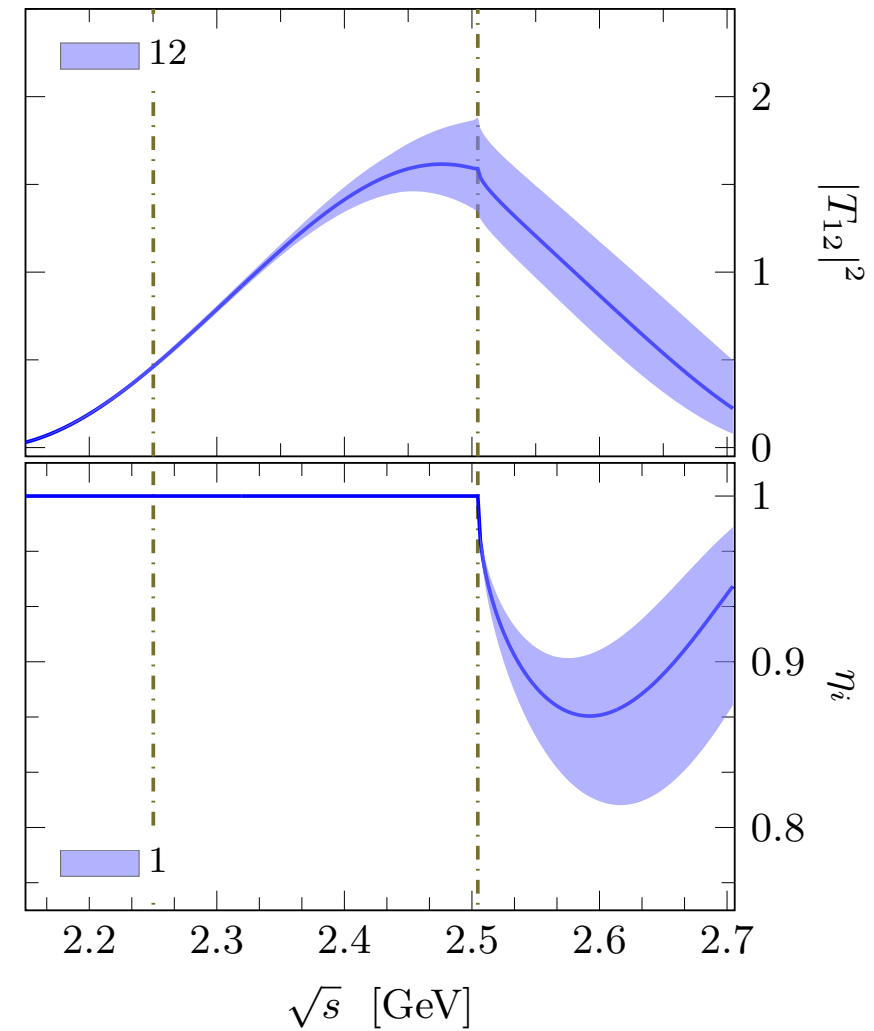
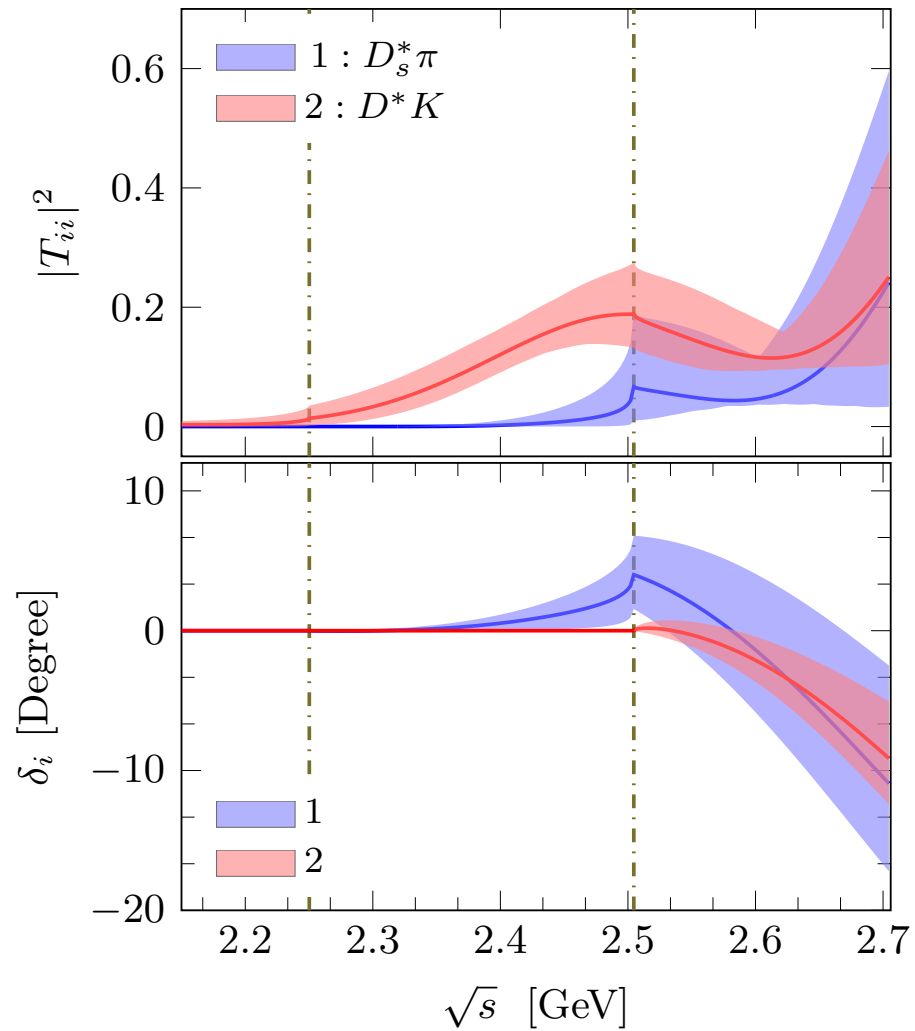
- Single channels:



The cusp structure at the $D^* \bar{K}$ ($I = 0$) threshold suggests the presence of a possible subthreshold resonance.

