Vacuum decay and its observables

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International Workshop on New Opportunities for Particle Physics 2026
2026/8/14-2026/8/16@IHEP

Outlines

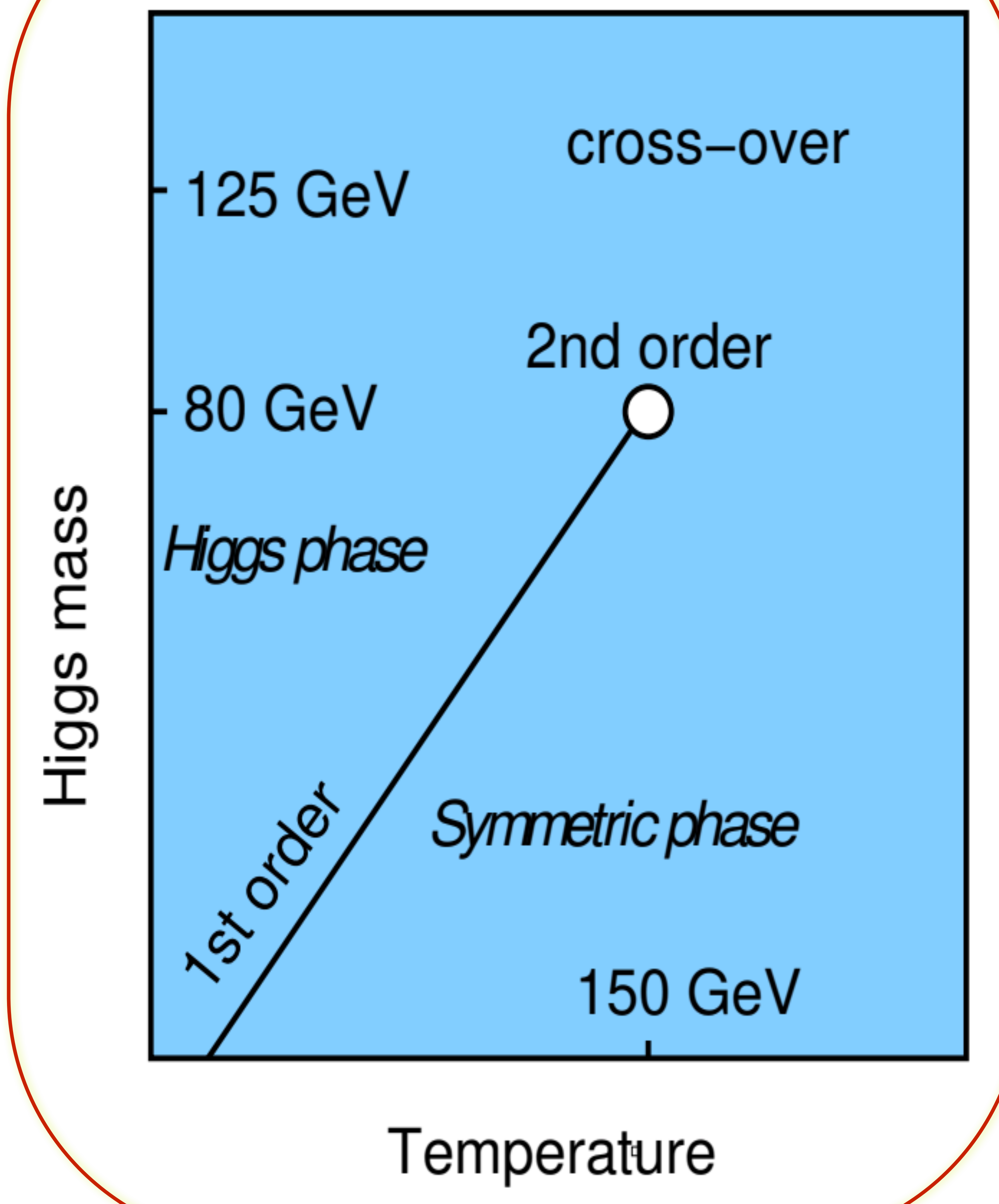
一 Background

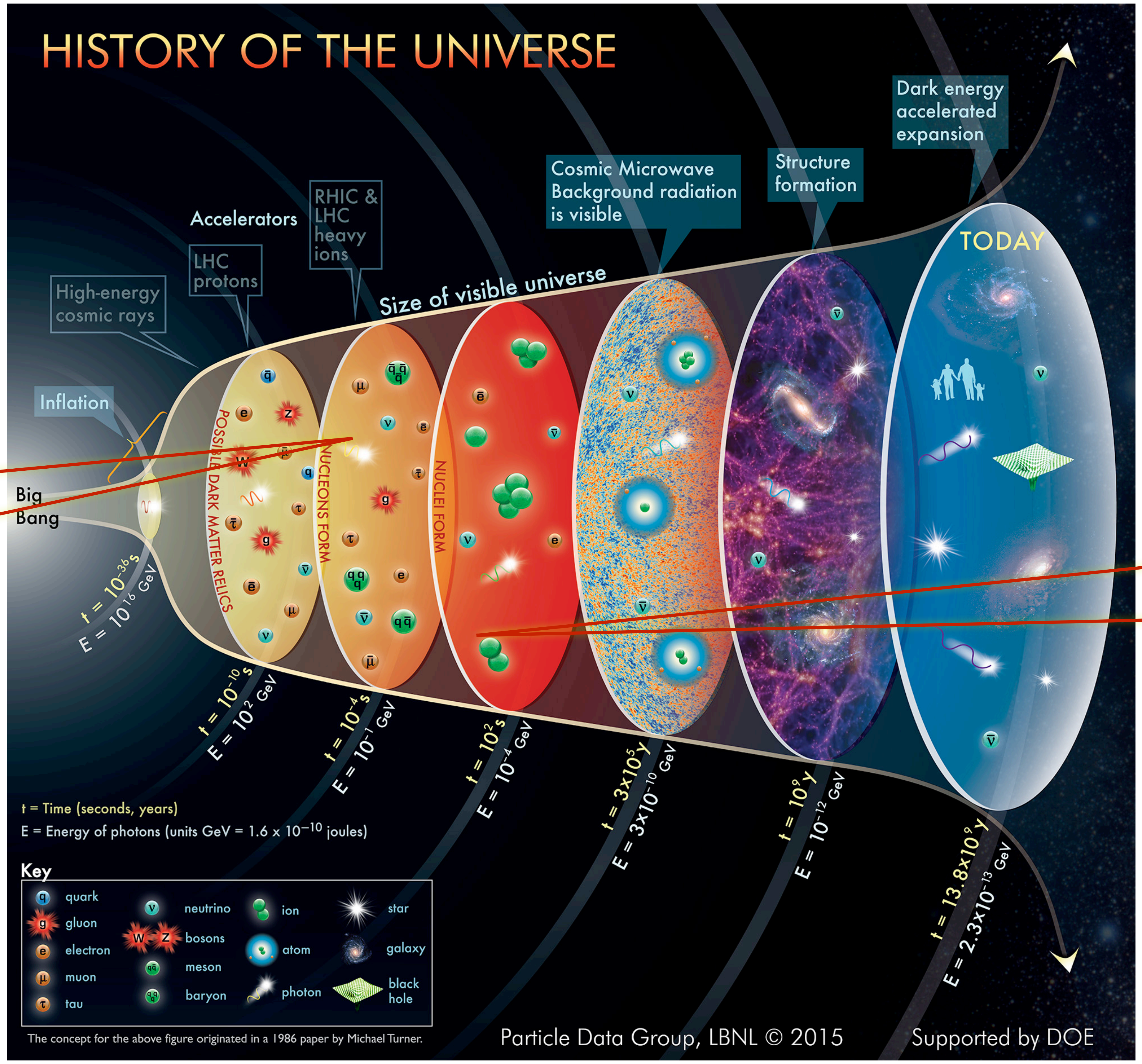
二 Quantum-thermal tunneling

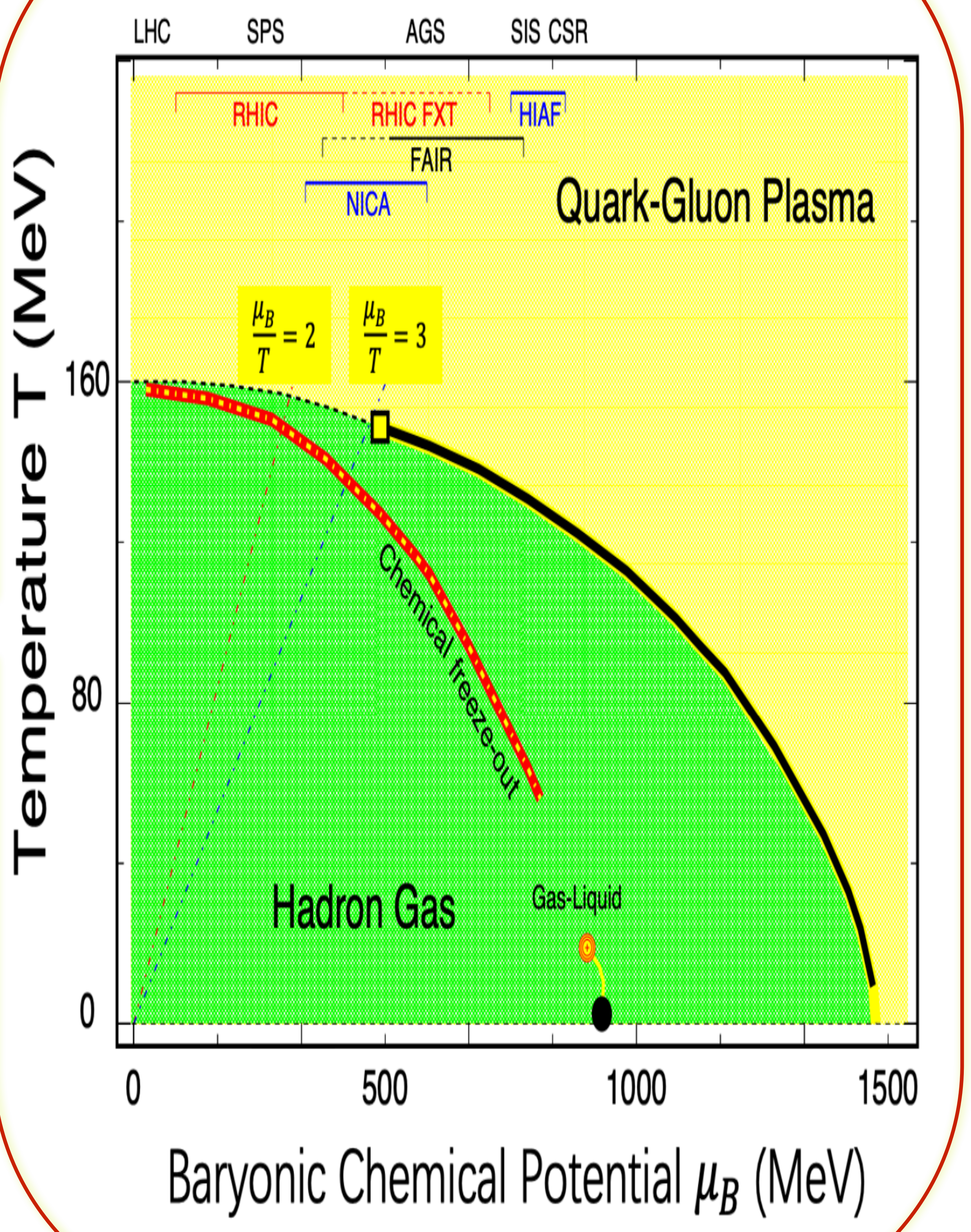
三 Electroweak Symmetry Breaking

四 Summary

Symmetry Breaking in the early Universe



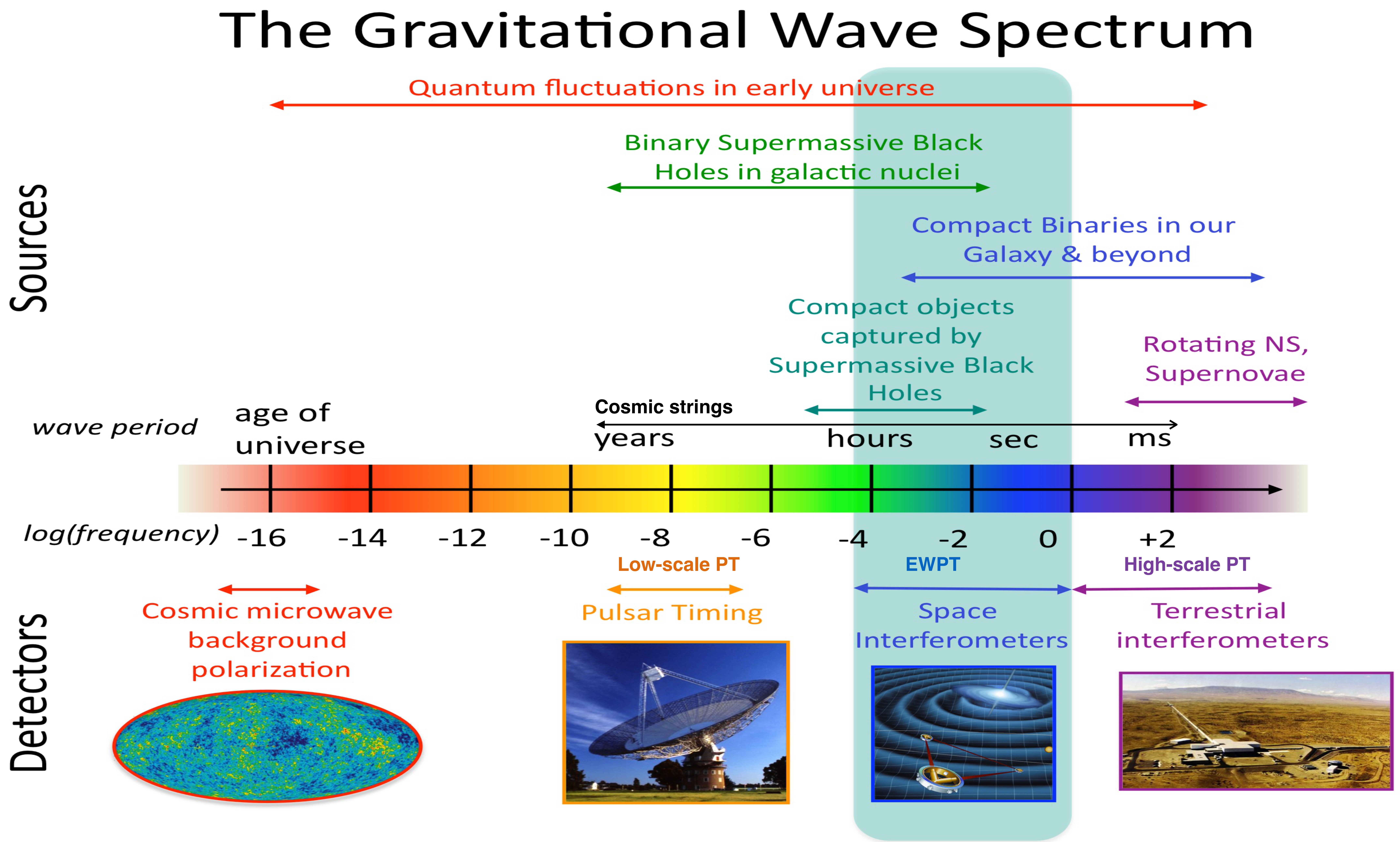




Electroweak Phase diagram

QCD Phase diagram

Stochastic gravitational wave background and New physics



▸ Collider&GW search for Electroweak symmetry breaking

■ Higgs&GWs

☑ **SM+Scalar Singlet**

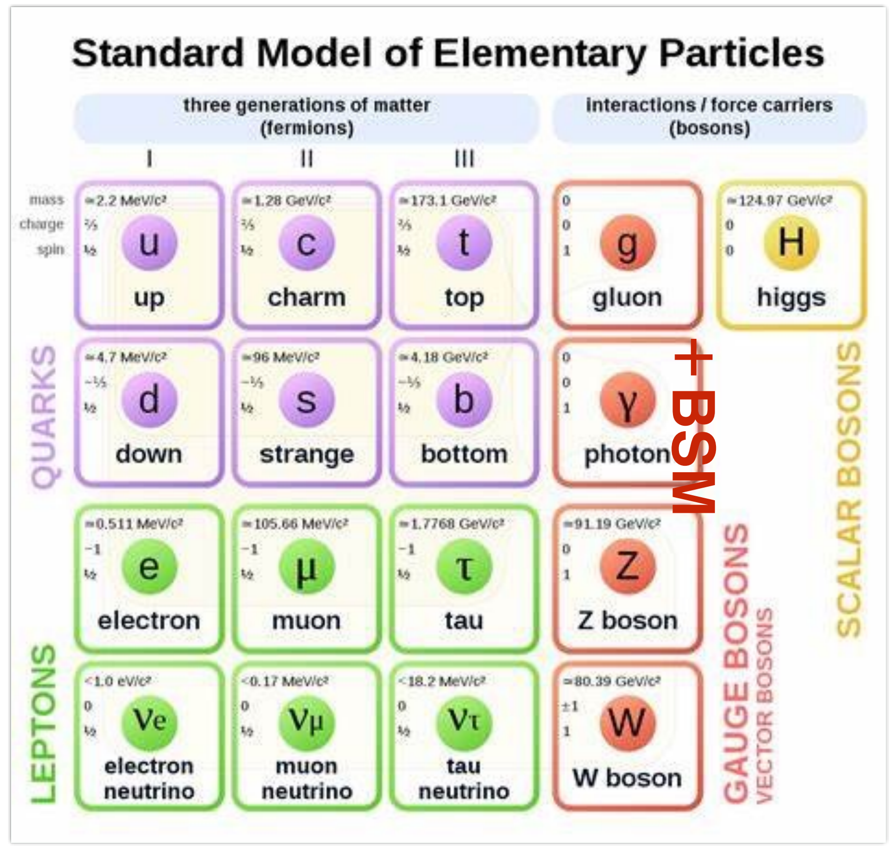
Profumo, Ramsey-Musolf, Wainwright, Winslow 14, **Bian**, Huang, Shu 15, Cheng, **Bian** 17, **Bian**, Tang 18,Chen, Li, Wu, **Bian**,19…

☑ **SM+Scalar Doublet**

Dorsch, Huber, Mimasu,No.14,Bernon, **Bian,** Jiang 17, **Bian**, Liu 18, Huang, Yu, 18,…

☑ **SM+Scalar Triplet**

Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19, Ramsey-Musolf etal 21, Zhou, **Bian**,Du,22,…



Symmetry breakdown

Symmetry breaking process

☑ **Composite Higgs**

Bruggisser, Harling, Matsedonskyi, Servant,18, **Bian**,Wu,Xie 19, **Bian**,Wu,Xie 20,…

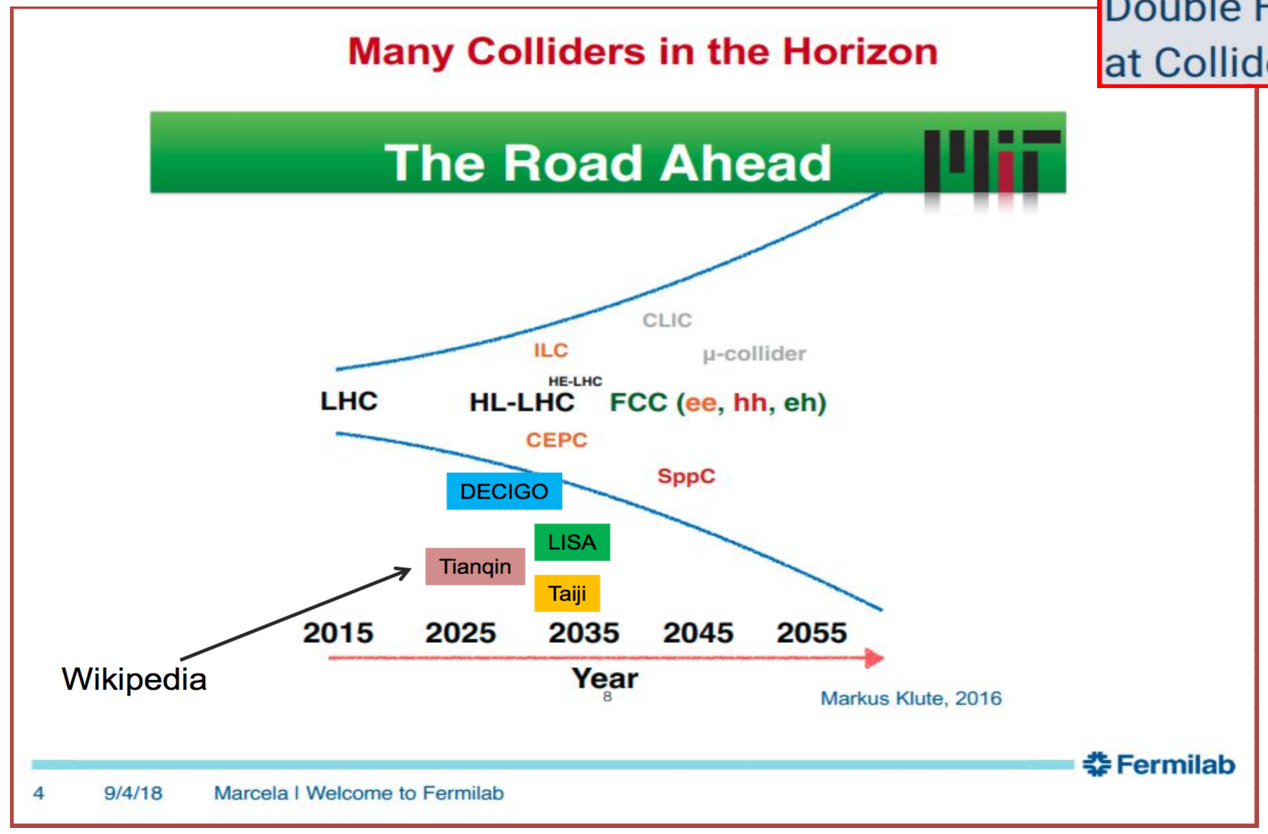
☑ **NMSSM**

Bi, **Bian**, Huang, Shu, Yin 15, **Bian,** Guo, Shu 17,Baum, Carena, Shah, Wagner, Wang 20, …

