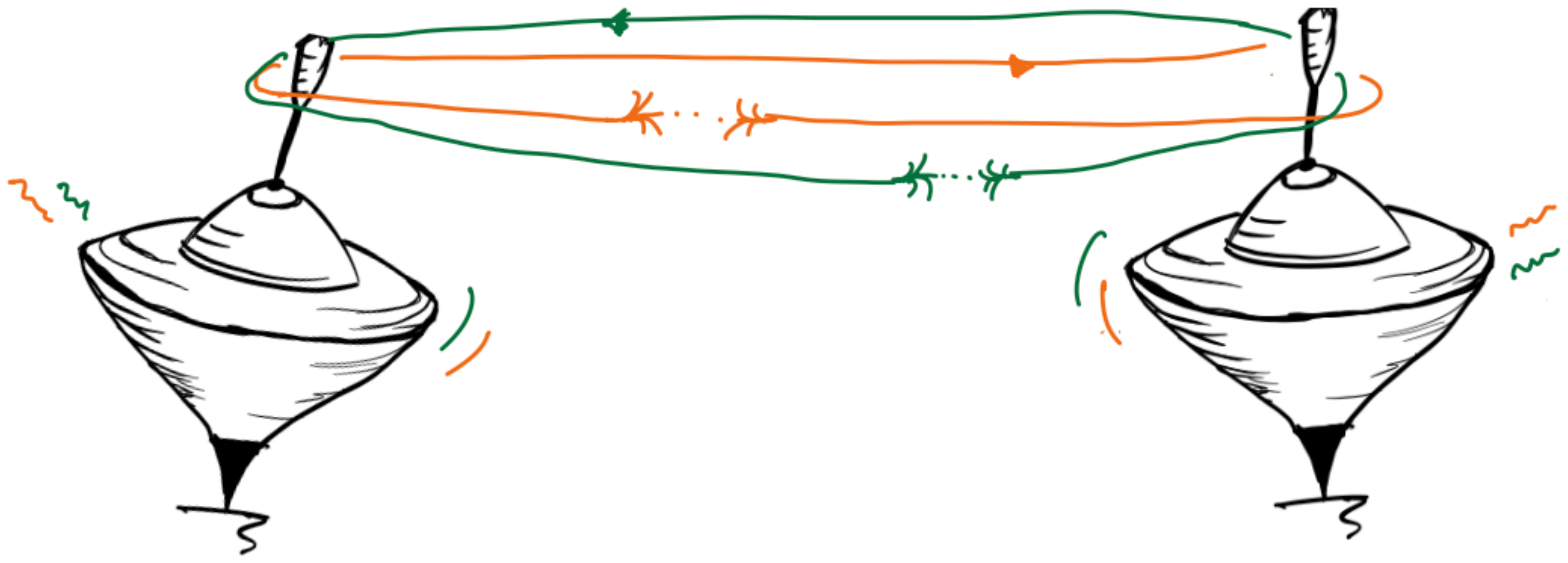
Entanglement and Decoherence at colliders

Rafael Aoude
DESY



w/ Alan Barr, José M. Camacho, Valentin Durupt, Guillermo G, Mir,
Fabio Maltoni, Maria. M. Llácer, Leonardo Satrioni, Kazuki Sakurai, Marcel Vos.

DESY.

International Workshop on New
Opportunities for Particle Physics
NOPP 2026

Outline

- Motivation

- Open Quantum systems

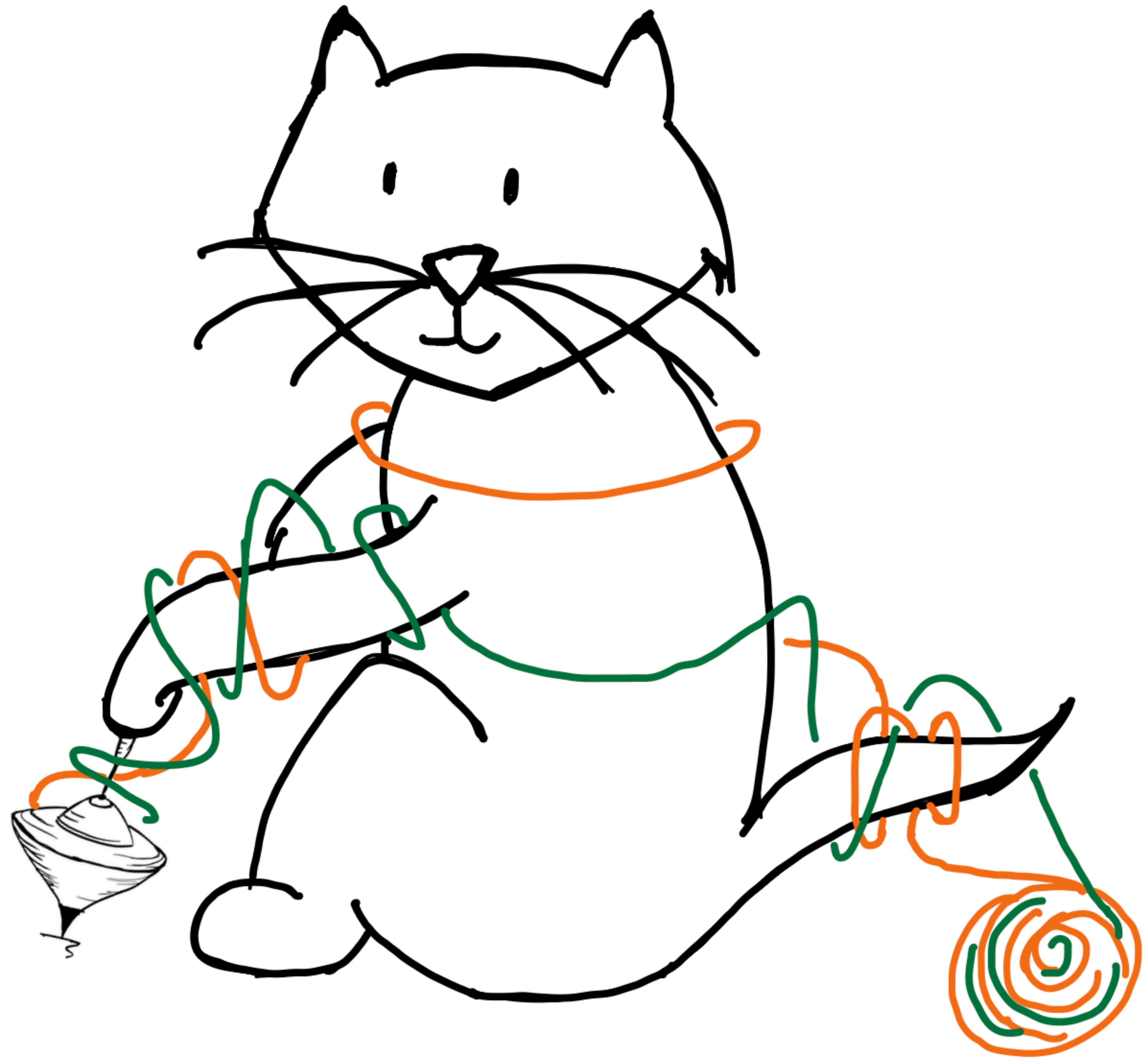
- Radiation as an Environment

- Examples:

  - Scalar decay

  - $e^+e^- \to t\bar{t} + g$

- Conclusion and prospects



Motivation

Talks by:
Tianjun Li, Jia Liu
Juan Ramon, Yoav,
Massimo Blasone

Entanglement has been measured by ATLAS and CMS

From the theory side, we should provide higher-order predictions

What are higher-orders?

Loops

Multiplicity

QCD/EW

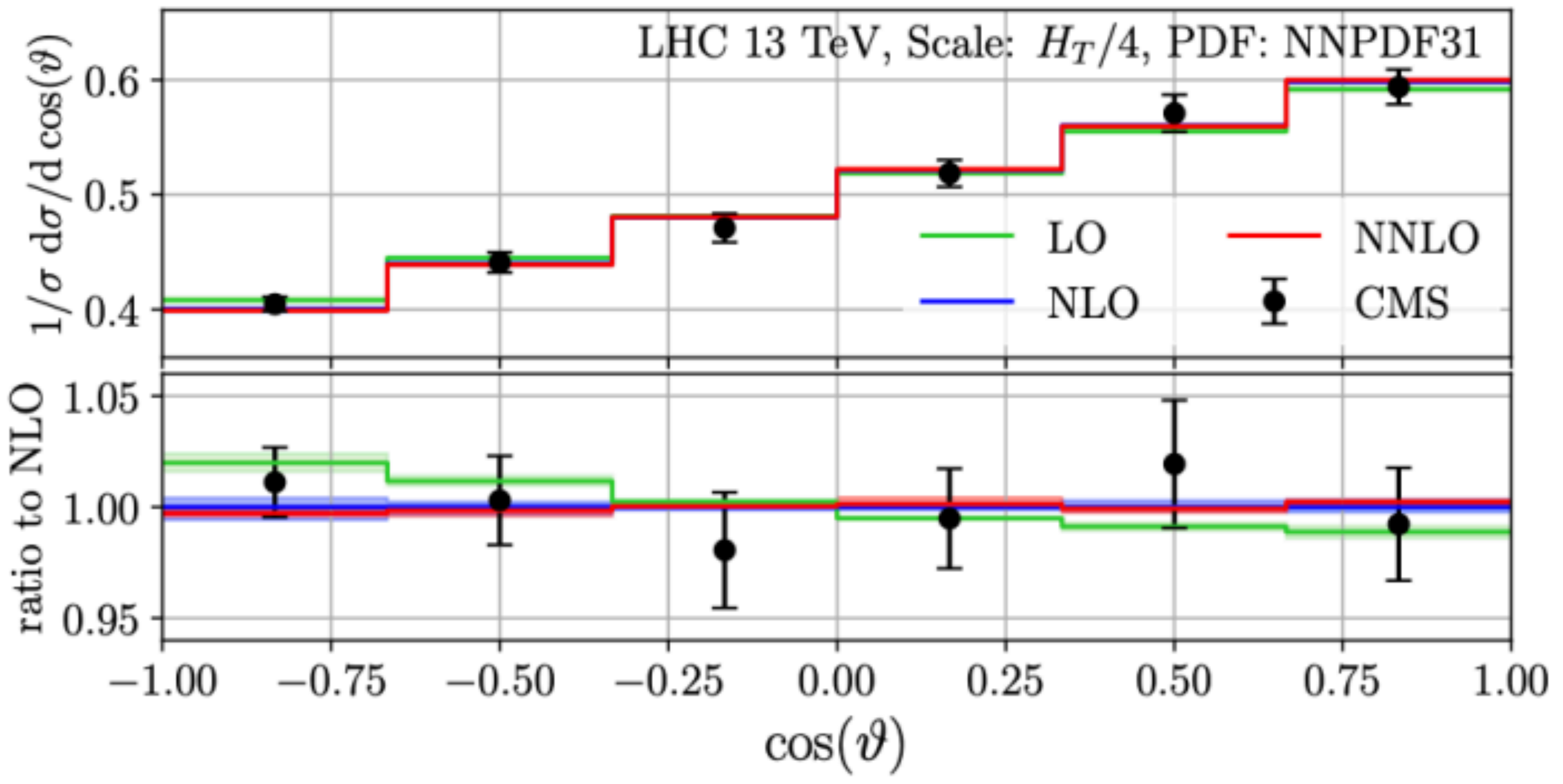
We should also sharpen and understand our QFT tools

Higher-order corrections

Top Pair

Coefficient	LO ($\times 10^3$)	NLO ($\times 10^3$)	NNLO ($\times 10^3$)	CMS ($\times 10^3$)
B_1^k	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 2 [mc]	-1^{+0}_{-1} [sc] ± 4 [mc]	5 ± 23
B_1^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+1}_{-0} [sc] ± 2 [mc]	0^{+1}_{-2} [sc] ± 2 [mc]	-23 ± 17
B_1^n	0^{+0}_{-0} [sc] ± 1 [mc]	3^{+1}_{-1} [sc] ± 1 [mc]	4^{+1}_{-0} [sc] ± 3 [mc]	6 ± 13
B_2^k	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+0}_{-1} [sc] ± 1 [mc]	-5^{+2}_{-3} [sc] ± 3 [mc]	7 ± 23
B_2^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+2}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 2 [mc]	-10 ± 20
B_2^n	0^{+0}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 1 [mc]	-3^{+1}_{-0} [sc] ± 3 [mc]	17 ± 13
C_{kk}	324^{+7}_{-7} [sc] ± 1 [mc]	330^{+2}_{-2} [sc] ± 3 [mc]	323^{+2}_{-5} [sc] ± 6 [mc]	300 ± 38
C_{rr}	6^{+5}_{-5} [sc] ± 1 [mc]	58^{+18}_{-12} [sc] ± 2 [mc]	69^{+8}_{-7} [sc] ± 3 [mc]	81 ± 32
C_{nn}	332^{+1}_{-0} [sc] ± 1 [mc]	330^{+1}_{-1} [sc] ± 2 [mc]	326^{+1}_{-1} [sc] ± 4 [mc]	329 ± 20
$C_{nr} + C_{rn}$	1^{+0}_{-0} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 3 [mc]	-4^{+4}_{-0} [sc] ± 6 [mc]	-4 ± 37
$C_{nr} - C_{rn}$	0^{+0}_{-1} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 2 [mc]	2^{+4}_{-2} [sc] ± 8 [mc]	-1 ± 38
$C_{nk} + C_{kn}$	0^{+0}_{-0} [sc] ± 1 [mc]	2^{+1}_{-0} [sc] ± 1 [mc]	3^{+4}_{-1} [sc] ± 3 [mc]	-43 ± 41
$C_{nk} - C_{kn}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+1}_{-1} [sc] ± 2 [mc]	6^{+0}_{-2} [sc] ± 7 [mc]	40 ± 29
$C_{rk} + C_{kr}$	-229^{+4}_{-4} [sc] ± 1 [mc]	-203^{+9}_{-7} [sc] ± 2 [mc]	-194^{+8}_{-6} [sc] ± 7 [mc]	-193 ± 64
$C_{rk} - C_{kr}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 4 [mc]	-1^{+1}_{-3} [sc] ± 5 [mc]	57 ± 46

[Czakon, Mitov, Poncelet ’08]



Diboson

γ_{1010} ($\times 10^3$)	QuTom (full)		direct calc. (NWA)	
M_H	LO	NLO	LO	NLO
183 GeV	$-1.5506(9)$	$-0.44(1)$	$-1.790(1)$	$-0.596(6)$
200 GeV	$-1.3085(5)$	$-0.417(9)$	$-1.364(1)$	$-0.469(5)$
225 GeV	$-0.8601(7)$	$-0.294(7)$	$-0.8776(7)$	$-0.315(3)$
250 GeV	$-0.559(1)$	$-0.200(3)$	$-0.5643(4)$	$-0.208(2)$

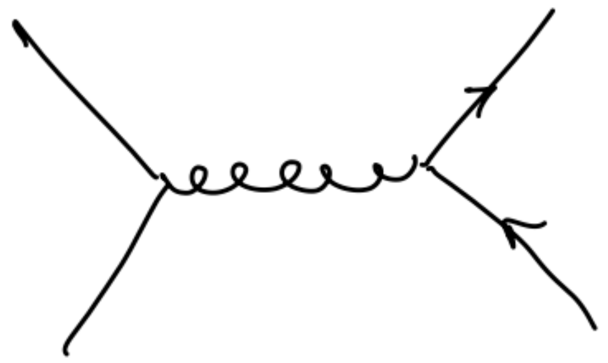
[Del Gratta Fabbri Grossi Maltoni Pagani Pellicioli Vicini ’25]

[Gonçalves, Kaladharan, Navarro ’25/’26]

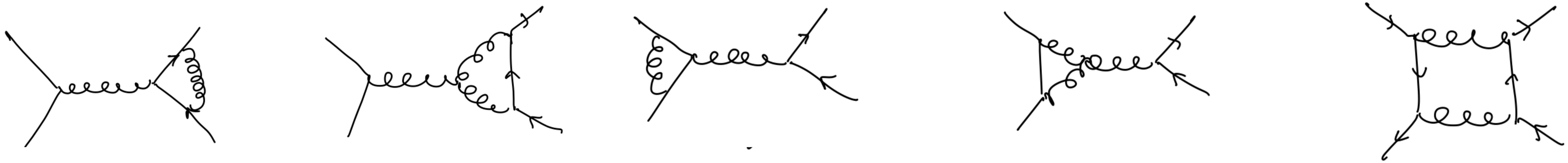
Higher-order corrections: Diagrammatic

Formally, for ttbar production at NLO (in QCD) we need...

