

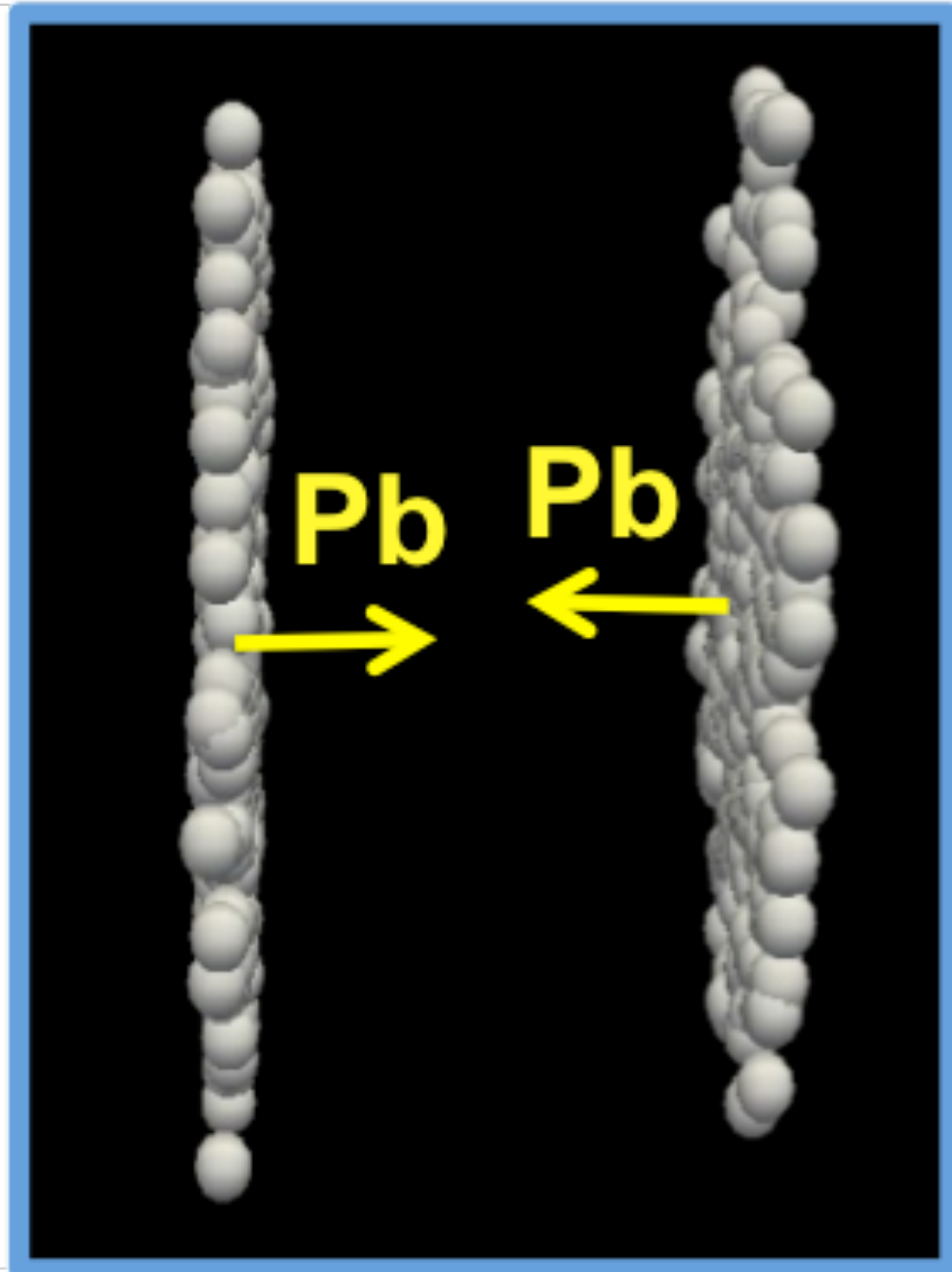
Apr-15-2026, Institute of High-Energy Physics, CAS

*Onset of Bjorken flow in a quantum many-body simulation
of the massive Schwinger model*

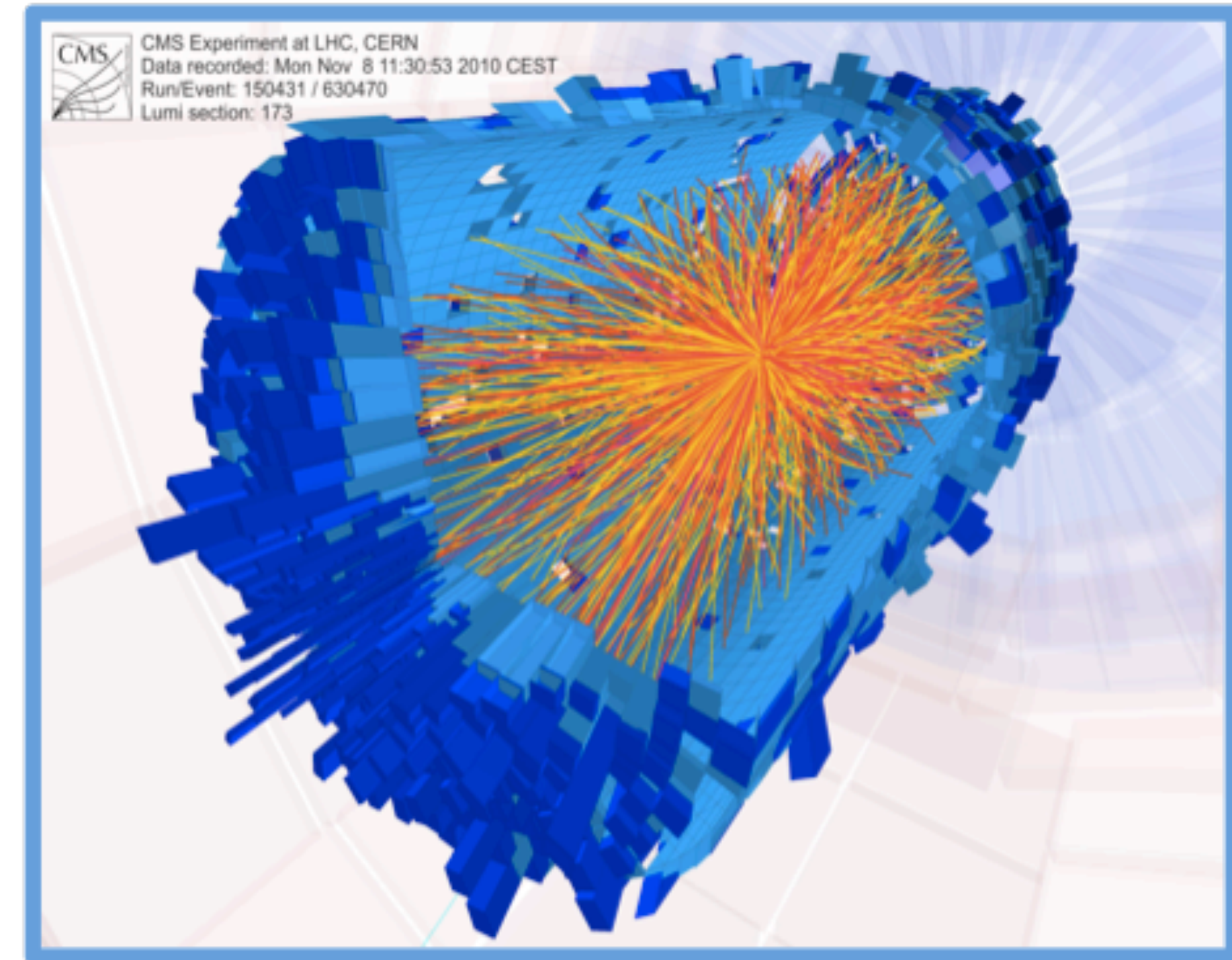
Shuzhe Shi (施舒哲), Tsinghua University

Haiyang Shao, Shile Chen, SS, 2509.10835 + 2509.10855

Pre-reaction



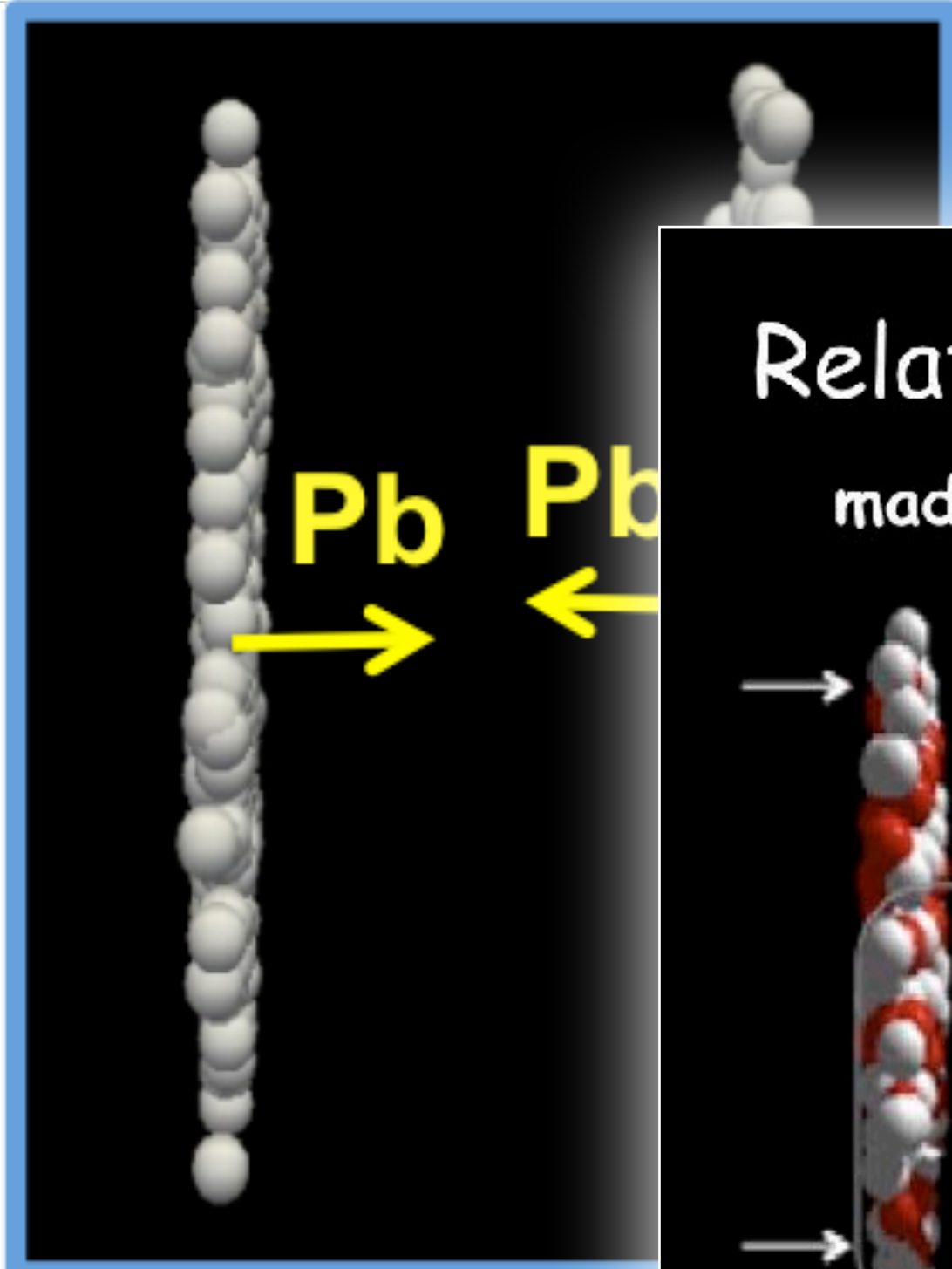
Detection



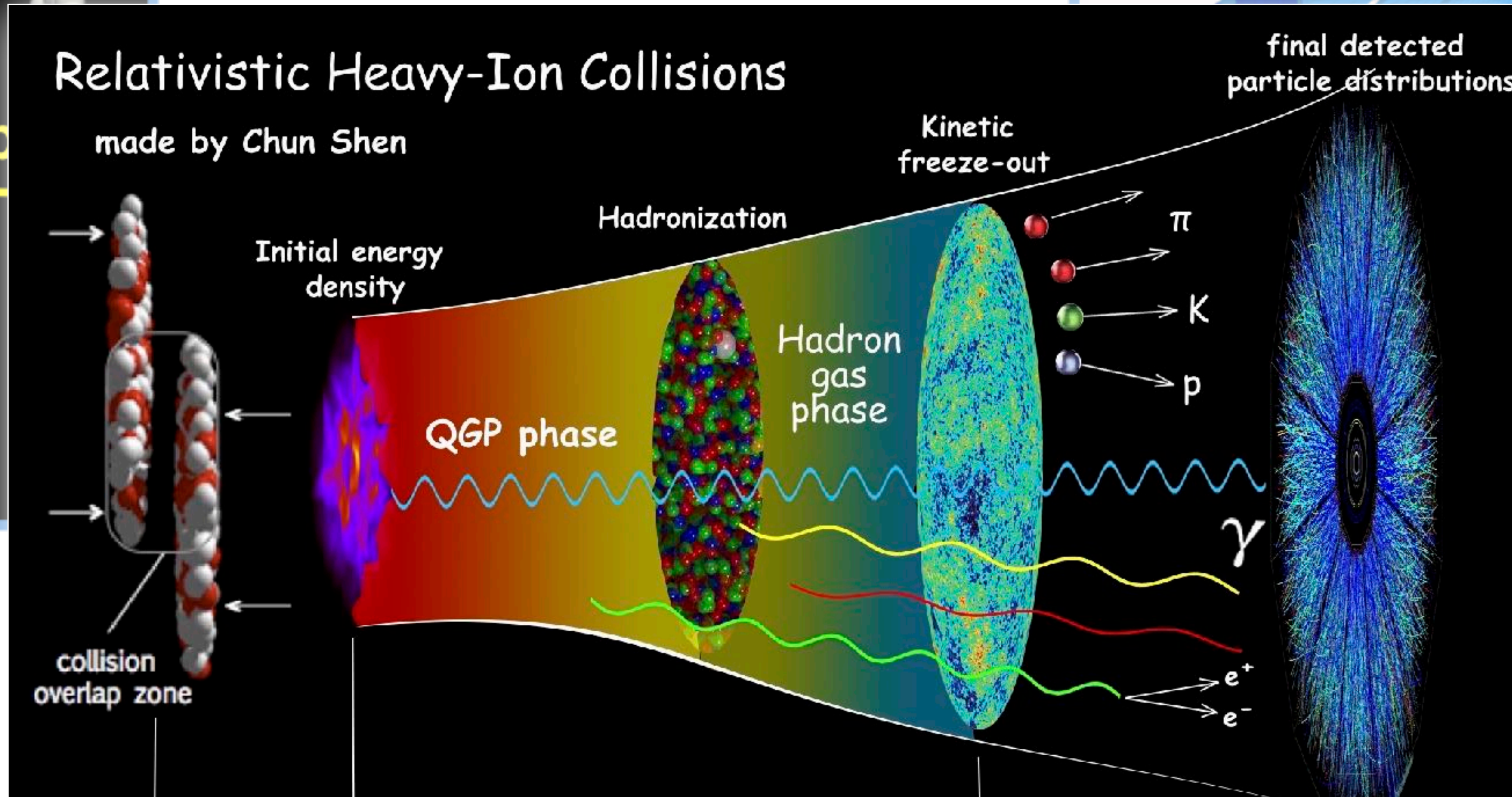
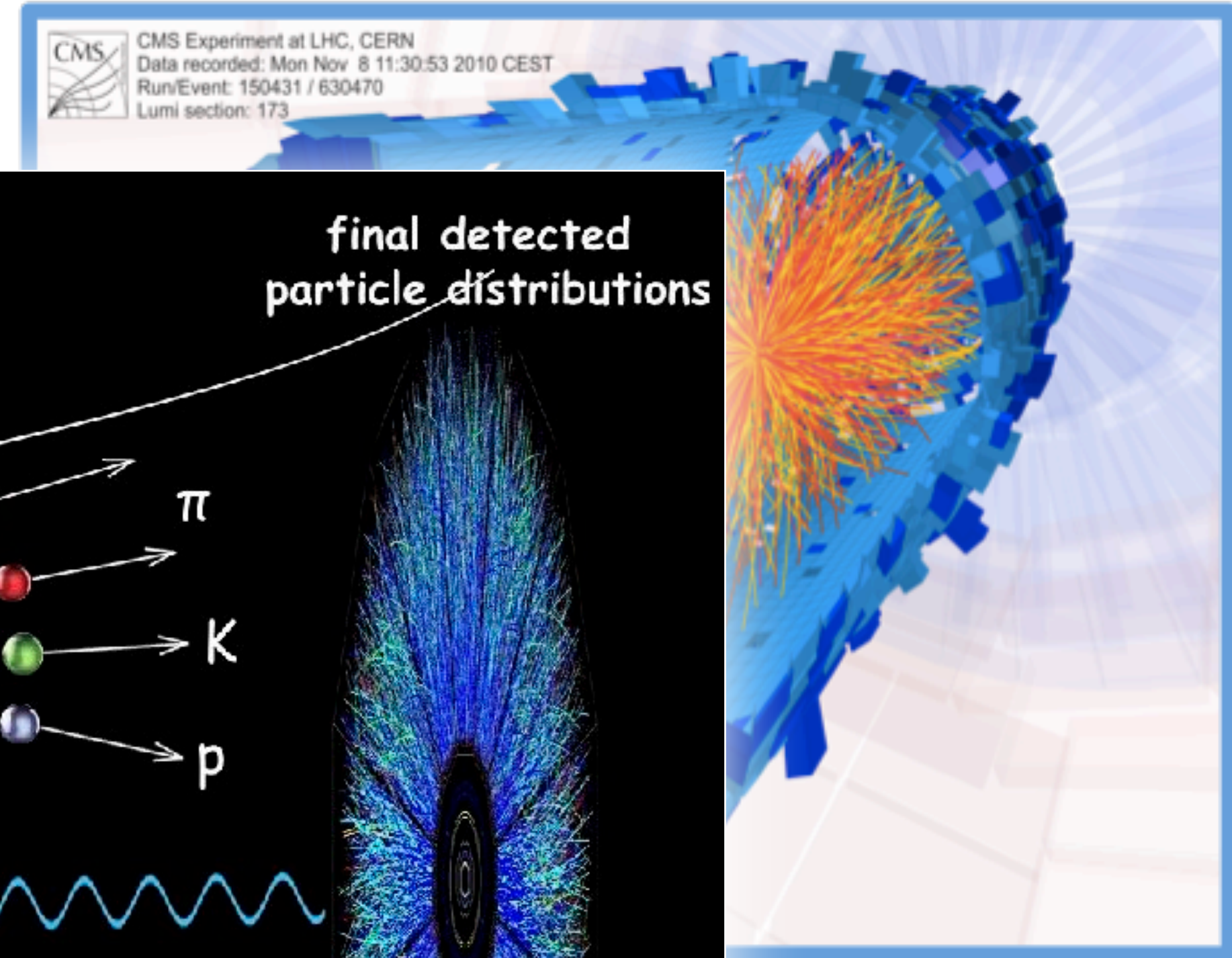
final state particles

