Quantumness in Field Theory

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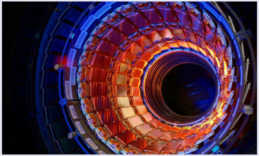
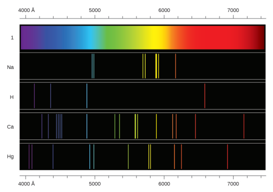
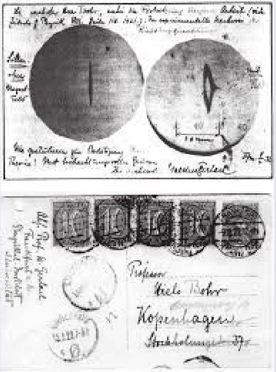


Quantum Theory: Quantization

- Quantum Mechanics originally named after quantized values:
  - Electromagnetic radiation (Black-body/Photoelectric effect)
  - Electron orbits (Atomic spectra)
  - Angular momentum (Stern-Gerlach)





Quantum Theory: Copenhagen Interpretation

- Copenhagen interpretation of Quantum Mechanics:
  - Particles $\longleftrightarrow$ Waves $\rightarrow$ Superposition
  - Outcomes of measurements: Observable eigenvalues $\rightarrow$ Quantization
  - Probabilities of outcomes $\rightarrow$ Wave Intensity $|\Psi|^2$ $\rightarrow$ Interference

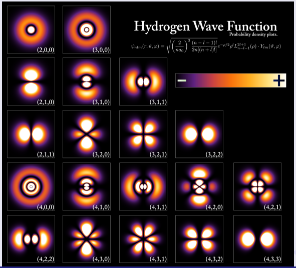
Pure states

- **Pure state** $\to$ Wave function $|\Psi\rangle$
  - Unit (ray) vector in Hilbert space $\mathcal{H}$
  - Coherent superpositions of states with well-defined phase:

$$|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle\,,\ \ \alpha_n \in \mathbb{C},\ \ \langle\Psi|\Psi\rangle = \sum_n |\alpha_n|^2 = 1$$

  - Born's rule: Projective measurements of observable A yield eigenvalue a_n with probability $|\alpha_n|^2$

$$A\,|\phi_n\rangle = a_n\,|\phi_n\rangle \implies \langle A\rangle = \langle\Psi|A|\Psi\rangle = \sum_n |\alpha_n|^2 a_n$$



Quantum vs. Classical

- Quantum Mechanics:
  - Probabilistic wave
  - *Fundamental* randomness
- Classical Mechanics:
  - Classical probability distributions
  - Randomness from *ignorance* (noise, experimental variations...)
- Is God *just* playing dice with the Universe? $\rightarrow$

Quantum vs. Classical

- Quantum Mechanics:
  1. Probabilistic wave
  2. *Fundamental* randomness
- Classical Mechanics:
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- Is God *just* playing dice with the Universe? $\rightarrow$ God well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.

Pure states: Entanglement

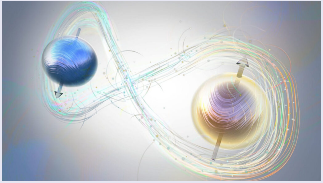
- Bipartite system A and B (Alice and Bob): $\mathcal{H}=\mathcal{H}_A\otimes\mathcal{H}_B$
- Separable states: Product states $\rightarrow$ Uncorrelated

$$|\Psi\rangle=|\psi\rangle\otimes|\chi\rangle$$

- Quantum superposition $\rightarrow$ **Entanglement**=Nonseparability

$$|\Psi\rangle=\sum_{n,m}\alpha_{nm}|\phi_n\rangle\otimes|\varphi_m\rangle\neq|\psi\rangle\otimes|\chi\rangle$$

- Separability $\Longleftrightarrow$ $\mathrm{rank}(\alpha_{nm})$=1
- Pure states: Classical and quantum correlations (entanglement) are **equivalent** $\rightarrow$ Important source of confusion!



Mixed states

- Real life is complicated: environments, subsystems, noise...
- **Mixed state** $\rightarrow$ Generalization of pure state $|\Psi\rangle$ to density matrix ρ
  - Hermitian non-negative operator with unit trace
  - Pure states $|\Psi\rangle \Longrightarrow$ Projectors $\rho = |\Psi\rangle\langle\Psi|$
  - Incoherent mixture of pure (eigen-)states $|\phi_n\rangle \rightarrow p_n$ are probabilities

$$\rho = \sum_n p_n \cdot |\phi_n\rangle\langle\phi_n|\,,\ \mathrm{Tr}\rho = \sum_n p_n = 1,\ p_n \geq 0$$

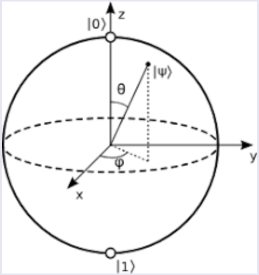
  - Expectation values: $\langle A\rangle = \mathrm{Tr}(\rho A) = \sum_n p_n \langle\phi_n|A|\phi_n\rangle$

Qubits: Pure state

- Qubit: Two-level quantum system $|0\rangle, |1\rangle \rightarrow$ Most simple!
- Paradigmatic example: Spin-1/2 particle. $|0\rangle \equiv |+\rangle$, $|1\rangle \equiv |-\rangle$
- General wave function: $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \equiv |\hat{\mathbf{n}}\rangle \rightarrow$ Eigenstate of spin projection: $\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}|\hat{\mathbf{n}}\rangle = |\hat{\mathbf{n}}\rangle$
- Projective measurements with ± 1 outcome:

$$\Pi_{\pm\hat{\mathbf{n}}} = |\pm\hat{\mathbf{n}}\rangle\langle\pm\hat{\mathbf{n}}| = \frac{1 \pm \boldsymbol{\sigma}\cdot\hat{\mathbf{n}}}{2}$$

- Unit vector $\hat{\mathbf{n}} = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta] \rightarrow$ Surface of Bloch sphere.



