The promise and challenges of photonic quantum metrology

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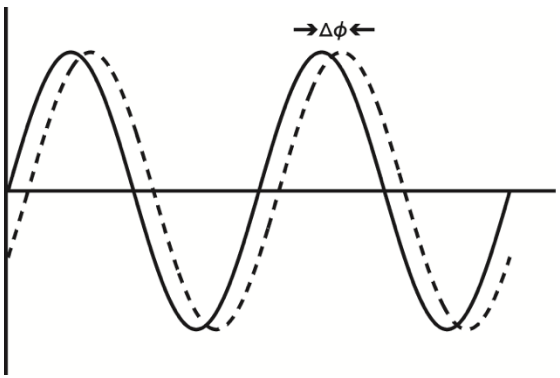
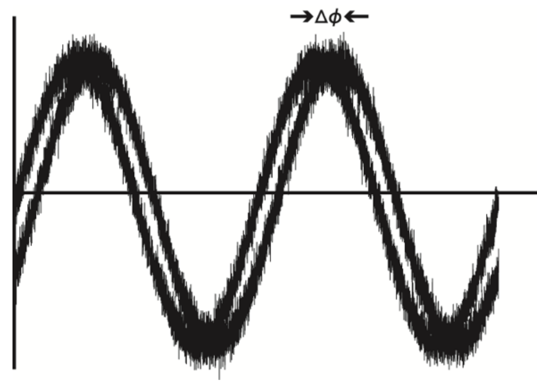
Precision metrology

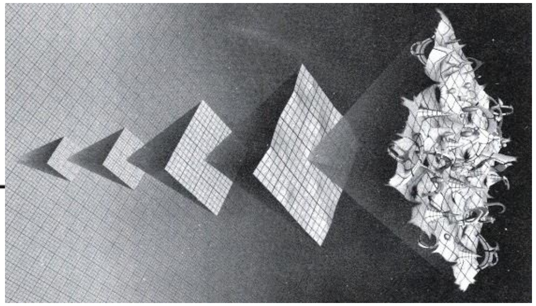
Ultimate precision is determined by quantum fluctuations

