Nuclear Science Computing Center at CCNU

Heavy Quarkonia from Lattice QCD:

In-Medium Properties and Mass Generation

Heng-Tong Ding (丁亨通)
Central China Normal University

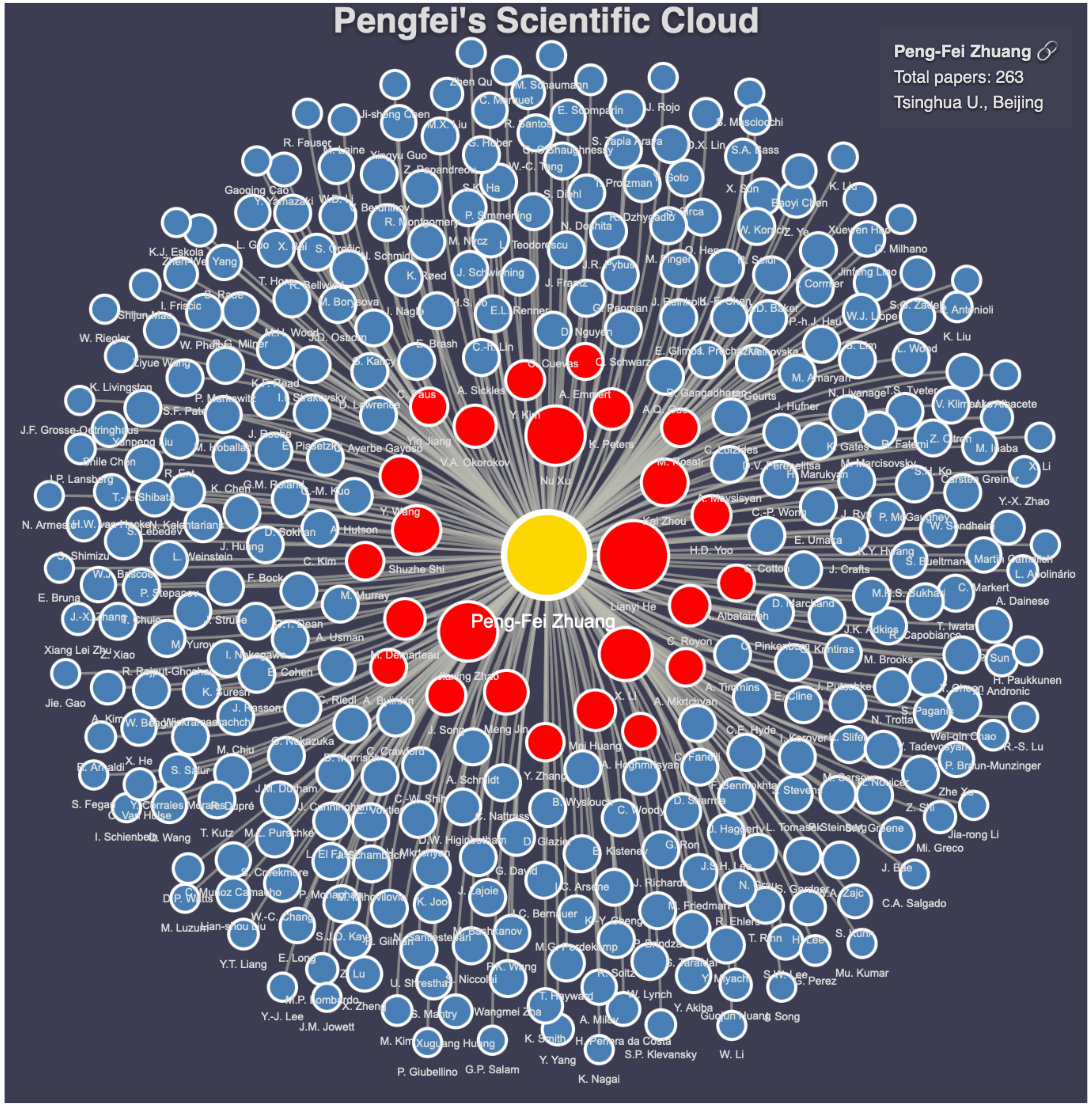
HTD, W.-P. Huang, R. Larsen et al., JHEP 05 (2025) 149
D. Bollweg, HTD, X. Gao et al., arXiv: 2601.13070 , PRL in press

Frontiers in high-energy nuclear physics
July 19-23,2026@Beijing

Alumni Story | Zhuang Pengfei:
Unveiling Mysteries, Empowering the Next Generation

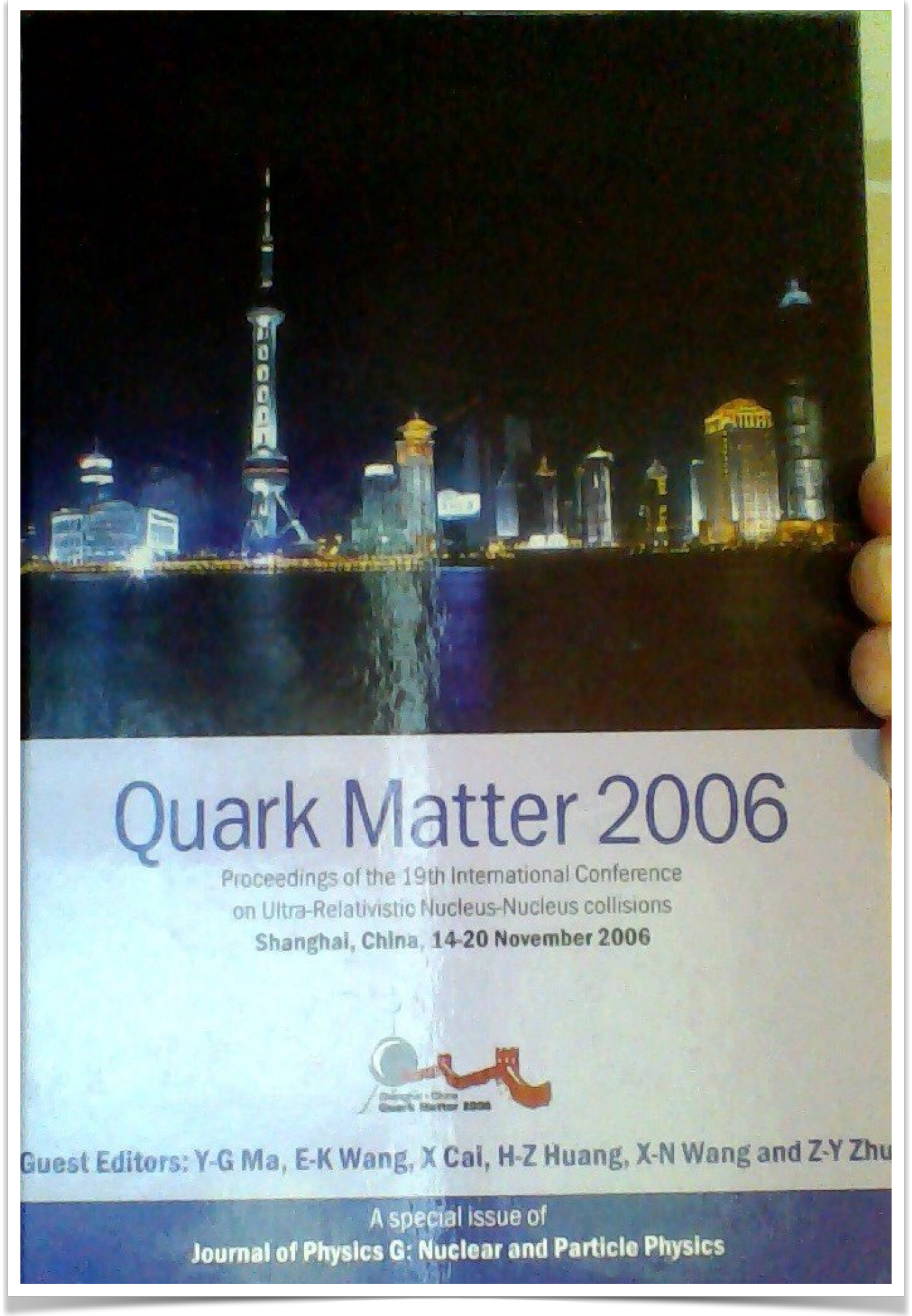
校友故事｜庄鹏飞：一手揭示神秘，一手托举韶华

From Physics Department, CCNU



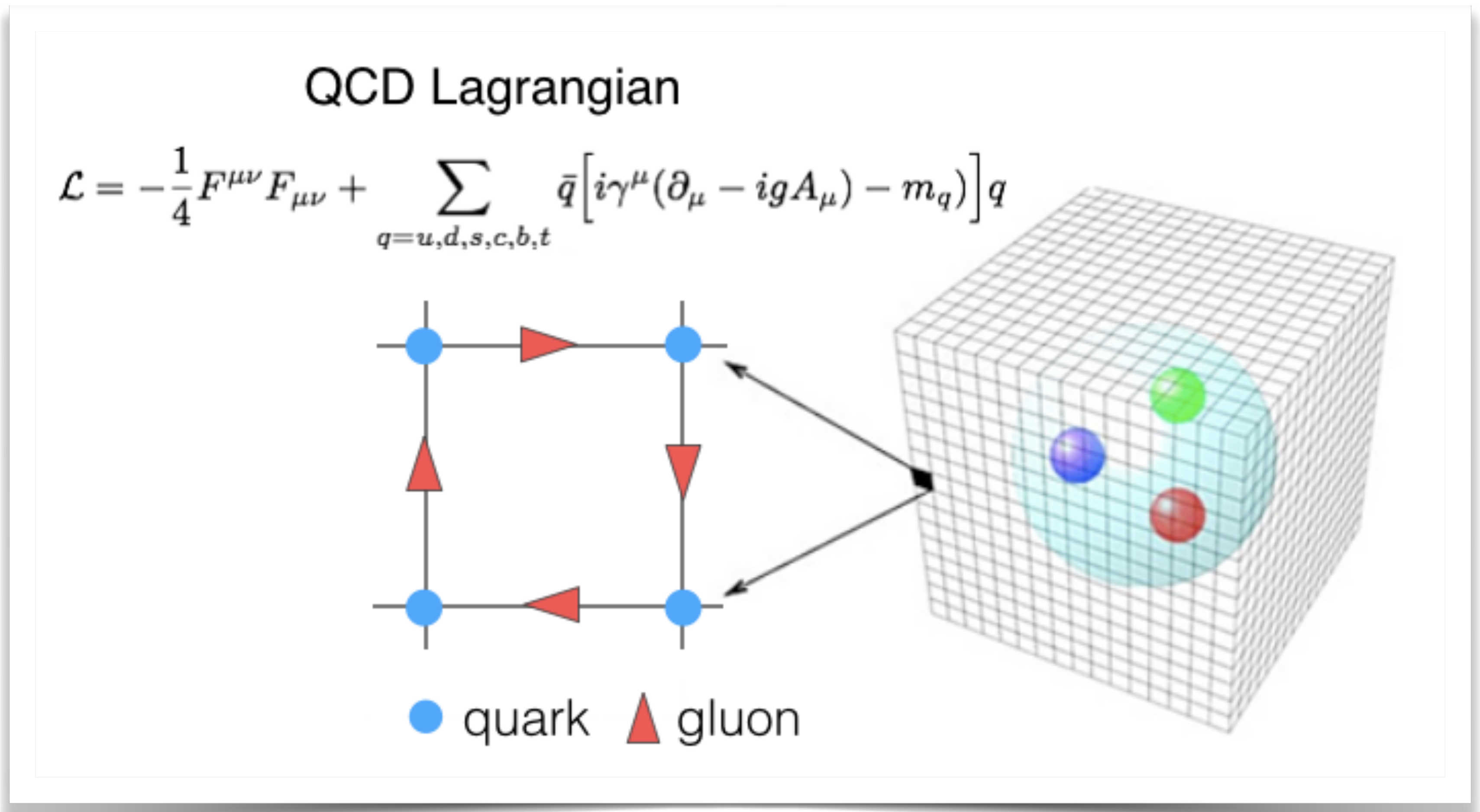
Workshop indico

Quark Matter 2006
Nov. 14 - 20, 2006, Shanghai, China



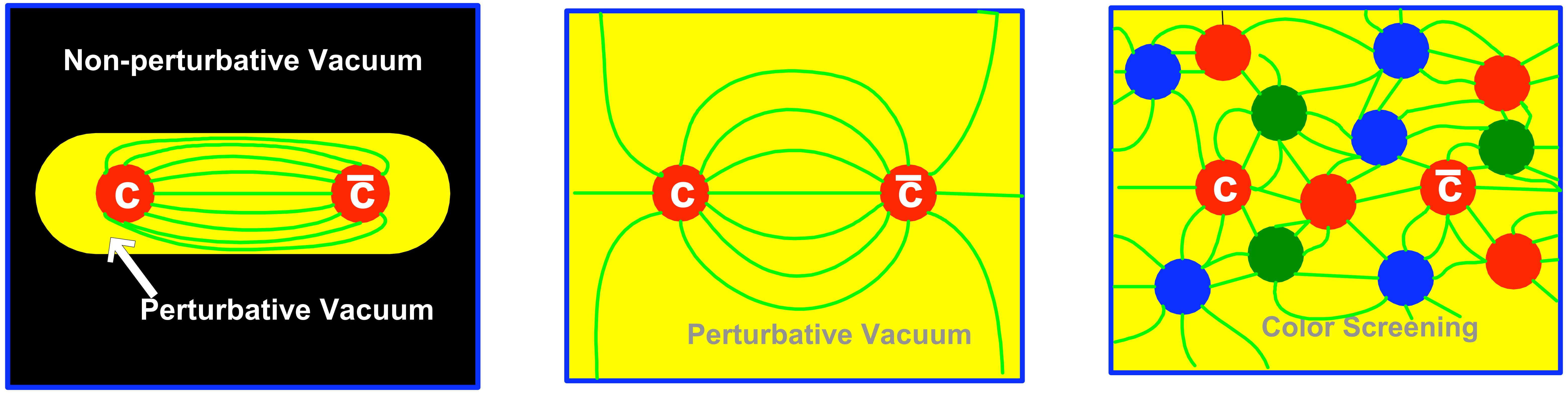
Selected Topics on
Heavy Flavor Production
in High-Energy Collisions
Nov. 22 - 23, 2006, Beijing, China

2007- now
Lattice QCD
Bielefeld University
Brookhaven National Lab
Columbia Univ.
CCNU



Quarkonia: probes of QGP

★ The suppression of the charmonium production can be a signature of the formation of the Quark Gluon Plasma [Matsui & Satz '86]



$$V(r) = -\frac{\alpha_{eff}}{r} + \sigma\, r$$

$$V(r,T) = -\frac{\alpha_{eff}}{r}\, e^{-r/\lambda_D(T)}$$

E(r) of the ccbar system should have a minimum wrt r if a bound state is possible

The condition for an extremum of E(r) is

$$x(1+x)\, e^{-x} = \frac{2}{m_c \alpha_{eff} \lambda_D} \quad \text{with} \quad x = r/\lambda_D$$

No solutions

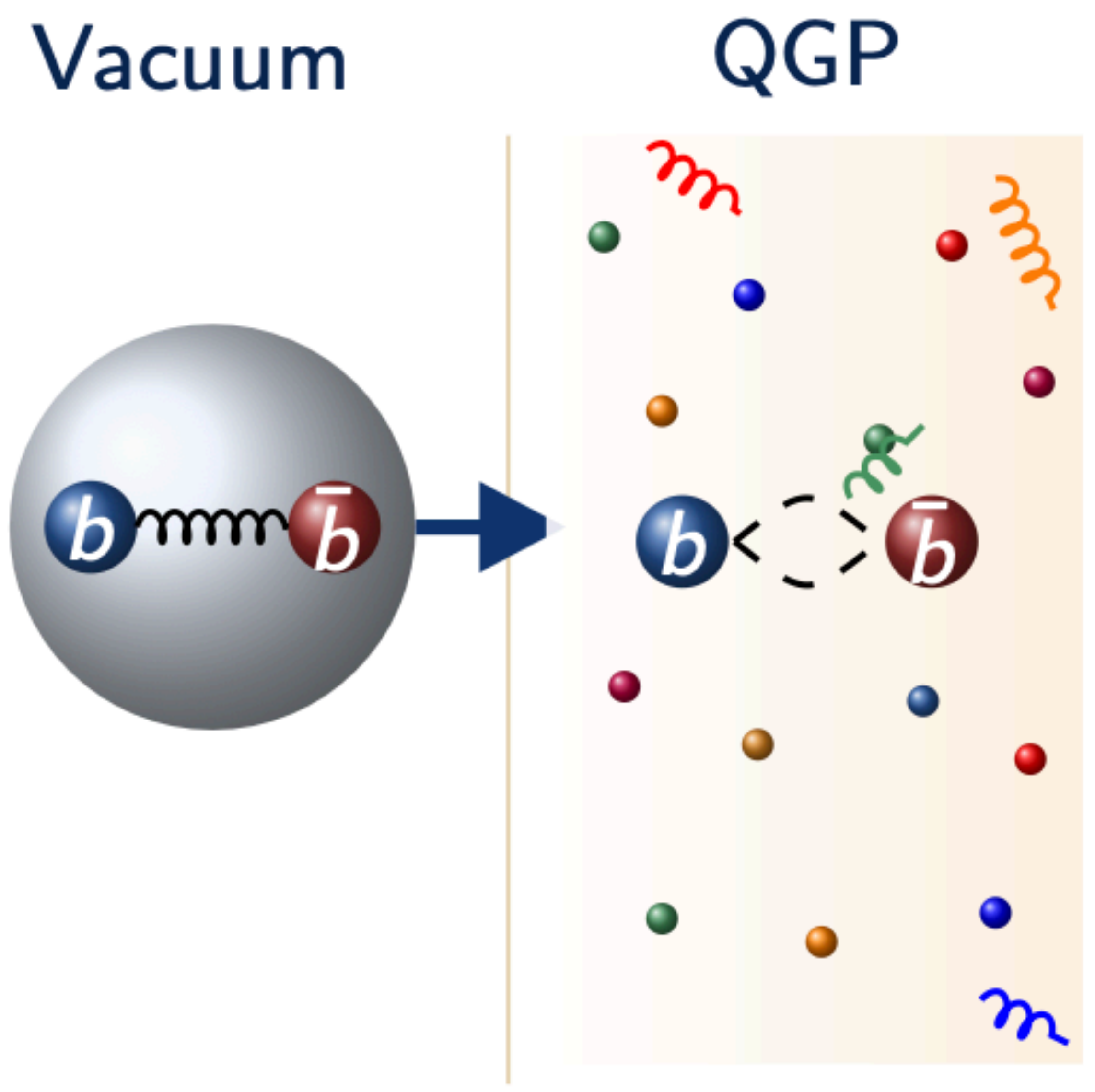
$\lambda_D < r$

the bound state will be "melted"

QGP thermometers

Beyond color screening

Classic picture

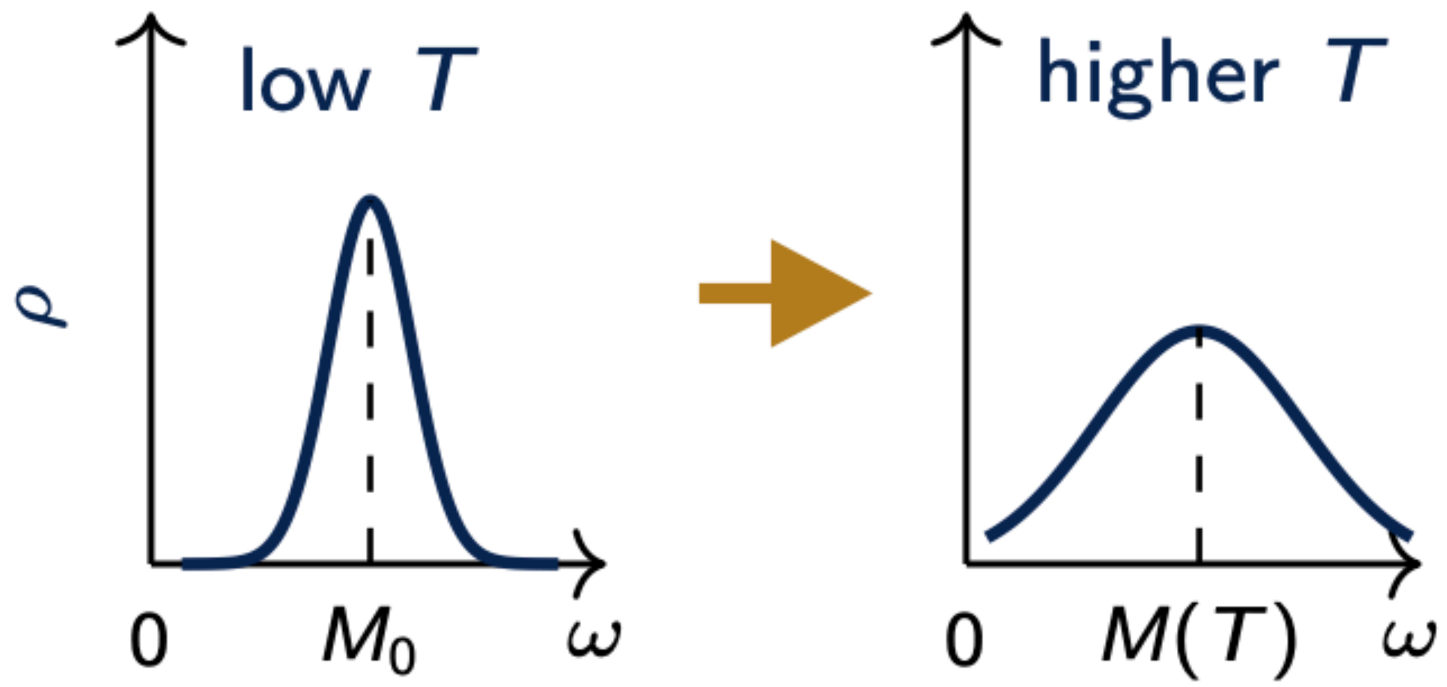


Screening weakens binding

Effective-theory picture

$$M \gg Mv \gg Mv^2,\ T,\ m_D$$

$$V(r,T) = \mathrm{Re}\,V(r,T) + i\,\mathrm{Im}\,V(r,T)$$



thermal scattering broadens the spectral peak

Lattice NRQCD

- Integrate out the heavy scale M
- no thermal wrap-around of heavy quarks

$$C(\tau,T) = \int_{-\infty}^{+\infty} d\omega\, \rho(\omega,T)\ K(\tau)$$

$$K(\tau) = e^{-\tau\omega},\ \ \tau_{max} = 1/T$$

more Euclidean-time information

Laine et al., JHEP03(2007)054,
Brambilla et al., PRD 78 (2008)014017,
Beraudo et al., NPA 806 (2008) 312,…

