Weyl anomaly induced transport and Late-Time Relaxation from Landau Singularities

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Outline

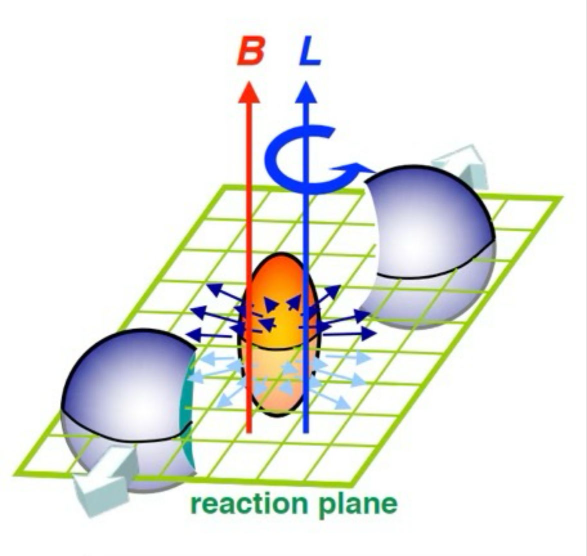
- Weyl anomaly induced transport
- Late-Time Relaxation from Landau Singularities
- Summary

Weyl anomaly induced transport in hydrodynamics

New development in novel transport induced by electromagnetic fields

Shi-Zheng Yang, J.H. Gao, Z.T. Liang, G.Y. Prokhorov, SP, O.V. Teryaev, arXiv: 2604.23849

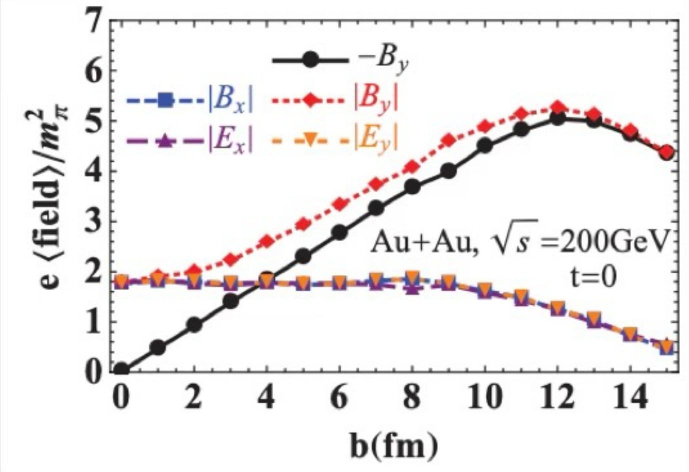
Strong electromagnetic fields

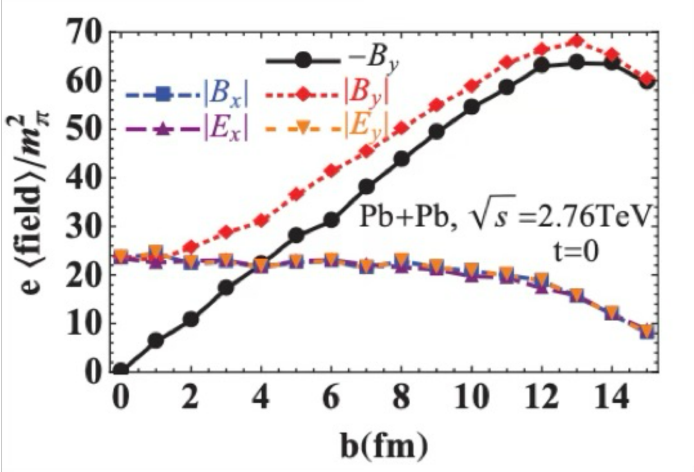


- **Theoretical estimation:**

  Lienard-Wiechert potential+ events-by-events

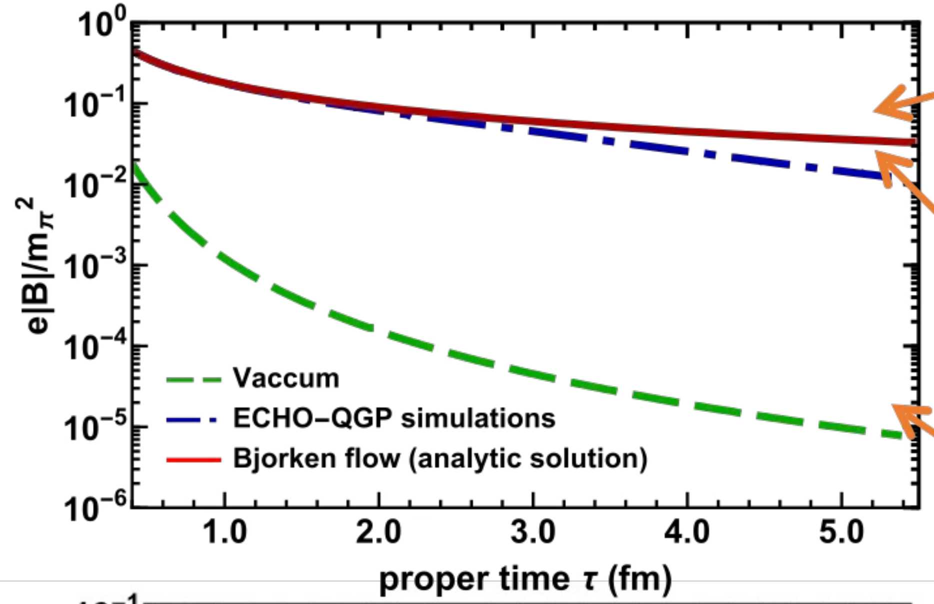
- **Strong B fields can be of order 10^{17} – 10^{18} Gauss, the strongest B fields discovered so far.**
- **HIC provides us the platform to study the physics under the extreme strong B fields.**