Ideal situation

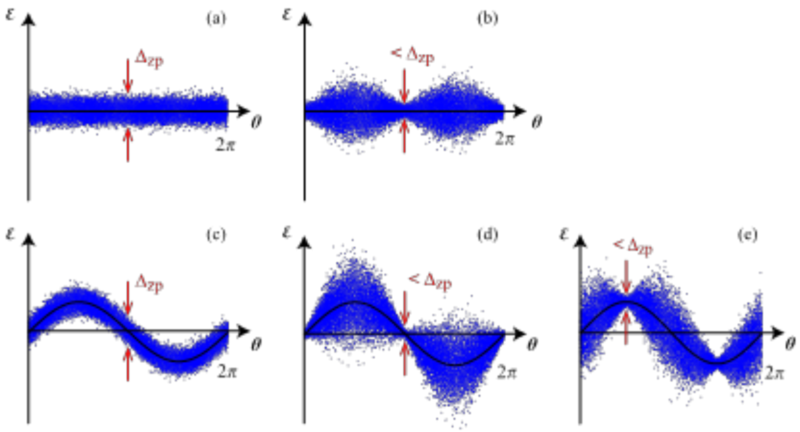


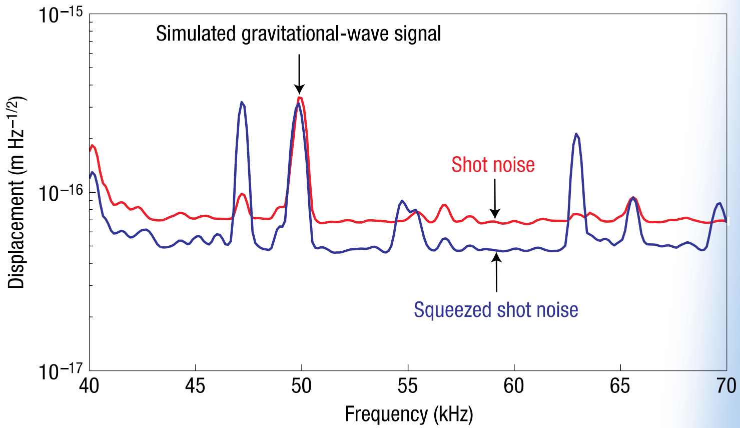
Light field is quatized





Squeezing the quantum noise





C. Caves, Phys. Rev. D, 23, 1693 (1981), K. Goda, Nature Phys. 4, 472 (2008)

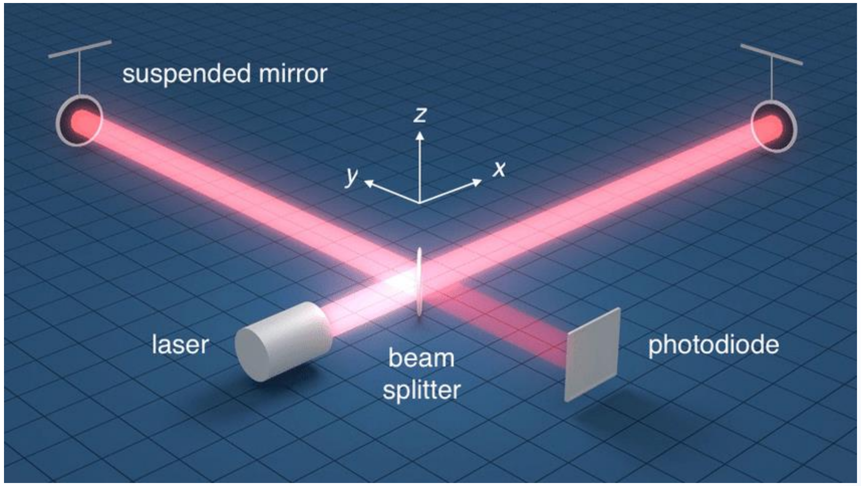
Quantum metrology and sensing

$$\hat{\rho} \longrightarrow \boxed{\mathcal{E}_\theta} \longrightarrow \hat{\rho}_\theta$$

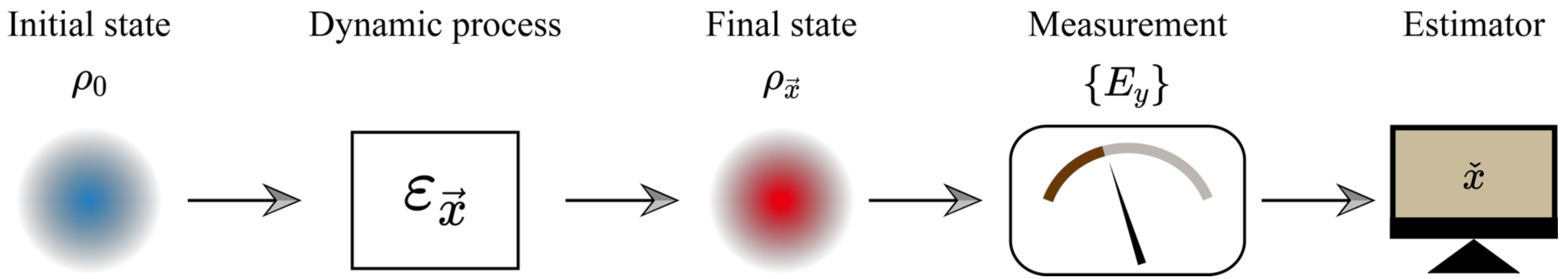
- Quantum hypothesis test

Nature Photon. 12, 724 (2018)
F. Xu et al, Phys. Rev. Lett. 127. 040504 (2021)

- Quantum parameter estimation



Quantum parameter estimation

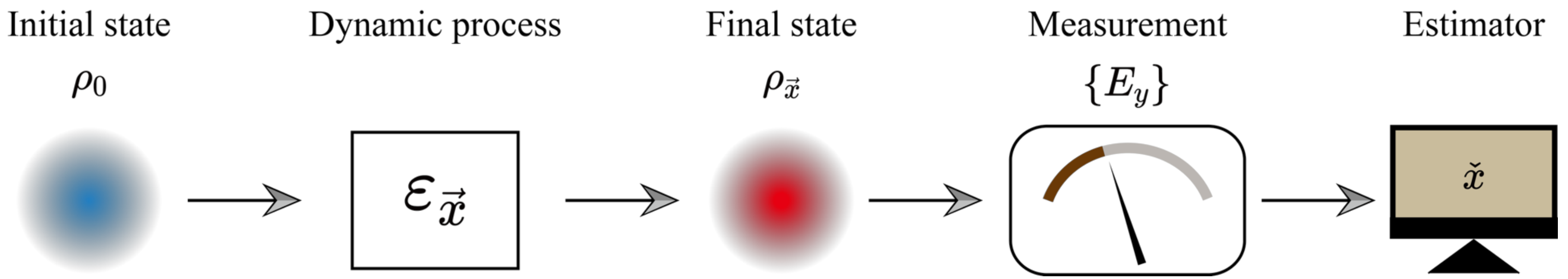


(Quantum) Cramér-Rao bound $$\mathrm{var}(\check{x}) \geq \frac{1}{mF(x)} \geq \frac{1}{mQ(\rho_x)}$$ m: number of trials