Non-Relativistic QCD on the lattice

- Bottom quark on the lattice:

S. Meinel, PRD 82, 114502 (2010)

R. Larsen, et.al., PRD 100, 074506 (2019)

R. Larsen, et.al., PLB 800, 135119 (2020)

– Tree-level tadpole-improved NRQCD action, with $\mathcal{O}(v^6)$ corrections

– Bare bottom mass tuning: matching kinetic mass M_{kin,η_b} to its PDG value

- Background gauge fields with (2+1)-flavor dynamical sea quarks:

– HISQ/tree action

– Quark mass: $m_s^{\mathrm{phy}}/m_l = 20$ $(m_\pi \approx 160$ MeV$)$

– Two fine lattice spacings used: $a = 0.0493$ fm and 0.0602 fm

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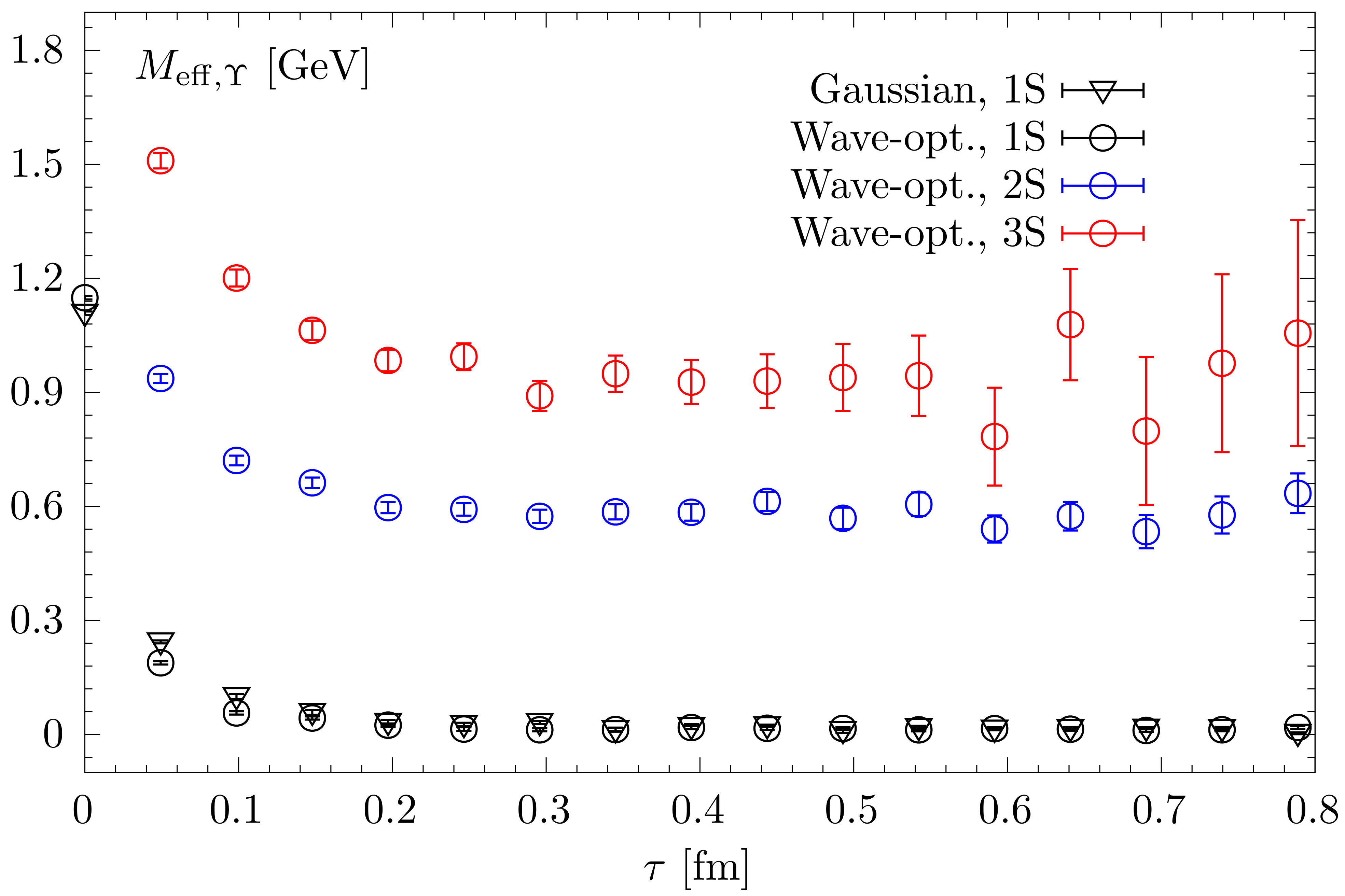
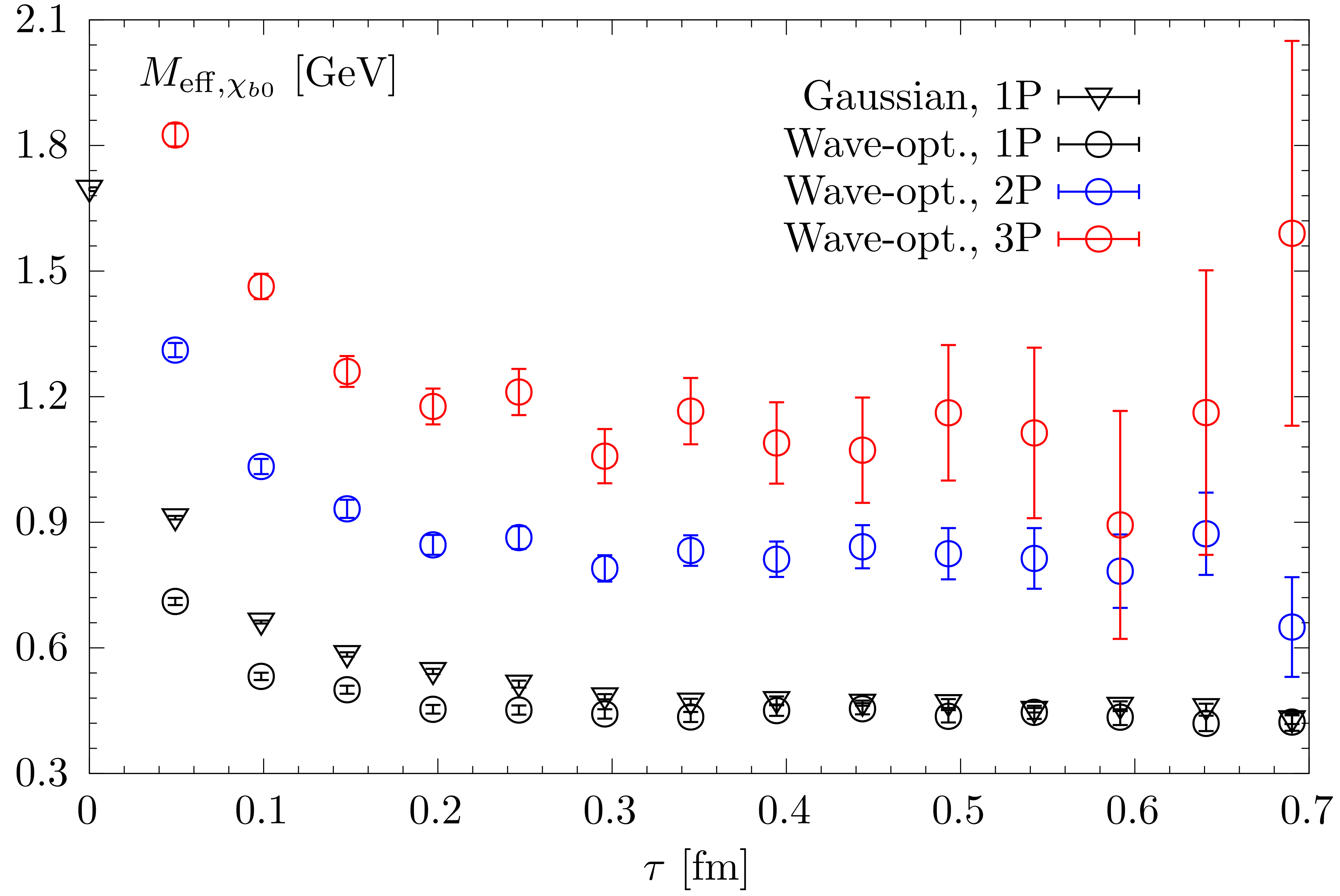
– Temperature is increased by reducing the temporal extent: $N_\tau \in [16, 30]$, $T \in (133, 250)$ MeV

HTD, W.-P. Huang, R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky and Z. Tang, JHEP 05 (2025) 149

Effective mass in the vacuum

$$M_{\mathrm{eff}}(\tau) = \log\left[\frac{C(\tau, T)}{C(\tau + a, T)}\right]$$





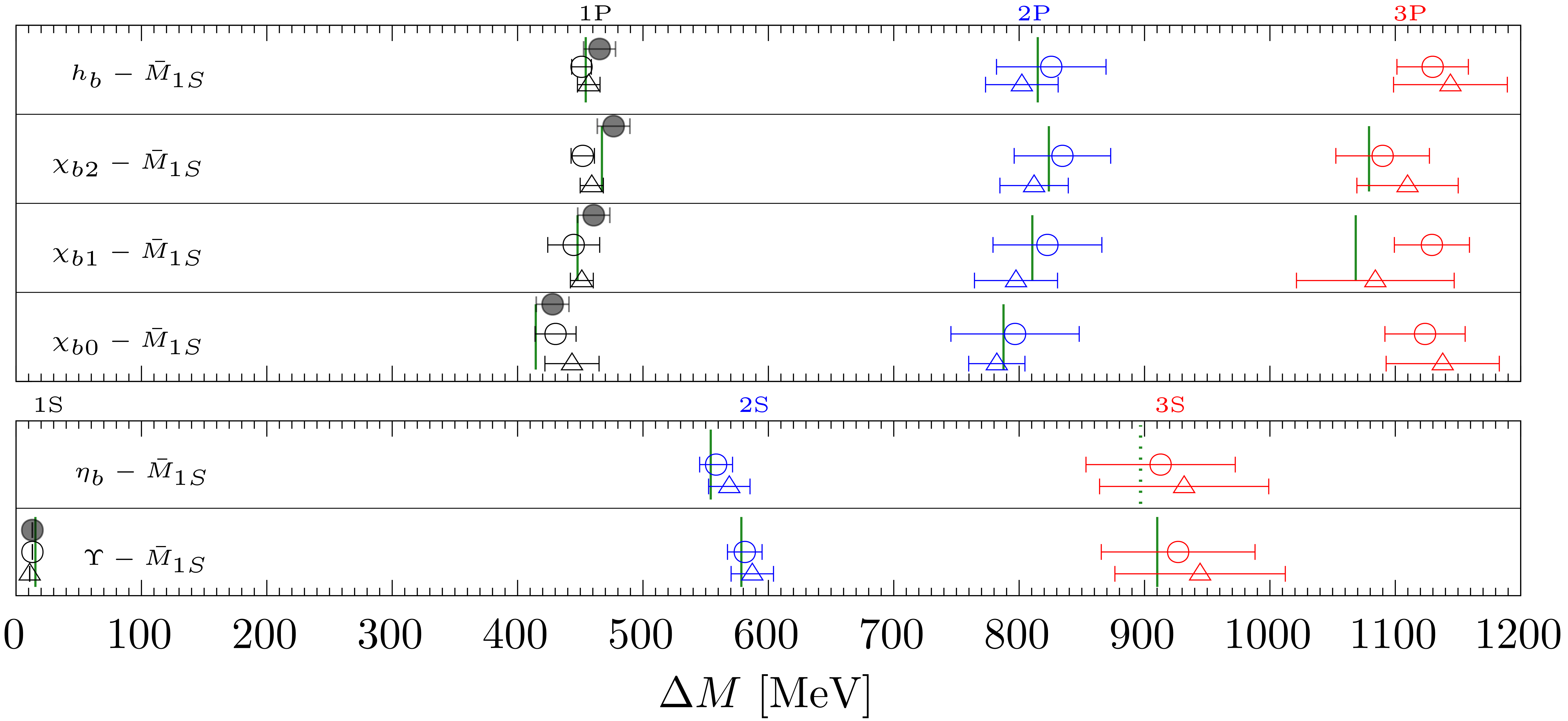
All vertical scales are calibrated with the spin-averaged mass of 1S bottomonium hereafter

- Mild effects arise from different extended sources for ground states at T = 0
- Plateau starts from t ~ 0.25 fm, but shorter for excited states with worse signal