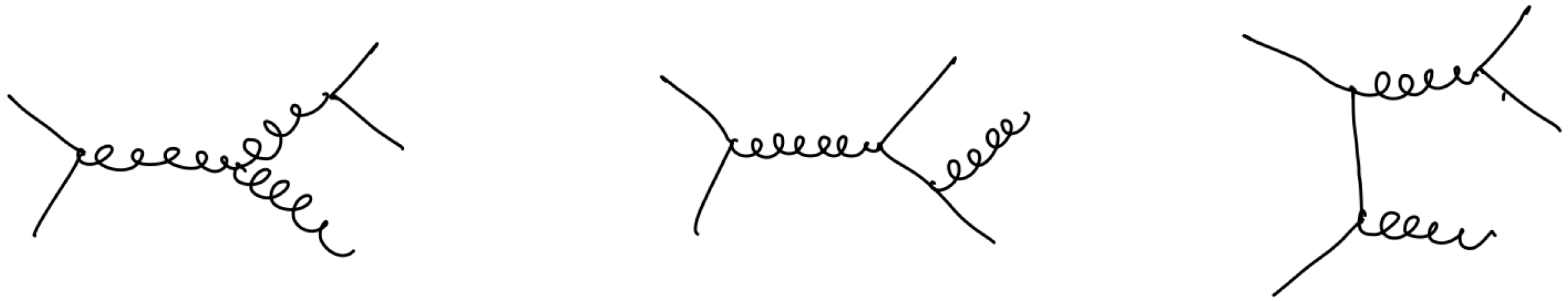
tree



virtual



real



Extra radiation?
Larger Hilbert space.

If the radiation is unresolvable, can we see this as a quantum map?

Decoherence!

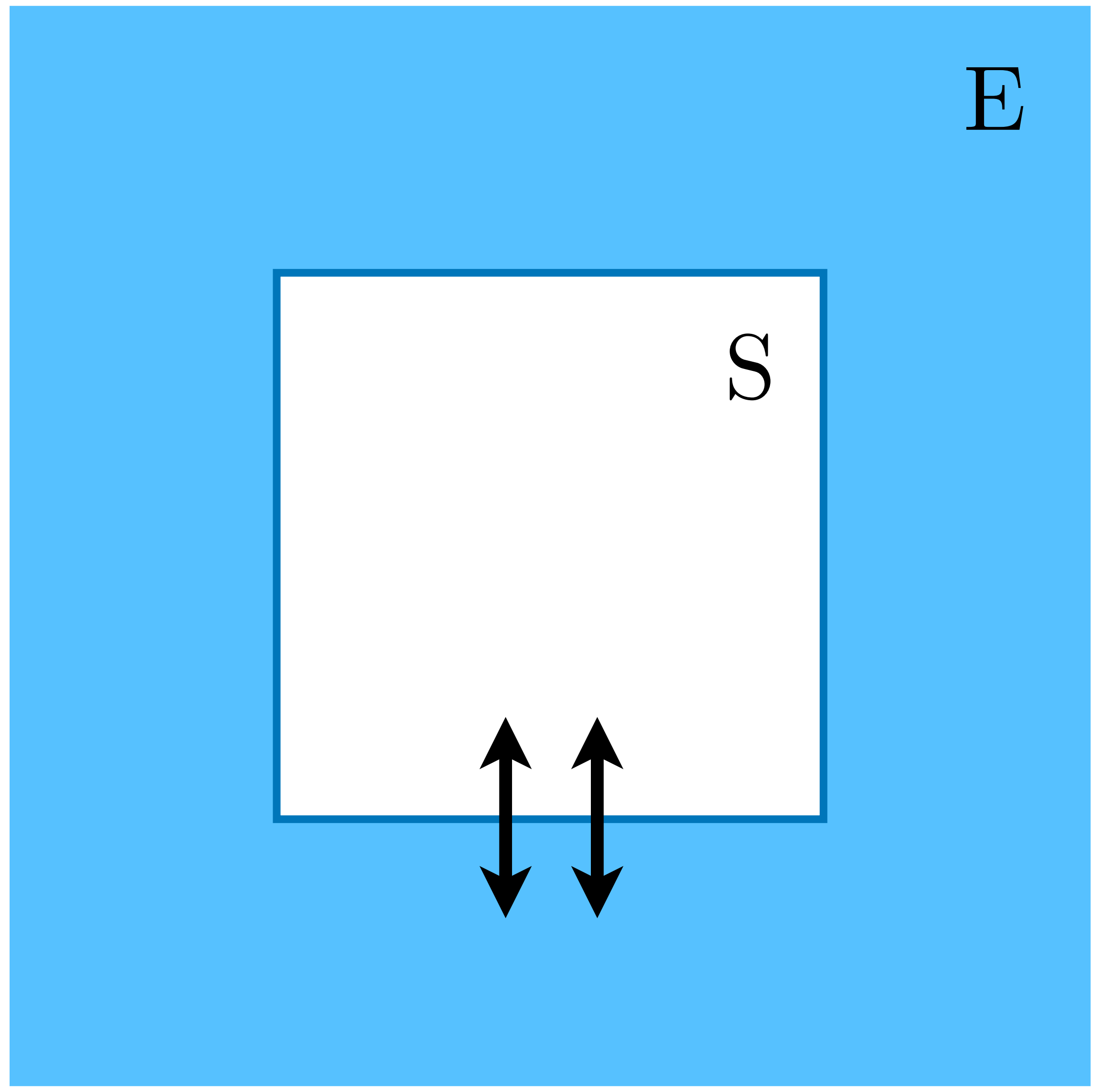
Open Systems: Quantum Maps

The evolution of a system+environment is unitary

$$\rho'(t) = U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)$$

Tracing over the environment subsystem

$$\rho_{\mathrm{S}}(t) = \mathrm{tr}_E\left[U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)\right]$$



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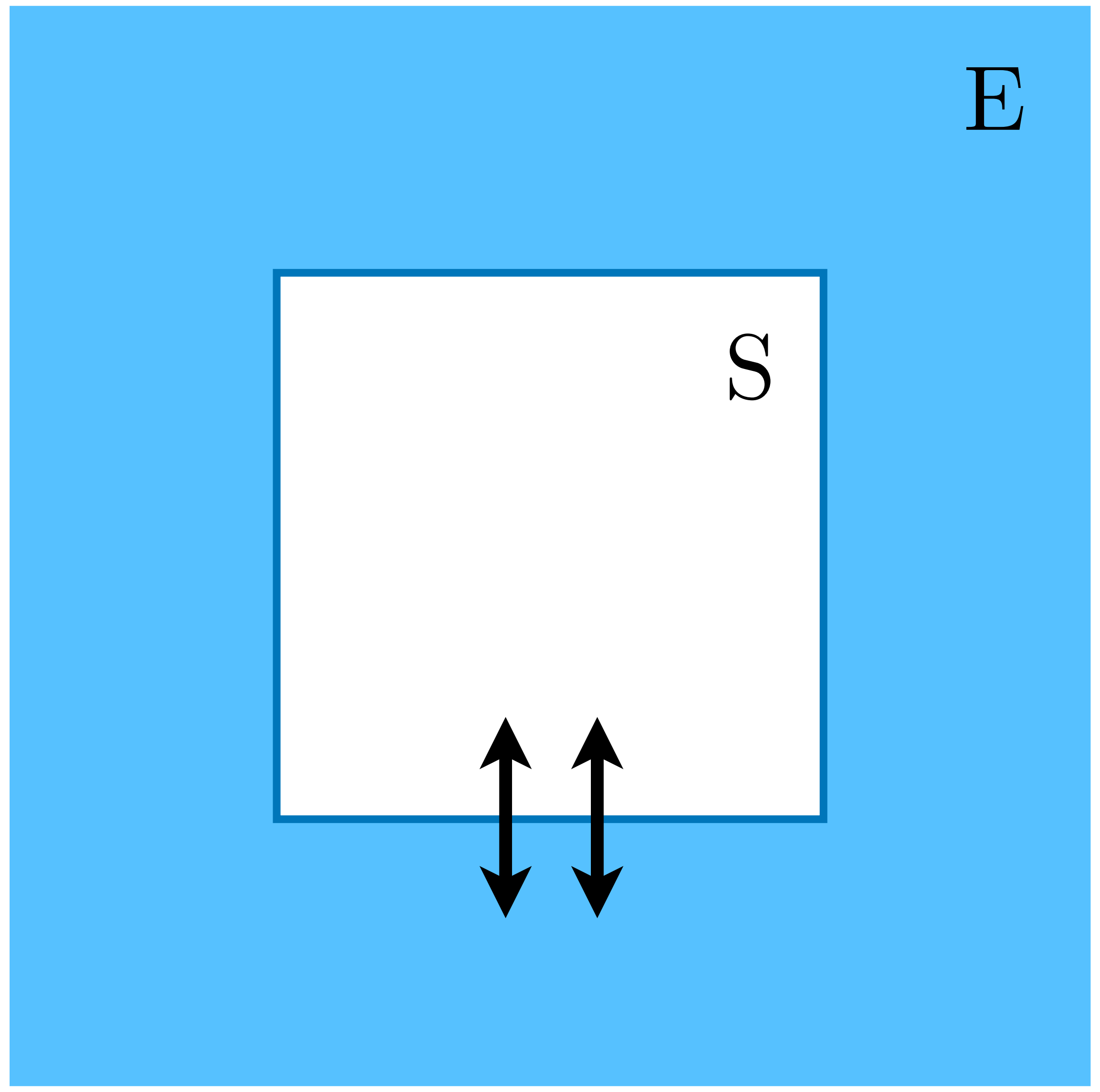


Tracing over the environment subsystem

$$\rho_{\mathrm{S}}(t) = \mathrm{tr}_E\left[U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)\right]$$

which we can write as a operator-sum representation (Kraus operators)

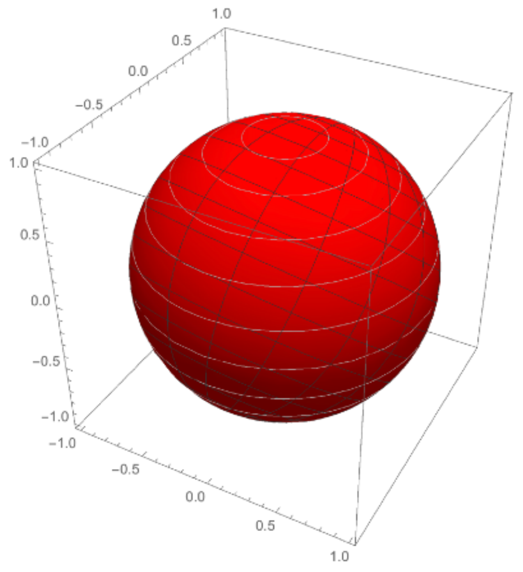
$$\rho_{\mathrm{S}}(t) = \sum_j K_j \rho_{\mathrm{S}}(0) K_j^\dagger \qquad \text{s.t} \qquad \sum_j K_j K_j^\dagger = 1$$

For bipartite qubits: K_j Tensor product of Pauli

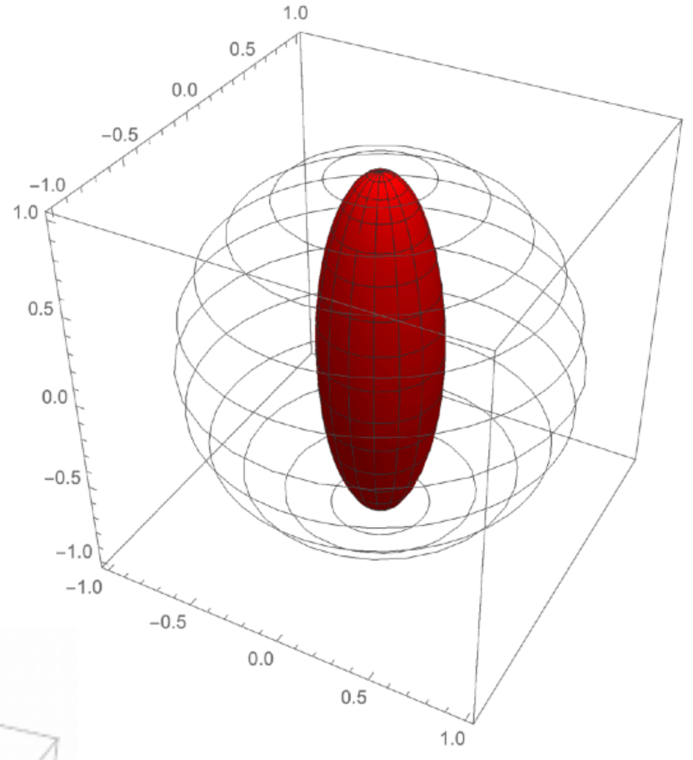
Types of Qubit Channels

[Jagadish, Petruccione ’19]