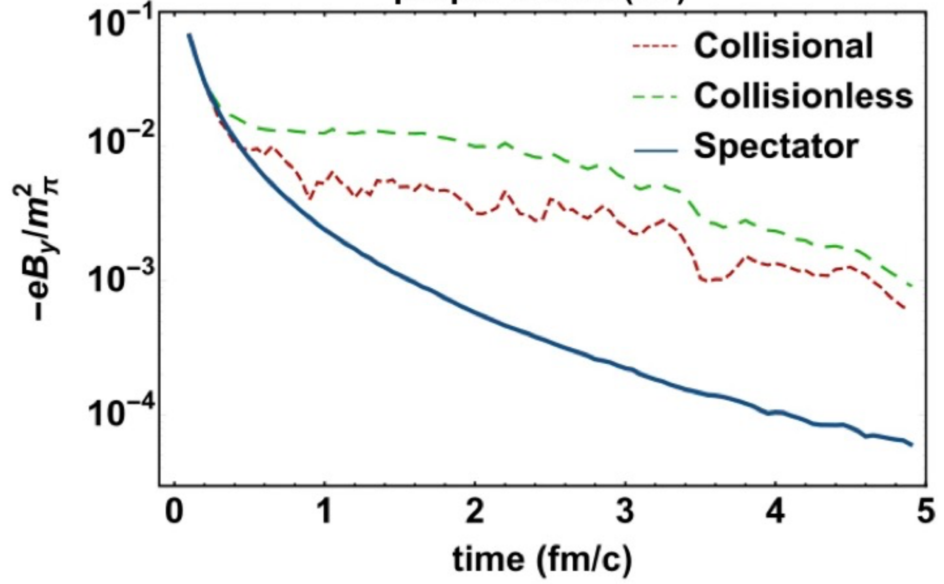




A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015;
H. Li, X.l. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Evolution of electromagnetic fields coupled with medium





Bjorken型理想磁流体(最慢衰减模式)
Pu, Roy, Rezzolla, Rischke, PRD(2015)
磁流体+手征磁效应+手征反常
Siddique, Wang, Pu, Wang, PRD (2019)
Peng, Wu, Wang, She, Pu, PRD(2023)

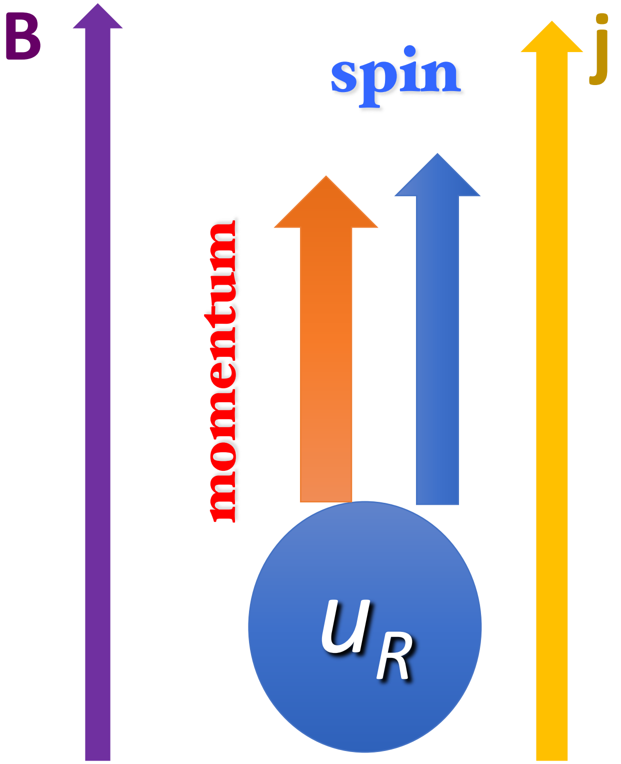
ECHO-QGP磁流体模拟
Inghirami, Zanna, Moghaddam,Becattini, Bleicher, EPJC(2016)