Mass spectra in the vacuum

Mass difference: $\Delta M = M - (M_{\eta_b} + 3M_{\Upsilon})/4$

Spin-averaged mass of 1S bottomonium



○ △ : wave-function optimized results at two lattice spacings

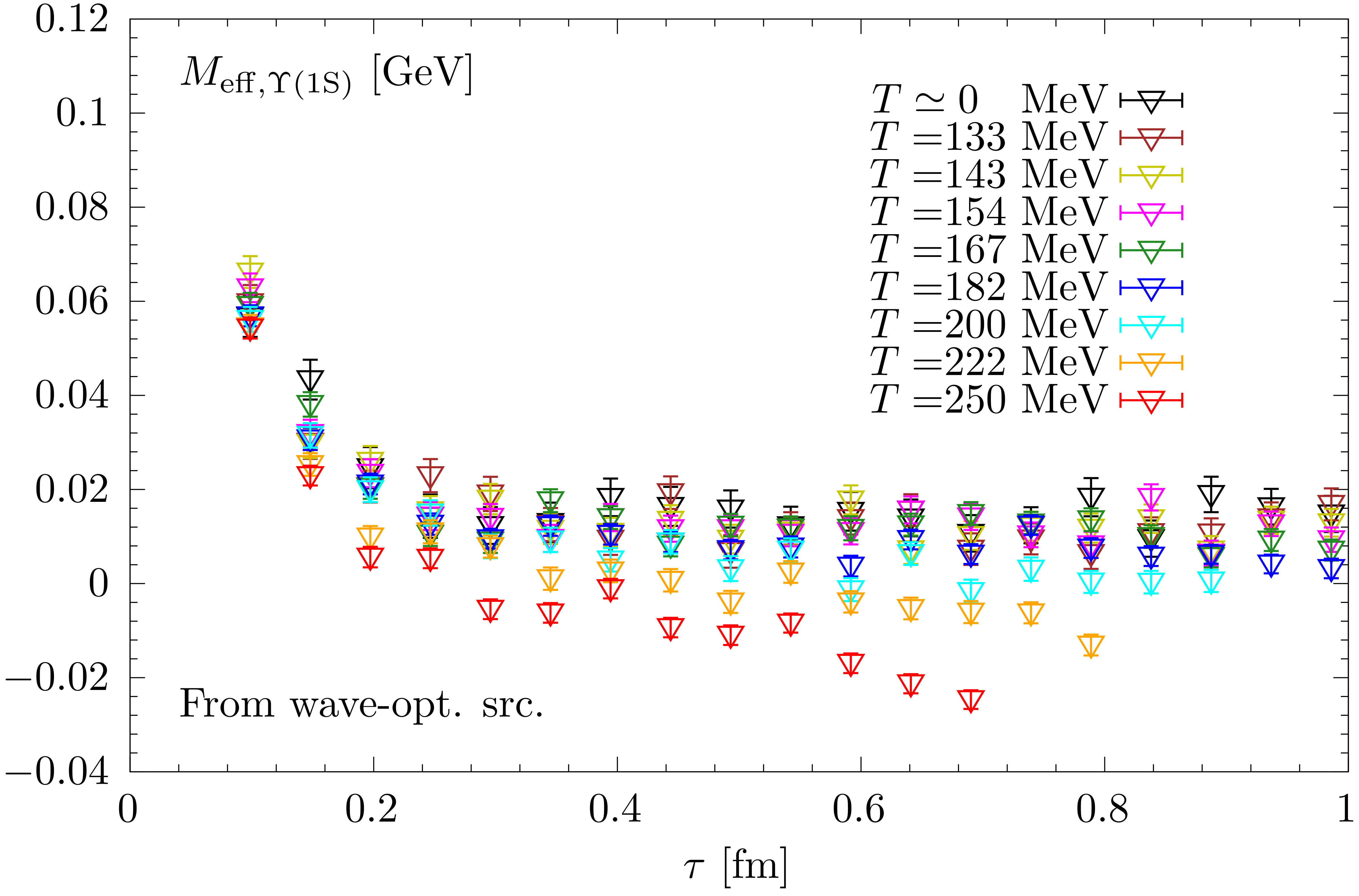
● : Gaussian-smeared results

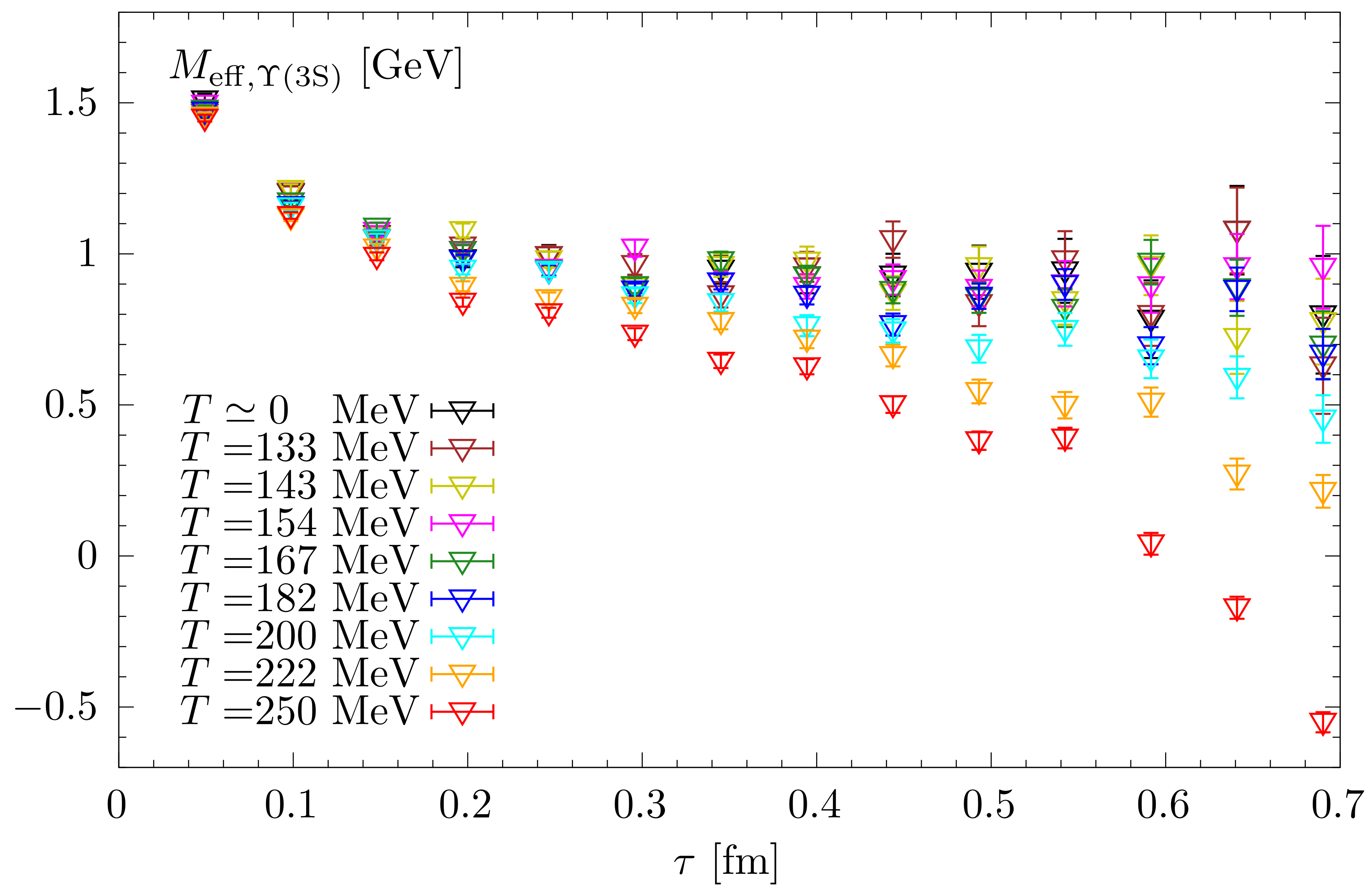
: PDG values

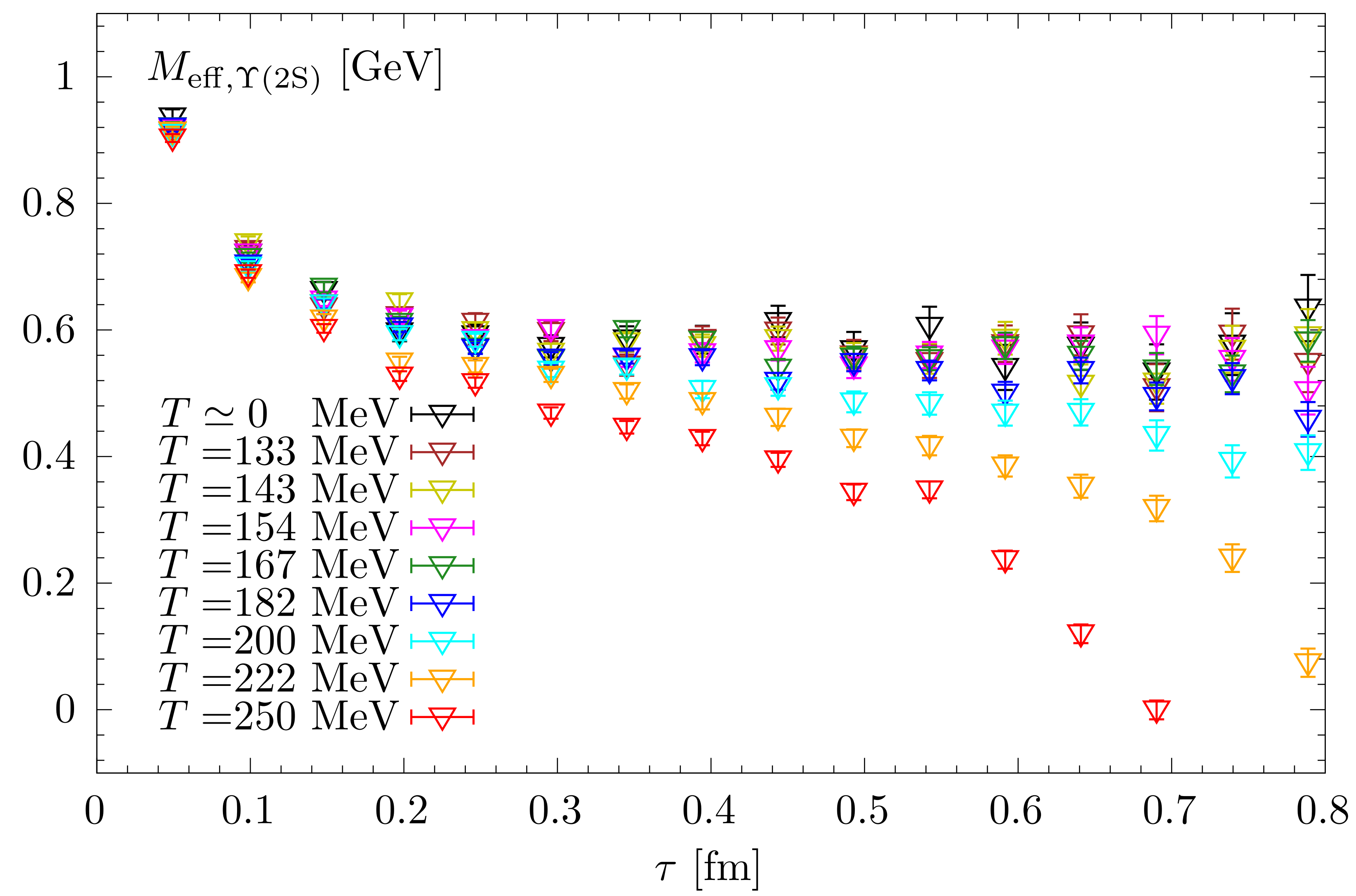
: lattice-predicted $M_{\eta_b(3S)}$

R. Larsen, et.al., PLB 800, 135119 (2020)

Effective mass at nonzero temperature

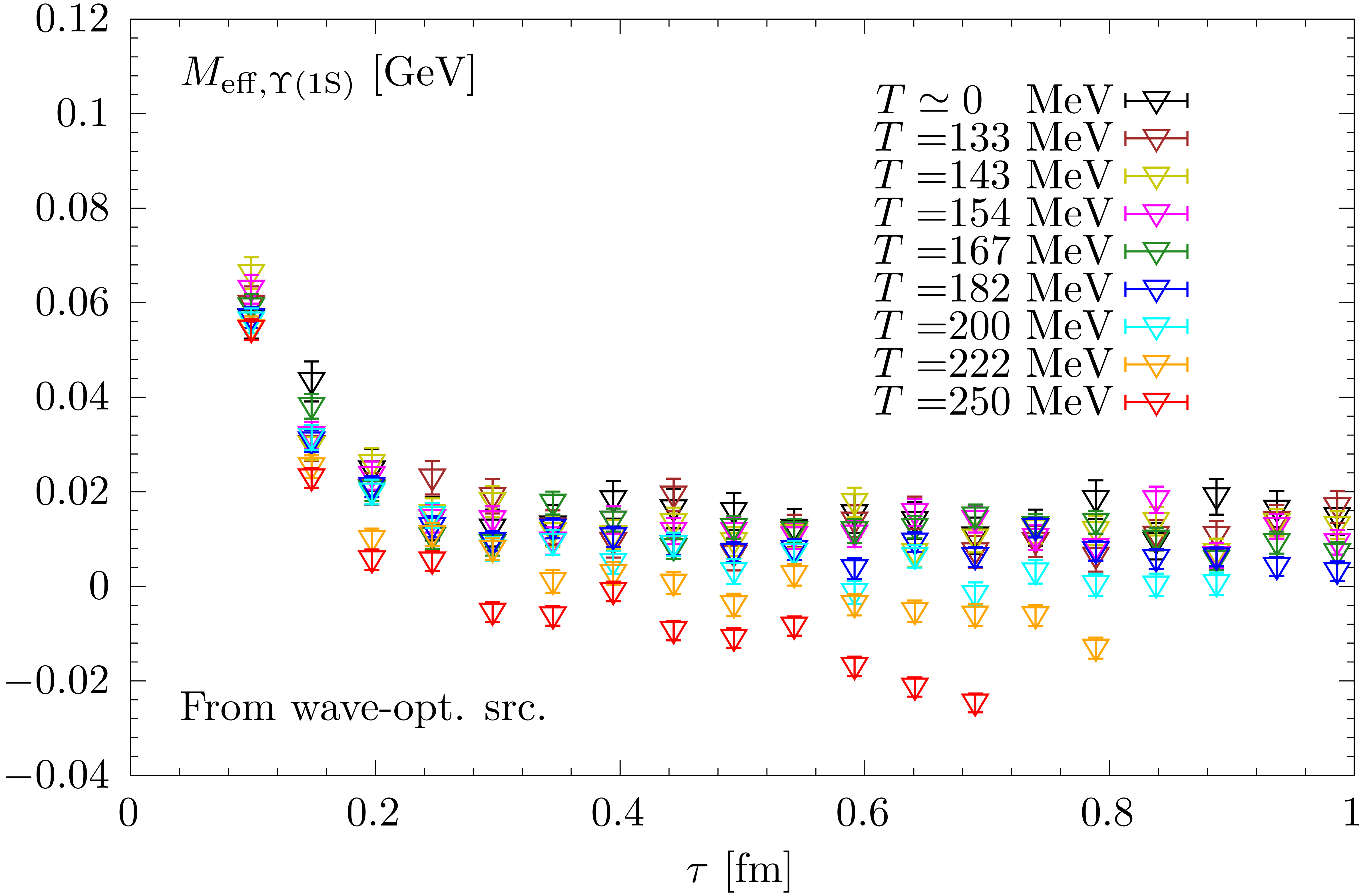


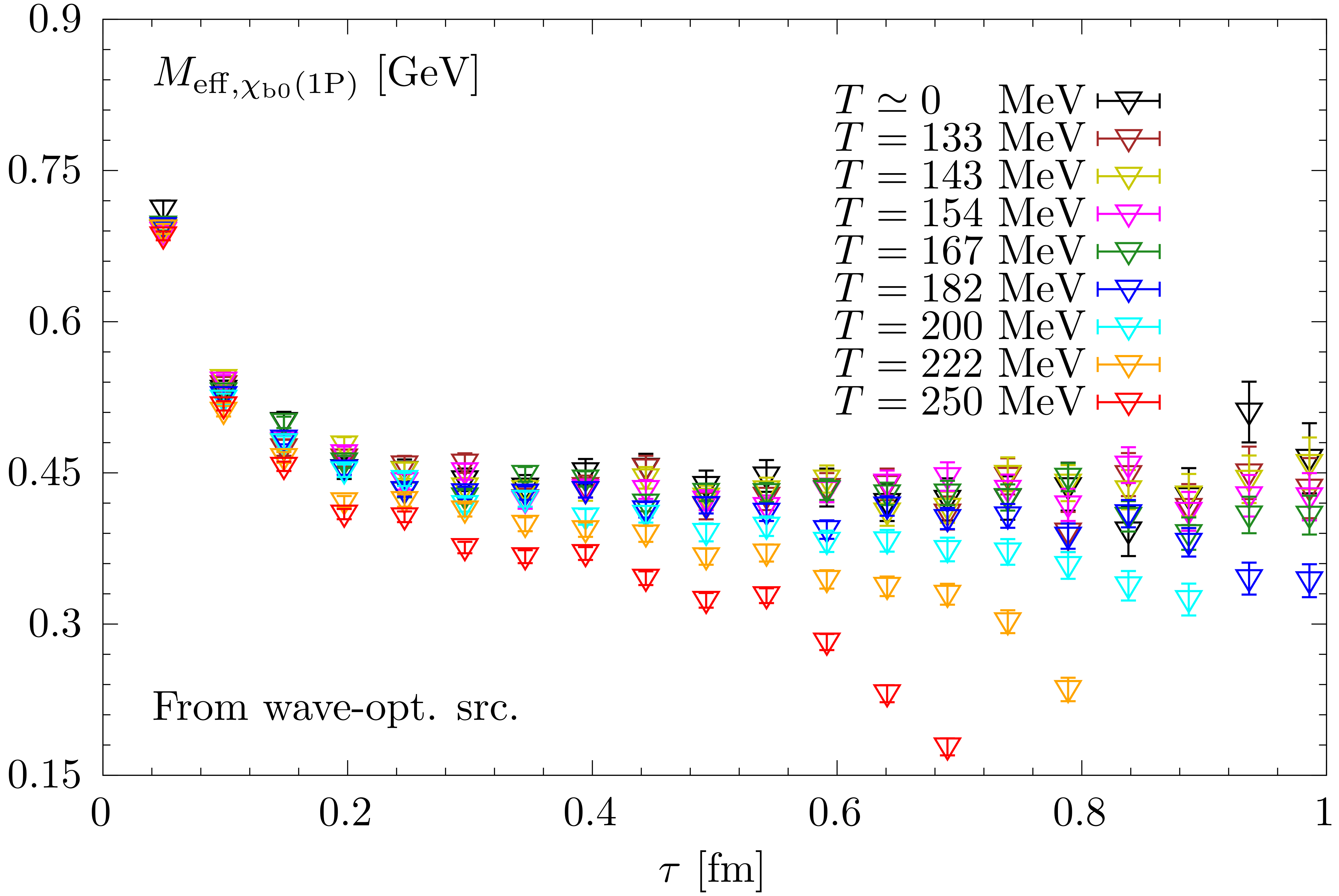




- Short distance: temperature independent
- $T \lesssim 170$ MeV: weak T dependence
- Higher T: clear in-medium effect, more in excited states

Effective mass at nonzero temperature





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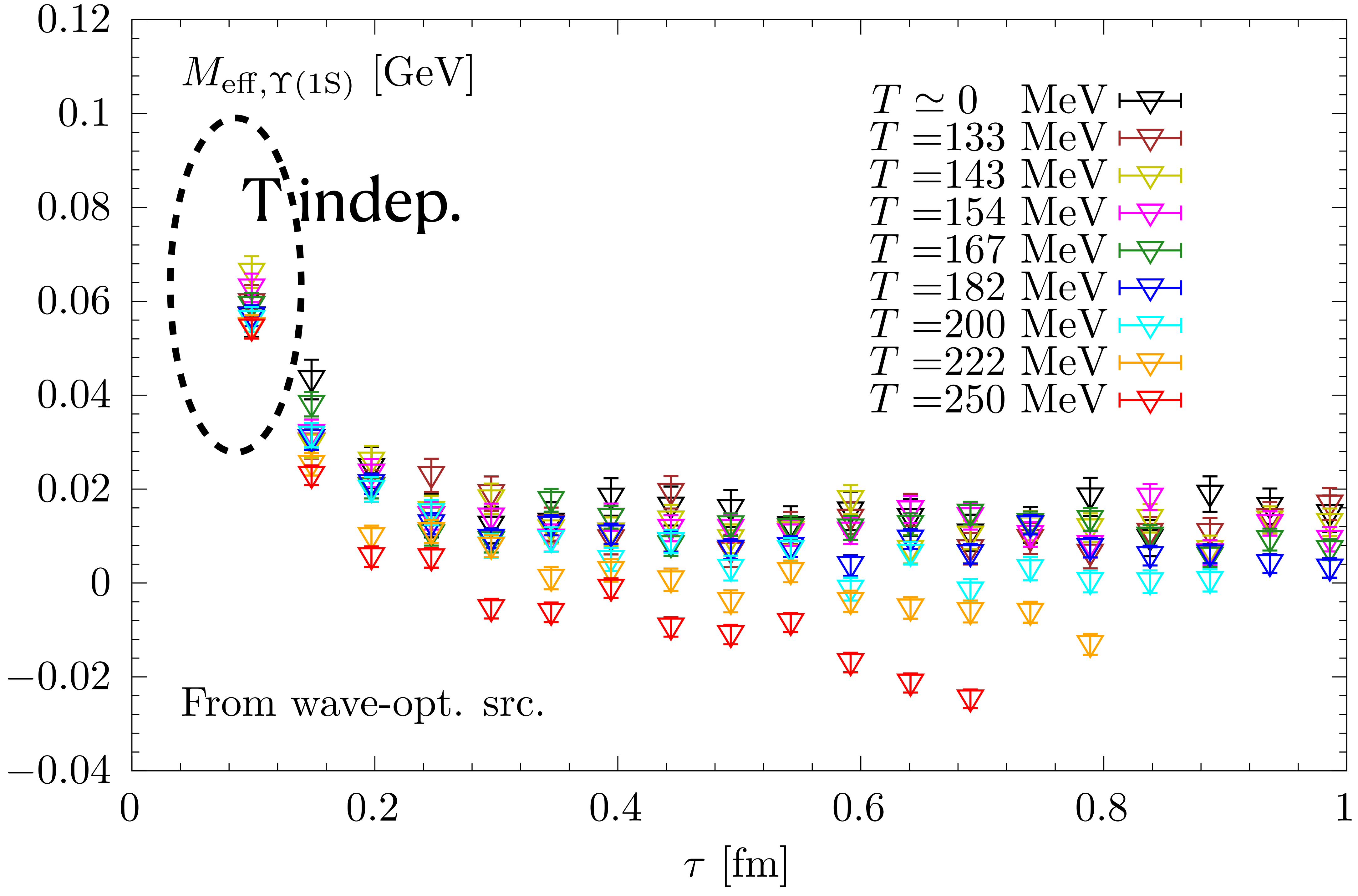
Disentangle the medium effects



Continuum-subtracted correlator

$$C_{\text{sub}}(\tau, T) \equiv C(\tau, T) - C_{\text{cont}}(\tau)$$

Sensitive to thermal effects

Temperature indep.
Extracted from T=0 as:
$C_{\text{cont}}(\tau) = C(\tau, T=0) - Ae^{-M\tau}$

$$C(\tau, T) = \int_{-\infty}^{+\infty} d\omega\, \rho(\omega, T)\, e^{-\tau\omega}$$

$$\rho_{\text{cont}}(\omega) = \rho(\omega, T=0) - A\delta(\omega - M)$$

$$\rho_{\text{med}}(\omega, T) \equiv \rho(\omega, T) - \rho_{\text{cont}}(\omega, T)$$

Zero T limit: $\rho_{\text{med}}(\omega, T=0) = A\delta(\omega - M)$

Continuum-subtracted effective mass of Υ states

$$M_{\text{eff}}^{\text{sub}}(\tau, T) = \log\left[\frac{C_{\text{sub}}(\tau, T)}{C_{\text{sub}}(\tau + a, T)}\right]$$

- No thermal effects:

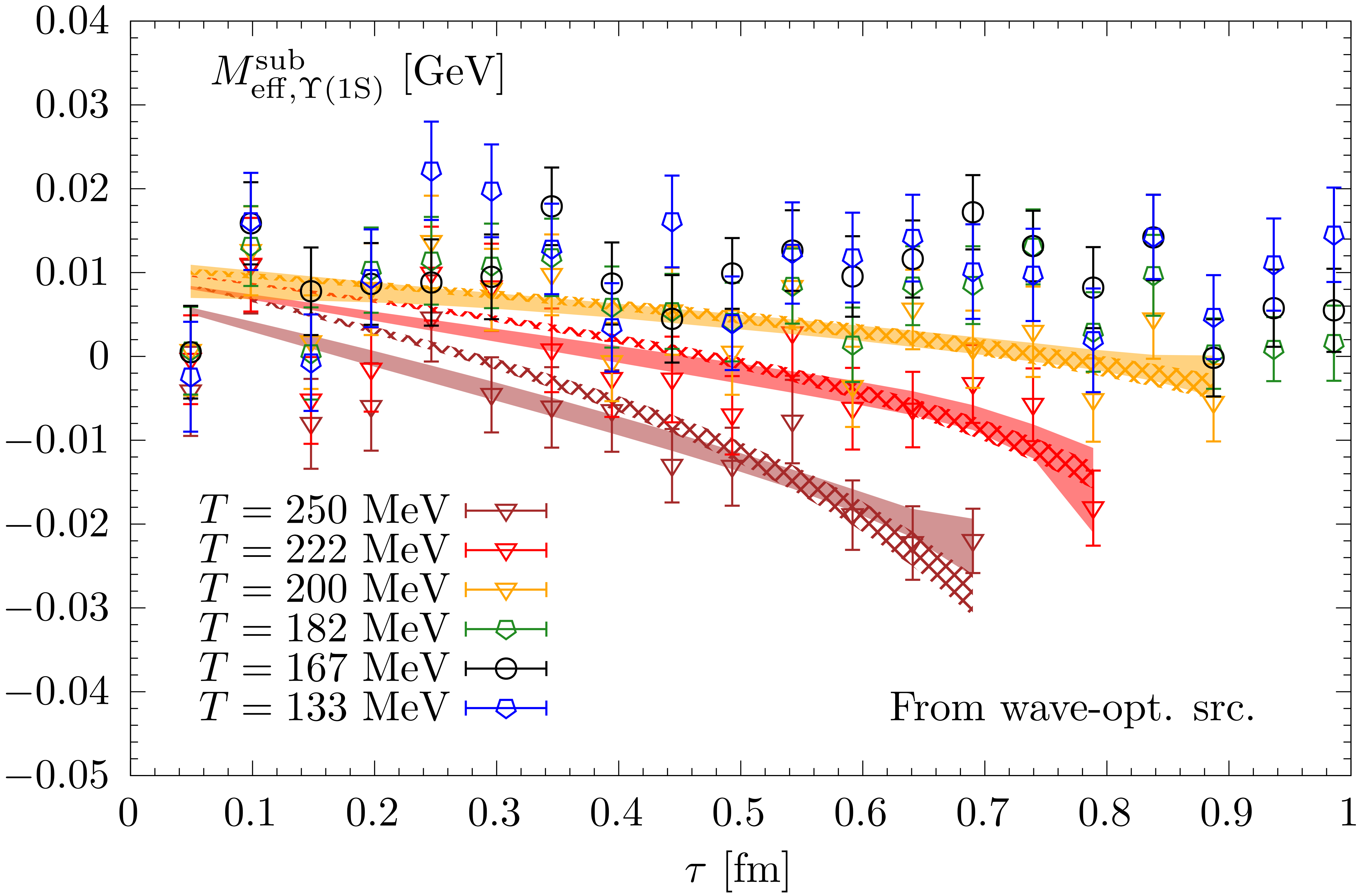
  $\rho_{\text{med}}(\omega) = A\delta(\omega - M)$

  $M_{\text{eff}}^{\text{sub}}(\tau, T)$: τ indep.

- Thermal effects:

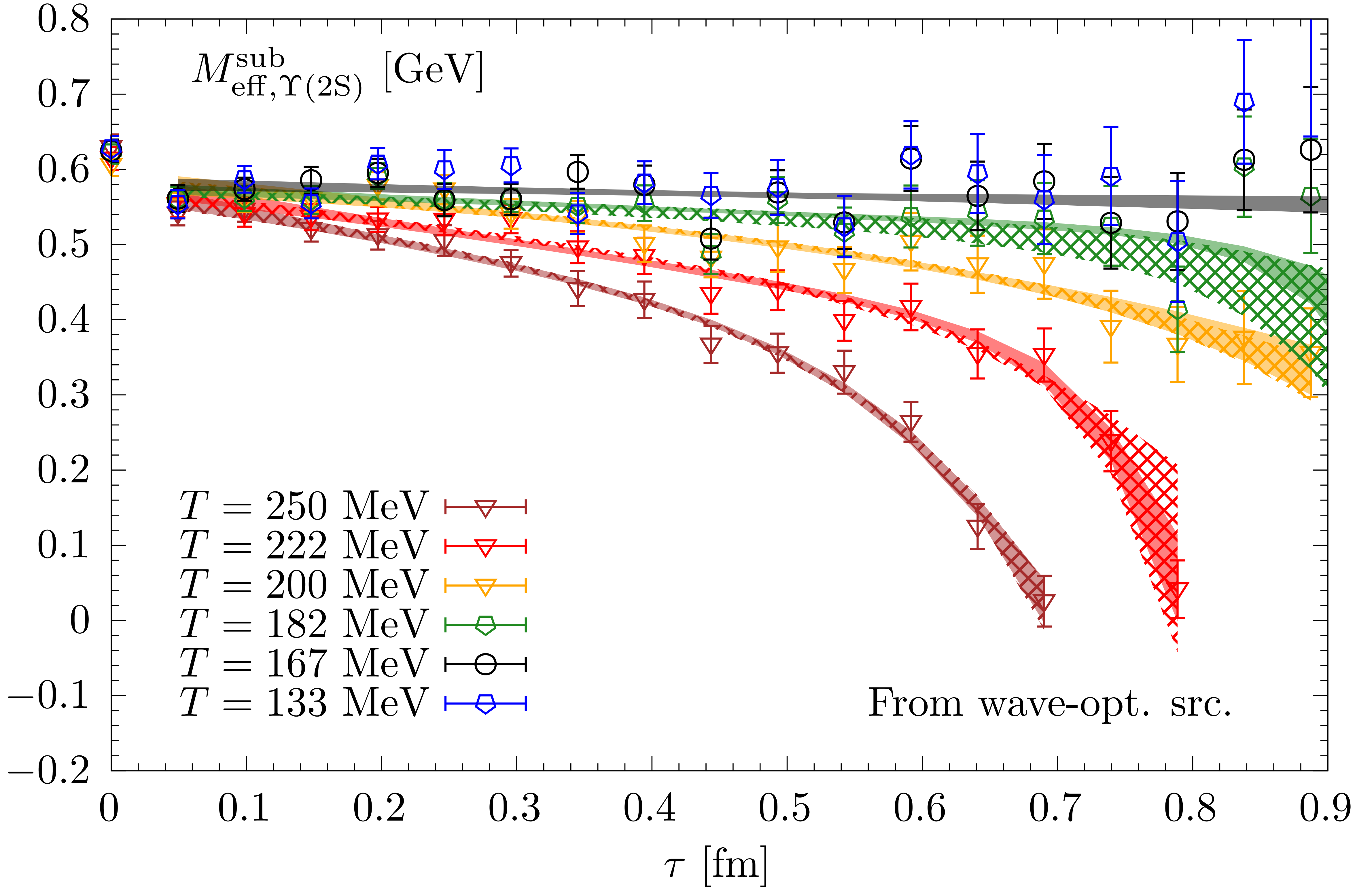
  $$\rho_{\text{med}}(\omega, T) = \rho_{\text{peak}}^{\text{low}}(\omega, T) + A_{\text{low}}(T)\,\delta\left(\omega - \omega_{\text{low}}(T)\right)$$

