Neutrinos in Cosmology: Some recent developments

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NOPP 2026
IHEP Beijing, China

Towards a precision calculation of N_{eff} in the Standard Model

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N_{eff} in the Standard Model

- N_{eff} is the "effective number of neutrino species"; parameterises expansion rate during BBN and CMB decoupling
- In the SM it is defined as
$$\left.\frac{\rho_\nu}{\rho_\gamma}\right|_{T/m_e \to 0} \equiv \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}^{\rm SM}$$
- $N_{eff} = 3$ in the SM if

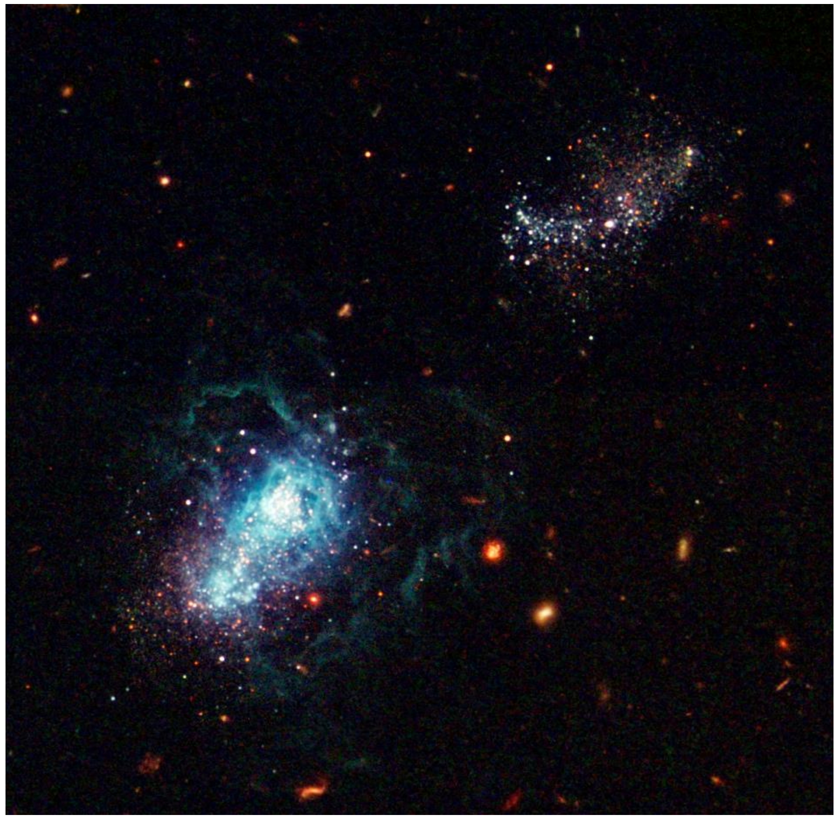
  i) primordial plasma is ideal gas,

  ii) neutrinos decouple instantaneously,

  iii) neutrinos decoupled at temperature $T \gg m_e$

  None of this is really true, leading to deviations from $N_{eff} = 3$
- BSM phenomena also lead to deviations from $N_{eff} = 3$ (extra light particles, modified expansion history, non-standard neutrino interactions…), making N_{eff} a powerful probe of BSM physics
- One can measure N_{eff} with percent accuracy

Observational Perspectives



Big Bang Nucleosynthesis

- Recently the primordial Helium abundance has been measured with unprecedented precision

Aver et al 2601.22238

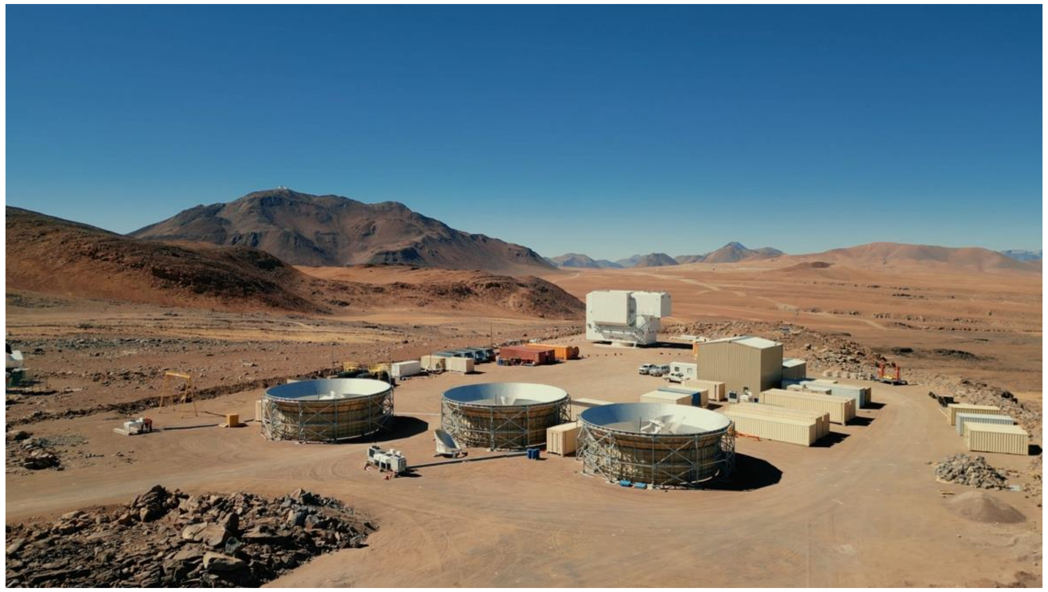
- This has — combined with various CMB and LSS measurement — yielded the currently strongest bound on $N_{eff} = 2.990 \pm 0.070$ Goldstein/Hill 2603.13226

Cosmic Microwave Background

- Planck alone imposes $N_{eff} = 2.99 \pm 0.17$

Planck 1807.06209

- Simons Observatory is foreseen to achieve $\pm\, 0.045$ Simons Observatory 2503.00636