Qubits: Density matrix

- General density matrix (2×2) for 1 qubit $\rightarrow$ 3 parameters B_i (Bloch vector):

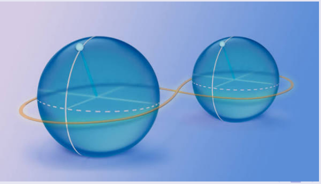
$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2},\ \mathbf{B} = \mathrm{Tr}[\boldsymbol{\sigma}\rho],\ |\mathbf{B}| \leq 1$$

- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix (4×4) for 2 qubits $\rightarrow$ 15 parameters $B_i^{\pm}, C_{ij}$

$$\rho = \frac{\mathbb{1} + \sum_i \left(B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- Polarization (Bloch) vectors $\mathbf{B}^{\pm}$ and correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle,\ B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle,\ C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$



Quantum Tomography

- **Quantum Tomography**: Reconstruction of quantum state from measurement of a *quorum* of observables.
- One-qubit tomography=3 parameters: polarization vector **B**

$$B_i = \langle \sigma^i \rangle$$

- Two-qubit tomography=15 parameters: polarization vectors $\mathbf{B}^{\pm}$ and correlation matrix **C**

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle\,,\ B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle\,,\ C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$

Coherence

- Coherence=Presence of non-diagonal density-matrix elements

$$\rho = \sum_{nm} \rho_{nm} \left| n \right\rangle \left\langle m \right|$$

- Coherence $\rightarrow$ Interference

$$\langle O \rangle = \sum_{n=m} \rho_{nn} \langle n|O|n \rangle + \sum_{n \neq m} \rho_{nm} \langle m|O|n \rangle$$

- Basis-dependent on situation of interest.
- Classical correlations emerge even for incoherent bipartite states:

$$\rho_{\text{class}} = \sum_{n,m} p_{nm} \left| n \right\rangle \otimes \left| m \right\rangle \left\langle n \right| \otimes \left\langle m \right|$$

- Where did the quantum go????

Quantum Discord

- Classically, two equivalent expressions for mutual information of bipartite system:

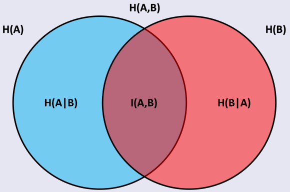
$$
\begin{aligned}
I(A,B) &= H(A)+H(B)-H(A,B) = H(A)-H(A|B) \\
H(A,B) &= -\sum_{x,y} p(x,y)\log_2 p(x,y) \\
H(A|B) &= \sum_{y} p(y)H(A|B=y)
\end{aligned}
$$

- Quantum Mechanics can introduce a "discord" between both expressions:

$$\mathcal{D}(A,B) \equiv H(B)-H(A,B)+H(A|B) \neq 0$$

- Most basic form of quantum correlations!
- Quantum discord is asymmetric: $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$

PRL 88, 017901 (2001)



Quantum Discord: Classical states

- OK...where is the Physics? $\rightarrow$ Only classical states have zero discord!

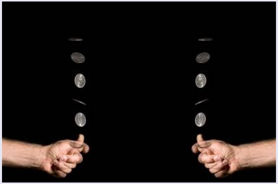
$$\rho_{\text{class}} = \sum_{n,m} p_{nm} |n\rangle \otimes |m\rangle \langle n| \otimes \langle m|$$

  - $|n\rangle, |m\rangle$: *orthonormal* basis for A, B
  - p_{nm}: Classical probability of $|n\rangle \otimes |m\rangle$

- Discord=Coherent state in any (separable) basis $\rightarrow$ Coherence: essential feature for quantum correlations!

- Qubits $\rightarrow$ Classical state=Heads and tails with two coins!

$$\begin{aligned} \rho_{\text{class}} &= p_{++} |++\rangle \langle ++| + p_{+-} |+-\rangle \langle +-| \\ &+ p_{-+} |-+\rangle \langle -+| + p_{--} |--\rangle \langle --| \end{aligned}$$



Entanglement

- What if we generalize the previous idea? $\rightarrow$ Separability:

$$\rho_{\rm sep} = \sum_{n,m} p_{nm} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right| = \sum_k p_k \rho_k^{(A)} \otimes \rho_k^{(B)}$$

- $\left|n\right\rangle, \left|m\right\rangle$ not necessarily orthonormal basis now $\rightarrow$ p_{nm} are *quasi-probabilities* (not disjoint events)
- Any classically correlated state (classical probability) is separable.
- **Entanglement**: Non-separability of a quantum state.