$$N \sim 10^{3-4}$$

Pre-reaction



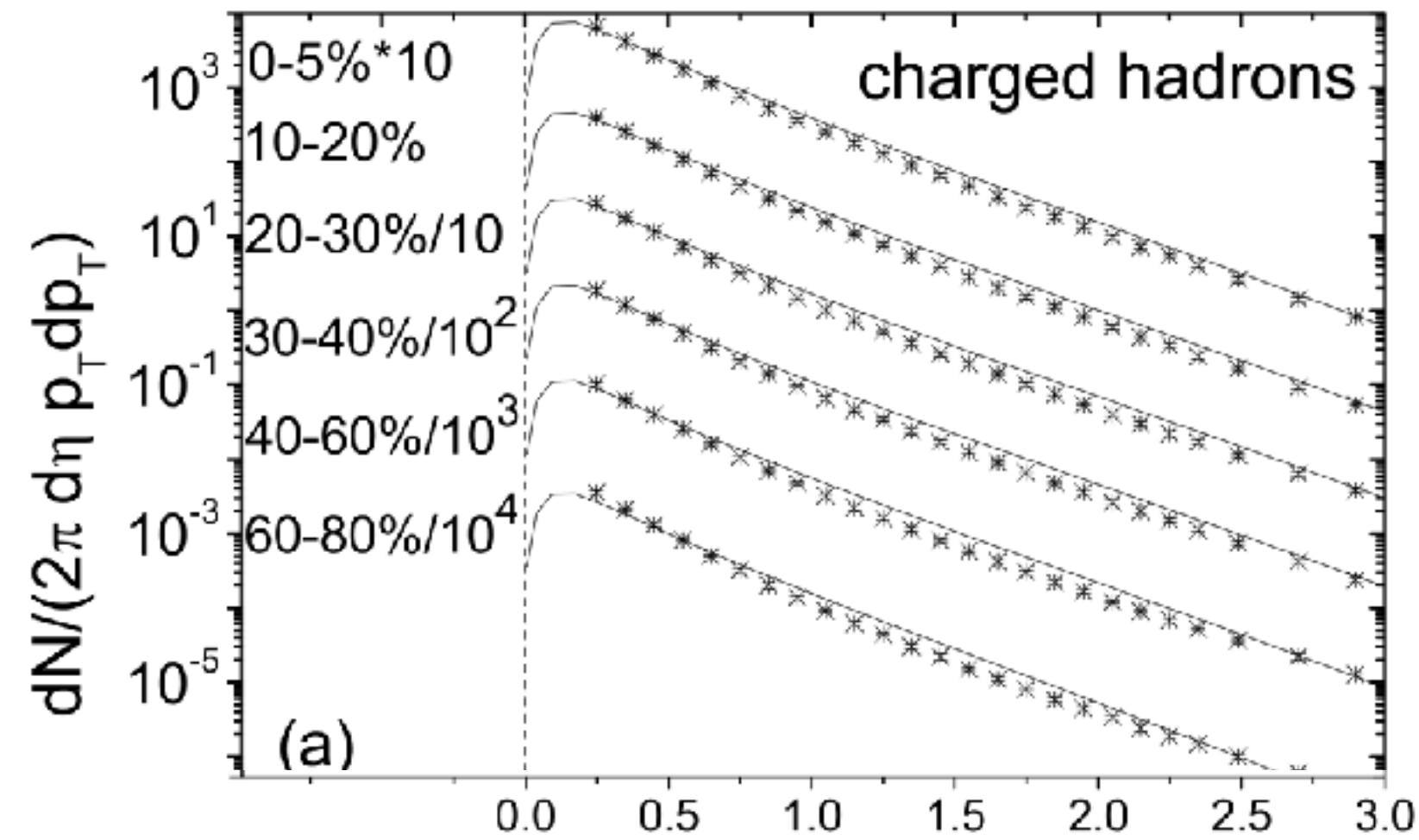
Detection



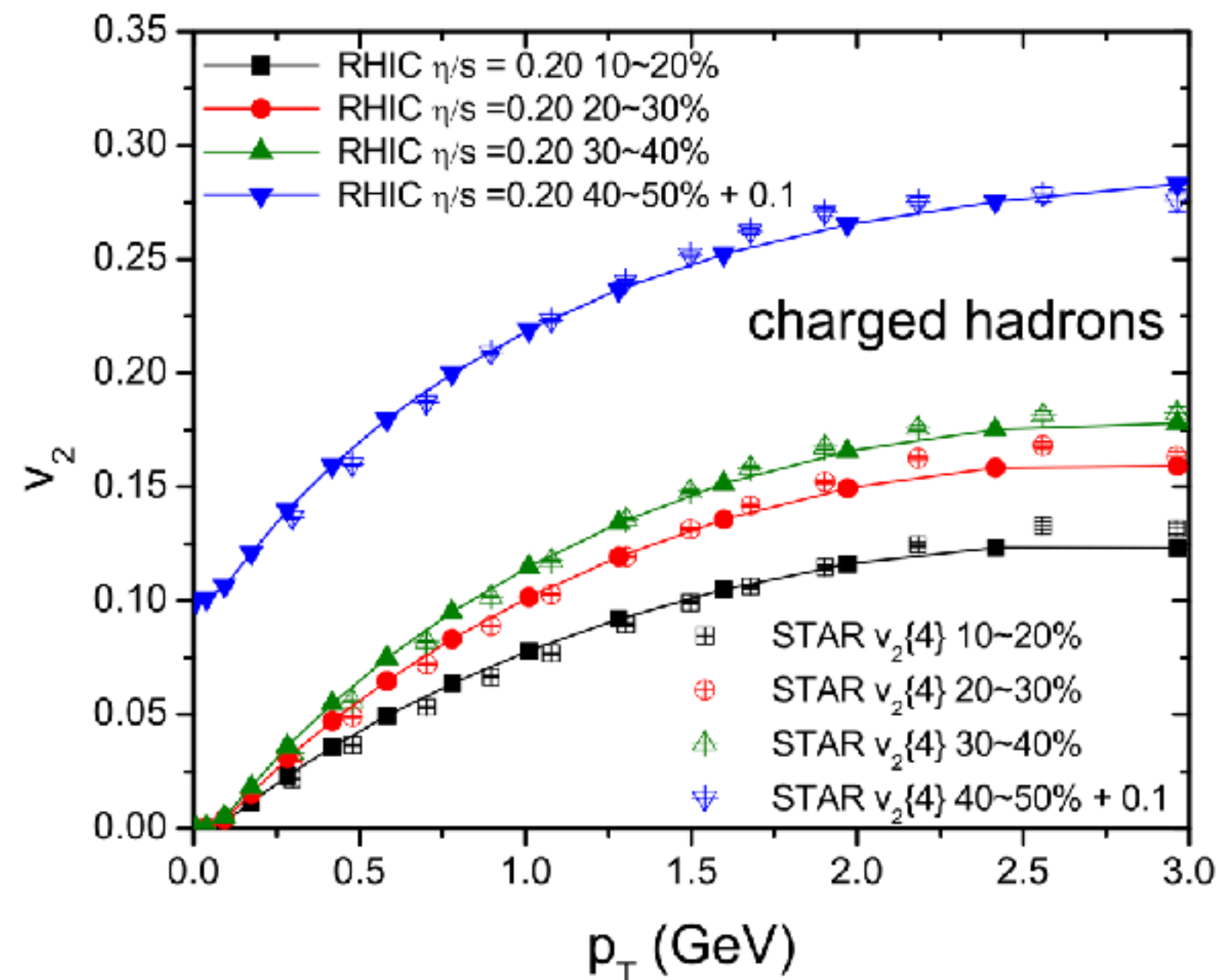
particles

-4

Collective Property of final state particles:



symbols: data
curves: hydrodynamics theory
(thermal w/ anisotropic blue-shift)



+ many independent probes
of the hot medium

$$\partial_{\mu} T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi) u^{\mu} u^{\nu} - (P + \Pi) g^{\mu\nu} + \pi^{\mu\nu}.$$

thermodynamic + kinetic quantities

(EOS: QCD physics validated)

viscous (non-equilibrium) corrections

$$\partial_{\mu} T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi) u^{\mu} u^{\nu} - (P + \Pi) g^{\mu\nu} + \pi^{\mu\nu}.$$

thermodynamic + kinetic quantities

viscous (non-equilibrium) corrections

Hydrodynamics \Leftarrow near local-equilibrium (many body)

for $\sim 10^{3-4}$ particles?

✓ mean free path (~ 0.5 fm) \ll system size (~ 10 fm)

? rapid equilibration? ($t \sim 0.5-1$ fm/c)

? small system?

Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

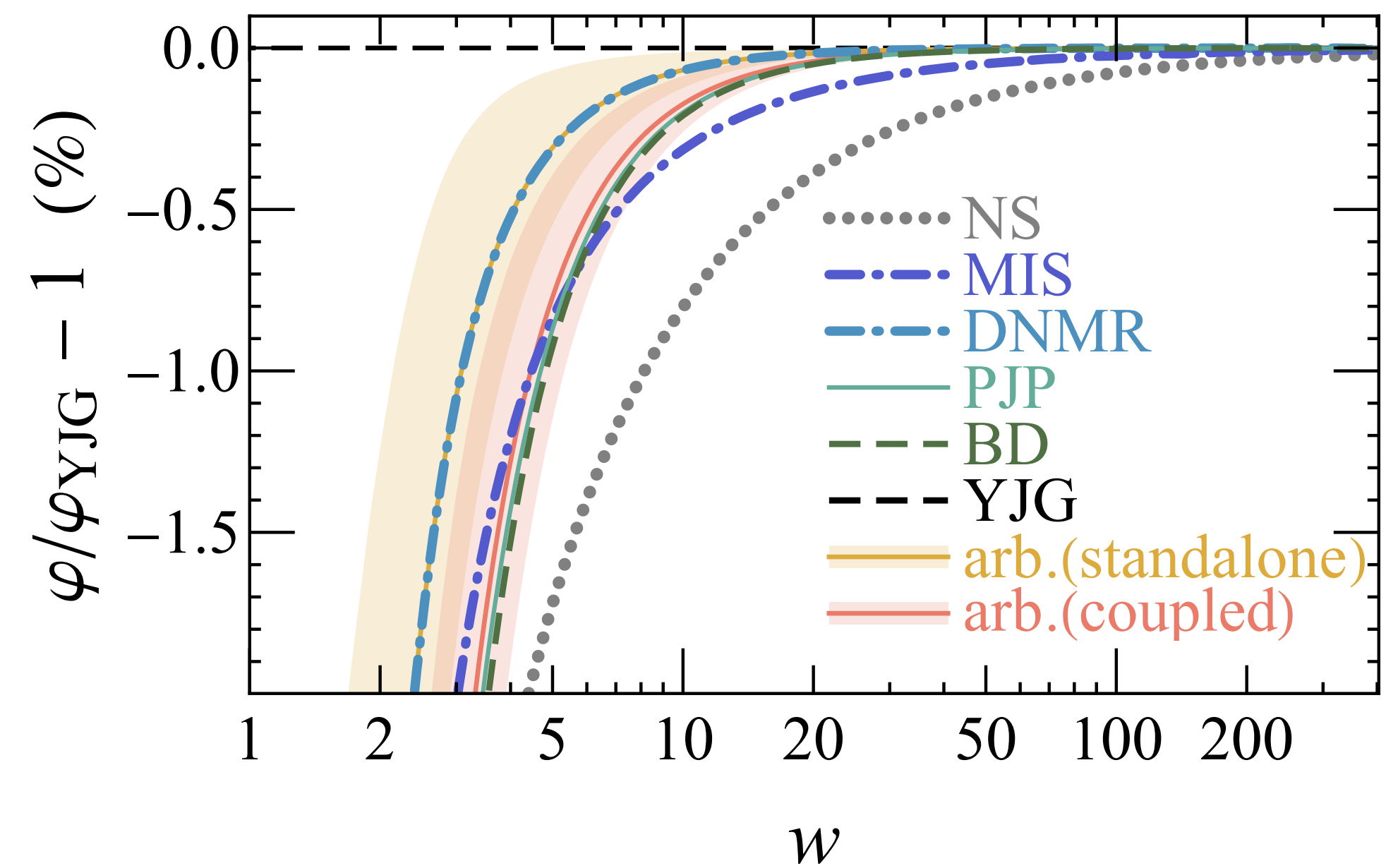
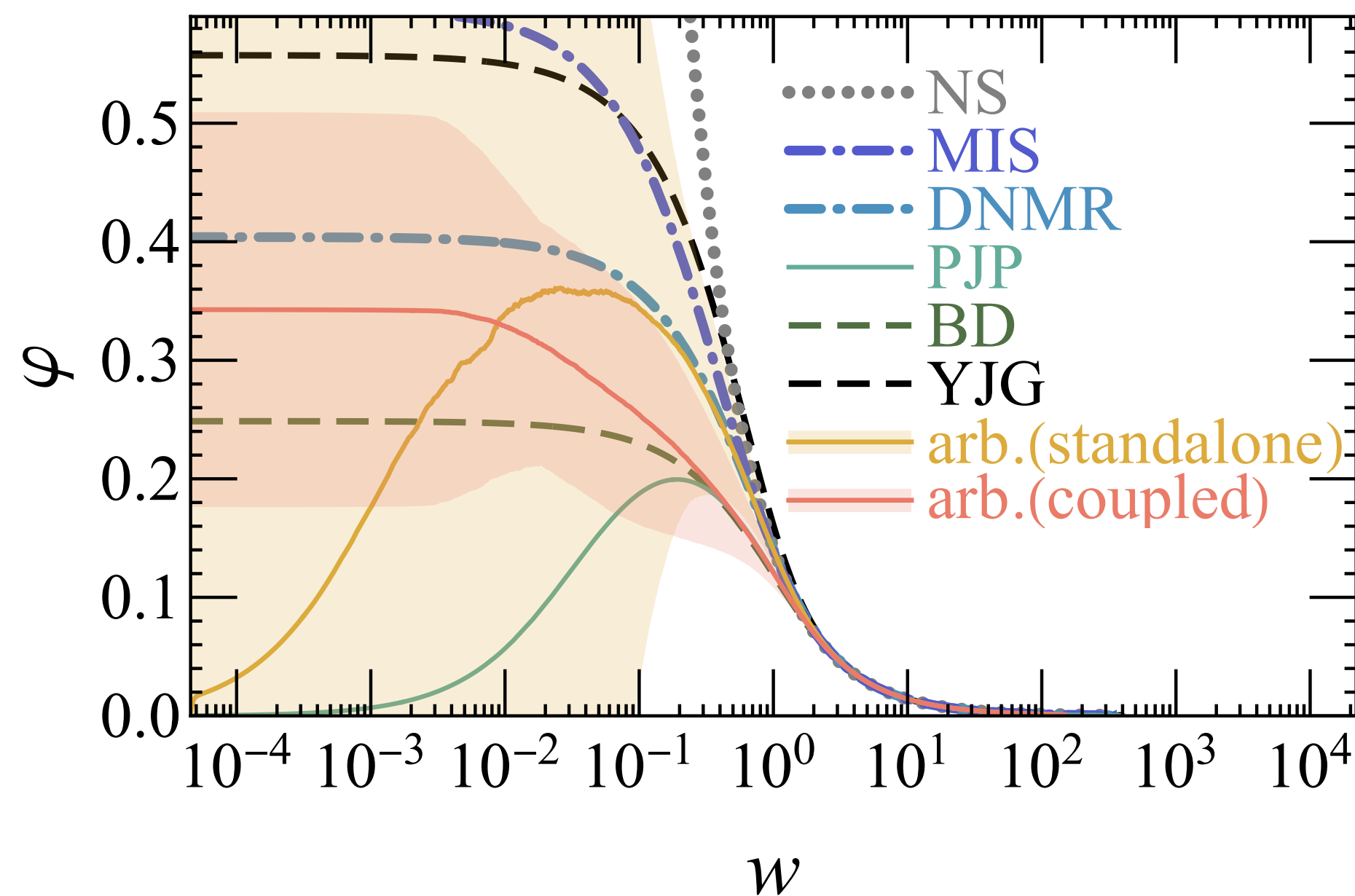
Heller, Spalinski, Phys.Rev.Lett. 115 (2015) 072501