  $M_{\text{eff}}^{\text{sub}}(\tau, T)$: τ dependent

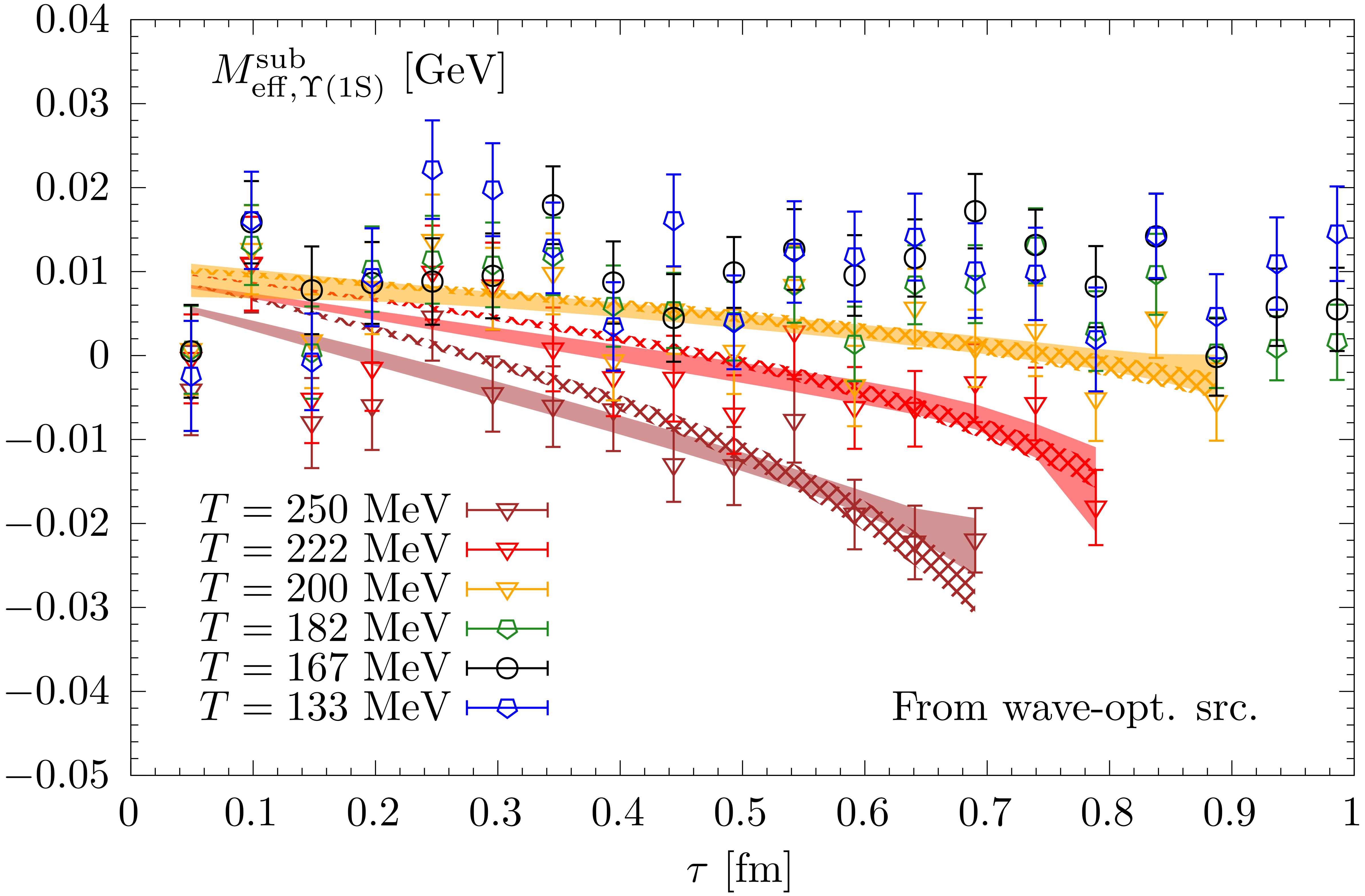


Thermal effects start to enter in 1S state of Υ at $T \gtrsim 200\,\text{MeV}$

Continuum-subtracted effective mass of Υ states



2S state of Υ at $T \gtrsim 182\,\mathrm{MeV}$



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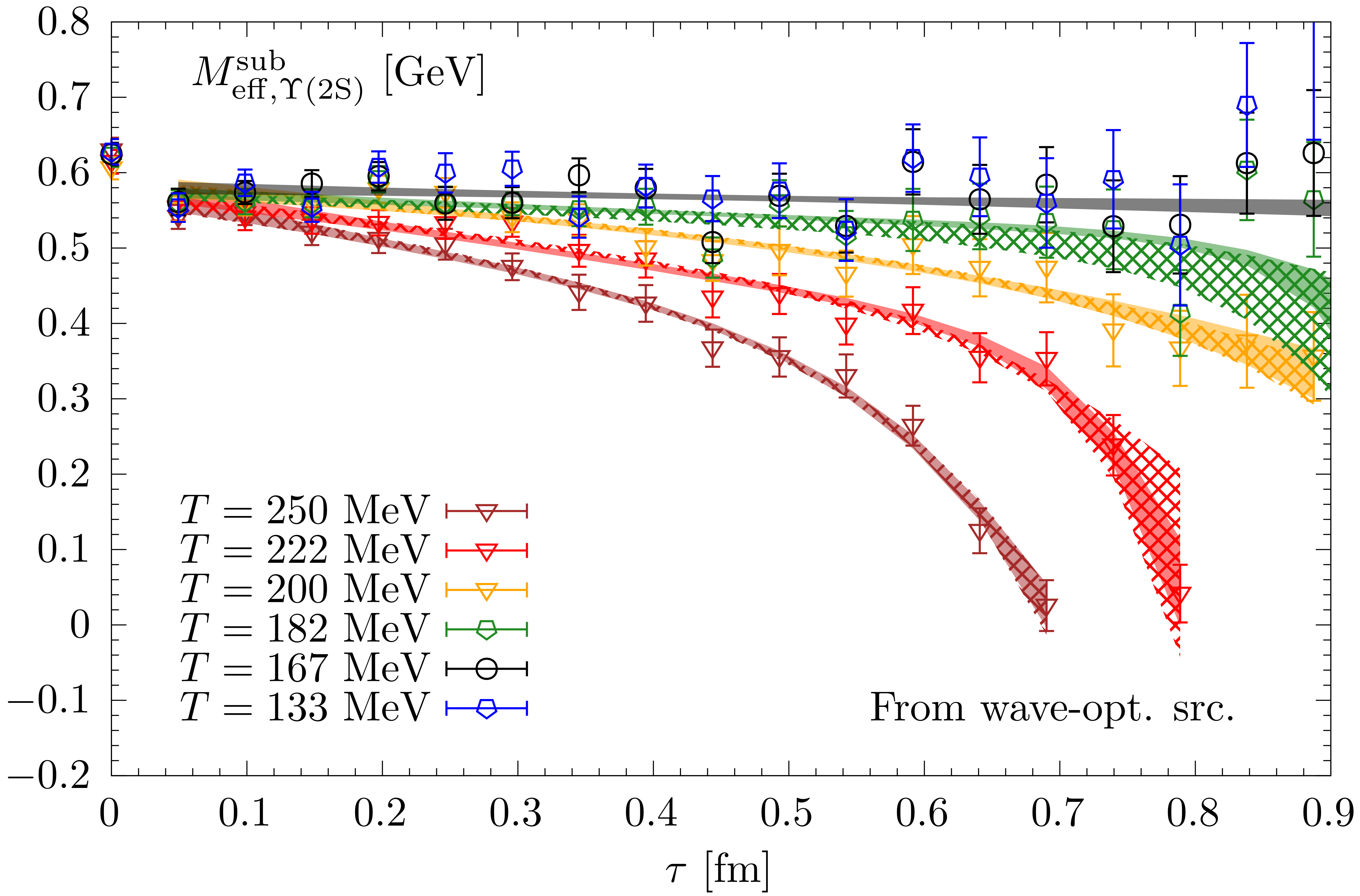
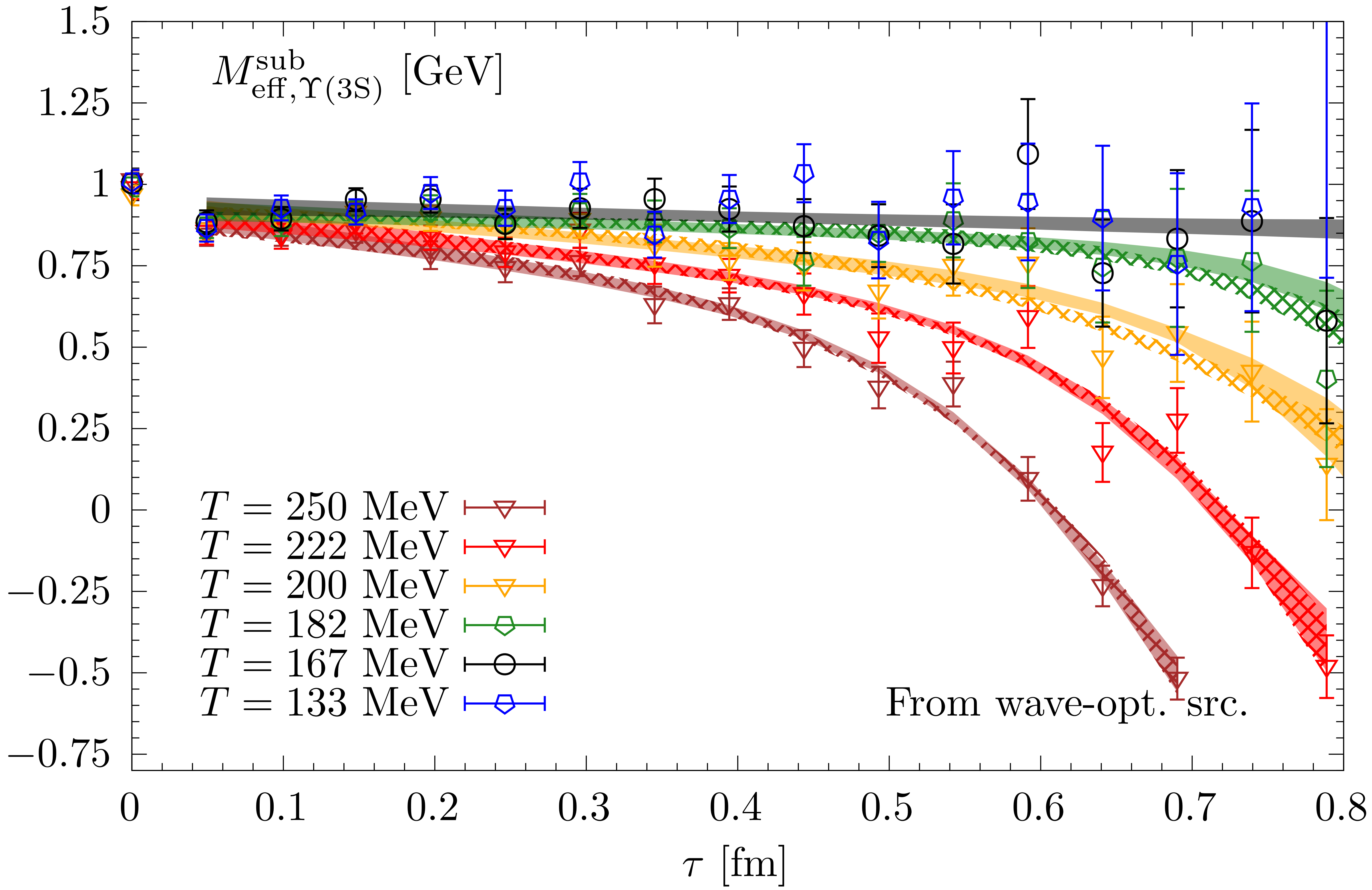
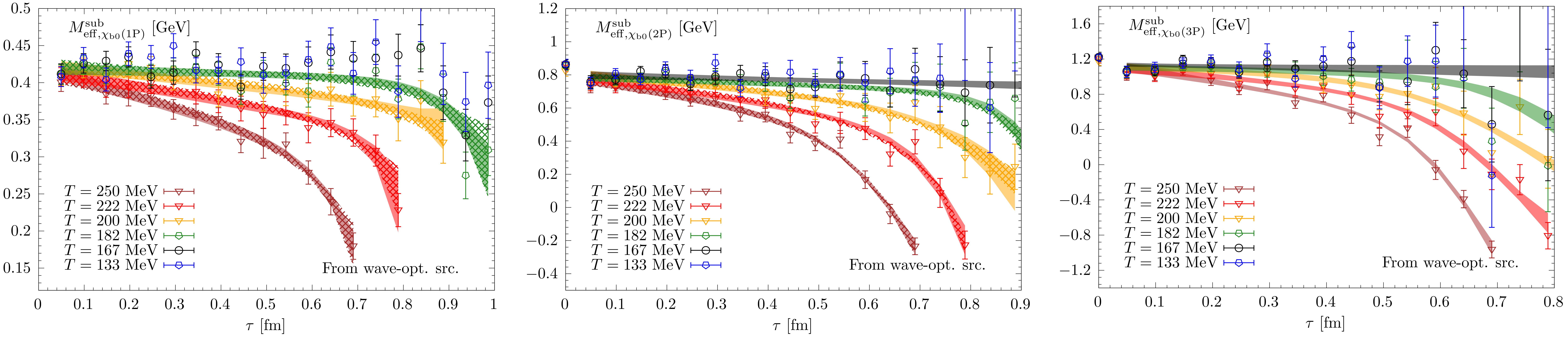
Continuum-subtracted effective mass of Υ states





- Short distance region: close to the case in the vacuum
- Middle distance region: nearly linear decrease, stepper slope at higher T
- Large distance region: sharp drop, obvious for higher T

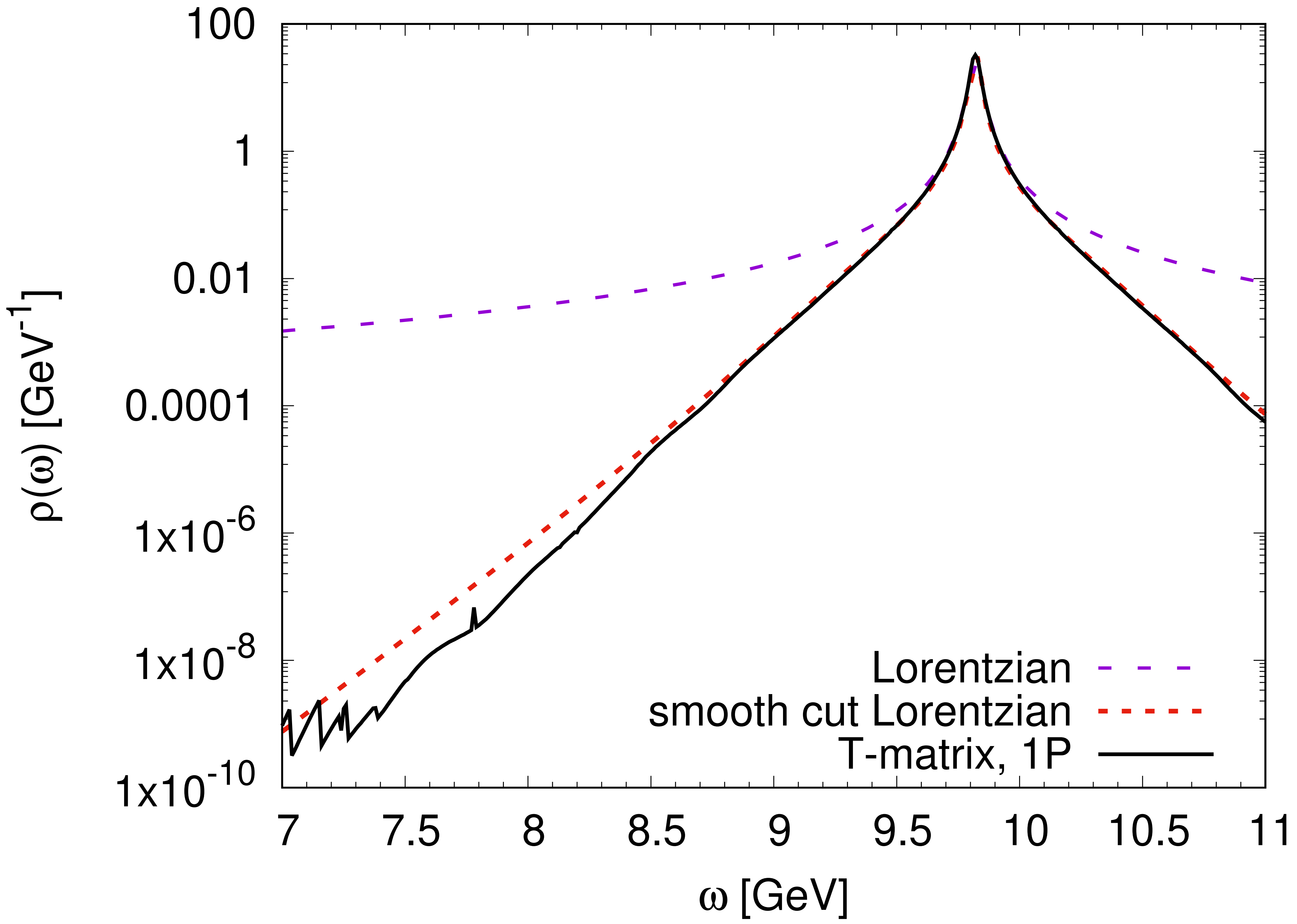
Continuum-subtracted effective mass of $\chi_{b0}(1P,2P,3P)$



- Similar observation as seen in Υ states
- Thermal effect already sets in at $T \gtrsim 182$ MeV for $\chi_{b0}(1P)$ state

Parameterization of spectral functions

$$\rho_{\rm med}(\omega,T) = A_{\rm low}(T)\delta\left(\omega-\omega_{\rm low}(T)\right) + A_{\rm med}(T)\frac{\Gamma(\omega,T)}{\left(\omega-M_{\rm med}(T)\right)^2+\Gamma^2(\omega,T)}$$



- Cut-Lorentzian

Bazavov, et al., PRD 109 (2024) 074504

$$\Gamma(\omega,T) = \Gamma_0\,\Theta(\mathrm{cut} - |\,\omega - \mathrm{M_{med}}\,|)$$

- Smooth-cut-Lorentzian

HTD, et al., JHEP 05 (2025) 149

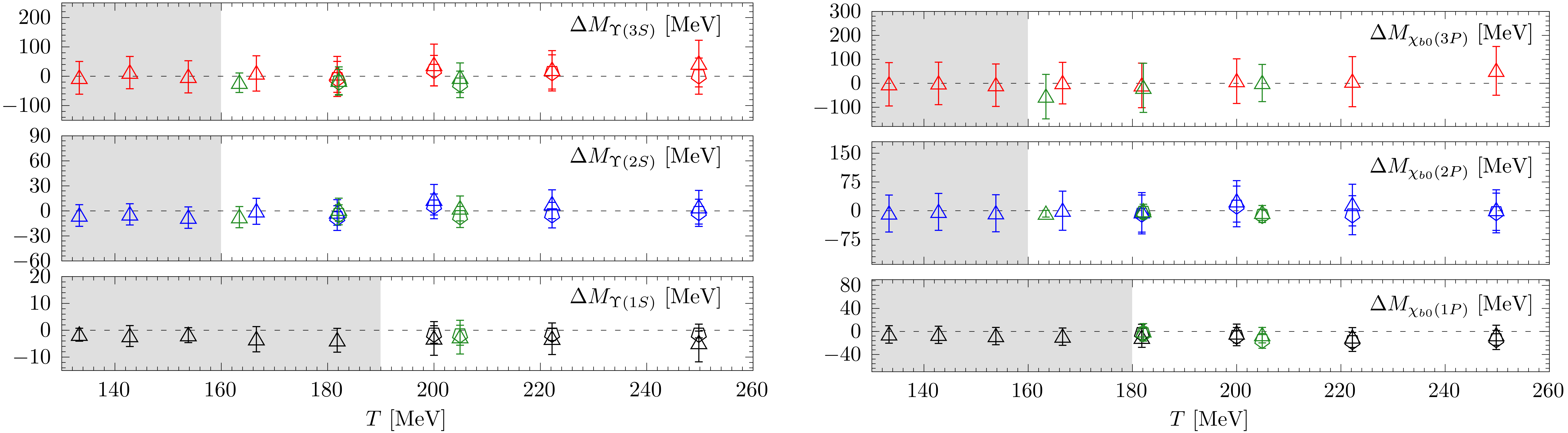
$$\Gamma(\omega) = \Gamma_0\,\Theta_\delta(\mathrm{cut}-(\omega-\mathrm{M}))\,\Theta_\delta(\mathrm{cut}+(\omega-\mathrm{M}))$$

$$\Theta_\delta(x) = \frac{1}{2}\left[1+\mathrm{sgn}(x)\tanh\left(\frac{x}{\delta}\right)\right]$$

Smooth-cut-Lorentian: Better description of the peak tail motivated from the T-matrix spf