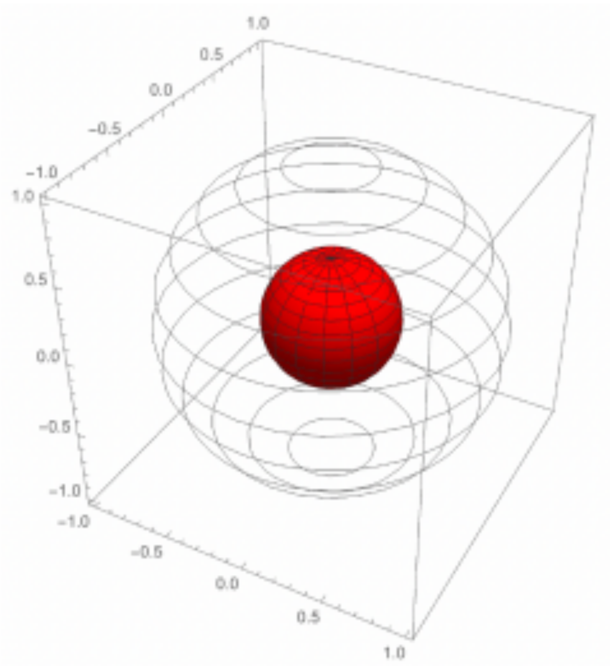
Consider the Kraus operator acting on the top



- Phase damping channel: $K_1 = \sqrt{1 - \frac{p}{2}}\mathbb{1} \qquad K_2 = \frac{p}{2}\sigma_3$

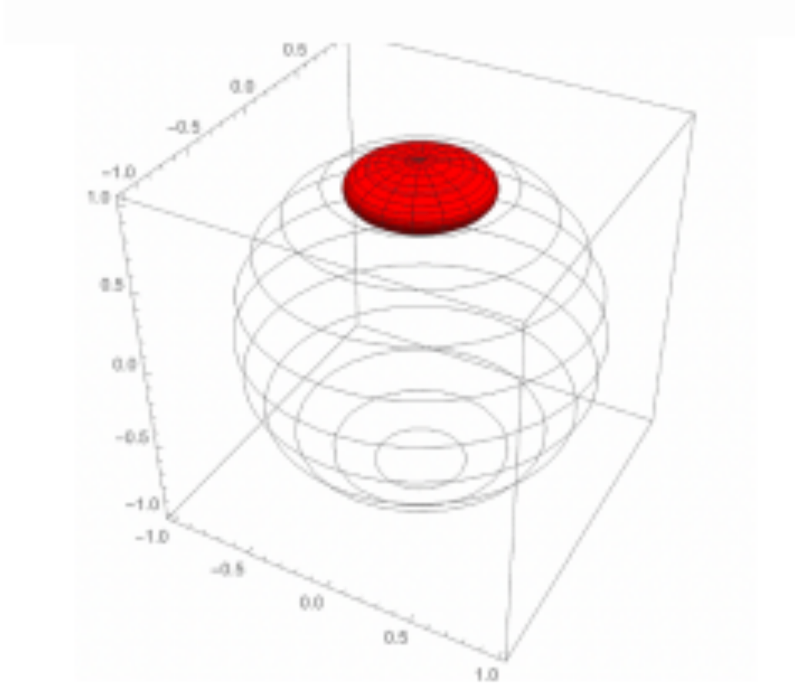


- Depolarising $K_1 = \sqrt{1 - 3p/4}\,\mathbb{1} \qquad K_j = \sqrt{p/4}\sigma_j$



- Spin flip $K_0 = \sqrt{1-p}\,I, \qquad K_1 = \sqrt{p}\,X$.

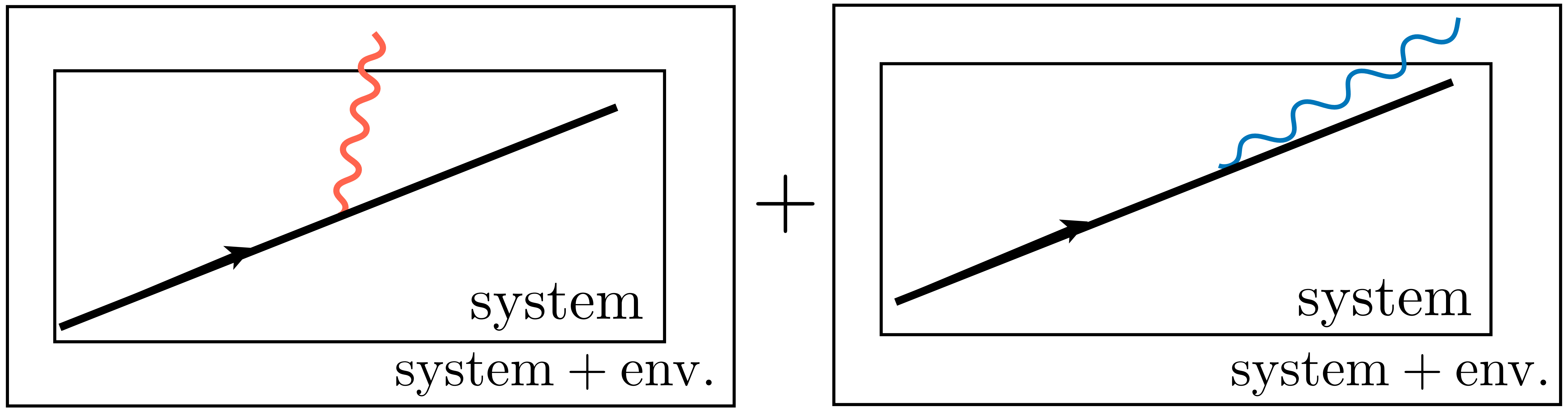
- Amplitude damping $K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \; K_2 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$



Radiation as environment

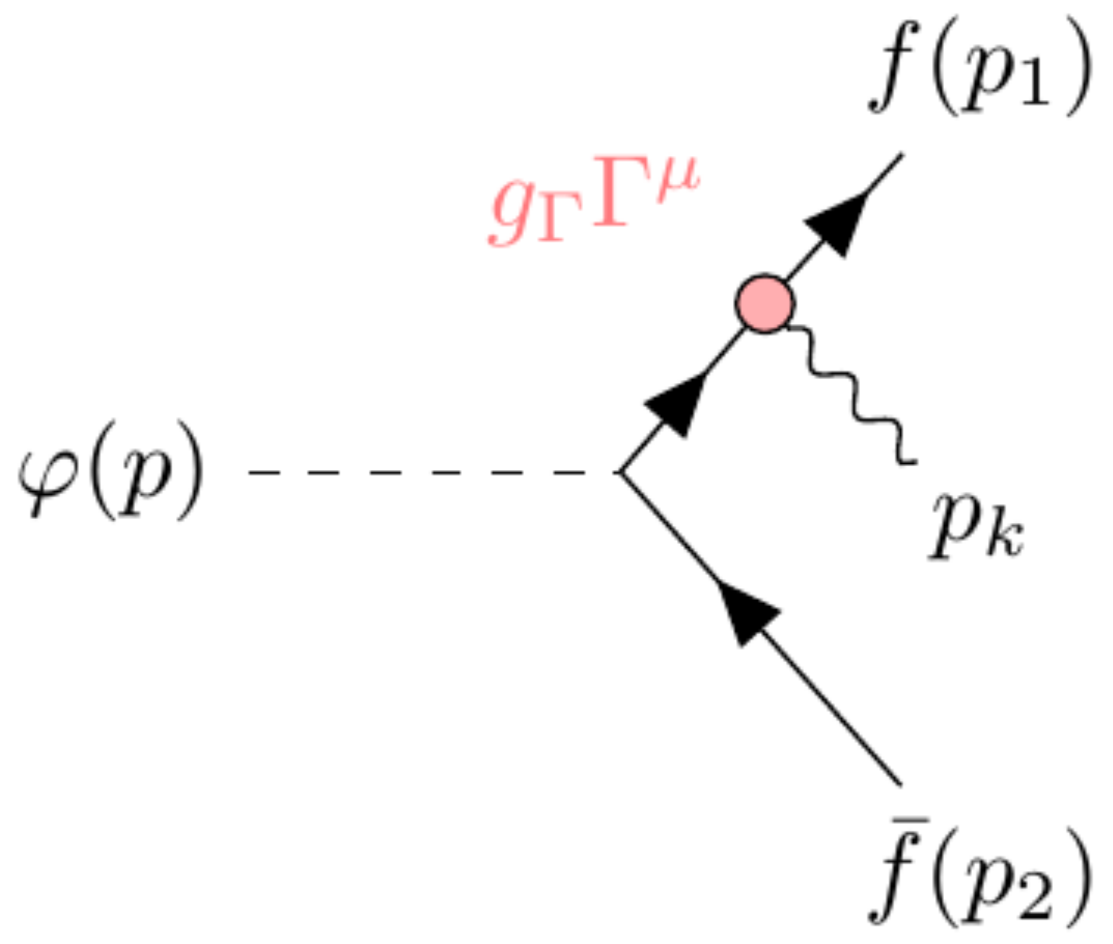
‘weak’ coupling between system and environment

soft collinear



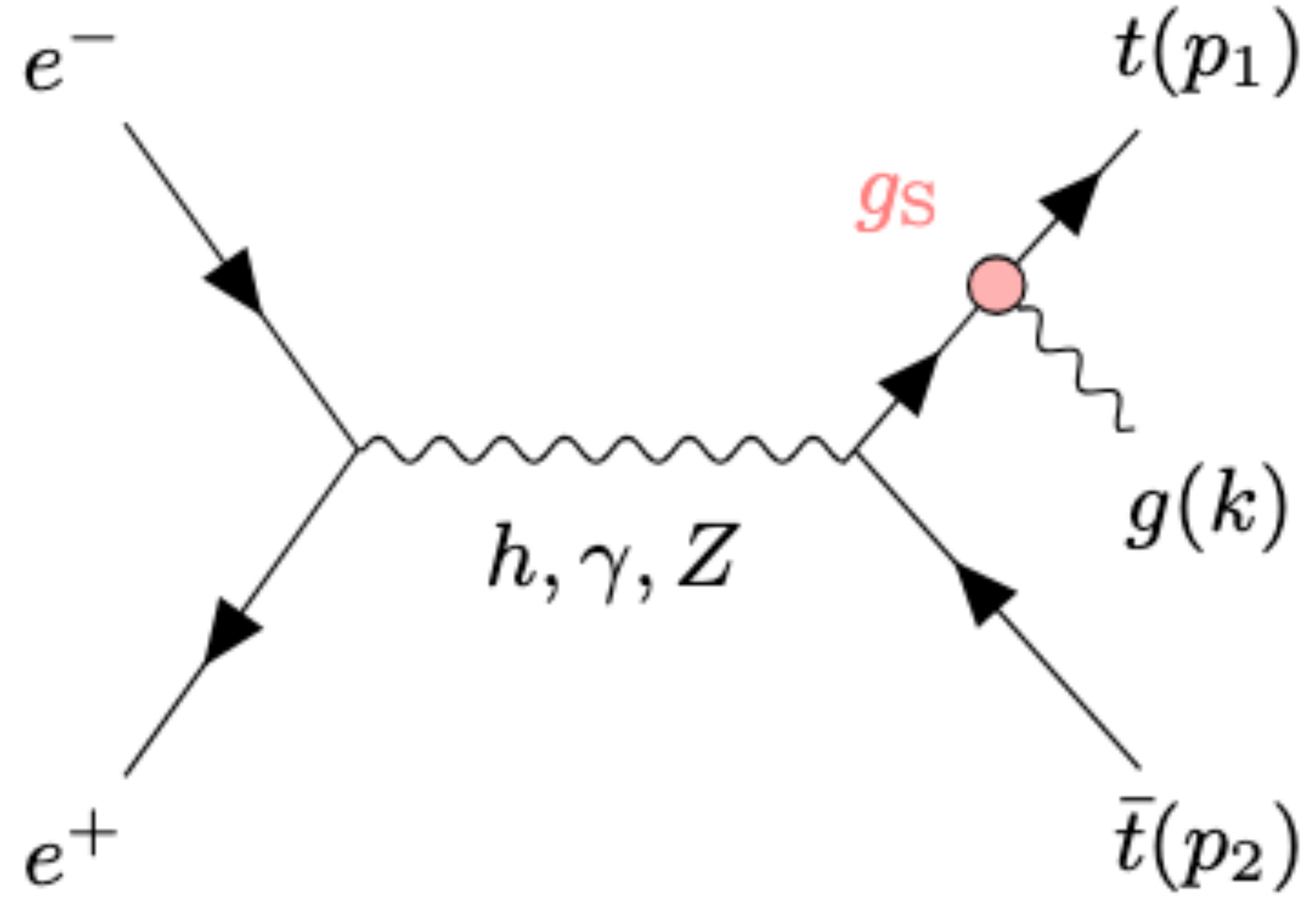
Applications

1. Scalar Decay



[RA, Alan Barr, Fabio Maltoni, **Leonardo Satrioni**]
Phys. Rev D. 113 (2026) 7, 076007

2. $e^+e^- \to t\bar{t} + g$



[RA, **José Manuel Camacho, Valentin Durupt, Guillermo García Mir**, F. Maltoni, M.M Llácer, **Leonardo Satrioni**, K. Sakurai, and M. Vos, '26]
[hep-ph/2604.16268]

Applications

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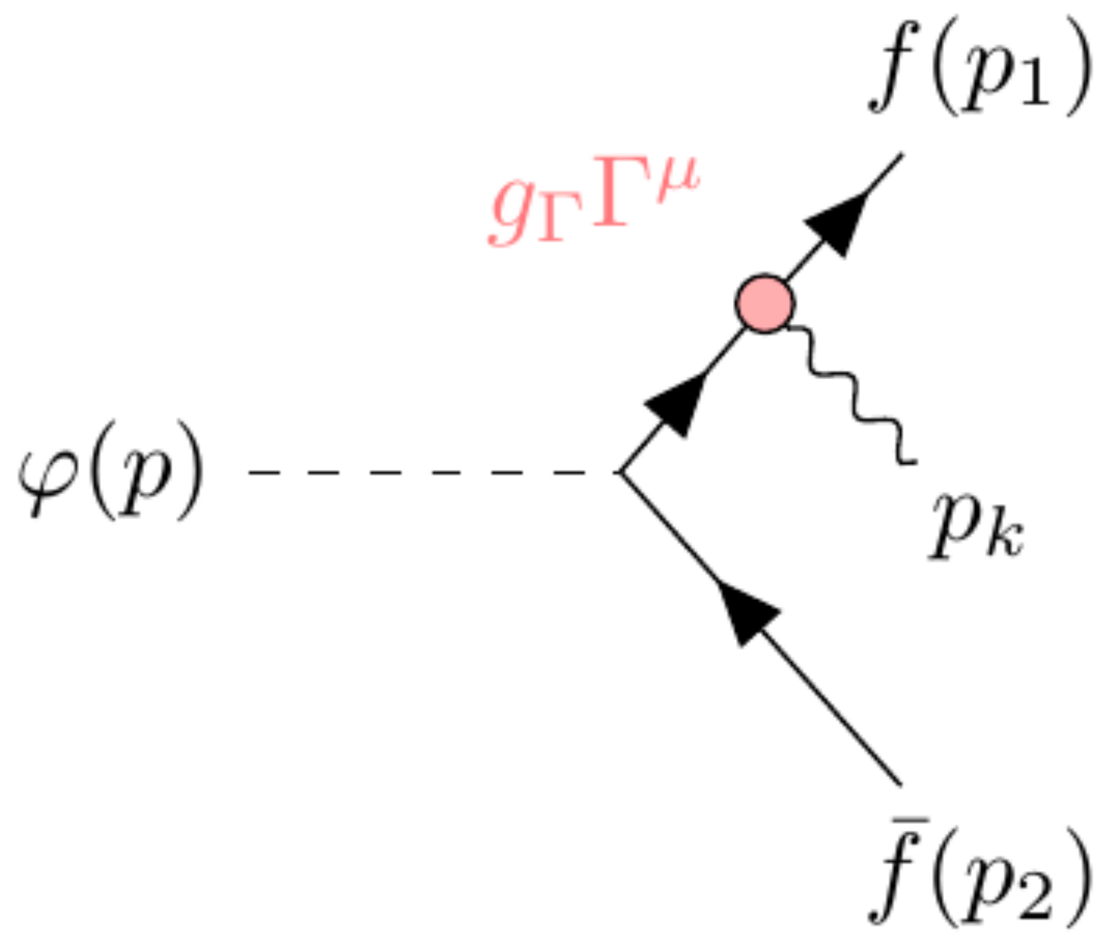