Hydrodynamic attractors

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

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convergence of attractors in 2nd- and high-order hydrodynamics

Shile Chen and SS, PhysRevC.111.L021902+PhysRevD.113.L071501

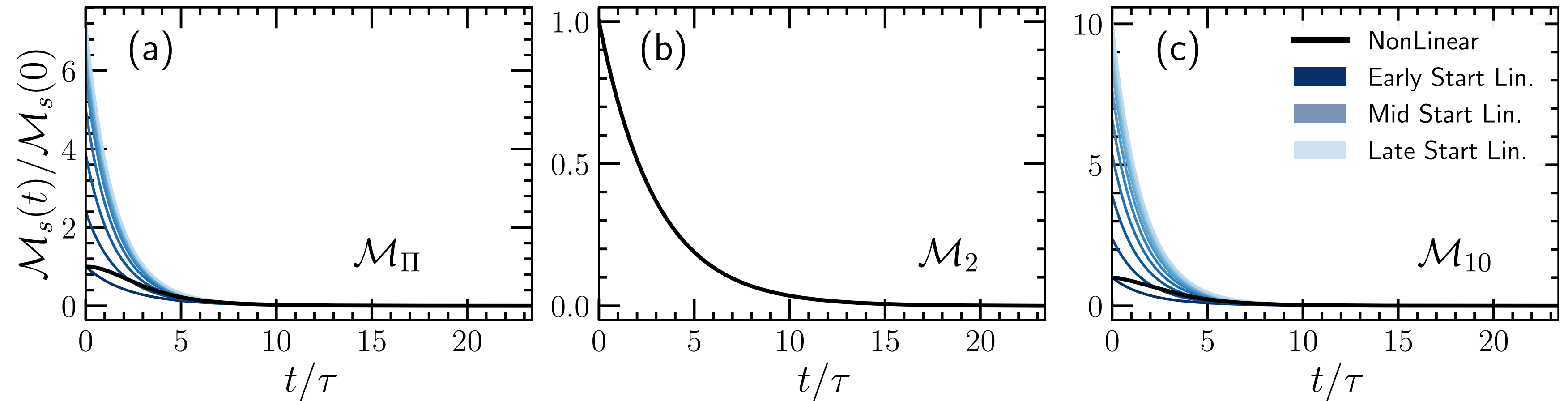
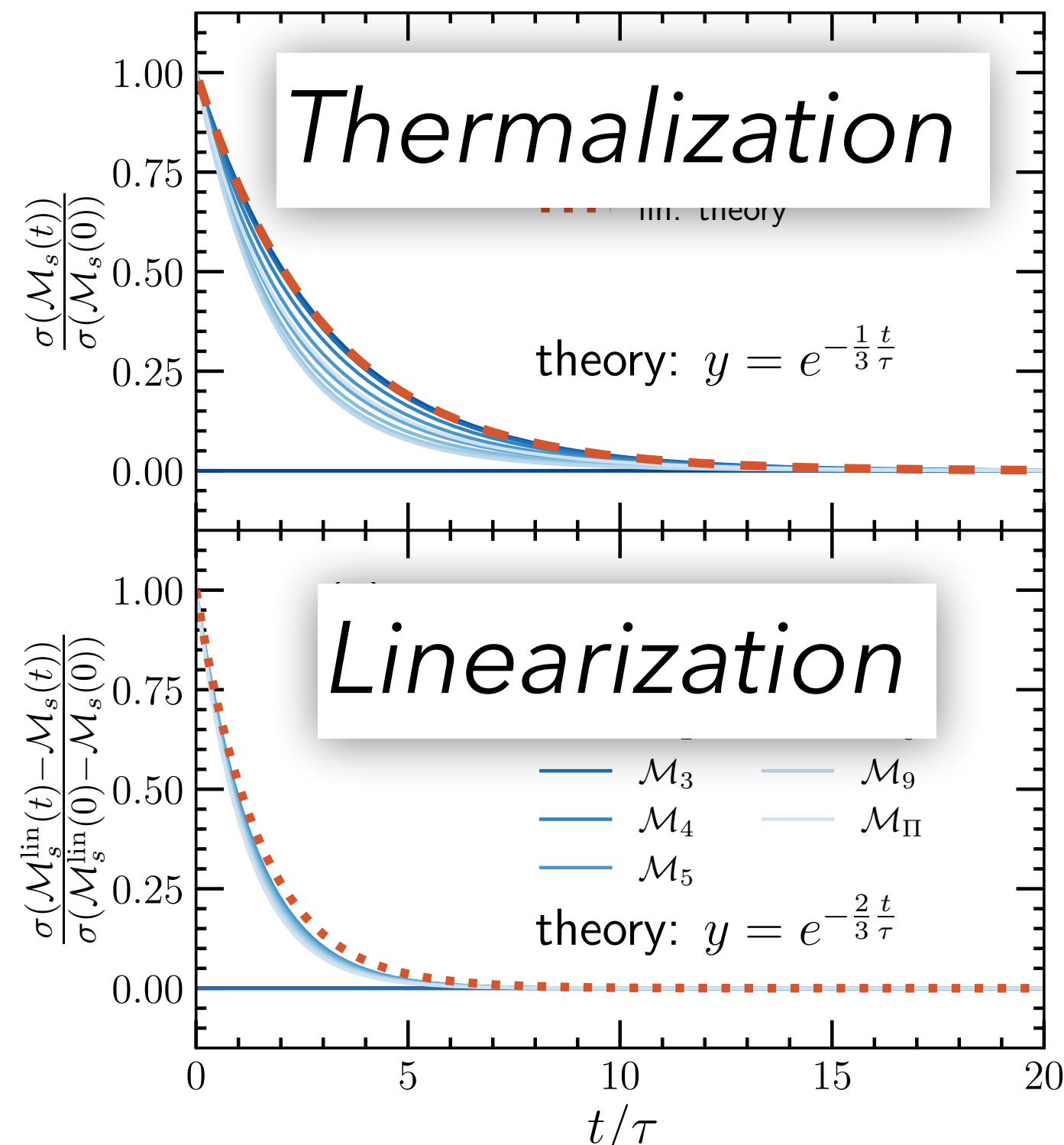
Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro
- usually *linearized* Boltzmann, e.g., Relaxation Time Approximation

see e.g. Strickland, JHEP 12 (2018) 128

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro
- usually *linearized* Boltzmann, e.g., Relaxation Time Approximation



Linearization (evolution well approximated by linearized Boltzmann) before thermalization.

Quark-Gluon Plasma:

A strongly coupled, quantum, (quasi)many-body system!

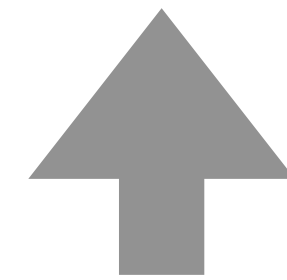
Ideally, full quantum simulation of QCD in 3+1D

Practically, strong coupling QED in 1+1D

– confinement, vacuum chiral condensate

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) dx.$$



E : electric field

A : electric potential

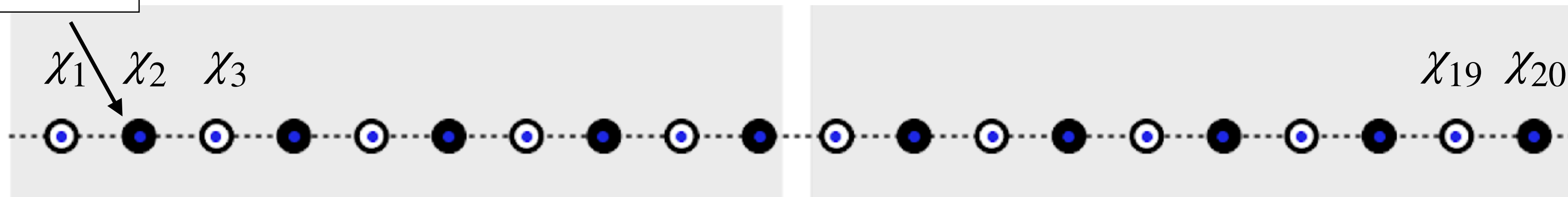
$\psi, \bar{\psi}$: fermion field

$$L(t) = \int \left(-\frac{F^{\mu\nu} F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\psi \right) dx.$$

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) dx.$$

$|0\rangle$:empty
 $|1\rangle$:occupied



$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b} \delta(x - y)$$

field operators
 represented by
 matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0.$$

Jordan-Wigner

$$\chi_n = \frac{S_n^-}{2} \prod_{m=1}^{n-1} (-iZ_m)$$

$$S_n^\pm \equiv I \otimes \dots \otimes I \otimes (\sigma_x \pm i\sigma_y) \otimes I \otimes \dots \otimes I$$

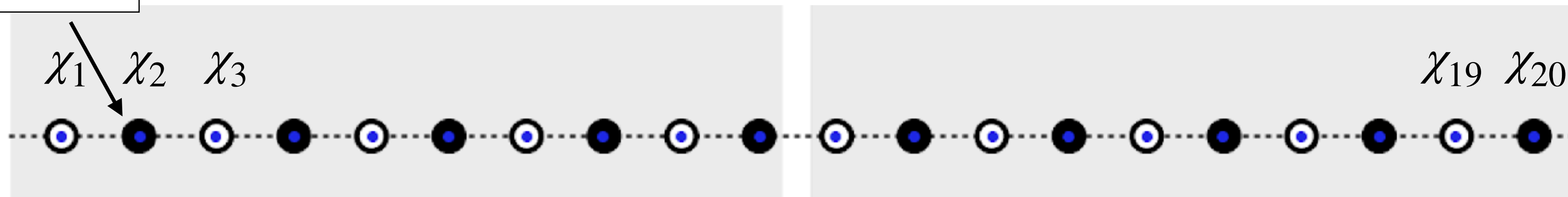
n^{th}

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) dx.$$

$|0\rangle$:empty
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$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b} \delta(x-y)$$



field operators
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$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0.$$

$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (S_n^+ U_n S_{n+1}^- + S_{n+1}^+ U_n^\dagger S_n^-) + \frac{m}{2} \sum_{n=1}^N (-1)^n Z_n + \frac{ag^2}{2} \sum_{n=1}^{N-1} L_n^2.$$

Eigenstates of H: vacuum and (quasi)-particles

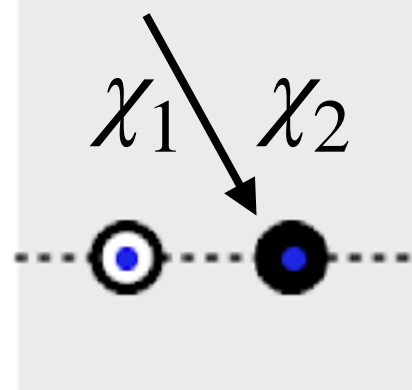
Time evolution:

$$\frac{\partial}{\partial t} |\psi(t)\rangle = -i H |\psi(t)\rangle$$

$$O(t) = \langle \psi(t) | \hat{O} | \psi(t) \rangle$$

$$H = \frac{1}{4a} \sum_{n=1} (S_n^+ U_n S_{n+1}^- + S_{n+1}^+ U_n^\dagger S_n^-) + \frac{m}{2} \sum_{n=1} (-1)^n Z_n + \frac{a g}{2} \sum_{n=1} L_n^2 .$$

$|0\rangle$:empty
 $|1\rangle$:occupied



$\delta_{a,b} \delta(x - y)$

and operators
 represented by
 matrices

$$\{\chi_n, \chi_m\} = 0 .$$

initial state: $|\Psi(t = 0)\rangle = e^{i\omega \int_{-w/2}^{+w/2} \bar{\psi}\psi(z)dz} |\text{vac}\rangle$

lattice sites: $N = 100$

vacuum + [excitation @ center]

spacing: $a = 1/2g$



$$|\Psi(t)\rangle = e^{-i\hat{H}_{\text{Sch}}t} |\Psi(t = 0)\rangle$$

$$\mathcal{L} \Rightarrow \hat{T}^{\mu\nu}$$

$$\langle\Psi(t)|\hat{T}^{\mu\nu}(x)|\Psi(t)\rangle = T^{\mu\nu}(t, x) = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$$

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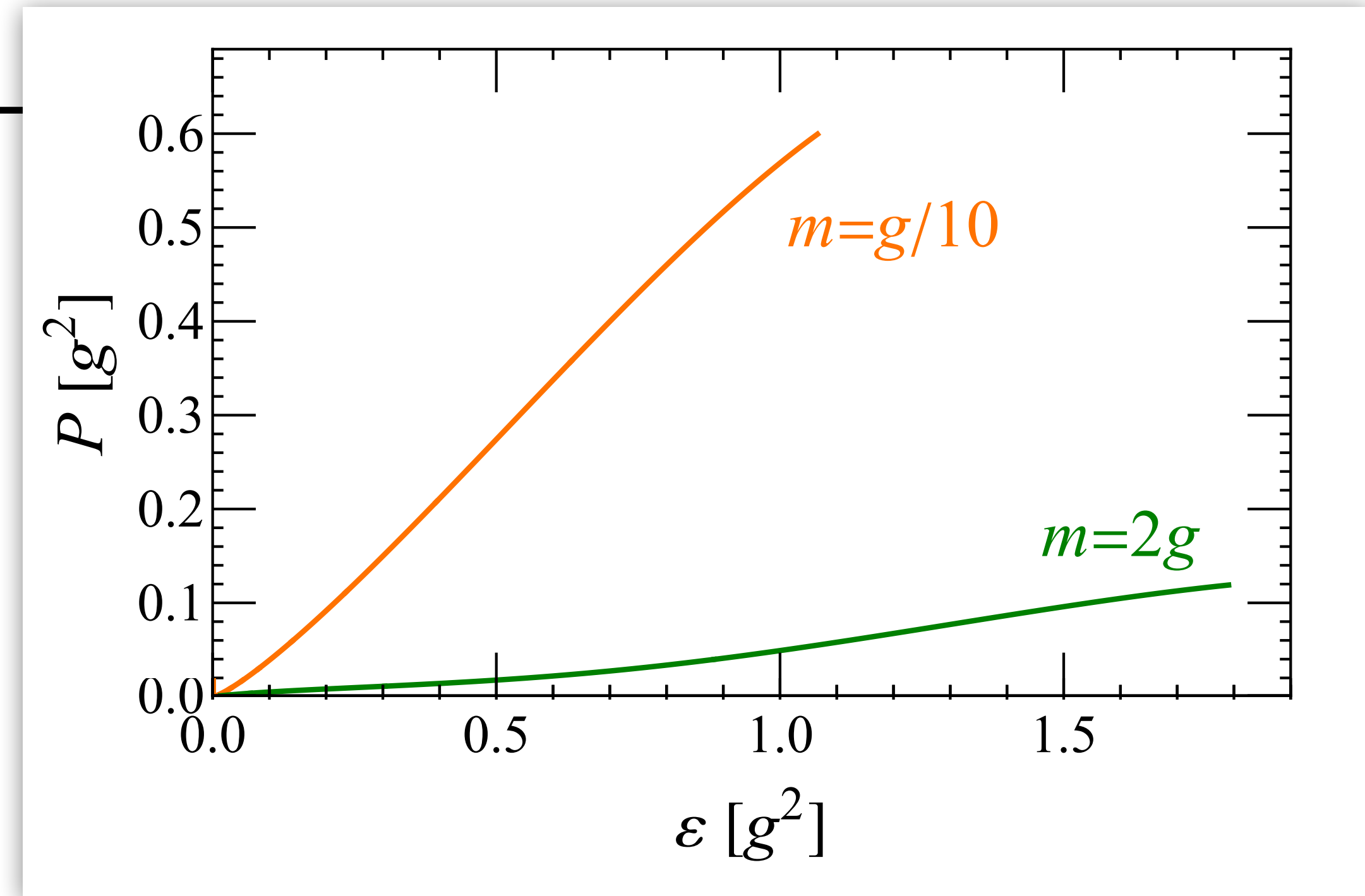
lattice sites: $N = 100$