Liu & Rapp, PRC 97 (2018) 034918, Tang et al,, Phys.Rev.D 112 (2025) 3, 034030

In-medium mass shift



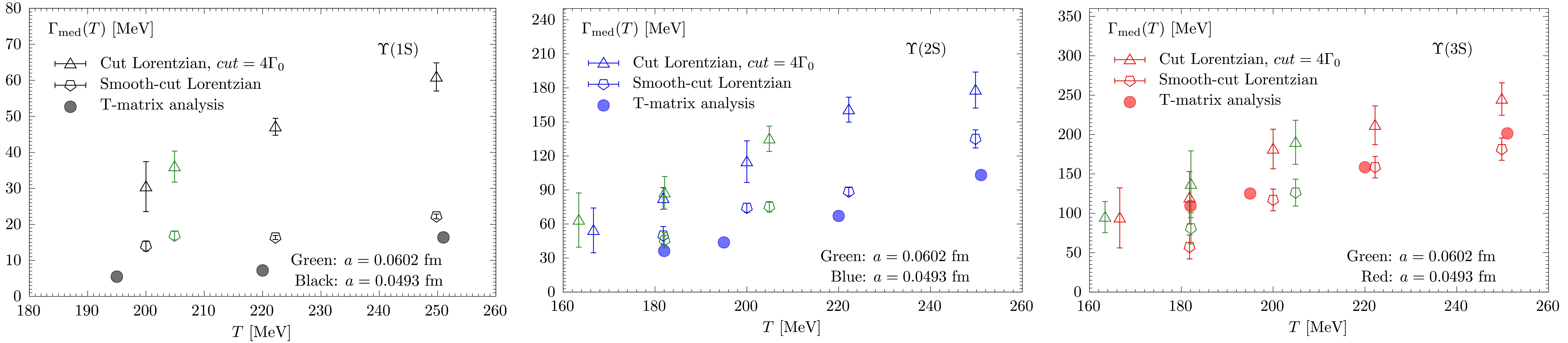
Triangle: cut-Lorentzian; Pentagon: Smooth-cut Lorentzian

Green points:a=0.0602fm, others: a=0.0493fm

- ΔM: consistent with zero: almost no change in the in-medium masses
- Unscreened real part of HQ potential seems to be supported up to 250 MeV

HTD, W.-P. Huang, R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky and Z. Tang, JHEP 05 (2025) 149

In-medium thermal width of Υ states



- Modeling tail of main peak matters
- Significant increase with rising temperatures
- Sequential hierarchy appears in the magnitudes of the thermal widths

HTD, W.-P. Huang, R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky and Z. Tang, JHEP 05 (2025) 149

In-medium thermal width of χ_{b0} states

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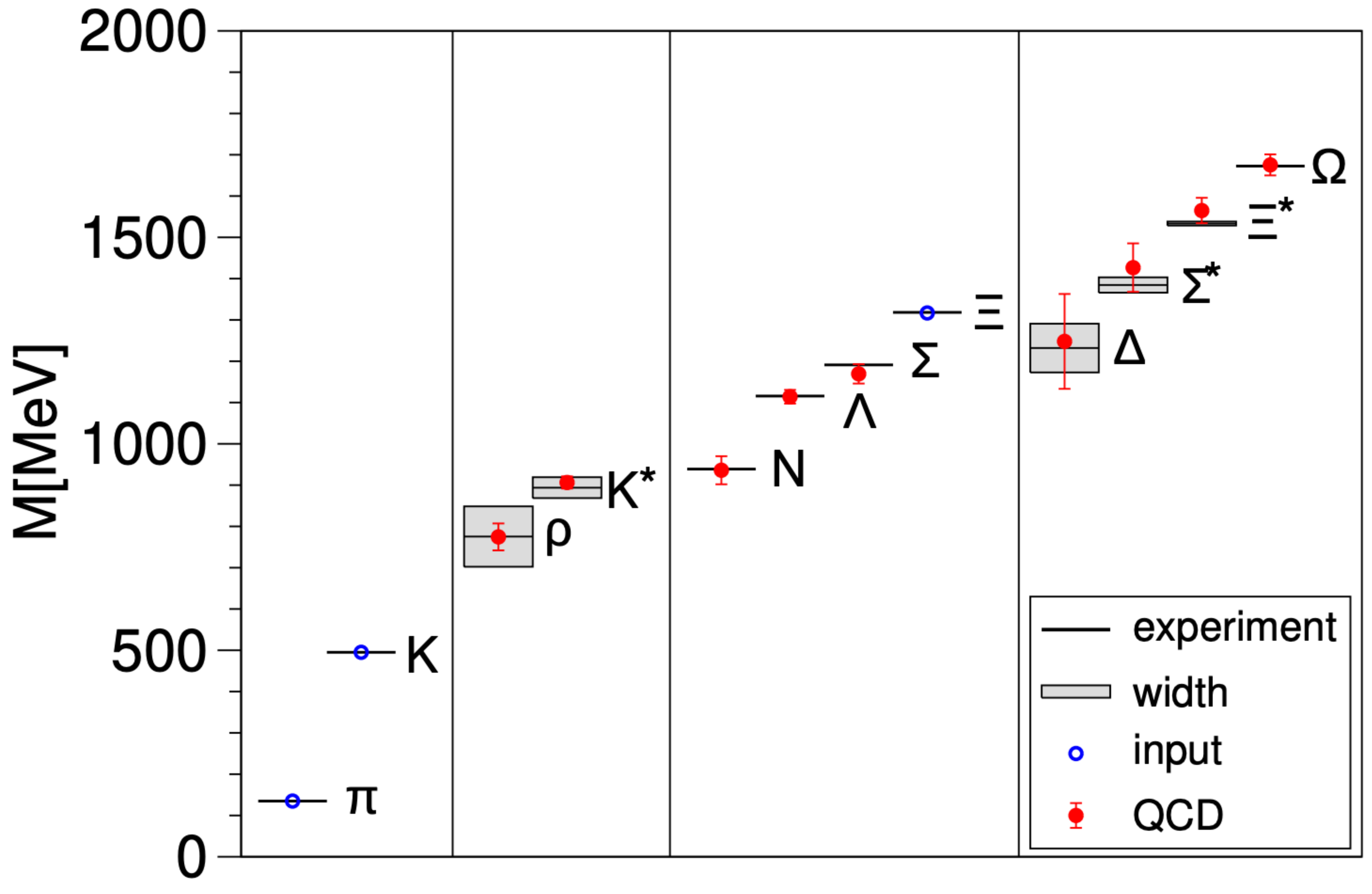
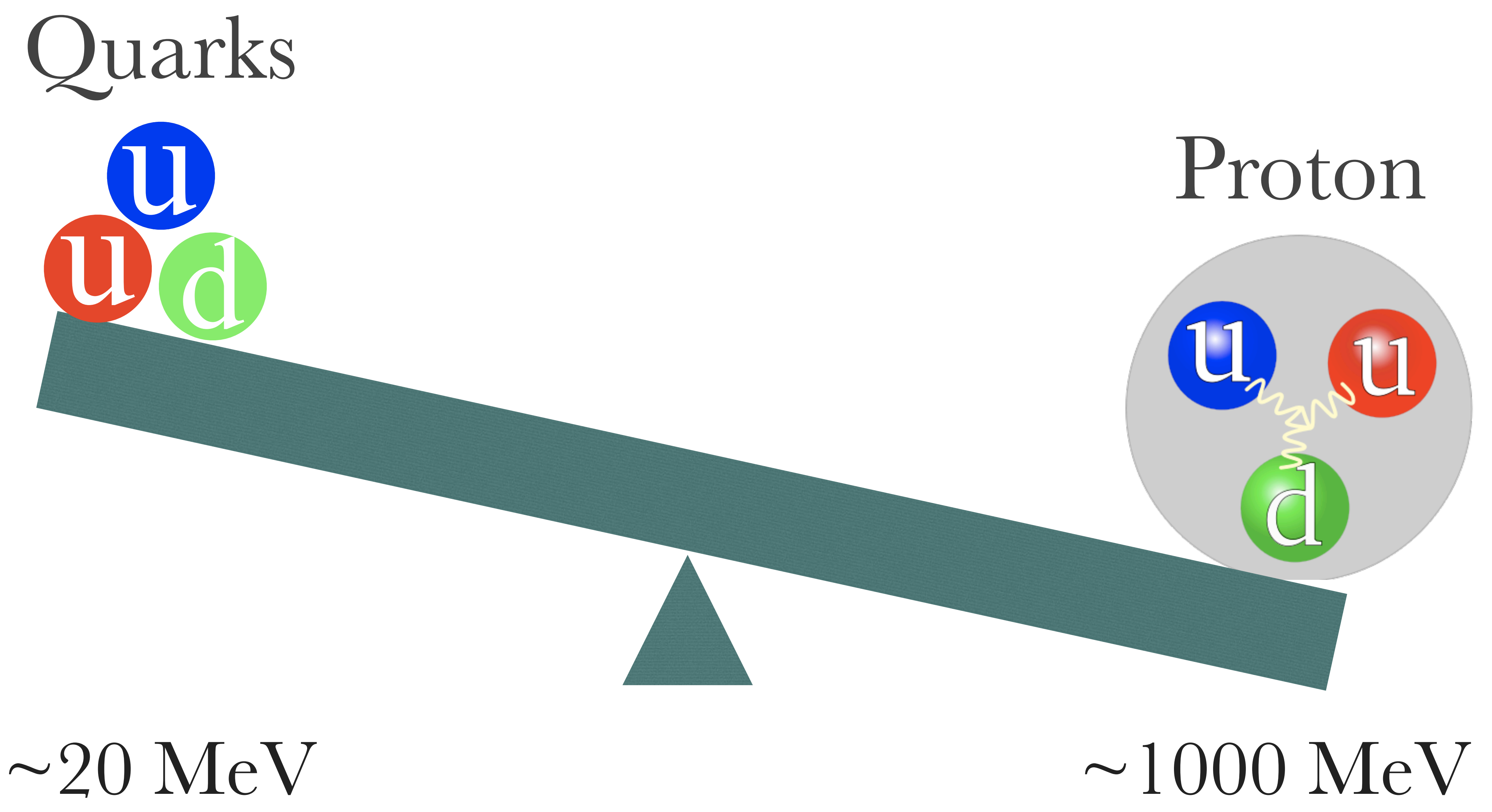
What generates the hadron mass?



Dürr et al., Science 322:1224-1227,2008



Most of the proton mass comes from strong force

Is J/ψ simply two charm quarks ?

$$M_{J/\psi} = 3.097 \text{ GeV}$$

Current quark mass: $m_c^{\overline{MS}}(m_c) \approx 1.27 \text{ GeV}$

Pole/Constituent-like mass:

$$m_c^{\text{pole}} = m_c^{\overline{\text{MS}}}(m_c)\left(1 + \frac{4}{3}\frac{\alpha_s}{\pi} + \cdots\right) \approx 1.5 \text{ GeV}$$

Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$\langle H(p)\,|\,T^{\mu\nu}\,|\,H(p)\rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}\right)}_{\text{Traceless}} + \underbrace{\frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}}_{\text{Trace}}$$

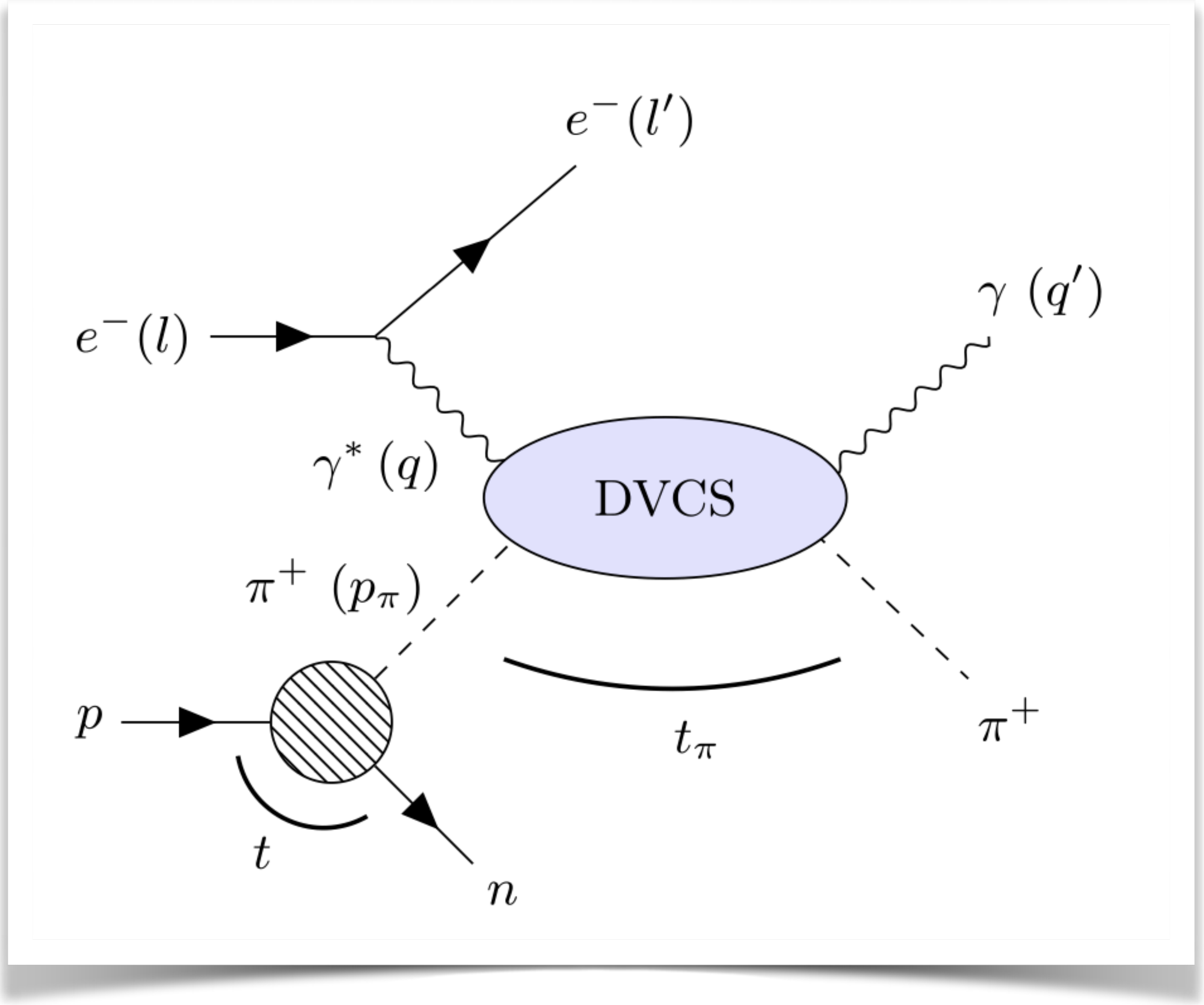
For QCD: $T^{\mu\nu}_{\mathrm{QCD}} = T^{\mu\nu}_q + T^{\mu\nu}_g = M_E + M_m + M_g + M_a$

Quark part: M_E quark kinetic energy, M_m explicit quark mass

Gluon part: M_g gluon energy, M_a anomaly

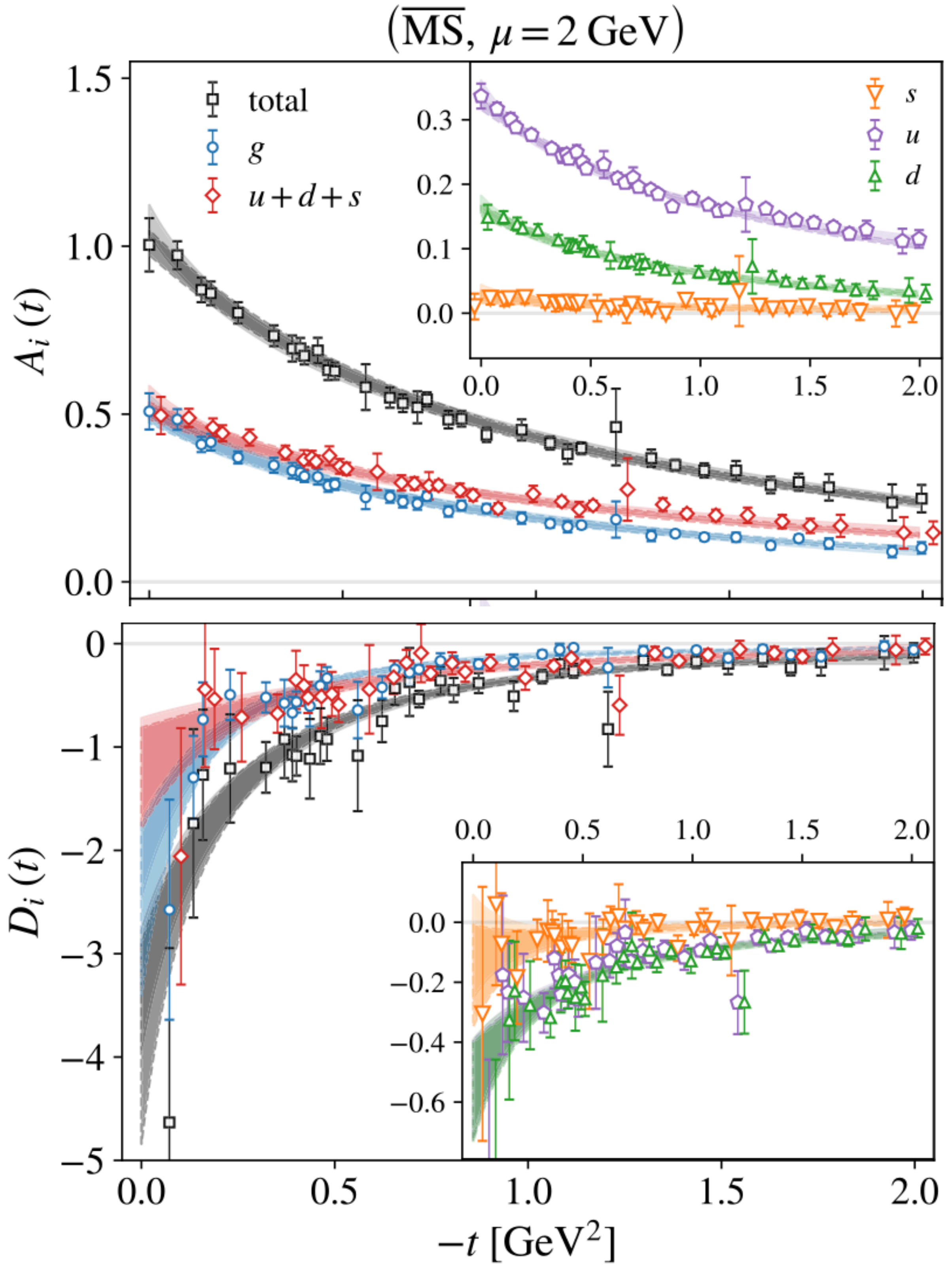
Current status: partial EMT access from experiment and lattice QCD

Deeply virtual exclusive processes
⇒ partial GFF / traceless EMT access



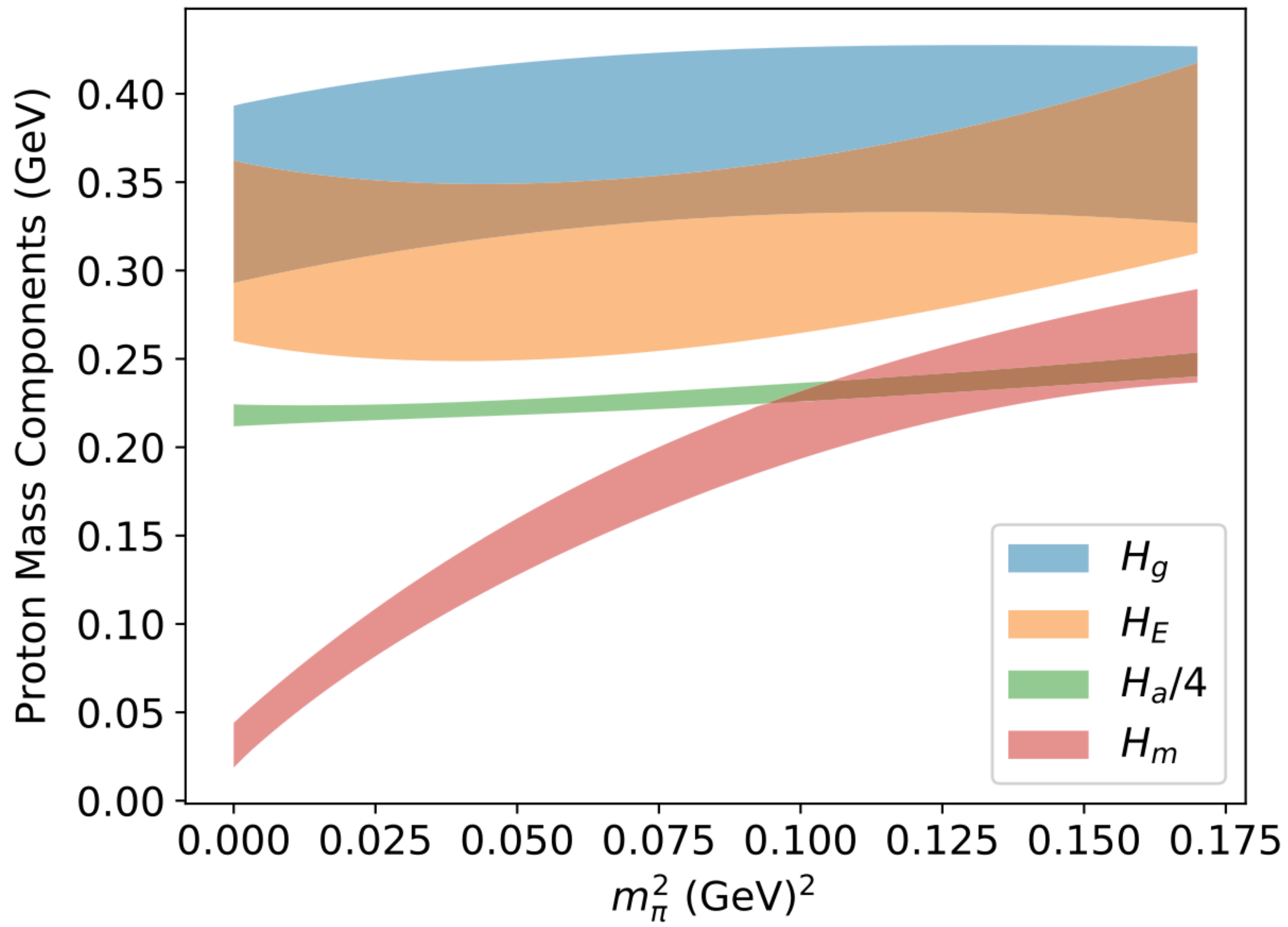
See EXP extracted pressure distribution in e.g.
Burkert, Elouadrhiri & Girod, Nature 557 (2018)396

Recent lattice QCD:
GFFs and spatial structure accessible
but the trace sector remained difficult



Hackett et al., PRL. 132 (2024) 251904

Earlier lattice QCD
proton mass decompositions:
relied on partial EMT input or
indirect closure