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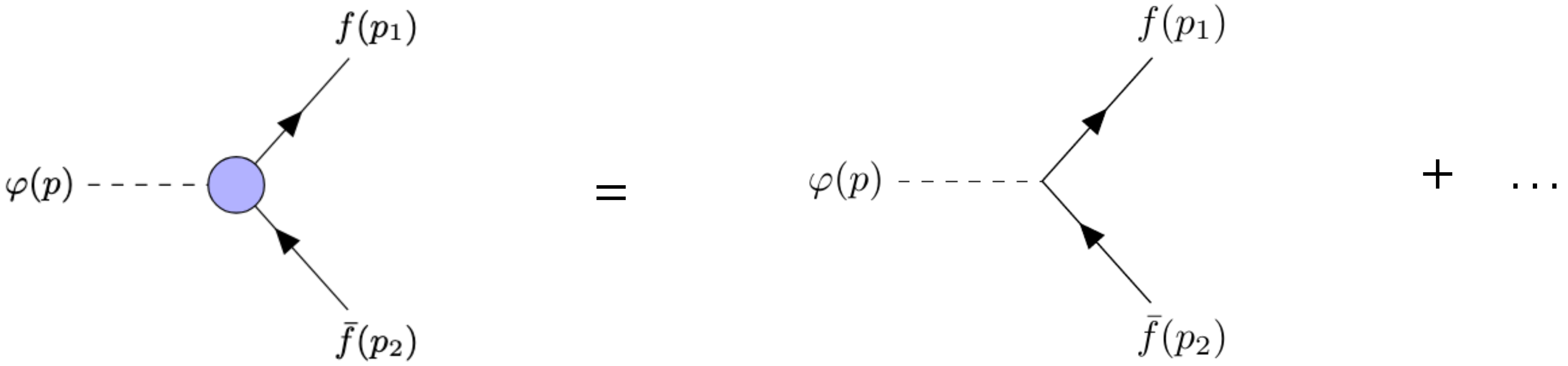
Scalar decay: working example



$$\mathcal{M}^{h_1,h_2} = y_f\, \bar{u}(p_1, h_1) v(p_2, h_2) \quad \longrightarrow \quad R_{\mathrm{LO}} = |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|$$

At tree-level: $R_{\mathrm{LO}} = \dfrac{4N_C y_f^2 m_f^2 \beta^2}{1-\beta^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\qquad \rho_{\mathrm{LO}} = \dfrac{1}{\mathrm{tr}[R_{\mathrm{LO}}]} R_{\mathrm{LO}} = |\Psi^+\rangle\langle\Psi^+|$

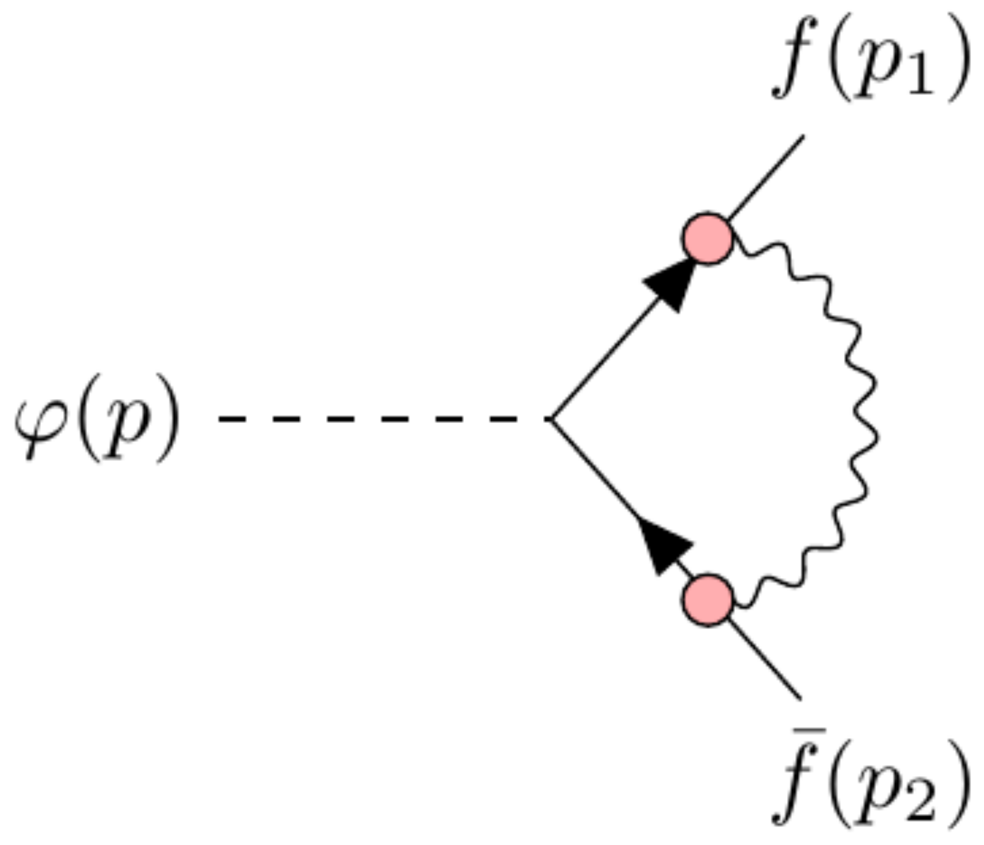
Maximally entangled: controlled place to study entanglement decrease

Scalar decay: working example

General interaction: Scalar, pseudo scalar, vector and axial

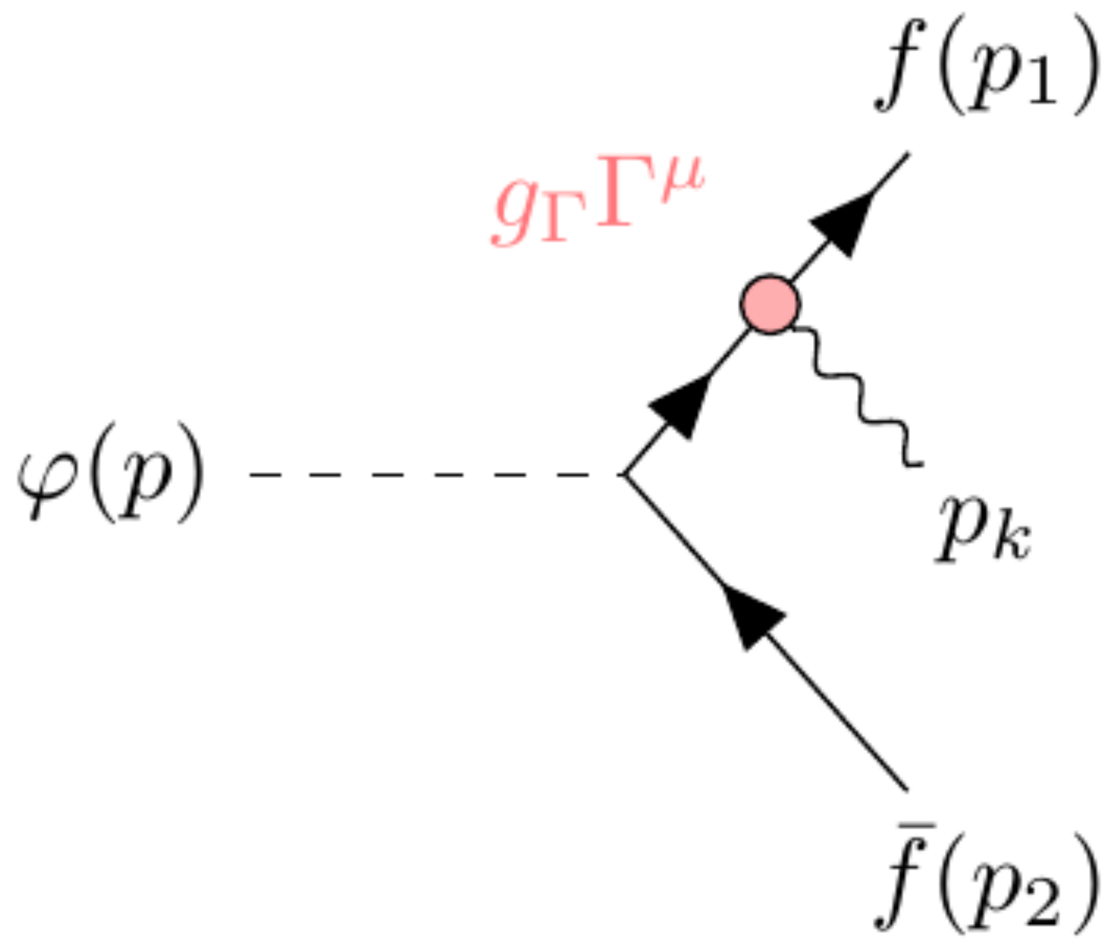
$$g_\Gamma \Gamma^\mu = \{g_S 1, g_P \gamma^5, g_V \gamma^\mu, g_A \gamma^\mu \gamma^5\}$$

Virtual correction: one-loop



+

Real emission



Same Hilbert space

Trace over the extra d.o.f (environment)

Quantum Map.
Open Quantum system

Virtual and Real corrections

$$\mathcal{M}_{\mathrm{LO}} = y_t \bar{u}(p_1, h_1) v(p_2, h_2)$$

$$\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt.}} = y_t \bar{u}(p_1, h_1)\, \mathbb{T}\, v(p_2, h_2)$$

$$\mathcal{M}_{\mathrm{NLO}}^{\mathrm{real}} = y_t\, g_s\, \varepsilon_\mu^\dagger(p_k)[\bar{u}(p_1, h_1) \mathbb{T}^\mu v(p_2, h_2)]$$

Virtual and Real corrections

$$\mathcal{M}_{\rm LO} = y_t \bar{u}(p_1, h_1) v(p_2, h_2)$$

$$\mathcal{M}_{\rm NLO}^{\rm virt.} = y_t \bar{u}(p_1, h_1)\, \mathbb{T}\, v(p_2, h_2) \xrightarrow[\text{Dirac Eq.}]{\text{Scalar current}} y_t \tilde{T}_{\rm virt.} \bar{u}(p_1, h_1) v(p_2, h_2)$$

$$\mathcal{M}_{\rm NLO}^{\rm real} = y_t\, g_s\, \varepsilon_\mu^\dagger(p_k) [\bar{u}(p_1, h_1) \mathbb{T}^\mu v(p_2, h_2)]$$

Virtual and Real corrections

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$$p_1^\mu, p_2^\mu \qquad\qquad \gamma^\mu \qquad\qquad p_\nu \sigma^{\mu\nu}$$

Virtual and Real corrections

$$\mathcal{M}_{\mathrm{LO}} = y_t \bar{u}(p_1, h_1) v(p_2, h_2)$$

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$$p_1^\mu, p_2^\mu \longleftarrow \cancel{\gamma^\mu} \longrightarrow p_\nu \sigma^{\mu\nu}$$

Gordon identity

Virtual and Real corrections

$$\mathcal{M}_{\mathrm{LO}} = y_t \bar{u}(p_1, h_1) v(p_2, h_2)$$

$$\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt.}} = y_t \bar{u}(p_1, h_1)\, \mathbb{T}\, v(p_2, h_2) \xrightarrow[\text{Dirac Eq.}]{\text{Scalar current}} y_t \tilde{T}_{\mathrm{virt.}} \bar{u}(p_1, h_1) v(p_2, h_2)$$

$$\mathcal{M}_{\mathrm{NLO}}^{\mathrm{real}} = y_t\, g_s\, \varepsilon_\mu^\dagger(p_k) [\bar{u}(p_1, h_1) \mathbb{T}^\mu v(p_2, h_2)]$$

p_1^μ, p_2^μ

Monopole

(IR-div)

$p_\nu \sigma^{\mu\nu}$

Dipole term

(IR-finite)

Density matrices

$$R_{\mathrm{LO}} = |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|$$

$$R_{\mathrm{NLO}}^{\mathrm{virt.}} = |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt}}| + |\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|$$

$$R_{\mathrm{NLO}}^{\mathrm{real}} = \sum_h |\mathcal{M}_{\mathrm{LO}}^{\mathrm{real}}(h)\rangle\langle\mathcal{M}_{\mathrm{NLO}}^{\mathrm{real}}(h)|$$

Density matrices

$$R_{\mathrm{LO}} = |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|$$

Scalar case

$$R_{\mathrm{NLO}}^{\mathrm{virt.}} = |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt}}| + |\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt}}\rangle\langle\mathcal{M}_{\mathrm{LO}}| = \tilde{p}_{\mathrm{virt.}}|\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|$$

$$R_{\mathrm{NLO}}^{\mathrm{real}} = \sum_h |\mathcal{M}_{\mathrm{LO}}^{\mathrm{real}}(h)\rangle\langle\mathcal{M}_{\mathrm{NLO}}^{\mathrm{real}}(h)|$$

$$= \tilde{p}_{\mathrm{real}}|\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}| + \tilde{q}\sum_j K_j|\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{LO}}|K_j^\dagger$$

Kraus operators

Soft and Collinear Kraus Operators

In the soft limit $\lim_{E_g \to 0} \boldsymbol{\rho}_{[\phi \to t\bar{t}g]} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = |\Psi^+\rangle\langle\Psi^+| = \boldsymbol{\rho}_{[\phi \to t\bar{t}]}.$ and the map is trivial