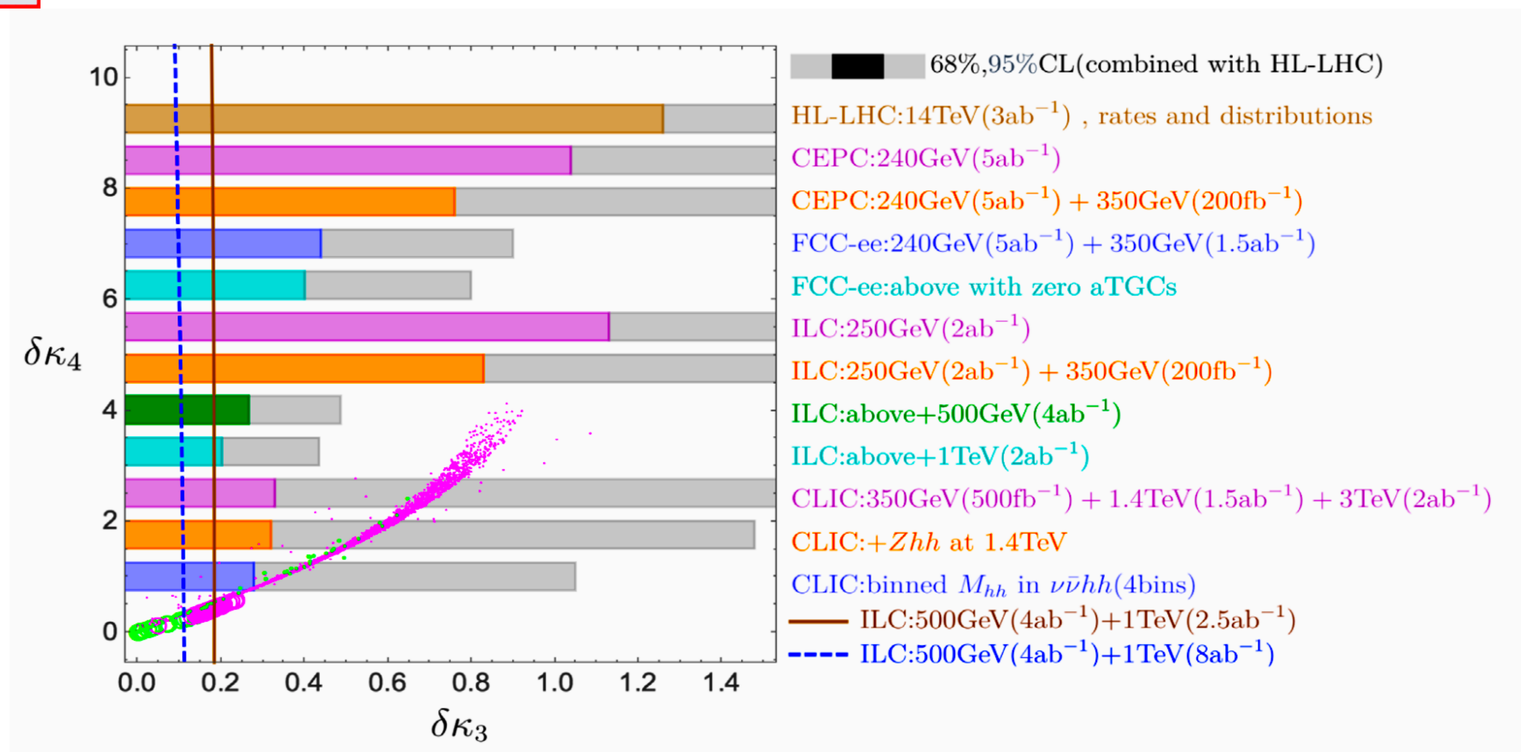
☑ **SMEFT**

Cao, Huang, Xie, & Zhang 17, Zhou, **Bian**, Guo 19, Cai,Hashino,Wang,Yu,22…

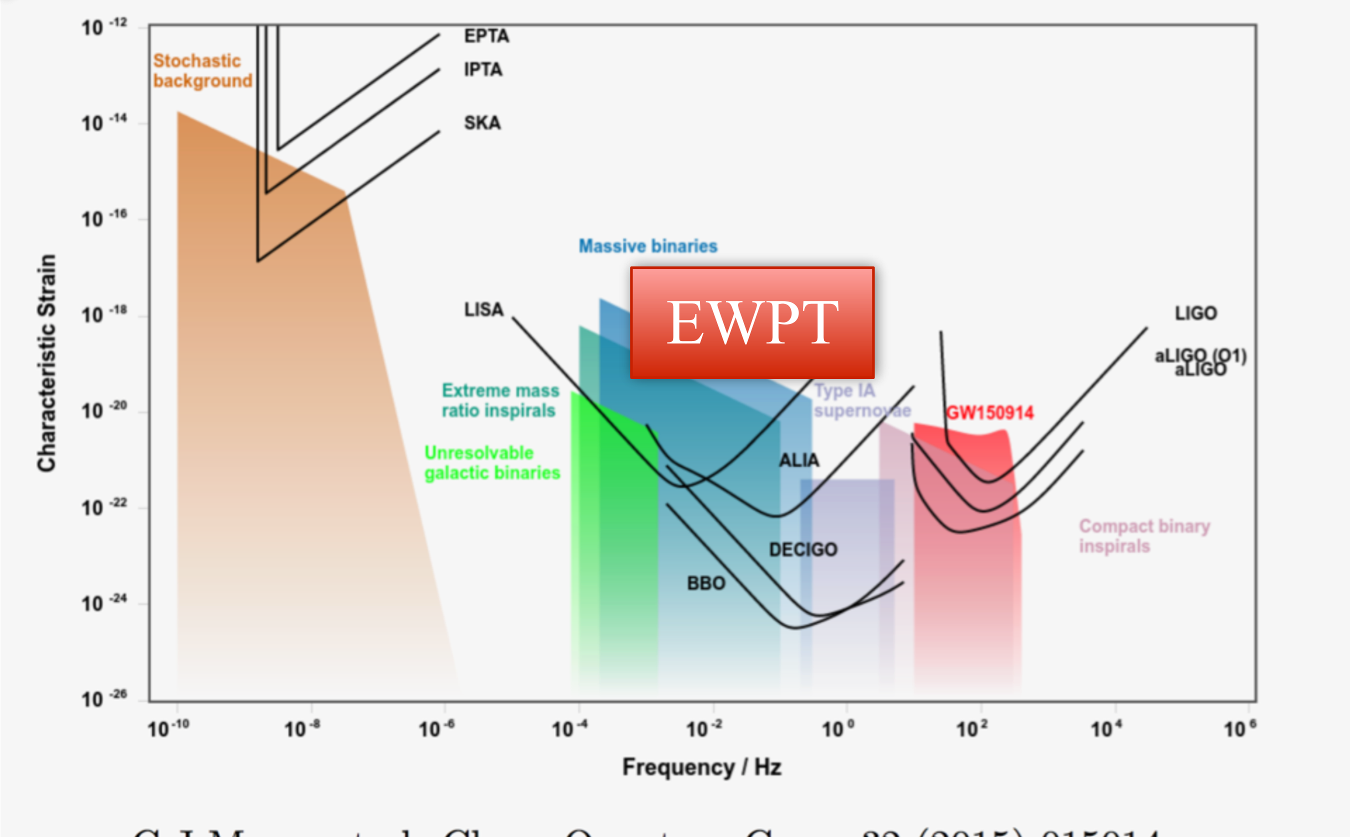
Double Higgs Production at Colliders Workshop



$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$



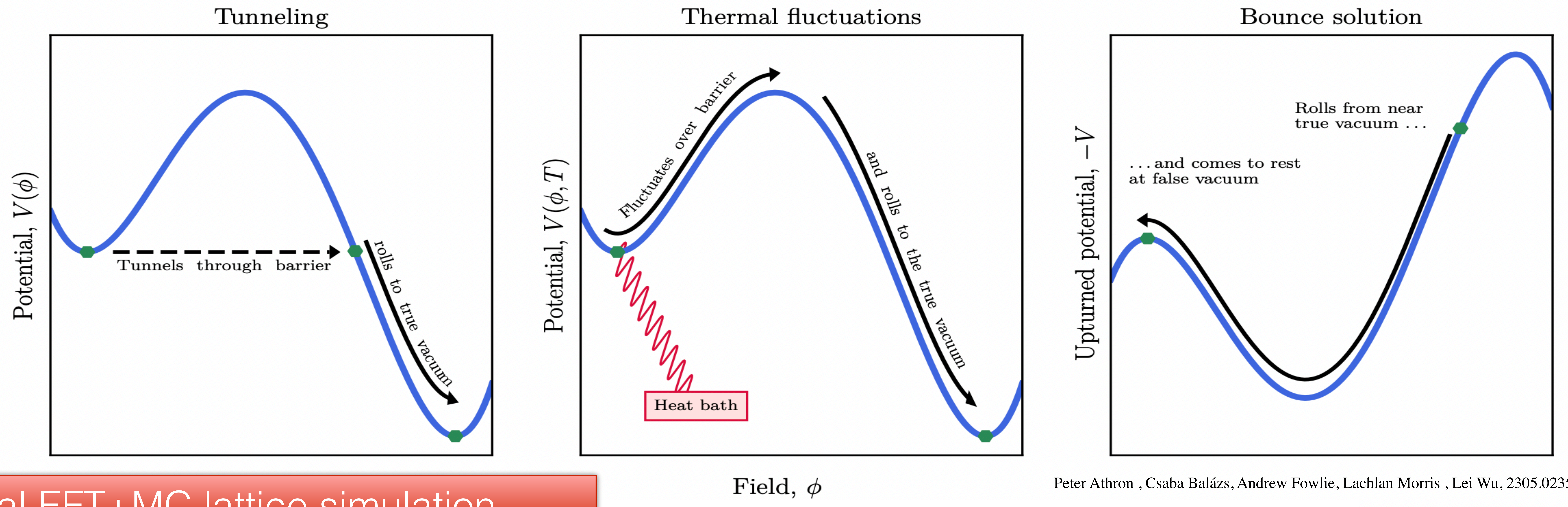
SNR > 10 for two-step and one-step SFOEWPT



C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

PTA,LIGO,LISA,TianQin,Taiji,⋯

Vacuum decay and FOPT



Peter Athron , Csaba Balázs, Andrew Fowlie, Lachlan Morris , Lei Wu, 2305.02357

Thermal EFT+MC lattice simulation

Classical lattice simulation

PT parameters

Effective action $\rightarrow \beta,\ H_*$

Energy budget $\rightarrow \alpha,\ \kappa(\alpha, v_w)$

Bubble wall dynamics $\rightarrow v_w$

$\rightarrow$

GW power spectrum

Numerical simulations $\rightarrow$

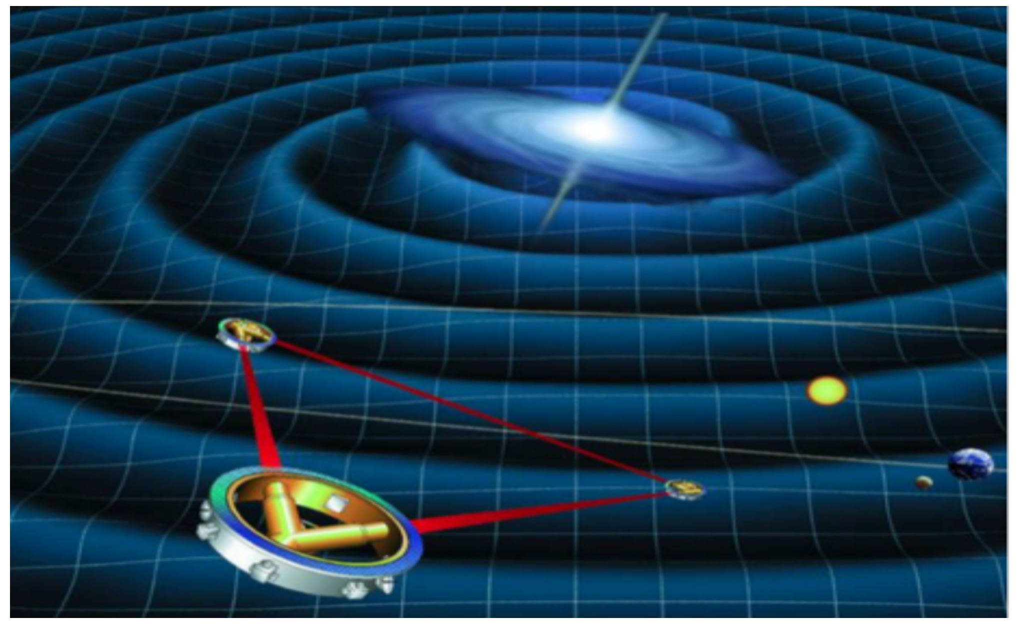
$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

$\rightarrow$

LISA sensitivity

Configuration + noise level $\rightarrow$

$h^2\Omega_{sens}(f)$



Chiara Caprini *et al* JCAP03(2020)024

LISA,TianQin,Taiji,$\cdots$

Bubble dynamics of FOPT

Thermal Field Theory

$$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

Tunneling

$$A(T) = T\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}$$

Fluctuation

$$A(T) = \frac{\sqrt{-\lambda_-}}{2\pi}\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2\pi}\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det^+[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}},$$

2305.02357

False vacuum fraction considering bubbles expansion (Kolmogorov–Johnson–Mehl–Avrami)

Bubble Volume

$$h(t) = \exp\left[-\int_{t_c}^{t} \mathrm{d}t' \, \frac{4\pi}{3} v_{\mathrm{w}}^3 (t-t')^3 \frac{\Gamma(t')}{\mathcal{V}}\right]$$

$$\frac{\Gamma}{\mathcal{V}} = \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t-t_f)}$$ Nucleation rate $$\beta \equiv \frac{\mathrm{d}}{\mathrm{d}t}\log\left(\frac{\Gamma(t)}{\mathcal{V}}\right)\Bigg|_{t=t_f}$$

$$-\log h(t) \simeq \int^{t} \mathrm{d}t' \frac{4\pi}{3} v_{\mathrm{w}}^3 (t-t')^3 \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t'-t_f)}$$
$$= \frac{4\pi}{3} v_{\mathrm{w}}^3 \frac{\Gamma_f}{\mathcal{V}} \frac{3!}{\beta^4} e^{\beta(t-t_f)}.$$

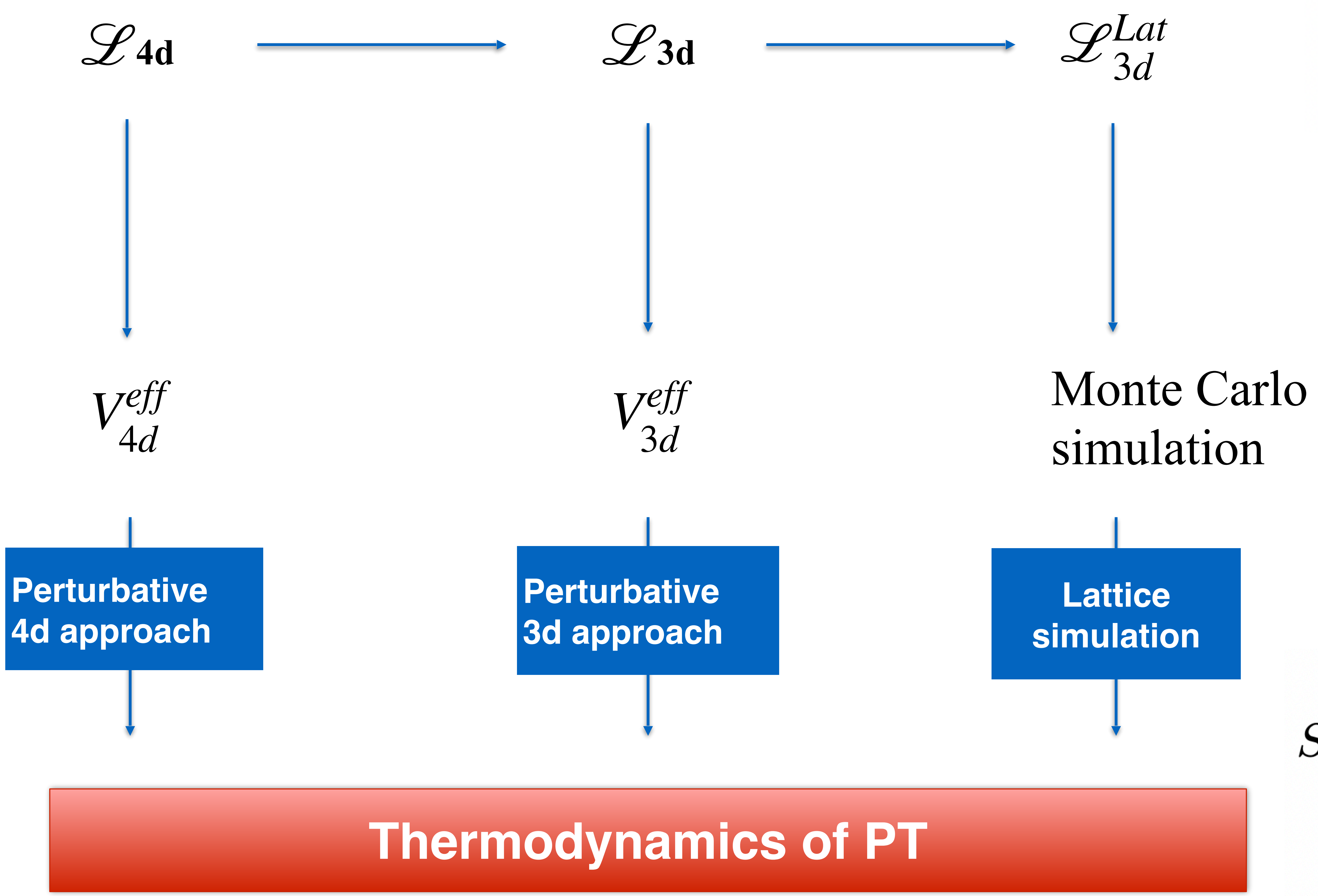
h(tf)=1/e

$$8\pi \frac{v_{\mathrm{w}}^3}{\beta^4}\frac{\Gamma_f}{\mathcal{V}} = 1$$

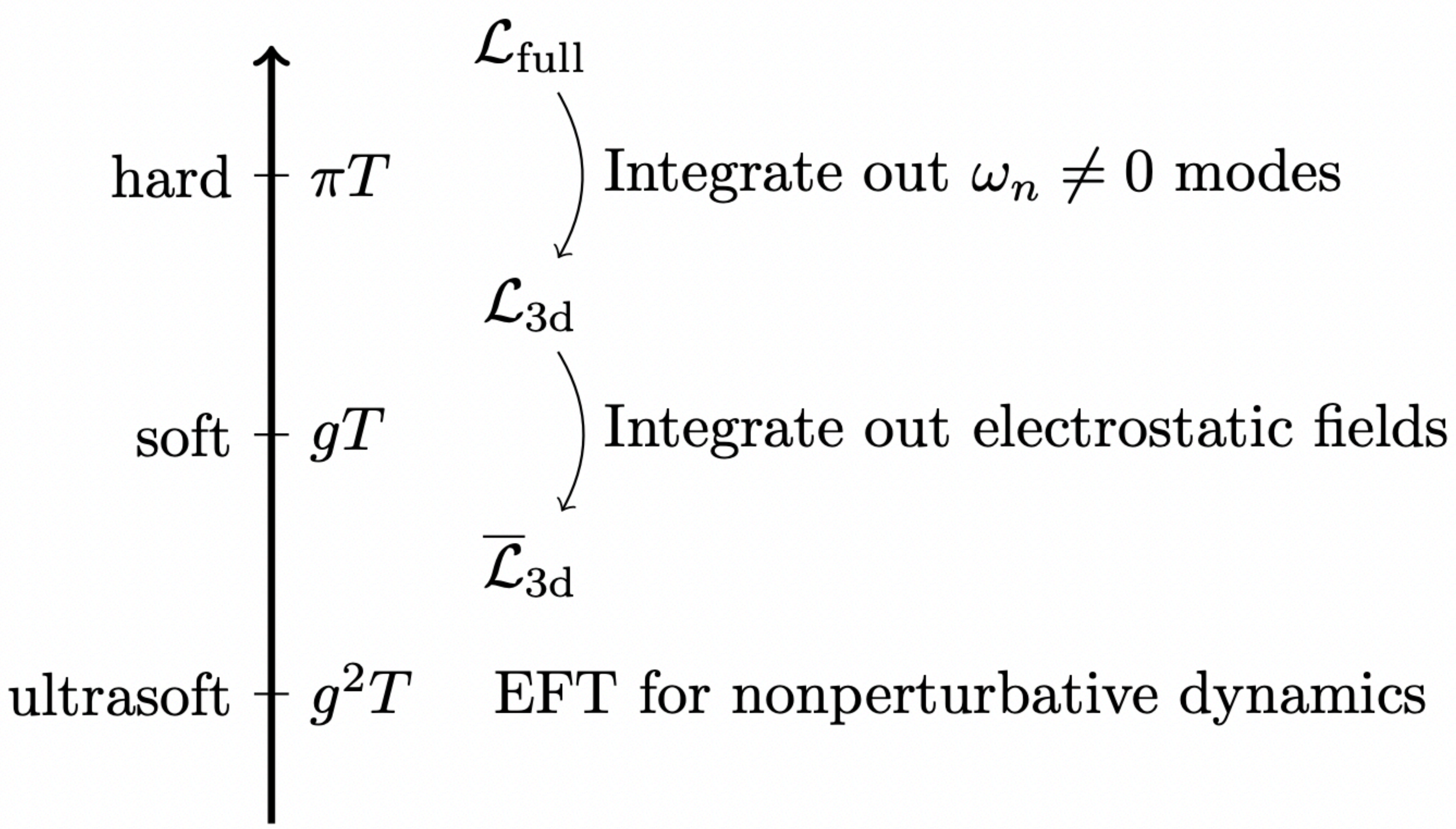
$$h(t) = \exp\left[-e^{\beta(t-t_f)}\right]$$

2008.09136

Perturbative theory for PT



$$S = \int d^4x \, [\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}]$$



$$S_{3\text{d}} = \int d^3x \left[\frac{1}{4}F_{ij}^a F_{ij}^a + (D_i\phi)^\dagger(D_i\phi) + \bar{m}^2\phi^\dagger\phi + \bar{\lambda}(\phi^\dagger\phi)^2 + \mathcal{L}_{\text{BSM}} + \text{higher-order operators}\right]$$

Gauge dependence of the PT parameters

$$\mathcal{L}_{3d}^{\text{ultrasoft}} = -\frac{1}{4}W^a_{ij}W^a_{ij} - \frac{1}{4}B_{ij}B_{ij} + (D_iH)^\dagger(D_iH) + \mathcal{L}_{\text{g.f}} + \mathcal{L}_{\text{ghost}} - V_{3d}^{\text{ultrasoft}}\ .$$