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$$|\Psi(t)\rangle = e^{-i\hat{H}_{\text{Sch}}t} |\Psi(t=0)\rangle$$

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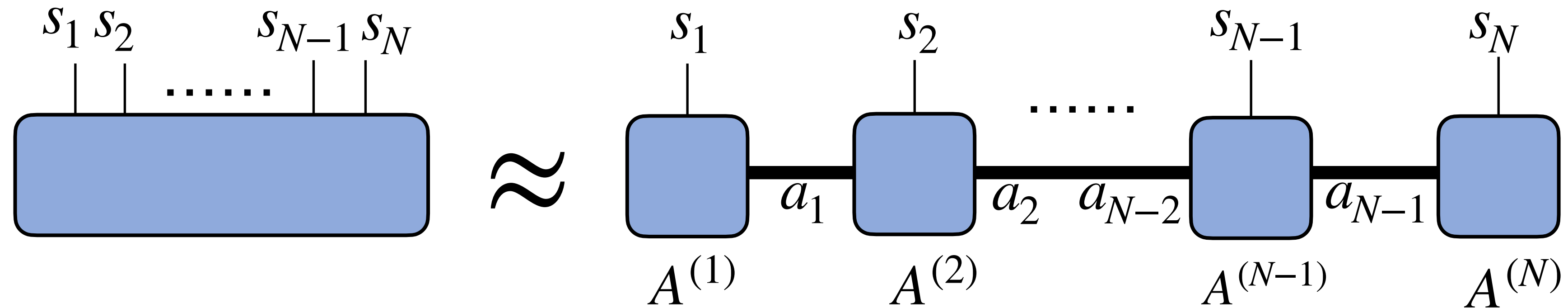


thermal pressure: $P = P_{\text{th}}(\epsilon)$

$$\langle \Psi(t) | \hat{T}^{\mu\nu}(x) | \Psi(t) \rangle = T^{\mu\nu}(t, x) = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$$

bulk pressure: non-equilibrium correction

$$|\Psi\rangle = \sum_{\{s\}} \psi_{s_1 s_2 \dots s_{N-1} s_N} |s_1\rangle \otimes |s_2\rangle \otimes \dots \otimes |s_{N-1}\rangle \otimes |s_N\rangle$$

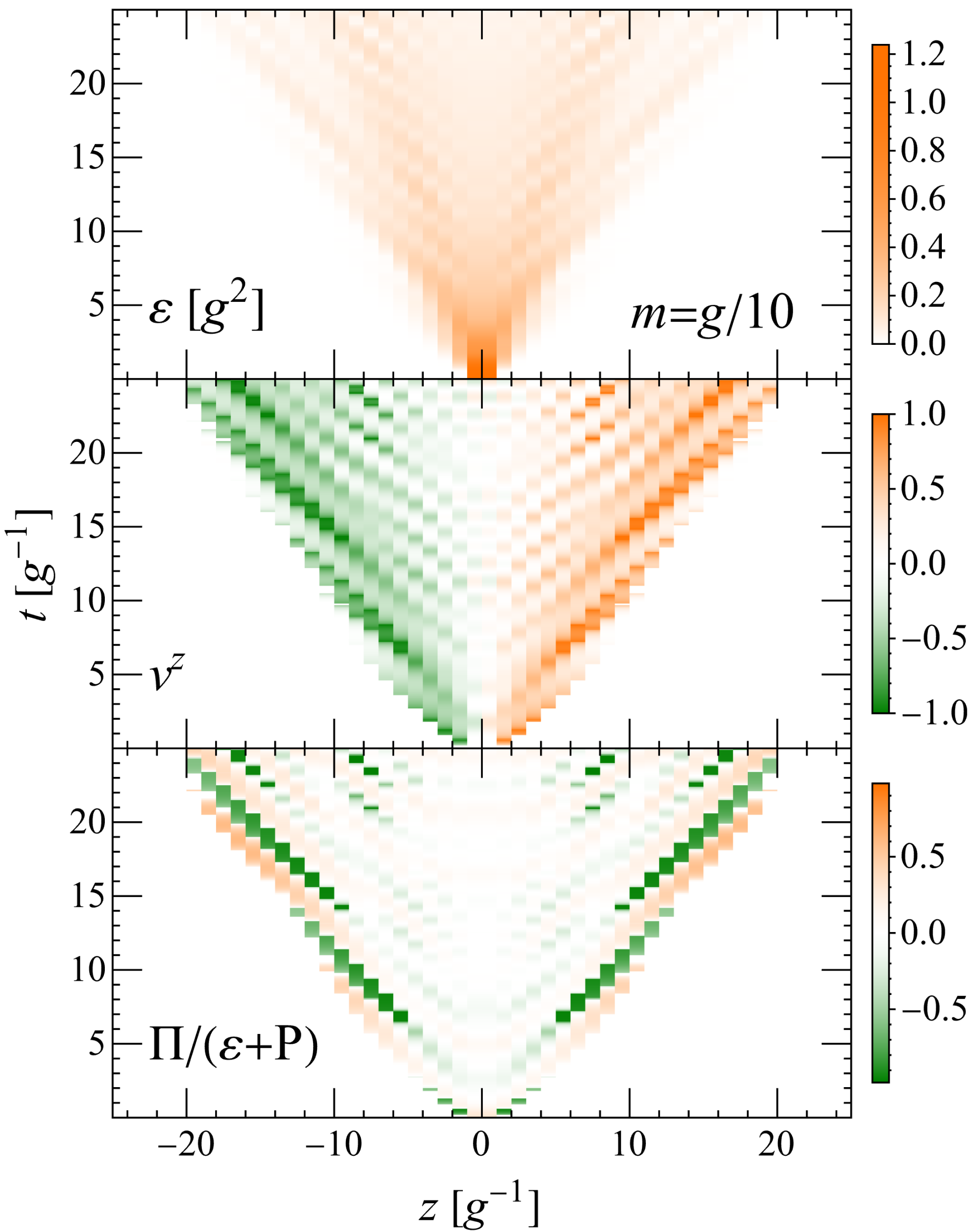


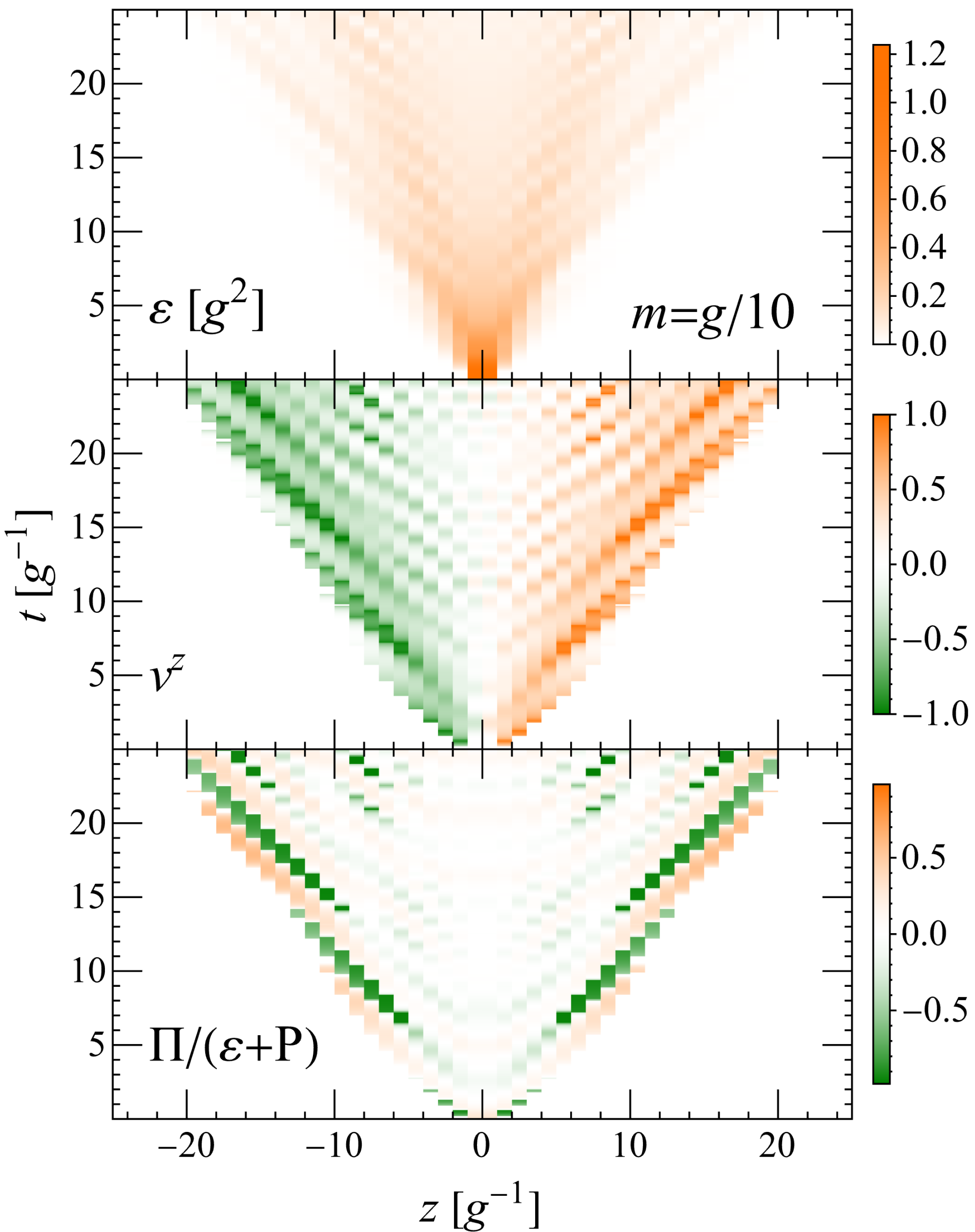
$$\psi_{s_1 s_2 \dots s_{N-1} s_N} \approx \sum_{a_1 a_2 \dots a_{N-2} a_{N-1}} A_{s_1 a_1}^{(1)} A_{s_2 a_1 a_2}^{(2)} \dots A_{s_{N-1} a_{N-2} a_{N-1}}^{(N-1)} A_{s_N a_{N-1}}^{(N)}$$

d.o.f.: D^N NDd^2

reproduces the full Hilbert space if d sufficiently large,
 otherwise, drop states with very high entanglement entropy

Our results converge over d





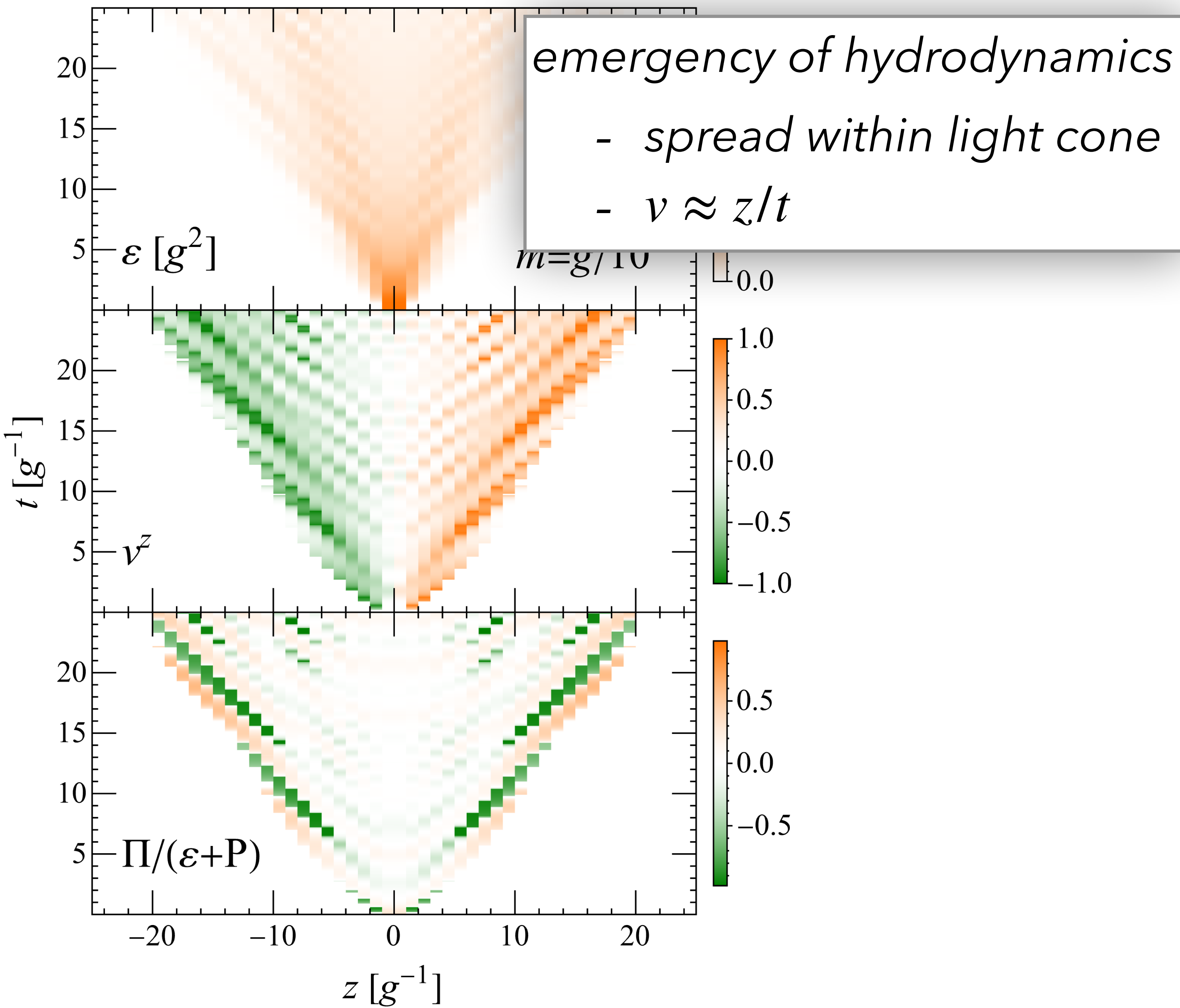
Bjorken flow – an analytical solution to the ideal hydrodynamic equations

$$\epsilon \propto \tau^{-1-c_s^2}, \quad \tau \equiv \sqrt{t^2 - z^2}.$$

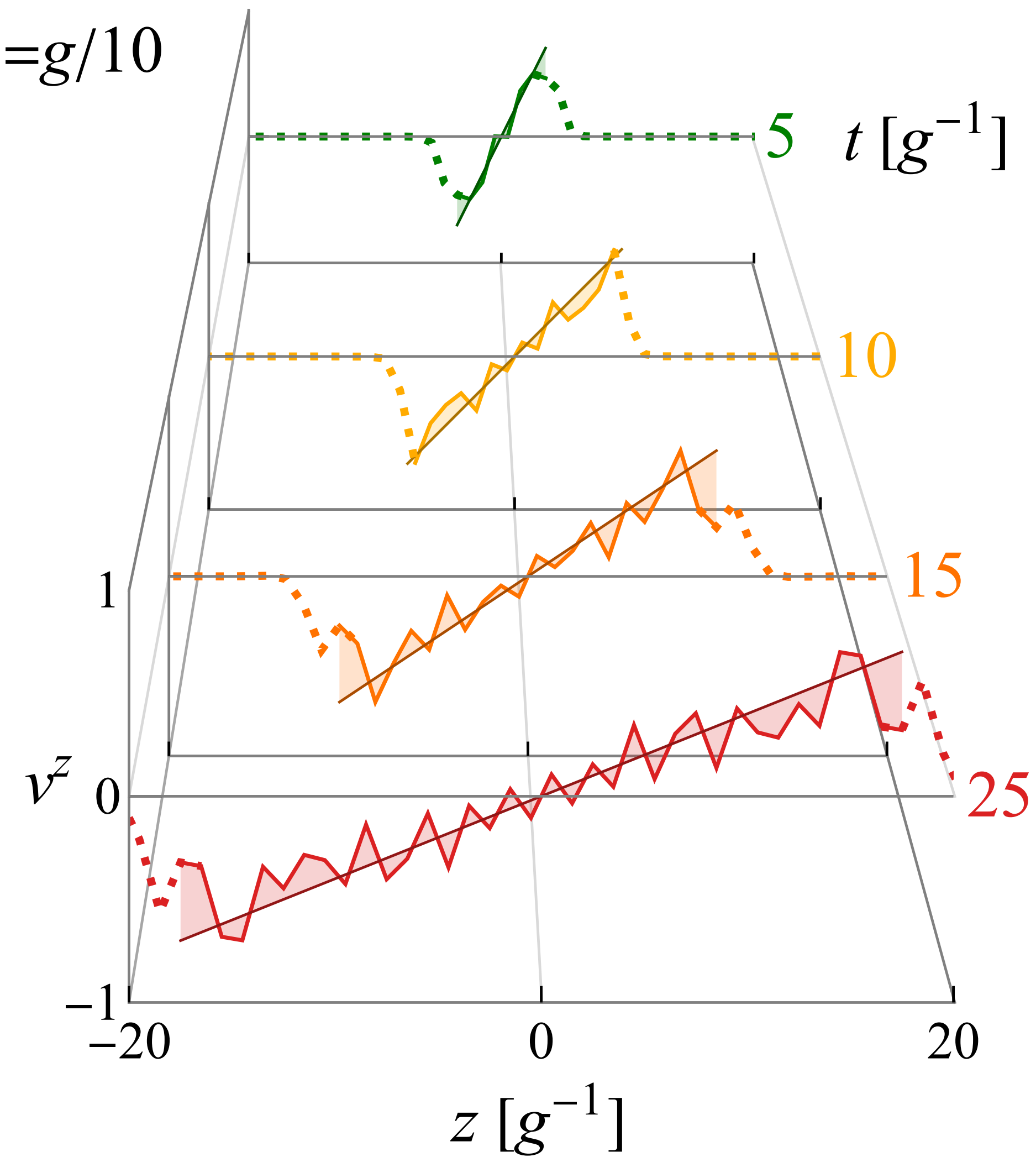
boost invariant (within a rapidity plateau)