In the (strictly) collinear limit $\lim_{\theta \to 0,\pi} \boldsymbol{\rho}_{[\phi \to t\bar{t}g]} = \frac{1}{2}\left(|\Phi^-\rangle\langle\Phi^-| + |\Phi^+\rangle\langle\Phi^+|\right) \equiv \boldsymbol{\rho}^{\text{coll}}_{[\phi \to t\bar{t}g]}.$

Kraus operators $K_j := \left\{ \frac{1}{\sqrt{2}}\,\sigma_1 \otimes \mathbb{I}_2, \quad \frac{1}{\sqrt{2}}\,\sigma_2 \otimes \mathbb{I}_2 \right\}$ Non-unique set

$$\boxed{\boldsymbol{\rho}_{[\phi \to t\bar{t}]} \to \boldsymbol{\rho}^{\text{coll}}_{[\phi \to t\bar{t}g]} = \mathcal{E}^{\text{coll}}_{[\phi \to t\bar{t}g]}\left[\boldsymbol{\rho}_{[\phi \to t\bar{t}]}\right]}$$

Full NLO map

$$\boxed{\mathcal{E}_{\mathrm{full}}[\rho_{\mathrm{LO}}] = \mathsf{p}_{\mathrm{id}}\, \mathbb{1}\rho_{\mathrm{LO}}\mathbb{1} + \mathsf{q} \sum_{j\neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^{\dagger}}$$

$$\mathsf{p}_{\mathrm{id}} = (\mathrm{p}_{\mathrm{LO}} + \mathrm{p}_{\mathrm{V}} + \mathrm{p}_{\mathrm{R}}^{\mathrm{soft}} + \mathrm{p}_{\mathrm{R}}^{\mathrm{col.}})$$

Identity part:
does not change entanglement

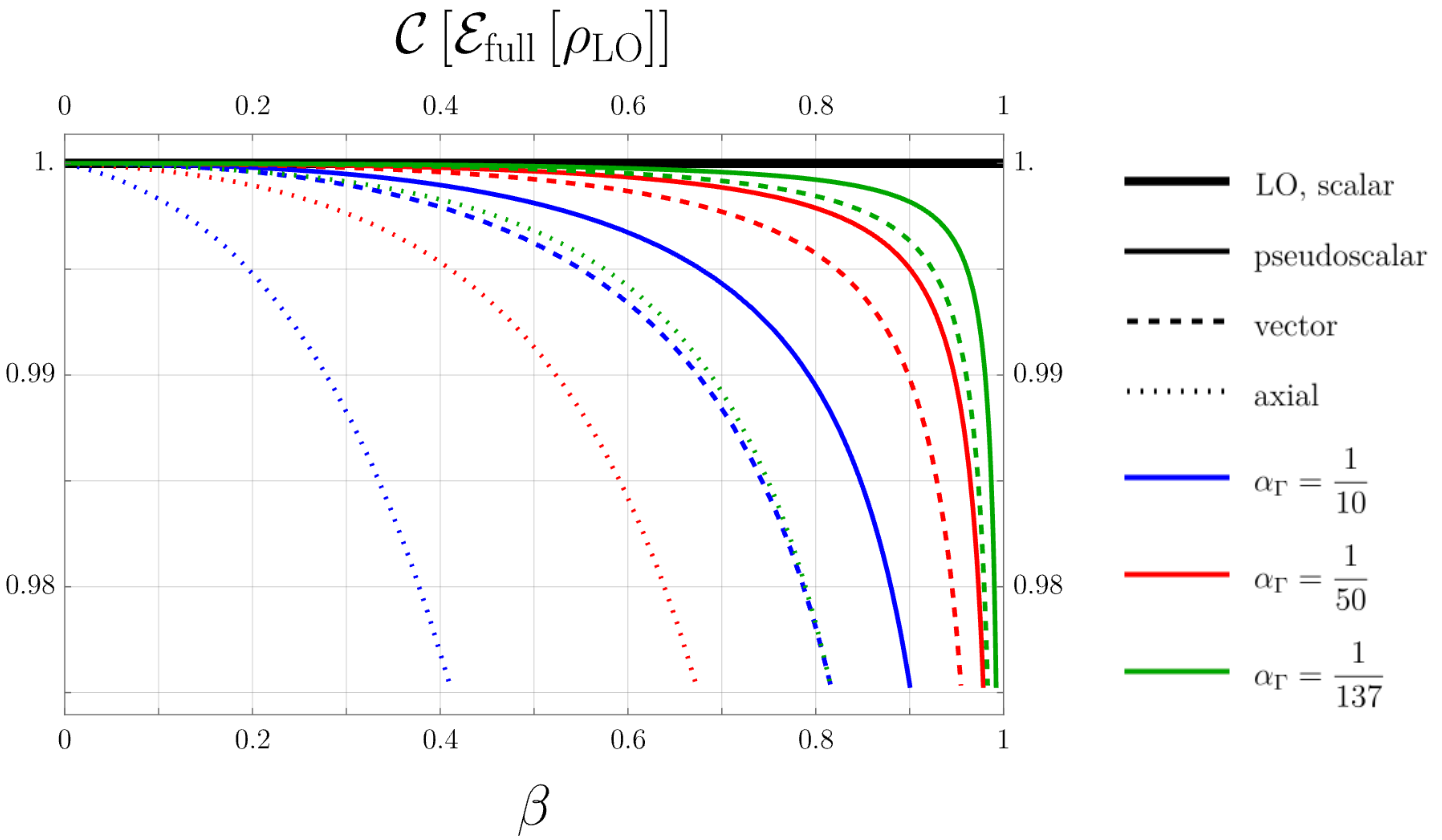
$$\mathsf{q} = \mathsf{q}^{\mathrm{col.}}$$

Non-trivial Kraus part:
Decoherence!

*in the leading soft/collinear limit

Decoherence

Concurrence:



$\beta = \sqrt{1 - 4m_f^2/m_\varphi^2}$

Entanglement is lost mainly due to collinear emission $\longrightarrow$ Small effects ~ 1%

Gluon energy dependence

At leading-order

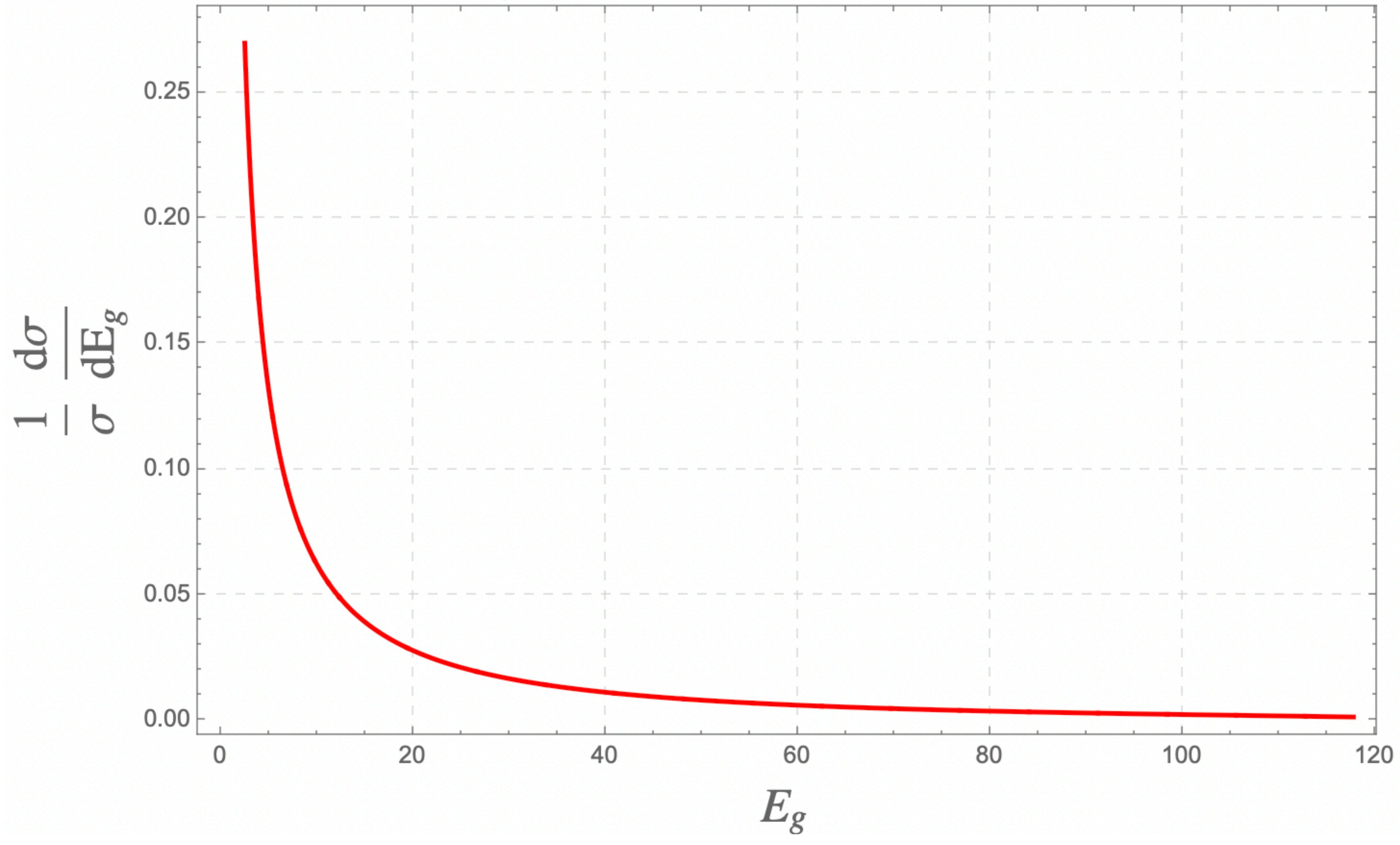
$$\rho_{\mathrm{LO}} = |\Psi^+\rangle\langle\Psi^+|$$

Just summing over the helicity

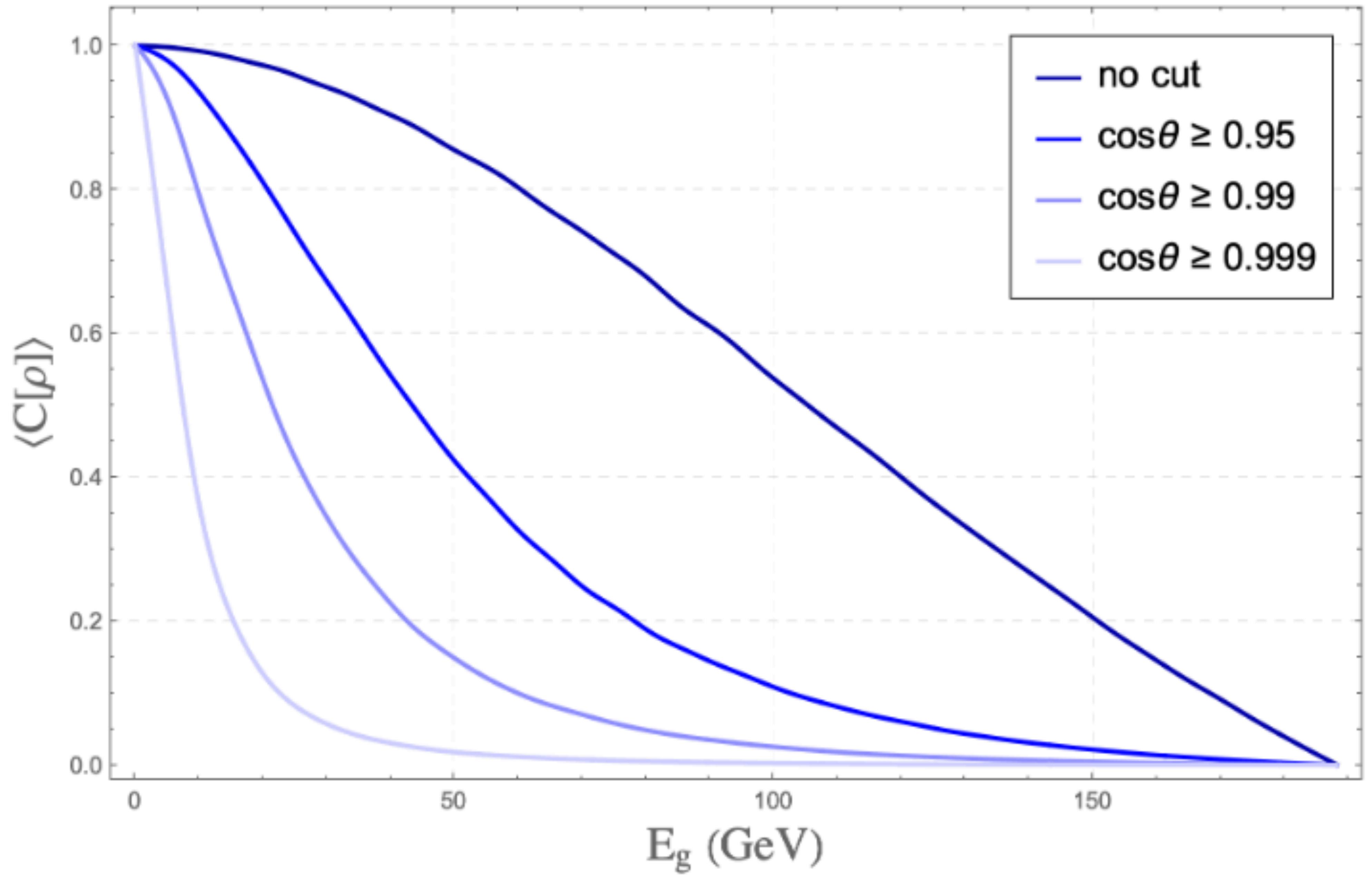
$$\rho^{\mathrm{red}}_{\mathrm{NLO}} = \mathrm{tr}_{\mathrm{hel}}[\rho_{\mathrm{NLO}}] = \rho^{\mathrm{red}}_{\mathrm{NLO}}(E_g, \cos\theta_g)$$

Collinear gluons - decoherence

$\frac{1}{\sigma}\frac{d\sigma}{dE_g}$ for $h \to t\bar{t}$, $\sqrt{s}$ = 500 GeV