Fisher information $$F(x) = \sum_y \frac{1}{p(y|x)} \left[\frac{\partial p(y|x)}{\partial x} \right]^2$$ $p(y|x) = Tr(\rho_x E_y)$

Quantum Fisher information $$Q(\rho_x) = \mathrm{Tr}[\rho_x \hat{L}_x^2]$$

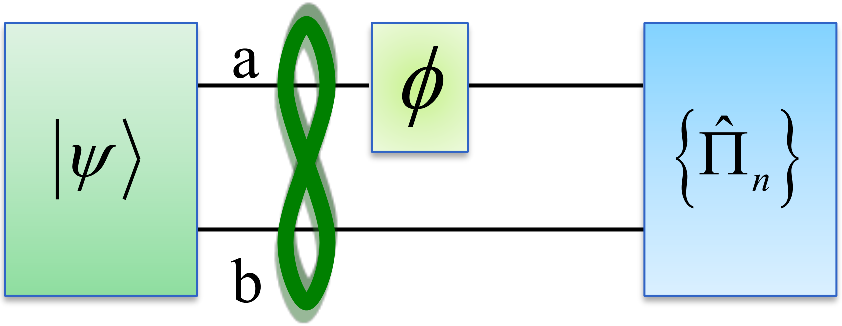
Classical and Heisenberg limits

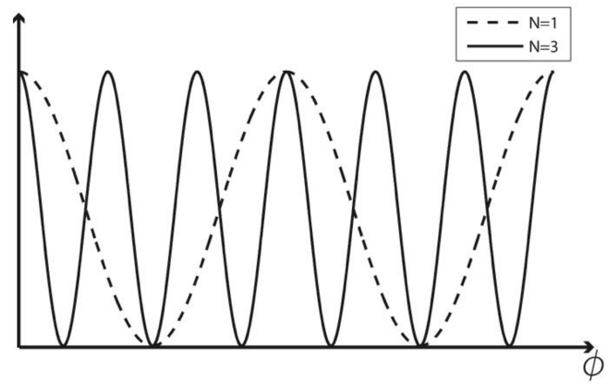


Standard quantum limit (SQL, classical limit): $\Delta^2 x \geq 1/2mN$
Heisenberg limit (HL): $\Delta^2 x \geq 1/2mN^2$

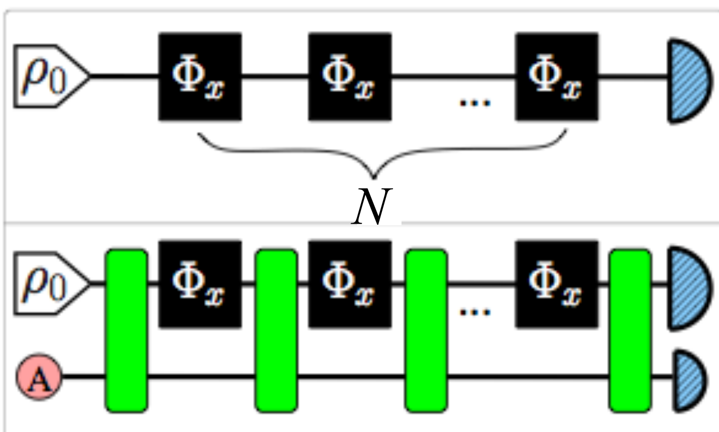
m: number of trials, N: number of particles per probe state
Total resources: mN

Parallel versus sequential strategies





$$|\psi(\phi)\rangle_{N00N} = \left(e^{iN\phi}|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b\right)/\sqrt{2}$$



$$|\psi(\phi)\rangle_{seq} = \left(e^{iN\phi}|1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b\right)/\sqrt{2}$$

In the **ideal** situation, both strategies achieve Heisenberg limit with resource N.