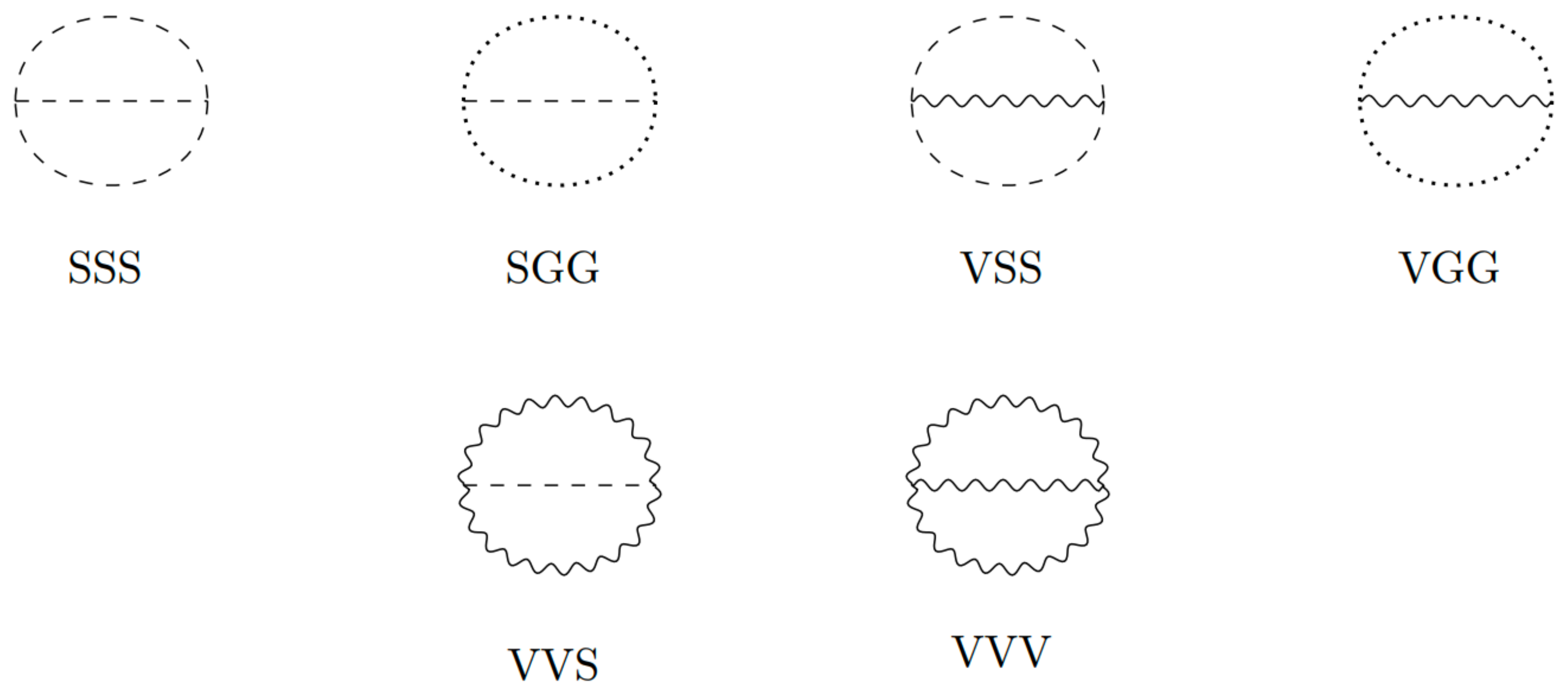
$$V^{\text{eff}}(\phi,T) = V_{\text{LO}} + V_{\text{NLO}}$$

$$V_{\text{LO}} = -\frac{1}{2}m^2\phi^2 + \frac{\lambda\phi^4}{4} + \frac{c_6\phi^6}{8} - \frac{1}{12\pi}\left(4m_W^3 + 2m_Z^3\right)$$

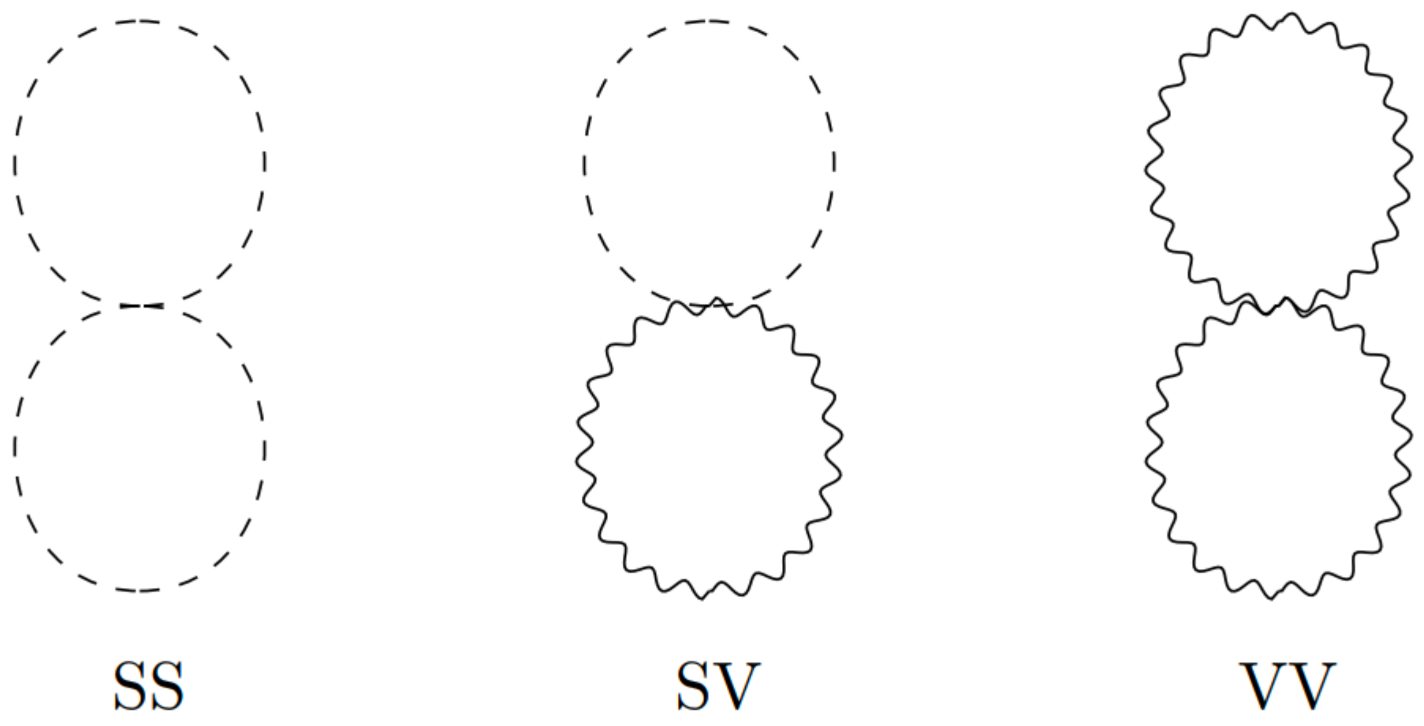
$$V_{\text{NLO}} = V_{\text{NLO}}^0 + \sqrt{\xi}V_{\text{NLO}}^\xi$$

$$V_{\text{NLO}}^0 = A\phi^2 + B\phi^2\ln\left(\frac{\phi}{\bar{\mu}}\right)$$

$$V_{\text{NLO}}^\xi = \frac{(2m_W + m_Z)(2m_W^3 + m_Z^3)}{16\pi^2\phi^2} - \frac{2m_W + m_Z}{8\pi}m_G^2$$



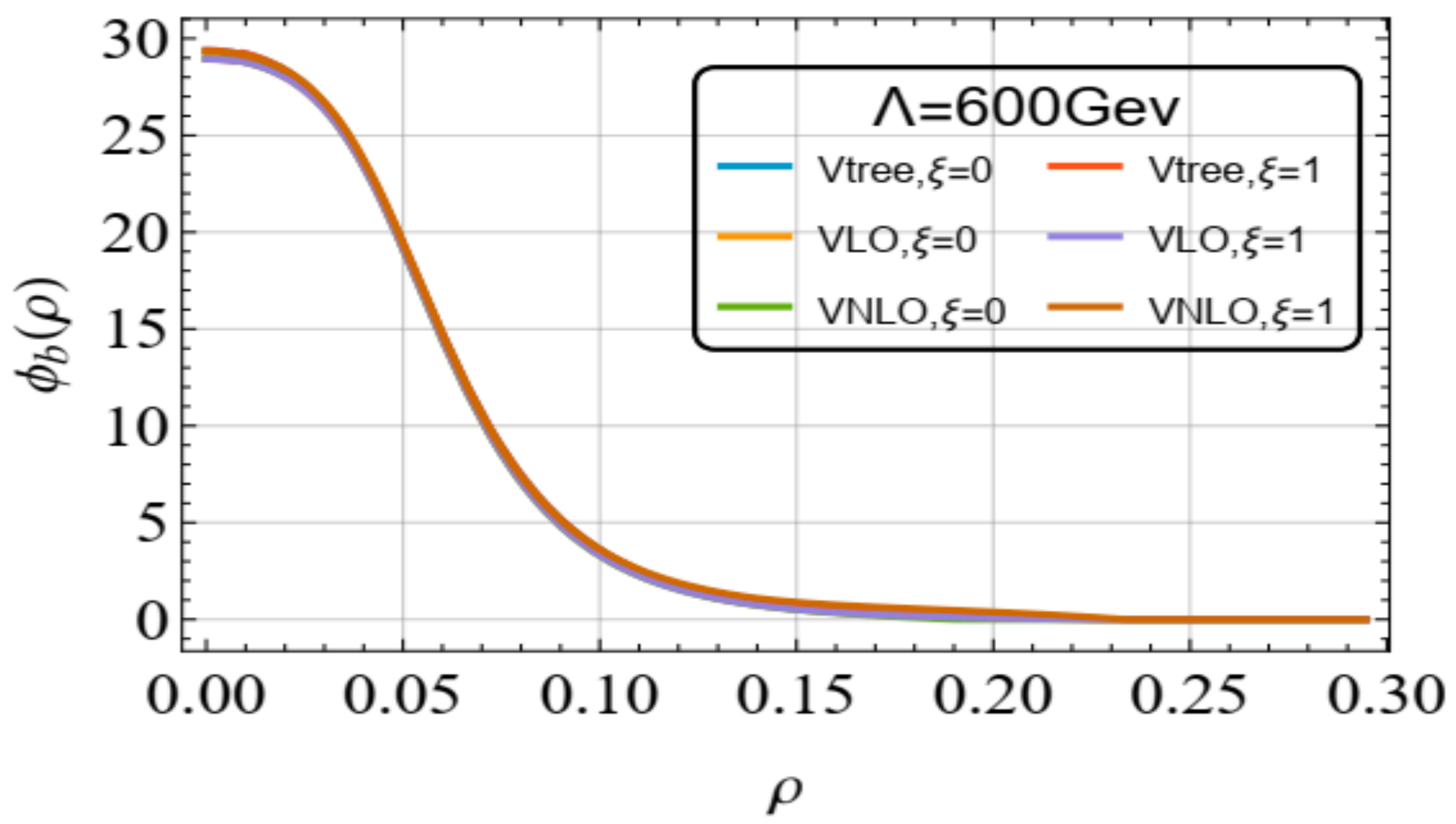
Sunset contributions

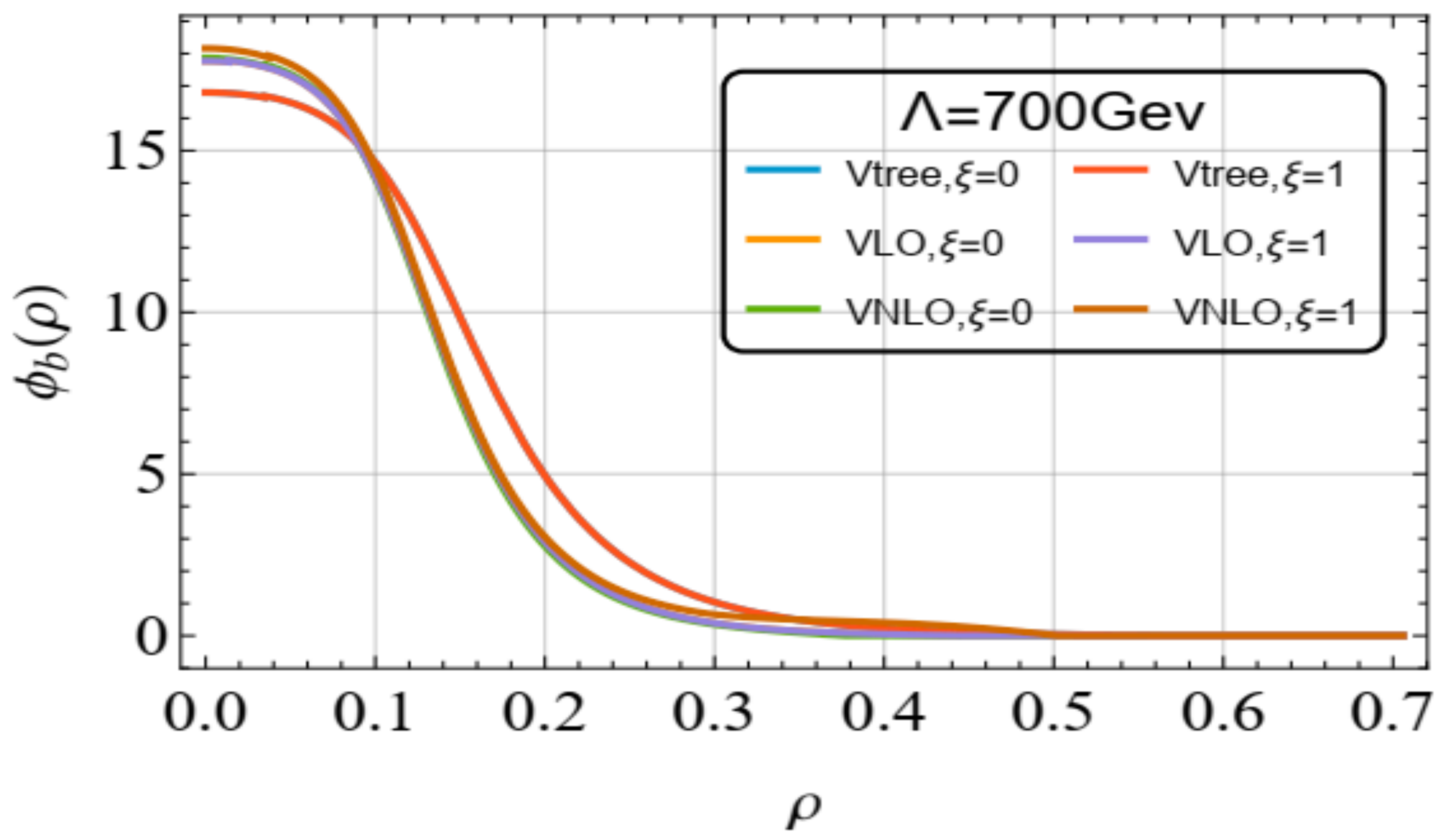


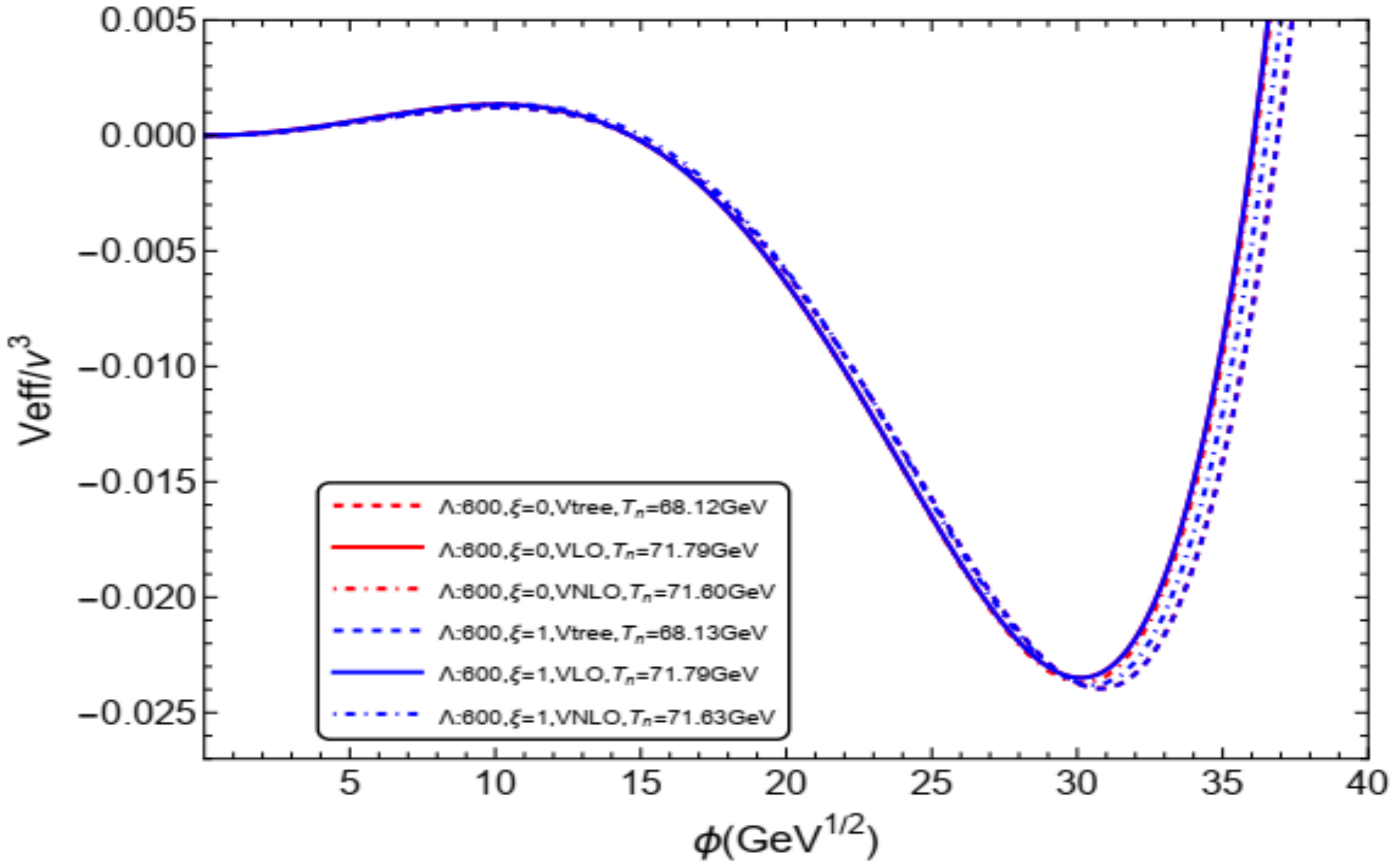
Double bubble contributions

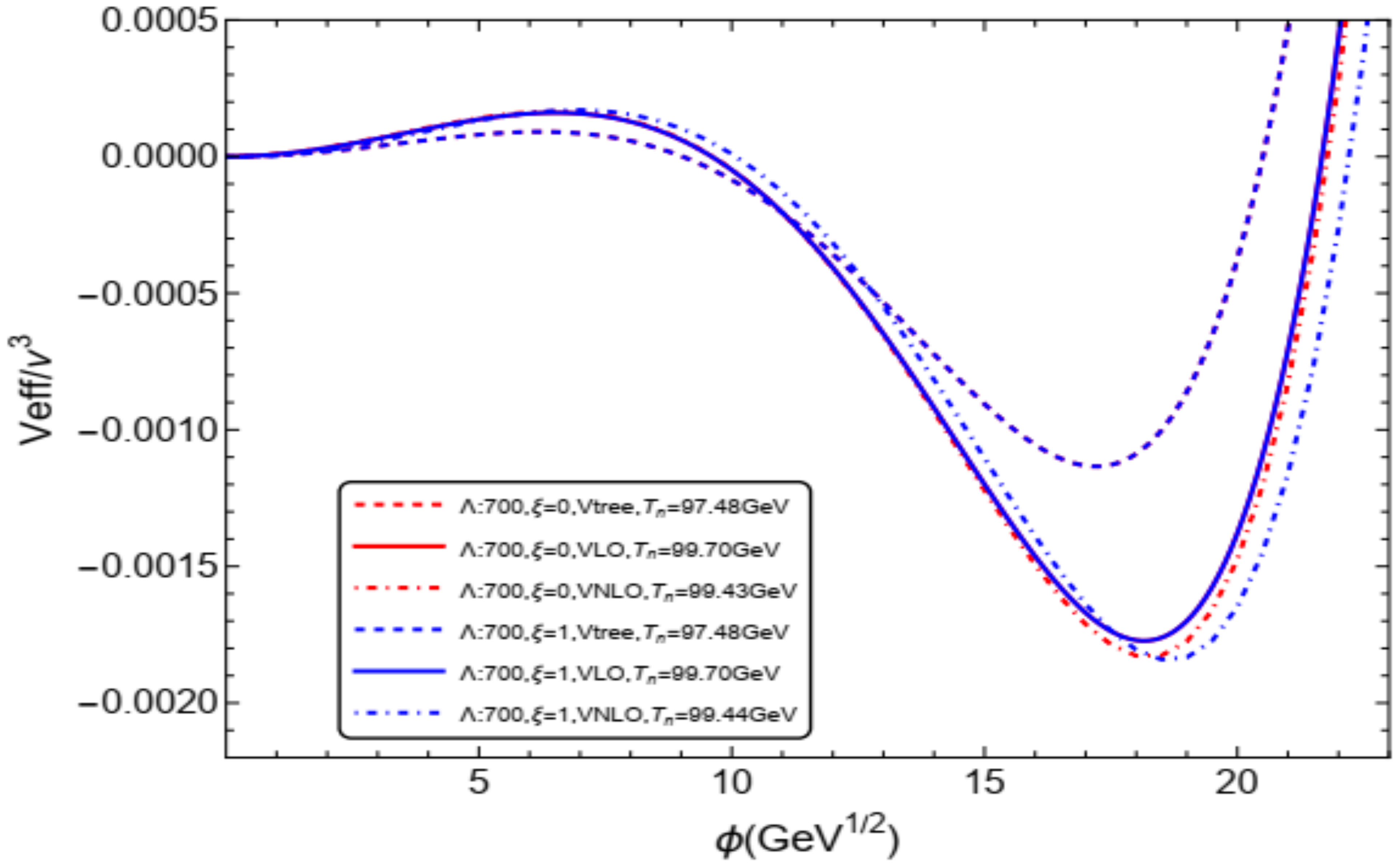
Jie Liu, Renhui Qin, **Ligong Bian**,2512.05565