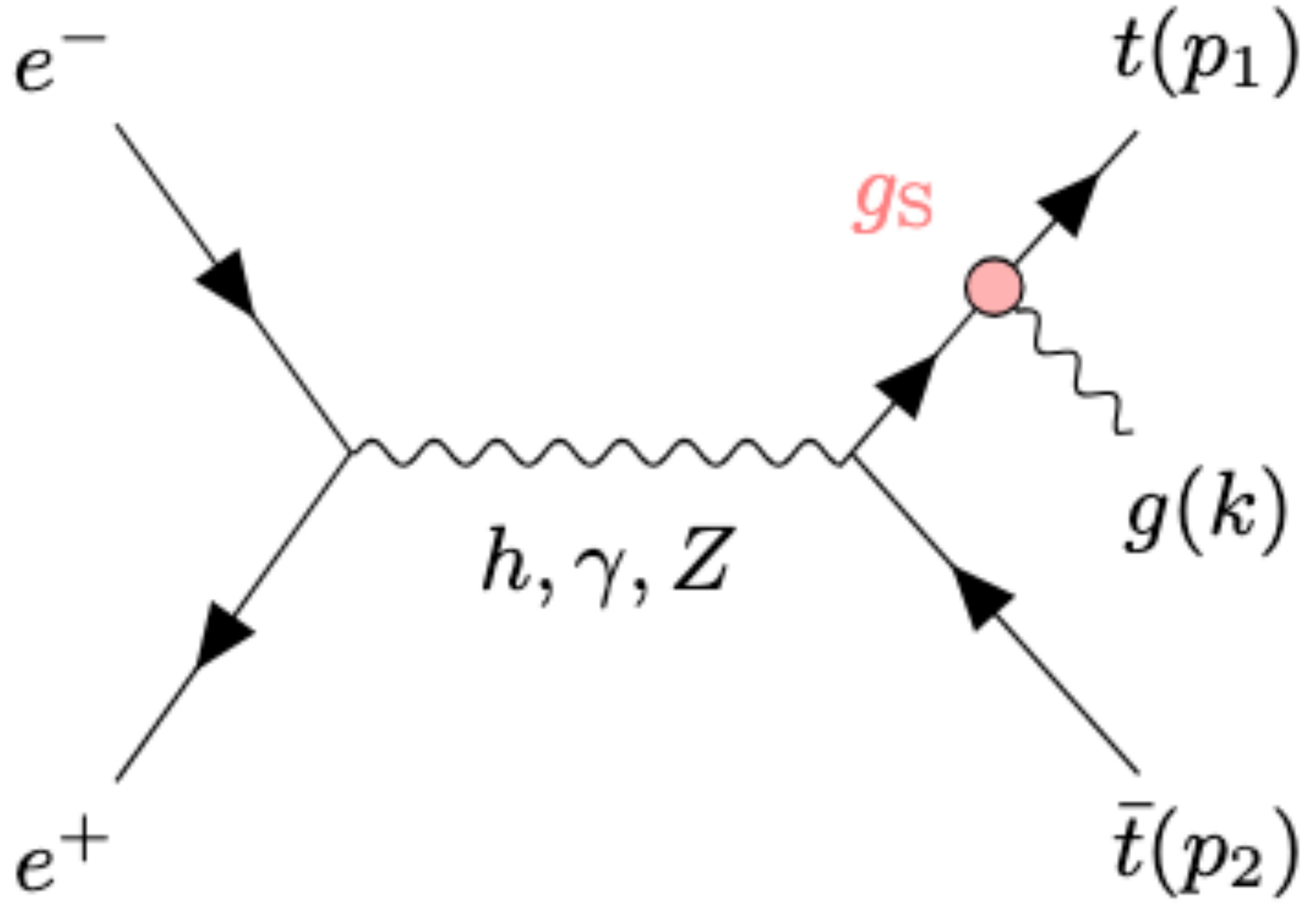


$\langle C[\rho]\rangle$ for $\phi \to t\bar{t}g$, $\sqrt{s}$ = 500 GeV, $t\bar{t}$ rest frame



Applications

2. $e^+e^- \to t\bar{t} + g$



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[hep-ph/2604.16268]

What changes?

- 5-point kinematics / summation over initial d.o.f

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- 5-point kinematics / summation over initial d.o.f
- Virtual has new structures compared to the LO.

$$R_{\mathrm{NLO}}^{\mathrm{virt.}} = \sum_{\text{in dof}} |\mathcal{M}_{\mathrm{LO}}\rangle\langle\mathcal{M}_{\mathrm{NLO}}^{\mathrm{virt.}}| + \text{h.c.}$$

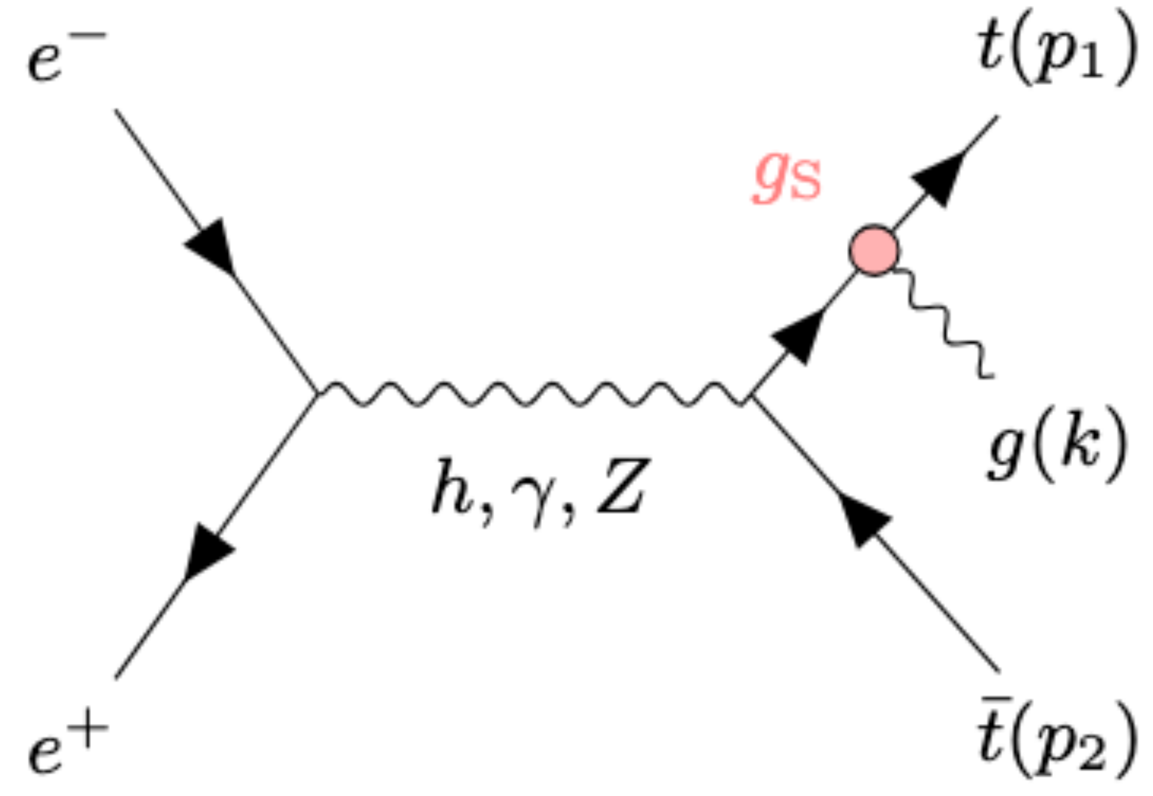
$$= -ig_S^2 C_A \left(J_1 R_{\mathrm{LO}} + J_0 R_{\mathrm{d}}\right)$$

$J_{1,0}$ linear combinations of PV functions (A-Ds)

J_0 is IR-finite $\qquad$ J_1 is IR-div $\longrightarrow$ cancels with real.

What changes?

- 5-point kinematics / summation over initial d.o.f
- Virtual has new structures compared to the LO.
- Real follows same approach (tensor operator)



production decoherence

$$\mathcal{M}_{\text{NLO}}^{\text{real}} = y_t\, g_s \underbrace{\frac{J_\nu^{ee}}{s - m_V^2}}_{\text{production}} \times \underbrace{[\bar{u}(p_1, h_1)\mathbb{T}^{\mu\nu} v(p_2, h_2)]\varepsilon_\mu^\dagger(p_k)}_{\text{decoherence}}$$

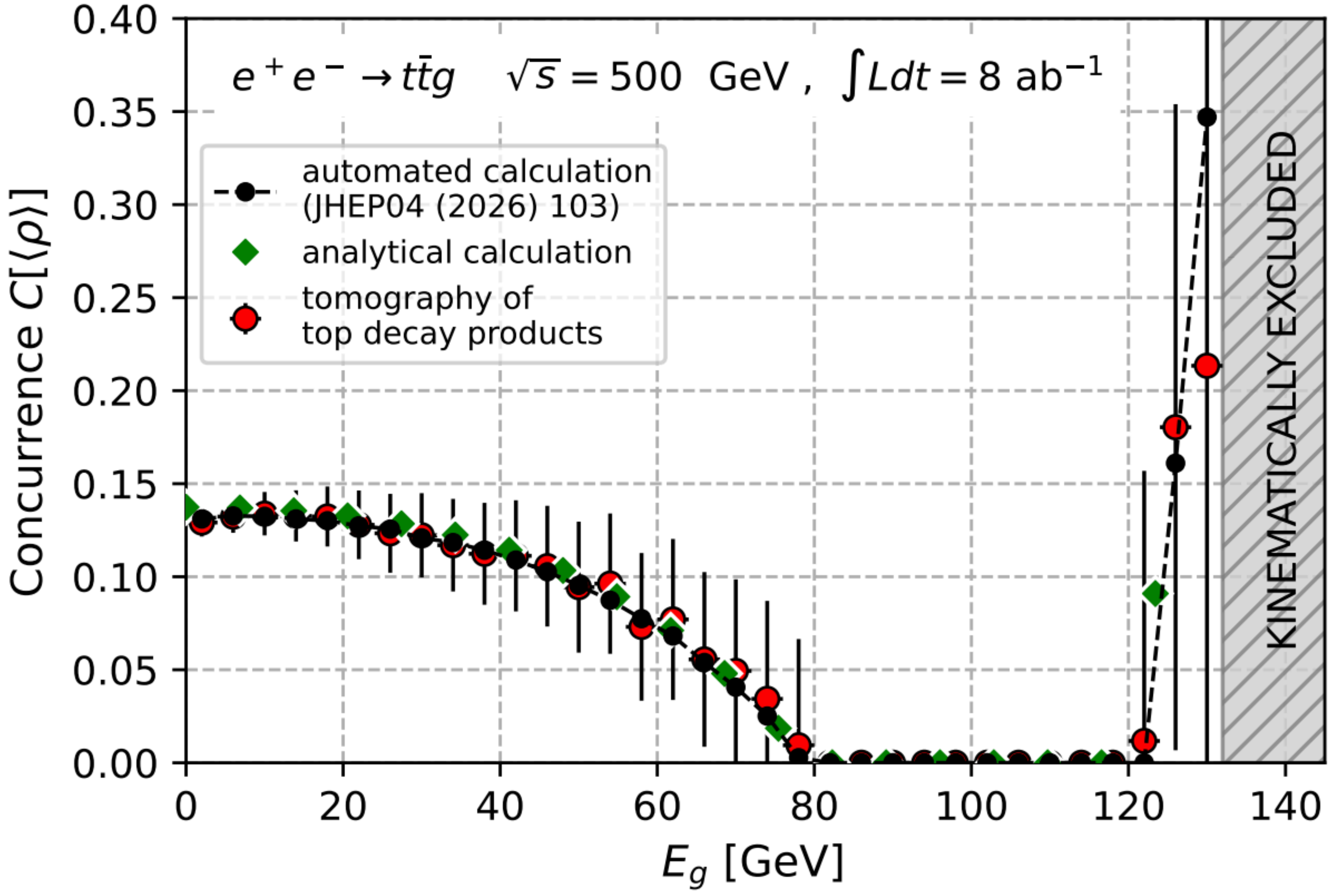
What changes?

- 5-point kinematics / summation over initial d.o.f
- Virtual has new structures compared to the LO.
- Real follows same approach (tensor operator)
- Final differential density matrix

$$\mathrm{d}\boldsymbol{\rho} = \frac{1}{\mathrm{tr}[\boldsymbol{R}]}\left[\left(\boldsymbol{R}_{\mathrm{LO}} + \boldsymbol{R}_{\mathrm{NLO}}^{\mathrm{virt.}}\right)\mathrm{d}\Phi_2 + \boldsymbol{R}_{\mathrm{NLO}}^{\mathrm{real}}\mathrm{d}\Phi_3\right]$$

$e^+e^- \to t\bar{t} + g$

For $E_g > 0$

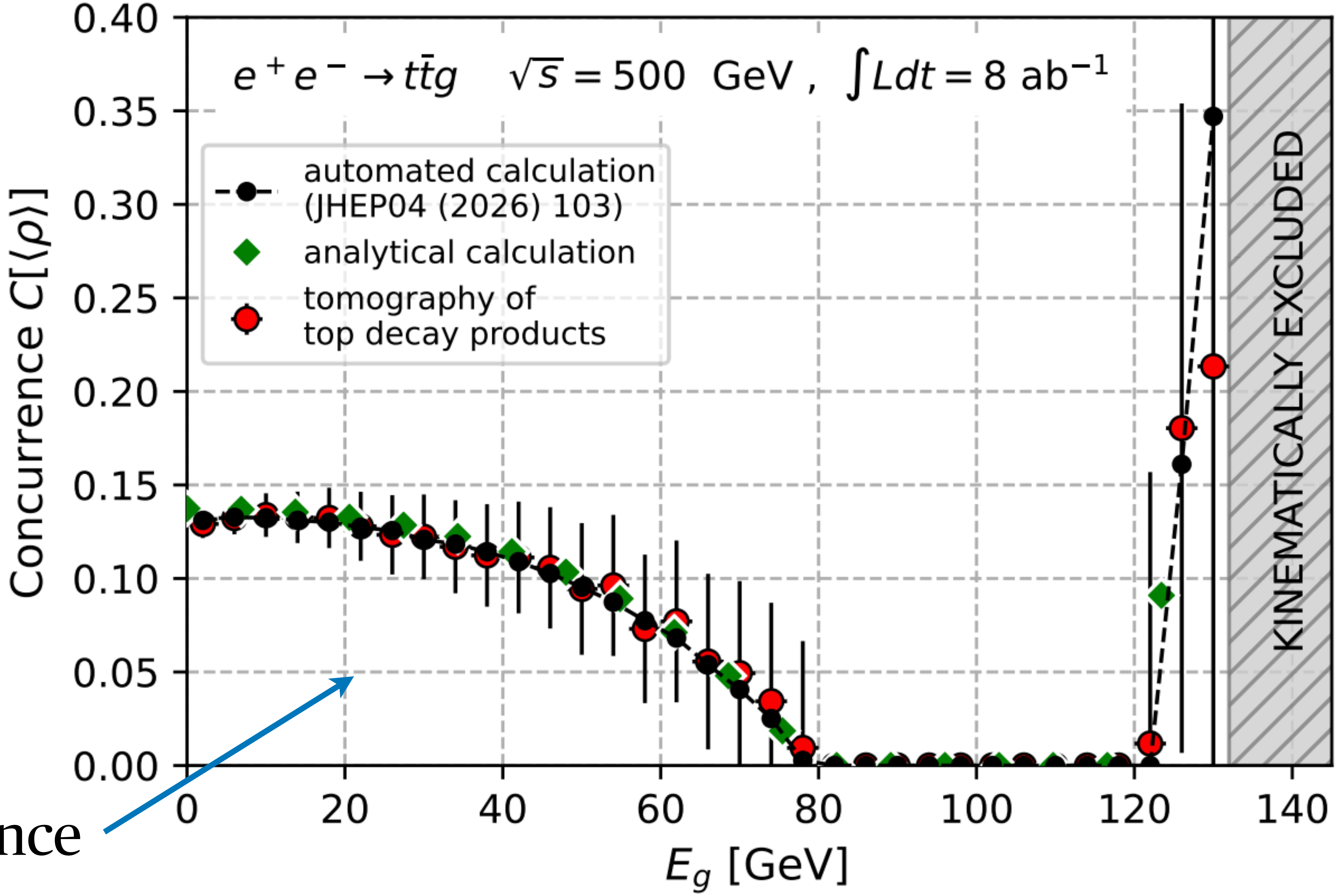


Tomography:
MadGraph+MadSpin
From decay products

For automated, see
[Durupt, Maltoni, Mattelaer]