Entanglement: Two qubits

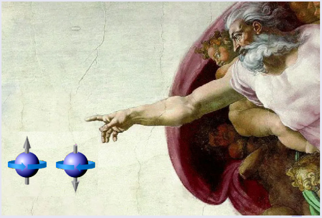
- Two qubits: Separability=$\exists$ Positive P-representation $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$:

$$\rho = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\, P(\mathbf{n}_A, \mathbf{n}_B) \left|\mathbf{n}_A \mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A \mathbf{n}_B\right|, \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\, P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- $P(\mathbf{n}_A, \mathbf{n}_B)$ is a quasi-probability: Overlap $|\langle \mathbf{n}_A | \mathbf{n}_B \rangle|^2 \neq 0$
- Separability=Purely classical spins pointing at directions $\mathbf{n}_A, \mathbf{n}_B$

$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\, P(\mathbf{n}_A, \mathbf{n}_B) n^i_A n^j_B$$

- Entanglement=NO positive P-representation $\rightarrow$ Genuine non-classical!



- Practical entanglement criteria:
  - Peres-Horodecki criterion: ρ separable $\rightarrow$ Partial tranpose $\rho^{T_B} = \sum_n p_n \rho_n^A \otimes (\rho_n^B)^T$ non-negative. Negative-valued $\rho^{T_B} \rightarrow \rho$ entangled!
    Very powerful: 2×2, $2 \times 3 \rightarrow$ Peres-Horodecki criterion=Entanglement
    Peres, PRL 77, 1413 (1996), Horodecki, PLA 232, 333 (1997).
  - Concurrence $0 \leq C[\rho] \leq 1$, $C[\rho] > 0$ iff ρ entangled. Wootters, PRL 80, 2245 (1998)
  - Entanglement witness: Negative-valued observable W such that $\langle W \rangle_{\text{sep}} = \mathrm{Tr}[W \rho_{\text{sep}}] \geq 0 \Longrightarrow \langle W \rangle < 0$ implies entanglement. Terhal, PLA 271, 319 (2000)


Steering

- Steering: Original conception of Schrödinger of EPR paradox $\rightarrow$ Only well-defined in 2007! (Wiseman, Jones, Doherty, PRL 98, 140402 (2007))
- *Local* Quantum Mechanics: Alice post-measurement (un-normalized) state described by local-hidden states

$$R_{\hat{\mathbf{n}}} = \Pi^B_{\hat{\mathbf{n}}}\rho\Pi^B_{\hat{\mathbf{n}}} = \int d\lambda\ p(1|\hat{\mathbf{n}}\lambda)p(\lambda)\rho_B(\lambda)$$

- If not, Bob can "steer" Alice quantum state $\rightarrow$ **Steerability**.

Bell nonlocality

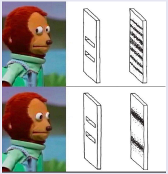
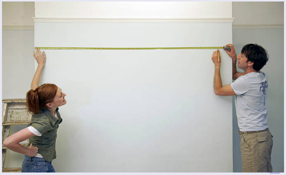
- EPR Paradox: Quantum Mechanics challenges local realism!
- Bell Test:
  - Alice/Bob perform causally disconnected projective spin measurements along alternate axes $(M_A, M'_A)/(M_B, M'_B)$ obtaining outputs $a, b = \pm 1$:

$$\text{Correlations} \Longrightarrow C(M_A, M_B) = \sum_{a,b=\pm 1} (a \cdot b) p(a, b | M_A M_B)$$

  - Local realistic theories $p(a, b|M_A M_B) = \int d\lambda \; p(a|M_A \lambda) p(b|M_B \lambda) p(\lambda)$ $\rightarrow$ **Bell (CHSH) inequality**:

$$\mathcal{B} \equiv |C(M_A, M_B) - C(M_A, M'_B) + C(M'_A, M_B) + C(M'_A, M'_B)| \leq 2$$

  - QM violates CHSH inequality up to Tsirelson bound $\mathcal{B} \leq 2\sqrt{2}$!
- Beauty: NO underlying model, just people enjoying measurements!

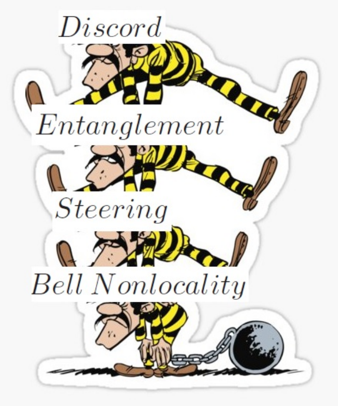



Hierarchy of Quantum Correlations

- Quantum Hierarchy (mixed states):

$$\textit{Bell Nonlocality} \subset \textit{Steering} \subset \textit{Entanglement} \subset \textit{Discord}$$

- Pure states: All correlations are equivalent to entanglement! N. Gisin, A. Peres, Phys. Lett. A 162, 15 (1992) $\rightarrow$ Another confusion source...



- Behavior under Alice/Bob exchange:
  1. Steering & Discord: Asymmetric.
  2. Bell Nonlocality & Entanglement: Symmetric.

Toast for Quantum Optics

- Light has motivated the major advancements in modern Physics:
  - EM radiation: Maxwell equations
  - Special Relativity: Michelson-Morley experiment
  - Quantum Mechanics: Black-body spectrum & Photoelectric effect
  - Quantum Field Theory: First quantized field

. . . and God said: "let there be light" . . .