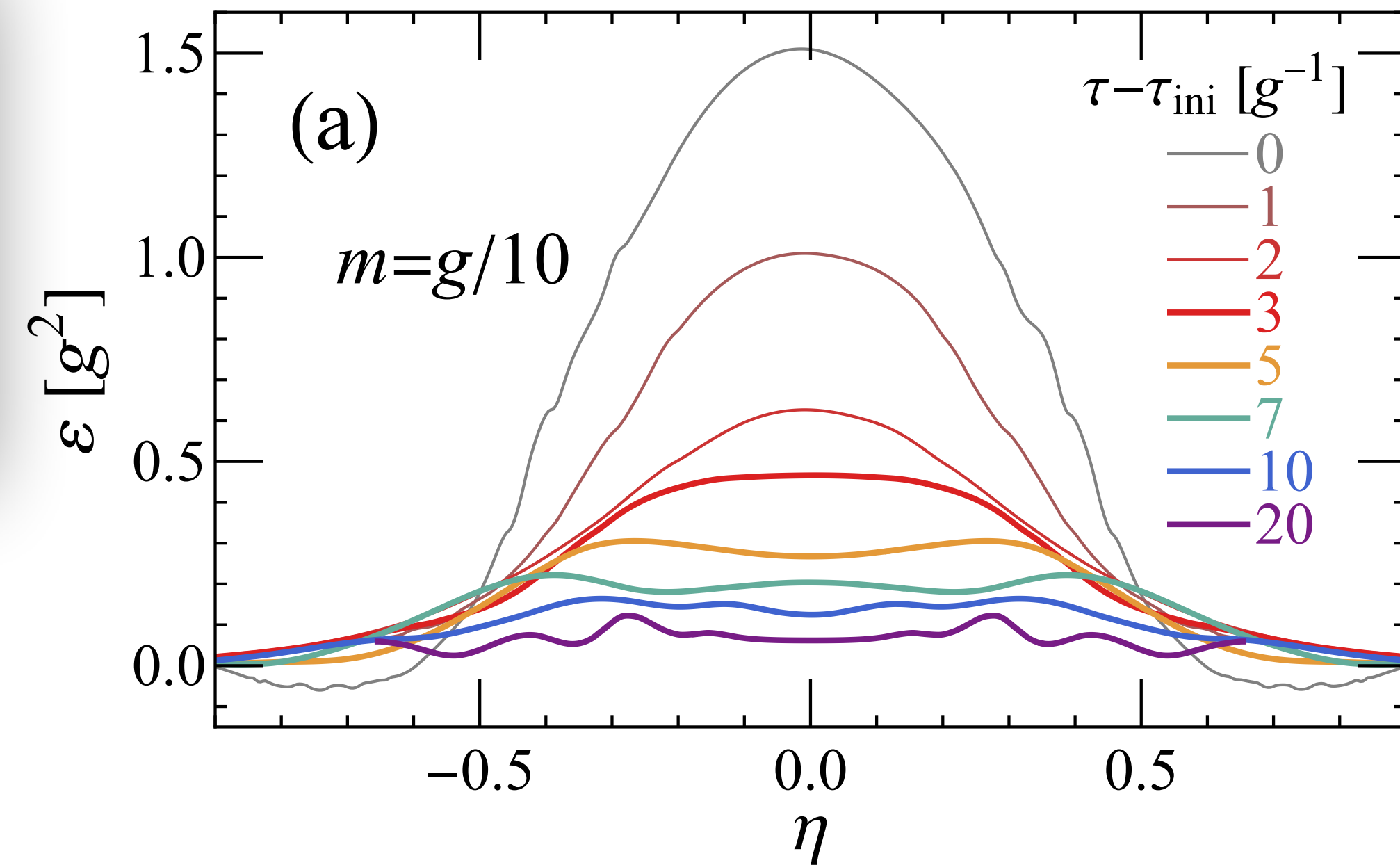
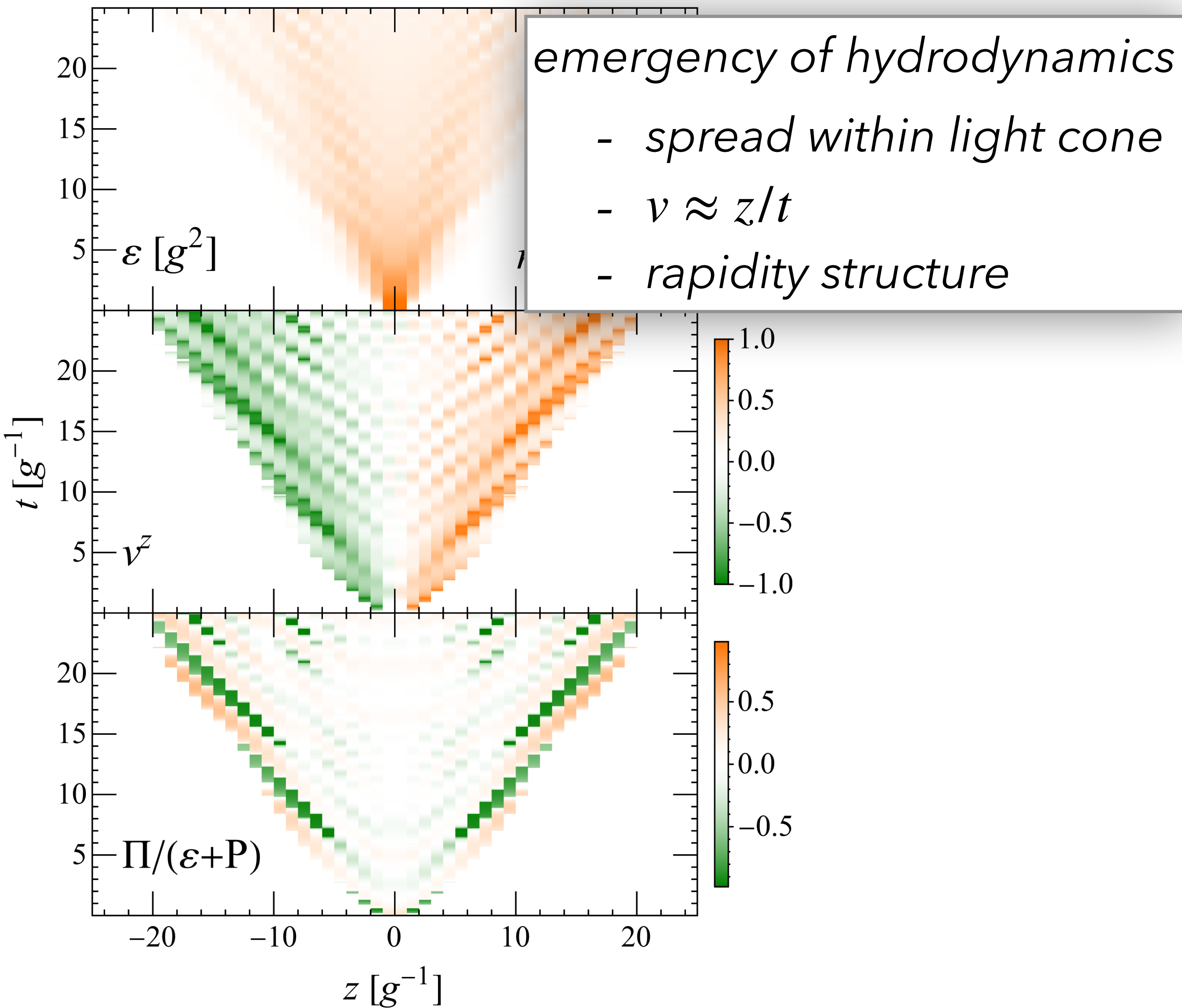
$$v^z = z/t.$$

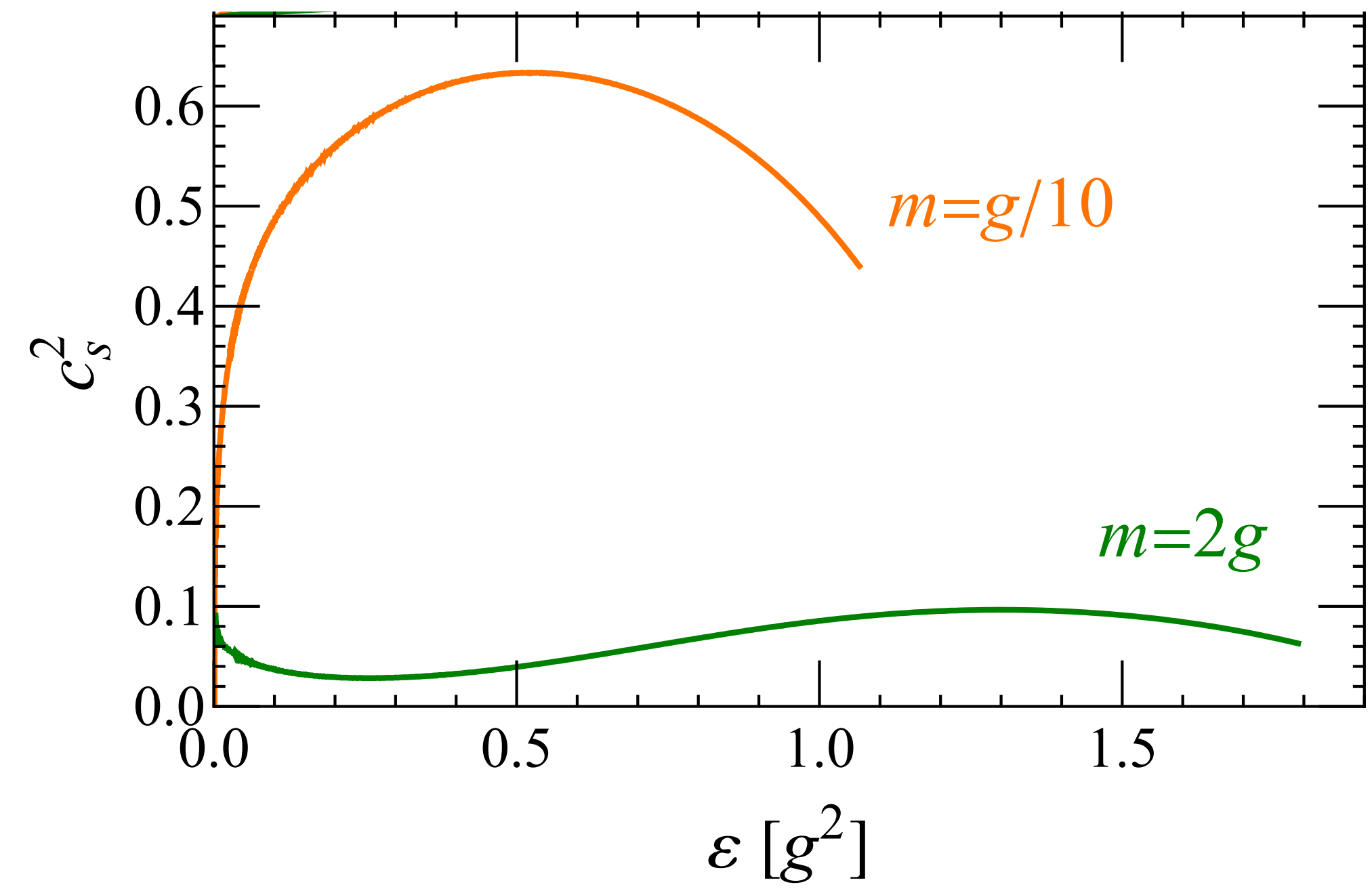
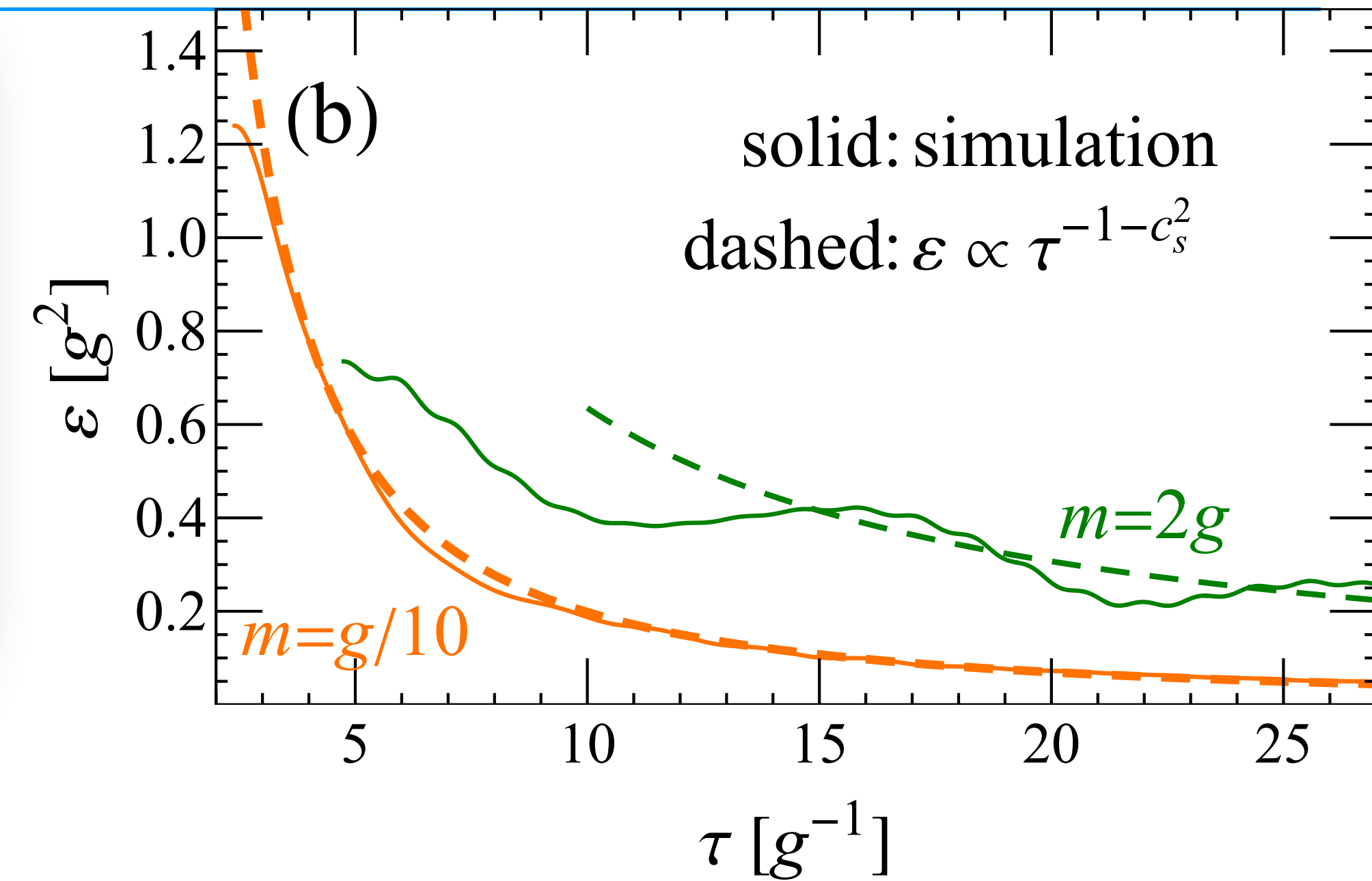
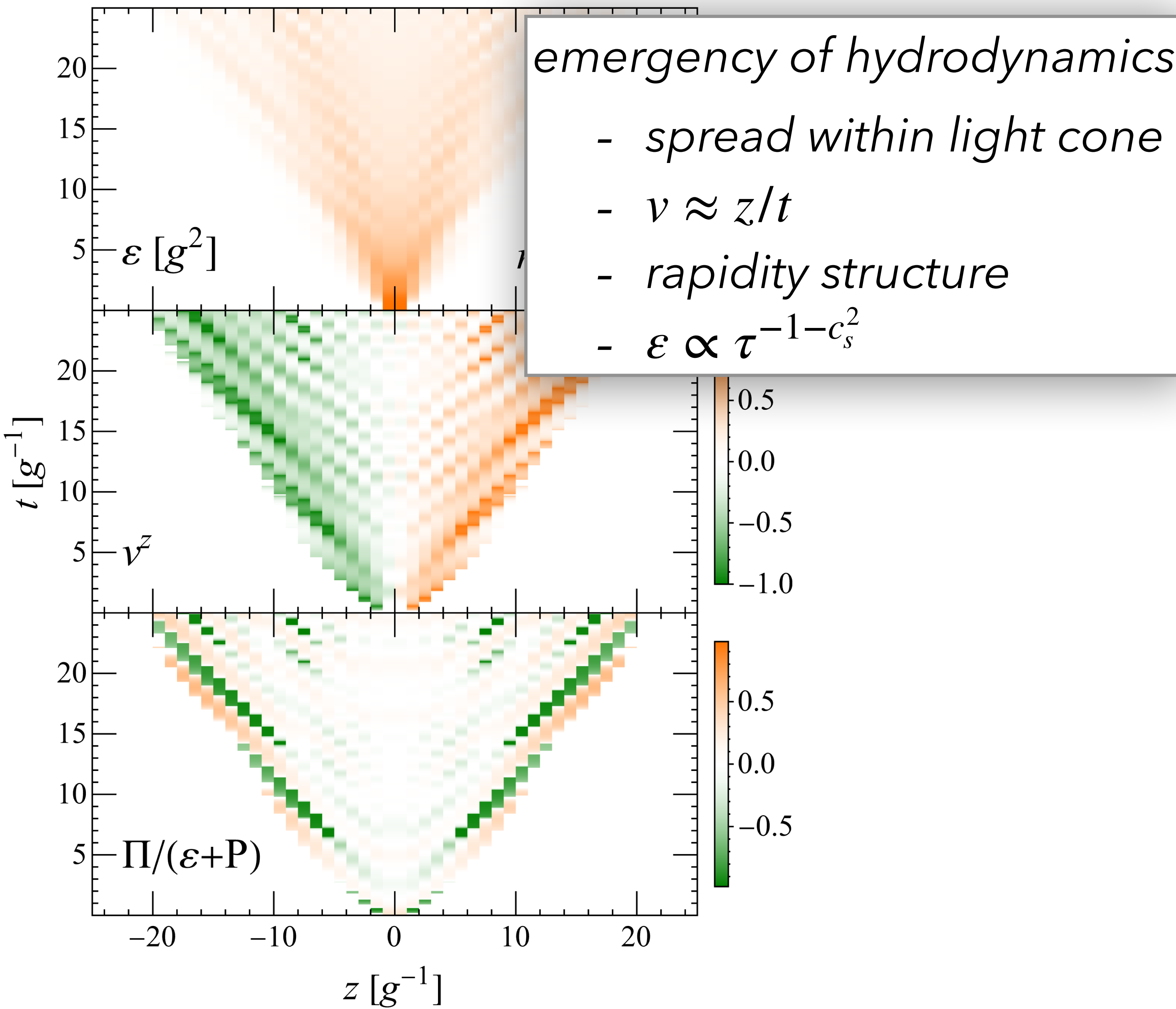
with viscosity: $\Pi \propto \tau^{-2}$