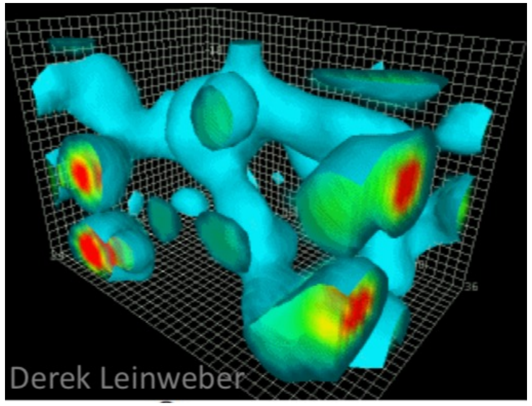
真空中衰减 (最快衰减模式)
Kharzeev, McLerran,Warringa,NPA(2008)

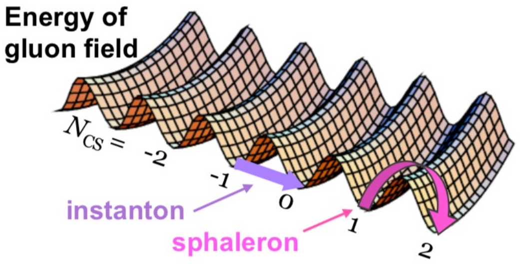
相对论玻尔兹曼方程与Maxwell方程耦合+ 2-2 领头Log阶pQCD散射矩阵元
Zhang, Sheng, Pu, Chen, Peng, Wang, Wang, PRR (2022)
Also see:
Wang, Zhao, Xu, Greiner, Zhuang, PRCL (2022); Li, Huang, PRD (2023);

Chiral magnetic effects

The CME is challenging to observe in heavy-ion collisions, but it has already been confirmed in other areas of physics.



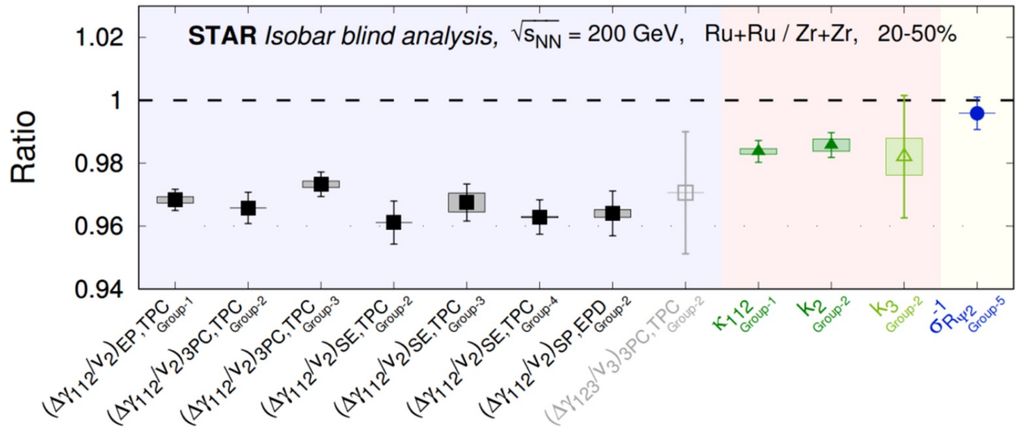




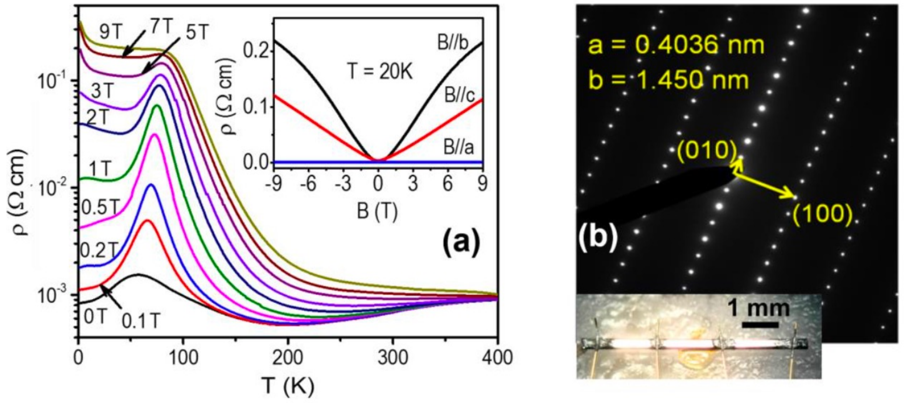
$$\boxed{\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},}$$

Kharzeev, Fukushima, Warrigna, (08,09), etc.

...



STAR, Phys. Rev. C, 2022, 105: 014901



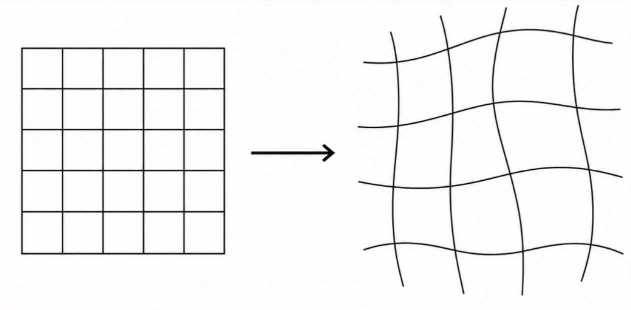
$ZrTe_5$: Nature Physics, 12 , 550–554, (2016)

Weyl anomaly

Question: **Can we observe macroscopic effects induced by the Weyl anomaly?**

Weyl symmetry

$$g_{\mu\nu}(x) \to e^{f(x)} g_{\mu\nu}(x)$$



For massless classical system

Trace of energy momentum tensor vanishes

$$T^{\mu}_{\mu} = 0$$

e.g. for a conformal fluid, the bulk pressure is zero.