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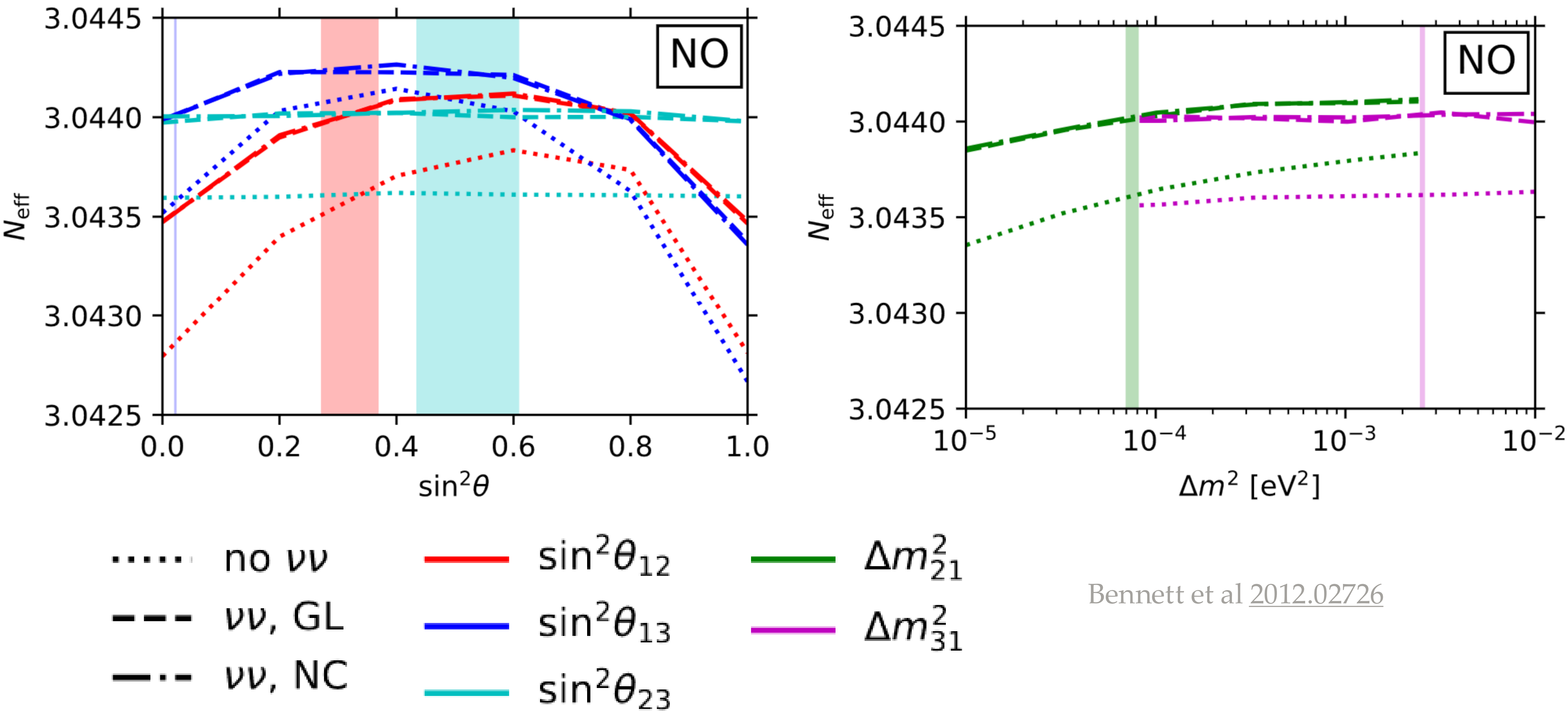
- **Status 2020 (current state-of-the-art value):**

$$N_{\rm eff}^{\rm SM} = 3.0440 \pm 0.0002$$

Akita/Yamaguchi al 2005.07047
Froustey/Pitrou 2008.01074
Bennett et al 2012.02726

(used by PDG, in CAMB and CLASS codes, major collaborations like DES, DESI…)

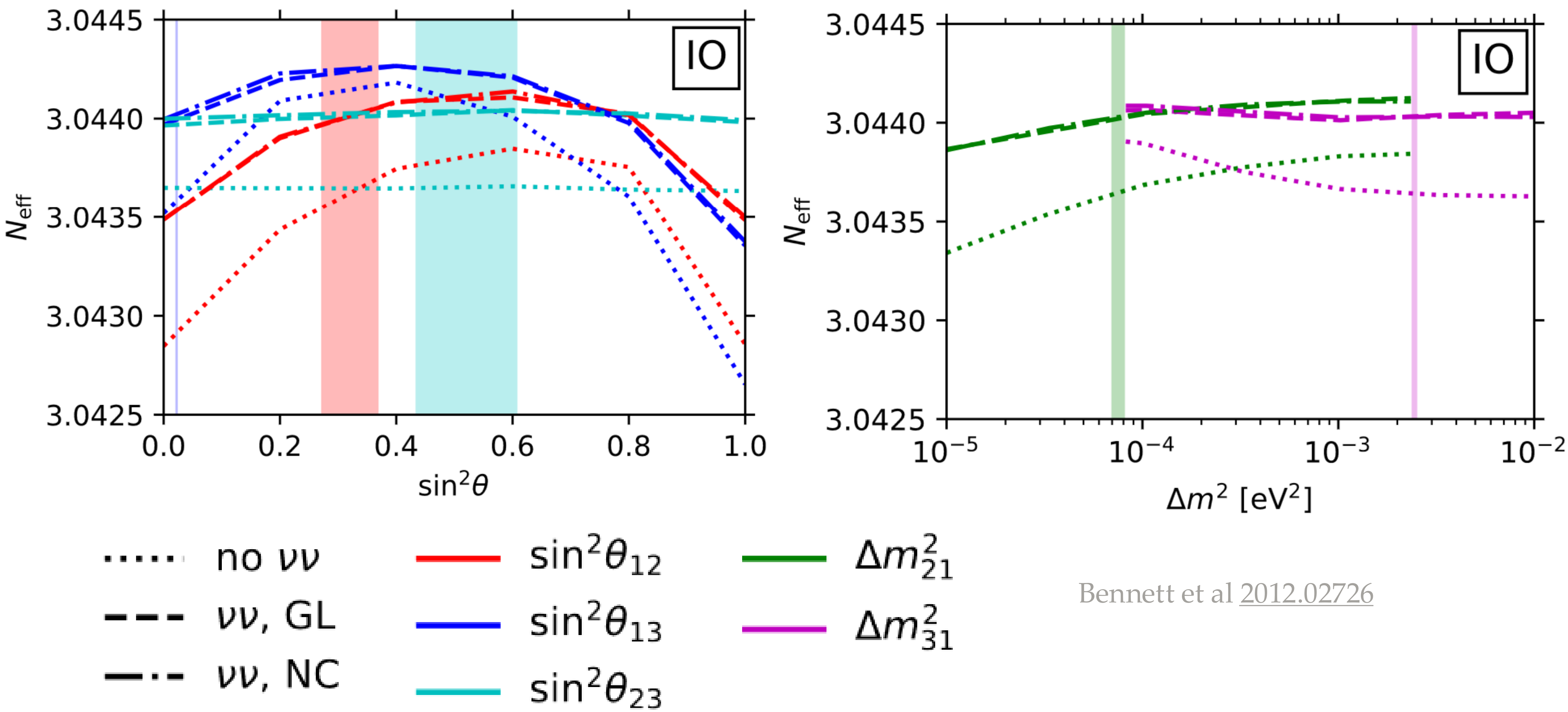
Experimental Uncertainties



Bennett et al 2012.02726

- Largest uncertainty from experiment: mixing angles
- Uncertainty from neutrino oscillation data is sub-dominant compared to numerical error (Gauss-Laguerre vs Newton-Cotes quadrature)

Experimental Uncertainties



Bennett et al 2012.02726

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Impact of NLO QED Corrections

- N_{eff} in the SM is found by solving

$$(\mathrm{d}/\mathrm{d}t)\rho_{\rm tot} + 3H(\rho_{\rm tot} + P_{\rm tot}) = 0, \qquad \partial_t \varrho - pH\partial_p = -\mathrm{i}[\mathbb{H}, \varrho] + \mathcal{I}[\varrho],$$

- O(e³) correction to equation of state is more important than neutrino oscillations!