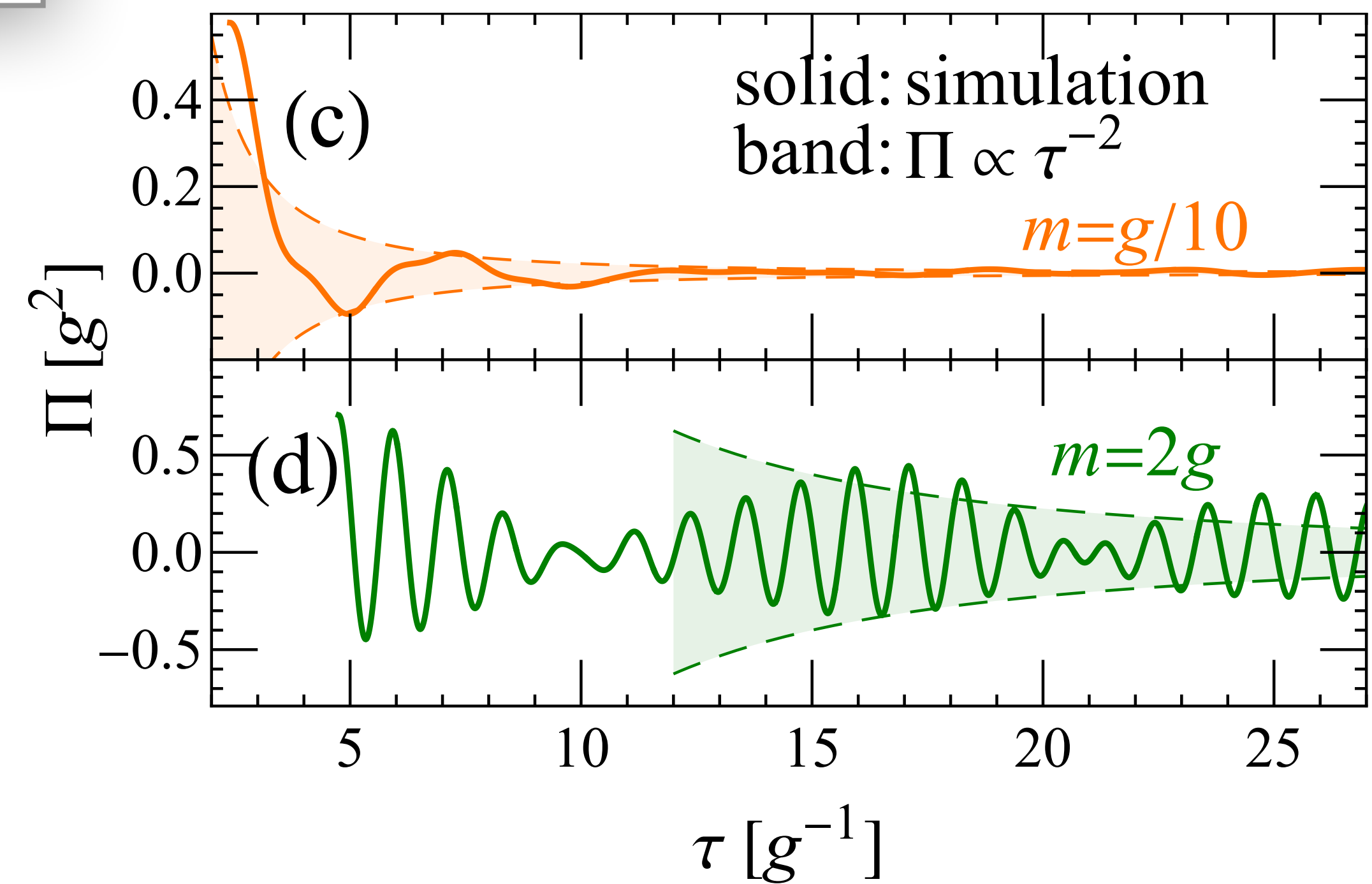
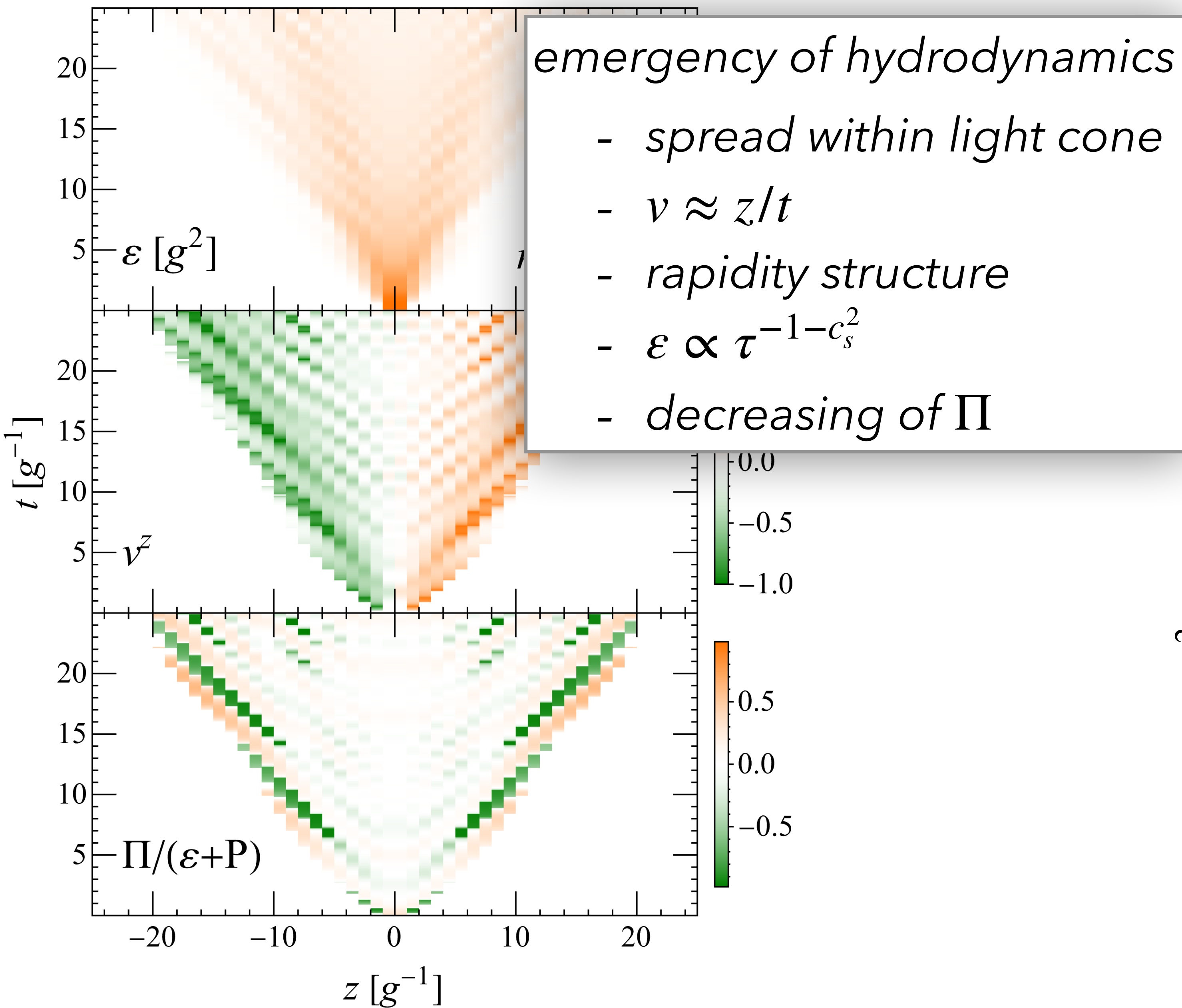


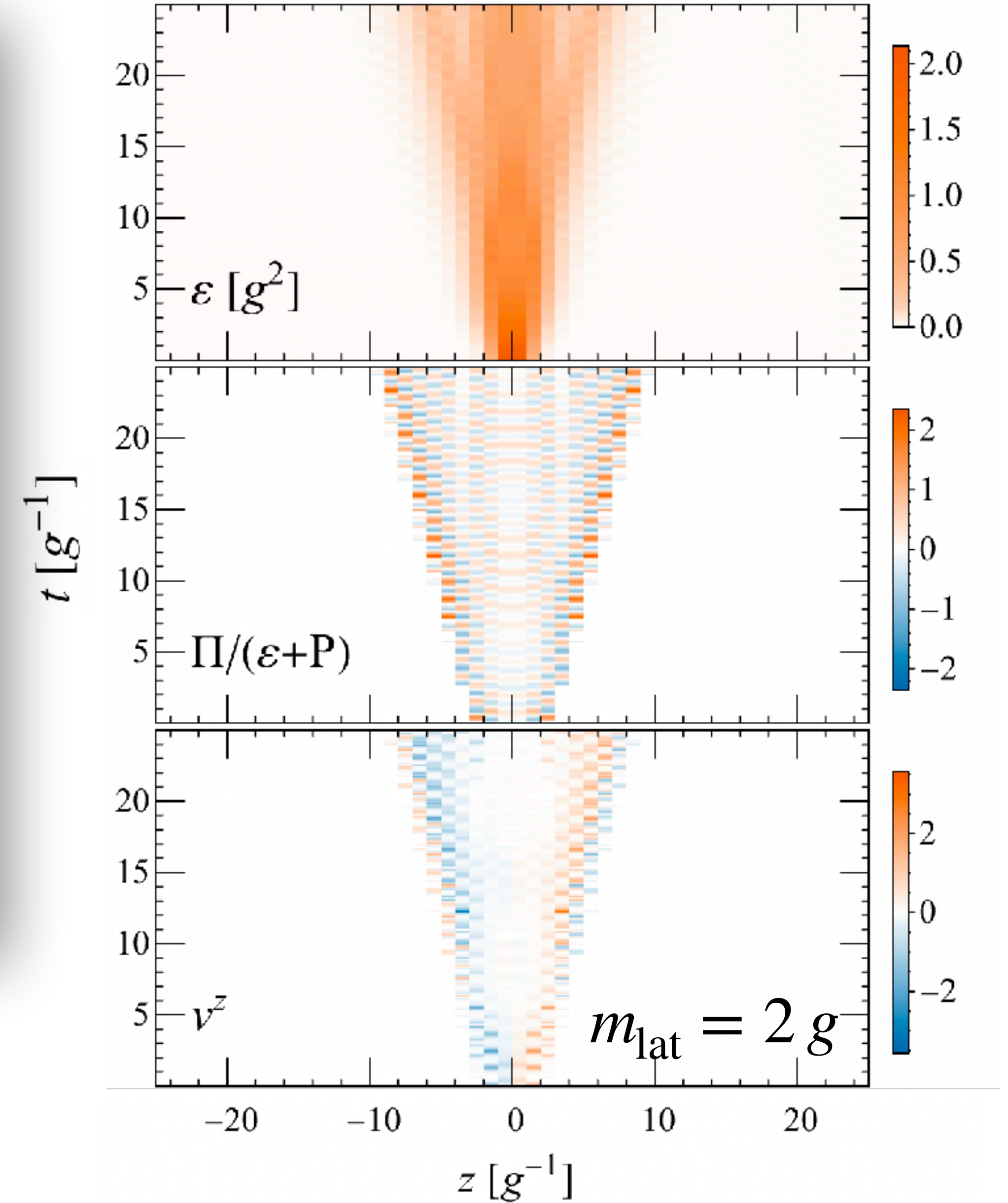
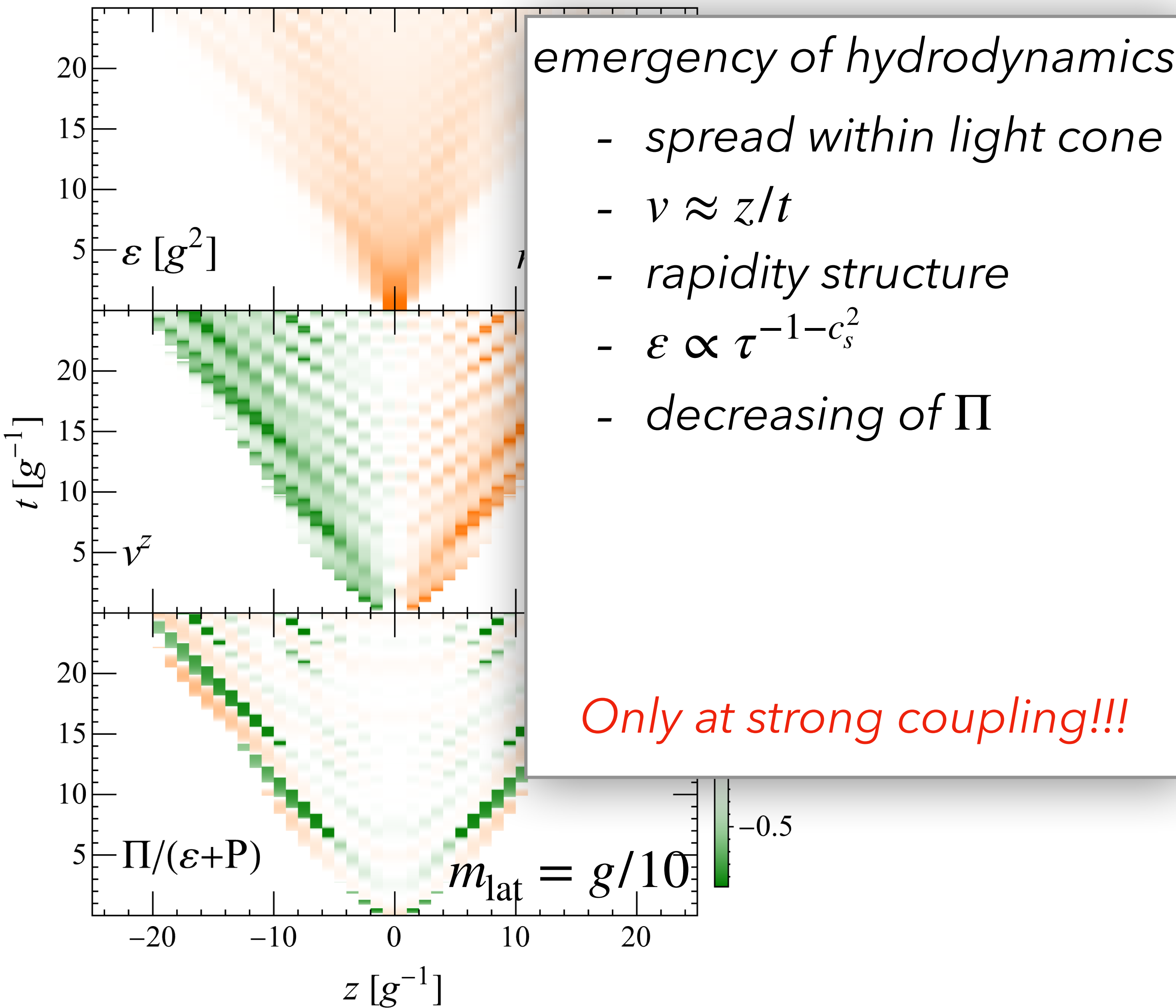
$m=g/10$