Weyl anomaly

But for massless QED,

$$\boxed{T^{\mu}_{\mu} = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}}$$

At one loop for fermion,

$$\beta(e) = \frac{e^3}{12\pi^2}$$

Simplification and Method

- **Assuming:**

Global equilibrium

(all effects are non-dissipative)

1. Write down all possible terms in hydrodynamics with unknown coefficients.

$$\begin{aligned} T^{\mu\nu}_{(2)} &= e_{(2)}u^\mu u^\nu + P_{(2)}(g^{\mu\nu} - u^\mu u^\nu) \\ &\quad +\kappa_1 a^\mu a^\nu + \kappa_2 E^\mu E^\nu + \kappa_3 a^{(\mu}E^{\nu)} \\ &\quad +\chi_5(u_\lambda \partial^\mu F^{\nu\lambda} + u_\lambda \partial^\nu F^{\mu\lambda}), \\ j^\mu_{(2)} &= n_{(2)}u^\mu + \xi_F \partial_\lambda F^{\lambda\mu}, \end{aligned}$$

$$\begin{aligned} e_{(2)} &= \lambda_1 a^2 + \lambda_2 a\cdot E + \lambda_3 E^2 \\ &\quad +(\chi_2+\chi_3)u^\rho \partial^\lambda F_{\lambda\rho}, \\ P_{(2)} &= \bar{\lambda}_1 a^2 + \bar{\lambda}_2 a\cdot E + \bar{\lambda}_3 E^2 + \chi_3 u^\rho \partial^\lambda F_{\lambda\rho}, \\ n_{(2)} &= \xi_1 a^2 + \xi a\cdot E + \xi_2 E^2. \end{aligned}$$

2. Using the conservation equations + Weyl anomaly to solve these coefficients.

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} j_\mu, \quad \partial_\mu j^\mu = 0, \quad g_{\mu\nu}T^{\mu\nu} = 2CE^2.$$

Currents induced by Weyl anomaly

$$j^\mu = -4\boxed{C}F^{\mu\nu}\boxed{a_\nu} = 4C[(a\cdot E)u^\mu - \epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha a_\beta]$$

Weyl anomaly coefficient

$$a_\mu = u^\nu \nabla_\nu u_\mu$$

Acceleration field

Vacuum magnetization current

$$j^\mu = -4\boxed{C}F^{\mu\nu}\boxed{a_\nu} = 4C[(a\cdot E)u^\mu - \boxed{\epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha a_\beta}]$$

Weyl anomaly coefficient

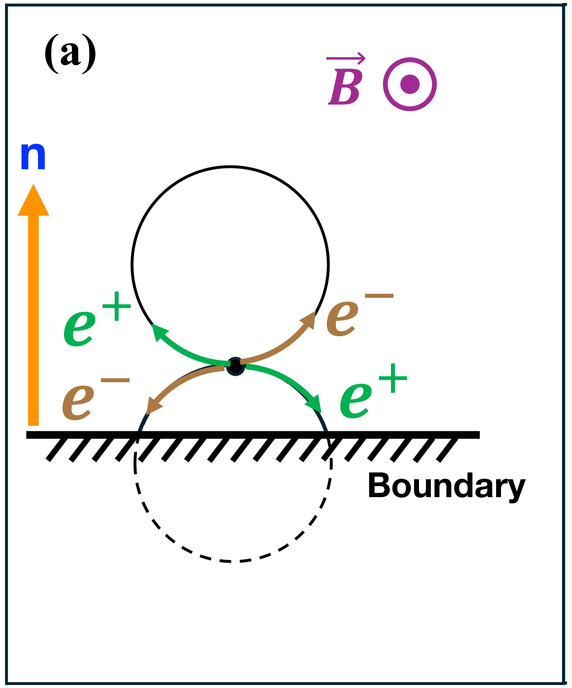
$$a_\mu = u^\nu \nabla_\nu u_\mu$$

Accelerate field

- **Boundary quantum field theory**

$$j^\mu = \langle \hat{j}^\mu \rangle = 4C\frac{F^{\mu\nu}\boxed{n_\nu}}{x}$$

Chu, Miao, PRL (2018) 121, 252602



Screening effects induced by electric fields

$$j^{\mu}=-4\boxed{C}F^{\mu\nu}\boxed{a_{\nu}}=4C[\boxed{(a\cdot E)u^{\mu}}-\epsilon^{\mu\nu\alpha\beta}u_{\nu}B_{\alpha}a_{\beta}]$$