Standard-model corrections to $N_{\rm eff}^{\rm SM}$	Leading-digit contribution
m_e/T_d correction	$+0.04$
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	$+0.01$
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	$+0.0005$
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

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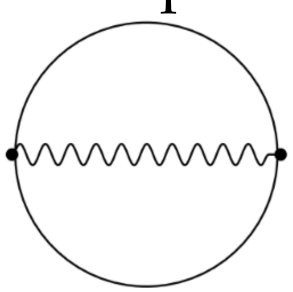
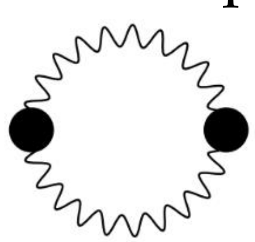
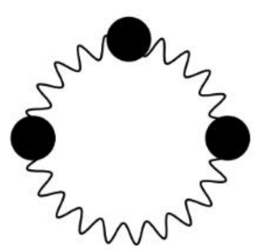
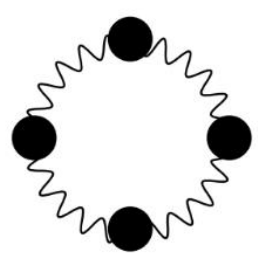
- The result $N_{eff} = 3.0440 \pm 0.0002$ obtained in Bennett et al 2012.02726 consistently included finite temperature QED corrections to the equation of state at NLO…

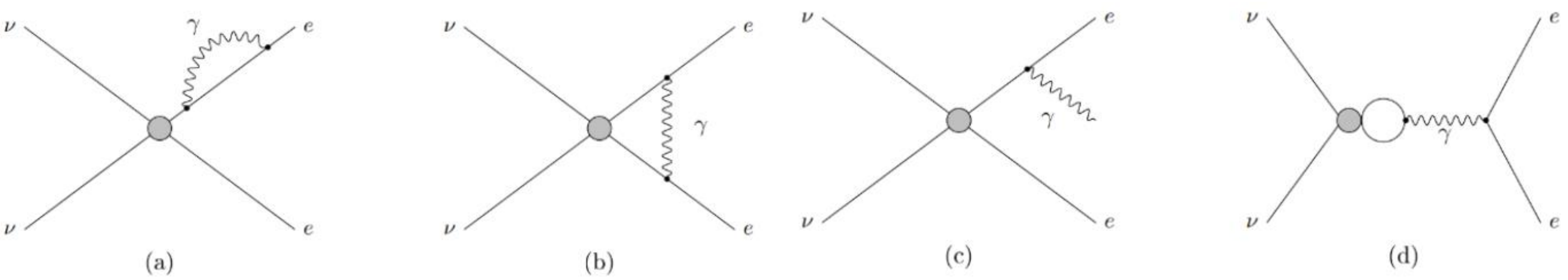
$\ln Z^{(2)} = -\frac{1}{2}$ 

$\ln Z^{(3)} = \frac{1}{2}\Big[\, \frac{1}{2}$  $-\frac{1}{3}$  $+\frac{1}{4}$  $+ \cdots \Big]$

- …but only roughly estimates NLO corrections to the collision term
- Cielo et al 2306.05460 found that NLO corrections to collision term lead to $N_{eff} = 3.043$

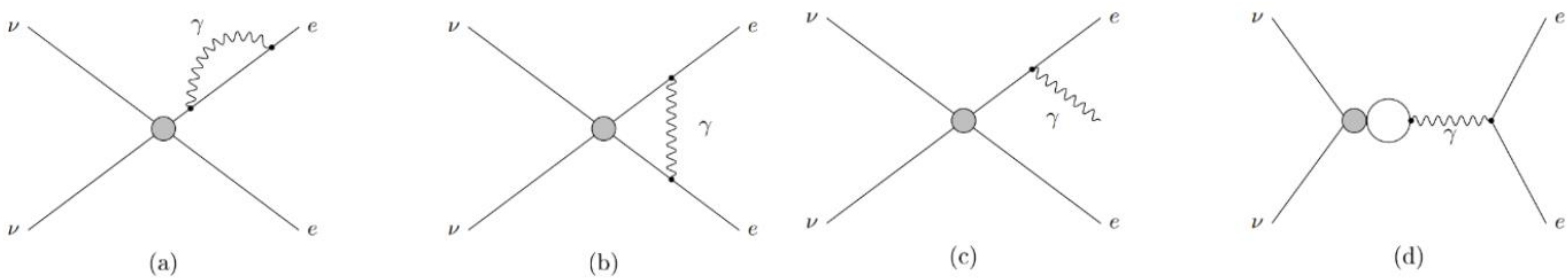
NLO Corrections to Collision Term

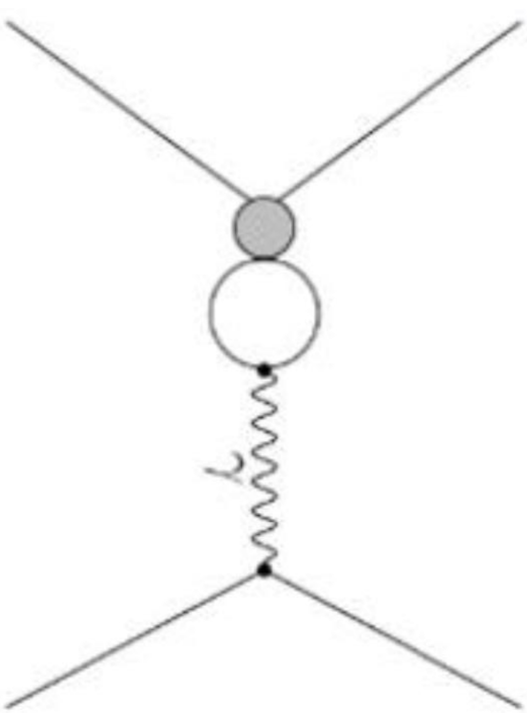
QED Corrections to Collision Term



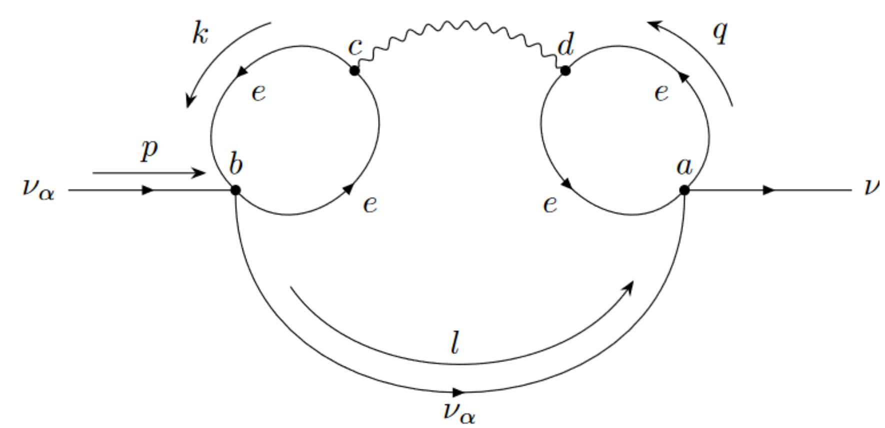
	Processes in Fig.1 included	Quantum statistics	Hadronic corrections	Non-linearities in QKE	Electron mass in NLO terms	Flavour dependence	First principles derivation
Bennett et al 2012.02726	(a) roughly estimated	partially (fermions)	no	yes	no	partially	no
Cielo et al 2306.05460	(a)-(c) partially	partially	no	no	partially	no	no
Jackson/Laine 2312.07015, 2412.03958	all	yes	partially	no	no	yes	yes
MaD et al 2402.18481	(d)	yes	no	no	yes	yes	yes