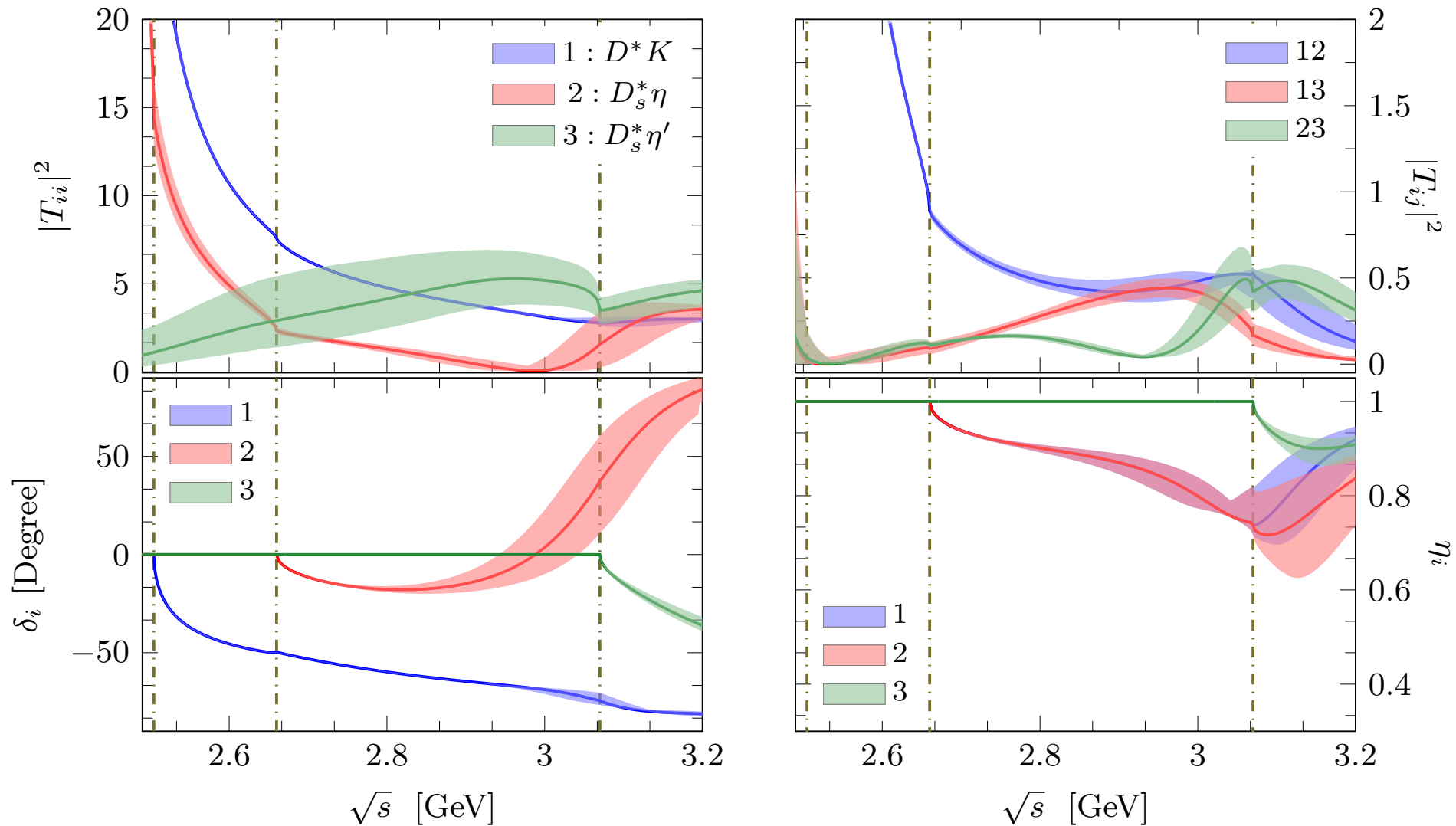
S-matrix: phase shifts and inelasticity

- The coupled channel with $(S, I) = (1, 1)$:



S-matrix: phase shifts and inelasticity

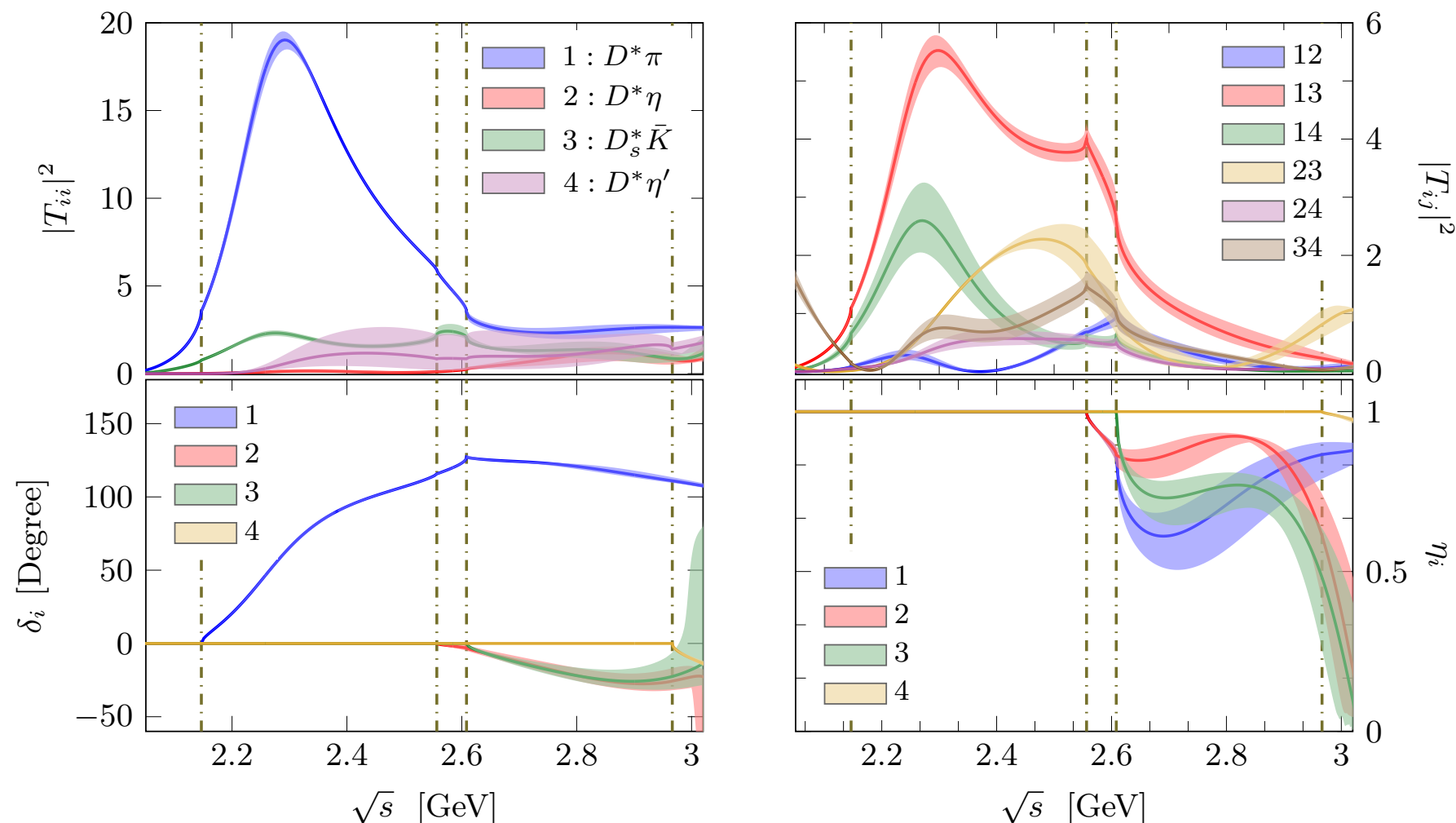
- The coupled channel with $(S, I) = (1, 0)$:



Signal of a subthreshold pole — the $D_{s1}(2460)$ bound state

S-matrix: phase shifts and inelasticity

- The coupled channel with $(S, I) = (0, 1/2)$:



The broad bump of T_{11} between the $D^*\pi$ and $D^*\eta$ thresholds, together with the sharp rise of the phase shift δ_1 past 90 degrees, provide a strong indication of a near-threshold resonance.

Poles and their trajectories

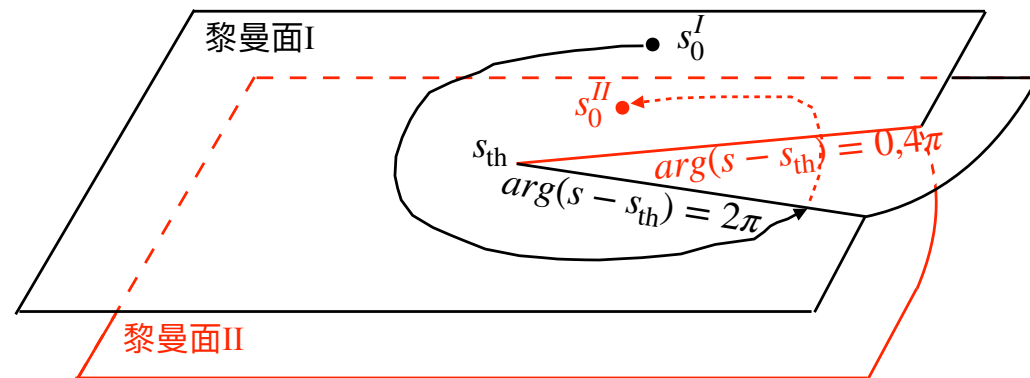
Dynamically generated poles

- Analyticity continuity:

$$g_i^{\text{II}} = g_i(s) - 2\xi_i \text{Im} g_i(s), \quad \xi_i \in \{0,1\}$$

- Notation of Riemann sheets:

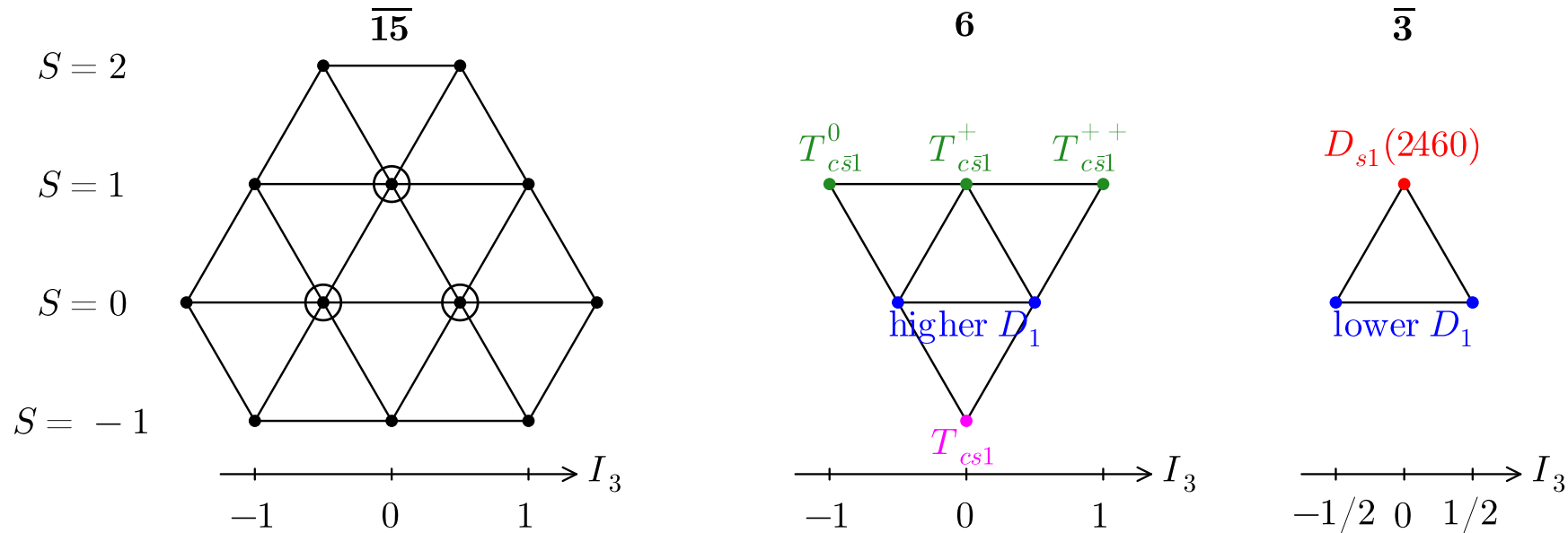
$$[\xi_1, \xi_2, \xi_3, \dots] \iff n = 1 + \sum_{i=1}^N \xi_i 2^{i-1}$$