Weyl anomaly coefficient

$$a_{\mu}=u^{\nu}\nabla_{\nu}u_{\mu}$$

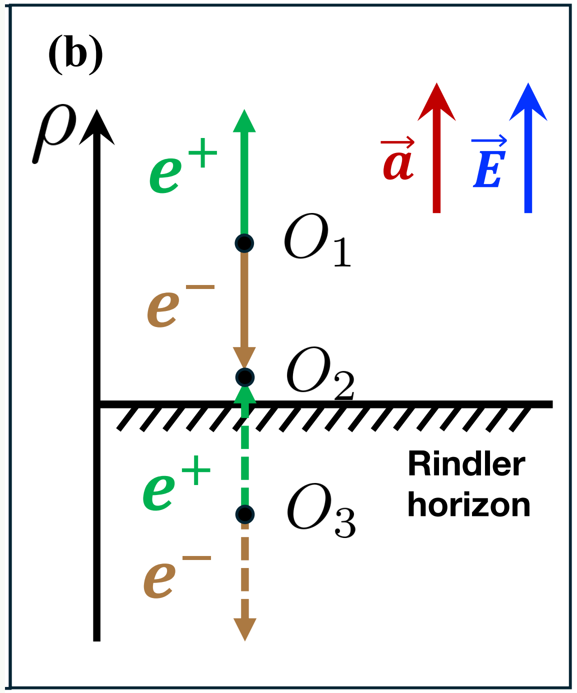
Accelerate field

- **Boundary quantum field theory**

$$j^{\mu}=\langle\hat{j}^{\mu}\rangle=4C\frac{F^{\mu\nu}n_{\nu}}{x}$$

If the boundary is moving, then

$$\frac{n^{\varepsilon}_{\mu}}{x}=-a_{\mu}\qquad j^{0}=-4C(\mathbf{E}\cdot\mathbf{a})$$



Applicantion to other fields

$$j^0 = -4C(\mathbf{E}\cdot\mathbf{a})$$

Screening effects

$$\mathbf{j} = 4C\mathbf{a}\times\mathbf{B}$$

Vacuum magnetization

Global equilibrium

(all effects are non-dissipative)