Yang, et al.,[χQCD], PRL 121 (2018) 212001

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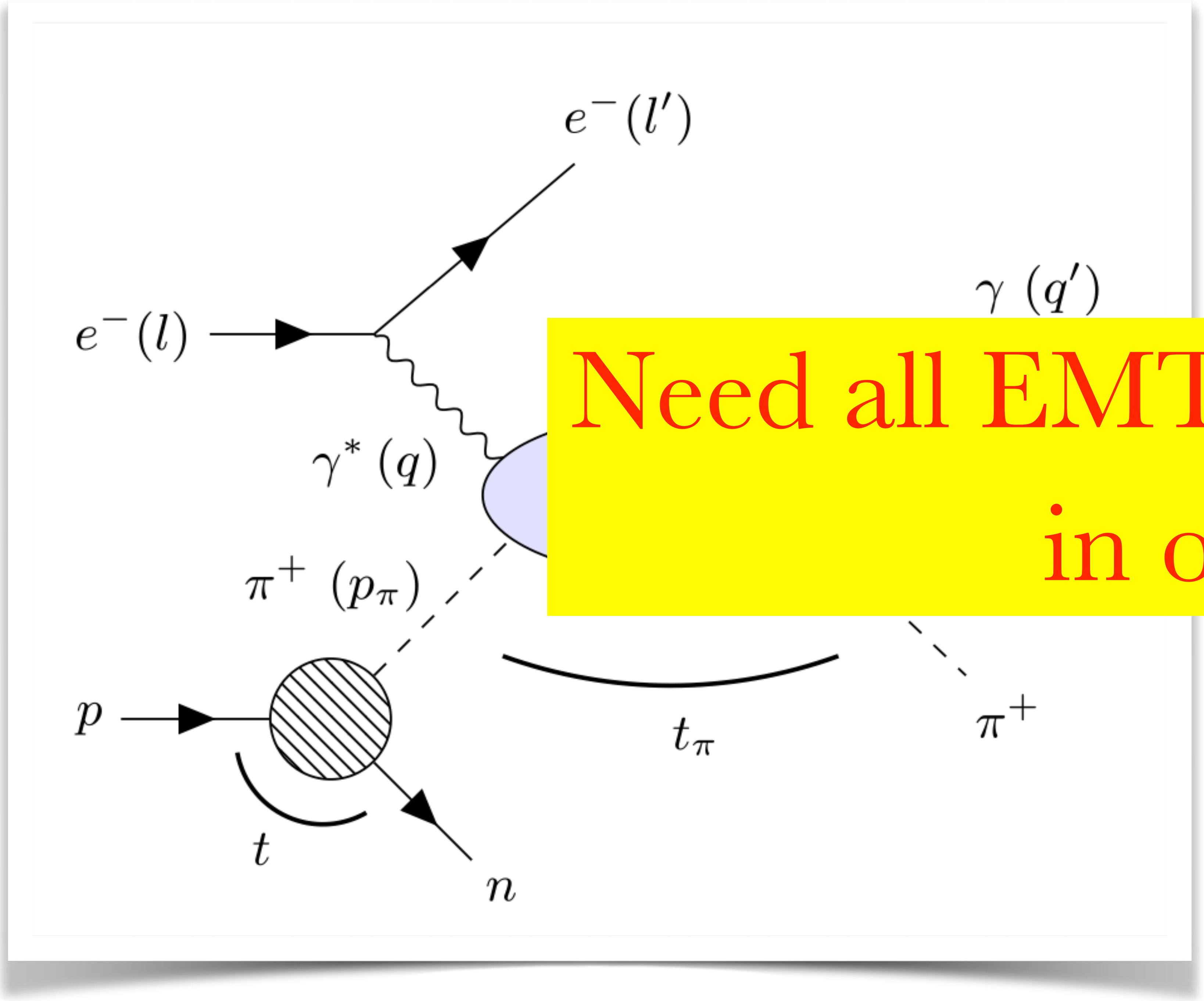

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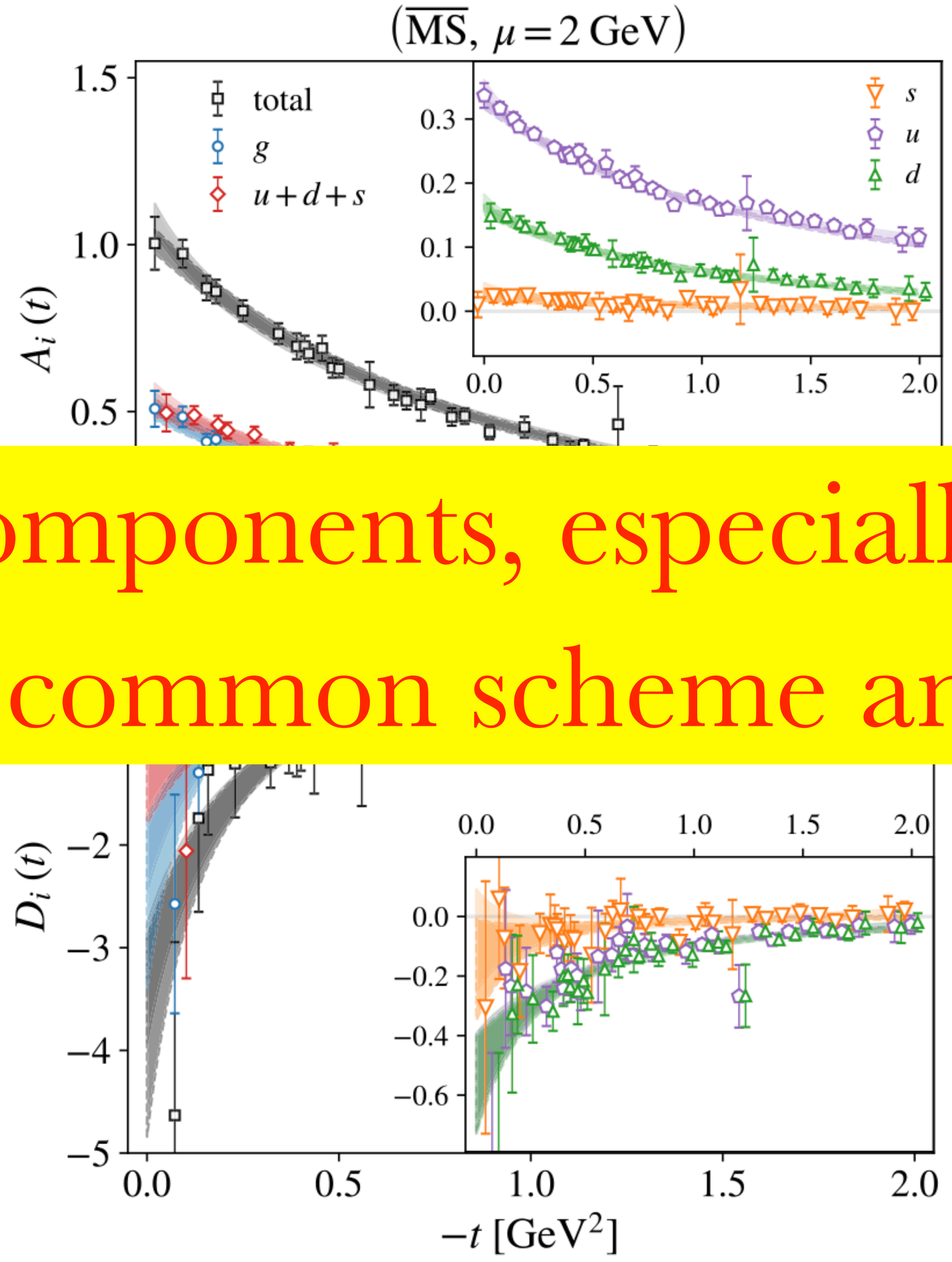

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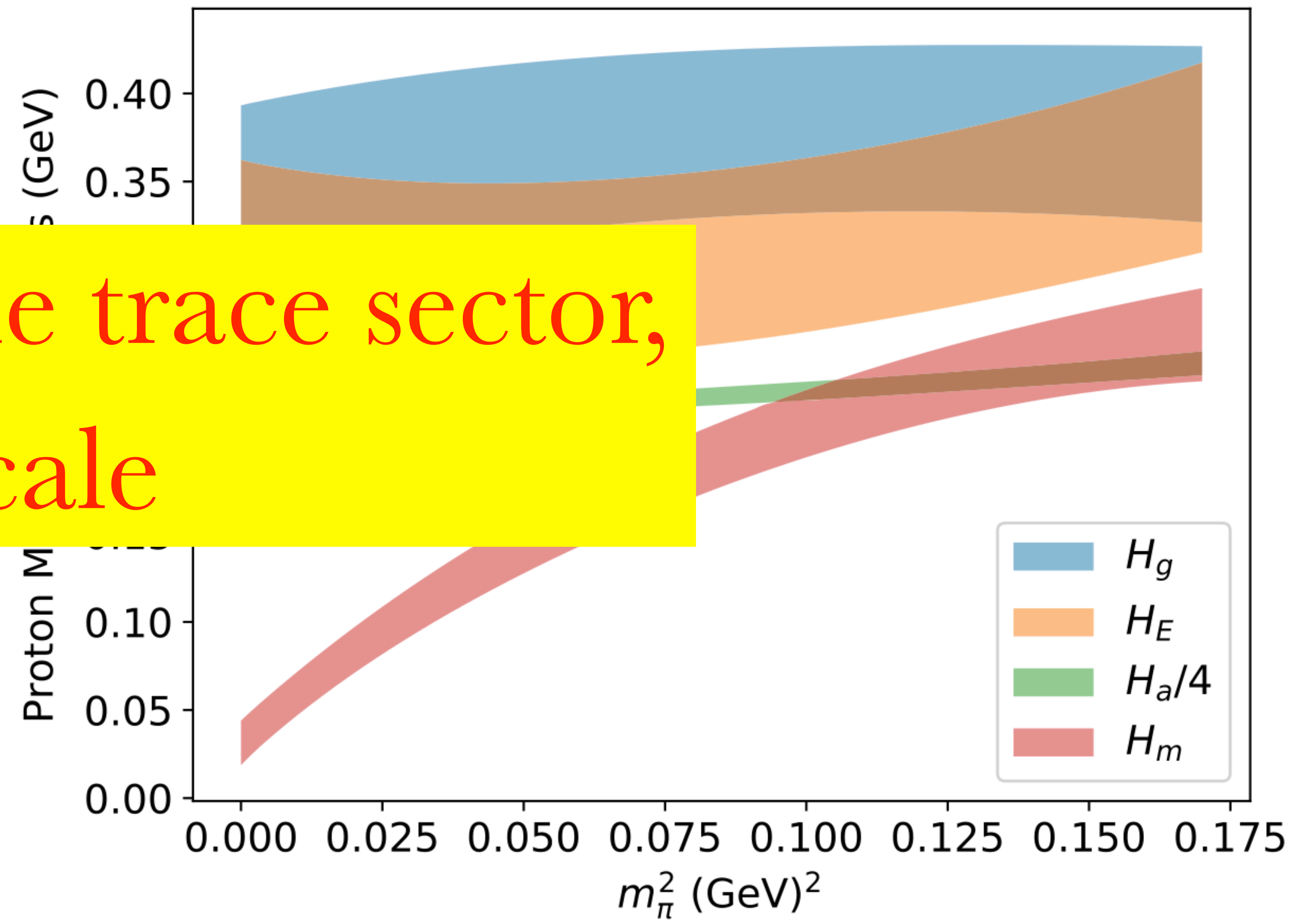


Yang, et al.,[χQCD], PRL 121 (2018) 212001

Need all EMT components, especially the trace sector, in one common scheme and scale

What was difficult before

$$T_{\rho\sigma}(x) = \mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4}\mathcal{O}_{2,\rho\sigma}(x) + \frac{1}{4}\mathcal{O}_{3,\rho\sigma}(x) - \frac{1}{2}\mathcal{O}_{4,\rho\sigma}(x) - \mathcal{O}_{5,\rho\sigma}(x)$$

Gluon part: $T_{g,\rho\sigma}$

Quark part: $T_{q,\rho\sigma}$

$$\mathcal{O}_{1,\rho\sigma} = \frac{2}{g_0^2}\mathrm{Tr}^c\left[F_{\rho\omega}F_{\sigma\omega}\right], \quad \mathcal{O}_{2,\rho\sigma} = \frac{2}{g_0^2}\delta_{\rho\sigma}\mathrm{Tr}^c\left[F_{\omega\xi}F_{\omega\xi}\right]$$

$$\mathcal{O}_{3,\rho\sigma} = \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f\left(\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho\right)\psi_f$$

$$\mathcal{O}_{4,\rho\sigma} = \delta_{\rho\sigma}\sum_f \bar{\psi}_f \overleftrightarrow{\not{D}}\psi_f, \quad \mathcal{O}_{5,\rho\sigma} = \delta_{\rho\sigma}\sum_f m_f\bar{\psi}_f\psi_f$$

Tensor operators: $\mathcal{O}_1, \mathcal{O}_3$ Pure-trace operators: $\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5$

- EMT trace lies in the scalar channel under lattice hypercubic symmetry $\Rightarrow$ mixing with lower-dimensional operators
- power-divergent subtraction is required
- conventional RI/MOM-type renormalization: gauge dependent/IR sensitive/ noisy

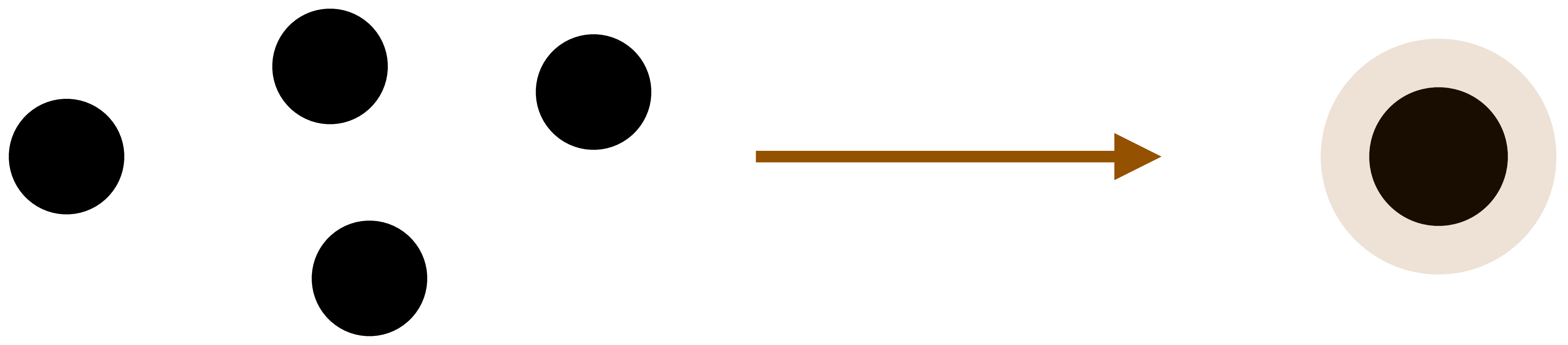
Belinfante, Phys. Rev. 128 (1962) 2832; Belitsky & Radyushkin, Phys. Rept. 418 (2005) 1; Freese, Phys. Rev. D 106 (2022) 125012,…

Our strategy: from gradient flow to a common $\overline{\mathrm{MS}}$ EMT scheme

Gradient Flow (GF): Lüscher et al., 10', 11', 14'

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(0,x) = A_\mu(x)$$

Flow time t_f smooth UV fluctuations over a radius of $\sqrt{8t_f}$



rough field — flowed/smeared field

Small-flow-time EMT reconstruction

$$T^{\overline{\mathrm{MS}}}_{\rho\sigma}(x,\mu) = \lim_{t_f\to 0} \Sigma_i c_i(t_f,\mu)\bar{\mathcal{O}}_{i,\rho\sigma}(t_f,x)$$

flowed operators are matched to the physical EMT; the coefficients c_i are known perturbatively

- controls mixing and avoids power-divergent subtractions
- puts traceless and trace sectors into one common framework

Operationally in this work

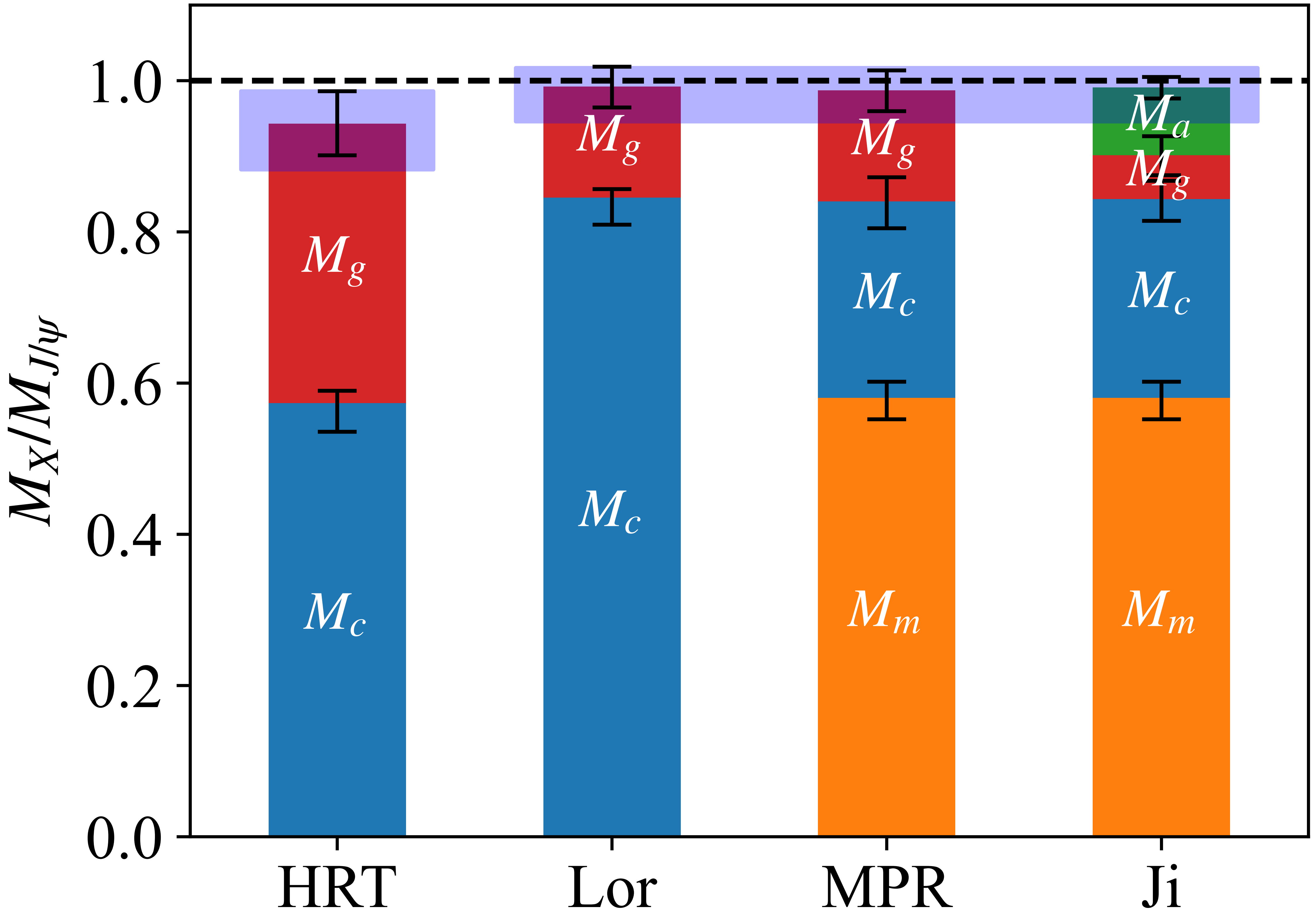
Flowed EMT at fixed t_f ⇨ Take $a \to 0$ at each fixed t_f ⇨ 2-loop matching GF → MSbar ⇨ Zero-Flow-time extrapolation $t_f \to 0$ ⇨ **All EMT ingredients in one common scheme**

common scheme-and-scale determination of traceless and trace contributions

Lattice QCD closes the J/ψ & η_c mass budget across independent sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split

J/ψ mass budget



Gluon energy is sizable

About 15% in Lorce/MPR,
About 30% in HRT

The Ji anomaly term is visible

277 MeV in J/ψ , close to 9%

Forward-limit GFF tests close

$$A_c + A_g = 1, \bar{C}_c + \bar{C}_g = 0$$

First direct results for $\bar{C}$

M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

Four Sum rules: Ji, PRL 74 (1995) 1071; Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , to appear in PRL

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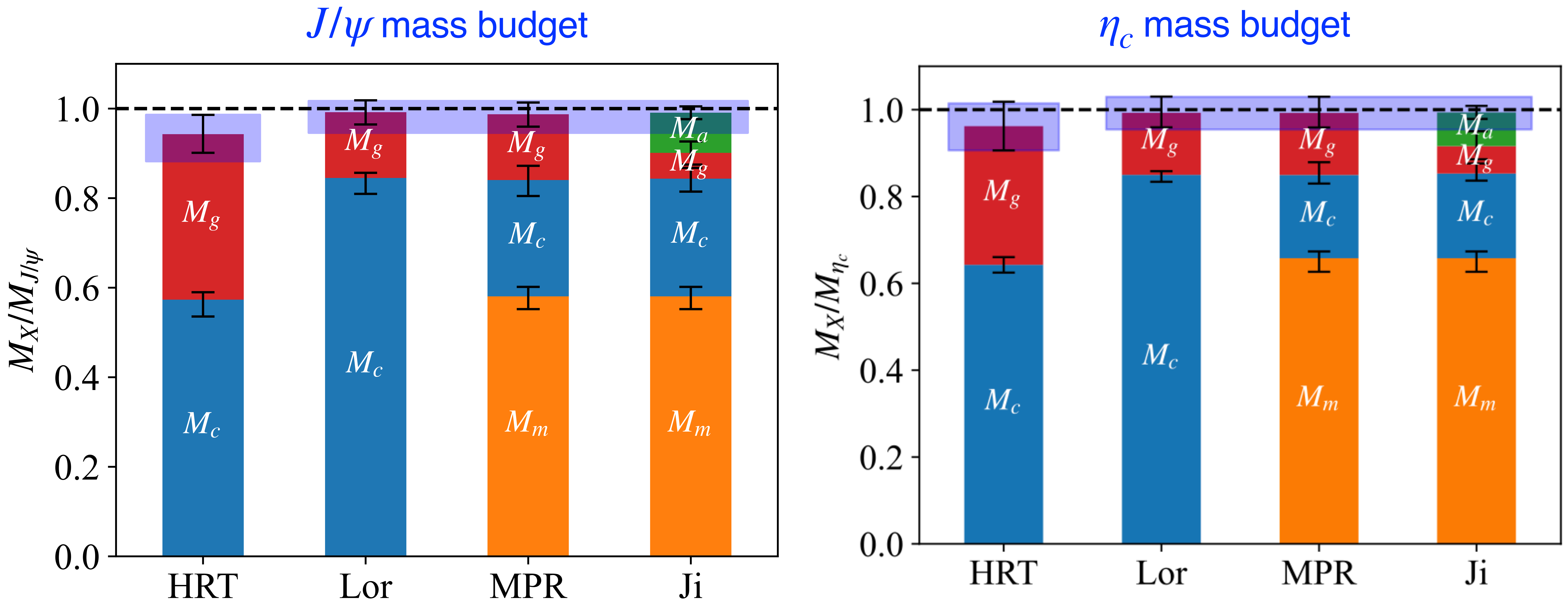
Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split