(S, I)	RS	poles $(M - i \frac{\Gamma}{2})$	Residues (GeV)			
			$ g_1 $	$ g_2 $	$ g_3 $	$ g_4 $
$(-1, 0)$	II	$2362.1_{-10.2}^{+13.6} - i 111.0_{-26.7}^{+25.1}$	$11.2_{-0.6}^{+1.0} (D^* \bar{K})$	—	—	—
$(1, 1)$	II	$2576.7_{-20.4}^{+30.5} - i 323.7_{-5.4}^{+3.0}$	$6.3_{-0.4}^{+0.6} (D_s^* \pi)$	$11.8_{-0.3}^{+0.3} (D^* K)$	—	—
	III	$2347.1_{-11.5}^{+15.1} - i 227.4_{-13.3}^{+11.5}$	$7.1_{-0.2}^{+0.3} (D_s^* \pi)$	$5.6_{-0.2}^{+0.2} (D^* K)$	—	—
$(1, 0)$	I	$2455.2_{-2.7}^{+3.2}$	$10.6_{-0.2}^{+0.2} (D^* K)$	$7.0_{-0.0}^{+0.0} (D_s^* \eta)$	$0.6_{-0.6}^{+0.6} (D_s^* \eta')$	—
$(0, \frac{1}{2})$	II	$2255.6_{-2.8}^{+3.3} - i 112.5_{-2.9}^{+2.6}$	$10.3_{-0.1}^{+0.1} (D^* \pi)$	$2.0_{-0.3}^{+0.3} (D^* \eta)$	$5.2_{-0.1}^{+0.1} (D_s^* \bar{K})$	$4.4_{-0.6}^{+0.6} (D^* \eta')$
	III	$2558.1_{-23.4}^{+31.0} - i 207.2_{-8.4}^{+7.8}$	$5.8_{-0.0}^{+0.1} (D^* \pi)$	$5.3_{-0.3}^{+0.4} (D^* \eta)$	$12.2_{-0.2}^{+0.2} (D_s^* \bar{K})$	$2.3_{-0.3}^{+0.4} (D^* \eta')$
	IV	$2307.1_{-12.1}^{+16.1} - i 186.4_{-14.5}^{+20.9}$	$7.4_{-0.4}^{+0.3} (D^* \pi)$	$5.0_{-0.0}^{+0.2} (D^* \eta)$	$5.0_{-0.3}^{+0.7} (D_s^* \bar{K})$	$4.7_{-1.2}^{+2.3} (D^* \eta')$

SU(3) analysis

- Weight diagrams of flavor SU(3) symmetry: $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$



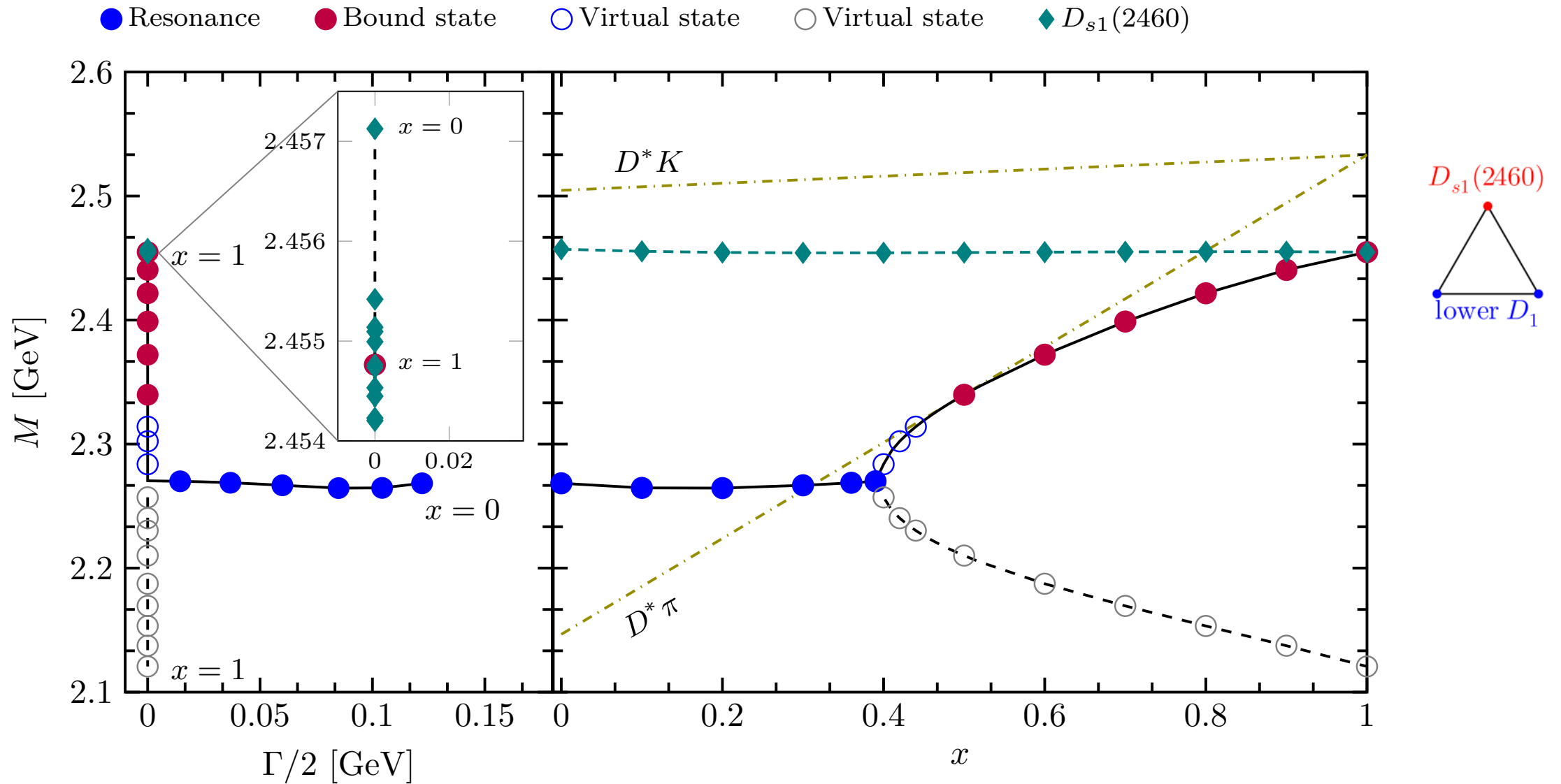
- Physical world vs SU(3) symmetry limit:

$$M_\phi = M_\phi^{\text{phy.}} + x(\bar{M}_0 - M_\phi^{\text{phy.}}), \quad \phi \in \{\pi, K, \bar{K}, \eta\},$$

$$M_{P^*} = M_{P^*}^{\text{phy.}} + x(\bar{M}_{P^*} - M_{P^*}^{\text{phy.}}), \quad P^* \in \{D^*, D_s^*\},$$