$$a_\mu = u^\nu\partial_\nu u_\mu = T^{-1}\partial_\mu T$$

$$\partial_\mu\overline{\mu} = -E_\mu$$

$$j^0 = -4C\boldsymbol{\nabla}\overline{\mu}\cdot\boldsymbol{\nabla}T$$

New effects

$$\mathbf{j} = 4CT^{-1}\boldsymbol{\nabla}T\times\mathbf{B}$$

Thermomagnetic Hall-like current

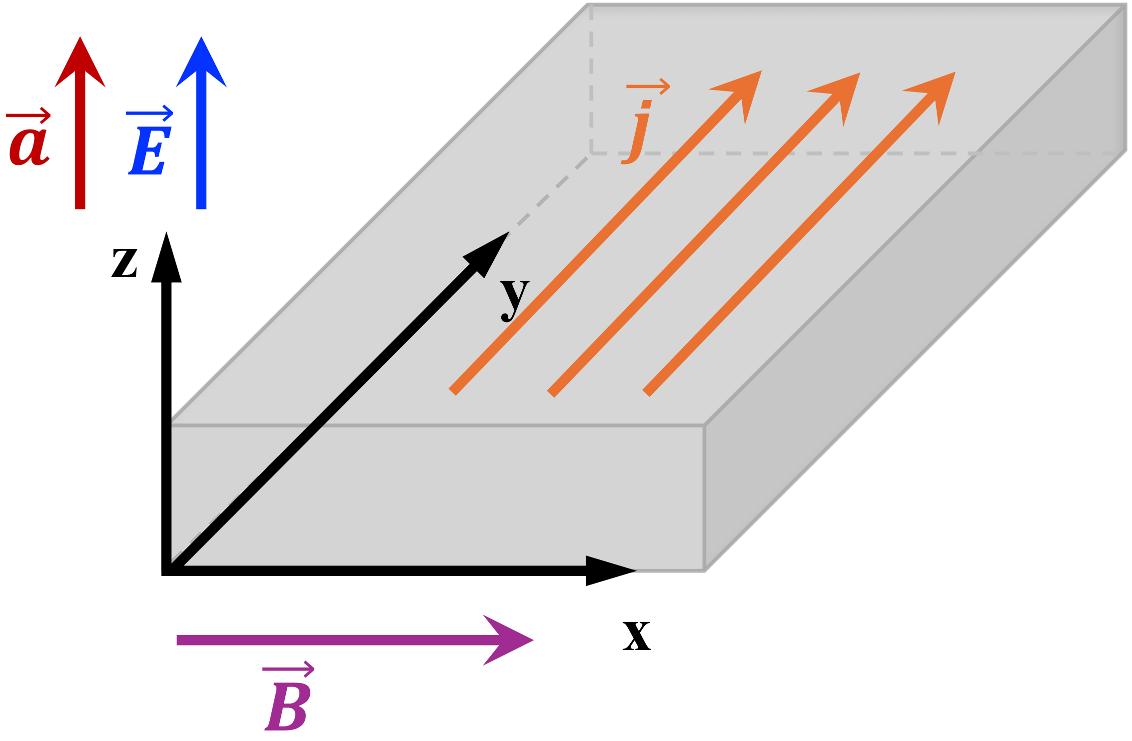
Applicantion to other fields

$$j^0 = -4C(\mathbf{E}\cdot\mathbf{a})$$

Screening effects

$$\mathbf{j} = 4C\mathbf{a}\times\mathbf{B}$$

Vacuum magnetization



Beyond Global equilibrium

Acceleration field (a^μ) is present everywhere, therefore, its coupling to the EB fields also occurs everywhere, i.e.

These two quantum effects can occur anywhere!

Universal: The coefficient C is hold due to Weyl anomaly.

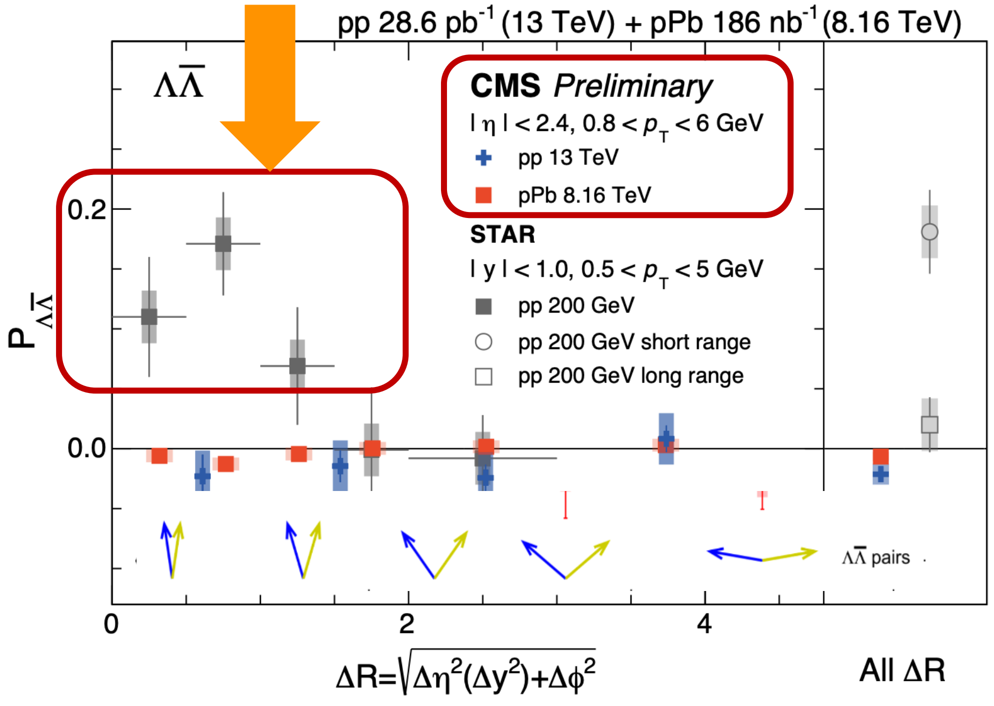
Spin hydrodynamics with conformal anomaly:
Zhang, Lv, Huang, arXiv:2603.17794

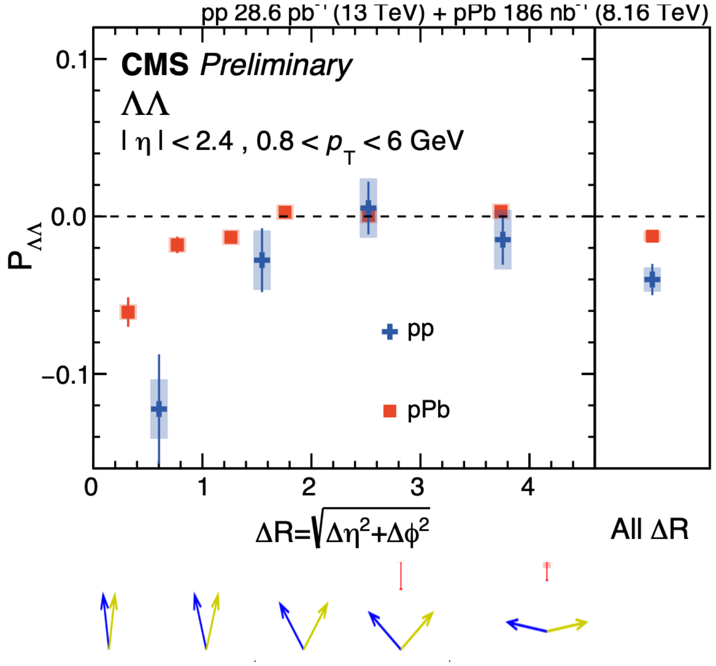
Late-Time Relaxation from Landau Singularities