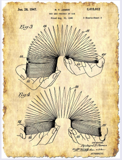
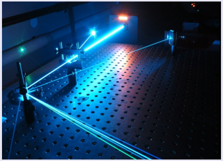
- Major paradigm in Quantum Information:
  - Discrete variables: Polarization, Orbital Angular Momentum
  - Continuous variables: Quantum amplitudes (Photons)

Harmonic oscillators

- EM field quantization $\sim$ Massless KG quantization:

$$\hat{\Phi}(\mathbf{x}) = \int \mathrm{d}^3\mathbf{k}\ [\hat{a}(\mathbf{k})\Phi_{\mathbf{k}}(\mathbf{x}) + \hat{a}^\dagger(\mathbf{k})\Phi^*_{\mathbf{k}}(\mathbf{x})]$$

- Canonical commutation rules $\rightarrow [\hat{a}(\mathbf{k}), \hat{a}^\dagger(\mathbf{k}')] = \delta(\mathbf{k}-\mathbf{k}')$
- QFT=Harmonic Oscillator ensemble $\rightarrow$ HO Textbook:
  - Phase space variables: $\hat{X} = \dfrac{\hat{a}+\hat{a}^\dagger}{\sqrt{2}}$, $\hat{P} = i\dfrac{\hat{a}^\dagger-\hat{a}}{\sqrt{2}}$
  - Number states: $\hat{a}^\dagger\hat{a}\,|n\rangle = n\,|n\rangle$
  - Coherent states: $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$, $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\displaystyle\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$, $\alpha \in \mathbb{C}$
  - Squeezed states: $\hat{b}\,|\lambda\rangle = 0$, $\hat{b} = (\cosh\lambda)\hat{a} + (\sinh\lambda)\hat{a}^\dagger$

Fock, P and Wigner representations

- Number states: complete orthonormal basis $I = \sum_n |n\rangle\langle n|$

$$\rho = \sum_{n,m} \rho_{nm} |n\rangle\langle m|$$

- P-representation: Normal-ordered expectation values

$$\rho = \int \mathrm{d}^2\alpha\, P(\alpha)\,|\alpha\rangle\langle\alpha|\,,\ \int \mathrm{d}^2\alpha\, |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a}\rangle\,,\ \int \mathrm{d}^2\alpha\, P(\alpha) = 1,$$

- Wigner representation ($\alpha = \frac{X+iP}{\sqrt{2}}$): *Symmetric* expectation values $\rightarrow$ Quantum Boltzmann distribution:

$$\begin{aligned} W(X,P) &= \frac{1}{2\pi}\int \mathrm{d}x\ \langle X + \frac{x}{2}|\rho|X - \frac{x}{2}\rangle\, e^{-iPx} \\ \int \mathrm{d}^2\alpha\ |\alpha|^2 W(\alpha) &= \langle \frac{\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger}{2}\rangle \end{aligned}$$

Quantum Correlations

- *Quasi-probability* distributions: $I = \frac{1}{\pi}\int \mathrm{d}^2\alpha \ \left|\alpha\right\rangle\left\langle\alpha\right|, \ |\left\langle\alpha|\beta\right\rangle|^2 \neq 0$
- General quantum state of EM field:

$$\rho = \int \mathrm{d}^2\{\alpha_{\mathbf{k}}\} \ P(\{\alpha_{\mathbf{k}}\}) \prod_{\mathbf{k}} \otimes \left|\alpha_{\mathbf{k}}\right\rangle\left\langle\alpha_{\mathbf{k}}\right|$$

- Classical EM field $\sim P(\alpha_{\mathbf{k}}) \geq 0, \ |\alpha_{\mathbf{k}}| \gg 1, |\Delta\alpha_{\mathbf{k}}/\alpha_{\mathbf{k}}| \ll 1$
- Measurements=Normal-ordered expectation values
  - Positive P-representation $\Longrightarrow$ Classical expectation value
  - Quantum correlations $\Longrightarrow$ Negative P-representations!
- Bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$: Product space of Fock spaces of two modes $\hat{a}_i, \hat{a}_j \rightarrow$ Infinite dimension!

$$\rho = \int \mathrm{d}^2\alpha_i \mathrm{d}^2\alpha_j \ P(\alpha_i, \alpha_j) \left|\alpha_i\alpha_j\right\rangle\left\langle\alpha_i\alpha_j\right|$$

Continuous Variable Entanglement

- Continuous system: Physical density matrix $\Longrightarrow$ Non-negative covariance matrix M (uncertainty principle)

$$\begin{aligned} M_{\alpha\beta} &= \langle \Delta\xi_\alpha \Delta\xi_\beta \rangle = \frac{\langle \{\Delta\xi_\alpha \Delta\xi_\beta\} \rangle}{2} + \frac{\langle [\Delta\xi_\alpha \Delta\xi_\beta] \rangle}{2} \equiv V_{\alpha\beta} + i\frac{\Omega_{\alpha\beta}}{2} \\ \Delta\xi_\alpha &= \xi_\alpha - \langle \xi_\alpha \rangle\,,\ \xi_\alpha = [X_1, X_2, P_1, P_2], \end{aligned}$$

- Transpose operation via Wigner representation is time inversion!

$$\begin{aligned} \rho \quad : \quad & W(X_1, X_2, P_1, P_2) \to \rho^{T_2}:\ W(X_1, X_2, P_1, -P_2) \\ \to \quad & V' = \Lambda V \Lambda,\ \Lambda \equiv \mathrm{diag}[1, 1, 1, -1] \end{aligned}$$

- Peres-Horodecki $\Longrightarrow$ ρ^{T_2} covariance matrix $M' = V' + i\frac{\Omega}{2}$ not non-negative $\to$ Entanglement!
- Gaussian state $\Longrightarrow$ Peres-Horodecki=Entanglement