Gauge dependence of the PT parameters

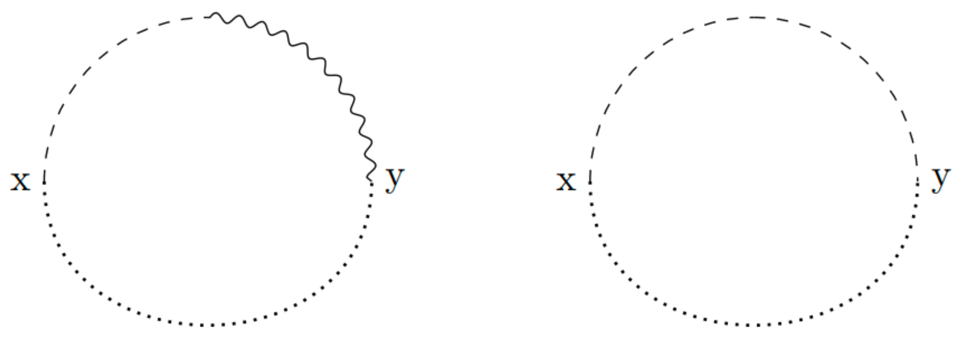




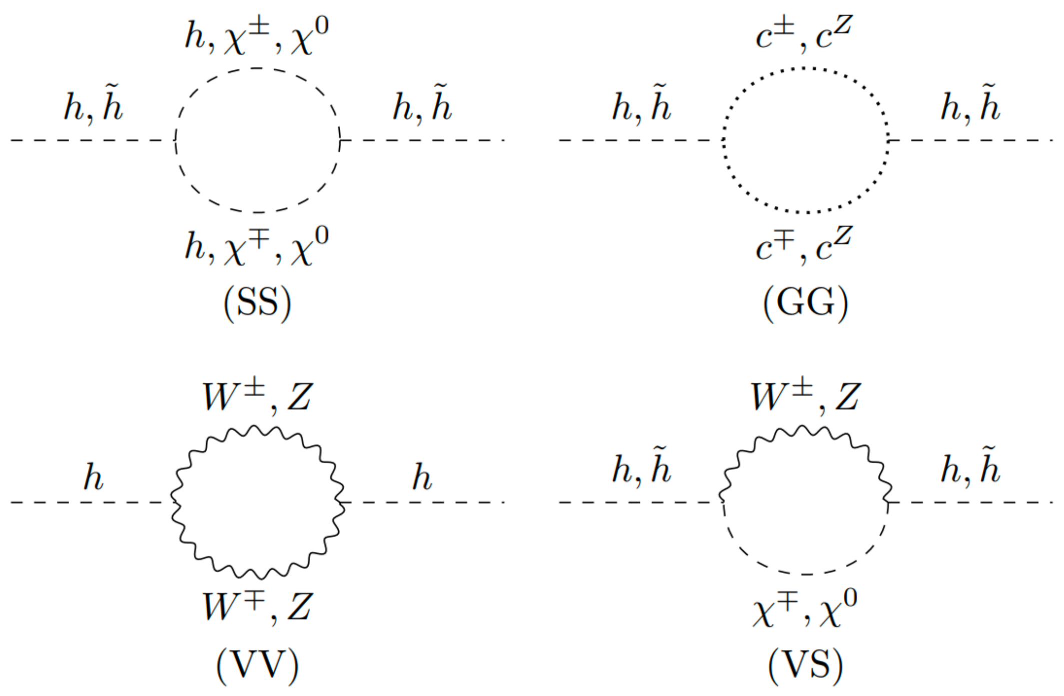




Gauge dependence of the PT parameters



$$C_{\rm LO} = \frac{\left(2g + \sqrt{g^2 + g'^2}\right)\sqrt{\xi}}{32\pi}$$



$$Z_{\rm NLO} = -\frac{11\left(\sqrt{g^2 + g'^2} + 2g\right)}{48\pi\phi}$$

$$\xi \frac{\partial}{\partial \xi} V_{\rm NLO} = -C_{\rm LO} \frac{\partial}{\partial \phi} V_{\rm LO} \ ,$$

$$\xi \frac{\partial}{\partial \xi} Z_{\rm NLO} = -2 \frac{\partial}{\partial \phi} C_{\rm LO} \ .$$

Nielsen identities

$$S_3 = \int d^3x \left[V^{\rm eff}(\phi, T) + \frac{1}{2} Z(\phi, T)\, (\partial_i \phi)^2 + \cdots \right]$$

$$Z = 1 + Z_{\rm NLO} + \mathcal{O}(g^2)$$

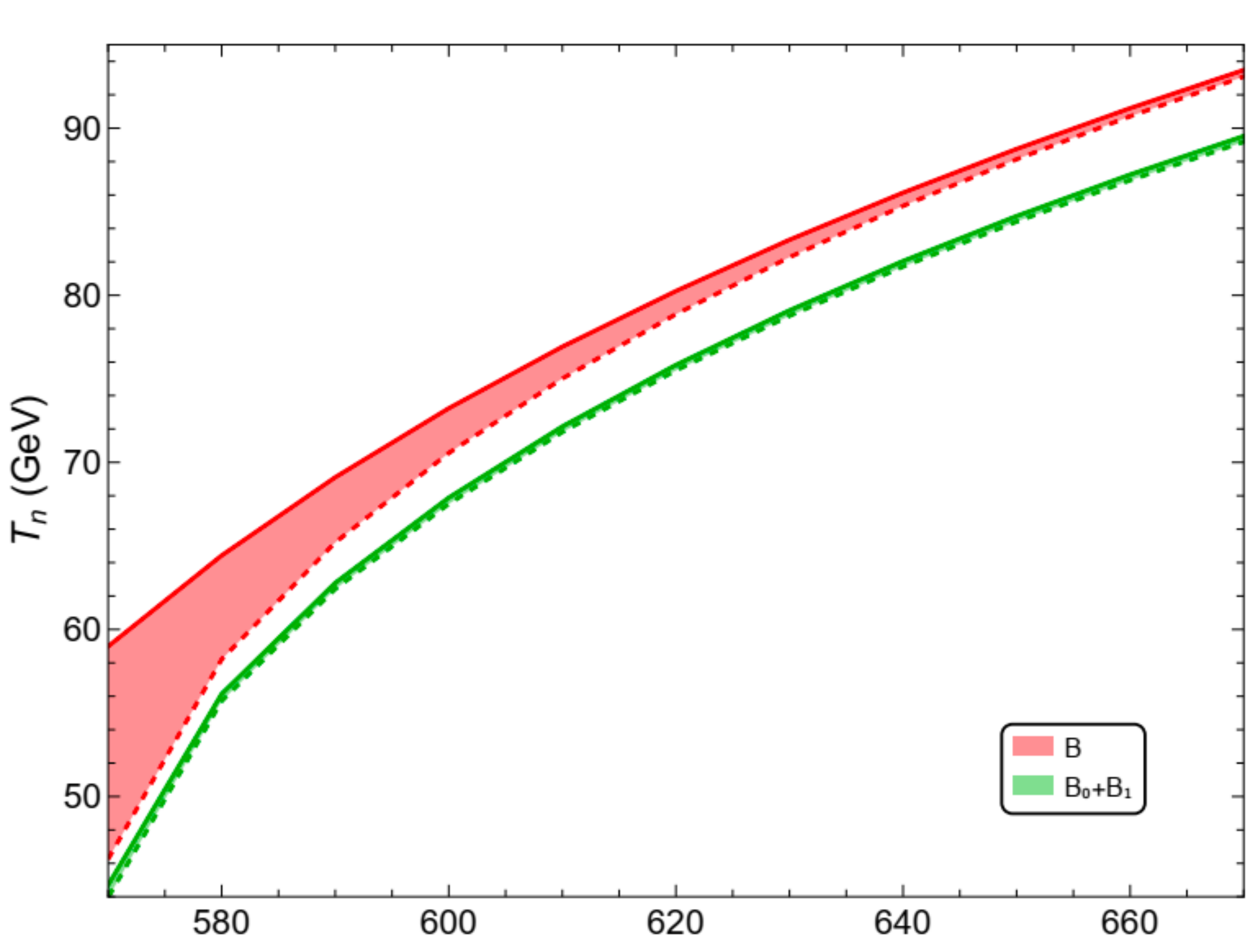
$$\mathcal{B}_0 = \int d^3x \left[V_{\rm LO}(\phi_b) + \frac{1}{2}\, (\partial_i \phi_b)^2 \right] \ ,$$

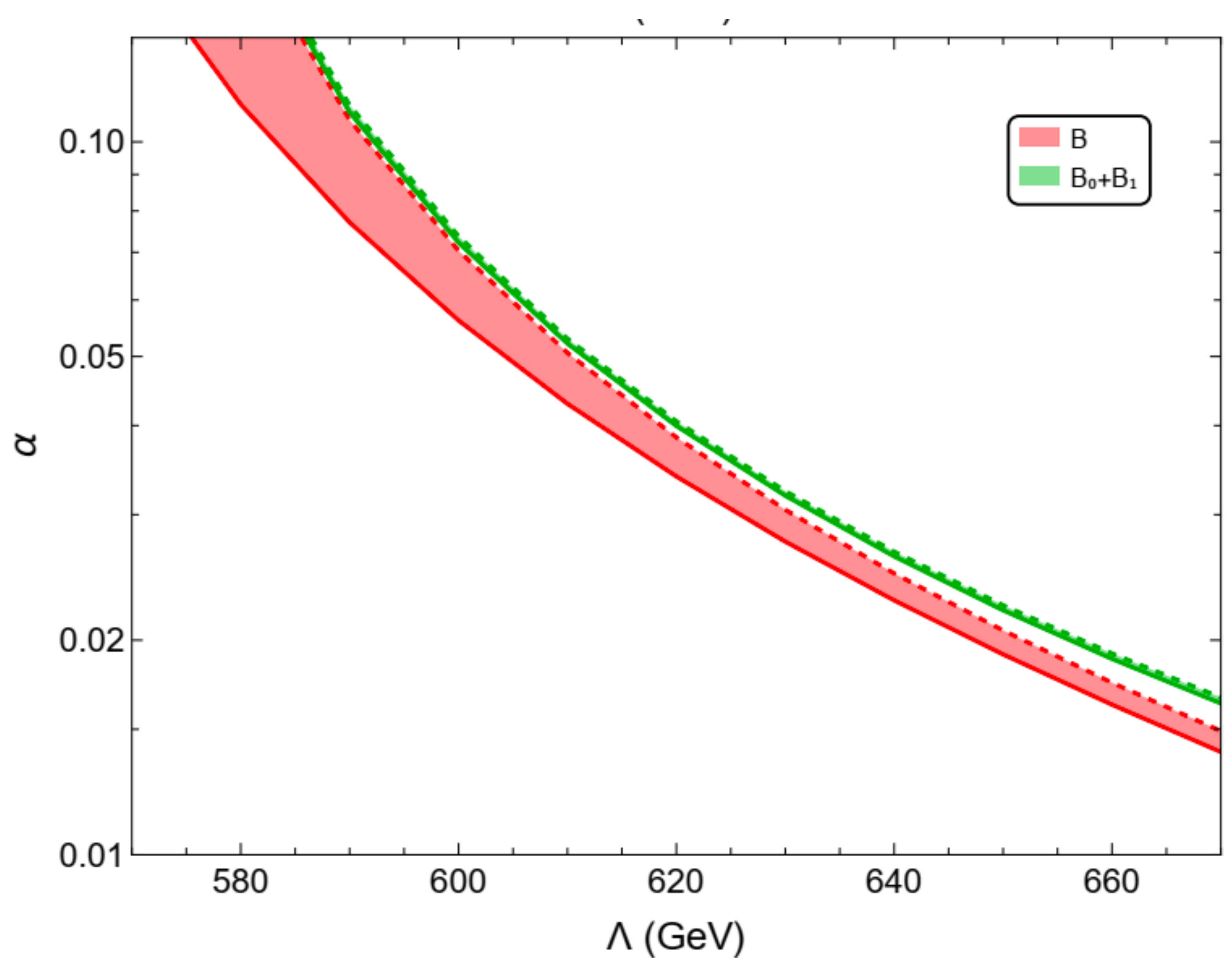
$$\mathcal{B}_1 = \int d^3x \left[V_{\rm NLO}(\phi_b) + \frac{1}{2} Z_{\rm NLO}(\phi_b)\, (\partial_i \phi_b)^2 \right]$$

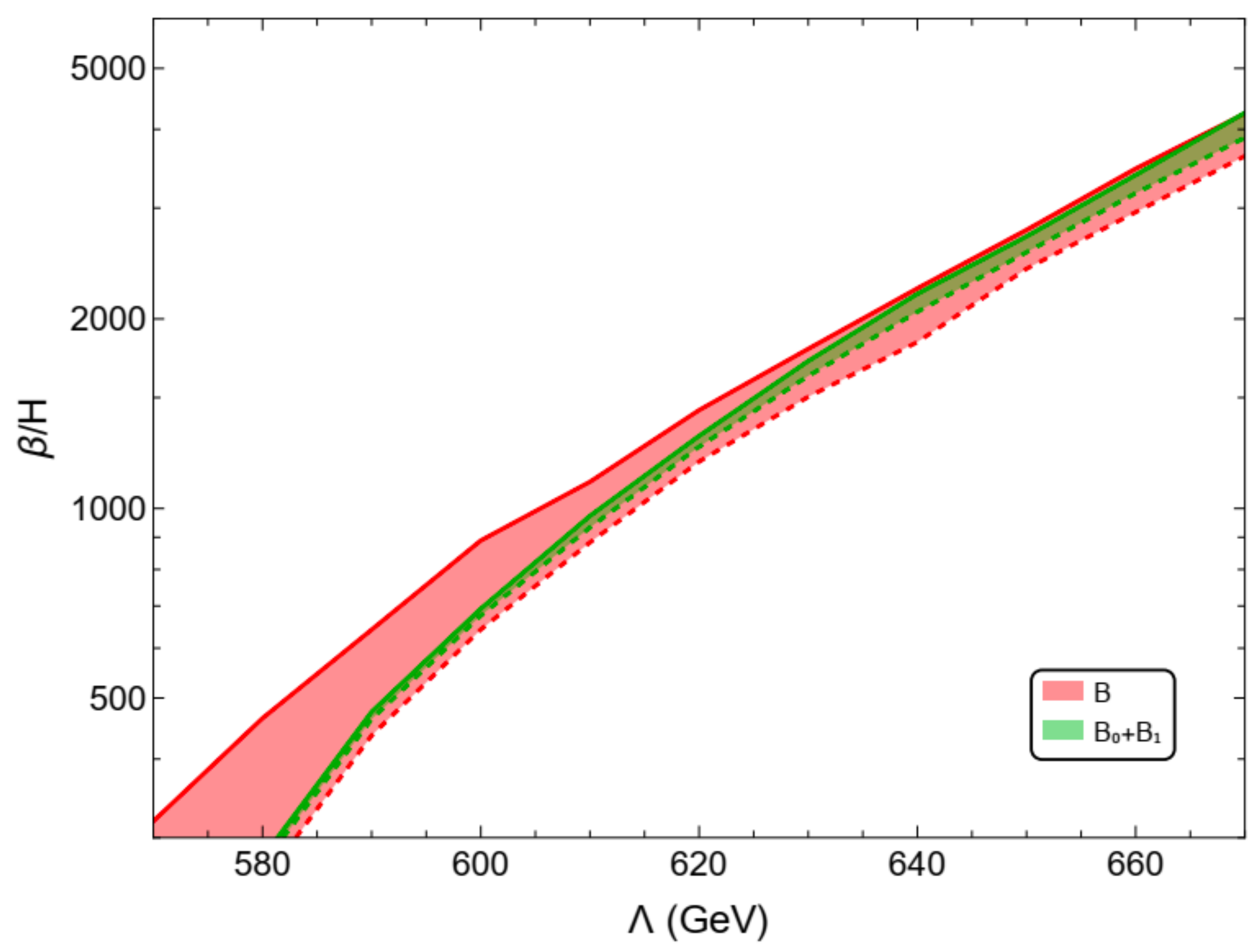
Gauge independence of the bounce action

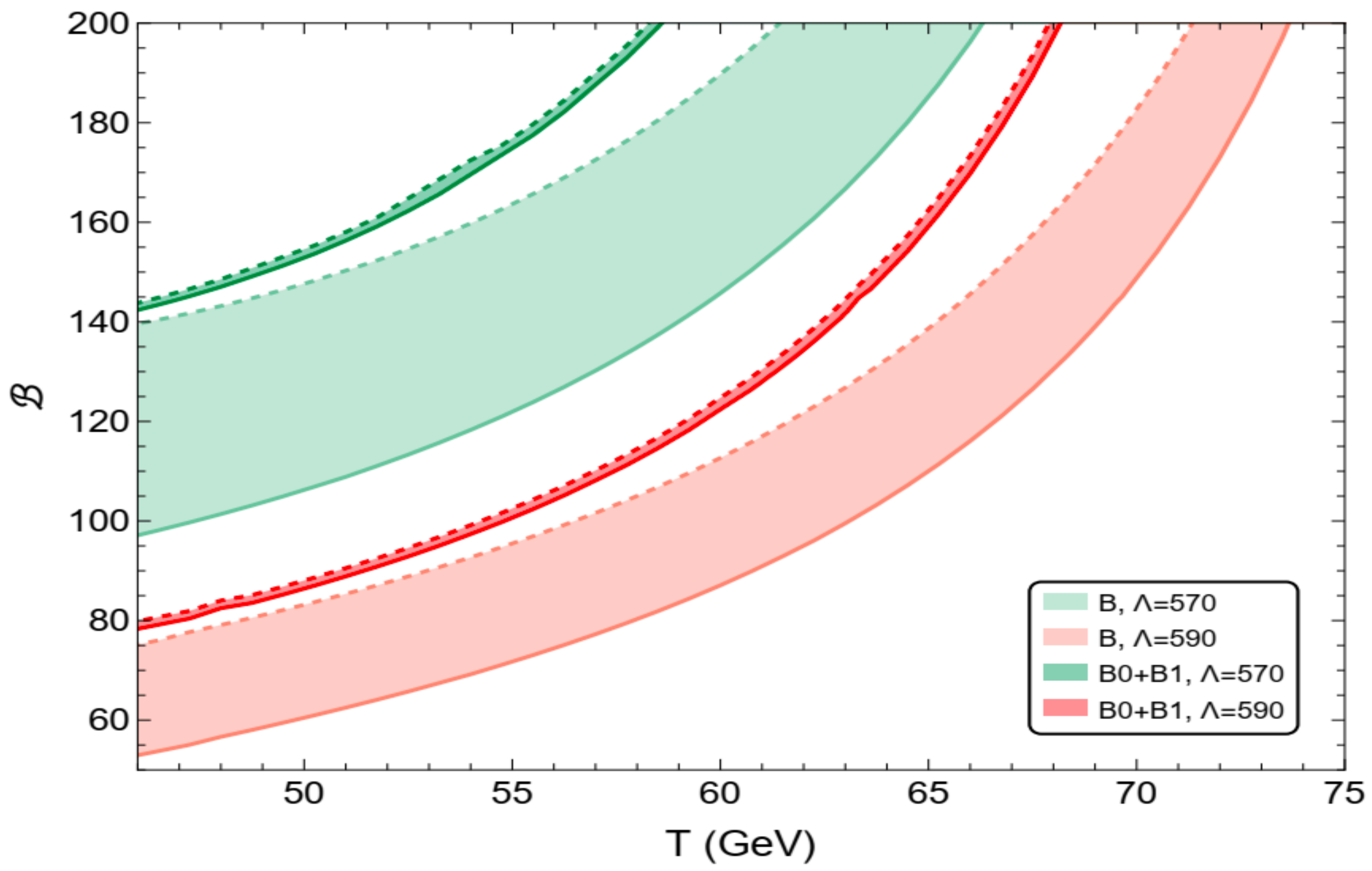
$$\begin{aligned}
\xi \frac{\partial}{\partial \xi} \mathcal{B}_1 &= \xi \frac{\partial}{\partial \xi} \int d^3x \left[V_{\rm NLO}(\phi_b) + \frac{1}{2} Z_{\rm NLO}(\phi_b)(\partial_\mu \phi_b)^2 \right] \\
&= \int d^3x \left[-C_{\rm LO} \frac{\partial V_{\rm LO}}{\partial \phi} - \frac{\partial C_{\rm LO}}{\partial \phi} (\partial_\mu \phi_b)^2 \right] \\
&= -\int d^3x \left[C_{\rm LO} \frac{\partial V_{\rm LO}}{\partial \phi} + \partial^\mu C_{\rm LO} (\partial_\mu \phi_b) \right] \\
&= -\int d^3x C_{\rm LO} \left[\frac{\partial V_{\rm LO}}{\partial \phi} - \Box \phi_b \right] = 0
\end{aligned}$$

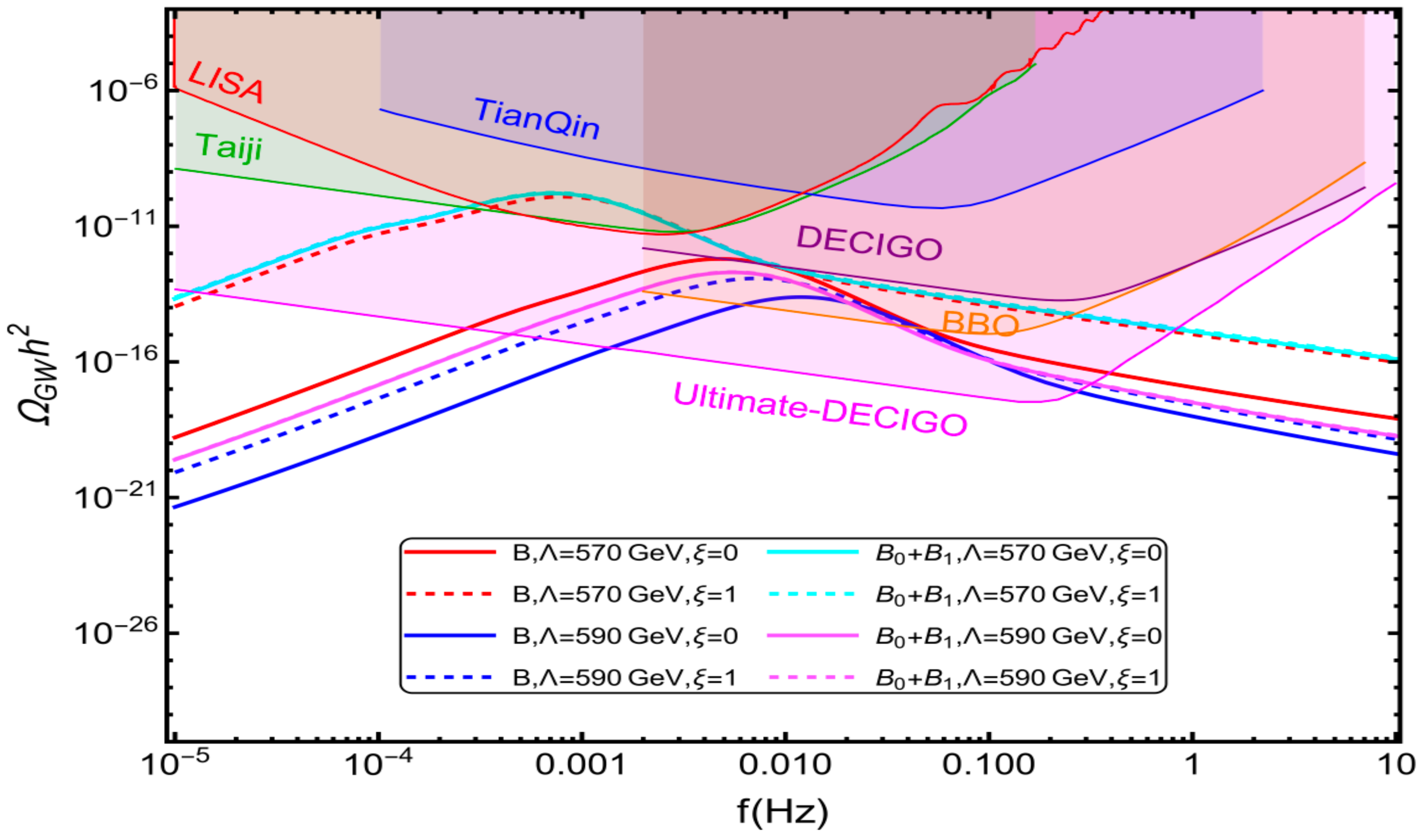
Gauge dependence of the PT parameters











1+1D simulation-across Quantum-Thermal crossover

The decay probability is formulated as a phase-space average over field configurations, with the false-vacuum basin specified by an indicator functional

$$P_{\rm FV}(t) = \frac{1}{Z}\int D\phi \frac{D\Pi}{2\pi}\Theta_{\rm FV}[\phi] W[\phi,\Pi;t].$$

$$W[\phi,\Pi;t] = \int D\varphi\, e^{-\frac{i}{\hbar}\int dx\, \Pi\varphi} \left\langle \phi + \frac{\varphi}{2}\middle|\hat{\rho}(t)\middle|\phi - \frac{\varphi}{2}\right\rangle.$$

Consider initial density matrix operator: $\hat{\rho} = \dfrac{1}{Z}e^{-\beta\hat{H}}$

The interacting thermal state is approximated by a self-consistent Gaussian quasi-free state.