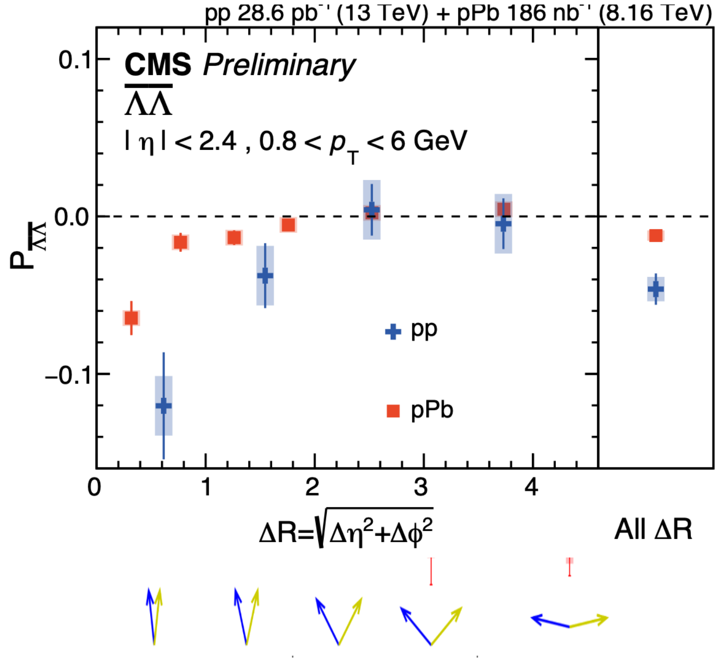
Dong-Lin Wang, SP, arXiv: 2605.04020

Spin-Spin correlation

STAR, Nature
650, 44-45 (2026)







CMS-PAS-HIN-26-002

Theory for spin correlation in QGP:

Lv, Yu, Liang, Wang, Wang, PRD109 (2024) 11, 114003
X.L. Sheng, X.Y. Wu, X.N. Wang, PRL136 (2026) 8, 082301

From initial state to final state

Can the (initial) spin correlation in QGP survive at late times?

Which mechanism governs the late-time behavior of a relativistic many-body system?

If the initial spin correlation originates from initial fluctuations,
what is the late-time behavior of these fluctuations?

Micrscopic theories
e.g. different theory can give different relaxation time for shear viscosity

Symmetries, conservation laws, constitutive relations

Must be univeral! But, how?

Landau, Lifshitz, Statistic Physics I

t

Early time dynamics

Late time dynamics

Example: Late time behavior in hydrodynamics

Relativistic hydrodynamics

Schwinger-Keldysh effective field theory

Haehl, Loganayagam, Rangamani, JHEP 04, 039
Crossly, Glorioso, Liu, JHEP 09, 095
Jensen, Pinzani, Yarom, JHEP 09, 127
Jain, Kovtun, PRL 128 (2022), 071601

$$\mathcal{L}_{\mathrm{eff}} = \varphi_a(\mathcal{R}\boxed{\varphi_r} + \mathcal{S}\boxed{\varphi_a}) + \mathcal{L}_{\mathrm{int}}$$

r: deviations from equilibrium

a: stochastic fluctuations

Singularities of "full" two point Green function → **Late time behavior of fluctuations**

$$G_{ij} = \langle \varphi_i \varphi_j \rangle$$

Two point Green function

$$\lim_{\mathbf{k}\to\mathbf{0}} G(t,\mathbf{k}) \propto \begin{cases} e^{-at} & \text{Rapid decay, "trivial mode"} \\ t^{-a} & \text{Slow decay, late time behavior} \end{cases}$$

Challenge and Landau singularity analysis

Dyson eq. for full two point Green function

$$G \approx \boxed{G^{(0)}} + G^{(0)}\boxed{\Sigma}G^{(0)} + \ldots,$$

Free two point Green function

Self energy

A typical loop integarl for self-energy: Challenge

$$I(k) = \int_p \Omega(k,p) \prod_{e=1}^{E} G_e^{(0)}(q_e)$$

Common way: derive the two-point Green's function within **a given theory at a specified perturbative order** by taking the complicated loop integrals.

Landau singularity analysis

Landau, Zh. Eksp. Teor. Fiz 37, 62 (1960)

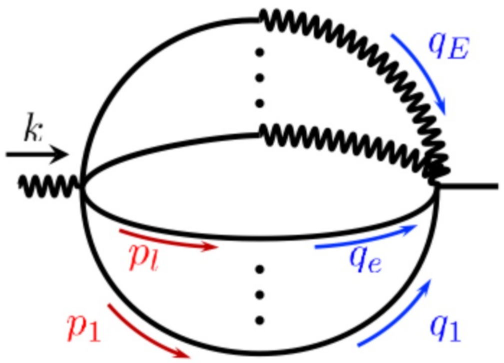
$$\alpha_e \boxed{D_e(q_e)} = 0, \quad e = 1, \ldots, E,$$

$$\sum_e \alpha_e \frac{\partial}{\partial p_l^\mu} \boxed{D_e(q_e)} = 0, \quad l = 1, \ldots, L,$$

Solving the linear equations for α_e, we can derive the information of the singularities of full two point Green function.