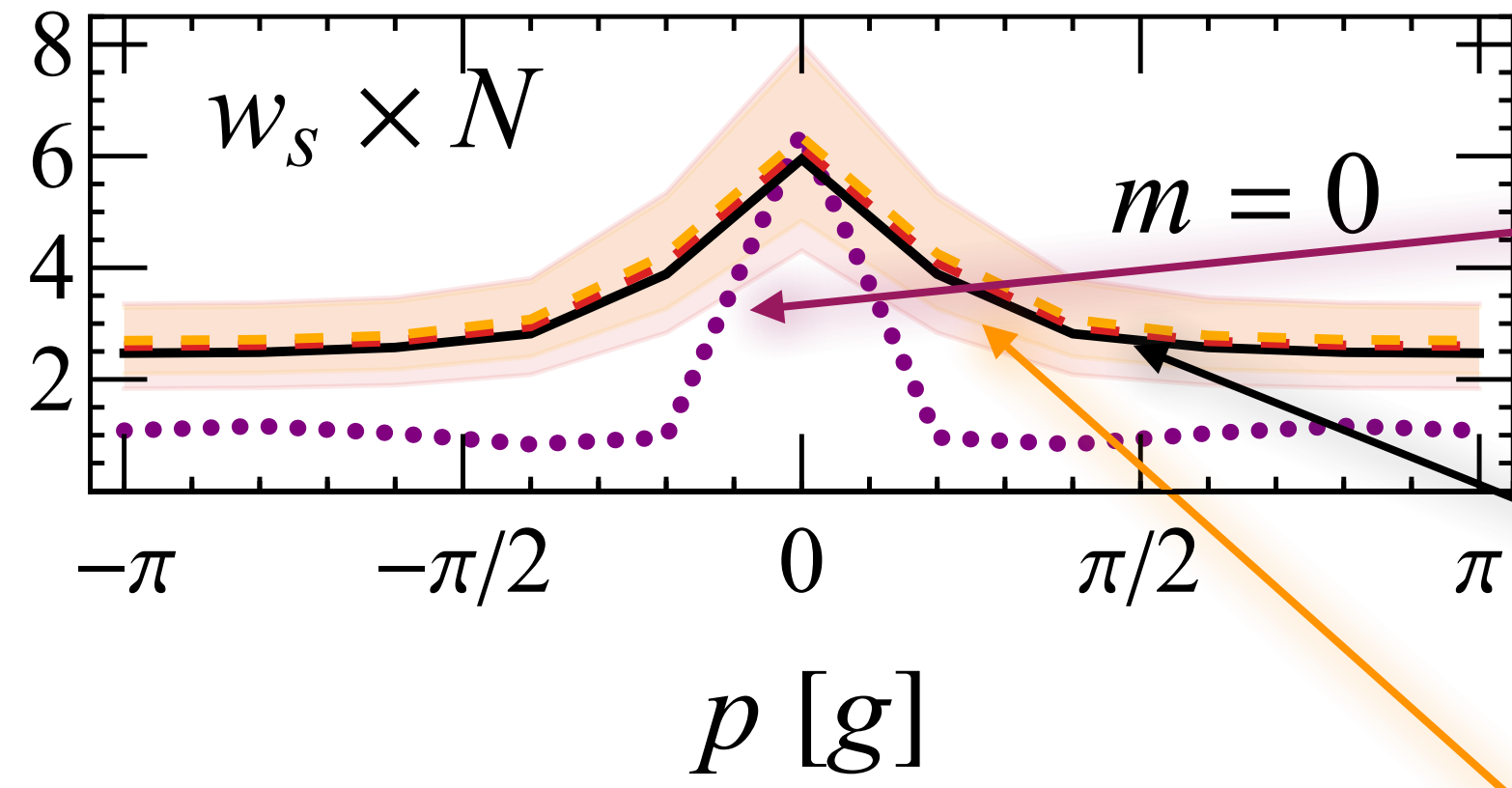


Quantum distribution function: Wigner function

$$\hat{W}_{\alpha\beta}(z, p) = \int \bar{\psi}_{\alpha}(z_{+}) U(z_{+}, z_{-}) \psi_{\beta}(z_{-}) e^{i\frac{py}{\hbar}} dy$$

$$W_{\alpha\beta}(t, p) = \int \langle \psi(t) | \hat{W}_{\alpha\beta}(z, p) | \psi(t) \rangle dz$$

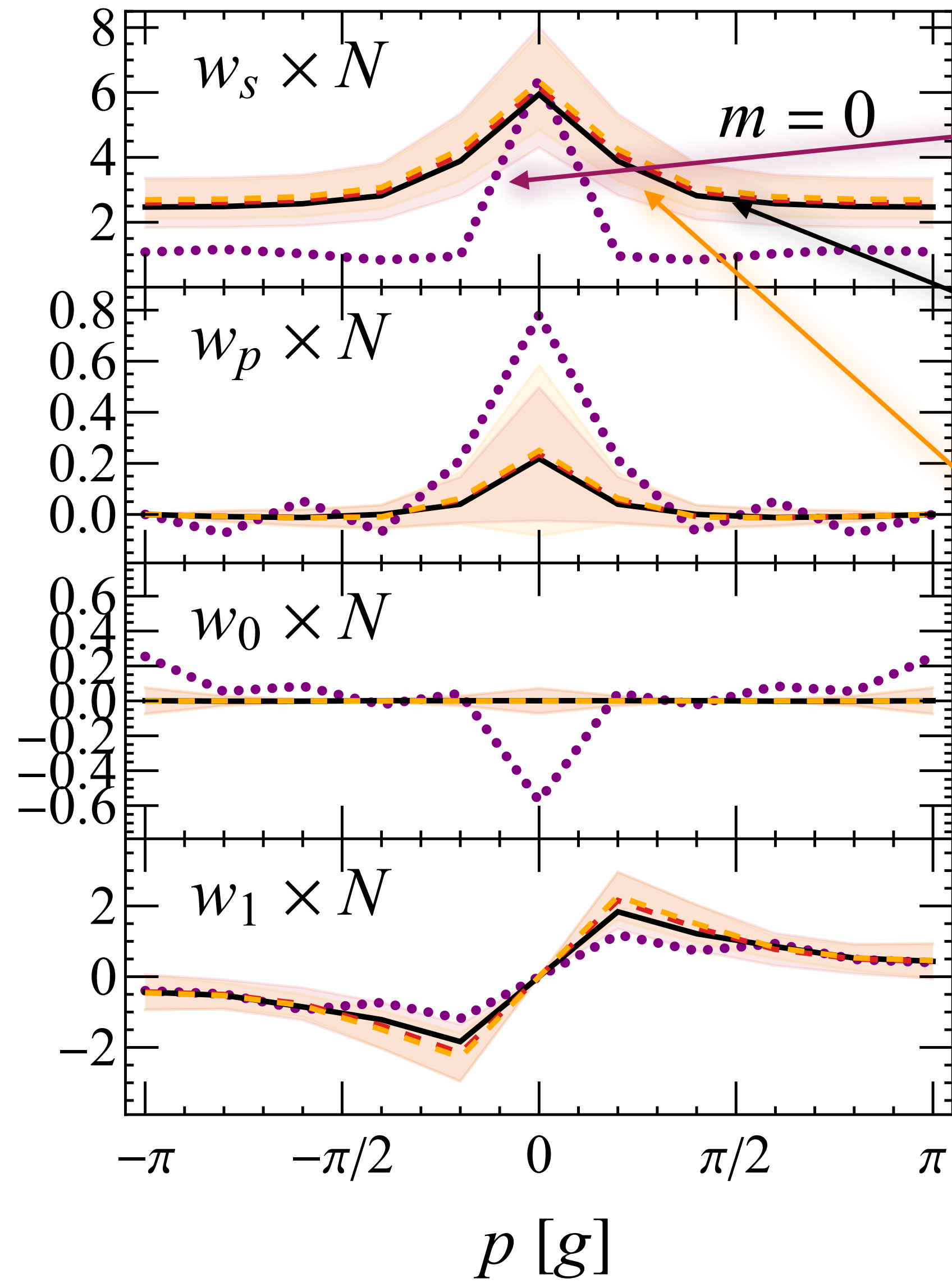
thermalization of quantum distribution function