Nature Photon. **5**, 222 (2011)

Outline

- Loss-tolerant quantum interferometry
- Precision metrology using weak measurements
- Multi-parameter quantum metrology

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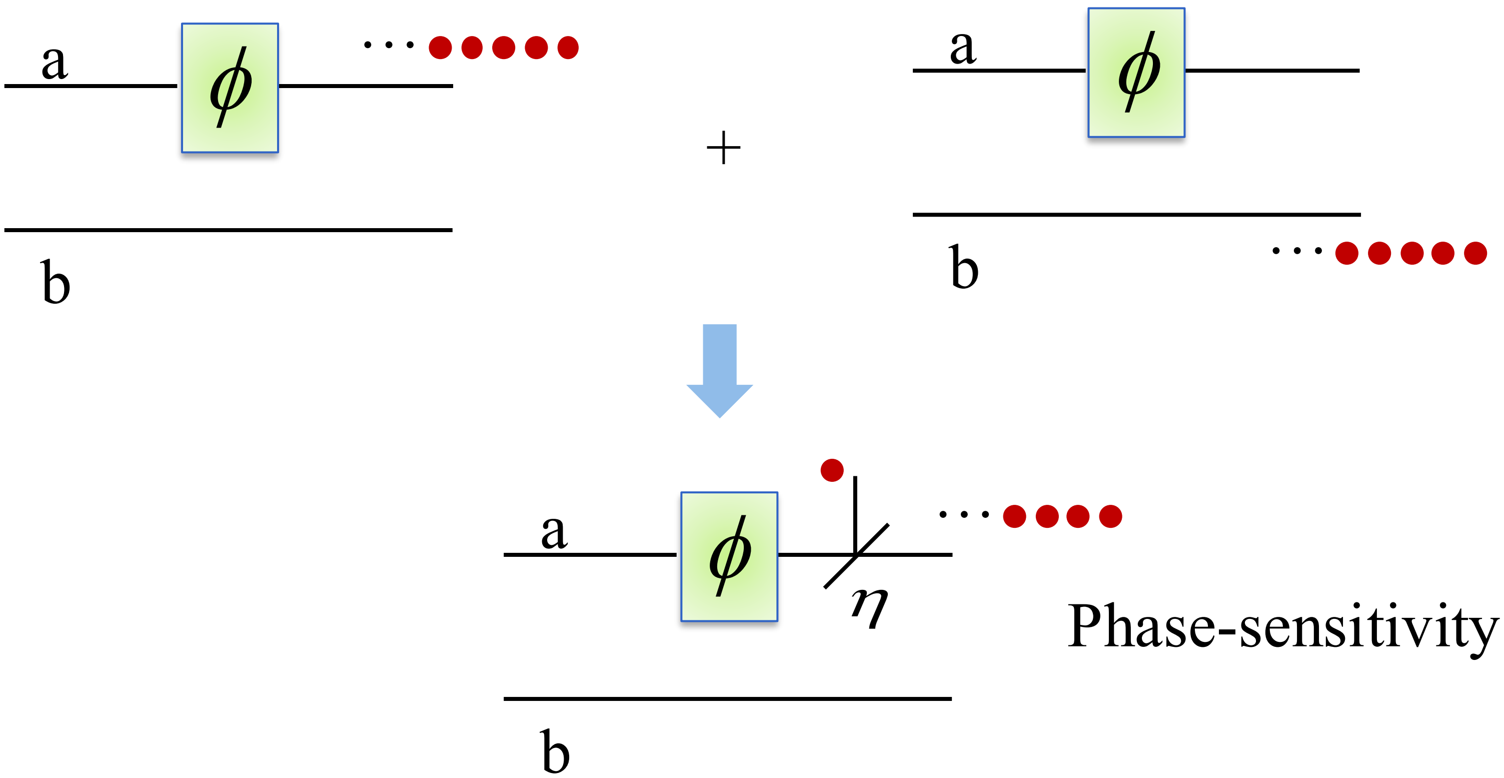
Problems with photonic N00N state

- Hard to generate: no deterministic way yet
- Very sensitive to photon loss



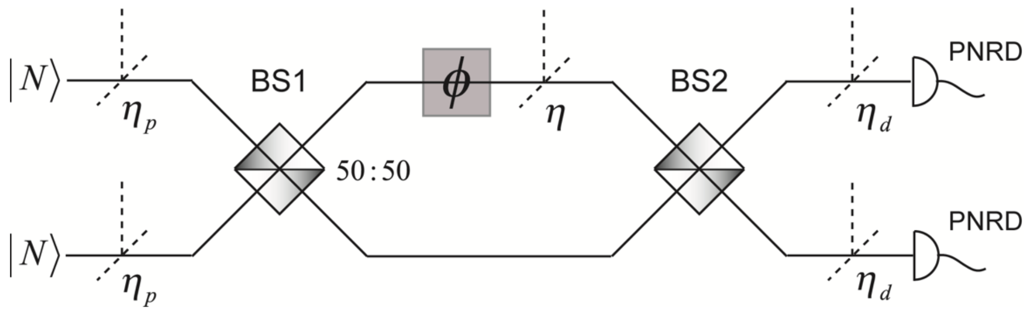
Phase-sensitivity lost

Probability of no-photon loss: η^N

Real-world quantum metrology: loss tolerance

Holland-Burnett state

- Heralded generation of N-photon Fock state
- Photon-number-resolving measurement