QED Corrections to Collision Term





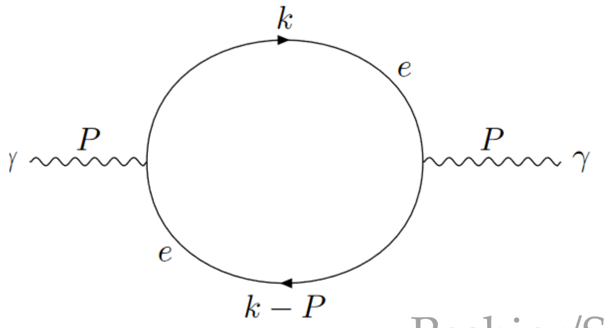
- t-channel version of diagram (d) is IR divergent in the t-channel,
- Hence we focussed on (d) in MaD et al 2402.18481
- In Closed-Time-Path formalism appears as a cut of



- requires usage of resumed finite temperature photon propagator

- But: annihilation channels transfer more energy than scatterings, see Jackson/Laine 2412.03958

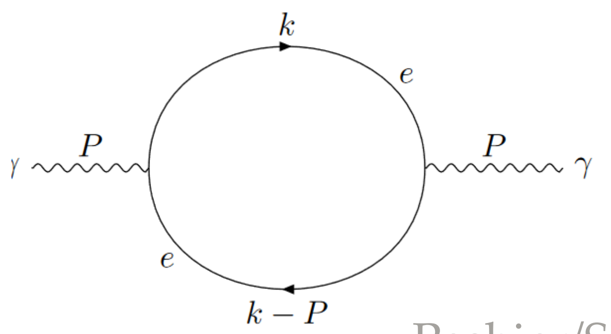
Resummed Photon Propagator



$$\Pi^{\mu\nu}_{ab}(P) = (-1)^{a+b} i e^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{Tr}\left[i S_e^{ab}(k) \gamma^\mu i S_e^{ba}(k-P) \gamma^\nu \right]$$

Peshier/Schertler/Thoma et 9708434 Thoma/Carrington 9708363 MaD et al 2402.18481

Resummed Photon Propagator



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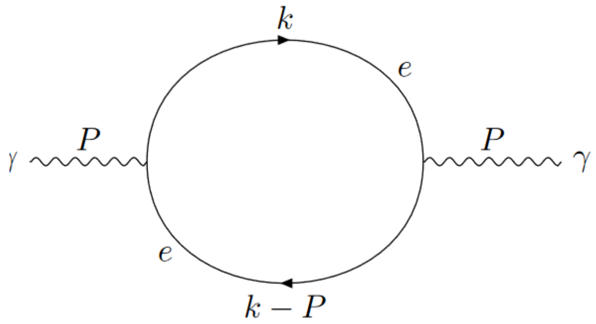
$$\begin{aligned}\mathrm{Re}\,\Pi^{T}_{11,T\neq 0} &= \frac{\alpha_{\mathrm{em}}}{\pi|\mathbf{P}|^3} \int_{m_e}^{\infty} \mathrm{d}\omega \left[2\ell_2(\omega,P) P_0 P^2 \omega - 4|\mathbf{k}||\mathbf{P}|(P_0^2 + |\mathbf{P}|^2) \right. \\ &\qquad \left. + \ell_1(\omega,P)(|\mathbf{P}|^2 P^2 + 2P_0^2\omega^2 - 2|\mathbf{k}|^2|\mathbf{P}|^2 + \frac{1}{2}P^4) \right] f_F(\omega) \\ &\overset{\mathrm{HTL}}{\approx} -\frac{3}{2} m_\gamma^2 \frac{P_0^2}{|\mathbf{P}|^2} \left[1 - \left(1 - \frac{|\mathbf{P}|^2}{P_0^2}\right) \frac{P_0}{2|\mathbf{P}|} \ln\left| \frac{P_0 + |\mathbf{P}|}{P_0 - |\mathbf{P}|} \right| \right] \longrightarrow 0, \quad \text{for } P_0 = 0\end{aligned}$$

$$\begin{aligned}\mathrm{Re}\,\Pi^{L}_{11,T\neq 0} &= \frac{\alpha_{\mathrm{em}} P^2}{\pi|\mathbf{P}|^3} \int_{m_e}^{\infty} \mathrm{d}\omega \left[8|\mathbf{k}||\mathbf{P}| - \ell_1(\omega,P)(P^2 + 4\omega^2) - 4\ell_2(\omega,P)P_0\omega \right] f_F(\omega) \\ &\overset{\mathrm{HTL}}{\approx} -3m_\gamma^2 \left(1 - \frac{P_0^2}{|\mathbf{P}|^2}\right) \left[1 - \frac{P_0}{2|\mathbf{P}|} \ln\left| \frac{P_0 + |\mathbf{P}|}{P_0 - |\mathbf{P}|} \right| \right] \longrightarrow -3m_\gamma^2 \quad \text{for } P_0 = 0,\end{aligned}$$