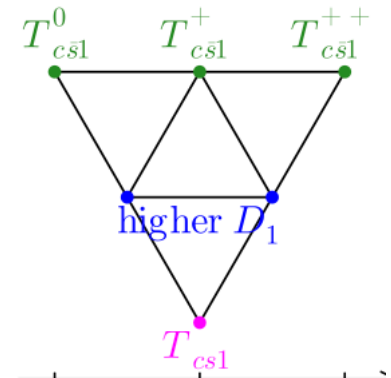
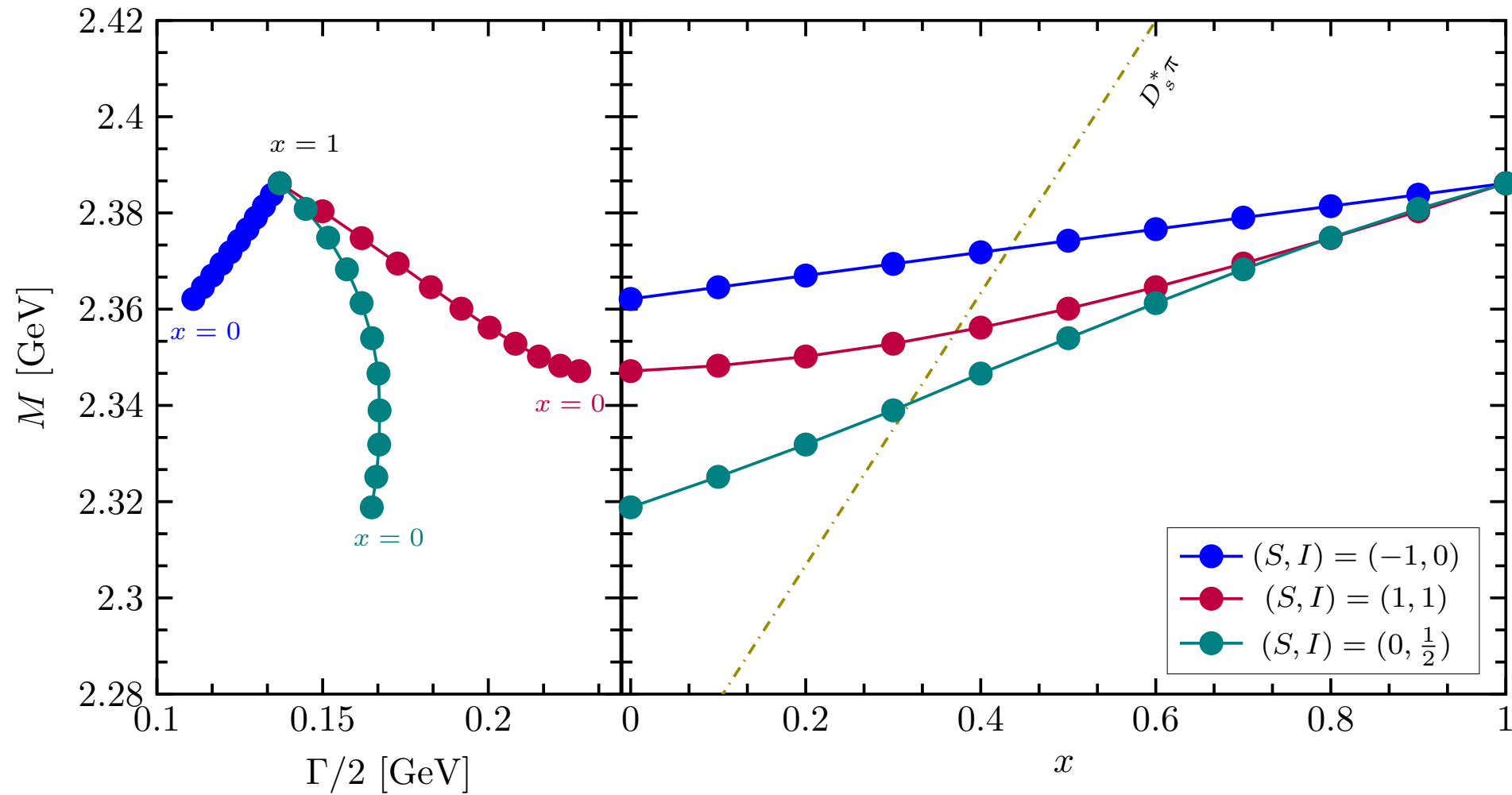
SU(3) analysis

- The SU(3) triplet:



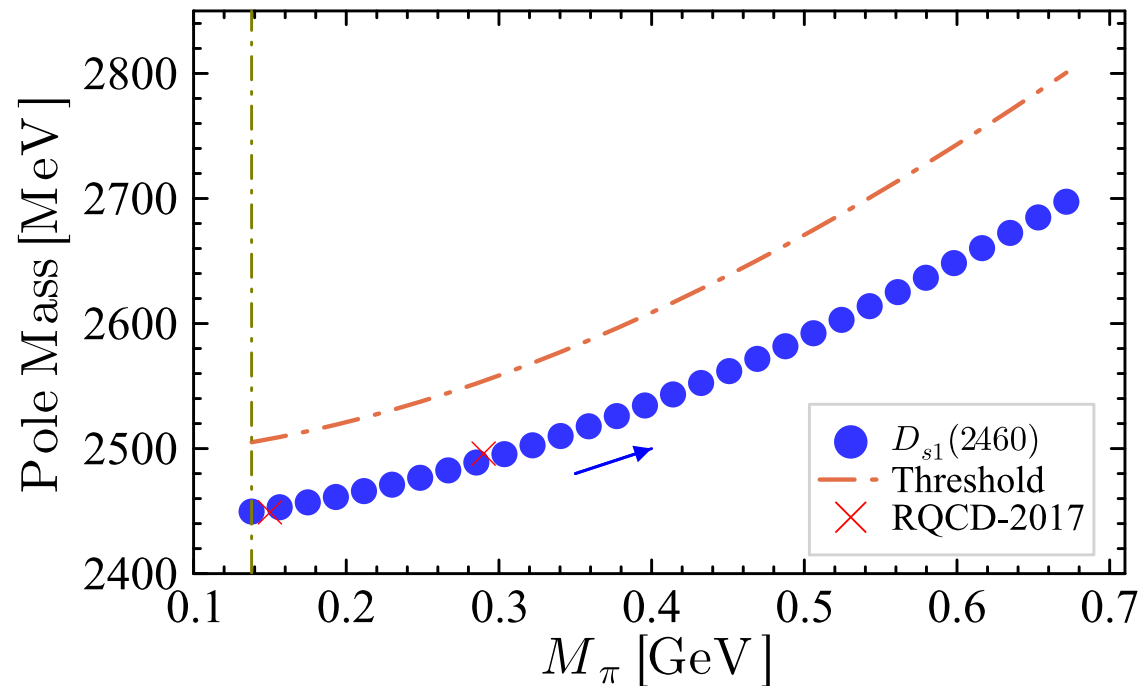
SU(3) analysis

- The SU(3) sextet:



M_π trajectory

- The bound-state pole: (1,0)



- Remains as a bound state as M_π increase
- In consistent with lattice QCD results