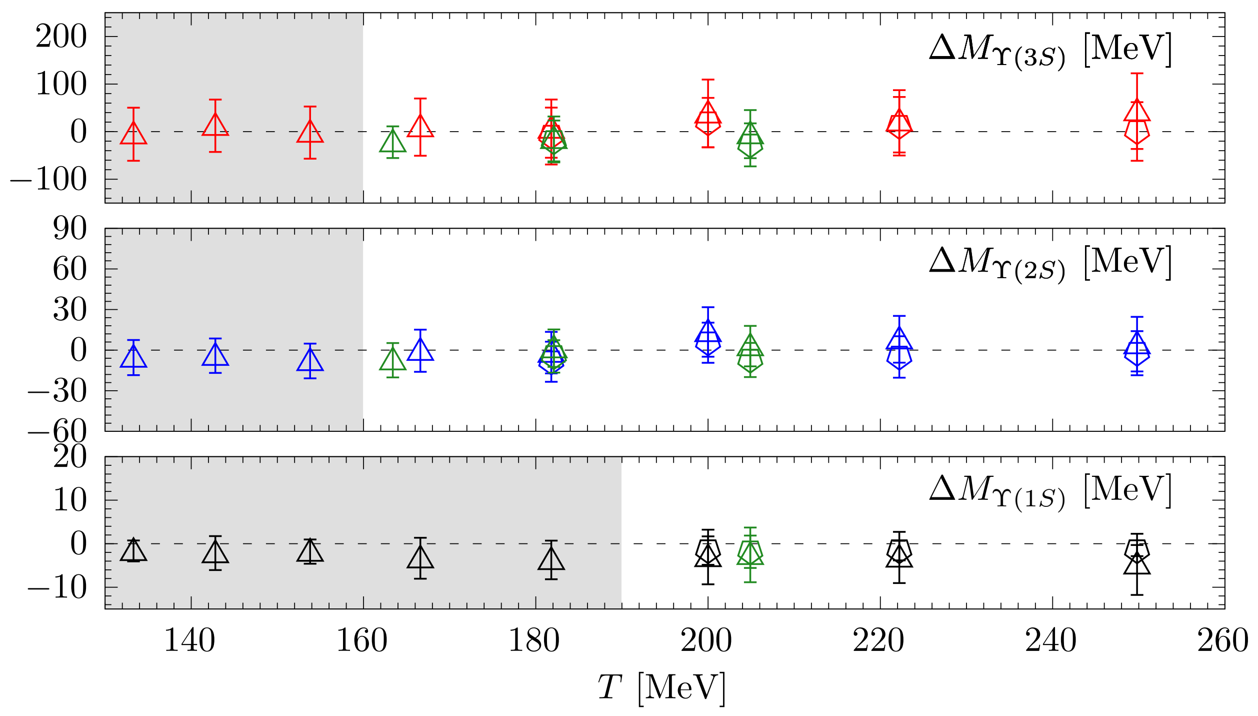
M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

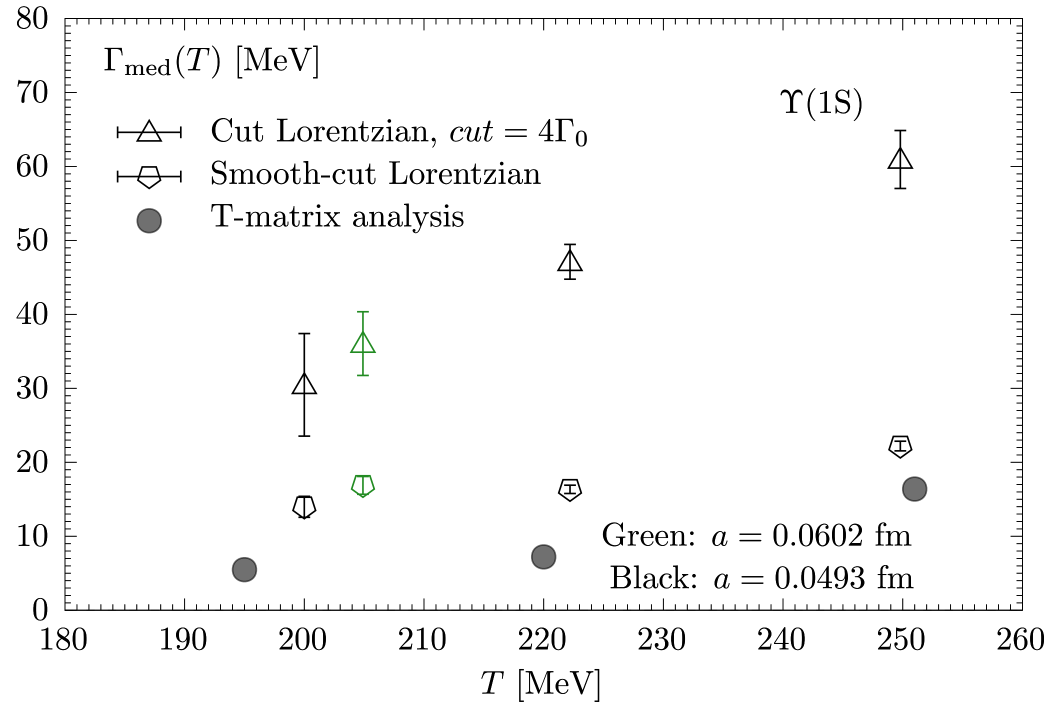
Four Sum rules: Ji, PRL 74 (1995) 1071; Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

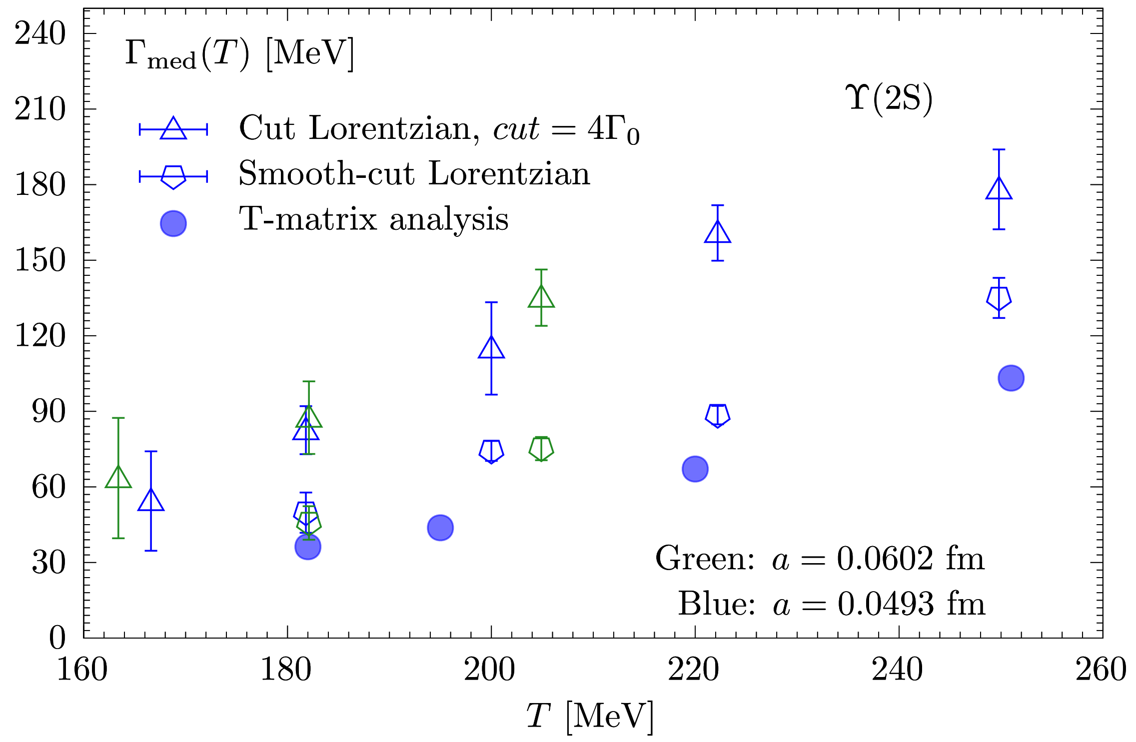
D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , to appear in PRL

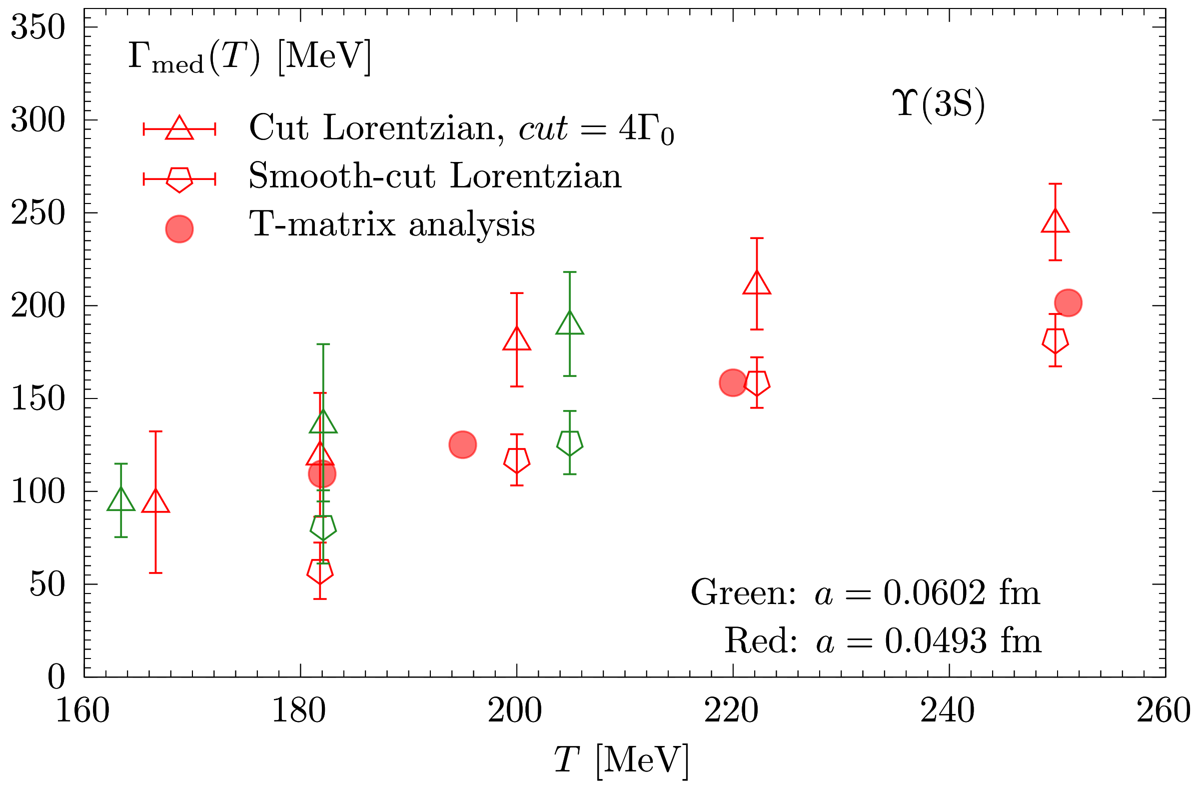
Summary

☑ Sequential thermal broadening & No significant changes in in-medium masses of $\Upsilon(1S,2S,3S)$ and $\chi_{b0}(1P,2P,3P)$

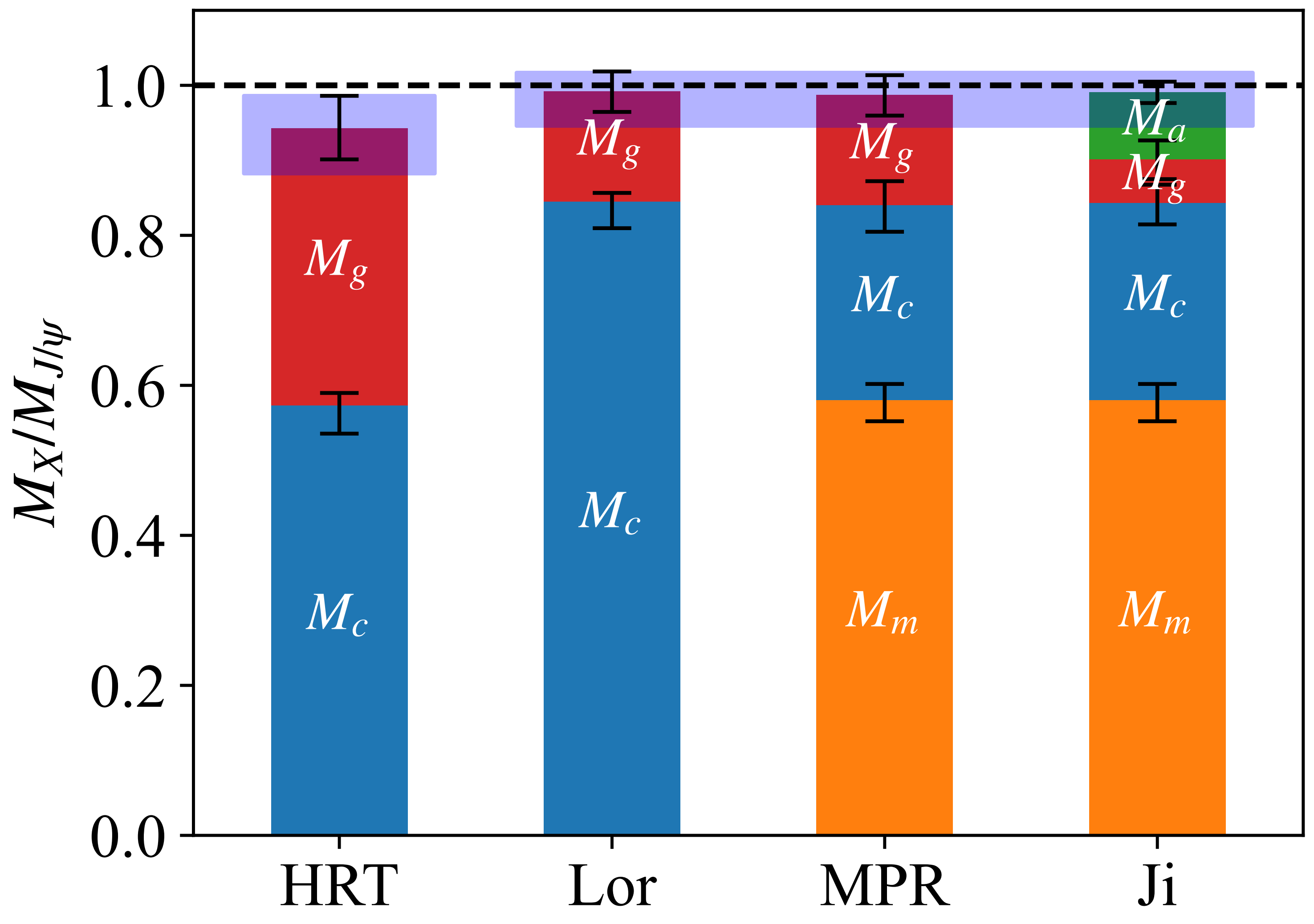


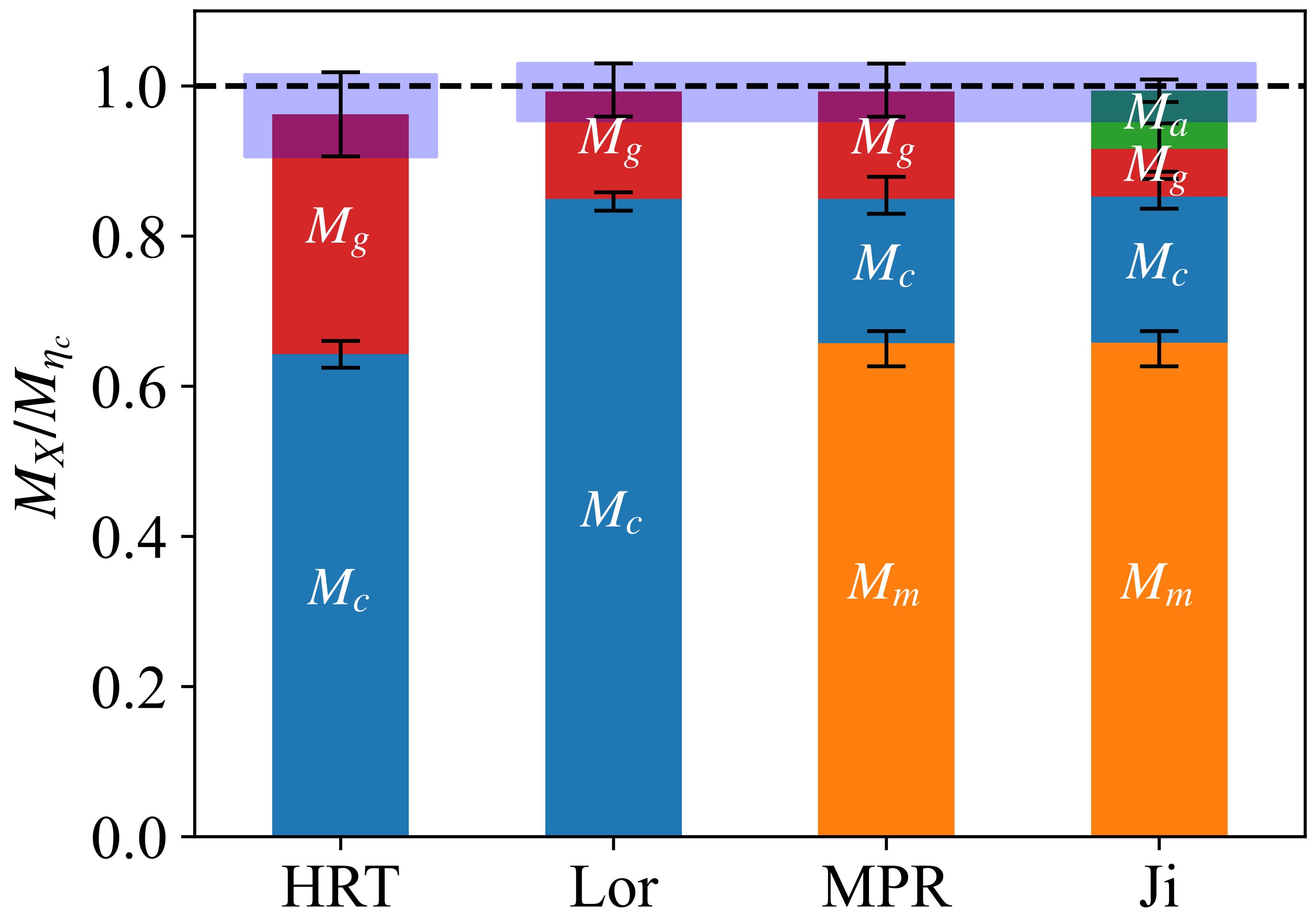






☑ A methodology to weigh every piece of hadrons: Sizable gluonic contribution in Jpsi & etac





Happy Birthday, Prof. Zhuang!

Wish you health, joy and many more inspiring moments!