Resummation: momentum-dependent photon mass and longitudinal photon mode

Neutrino Decay



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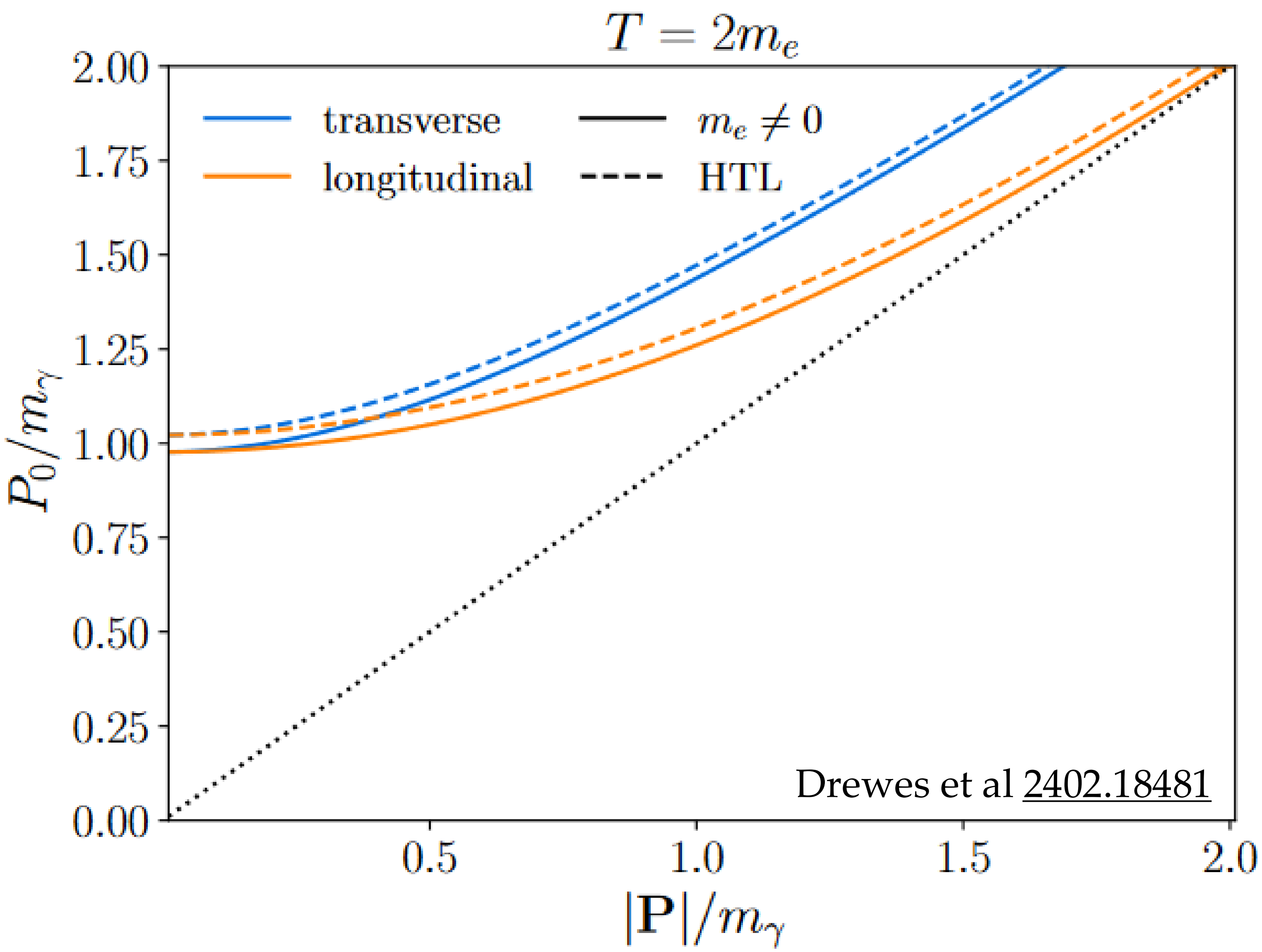
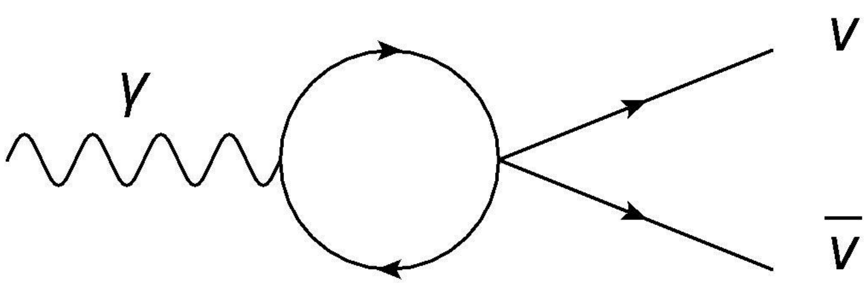


Drewes et al 2402.18481

Gives rise to well-known plasmon process that is relevant for White Dwarf cooling



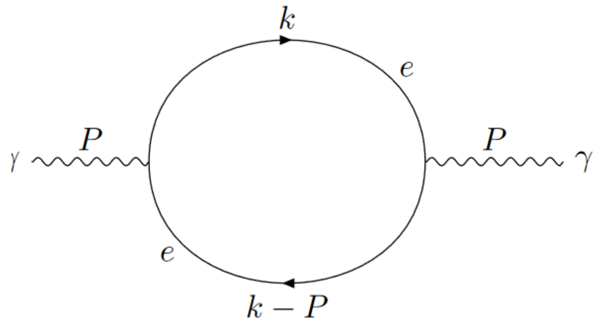
Braaten/Segel 9302213

Haft/Raffelt/Weiss 9309014

MaD et al 2109.06158

Impact on N_{eff} is negligible

Correction to Neutrino Scattering



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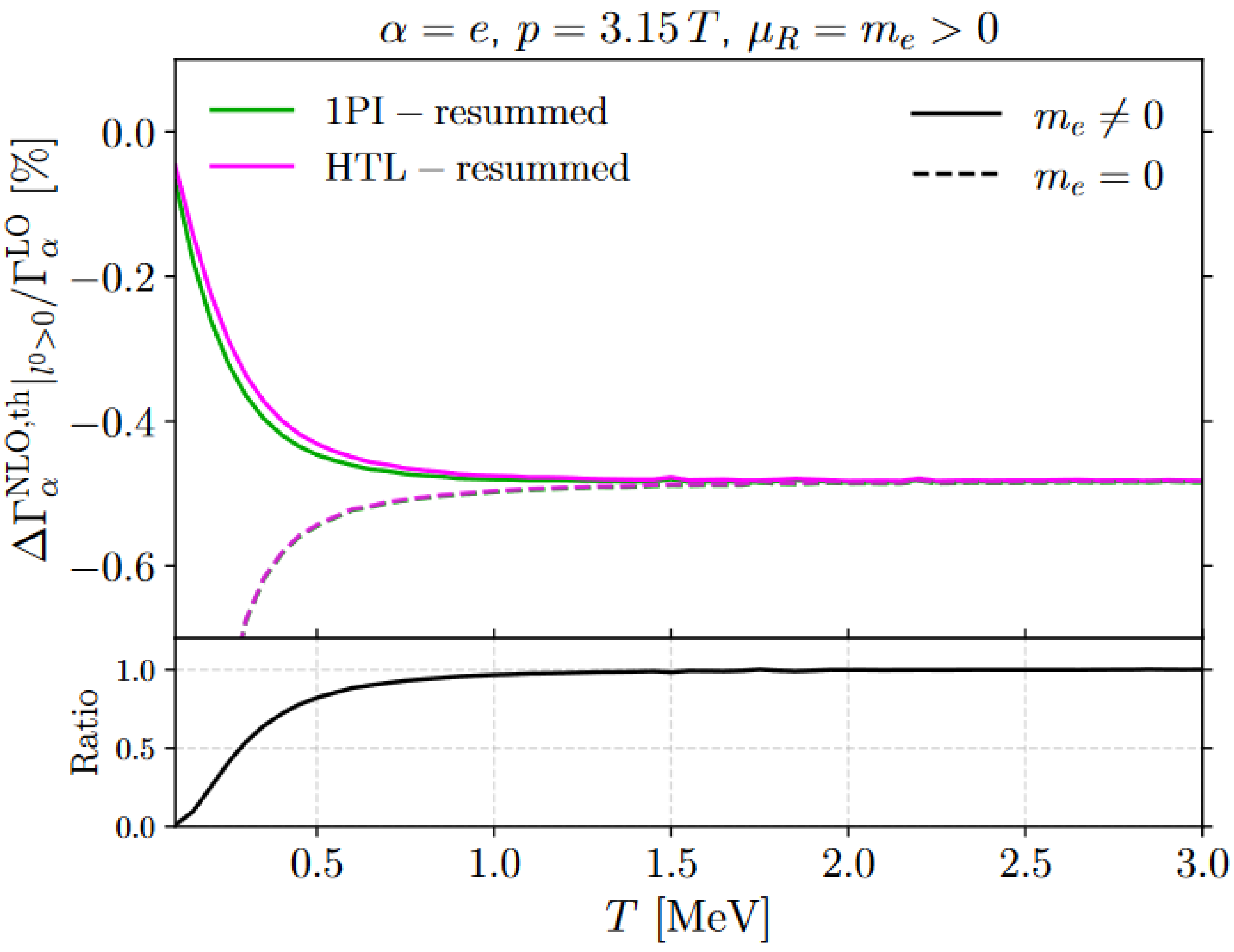
MaD et al 2402.18481