- Pion mass dependence

$$M_K^2 = \dot{M}_K^2 + \frac{1}{2}M_\pi^2$$

$$M_{\eta'/\eta}^2 = M_K^2 + \frac{M_0^2}{2} \pm \frac{R}{2}$$

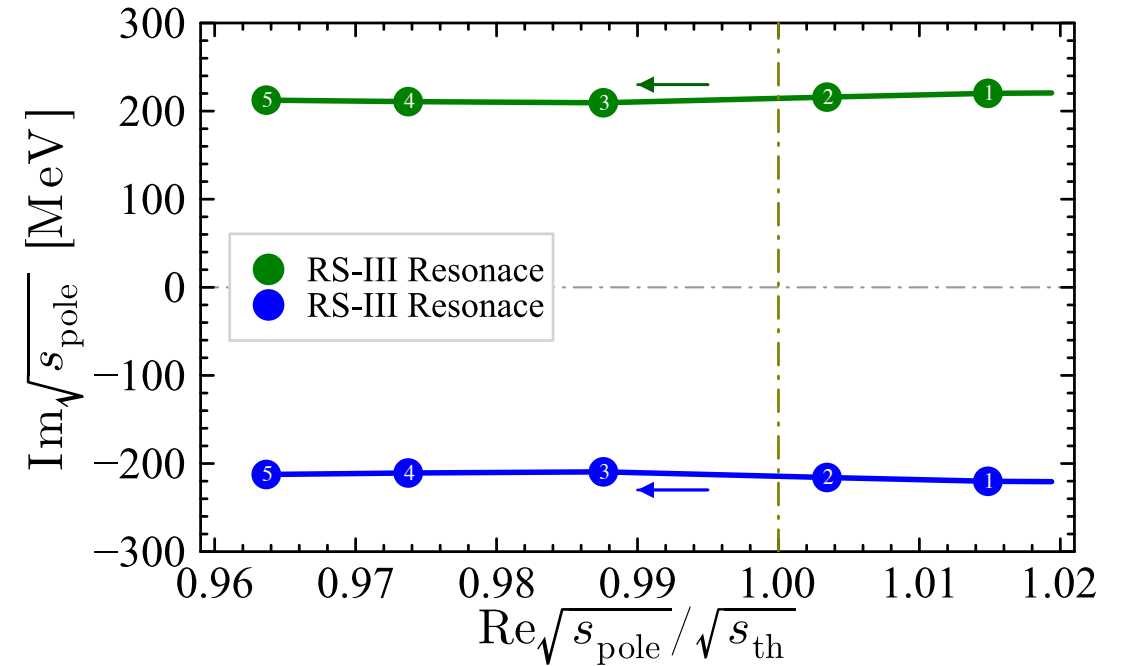
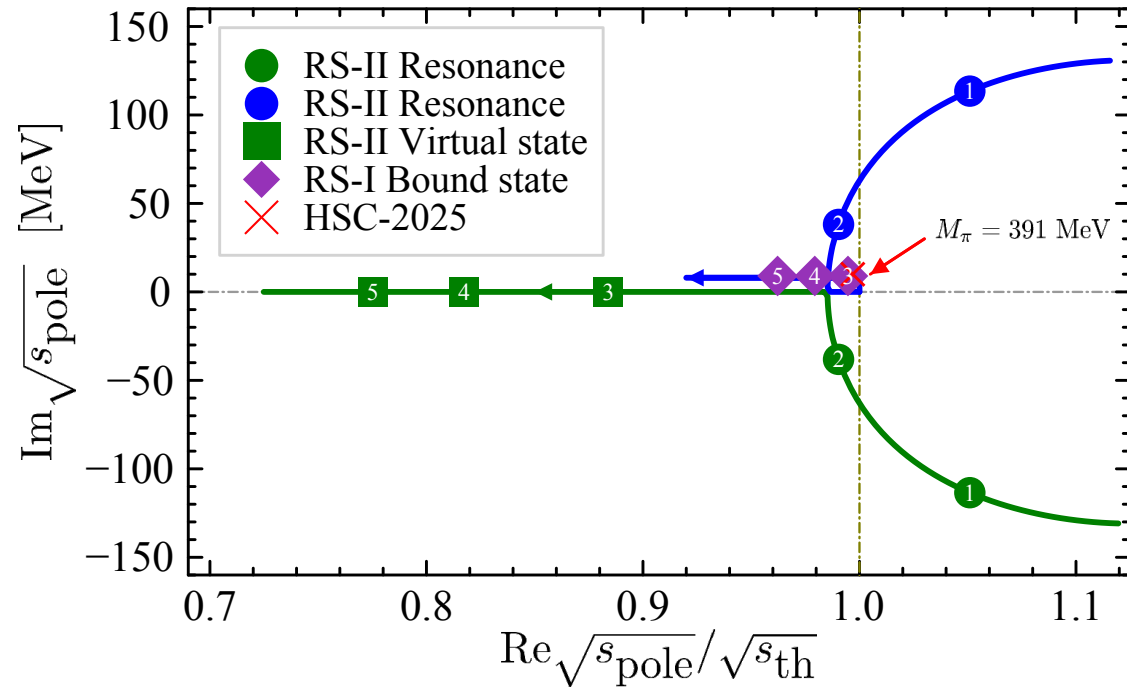
$$\sin \theta = - \left(\sqrt{1 + \frac{S^2}{32\Delta^4}} \right)^{-1}$$