Backup

Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$T^{\mu\nu}_{\mathrm{QCD}} = T^{\mu\nu}_q + T^{\mu\nu}_g$$

$$\langle H(p)\,|\,T^{\mu\nu}\,|\,H(p)\rangle = 2p^\mu p^\nu$$

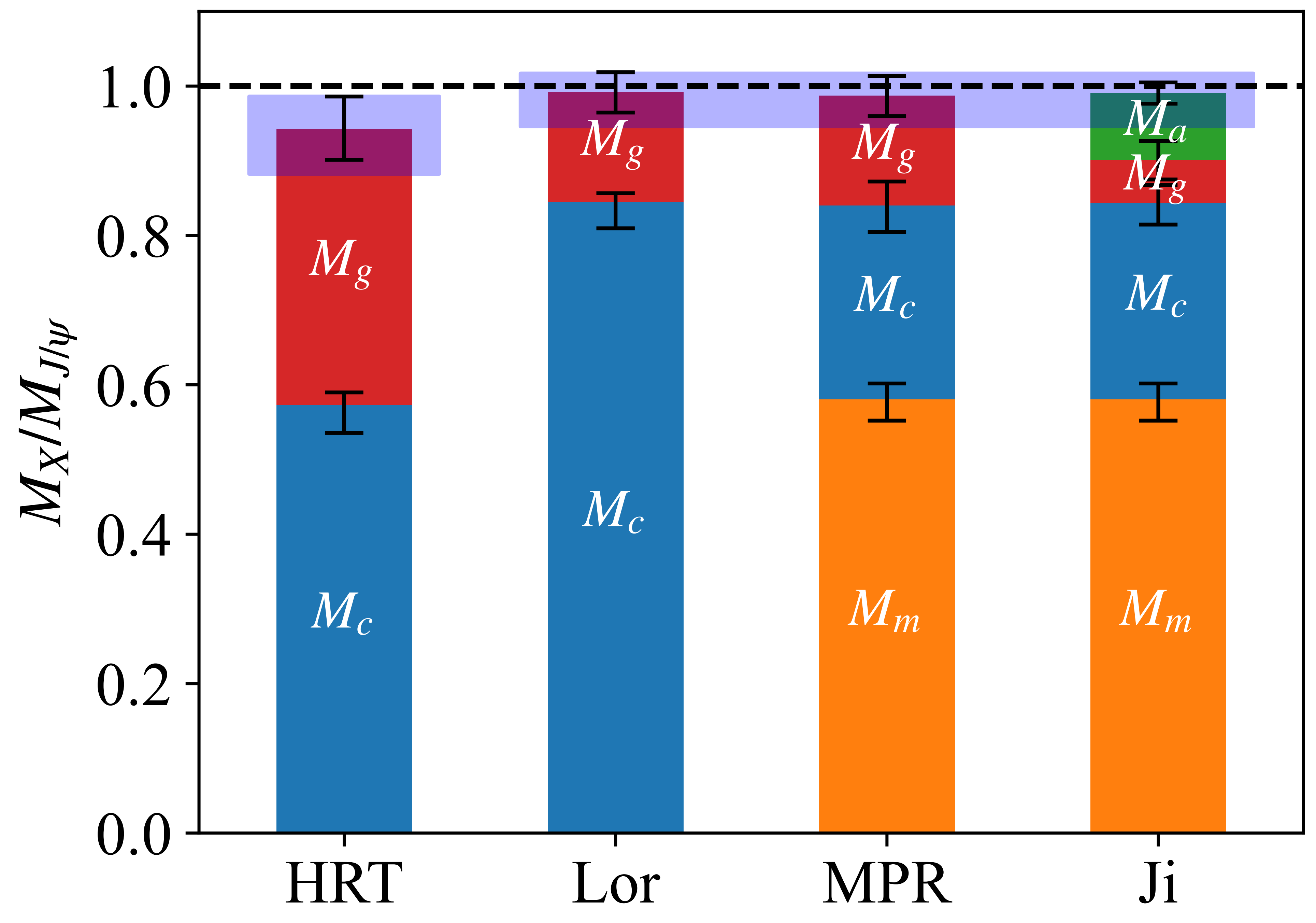
$$T^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}\right) + \frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}$$

same EMT information$\longrightarrow$ different organizations $\longrightarrow$ different hadron-mass sum rules

Ji | HRT | Lorcé | MPR

Ji, PRL 74 (1995) 1071 ;Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

Four sum rules



$$M_H = \sum_{X=q,g} M_X^{\rm HRT}, \ \text{with} \ M_X^{\rm HRT} = \frac{\langle H| \left(T_X^{\overline{\rm MS}}\right)^{\rho}_{\ \rho} |H\rangle}{2M_H}.$$

$$M_H = \sum_{X=q,g} M_X^{\rm Lor}, \quad \text{with} \quad M_X^{\rm Lor} = \frac{\langle H| T_{X,00}^{\overline{\rm MS}} |H\rangle}{2M_H}.$$

$$M_H = \sum_{X=q,g,m} M_X^{\rm MPR}, \quad \text{with} \quad M_m^{\rm MPR} = \sum_f \sigma_f,$$

$$M_q^{\rm MPR} = M_q^{\rm Lor} - \sum_f \sigma_f, \quad M_g^{\rm MPR} = M_g^{\rm Lor}.$$

$$M_H = \sum_{X=q,g,m,a} M_X^{\rm Ji}, \quad \text{with} \quad M_m^{\rm Ji} = M_m^{\rm MPR},$$

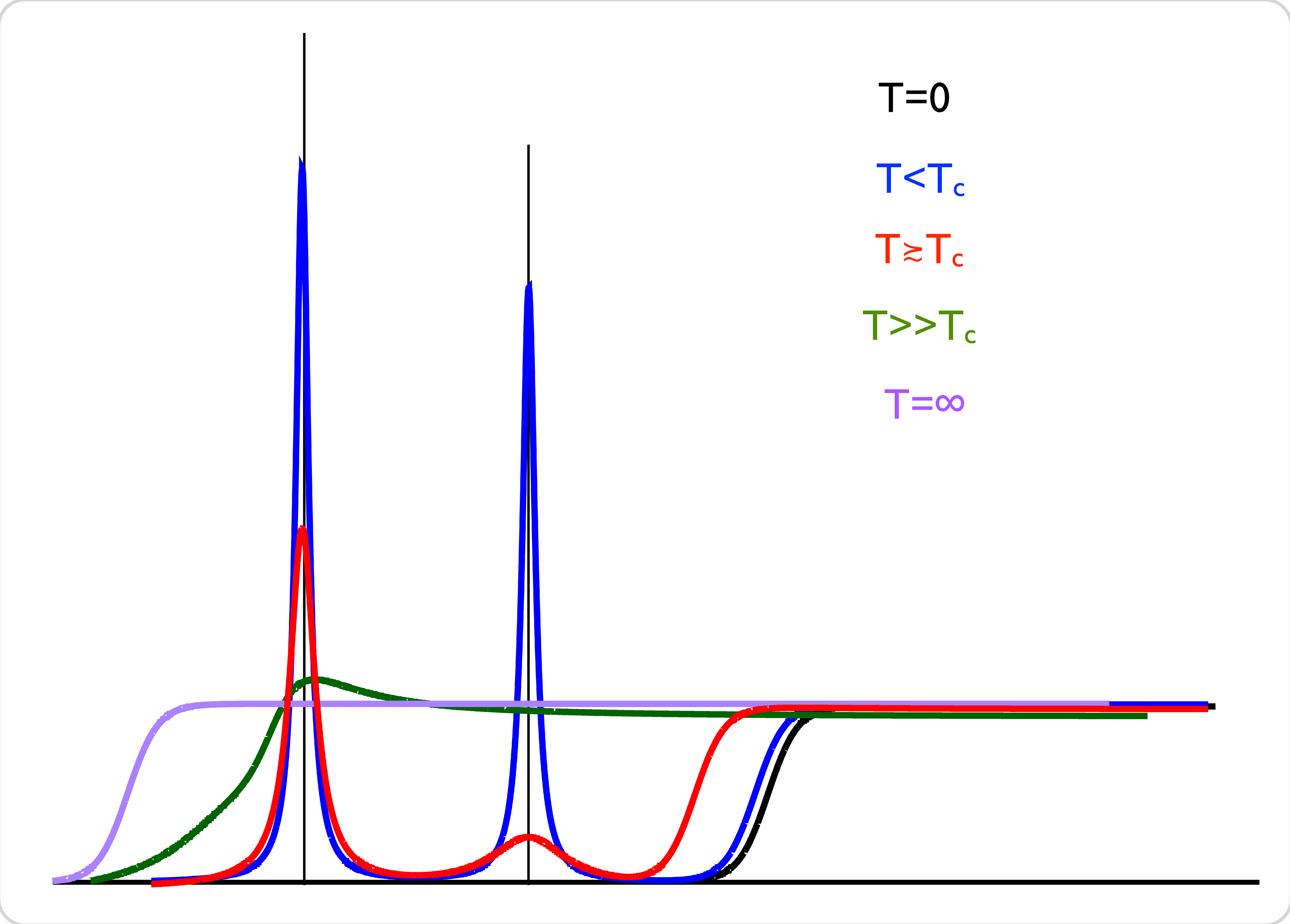
$$M_q^{\rm Ji} = M_q^{\rm Lor} - \frac{1}{4} M_q^{\rm HRT} - \frac{3}{4} M_m^{\rm MPR},$$

$$M_g^{\rm Ji} = M_g^{\rm Lor} - \frac{1}{4} M_g^{\rm HRT},$$

$$M_a^{\rm Ji} = \frac{1}{4}\left[M_q^{\rm HRT} + M_g^{\rm HRT} - M_m^{\rm MPR}\right].$$

Ji, PRL 74 (1995) 1071 ;Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

Challenges on the lattice

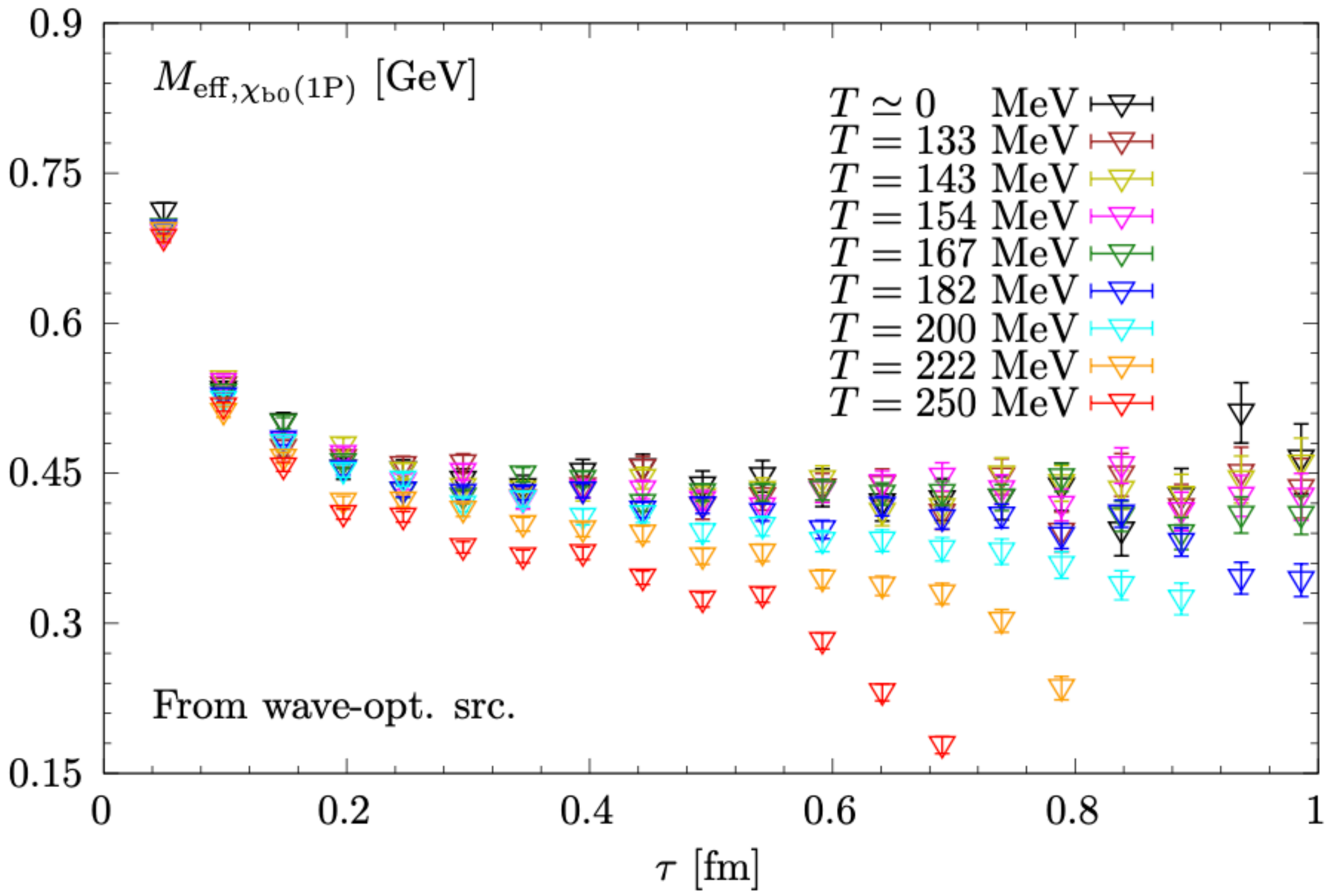


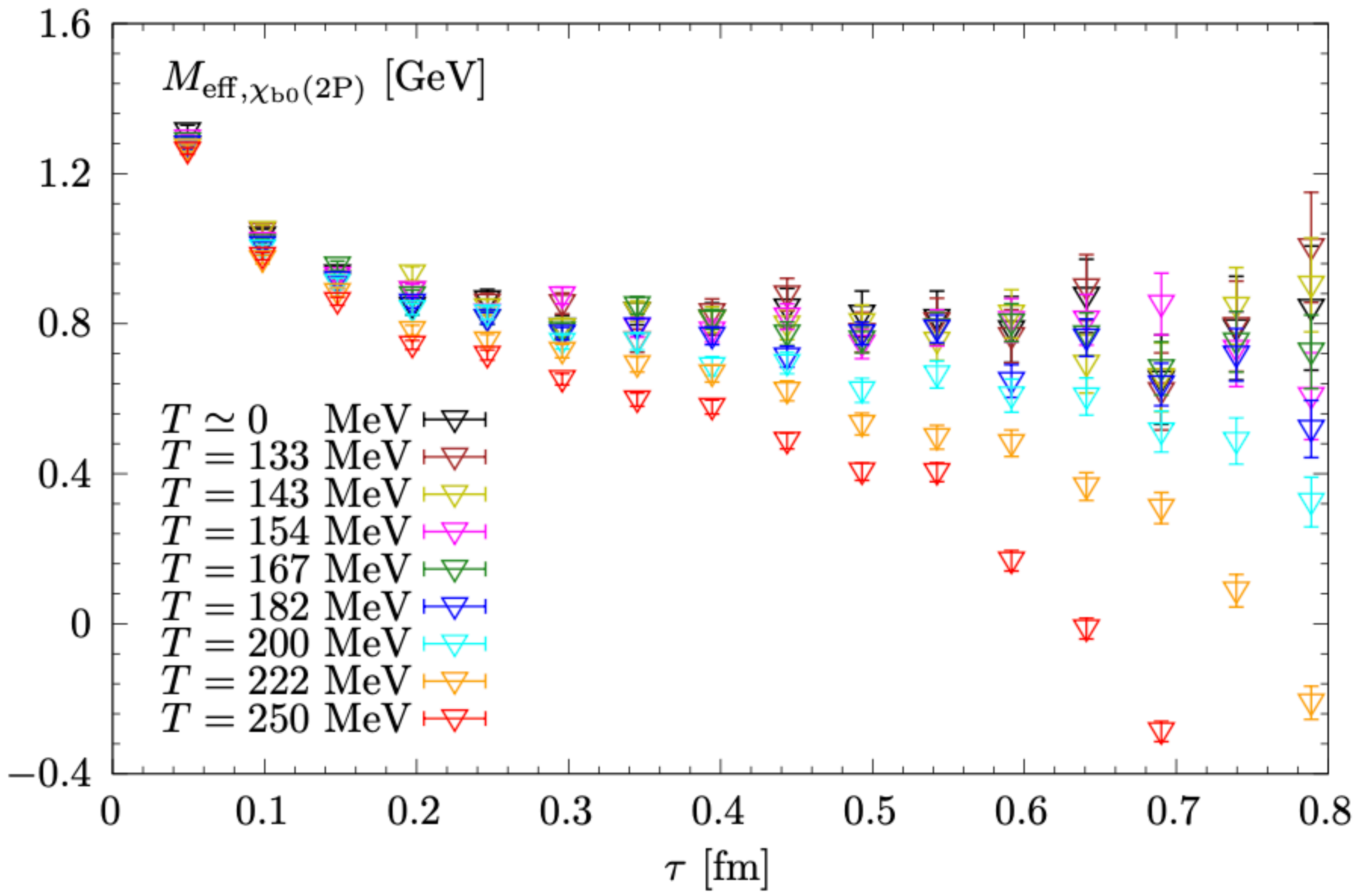
- Fine lattices spacing, $a^{-1} \gg m_Q$ to accommodate heavy quark on the lattice

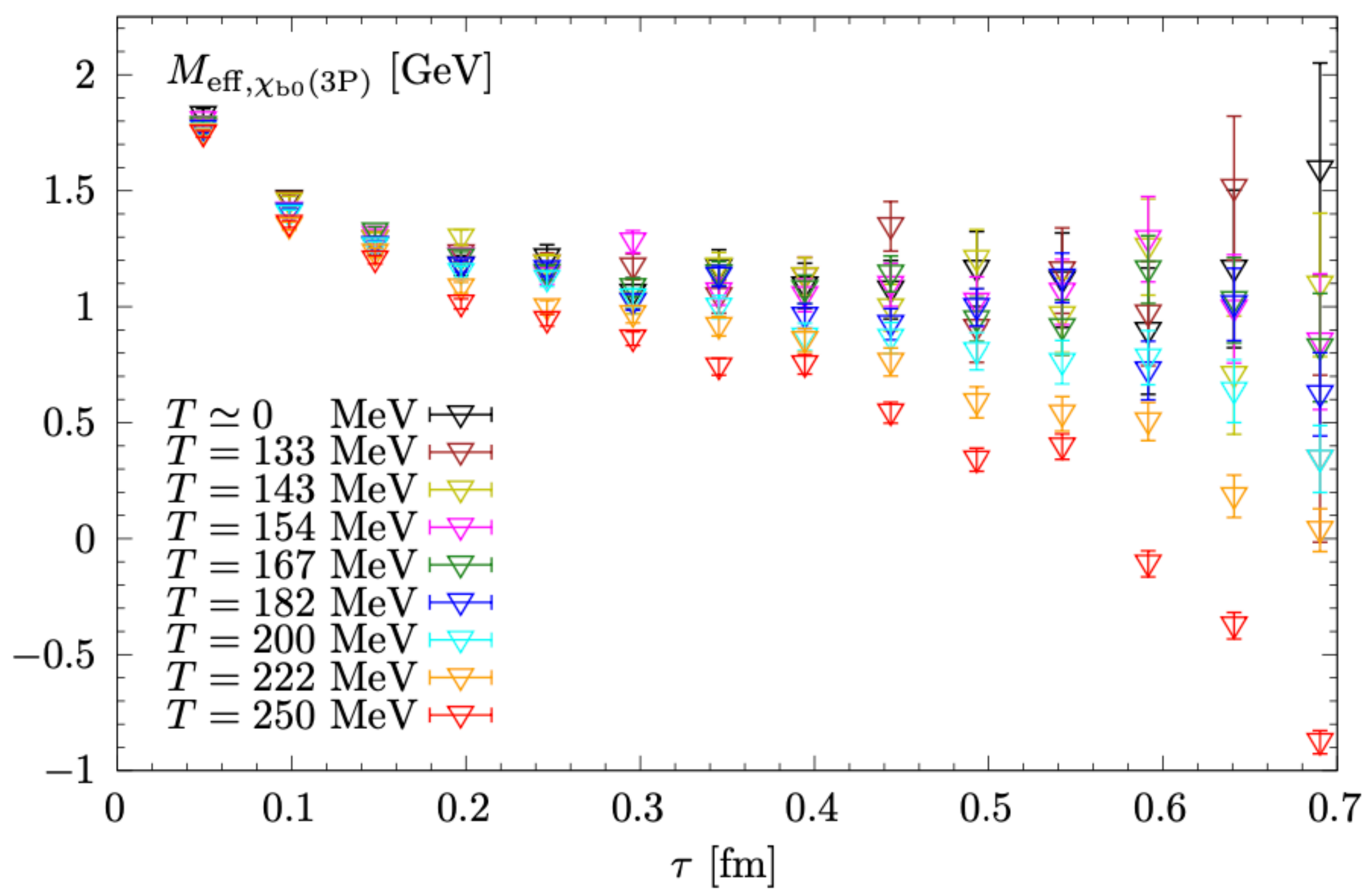
- Inverse problem to extract spectral functions from Euclidean two point correlation functions

$$G(\tau,T) = \int \frac{\mathrm{d}\omega}{2\pi}\, \rho(\omega,T)\, K(\omega,\tau,T); \qquad K(\tau,\omega,T) = \frac{\cosh(\omega(\tau-\frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

Effective mass of χ_{b0} 1P, 2P & 3P states at at nonzero temperature







Correlators with extended sources

$$C(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x},\tau) O^{\dagger}(\mathbf{0},\, 0)\rangle$$

Gaussian-smeared source

R. Larsen, et.al., PRD 100, 074506 (2019)

$$O(\mathbf{x},\tau) = \bar{\tilde{q}}(\mathbf{x},\tau)\Gamma\tilde{q}(\mathbf{x},\tau)$$

Bottomonium interpolators

Smeared quark and antiquark fields: $\tilde{q} = Wq$

Width

Gaussian-shaped factor: $W = \left(1 + \frac{\sigma^2}{4N}\Delta^{(2)}\right)^N$

Smearing step

Wave-function optimized source

R. Larsen, et.al., PLB 800, 135119 (2020)

$$O_{\alpha}(\mathbf{x},\tau) = \sum_{\mathbf{r}} \Psi_{\alpha}(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r},\tau)\Gamma q(\mathbf{x},\tau)$$

Solved from discretized 3D Schrodinger eq.

S. Meinel, PRD 82, 114502 (2010)

$$\left[-\frac{\Delta}{m_b} + V(\mathbf{r})\right]\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

$O(a^4)$-improved discretized Laplacian

Cornell potential

Correlators from lattice NRQCD

NRQCD formalism

- Scale separation: $\underbrace{M_b}_{\text{mass}} \gg \underbrace{M_b v}_{\text{momentum}} \gg \underbrace{M_b v^2}_{\text{energy}}$

Heavy-quark velocity, $v^2 \sim 0.1$ for bottomonium

E. Eichten, et.al., PRD 21 (1980) 203

- Hamiltonian expansion in v :

$$H = \boxed{H_0} + \boxed{\delta H}$$

Leading order $\mathcal{O}(v^2)$: $H_0 = -\dfrac{\Delta^{(2)}}{2M_b}$

Corrections up to $\mathcal{O}(v^6)$: spin-dependent terms included

B.A. Thacker, G.Peter Lepage, PRD 43 (1991) 196-208
G.Peter Lepage, et.al., PRD 46 (1992) 4052-4067
S. Meinel, PRD 82 (2010) 114502

Temporal correlator calculation

- Initial value problem: No $q\bar{q}$ pair creation

- Discrete evolution along Euclidean time:

$$\boxed{G(\mathbf{x}, \tau + a)} = \boxed{K(\tau)}\, G(\mathbf{x}, \tau)$$

Heavy-quark propagator: $G(\mathbf{x}, \tau) = \langle q(\mathbf{x}, \tau) q^\dagger(\mathbf{0}, 0)\rangle$

Evolution operator: $K(\tau) = \left(1 - \dfrac{a\,\delta H|_\tau}{2}\right)\left(1 - \dfrac{aH_0|_\tau}{2n}\right)^n U_4^\dagger(\tau)\ \cdots$

Lepage parameter: discretization time step

- Meson two-point function:

$$C(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) O^\dagger(0,0)\rangle = \sum_{\mathbf{x}} \mathrm{Tr}\left[\tilde{G}^\dagger(\mathbf{x}, \tau)\Gamma \tilde{G}(\mathbf{x}, \tau)\Gamma^\dagger\right]$$

Meson operator with smeared field: $O(\mathbf{x}, \tau) = \tilde{\bar{q}}(\mathbf{x}, \tau)\Gamma \tilde{q}(\mathbf{x}, \tau)$

Smeared (extended) field: $\tilde{q}(\mathbf{x}, \tau) = W q\,, \tilde{G} = WGW^\dagger$

Correlators with extended sources

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