Impact on *Neff*

- In principle: need to solve density matrix equation for all neutrino modes
- Large set of coupled equations, highly non-linear due to neutrino self-interactions (the Γ are functionals of ϱ)
- We use two simpler methods to estimate impact on Neff:
  - entropy conservation
  - solve density matrix equation in "damping approximation"

Method	$\delta N_{\rm eff}/N_{\rm eff}^{\rm LO}$	$\delta N_{\rm eff}^{m_e=0}/N_{\rm eff}^{\rm LO}$
Entropy conservation (common decoupling)	-1.1×10^{-5}	-1.2×10^{-5}
Entropy conservation (flavour-dependent decoupling)	-5.4×10^{-6}	-6.1×10^{-6}
Boltzmann damping approximation (mean momentum)	-7.8×10^{-6}	-1.6×10^{-5}
Boltzmann damping approximation (full momentum)	-7.9×10^{-6}	-2.6×10^{-5}

- Ultimately: full density matrix needs to be solved with all contributions
- Our results indicate that NLO corrections to collision integral do not modify Neff considerably , indicating that $N_{eff} = 3.044$ in the SM

MaD et al 2402.18481

- Independent computations by Jackson and Laine including all annihilation channels come to the same conclusion

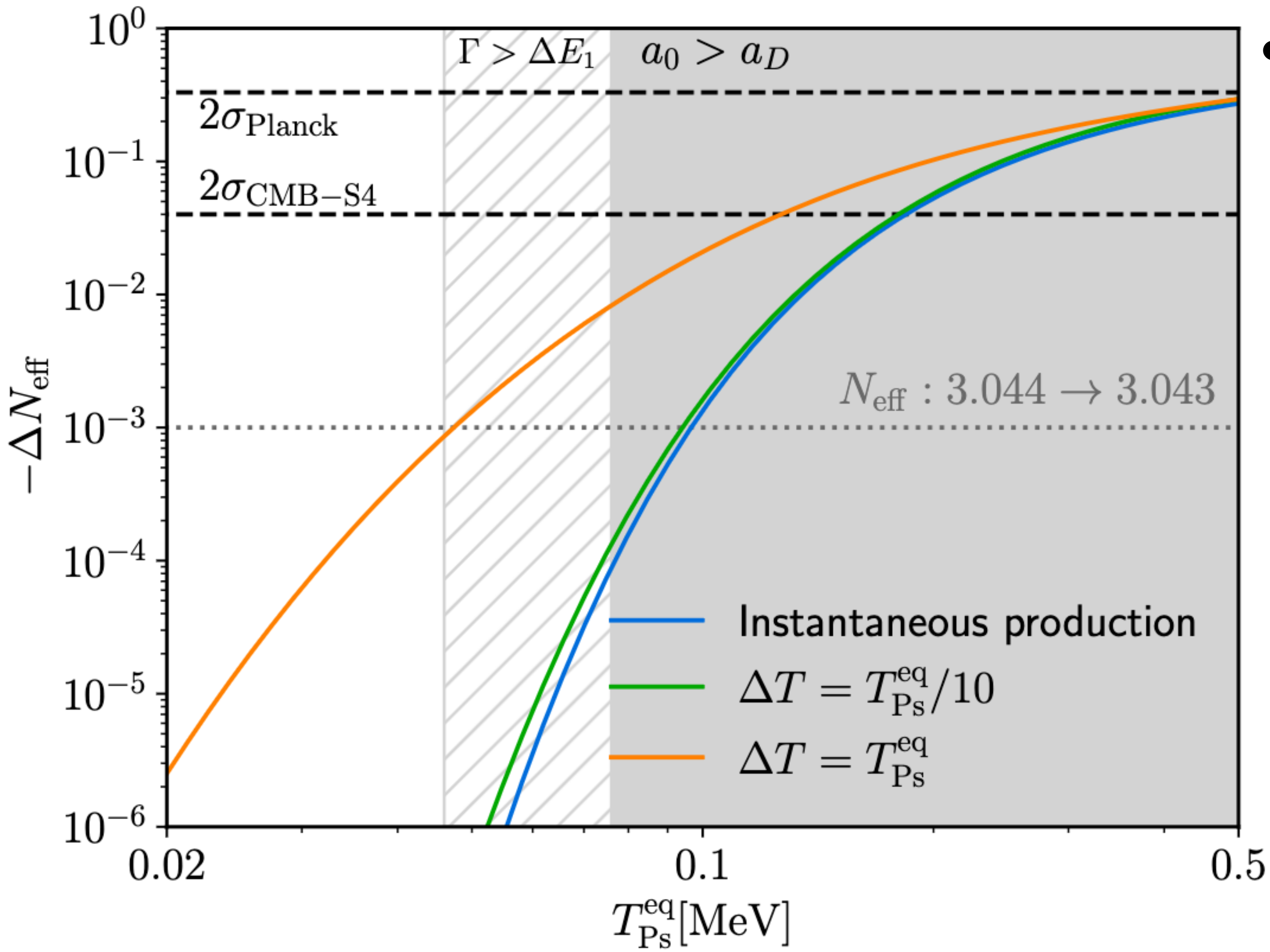
Jackson/Laine 2312.07015, 2412.03958

Positronium Formation

Bound State Formation

- Transient population of positronium affects QED equation of state

Approach I: Estimate impact of entropy change from adding bound states (assuming energy conservation)



- Bound state properties: heuristic description by finite T effective potential

$$V(r) = -\alpha m_D - \frac{\alpha}{r} e^{-m_D r} - \mathrm{i}\alpha T \phi(m_D r)$$

$$\phi(y) = 2\int_0^{+\infty} \mathrm{d}u \frac{u}{(u^2+1)^2}\left(1 - \frac{\sin(uy)}{uy}\right)$$

- Formation occurs when Yukawa potential accommodates bound states that are separated from the continuum by a gap exceeding their thermal width