$$M_{D^*}^2 = \dot{M}_{D^*}^2 + 2(2h_0^* + h_1^*)M_\pi^2$$

$$M_{D_s^*}^2 = \dot{M}_{D_s^*}^2 + 4h_0^*M_\pi^2$$

M_π trajectory

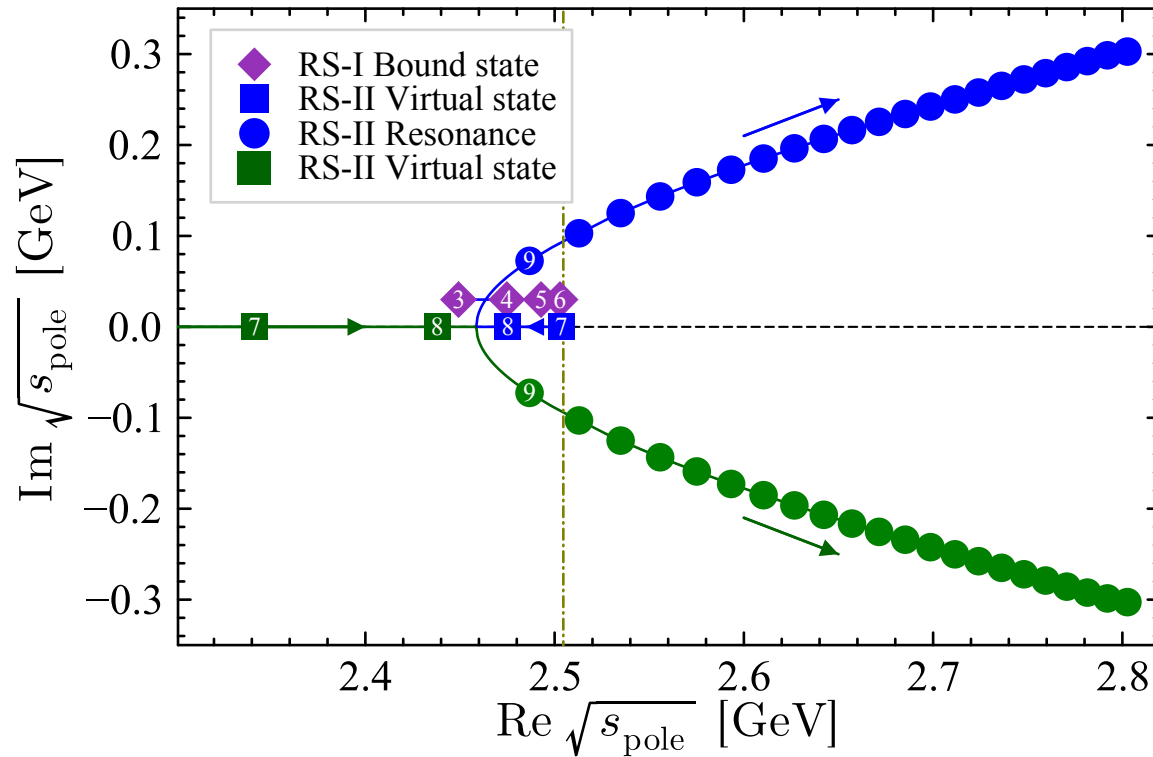
- The two poles in (0,1/2)



- Lower pole (left): a pair of resonances $\longrightarrow$ two virtual states $\longrightarrow$ a bound state + a virtual state
- Higher pole (right): always a pair of resonances

N_C trajectory

- The bound-state pole in (1,0)



- N_C dependence

$$M_\pi, M_K, \bar{M}_{P^*} \sim \mathcal{O}(1)$$

$$F_0 \sim \mathcal{O}(\sqrt{N_C})$$

$$M_0^2 \sim 1/N_C$$

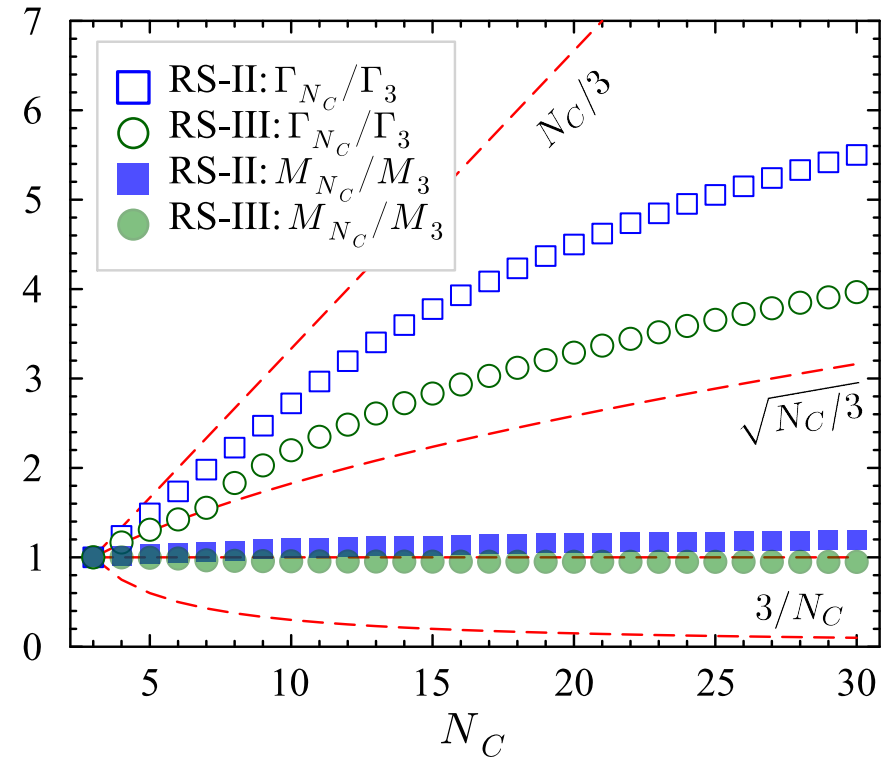
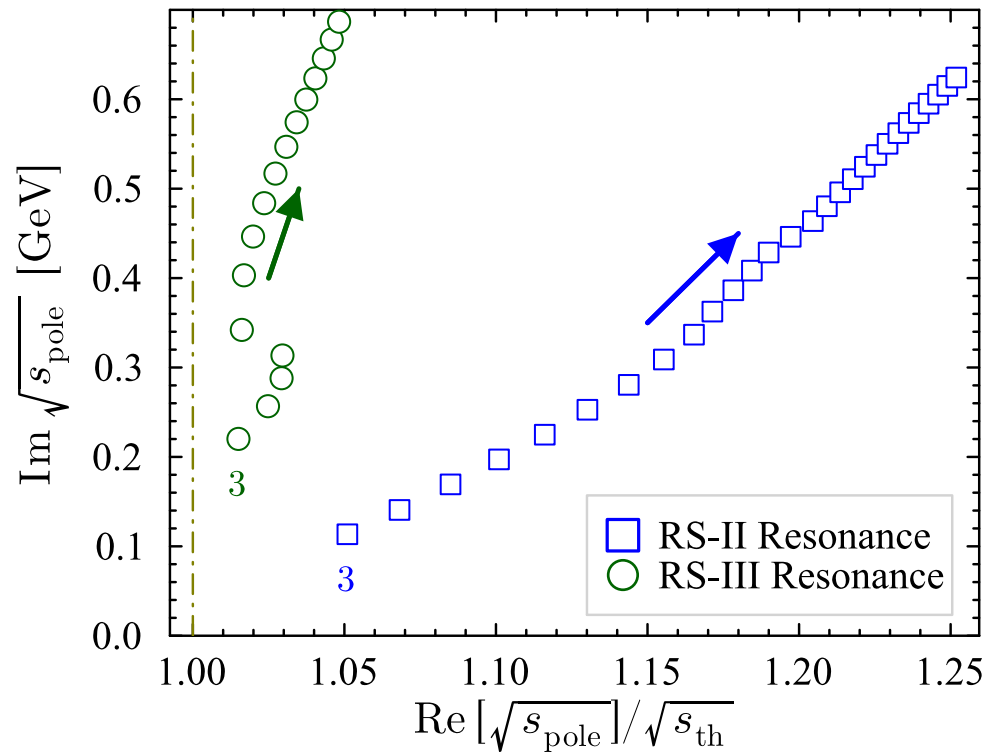
$$h_{1,3,5}^* \sim \mathcal{O}(1) \quad h_{2,4,5}^* \sim \mathcal{O}(1/N_C)$$

$$a(\mu) \sim \mathcal{O}(1)$$

- Evolution of the pole: **bound state** $\longrightarrow$ **virtual state** $\longrightarrow$ **resonance**, to complex infinity as $N_C \rightarrow \infty$
- $D_{s1}(2460)$ is of non- $\bar{q}q$ nature