Mode-by-Mode Positive Wigner Sampling:

$$W_k(\eta_k,\xi_k) = \exp\left[-\tanh\frac{\beta\omega_k}{2}\left(\frac{|\xi_k|^2}{\omega_k} + \omega_k|\eta_k|^2\right)\right],$$

$$\omega_k = \sqrt{k^2 + M_H^2}.$$

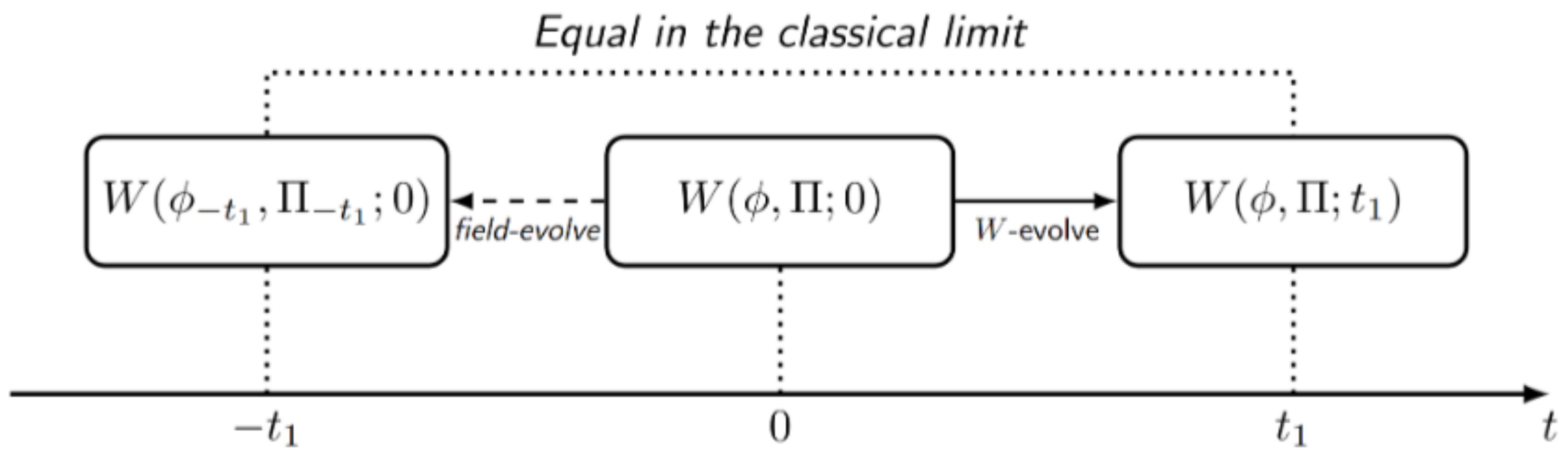
$$\langle|\eta_k|^2\rangle = \frac{1}{2\omega_k}\coth\frac{\beta\omega_k}{2}, \qquad \langle|\xi_k|^2\rangle = \frac{\omega_k}{2}\coth\frac{\beta\omega_k}{2}.$$

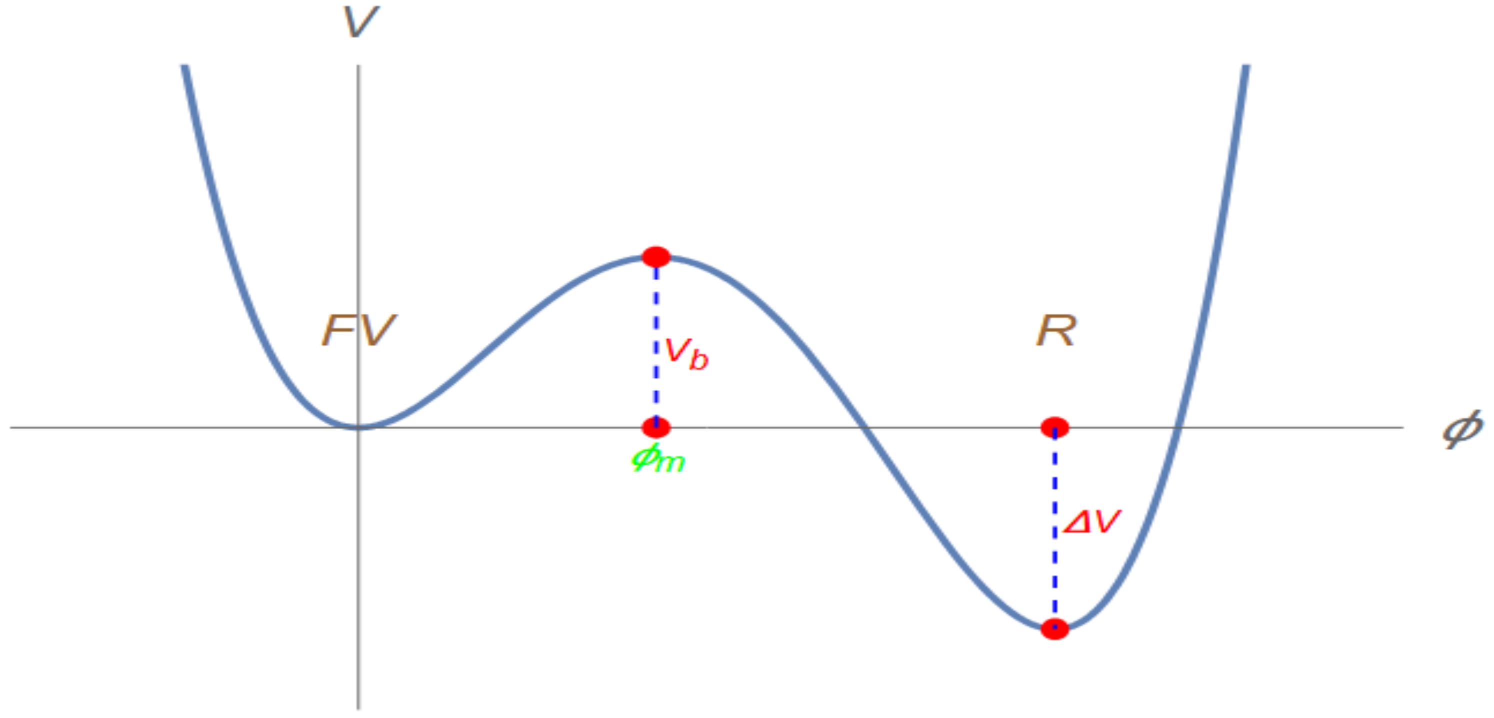
This positive Hartree-Gaussian Wigner functional allows direct Monte Carlo sampling of initial field and momentum modes, including both zero-point and thermal fluctuations.

Haiyang Wang, Renhui Qin, Ligong Bian, arXiv: 2506.18334

1+1D simulation-across Quantum-Thermal crossover

The initial quantum-thermal ensemble is evolved by classical Hamiltonian trajectories after neglecting higher-order $\hbar^2$terms in the Wigner equation.





$$\hat{L} = \int d^3x \left(\frac{1}{2}\partial^\mu \hat{\phi}\partial_\mu \hat{\phi} - V(\hat{\phi})\right) \qquad \hat{H} = \int d^3x \hat{\mathcal{H}} = \int d^3x \left[\frac{\hat{\Pi}^2}{2} + \frac{(\nabla\hat{\phi})^2}{2} + V(\hat{\phi})\right]$$

$$i\hbar \frac{\partial}{\partial t}\hat{\rho}(t) = [\hat{H}, \hat{\rho}(t)] \qquad \frac{\partial}{\partial t} W[\phi, \pi; t] = -2H\frac{1}{i\hbar}\sin\left(\frac{i\hbar}{2}\Lambda\right) W[\phi, \pi; t]$$

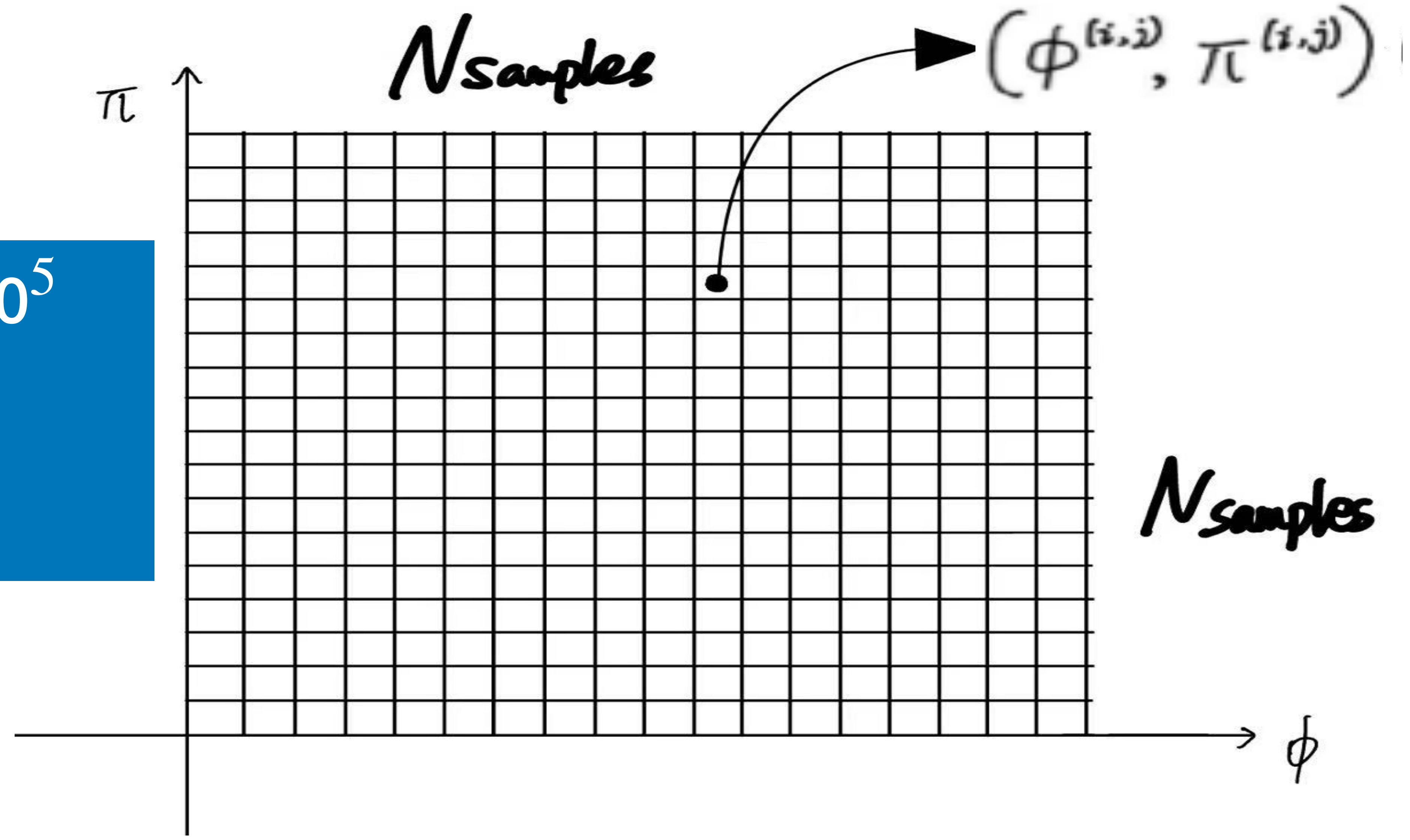
$$\Lambda = \frac{\overleftarrow{\delta}}{\delta \Pi}\frac{\overrightarrow{\delta}}{\delta \phi} - \frac{\overleftarrow{\delta}}{\delta \Phi}\frac{\overrightarrow{\delta}}{\delta \Pi}$$ is the Poisson bracket operator

Ignore $O(\hbar^2)$

$$\left[\frac{\partial}{\partial t} + \int d^3x \left(\frac{\delta H}{\delta \Pi}\frac{\delta}{\delta \phi} - \frac{\delta H}{\delta \phi}\frac{\delta}{\delta \Pi}\right)\right] W[\phi, \Pi; t] = 0$$

$$\begin{cases} \dfrac{\delta H}{\delta \Pi} = \dfrac{d\phi}{dt} = \Pi \\ -\dfrac{\delta H}{\delta \phi} = \dfrac{d\Phi}{dt} = \nabla^2\phi - \dfrac{\delta V(\phi)}{\delta \phi} \end{cases} \qquad W[\phi, \Pi; t] = W[\phi_{-t}, \Pi_{-t}; 0]$$

Nsamples = 10^5
Δx = 0.25
Δt = 0. 01
Nx = 1024



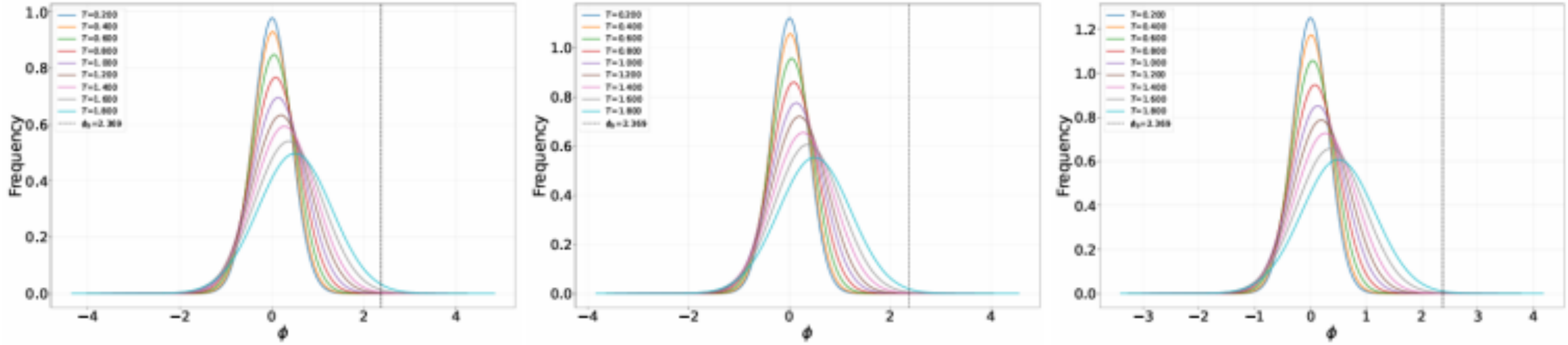
1+1D simulation-across Quantum-Thermal crossover

Coarse-Graining: A method for eliminating high-k noise

Short-wavelength high-k modes introduce local noise irrelevant to large-scale decay dynamics;

we thus adopt coarse-grained fields to filter short-range fluctuations, extract bubble-nucleation-dominated low-energy physics, and enhance numerical stability and physical clarity.

$$\phi_\ell(x,t) = \int dy\, W_\ell(x-y)\phi(y,t) \qquad \ell = c_1\xi_{\rm FV}, \quad \xi_{\rm FV} \sim M_H^{-1}.$$



Coarse graining suppresses UV pointwise noise and isolates the long-wavelength structures relevant for bubble nucleation.

1+1D simulation-across Quantum-Thermal crossover

Indicator Functional: How to Define a field configuration Decay?

$$P_{\mathrm{FV}}(t) = \frac{1}{Z}\int D\phi \frac{D\Pi}{2\pi}\Theta_{\mathrm{FV}}[\phi]W[\phi,\Pi;t] \xrightarrow{\text{Monte Carlo average}} P_{\mathrm{FV}}(t) \simeq \frac{1}{N_s}\sum_{i=1}^{N_s}\Theta_{\mathrm{FV}}\left[\phi_t^{(i)}\right].$$

The main ambiguity is not the Monte Carlo average itself, but how Θ_{FV} is implemented for an extended field configuration. Different choices define different real-time observables.

- Global survival:

  $\bar{\phi}_\ell(t) > \phi_{\mathrm{th}}$. Whole-volume average crossing.

- Connected-cluster survival:

  $\phi_\ell(x,t) > \phi_{\mathrm{th}}$ over a connected interval with $L > L_c, \Delta t > \tau_{\mathrm{hold}}$. Persistent supercritical domain.

- False-vacuum fraction:

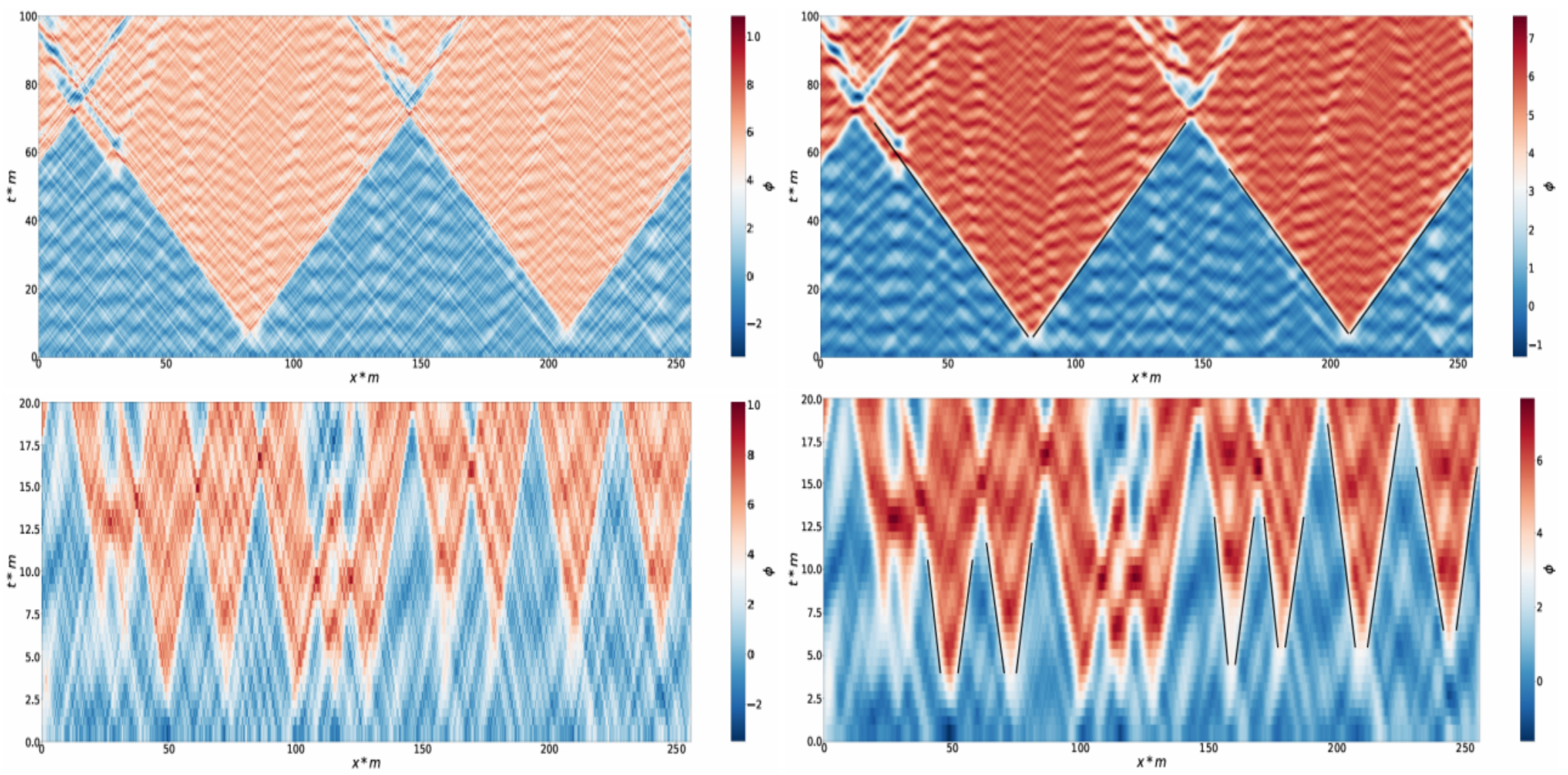
$$f_{\mathrm{FV}}^{(i)}(t) = \frac{1}{L}\int dx\, \Theta\left(\phi_{\mathrm{th}} - \phi_\ell^{(i)}(x,t)\right),$$

$$P_{\mathrm{frac}}(t) = \frac{1}{N_s}\sum_i f_{\mathrm{FV}}^{(i)}(t).$$

Global and connected-cluster criteria are sample-level survival observables, while P_{frac} measures spatial conversion.

1+1D simulation-across Quantum-Thermal crossover

Real-Time Dynamics: Low vs High Temperature



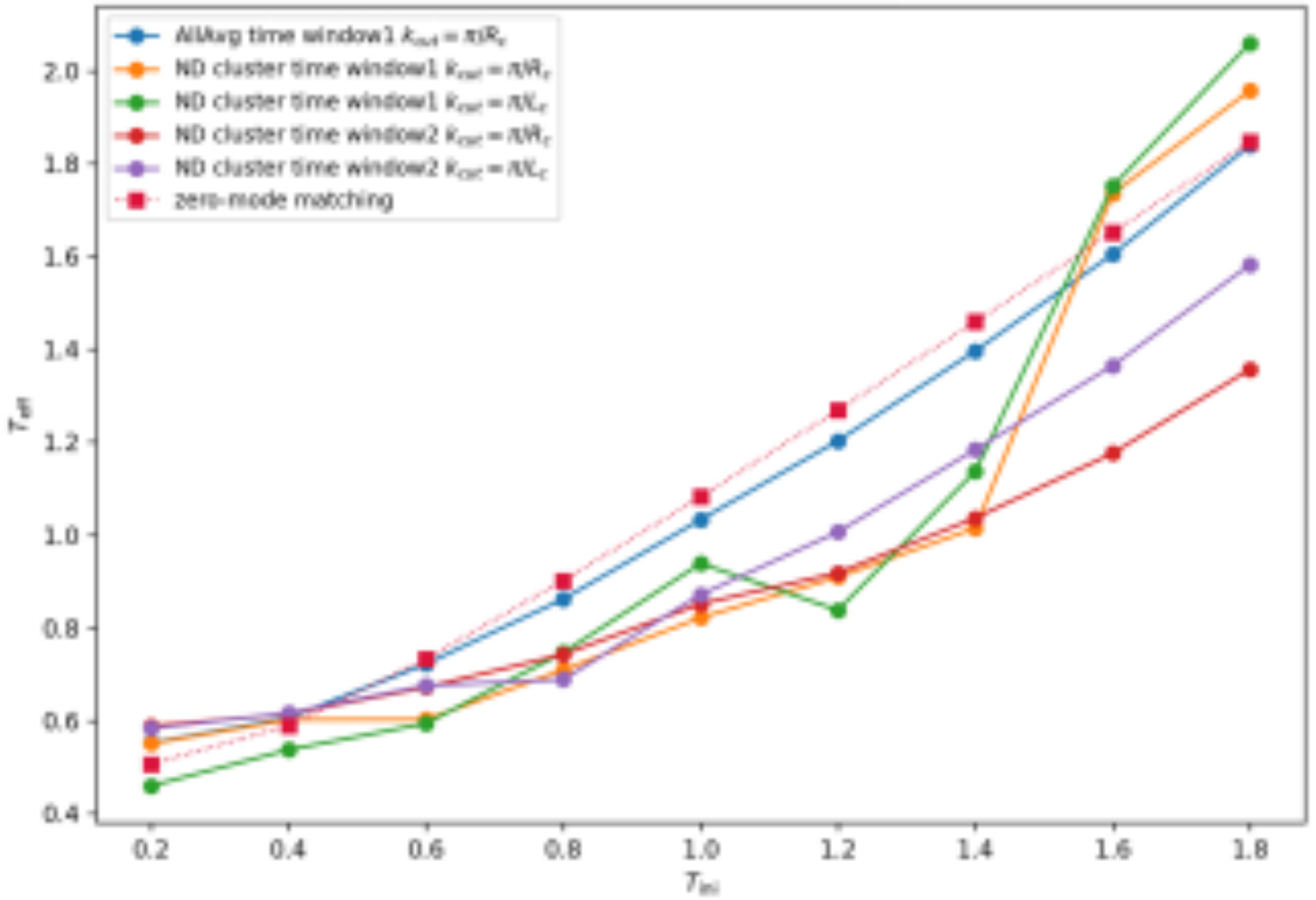
Low Tem:
Dilute kink–antikink-like domains; partial reflection or return can occur.