- Magnitude of effect strongly depends on when positronium forms

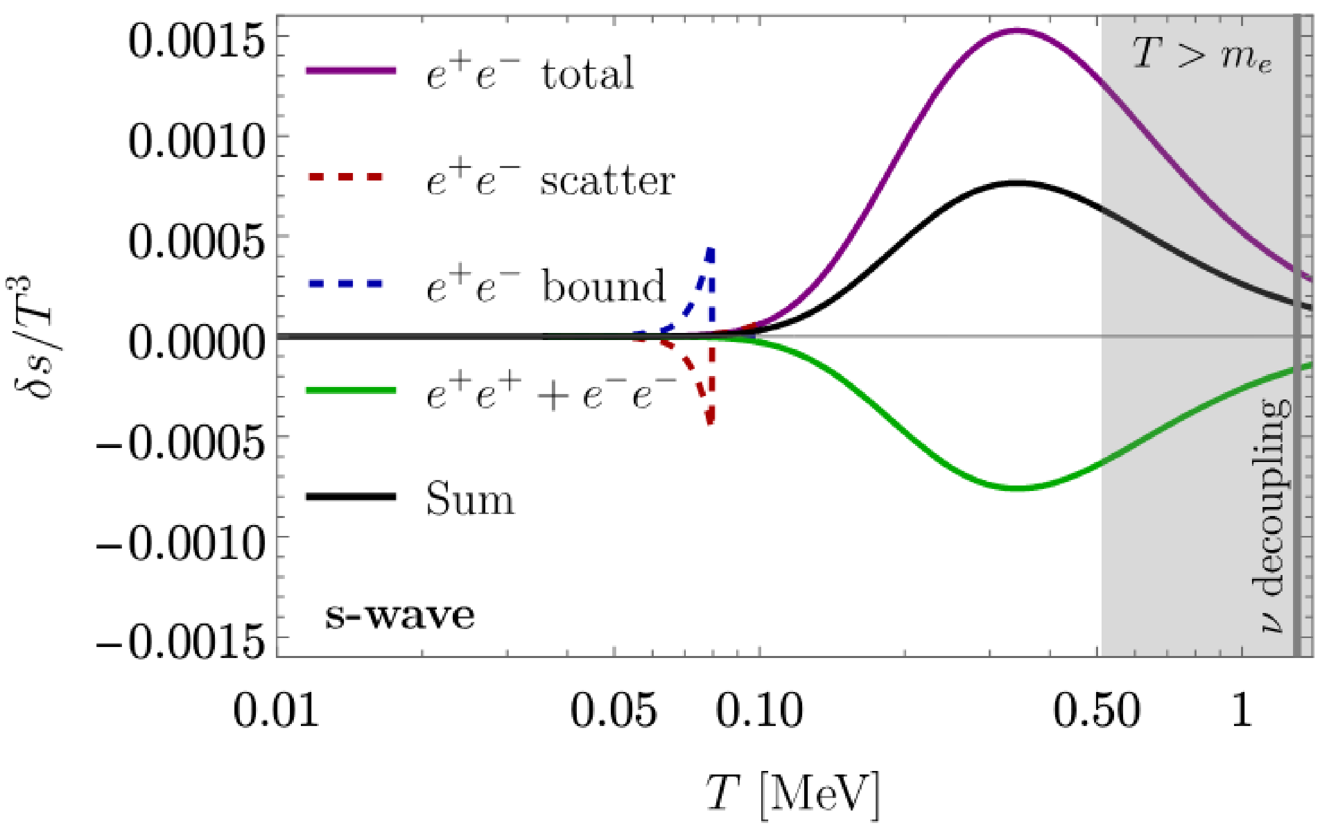
Binder et al 2411.14091

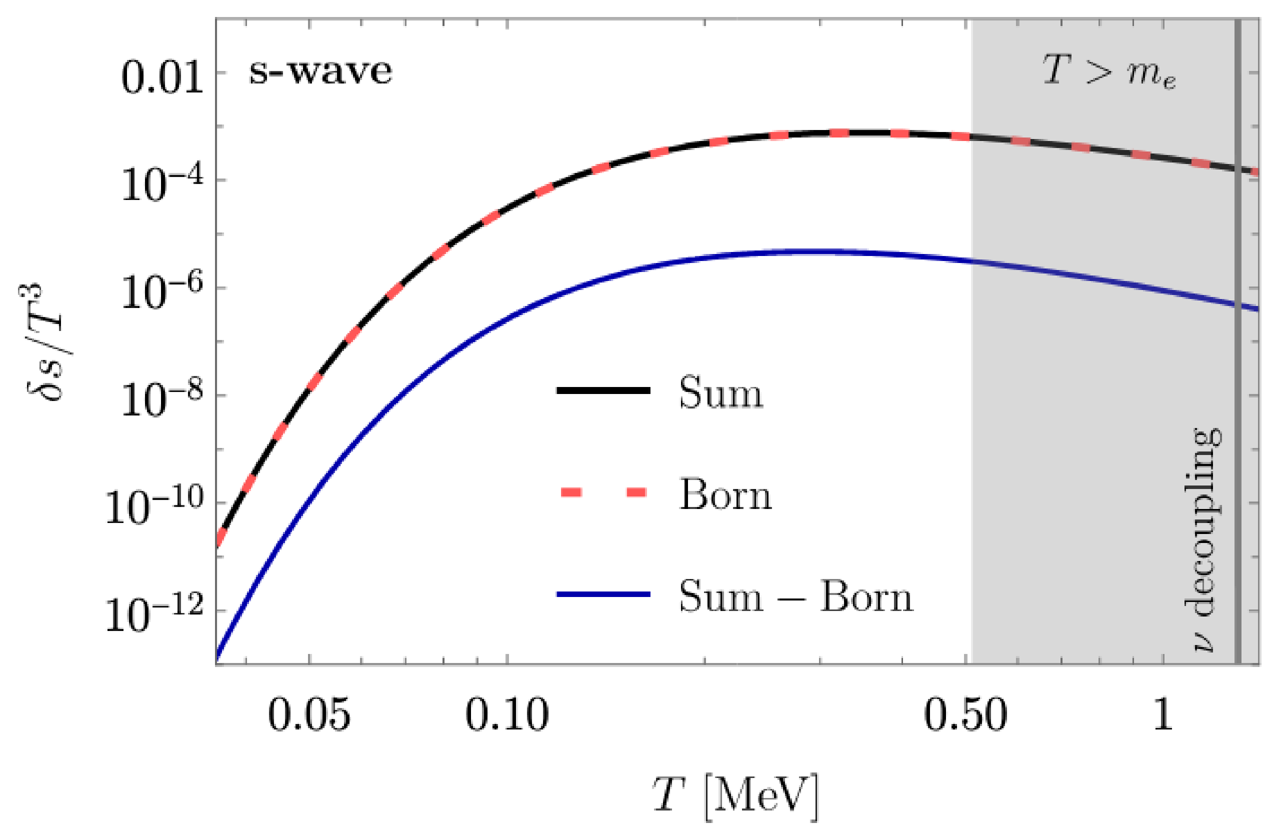
Bound State Formation

- Transient population of positronium affects QED equation of state

Approach II: Quantify deviation from ideal gas assuming equilibrium
(assuming energy and entropy conservation)