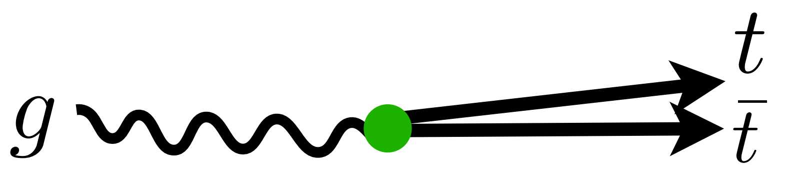
$e^+e^- \to t\bar{t} + g$

For $E_g > 0$

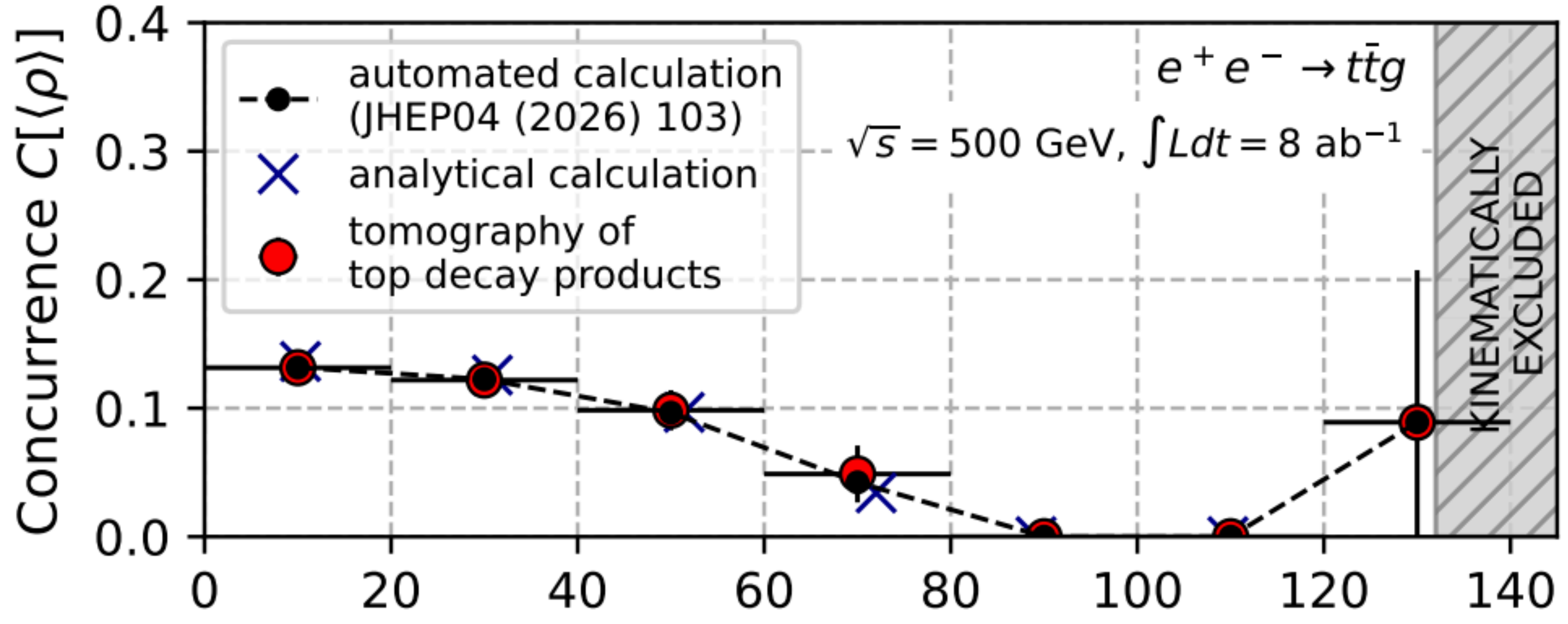


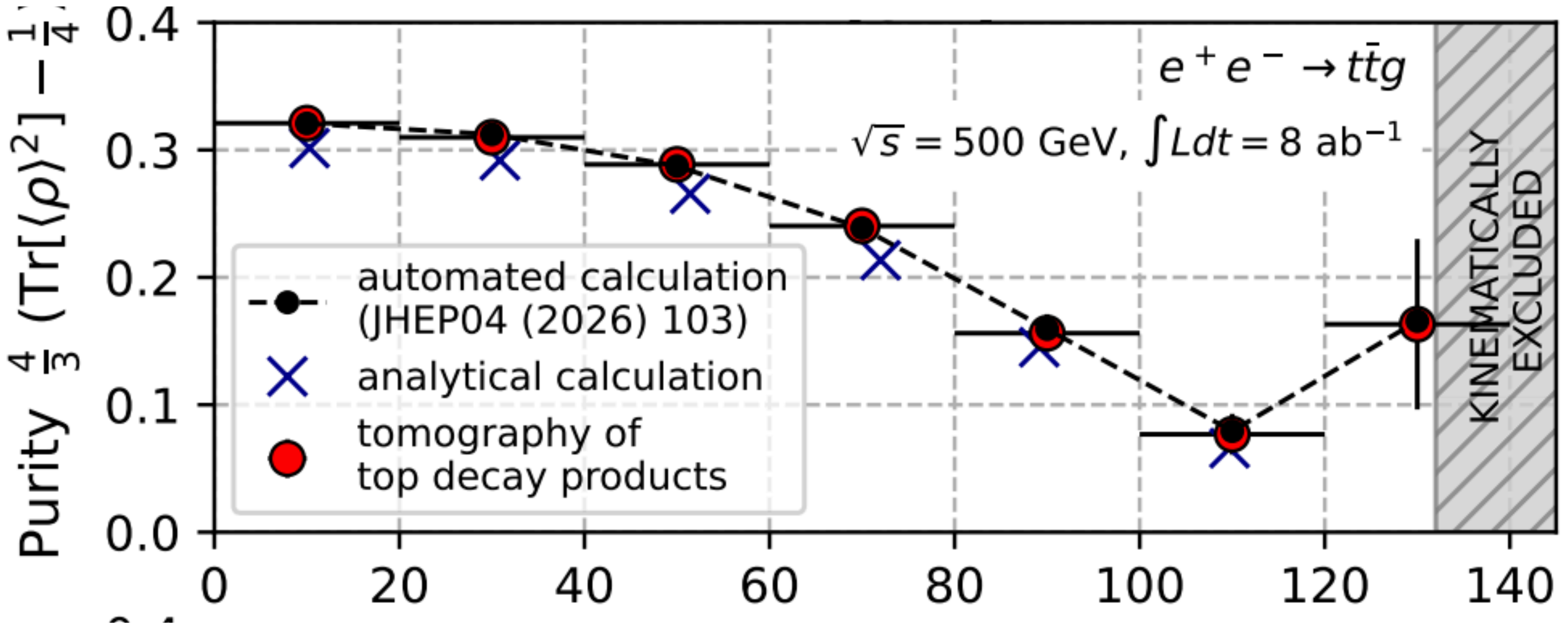
Decoherence

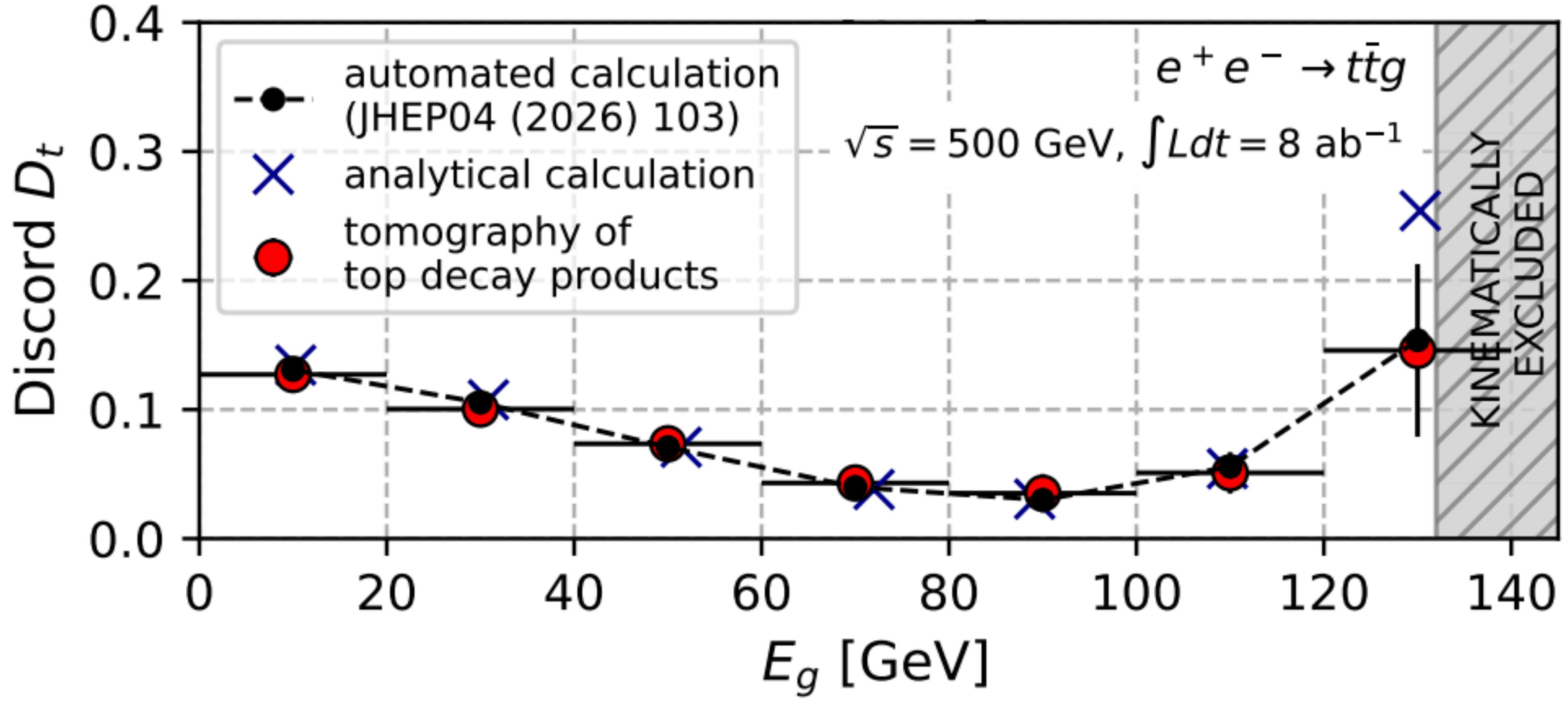
End-point kin.



Other measures: Purity and Discord







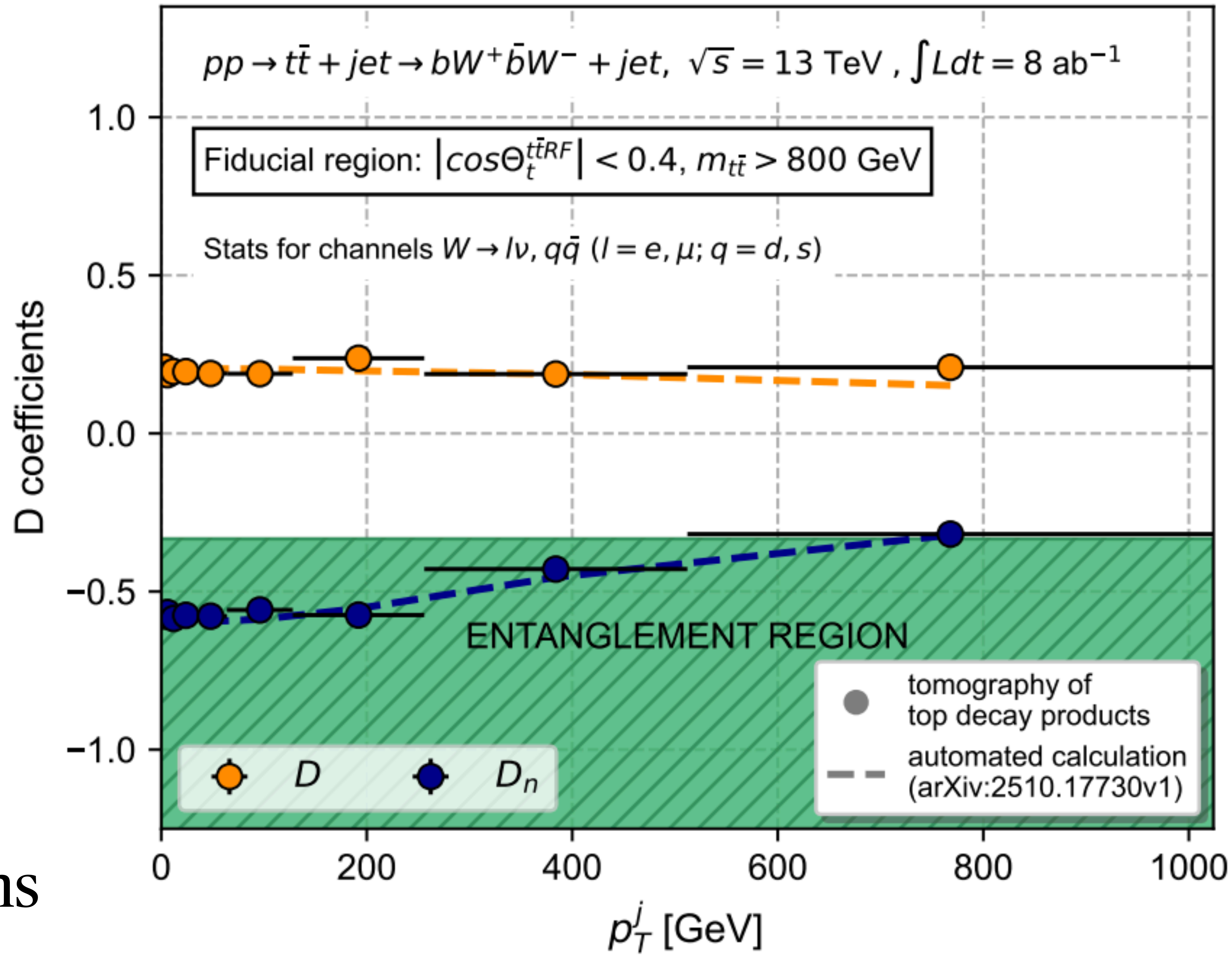
LHC

New complications:

- gg initial state (mixed)
- boosted vs. threshold regions

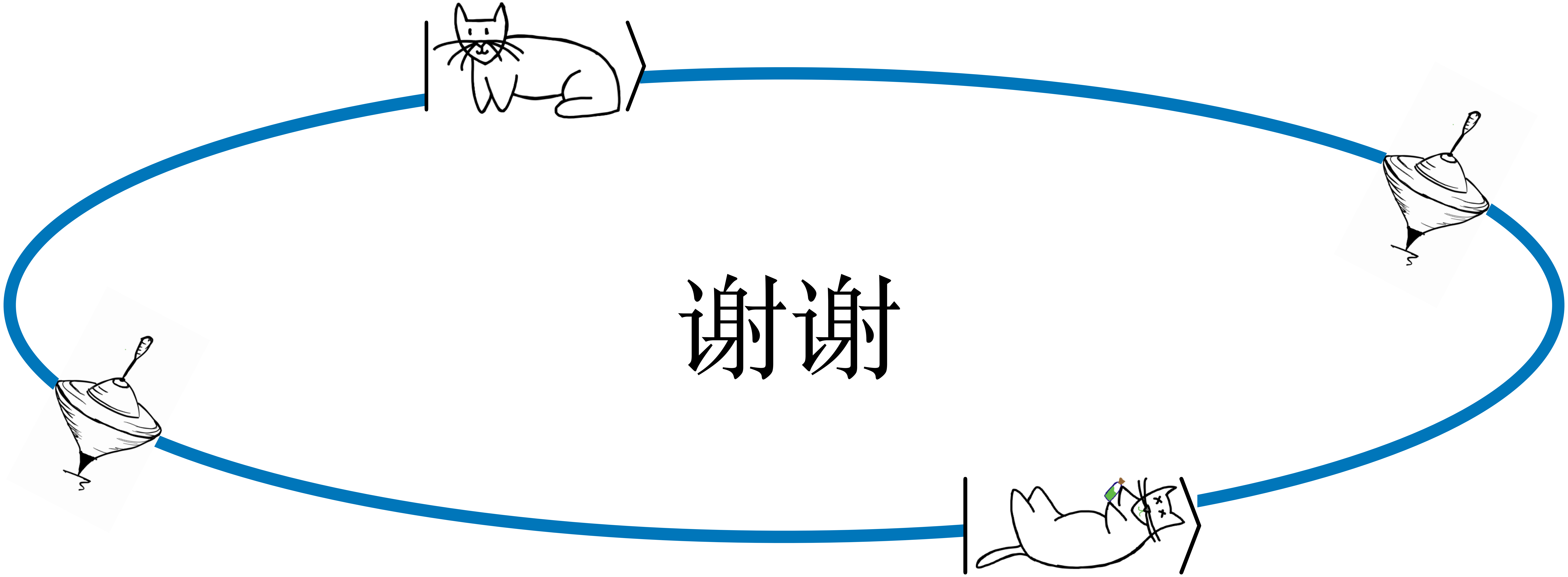
Caveats:

- no ISR
- no systematics
- improve system/env. definitions



Conclusions

- Open quantum systems offer us new insights
- Collinear radiation is responsible to flip the top *spin* (dipole operator)
- Soft radiation, long-range, effects do not flip*
- Experimental prospects
- Further studies:
    - Resummation for massless quarks using SCET [Gu, Lin, Shao, Wang, Yang '25]
    - Decoherence in the Radiative Z decay [Cheng, Han, Singh, Zu '26]
- Many more to explore! Qutrits, N^2LO effects, next-to-soft, ...


谢谢

Backup slides

IR safe?

- IR cancellations are in the “identity part” of the map

Cancel as in the cross-section/decay: KLN theorem

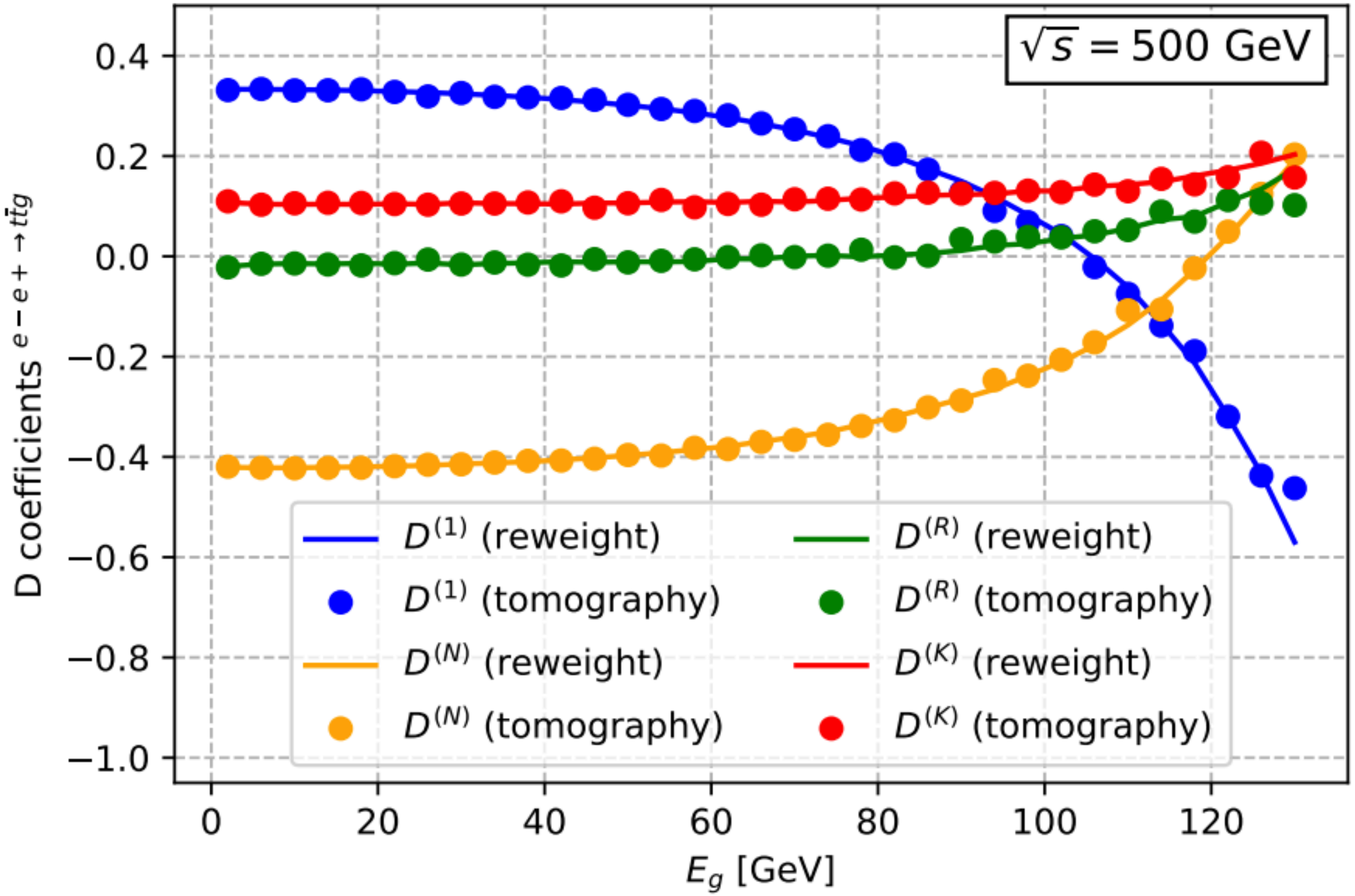
- By power counting, one can see that the identity part contains integrals of

$$\int \mathrm{d}\Phi(k) \frac{1}{(p_i \cdot k)(p_j \cdot k)} \qquad i, j = 1, 2 \longrightarrow \text{IR divergent}$$

- While the non-trivial Kraus has (rank-n tensor integral) $$\int \mathrm{d}\Phi(k) \frac{k^{\mu_1} \dots k^{\mu_n}}{(p_i \cdot k)(p_j \cdot k)}$$ IR finite for n > 0

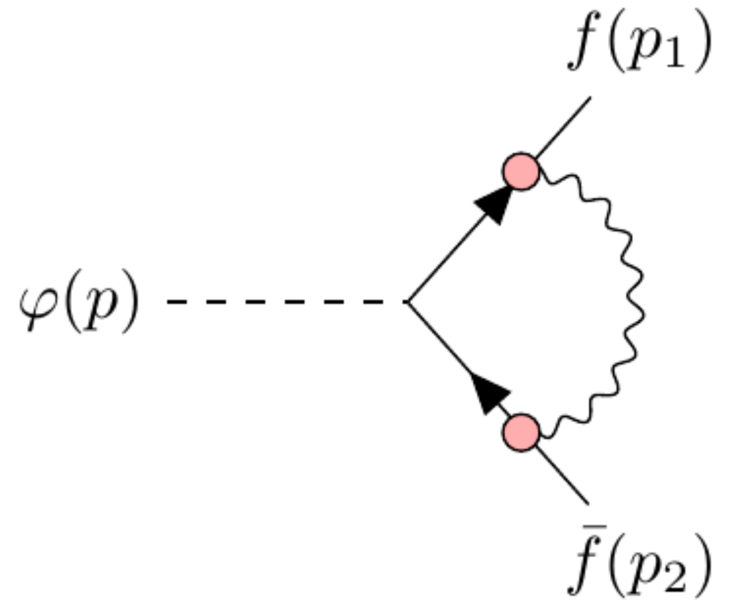
Entanglement markers

$$D=\frac{1}{3}\left(C_{kk}+C_{rr}+C_{nn}\right)\ ,$$
$$D_n=\frac{1}{3}\left(C_{kk}+C_{rr}-C_{nn}\right)\ ,$$
$$D_r=\frac{1}{3}\left(C_{kk}-C_{rr}+C_{nn}\right)\ ,$$
$$D_k=\frac{1}{3}\left(-C_{kk}+C_{rr}+C_{nn}\right).$$



Loop Corrections argument

$$\mathcal{M}_{\mathrm{LO}} = y_t \bar{u}(p_1, h_1) v(p_2, h_2)$$



$$\mathcal{M}^{\mathrm{virt.}}_{\mathrm{NLO}} = y_t \bar{u}(p_1, h_1)\, \mathbb{T}\, v(p_2, h_2)$$

$\mathbb{T}$ Most general structure

p_1^μ, p_2^μ $\quad\quad \gamma_\mu, \quad \gamma_\mu\gamma_\nu, \quad \gamma_\mu\gamma_\nu\gamma_\alpha, \cdots \longrightarrow$ Chains of $\not{p}_1$, $\not{p}_2$

Particles on-shell: Dirac Equation

All structures reduce down to identity

True to all loop orders.

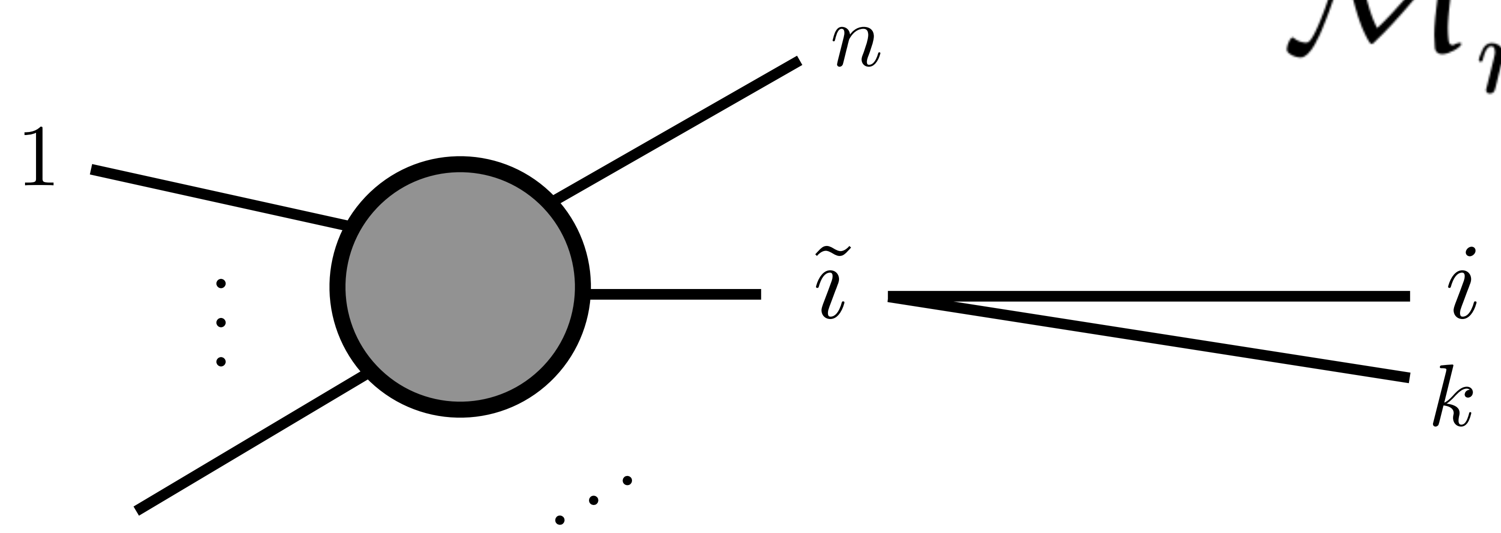
Special for scalar.

$$\longrightarrow \quad \bar{u}(p_1, h_1)\mathbb{T}_{\mathrm{virt.}} v(p_2, h_2) = \tilde{T}_{\mathrm{virt.}} \bar{u}(p_1, h_1) v(p_2, h_2)$$

Splitting functions as Kraus operators

[Richardson, Webster '18]
[Hamilton, Karlberg, Salam, Scyboz, Verheyen '21]

In the collinear limit, a n-parton system undergoes a splitting $\tilde{\imath} \to ik$



$$\mathcal{M}_{n+1}^{\lambda_i \lambda_k}(\cdots, p_i, p_k, \cdots) = \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}} \lambda_i \lambda_k} \; \mathcal{M}_n^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)$$

Helicity dependent AP splitting function

Tracing over the unresolved d.o.f

$$\rho_{\rm red}^{\lambda_i \lambda_i'} = \sum_{\sigma=\pm} \int_{p_k} \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}} \lambda_i \sigma} \cdot \rho^{\lambda_{\tilde{\imath}} \lambda_{\tilde{\imath}}'} \cdot \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}}' \lambda_i' \sigma} \quad = \quad \mathsf{q}^{\rm col} \sum_{j \neq {\rm id}} K_j \rho_{\rm LO} K_j^\dagger$$

Splitting functions as Kraus (here: one emission)