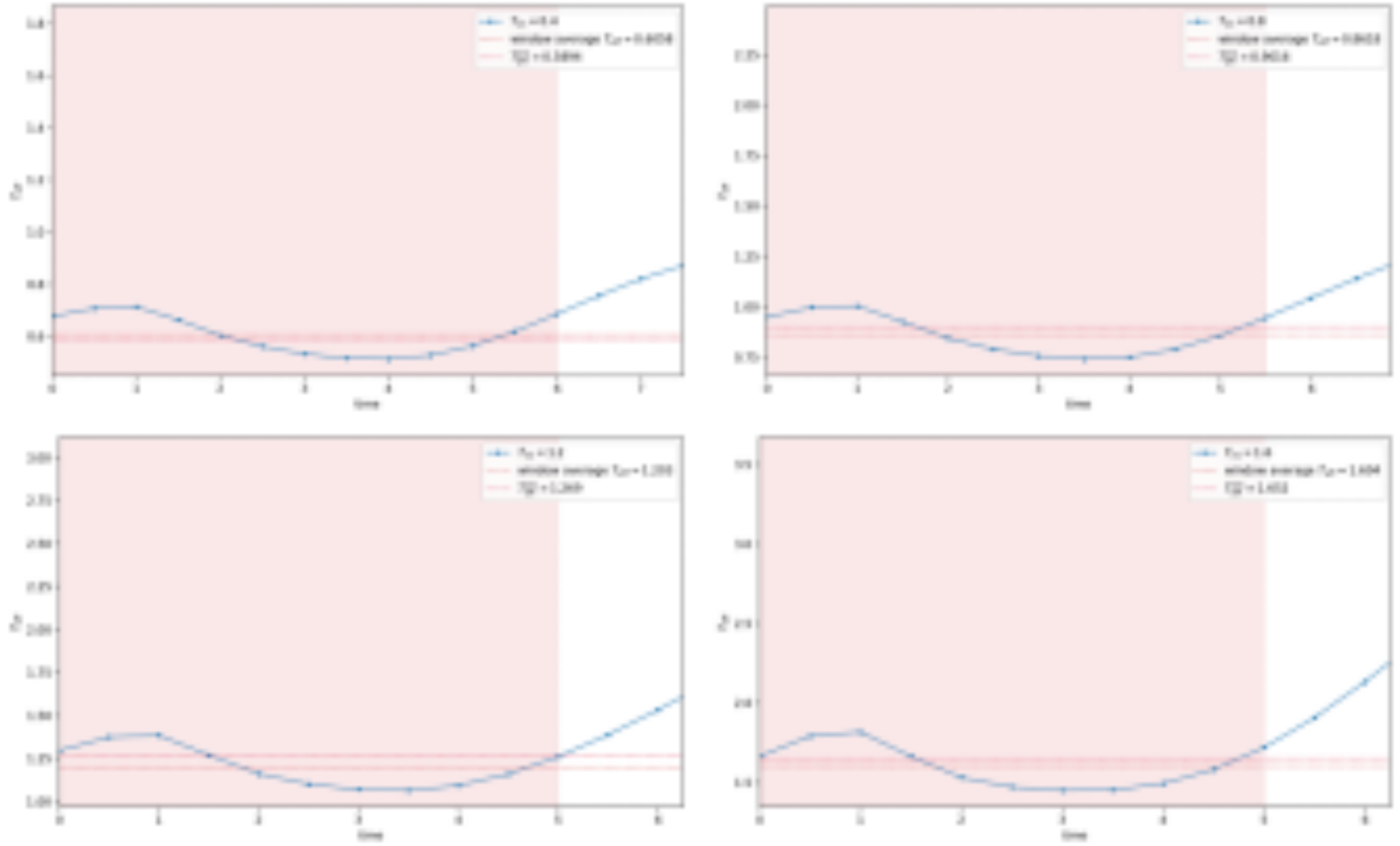
High Tem:
Multiple local transformed regions appear and grow rapidly; dynamics is closer to nucleation-and-growth.

Effective Temperature:

Since the sampled ensemble contains both thermal and zero-point fluctuations, the comparison with thermal nucleation theory uses a low-k effective temperature $T_{\rm eff}$, not simply $T_{\rm ini}$

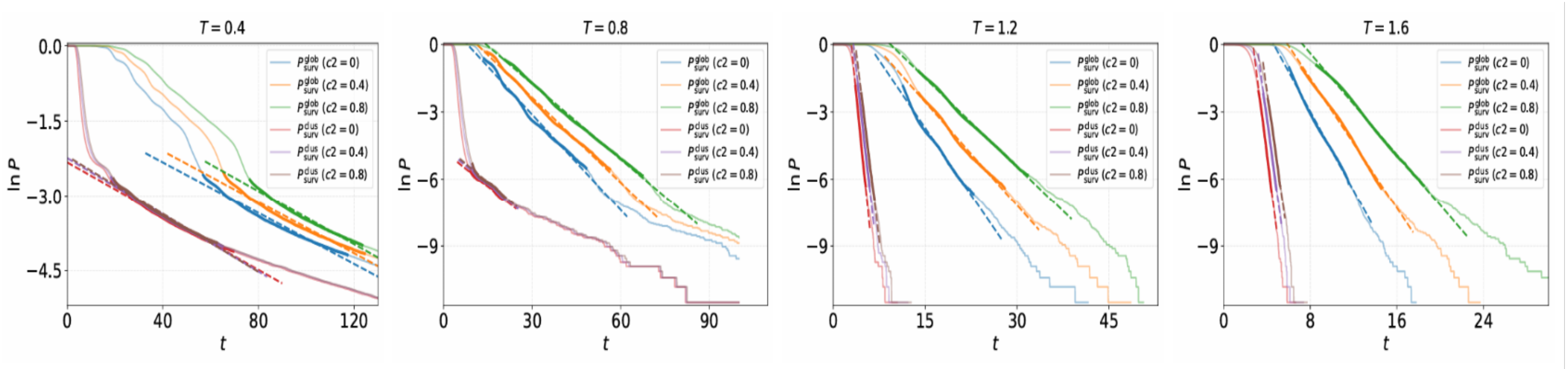
$$P_{\Pi}(k,t) \simeq T_{\rm eff}, \qquad k < k_{\rm cut}.$$

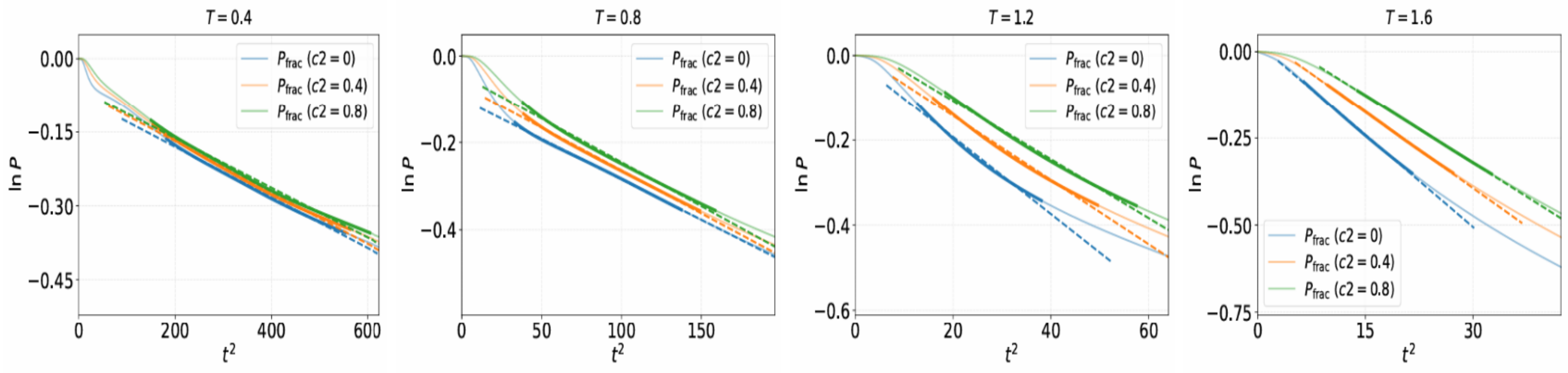




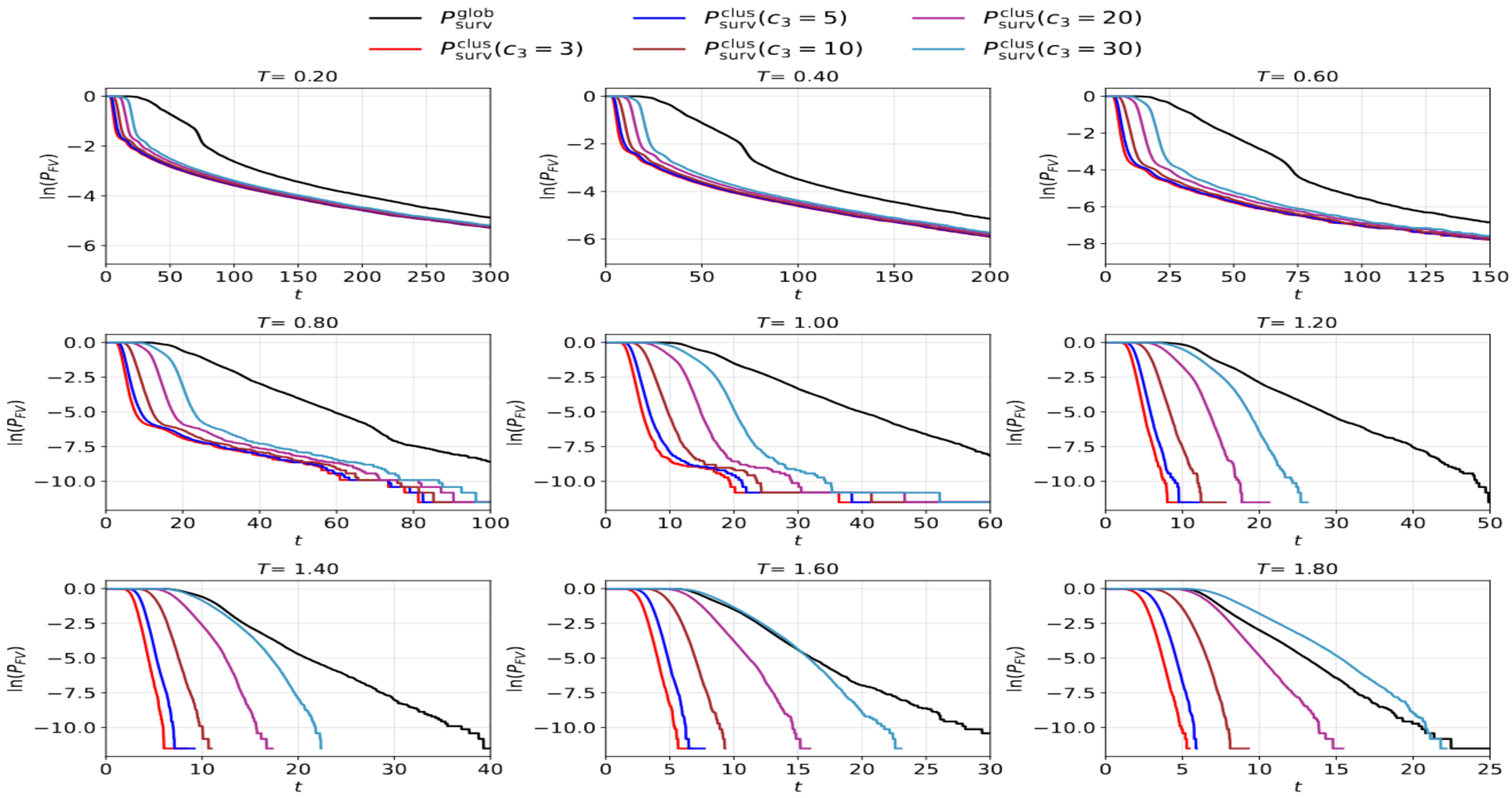
In our study, we use “AllAvg” result, is obtained by averaging the low-k spectrum over all samples.

1+1D simulation-across Quantum-Thermal crossover





1+1D simulation-across Quantum-Thermal crossover



1+1D simulation-across Quantum-Thermal crossover

For the benchmark potential, the uniform background ϕ is treated as a scanning variable. At each fixed ϕ, we solve the same renormalized Hartree gap equation as in the initial-state construction.

$$M_H^2(\phi, T) = m^2 + 2g\phi + 3\lambda\phi^2 + 3\lambda G_{\text{eff}}(\phi, T),$$

$$G_{\text{eff}}(\phi, T) = \frac{1}{L}\sum_k \left[\frac{n_B(\omega_k) + 1/2}{\omega_k}\right]_{\text{sub}},$$

$$\omega_k^2 = \hat{k}^2 + M_H^2(\phi, T).$$

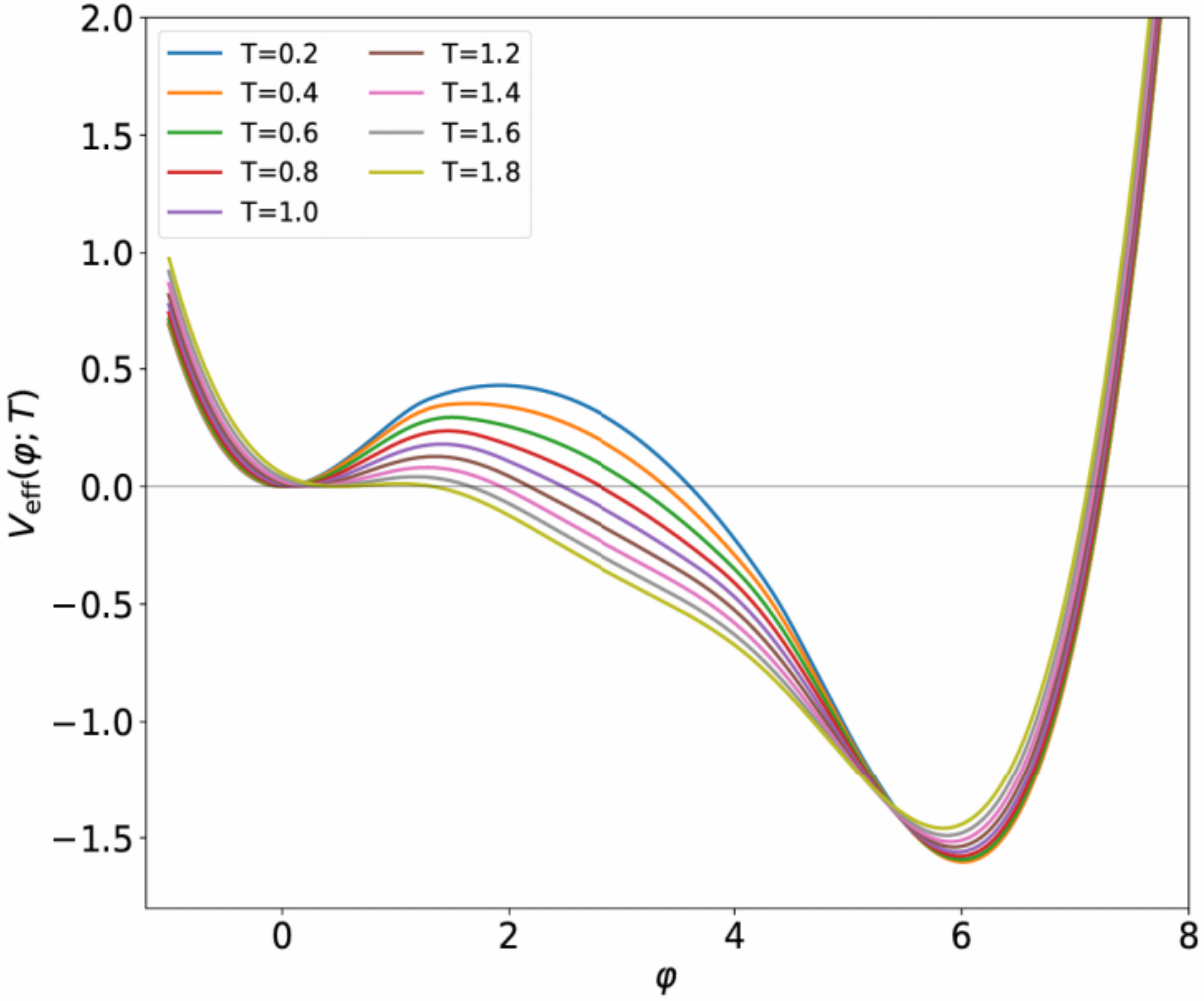
$$\frac{\partial V_{\text{eff}}(\phi; T)}{\partial \phi} = m^2\phi + g\phi^2 + \lambda\phi^3 + gG_{\text{eff}}(\phi, T) + 3\lambda\phi G_{\text{eff}}(\phi, T).$$

$$V_{\text{eff}}(\phi; T) = \int^{\phi} d\phi' \, \frac{\partial V_{\text{eff}}(\phi'; T)}{\partial \phi'} + \text{const.}$$

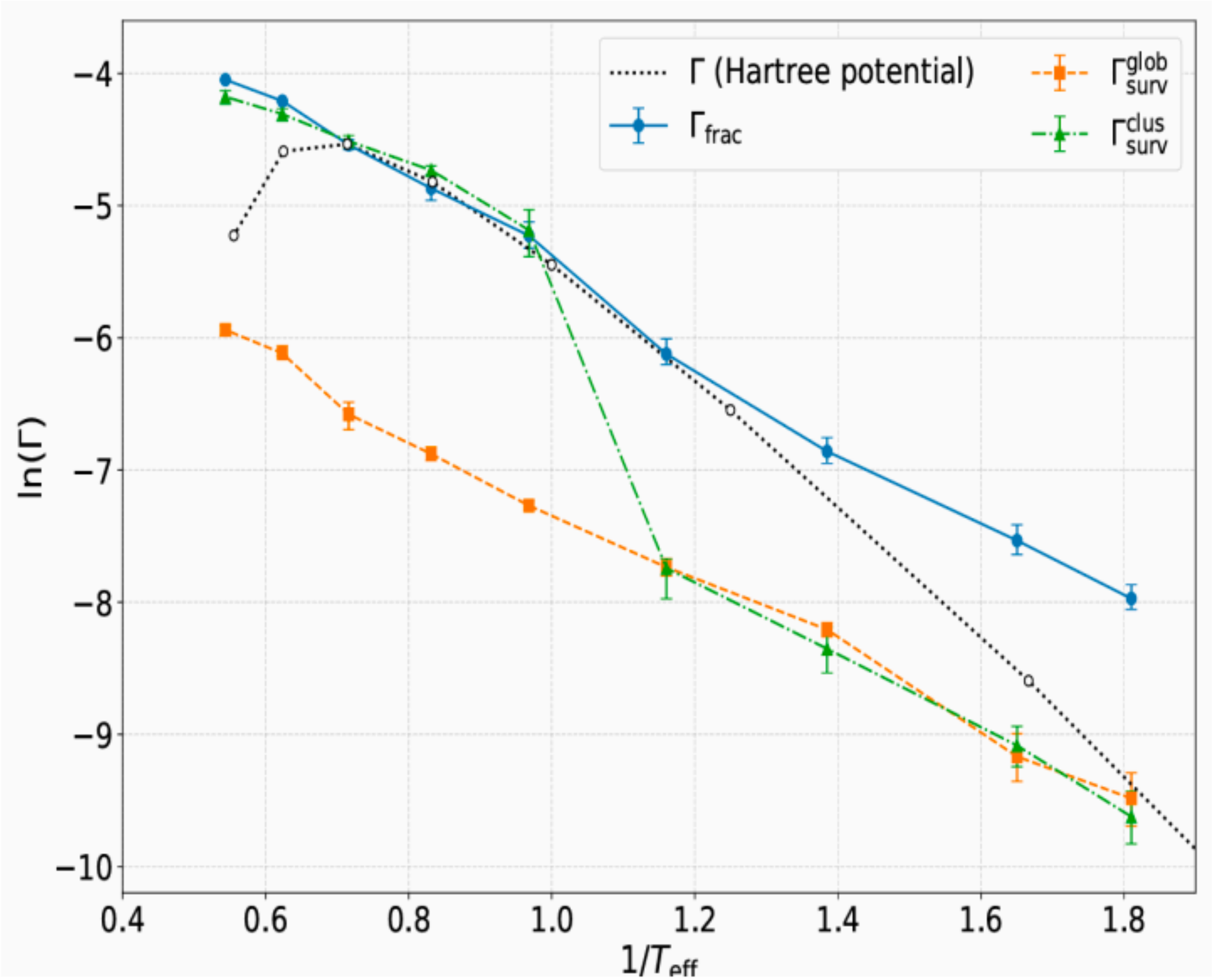
$$\frac{d^2\phi_B}{dr^2} - \frac{\partial V_{\text{eff}}}{\partial \phi} = 0,$$

$$S = \int dr \left[\frac{1}{2}\left(\frac{d\phi_B}{dr}\right)^2 + V_{\text{eff}}(\phi_B)\right],$$

$$\Gamma_{\text{th}} = A_{\text{stat}} A_{\text{dyn}} e^{-S/T}.$$



1+1D simulation-across Quantum-Thermal crossover



Low Tem:

Events are dilute.

Global and connected-cluster identify similar events.

Fraction observable can be distorted by transient conversion and kink–antikink reflection.

High Tem:

Many local seeds.

Connected-cluster detects persistent domains.

Global average waits until a large part of the box converts.

Therefore global survival underestimates the instantaneous nucleation rate.