- For $T \gg m_e$: system is perturbative, use results from Bennett et al 1911.04504
- For $T \ll m_e$: non-perturbative treatment with Beth-Uhlenbeck formula





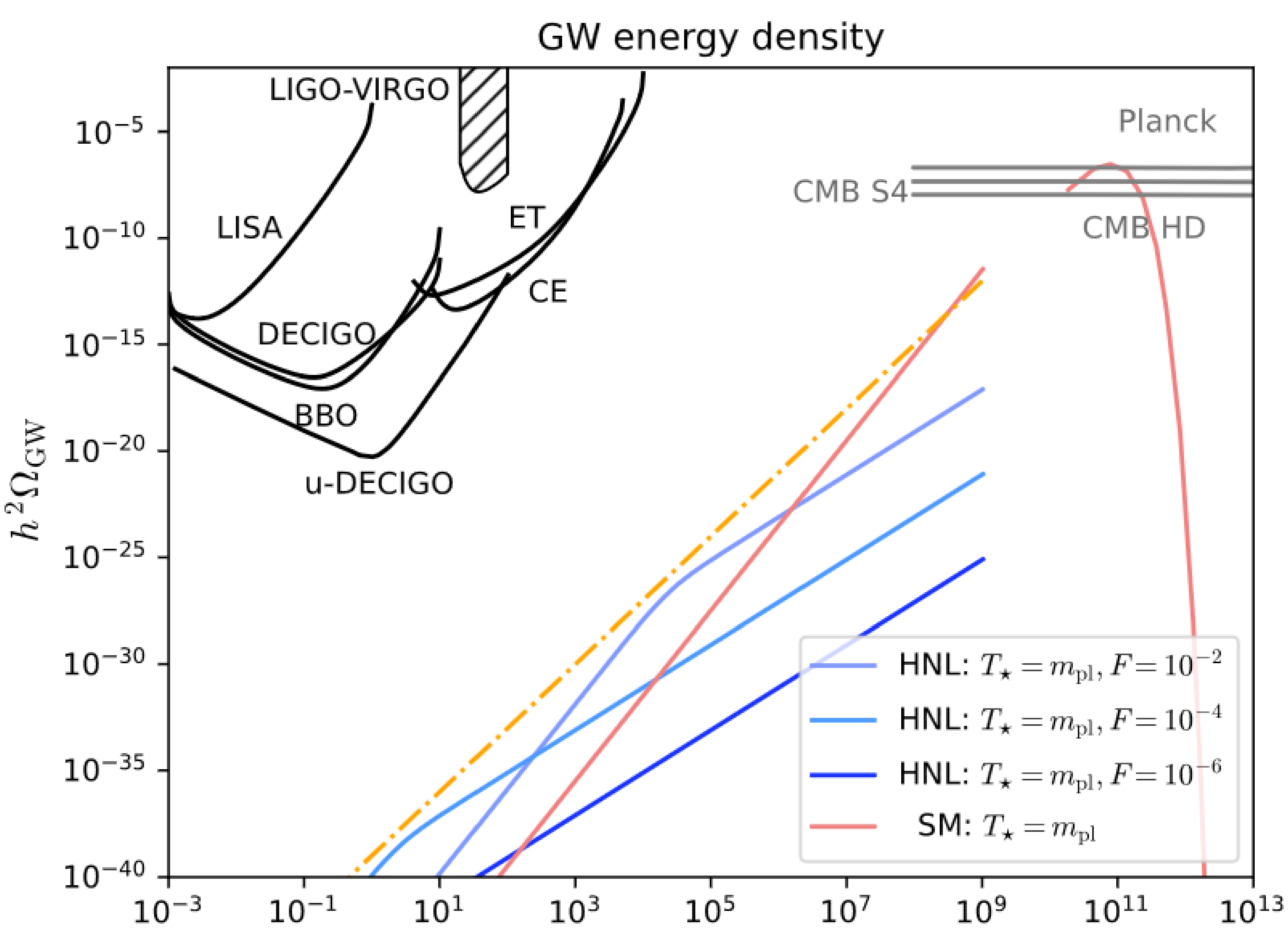
- If the system remains in local equilibrium, impact on N_{eff} is below 10^{-6}

Binder et al 2411.14091

GW Contribution

GWs as a Big Bang Thermometer

- Any plasma in thermal equilibrium emits a black body spectrum of GWs, the graviton equivalent to the CMB and CνB Ghiglieri/Laine 1504.02569
- This irreducible GW background contributes to N_{eff} in the SM, but depends on T_{re}



MaD/Georis/Klaric/Klose 2312.13855

Ringwald/Schütte-Engel/Tamarit 2011.04731
Ghiglieri/Jackson/Laine/Zhu 2004.11392
MaD/Georis/Klaric/Klose 2312.13855

- Direct detection is extremely difficult
- Contribution to N_{eff} imposes the first "direct" constrains on the reheating temperature T_{re}

The Need for Speed
(if you wanna go BSM)

Speeding up N_{eff} Computations

- Suppose the conclusions from Jackson/Laine 2312.07015 & 2412.03958 and MaD et al 2402.18481 are correct and NLO corrections from the collision integral are negligible, so that $N_{eff} = 3.044$ in the SM… then, what next?

- Precision computation sampling the ν–momentum distributions in the non-linear collision term are numerically demanding… not suitable to scan parameter spaces of BSM models

- Using moments (instead of sampling momenta) massively reduces the computation time and only leads to errors around 0.04%, recovering $N_{eff} = 3.044$; a public code is provided in Escudero/Jackson/Laine/Sandner 2511.04747

- Direct Simulation Monte Carlo methods can further accelerate the computations, Ihnatenko/Ovchynnikov 2508.08379 recover $N_{eff} = 3.0439 \pm 0.0006$ in the SM (compared to $N_{eff} = 3.0440 \pm 0.0002$ in Bennett et al 2012.02726)

Summary & Conclusions

- In 2020 a new benchmark value $N_{eff} = 3.044$ in the SM was established that includes neutrino oscillations and NLO corrections to the QED EoS
  Akita/Yamaguchi al 2005.07047, Froustey/Pitrou 2008.01074, Bennett et al 2012.02726

- NLO corrections to the collision term have been studied by different authors, suggesting
  $N_{eff} = 3.043$ in Cielo et al 2306.05460
  $N_{eff} = 3.044$ in Jackson/Laine 2312.07015 & 2412.03958 and MaD et al 2402.18481
  The discrepancy has not (yet) been resolved, my personal working assumption remains
  $N_{eff} = 3.0440 \pm 0.0002$ based on Bennett et al 2012.02726

- Bound state formation is unlikely to have a sizeable impact Binder et al 2411.14091

- GW contribution to N_{eff} in the SM is detectable for very high reheating temperature
  Ghiglieri et al 1504.02569 & 2004.11392, Ringwald et al 2011.04731, MaD et al 2312.13855

- Beyond the SM increasing speed may be more important than increasing precision; e.g.
  using moments instead of momenta Escudero/Jackson/Laine/Sandner 2511.04747
  using Direct Simulation Monte Carlo Ihnatenko/Ovchynnikov 2508.08379