Simon, PRL 84, 2726 (2000)

Cauchy-Schwarz violation

- Classical light (non-negative P-representation) satisfies

$$|\langle\hat{a}^2\rangle| = \left|\int \mathrm{d}^2\alpha\ P(\alpha)\alpha^2\right| \leq \int \mathrm{d}^2\alpha\ P(\alpha)|\alpha|^2 = \langle\hat{a}^\dagger\hat{a}\rangle$$

- Proof explicitly based on positivity of $P \rightarrow$ Effective scalar product $\rightarrow$ Cauchy-Schwarz inequalities!
- Operators A, B do satisfy mathematical Cauchy-Schwarz inequality:

$$\langle A|B\rangle \equiv \mathrm{Tr}(\rho A^\dagger B) = \langle A^\dagger B\rangle$$

- *Quantum* CS inequality: $|\langle\hat{a}^2\rangle| \leq \sqrt{\langle\hat{a}\hat{a}^\dagger\rangle\langle\hat{a}^\dagger\hat{a}\rangle} = \sqrt{\langle\hat{a}^\dagger\hat{a}\rangle(\langle\hat{a}^\dagger\hat{a}\rangle+1)}$
- There is room for violation of *classical* CS inequality!
- Explicit example: Squeezed vacuum $|\lambda\rangle = e^{\lambda\frac{(\hat{a}^\dagger)^2-\hat{a}^2}{2}}|0\rangle$:

$$|\langle\hat{a}^2\rangle| = \cosh\lambda|\sinh\lambda| > \sinh^2\lambda = \langle\hat{a}^\dagger\hat{a}\rangle,\ \lambda \neq 0 \in \mathbb{R}$$

Cauchy-Schwarz violation: Bipartite case

- Quantum correlations tested via

$$g_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle \,,\ c_{ij} = \langle \hat{a}_i \hat{a}_j \rangle \,,\ \Gamma_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i \rangle$$

- Classical vs. Quantum CS inequality
  - $|c_{ij}|^2 \leq g_{ii} g_{jj}$ **vs.** $|c_{ij}|^2 \leq g_{ii}(g_{jj} + 1)$
  - $\Gamma_{ij} \leq \sqrt{\Gamma_{ii}\Gamma_{jj}}$ **vs.** $\Gamma_{ij} \leq \sqrt{(\Gamma_{ii} + g_{ii})(\Gamma_{jj} + g_{jj})}$
- CS violation *independent* from entanglement $\rightarrow$ Complementary test of quantum behavior!
- Explicit example: Product number state $\rho = |nm\rangle \langle nm|$ with $n, m \neq 0$

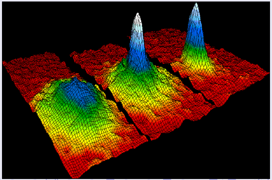
$$\Gamma_{ij} = nm > \sqrt{n(n-1)m(m-1)} = \sqrt{\Gamma_{ii}\Gamma_{jj}}$$

- Number states: Negative Wigner representations $\rightarrow$ Stronger quantumness than negative P-representation $\rightarrow$ Entanglement is not the whole story...

Analogue gravity & Universal Hawking Effect

- Hawking (1974): Event horizon of black hole emits radiation with thermal spectrum $\rightarrow$ Hawking radiation
- Unruh (1981): Fluctuations of potential ideal barotropic flow behave as massless scalar field in curved spacetime $\rightarrow$ Analogue gravity
- Garay, Anglin, Cirac & Zoller (2001): Bose-Einstein condensates behave as quantum fluid $\rightarrow$ *Universal* Hawking effect
- Intuitive:
  - Light cannot escape from black hole=Sound cannot travel upstream in supersonic flow
  - Subsonic/supersonic flows = Exterior/interior black hole
  - Subsonic-supersonic interface = Event horizon





CS Violation & Entanglement in Hawking effect

- What makes Hawking radiation quantum? $\rightarrow$ Competition: Zero-point quantum effect **vs.** Thermal/coherent stimulation
- Only genuinely quantum Hawking effect contributes to CS violation & entanglement witnesses.
- Quantum correlations=Smoking gun of Hawking effect!
- In typical analogue scenarios, CS violation=entanglement.

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Violation of Cauchy-Schwarz inequalities by spontaneous Hawking radiation in resonant boson structures

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PAPER

Entanglement and violation of classical inequalities in the Hawking radiation of flowing atom condensates

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JRMdN, F. Sols, I. Zapata, PRA 89, 043808 (2014).
X. Busch, R. Parentani, PRD 89, 105024 (2014).
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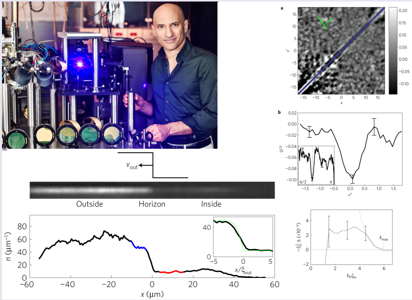
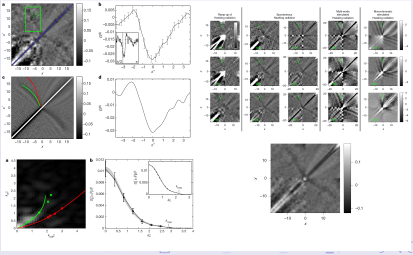
Experimental Entanglement

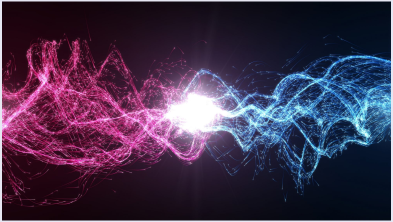
- First experimental evidence of Hawking effect: Entanglement measurement!
  Jeff Steinhauer, Nature Physics 12, 959 (2016).
- Basis for later conclusive observations of Hawking effect:
  JRMdN, K. Golubkov, V. I. Kolobov, J. Steinhauer, Nature 569, 688 (2019).
  V. I. Kolobov, K. Golubkov, **JRMdN**, J. Steinhauer, Nature Physics 17, 362 (2021).