Lattice electroweak theory-classical simulation

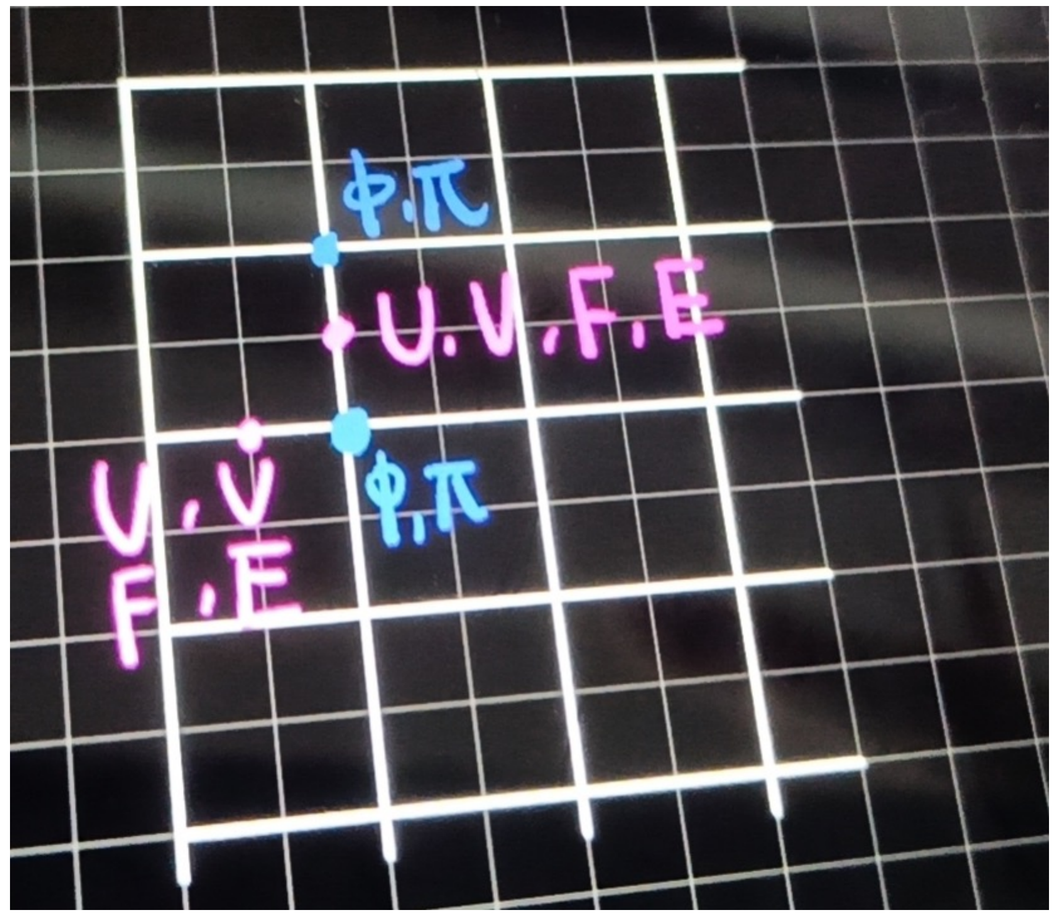
$\Phi(t, x)$: Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g\Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g\Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g\Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g\Delta t B_0\right).$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$
$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t\Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2}g'\Delta x\Delta t E_i(t+\Delta t/2,x)V_i(t,x)$$
$$U_i(t+\Delta t,x) = g\Delta x\Delta t F_i(t+\Delta t/2,x)U_i(t,x),$$

leapfrog

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1st order EWPT simulation-quantum tunneling

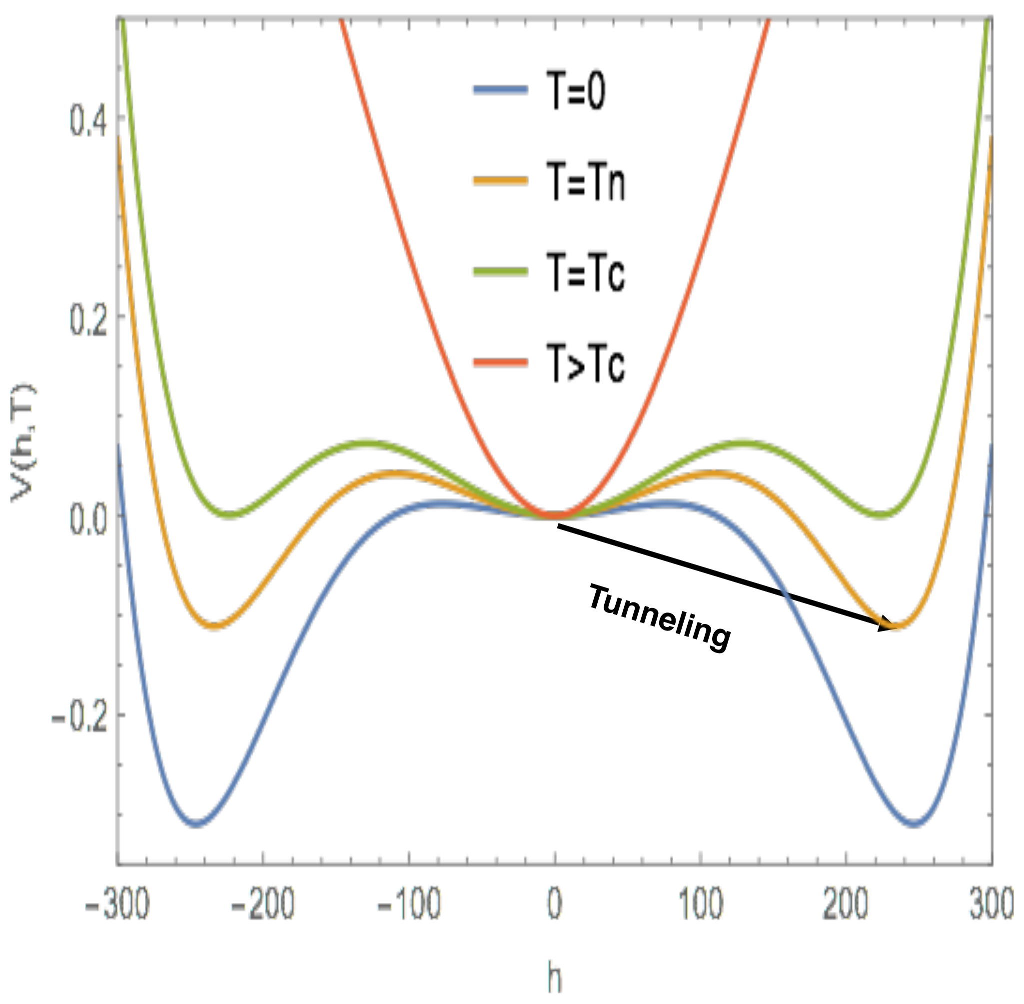
Field basis equation of motion

$$
\begin{aligned}
\partial_0^2 \Phi =& D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\
\partial_0^2 B_i =& -\partial_j B_{ij} + g' \,\mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a =& -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi]\,, \\
\partial_0 \partial_j B_j - & g' \,\mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0, \\
\partial_0 \partial_j W_j^a + & g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.
\end{aligned}
$$

Lattice implementation

$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i \big[U_i(t,x)V_i(t,x)\Phi(t,x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)\big] - \frac{\partial U}{\partial \Phi^\dagger}\Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2, x)] =& \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)] \\
& - \frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x) \\
& + V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)] \\
& - \frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}
$$

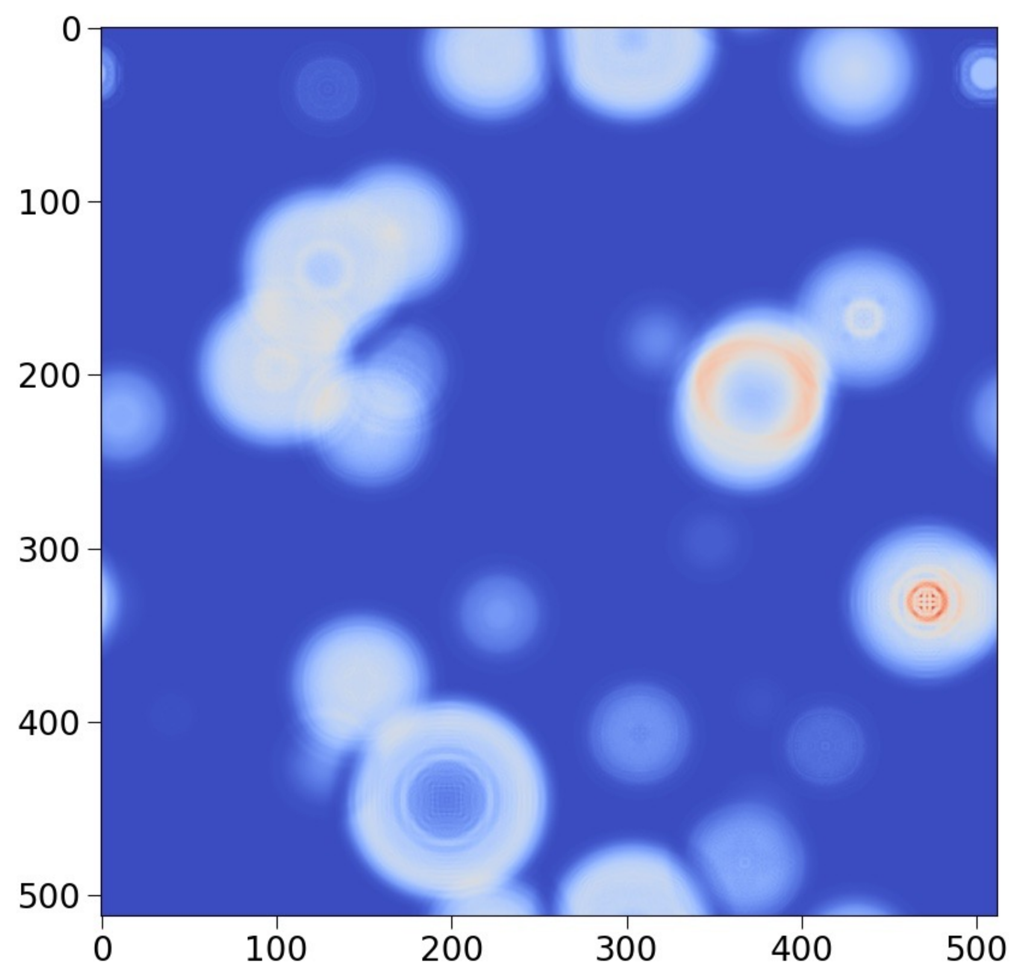
Finite-T Veff



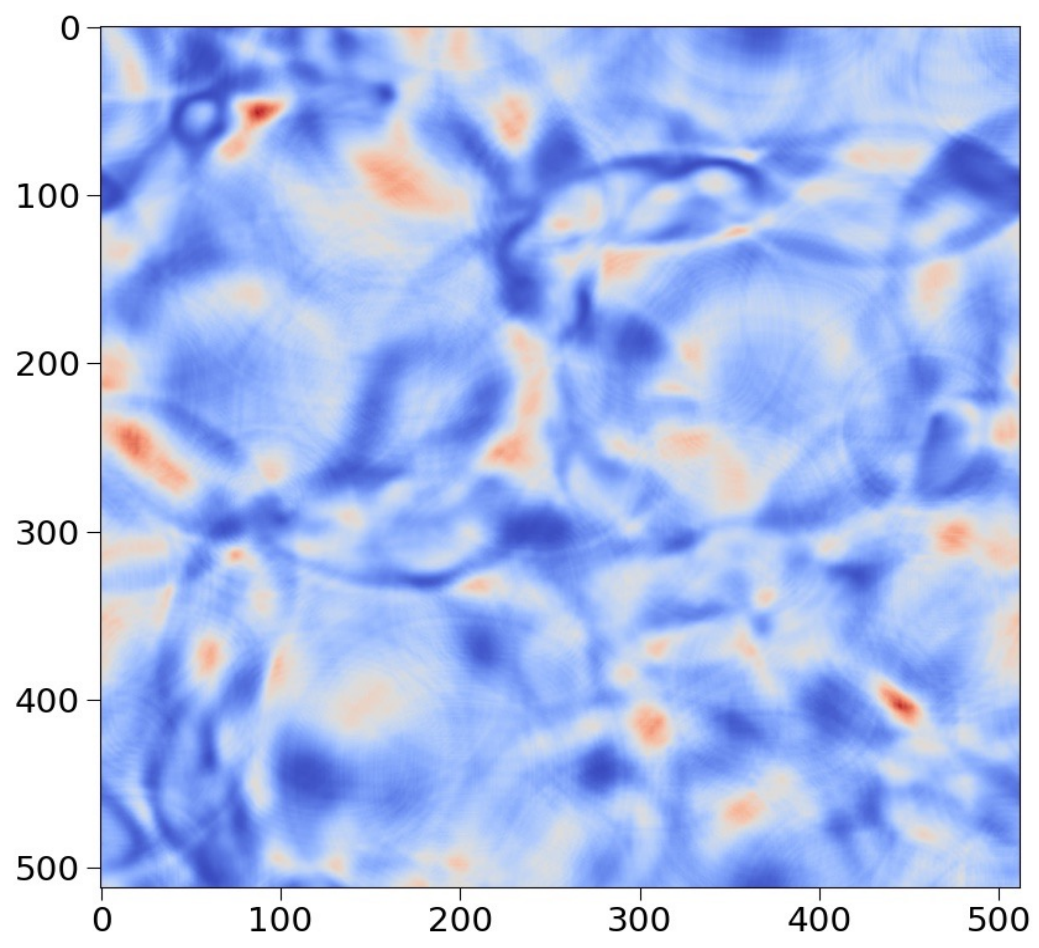
Finite-T calculation

temperature (T_*)
duration (β)
strength (α)

Nucleation



Expansion&Percolation



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Bubble collision and PMF

1 st Phase transition + Magnetic field = Matter-antimatter asymmetry ?

Higgs doublet   U(1) gauge field strength

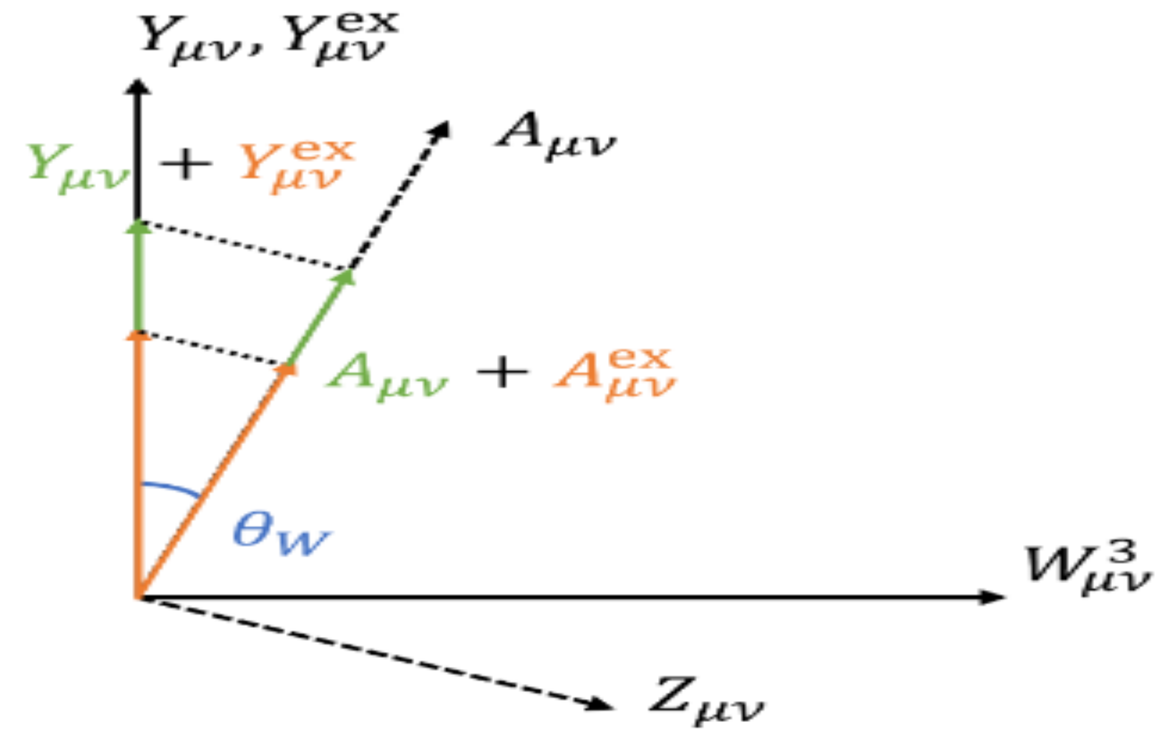
$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} Y^{\rm ex}_{\mu\nu} Y^{\mu\nu} + \mathcal{V}(\Phi)$$

SU(2) gauge field strength   External U(1) gauge field strength
Dose not change with time

$$\mathrm{SU}(2)_{\rm L} \times \mathrm{U}(1)_{\rm Y} \quad \Phi,\ W^a_\mu,\ \boxed{Y_\mu} \xrightarrow[\text{symmetry breaking}]{\text{Spontaneous}} \mathrm{U}(1)_{\rm em} \quad A_\mu,\ H,\ Z_\mu,\ W^\pm_\mu$$

$$D_\mu = \partial_\mu - ig\frac{\sigma^a}{2} W^a_\mu - ig' \frac{1}{2}(Y_\mu + Y^{\rm ex}_\mu)$$ is the covariant derivative.

$$\mathcal{V}(\Phi) = -\mu^2(\Phi^\dagger\Phi - \eta^2) + A(\Phi^\dagger\Phi - \eta^2)^{3/2} + \lambda(\Phi^\dagger\Phi - \eta^2)^2$$



$$\boxed{\partial_\mu j^\mu_{\rm B} = N_{\rm g}\left[\frac{g^2}{16\pi^2}\mathrm{Tr}\,(W_{\mu\nu}\tilde{W}^{\mu\nu}) - \frac{g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right]}$$

Adler, PR 177, 2426 (1969)
Bell, Jackiw, NCA 60, 47 (1969)
't Hooft, PRL 39, 8 (1976)
't Hooft, PRD 14, 3432 (1976)

$$\Delta n_{\rm B} = n_{\rm B}(t) - n_{\rm B}(0) = N_{\rm g}\frac{\Delta N_{\rm CS}(t)}{V} = N_{\rm g}\frac{N_{\rm CS}(t) - N_{\rm CS}(0)}{V}$$

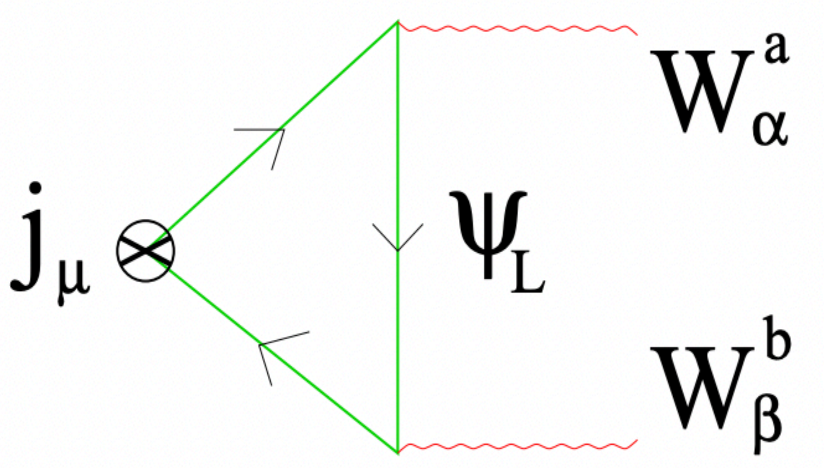
Number of families of fermions $N_{\rm g} = 3$

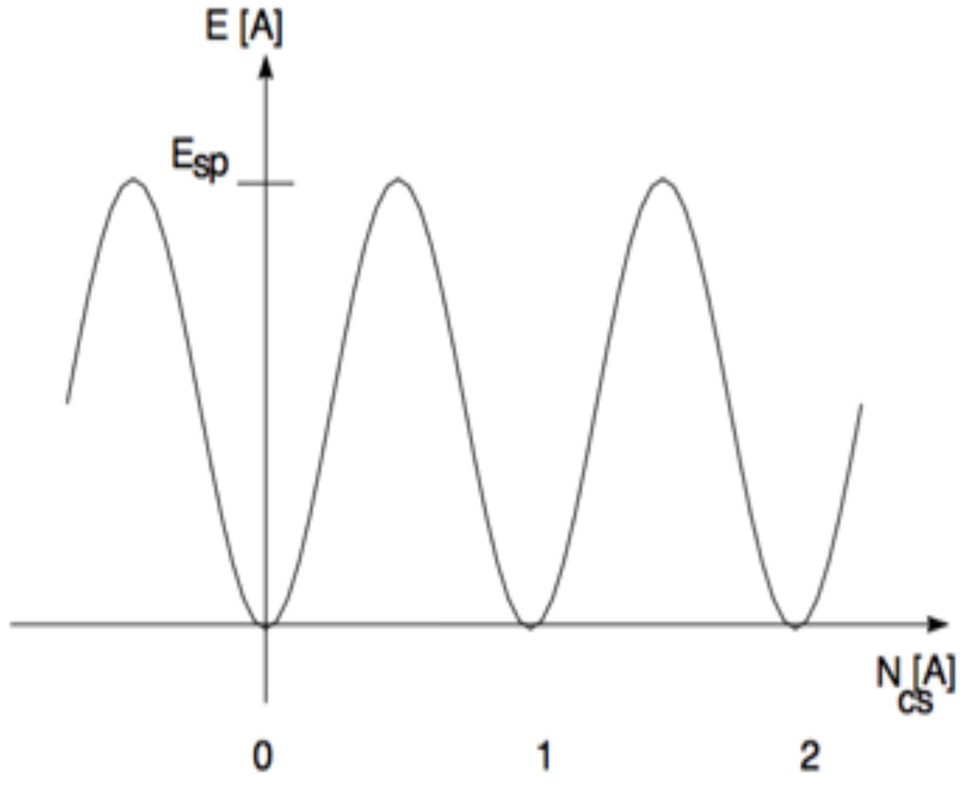
Chern-Simons Number density:

$$\frac{N_{\rm CS}(t)}{V} = \frac{1}{V}\frac{1}{32\pi^2}\varepsilon^{ijk}\int {\rm d}^3x \left[-g'^2 \boxed{(Y_i + Y^{\rm ex}_i)(Y_{jk} + Y^{\rm ex}_{jk})} + g^2\left(W^a_i W^a_{jk} - \frac{g}{3}\varepsilon^{abc} W^a_i W^b_j W^c_k\right)\right]$$

Helicity density:

$$h_Y = \frac{H_Y}{V} = \frac{1}{V}\varepsilon^{ijk}\int {\rm d}x^3\, \boxed{(Y_i + Y^{\rm ex}_i)(Y_{jk} + Y^{\rm ex}_{jk})} = \frac{1}{V}\int {\rm d}x^3\, (\boldsymbol{Y} + \boldsymbol{Y}^{\rm ex})\cdot(\boldsymbol{B}_Y + \boldsymbol{B}^{\rm ex}_Y)$$