De: denominator for G(0)

Banana digram and late time behavior



$$G(t,\mathbf{k}) \approx \sum_{n=1}^{N_0} [C_0(\mathbf{k}) + C_1(\mathbf{k})t] e^{-i\mathfrak{w}_n(\mathbf{k})t}$$

$$+ \sum_{L,V,\kappa,\omega_{(s)}} \sum_{j=0}^{\infty} D(\mathbf{k}) \boxed{t^{-1-j-\sigma/2}} \boxed{e^{-i\omega_{(s)}(\mathbf{k})t}},$$

Competition between the two terms

Leading singularities:

$$\omega_{(s)} = \underbrace{\boxed{-i\gamma_{(s)}}}_{} - c|\mathbf{k}| - i\Gamma_{(s)}|\mathbf{k}|^2$$

$\gamma_s \neq 0$

Exponential decay dominates.

$\gamma_s = 0$

Two point Green function at late time

$$\lim_{\mathbf{k}\to\mathbf{0}} G(t,\mathbf{k}) \approx \text{const} + D(\mathbf{0}) t^{-L_{\min} d/2}.$$

Lmin: minimum number of loops
d: space dimension

This late-time behavior of fluctuations is generic in hydrodynamics.
Correlations of spin fluctuations exhibit the same behavior.

Summary

Summary

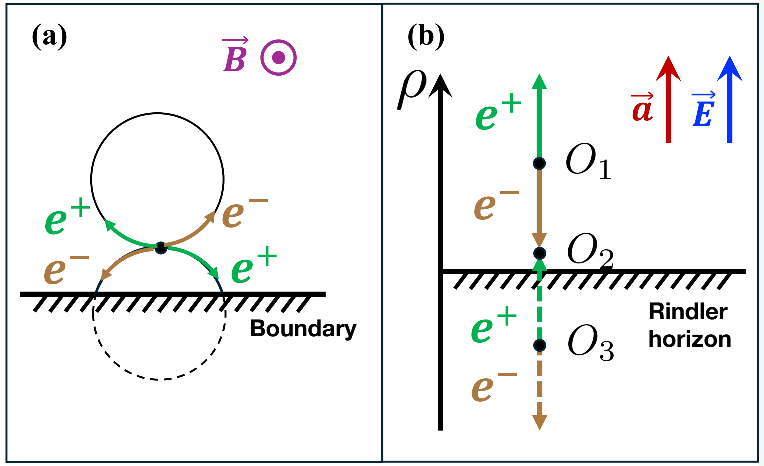
Novel transport induced by Weyl anomaly

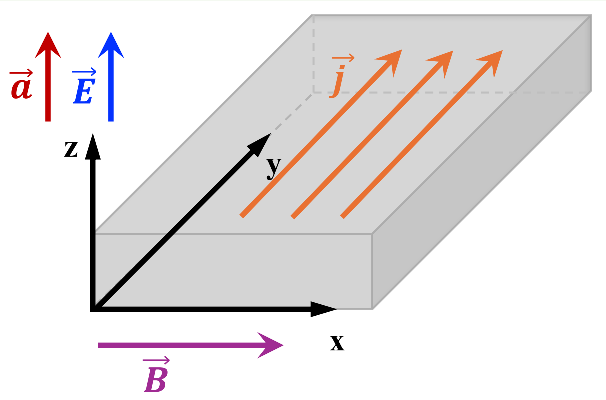
$$j^0 = -4C(\mathbf{E}\cdot\mathbf{a})$$

Screening effects

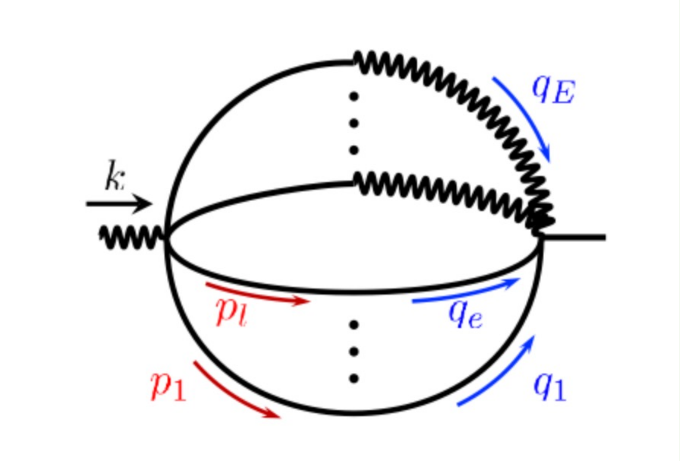
$$\mathbf{j} = 4C\mathbf{a}\times\mathbf{B}$$

Vacuum magnetization





Late time relaxation from Landau singularity



$$\lim_{\mathbf{k}\to\mathbf{0}} G(t,\mathbf{k}) \approx \text{const} + D(\mathbf{0})t^{-L_{\min}d/2}.$$

Lmin: minimum number of loops
d: space dimension

Thank you for your time!