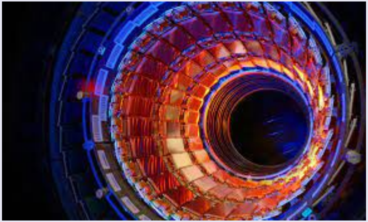




Quantum High-Energy Colliders?

- Outside optics, condensed matter, analogues... Which other experiments are described by QFT? $\rightarrow$ High-energy colliders $\rightarrow$ Genuinely relativistic QFT!
- Naively, Quantum Correlations should easily emerge there...right? $\rightarrow$ Not so fast!
  - Momentum measurement $\rightarrow$ Decoherence
  - Lack of control of internal d.o.f. in initial state $\rightarrow$ Decoherence
  - Collider observables: cross-sections, lifetimes. . . $\rightarrow$ Classical objects
  - QFT cares about expectation values, not quantum states!





Quantum shortlist

- Where should we look for entanglement in a collider?
  - Continuous variable entanglement → Problem: No squeezed states, low number states
  - Orbital degrees of freedom → Problem: Colliders perform projective momentum measurements of individual particles → Decoherence
  - Discrete degrees of freedom:
    1. Spin: fermion (qubit) or massive vector boson (qutrit)
    2. Flavor: mesons PRL 99, 131802 (2007) or neutrinos EPL 85, 50002 (2009) (see Massimo's talk)

Spin quantum state

- Most natural entangled system: Particle-antiparticle ($P\bar{P}$) pair produced from initial state I with internal degrees of freedom λ
- For given momenta $\mathbf{P}$, $P\bar{P}$ spins described by production spin density matrix (R-matrix):

$$R^{I\lambda}_{\alpha\beta,\alpha'\beta'}(\mathbf{P}) \equiv \langle \mathbf{P}\alpha\beta|\, T\, |I\lambda\rangle \langle I\lambda|\, T^{\dagger}\, |\mathbf{P}\alpha'\beta'\rangle$$

- Experiment: Momentum measurements+Event averaging $\rightarrow$ Genuine density-matrix description!

$$R^{I}(\mathbf{P}) = \frac{1}{N_{\lambda}} \sum_{\lambda} R^{I\lambda}(\mathbf{P})$$

$$\rho^{I}(\mathbf{P}) = \frac{R^{I}(\mathbf{P})}{\mathrm{Tr}\left[R^{I}(\mathbf{P})\right]}$$

The End...Or The Beginning?

- But how can we measure the spin quantum state?
- Which high-energy systems are good candidates for such experiment?

The End...Or The Beginning?

- But how can we measure the spin quantum state?
- Which high-energy systems are good candidates for such experiment?
- This is where the real story begins...

Backup

Quantum optics representations

- Quantum state completely determined by characteristic function

$$\chi(\eta) \equiv \langle e^{\eta \hat{a}^\dagger - \eta^* \hat{a}} \rangle = \mathrm{Tr}[e^{\eta \hat{a}^\dagger - \eta^* \hat{a}} \hat{\rho}].$$

- Normal and antinormal versions

$$\begin{aligned} \chi_N(\eta) &\equiv \langle e^{\eta \hat{a}^\dagger} e^{-\eta^* \hat{a}} \rangle , \\ \chi_A(\eta) &\equiv \langle e^{-\eta^* \hat{a}} e^{\eta \hat{a}^\dagger} \rangle . \end{aligned}$$

- Fourier transforms $\rightarrow P, Q, W$ representations for computing normal, antinormal or symmetric expectation values:

$$\begin{aligned} P(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_N(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a} \rangle \\ Q(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_A(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 Q(\alpha) = \langle \hat{a} \hat{a}^\dagger \rangle \\ W(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} \; e^{(\eta^*\alpha - \eta\alpha^*)} \chi(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 W(\alpha) = \langle \frac{\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger}{2} \rangle \end{aligned}$$

Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy $\rightarrow$ Von Neumann entropy ($p_n \geq 0$, ρ eigenvalues)

$$H(A,B) \rightarrow H(\rho) = -\sum_n p_n \log_2 p_n$$

$$H(A) \rightarrow H(\rho_A),\ H(B) \rightarrow H(\rho_B),\ \rho_{A,B} = \mathrm{Tr}_{B,A}\rho$$

- Conditional probability $\rightarrow$ Conditional state $\rho_{A|B}$ = One-qubit state after Bob's spin measurement along $\hat{\mathbf{n}}$:

$$H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})$$

$$\rho_{\hat{\mathbf{n}}} = \frac{\Pi^B_{\hat{\mathbf{n}}}\rho\Pi^B_{\hat{\mathbf{n}}}}{p_{\hat{\mathbf{n}}}} = \frac{1+\mathbf{B}^+_{\hat{\mathbf{n}}}\cdot\sigma}{2},\ \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C}\cdot\hat{\mathbf{n}}}{1+\hat{\mathbf{n}}\cdot\mathbf{B}^-},\ p_{\hat{\mathbf{n}}} = \frac{1+\hat{\mathbf{n}}\cdot\mathbf{B}^-}{2}$$