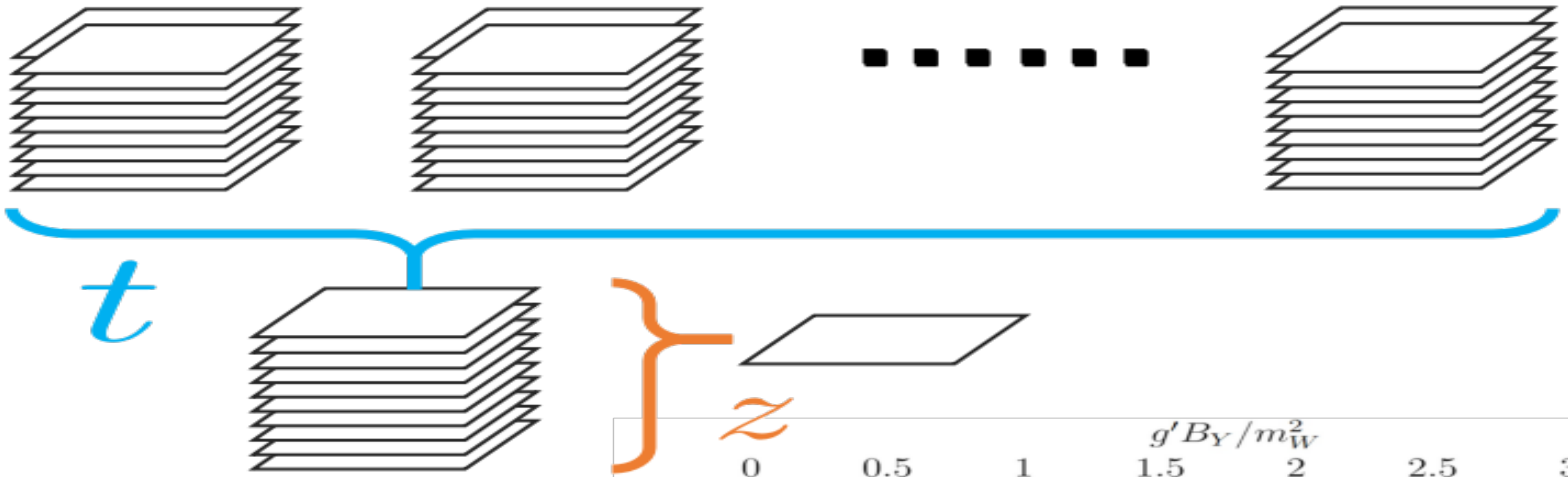
Net Baryon density: $$\boxed{\eta_{\rm B}(t) = \frac{n_{\rm B}}{s} = \frac{45 N_{\rm g}}{2\pi^2 g_{*S} T^3}\frac{\Delta N_{\rm CS}(t)}{V}}$$

Di, **Bian***, Cai*,2409.16124, 2508.07416

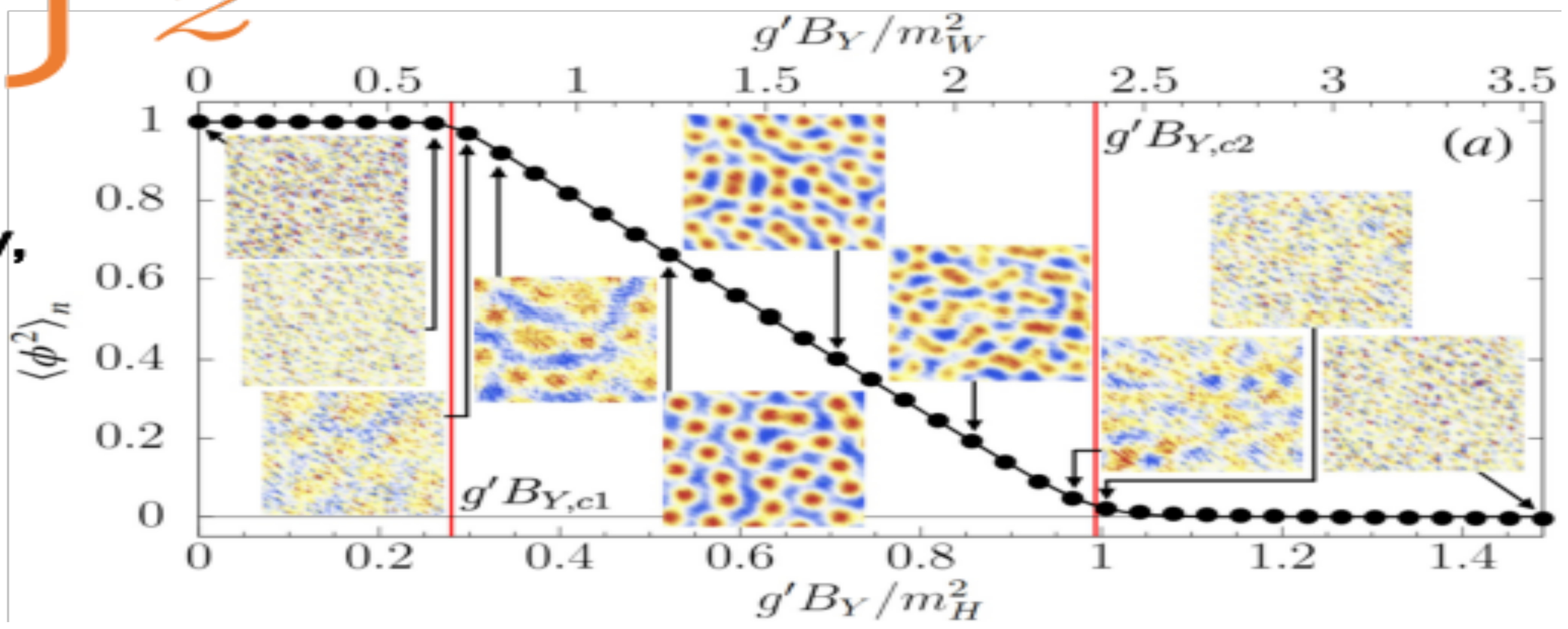
Bubble collision and PMF

When $\frac{g' B_Y}{m_W^2} > 1$, the Higgs field will form a hexagonally arranged vortex, which is called Ambjørn-Olesen condense.

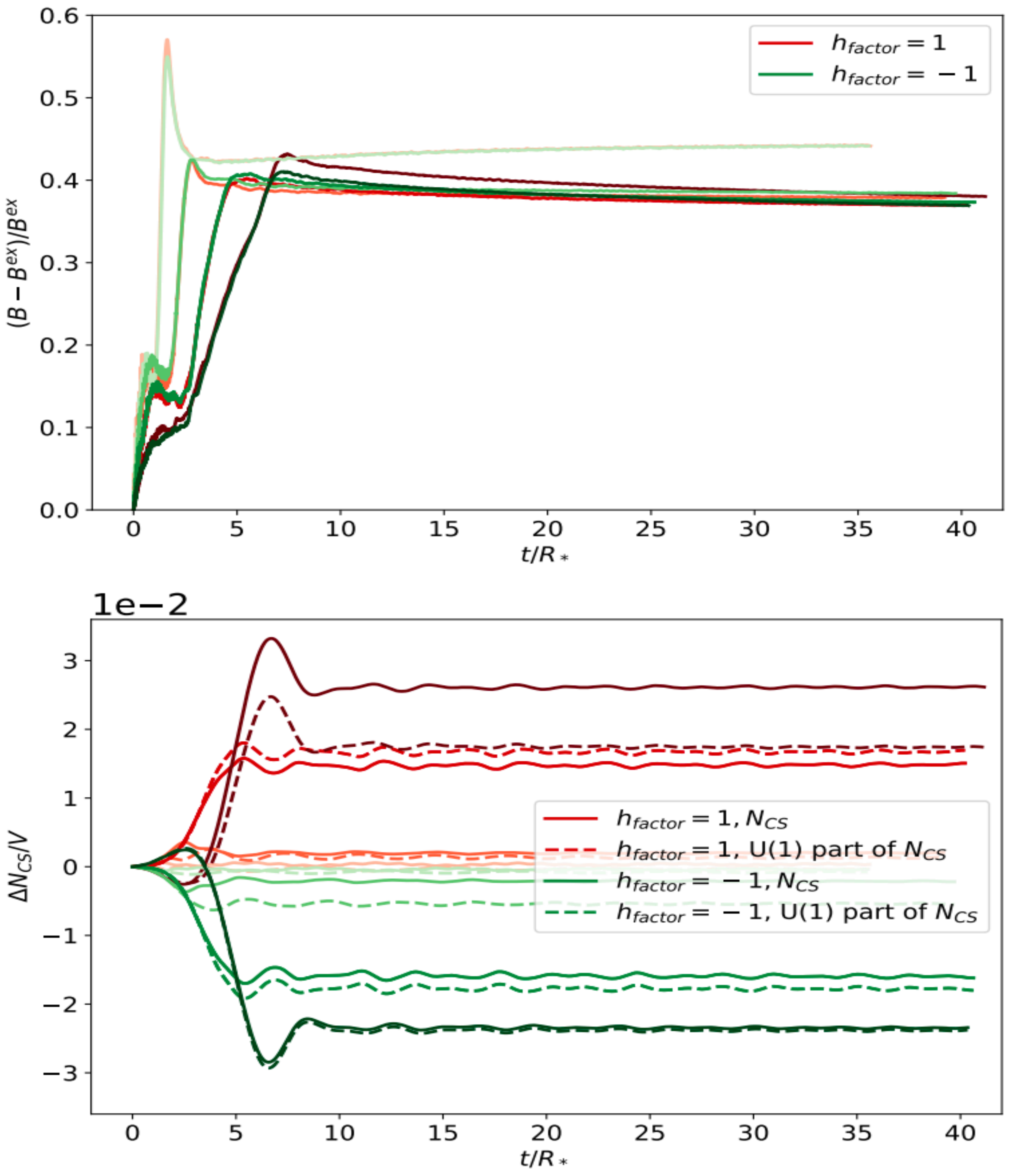
Salam, Strathdee, NPB 90, 203 (1975)
Linde, PLB 62, 435 (1976)
Nielsen, Olesen, NPB 144, 376 (1978)
Ambjørn. PLB 220, 659 (1989)



Chernodub, Goy, Molochkov, PRL 130 (2023)

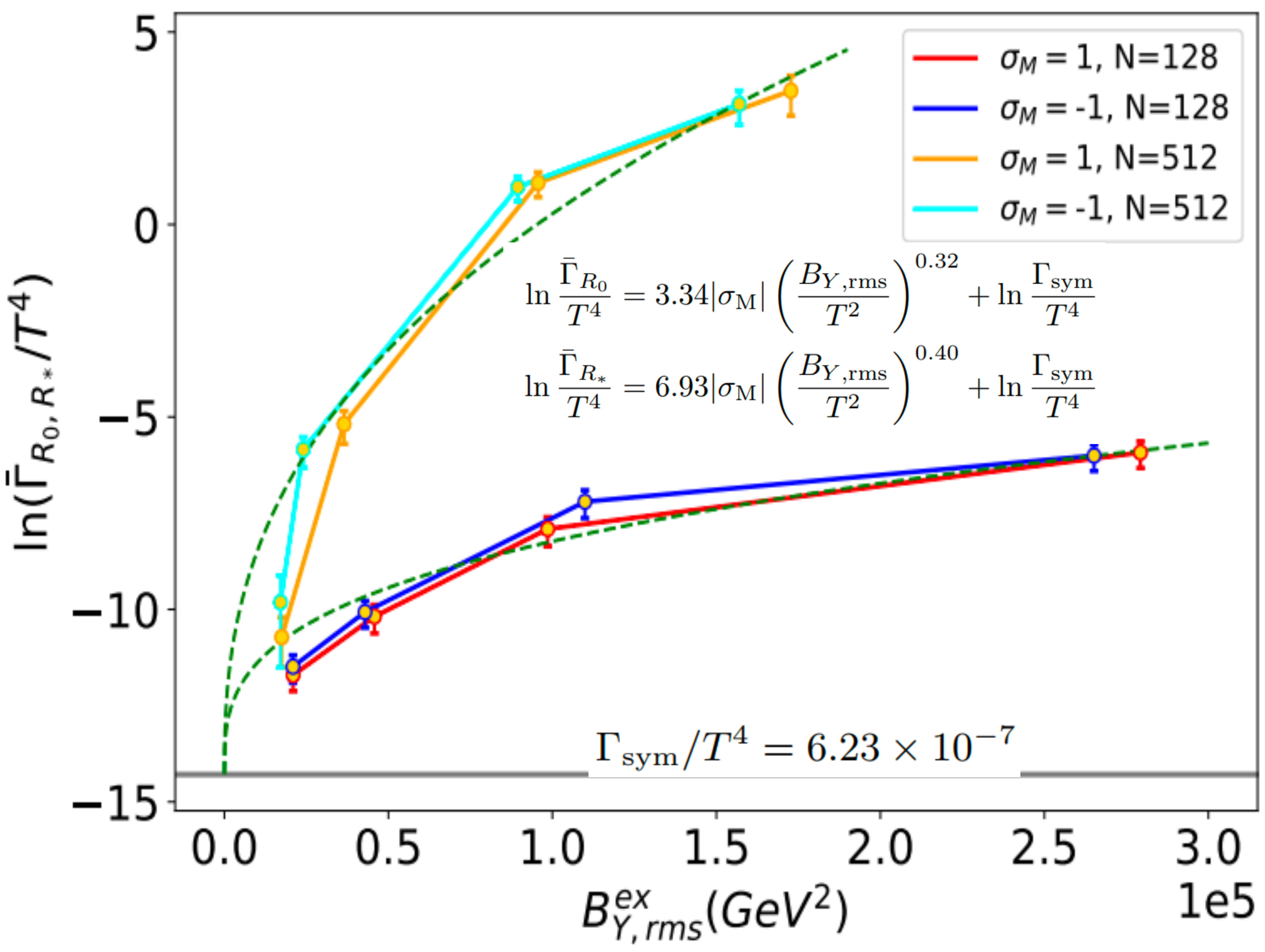


Bubble collision and PMF



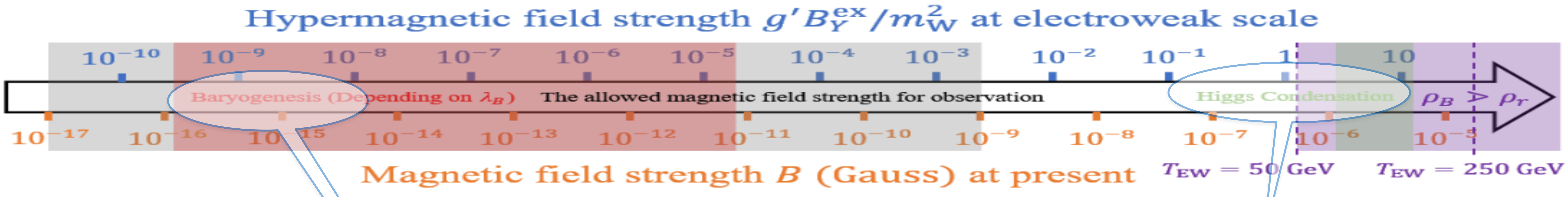
Sphaleron rate

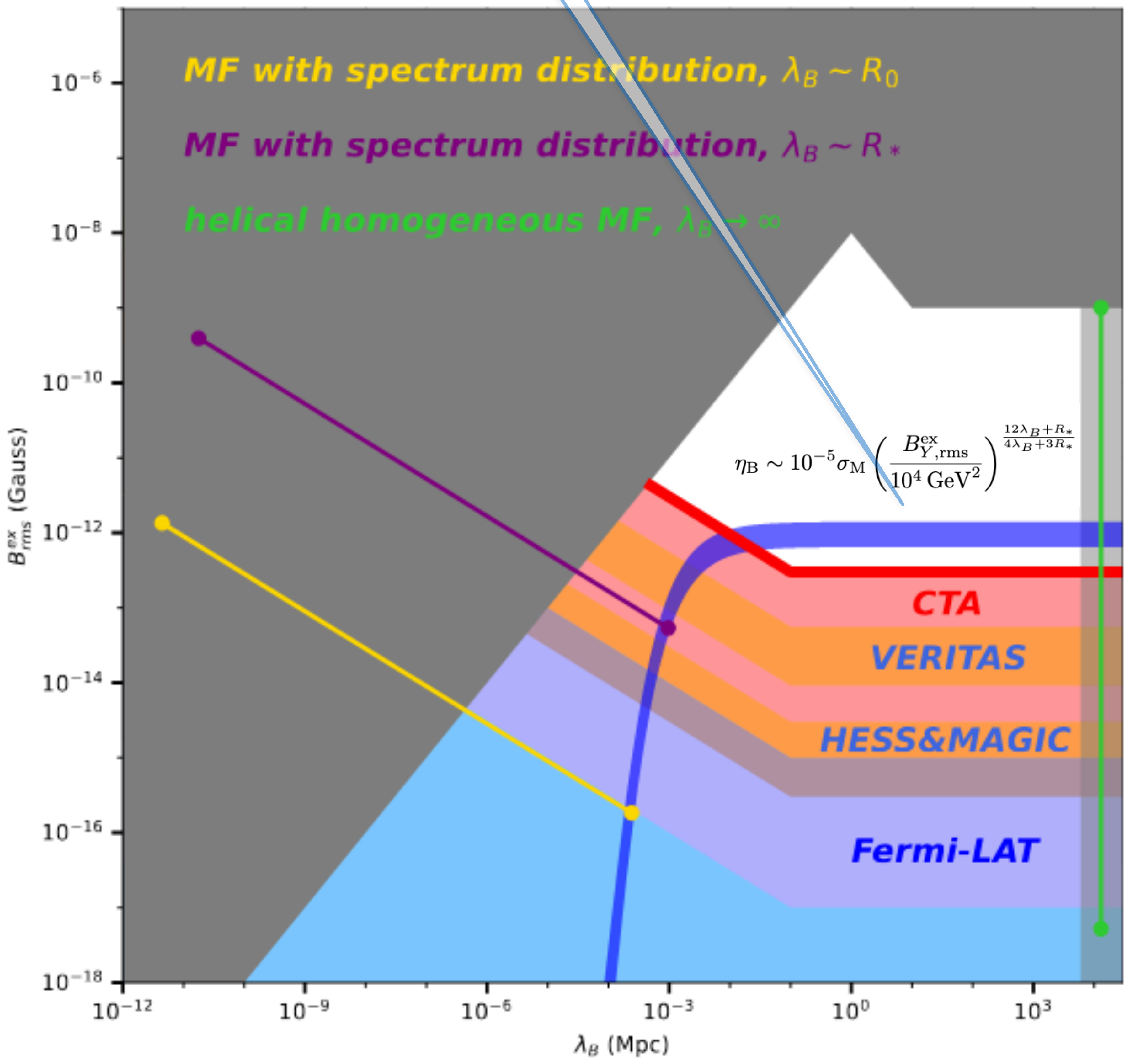
$$\Gamma(t) = \frac{1}{V}\frac{\mathrm{d}}{\mathrm{d}t}[\langle(\Delta N_{\mathrm{CS}}(t))^2\rangle - \langle\Delta N_{\mathrm{CS}}(t)\rangle^2]$$

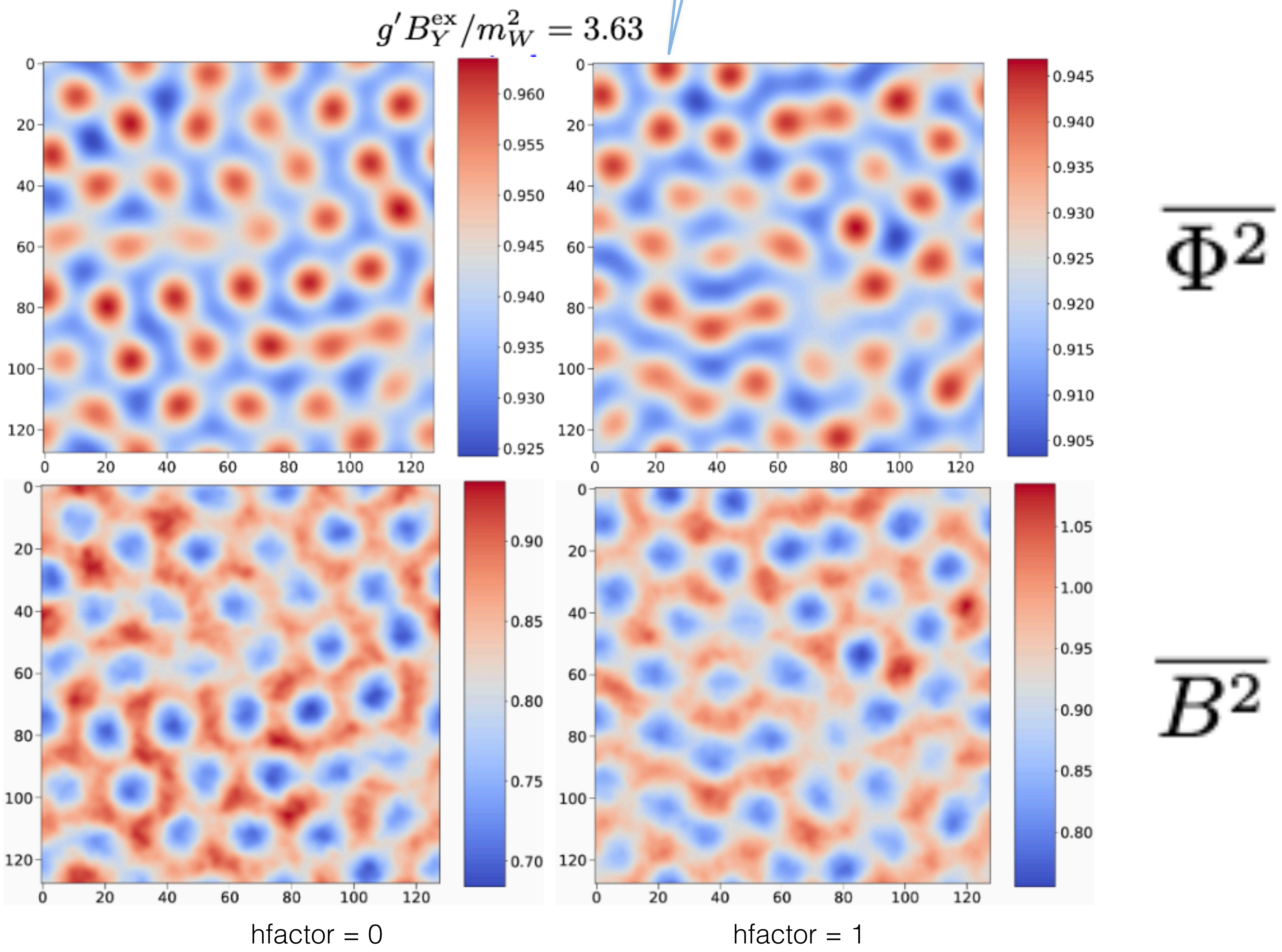


Di, **Bian***, Cai*,2409.16124,

Bubble collision and PMF







Di, **Bian***, Cai*,2409.16124, 2508.07416

❖ Summary

- Gauge invariant calculation of the nucleation rate reduces uncertainty for GW prediction
- Connected-cluster survival provides a local, sample-level decay diagnostic across the quantum-to-thermal crossover
- global survival is delayed by spatial averaging at high T, while false-vacuum fraction can be contaminated by transient spatial conversion at low T
- Primordial helical MF during EWPT can generate BAU through Chiral Anomaly

❖ Future

- Dark matter and PT
- Lattice/Quantum simulation on topological defects
- Magnetic fields, baryogenesis, and Axion dynamics

Thanks

谢谢！

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