N_C trajectory

- The two poles in (0,1/2)



- Both the two poles of $D_1(2430)$ flow to complex infinity as $N_C \rightarrow \infty$;
- Scale as N_C^α with $1/2 \leq \alpha < 1$;
- $D_1(2430)$ is of non- $\bar{q}q$ nature

Summary and outlook

Summary and outlook

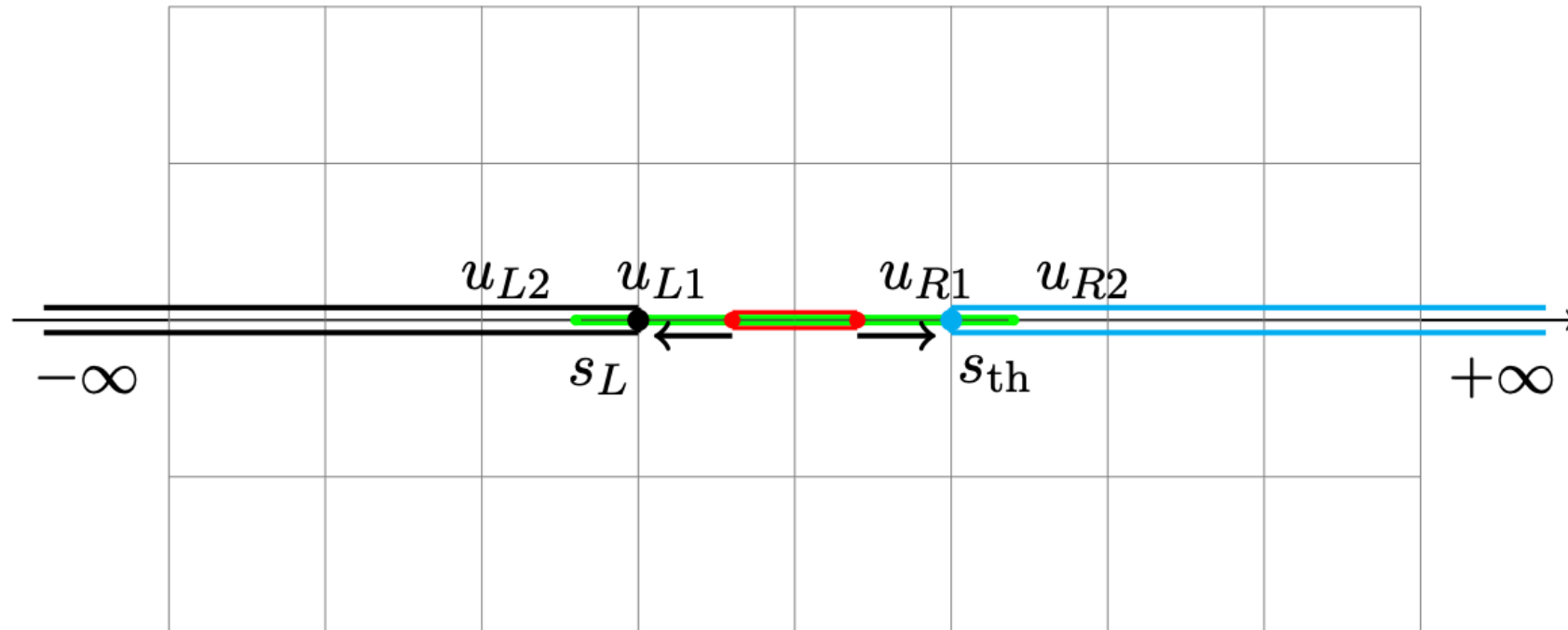
- **Chiral potentials of $P^*\phi$ interaction up to NLO**
 - Relativistic kinematics: polarization vectors
 - U(3) framework: the contribution of η_0 singlet
 - Both contact and meson-exchange (EX) contributions: the S-wave EX is negligible
- **Systematical Generation of axial-vector charmed mesons**
 - $D_{s1}(2460)$: bound-state pole, member of SU(3) triplet
 - $D_1(2430)$: two resonance poles, lower/higher pole belonging to SU(3) triplet/sextet
 - Inner structure: both of them are of non- $\bar{q}q$ nature
- **Outlook**
 - Correlation functions for femtoscopy studies in experiments
 - Off-shell treatment of the BSE, free of unphysical cuts of on-shell artifact

Thanks!

Backup

Left-hand cut

- The left-hand cut induced by u -channel meson exchange
 - ▶ s -channel cuts: $s_L = (M_D^* - M_\pi)^2$, $s_R = (M_D^* + M_\pi)^2$
 - ▶ Segment cut (green) from u -channel D^* exchange: $u_{L1} = (M_{D^*}^2 - M_\pi^2)^2 / M_{D^*}^2$, $u_{R1} = 2M_\pi^2 + M_{D^*}^2$.
 - ▶ Segment cut (red) from u -channel D exchange: $u_{L2} = (M_{D^*}^2 - M_\pi^2)^2 / M_D^2$, $u_{R2} = 2M_\pi^2 + 2M_{D^*}^2 - M_D^2$.



NLO potential

- **Explicit expressions of the potential functions:**

- ▶ NLO Contact

$$\mathcal{H}_{24}(s, t) = 2h_2^* p_2 \cdot p_4 + h_4^* (p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3) ,$$

$$\mathcal{H}_{35}(s, t) = h_3^* p_2 \cdot p_4 + h_5^* (p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3) .$$

- ▶ LO meson exchange

$$\mathcal{G}_P(s, t) = \frac{p_2 \cdot \epsilon_1 p_4 \cdot \epsilon_3^*}{(p_1 + p_2)^2 - M_{h^2}} ,$$

$$\begin{aligned} \mathcal{G}_{P^*}(s, t) = & \frac{1}{(p_1 + p_2)^2 - M_{h^*}^2} \left\{ p_1 \cdot \epsilon_3^* [p_2 \cdot p_4 p_3 \cdot \epsilon_1 - p_2 \cdot p_3 p_4 \cdot \epsilon_1] + p_2 \cdot \epsilon_3^* \right. \\ & \left. \times [p_1 \cdot p_3 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_3 \cdot \epsilon_1] + \epsilon_1 \cdot \epsilon_3^* [p_2 \cdot p_3 p_1 \cdot p_4 - p_1 \cdot p_3 p_2 \cdot p_4] \right\} . \end{aligned}$$