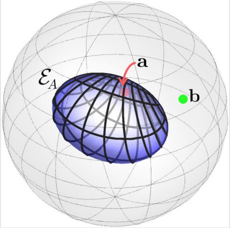
- Genuine quantumness $\rightarrow$ Minimization over all spin directions to exclude quantization effects:

$$\mathcal{D}(A,B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \neq 0$$

- 1-qubit Alice post-measurement state → Conditional Bloch vector:

$$\rho_{\hat{\mathbf{n}}} = \frac{R_{\hat{\mathbf{n}}}}{\mathrm{Tr}[R_{\hat{\mathbf{n}}}]} = \frac{1 + \mathbf{B}^{+}_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \quad \mathbf{B}^{+}_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^{+} + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^{-}}$$

- Set of conditional polarizations $\mathbf{B}^{+}_{\hat{\mathbf{n}}}$ → **Steering ellipsoid**: Fundamental QI object, containing all information about the system.
- Similar for Bob → Steering: also asymmetric between Alice and Bob.



Jevtic, Pusey, Jennings, Rudolph
PRL 113, 020402 (2014)

Quantum Many-Body Field Theory

- Second-quantization many-body Hamiltonian (fermions and bosons):

$$\begin{aligned}\hat{H} &= \int \mathrm{d}\mathbf{x}\ \hat{\Psi}^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{x})\right]\hat{\Psi}(\mathbf{x}) \\ &+ \frac{1}{2}\int \mathrm{d}\mathbf{x}\int \mathrm{d}\mathbf{x}'\hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}^\dagger(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\hat{\Psi}(\mathbf{x}')\hat{\Psi}(\mathbf{x})\end{aligned}$$

- Field operator: $\hat{\Psi}(\mathbf{x})=\sum_n \hat{a}_n\phi_n(\mathbf{x})$, $\hat{a}_n=\langle\phi_n|\hat{\Psi}\rangle$
- Fermionic or bosonic annihilation operators: $[\hat{a}_n,\hat{a}^\dagger_m]_\pm=\delta_{nm}$
- Heisenberg equation of motion of field operator:

$$\begin{aligned}& i\hbar\partial_t\hat{\Psi}(\mathbf{x},t)=[\hat{\Psi}(\mathbf{x},t),\hat{H}] \\ &= \left[-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{x})+\int \mathrm{d}\mathbf{x}'\hat{\Psi}^\dagger(\mathbf{x}',t)V(\mathbf{x}-\mathbf{x}')\hat{\Psi}(\mathbf{x}',t)\right]\hat{\Psi}(\mathbf{x},t)\end{aligned}$$

- Same equation of motion derived from a classical Lagrangian:

$$L=\int \mathrm{d}\mathbf{x}\ i\Psi^*\partial_t\Psi - H$$

Non-relativistic vs. relativistic QFT

- Differences with respect relativistic QFT:
  - Not Lorentz invariant
  - Fixed cutoffs (IR $\rightarrow$ System size; UV $\rightarrow$ Microscopic physics)
  - Interactions are (typically) classical
  - Effective and approximate theories
  - Second-quantization is not fundamental (trick to rewrite symmetric/antisymmetric wavefunctions)
- Features:
  - Highly tunable (interactions, external potential, mass)
  - Non-trivial spatiotemporal dependence leads to rich physics (Analogue Gravity, Floquet systems)
  - Variety of internal degrees of freedom (high spin, isospin, mixtures)
  - Many-body systems lead to spontaneous symmetry breaking (supersolids, time crystals)
  - Topological physics and synthetic dimensions

Condensates: GP mean-field

- Bose-Einstein condensate close to $T = 0$
- Field operator: $\hat{\Psi}(\mathbf{x}, t) = \Psi(\mathbf{x}, t) + \hat{\varphi}(\mathbf{x}, t) \rightarrow$ Higgs-mechanism
- Contact interaction $V(\mathbf{x} - \mathbf{x}') = g\delta(\mathbf{x} - \mathbf{x}') \rightarrow$ Non-relativistic Higgs boson!

$$\mathcal{L} = i\Psi^*\partial_t\Psi - \frac{|\nabla\Psi|^2}{2m} - V(\mathbf{x})|\Psi|^2 - g\frac{|\Psi|^4}{2}$$

- Heisenberg e.o.m. $\rightarrow$ Gross-Pitaevskii equation:

$$i\hbar\partial_t\Psi(\mathbf{x}, t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g|\Psi(\mathbf{x}, t)|^2\right]\Psi(\mathbf{x}, t)$$

- Condensate wavefunction=Coherent state spontaneously breaking $U(1)$ symmetry

Condensates: Bogoliubov approximation

- Stationary condensate: $\hat{\Psi}(\mathbf{x},t)=[\Psi_0(\mathbf{x})+\hat{\varphi}(\mathbf{x},t)]e^{-i\mu t/\hbar}$+Quantum fluctuations at linear order $\rightarrow$ Bogoliubov-de Gennes equations:

$$M\hat{\Phi} = i\hbar\partial_t\hat{\Phi},\ M=\begin{bmatrix} N & A \\ -A^* & -N^* \end{bmatrix},\ \hat{\Phi}\equiv\begin{bmatrix}\hat{\varphi}(\mathbf{x},t) \\ \hat{\varphi}^\dagger(\mathbf{x},t)\end{bmatrix}$$

$$N = -\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{x})+2g|\Psi_0(\mathbf{x})|^2-\mu,\ A=g\Psi_0^2(\mathbf{x})$$