initial condition

long-time average

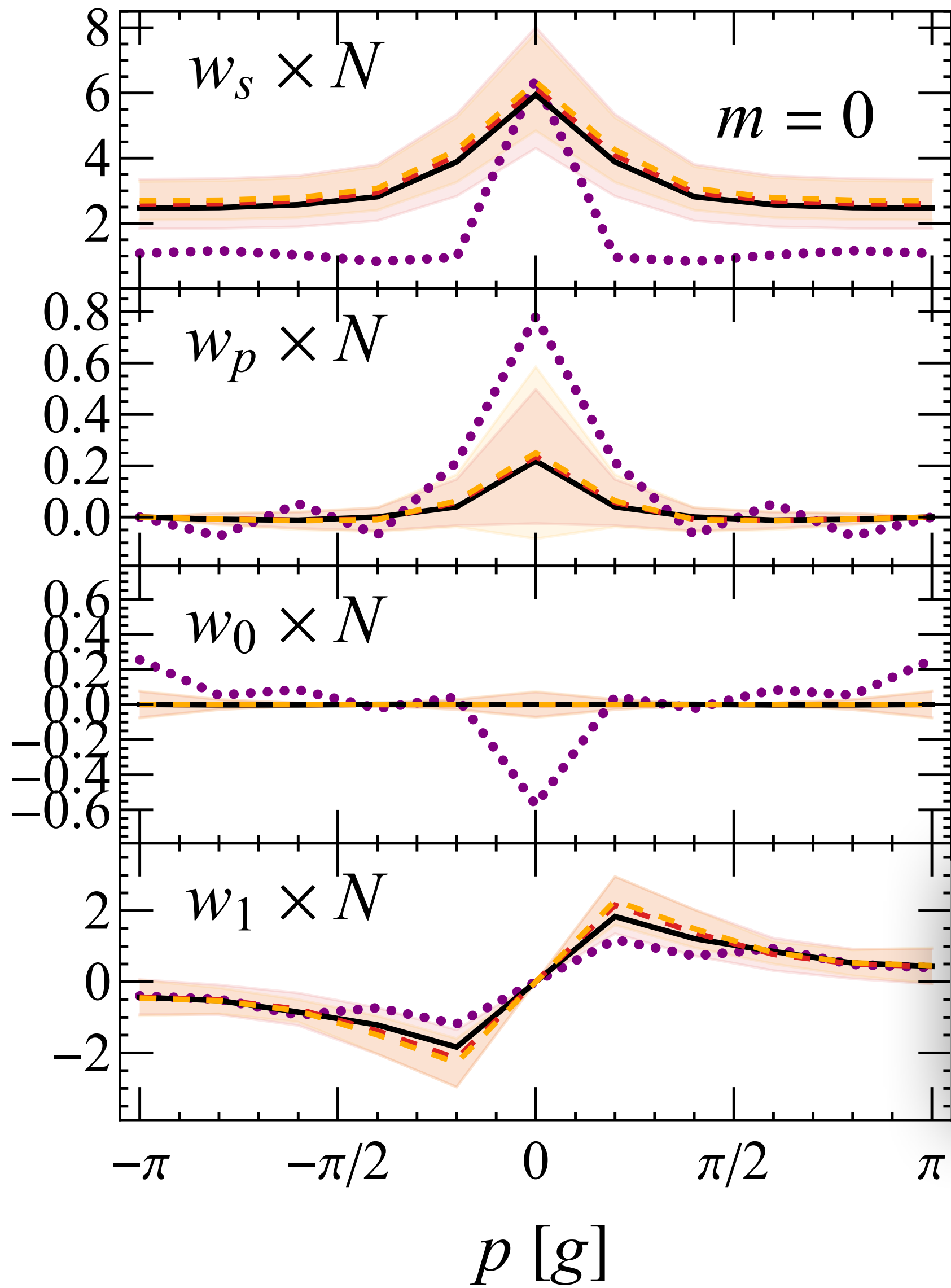
MCE/CE average



initial condition

long-time average

MCE/CE average

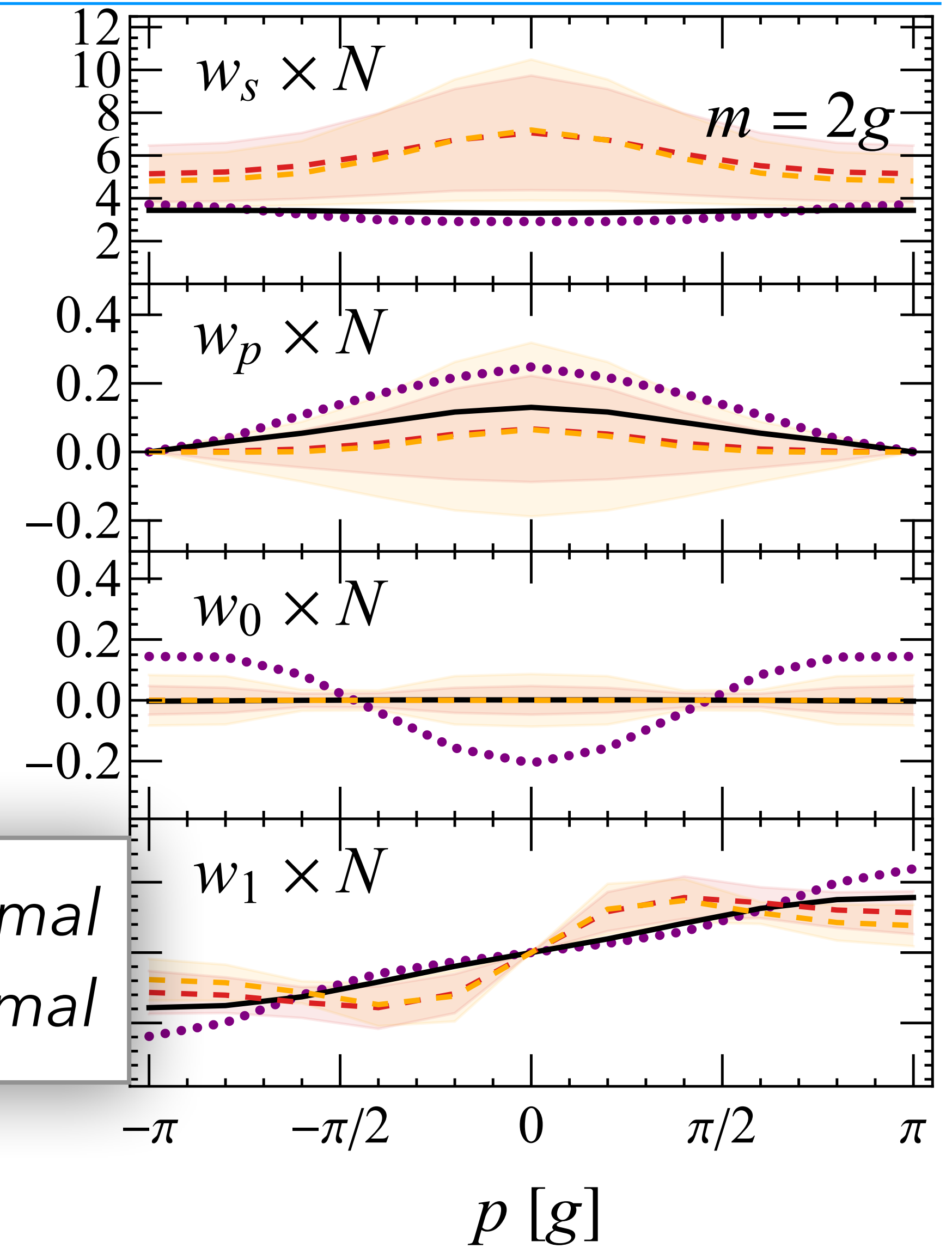


initial condition

long-time average

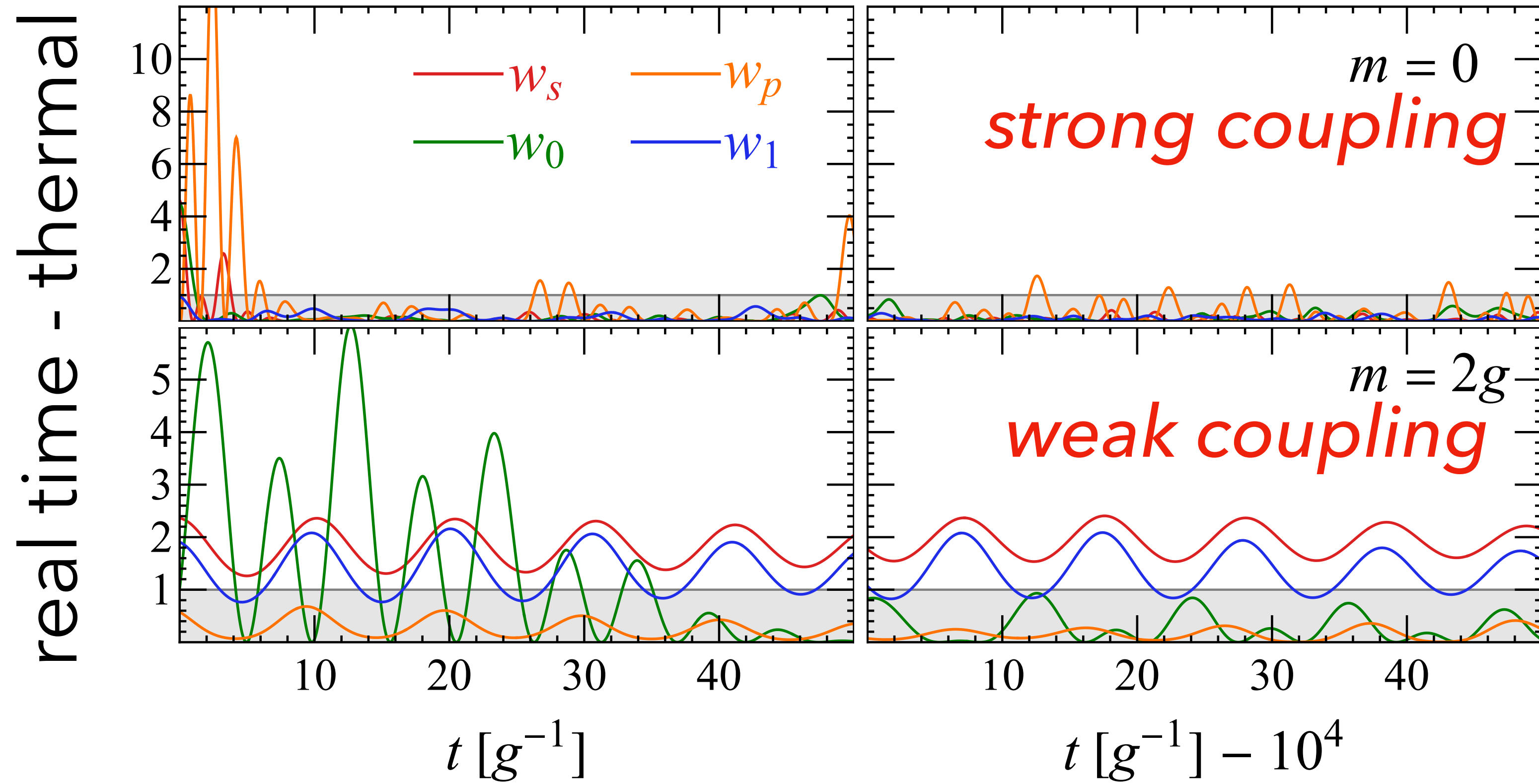
MCE/CE average

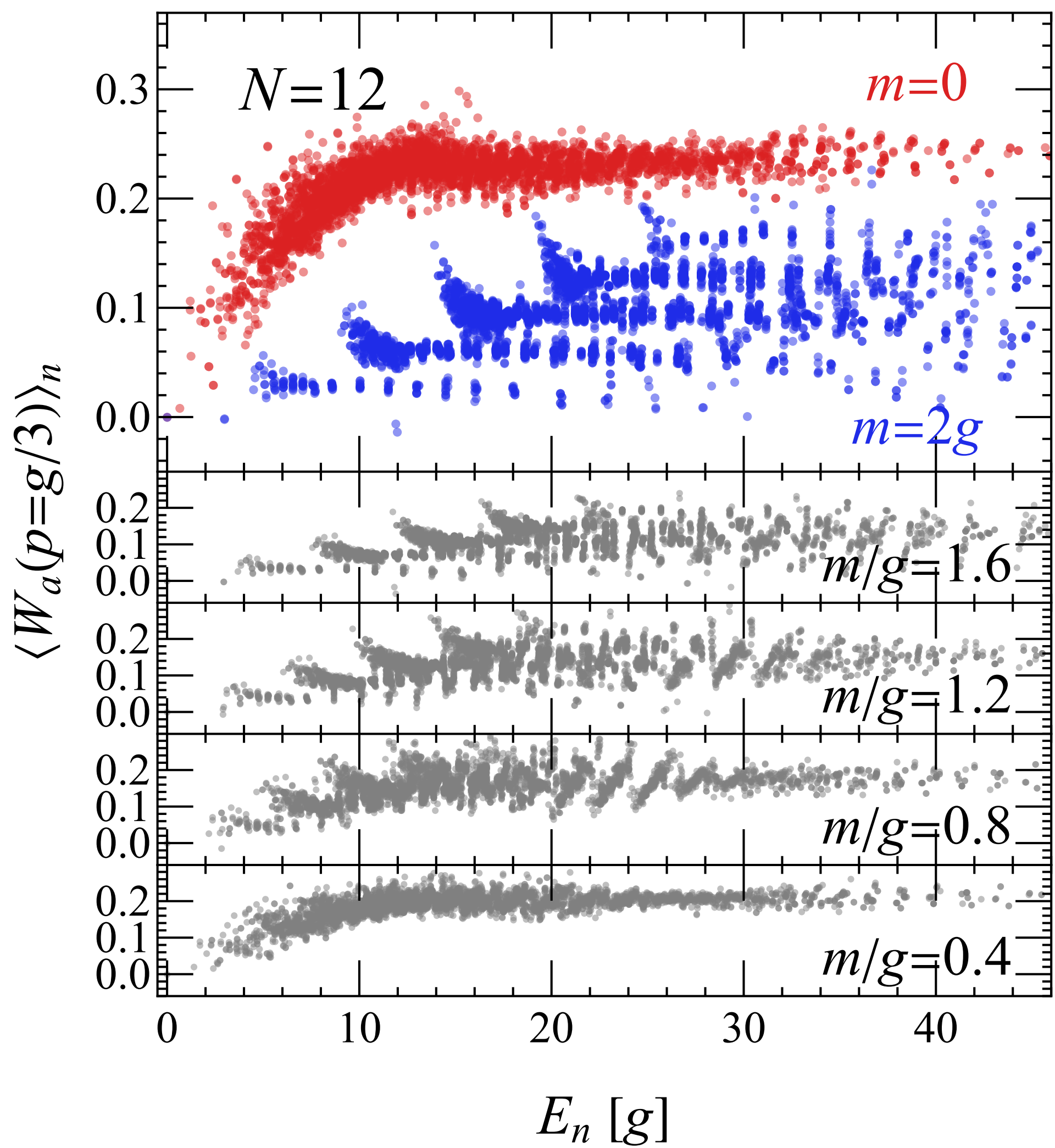
strong coupling: LTA = thermal
weak coupling: LTA \neq thermal



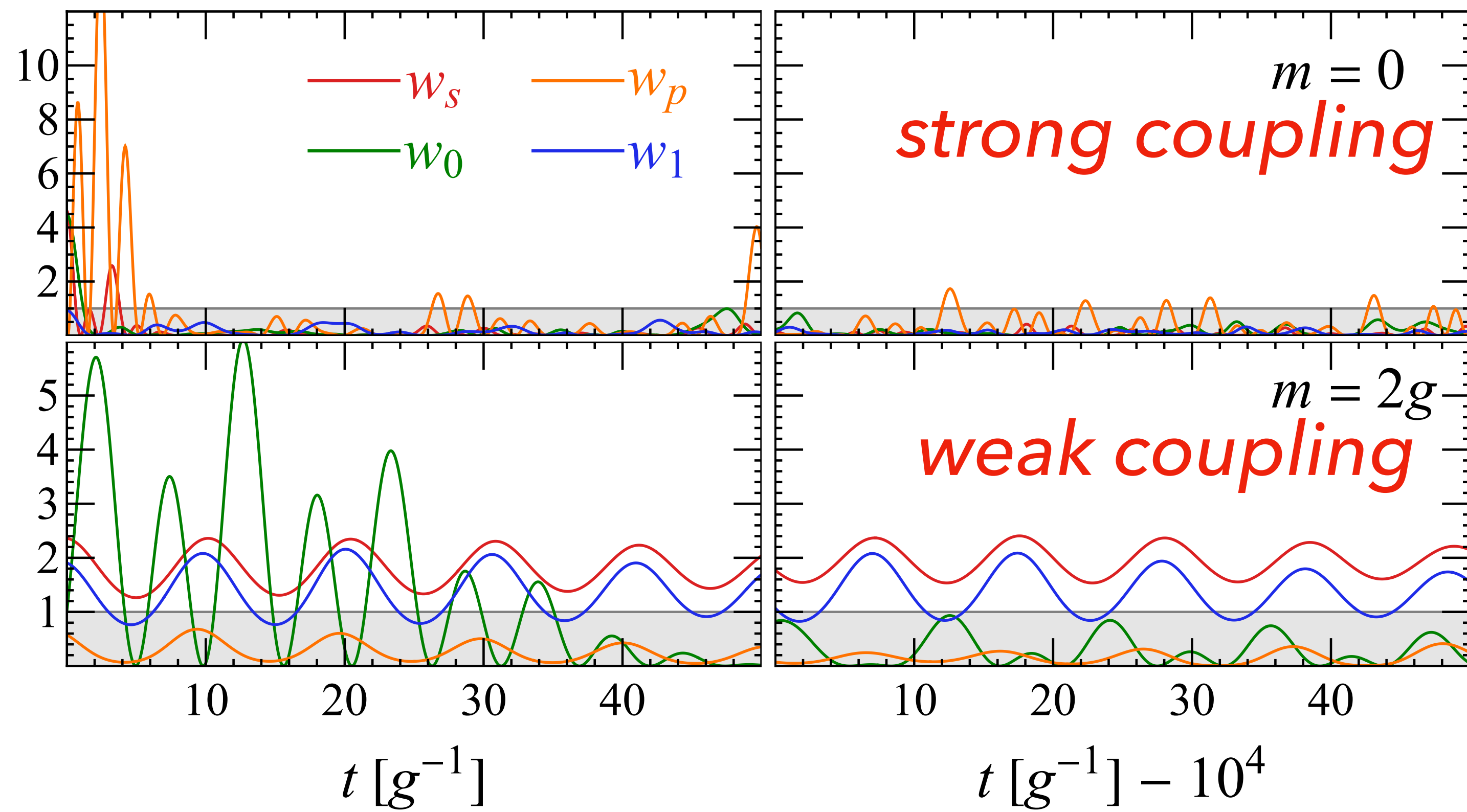
strong coupling

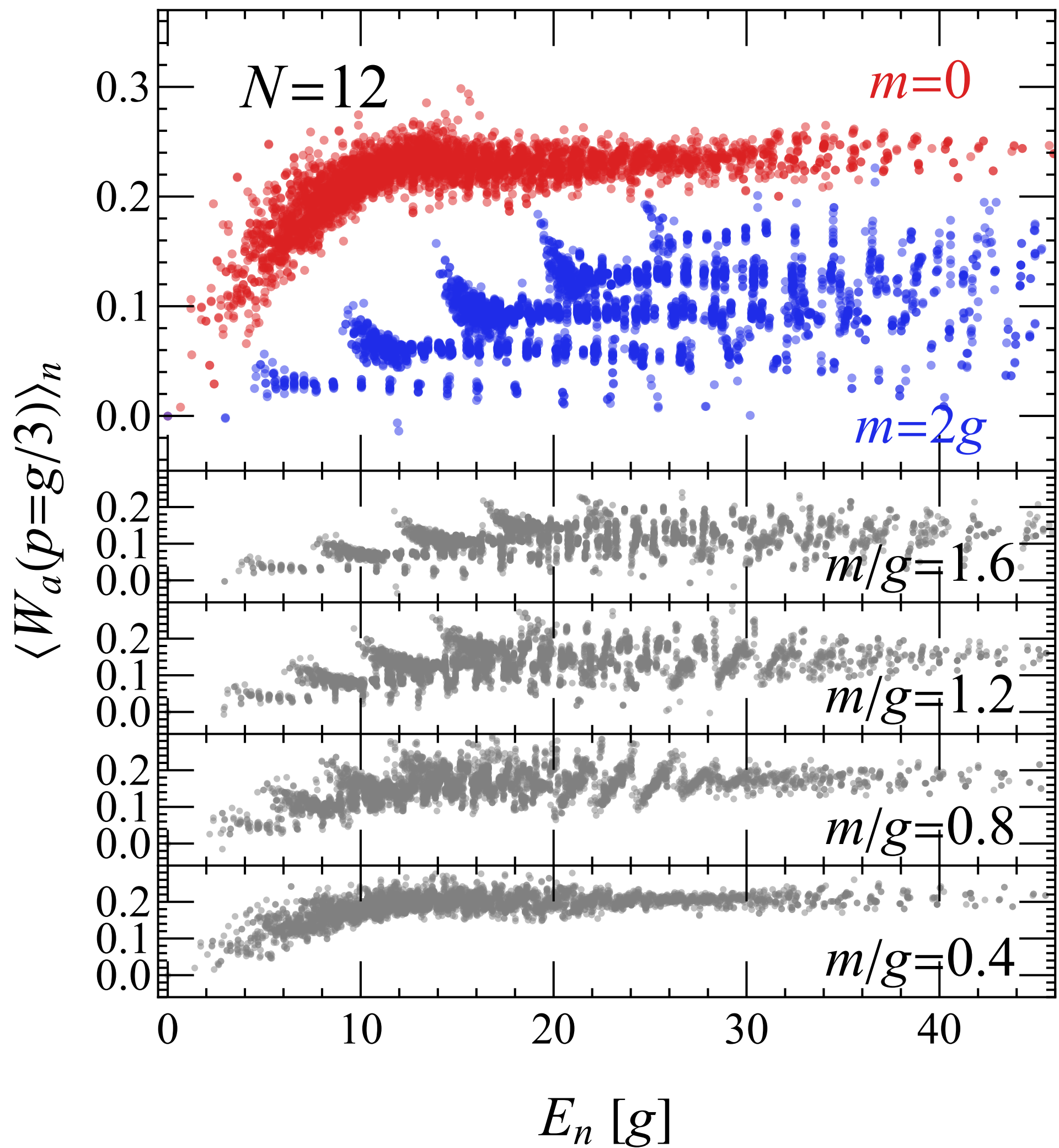
weak coupling





real time - thermal





eigenstate thermalization hypothesis

← $\langle n | \hat{O} | n \rangle \approx f_O(E_n)$

$$\sum_n p_n \langle n | \hat{O} | n \rangle \approx f_O\left(\sum_n p_n E_n\right)$$

general pure state

$$|c_n|^2$$

thermal

$$e^{-\beta E_n} / Z$$

$$\langle O \rangle_{PS} \approx \langle O \rangle_{th}$$

$$\text{if } \langle E \rangle_{PS} = \langle E \rangle_{th}$$

Real-time non-perturbative quantum evolution

- emergence of hydrodynamics
 - light-cone structure/rapidity structure
 - energy density, bulk pressure, velocity
- thermalization of quantum distribution function
 - time scale consistent with hydrodynamics
- full quantum state accessible, ready to test different microscopic derivations of quantum hydrodynamics — ideals are welcome!