- Eigenmodes: $Mz_n=\epsilon_n z_n,\ z_n=\begin{bmatrix} u_n \\ v_n\end{bmatrix},\ M\bar{z}_n=-\epsilon_n^*\bar{z}_n,\ \bar{z}_n=\sigma_x z_n^*$
- Conserved inner product:

$$(z_n|z_m)\equiv\langle z_n|\sigma_z|z_m\rangle=\int \mathrm{d}\mathbf{x}\ z_n^\dagger\sigma_z z_m=\int \mathrm{d}\mathbf{x}\ u_n^* u_m - v_n^* v_m$$

- Complete basis $(z_n|z_m)=\delta_{nm}\implies$ Quantization mimics KG field:

$$\hat{\Phi}=\sum_n z_n\hat{b}_n+\bar{z}_n\hat{b}_n^\dagger$$

Bogoliubov vs. Higgs

- Amplitude-phase (Madelung) decomposition:

$$\hat{\Psi}(\mathbf{x},t)=\sqrt{\hat{n}(\mathbf{x},t)}e^{i\hat{\theta}(\mathbf{x},t)}=\sqrt{n(\mathbf{x},t)+\delta\hat{n}(\mathbf{x},t)}e^{i(\theta(\mathbf{x},t)+\delta\hat{\theta}(\mathbf{x},t))}$$

- Density/phase fluctuations=Higgs/Goldstone modes:

$$\begin{aligned}\delta\hat{n}(\mathbf{x},t) &= \Psi_0^*(\mathbf{x})\hat{\varphi}(\mathbf{x},t)+\Psi_0(\mathbf{x})\hat{\varphi}^\dagger(\mathbf{x},t)\\ \delta\hat{\theta}(\mathbf{x},t) &= i\frac{\Psi_0(\mathbf{x})\hat{\varphi}^\dagger(\mathbf{x},t)-\Psi_0^*(\mathbf{x})\hat{\varphi}(\mathbf{x},t)}{2|\Psi_0(\mathbf{x})|^2}\end{aligned}$$

- Homogeneous condensate: $\Psi_0(\mathbf{x})=\sqrt{n_0}\rightarrow$ Massless superluminal dispersion relation for both Higgs and Goldstone modes:

$$\omega^2=c^2k^2\left(1+\frac{k^2}{4\Lambda^2}\right),\ c^2=\frac{gn_0}{m},\ \Lambda=\frac{1}{\xi_0}=\frac{mc}{\hbar}$$

- Actual Higgs: $\omega^2=c^2k^2$ (Goldstone), $\omega^2=\frac{m_H^2c^4}{\hbar^2}+c^2k^2$ (Higgs)

- GP/Schrödinger equation similar to Euler equation for potential flow!

$$\partial_t n + \nabla(n\mathbf{v}) = 0,\ \ \mathbf{v}(\mathbf{x},t) = \frac{\hbar\nabla\theta(\mathbf{x},t)}{m} \rightarrow \text{Potential flow!}$$

$$\hbar\partial_t\theta = \frac{\hbar^2\nabla^2\sqrt{n}}{2m\sqrt{n}} - \frac{1}{2}m\mathbf{v}^2 - gn - V$$

- Neglecting quantum ($\hbar$) potential in smooth background $\rightarrow$ Ideal potential flow $\rightarrow$ Analogue gravity (massless Hermitian field in curved spacetime)

$$\begin{aligned} \Box\delta\hat{\theta} &\equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\hat{\theta}) = 0 \\ g_{\mu\nu}(x) &= \frac{n(x)}{c(x)}\left[\begin{array}{cc} -[c^2(x)-v^2(x)] & -\mathbf{v}^T(x) \\ -\mathbf{v}(x) & \delta_{ij} \end{array}\right], \end{aligned}$$

- Gravitational phenomena (e.g., Hawking radiation) in the lab!

W. G. Unruh, PRL 46, 1351 (1981)

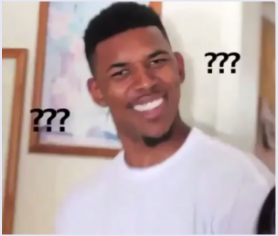
L. J. Garay, J. R. Anglin, J. I. Cirac, P. Zoller, PRL 85, 4643 (2000)

Cauchy-Schwarz violation

- Simple criterion of entanglement: Cauchy-Schwarz violation

$$|\mathrm{Tr}\ \mathbf{C}| = |\langle \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \rangle| = \left| \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B)\mathbf{n}_A \cdot \mathbf{n}_B \right| \leq 1$$

- Wait a minute...Cosine average larger than one??? $\rightarrow$ CS Violation
- $\rightarrow$ **Entanglement witness**: $D \equiv \frac{\mathrm{Tr}[\mathbf{C}]}{3}$
- Genuine quantum range: $-1 \leq D < -1/3$



- D directly measurable from the distribution of angular separation:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi),\ \cos\varphi \equiv \hat{\ell}_+ \cdot \hat{\ell}_-$$

- Entanglement detection from one single magnitude $\rightarrow$ No need for Quantum Tomography!