LETTERS

https://doi.org/10.1038/s41566-017-0011-5

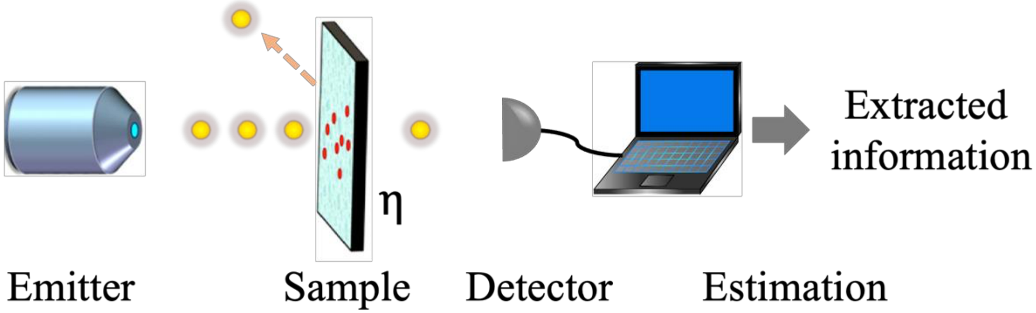
nature photonics

Unconditional violation of the shot-noise limit in photonic quantum metrology

Sergei Slussarenko[1], Morgan M. Weston[1], Helen M. Chrzanowski[1,2], Lynden K. Shalm[3], Varun B. Verma[3], Sae Woo Nam[3] and Geoff J. Pryde[1*]

Dose-limited quantum metrology

Resource: total number of photons (**dose**) that interact with the sample.



Maximize the precision under a dose constraint $d \leq d_{th}$ is equivalent to maximize Fisher information per dose:

$$\xi = \frac{F}{d} \qquad \delta\phi \geq \frac{1}{\sqrt{d_{th}\,\xi}}$$

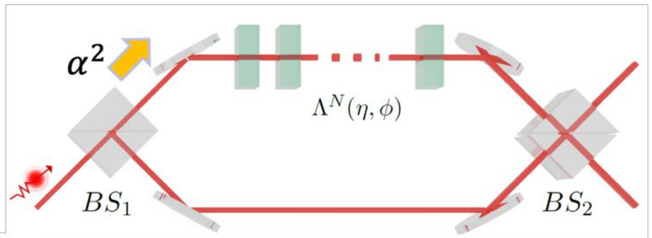
In the presence of loss η, the quantum limit is

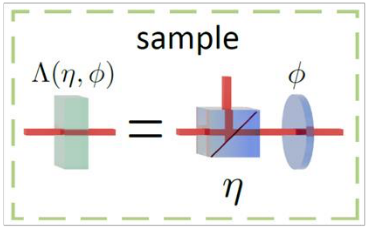
$$\xi_{QL} = \frac{4\eta}{1-\eta}$$

Nature Physics 7, 406 (2011)

Sequential strategy

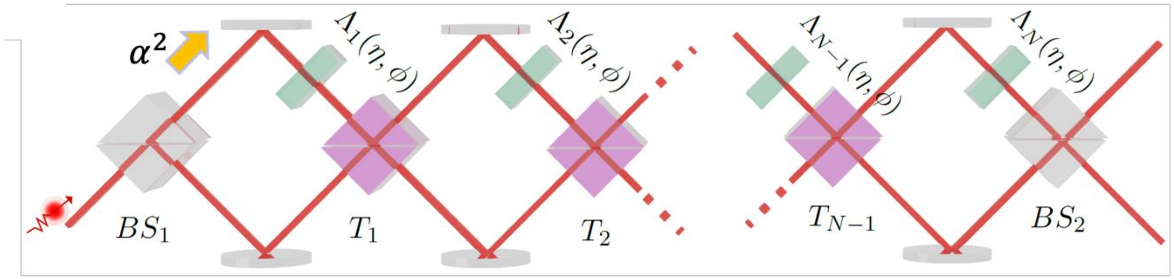
Standard multi-pass (MP) strategy



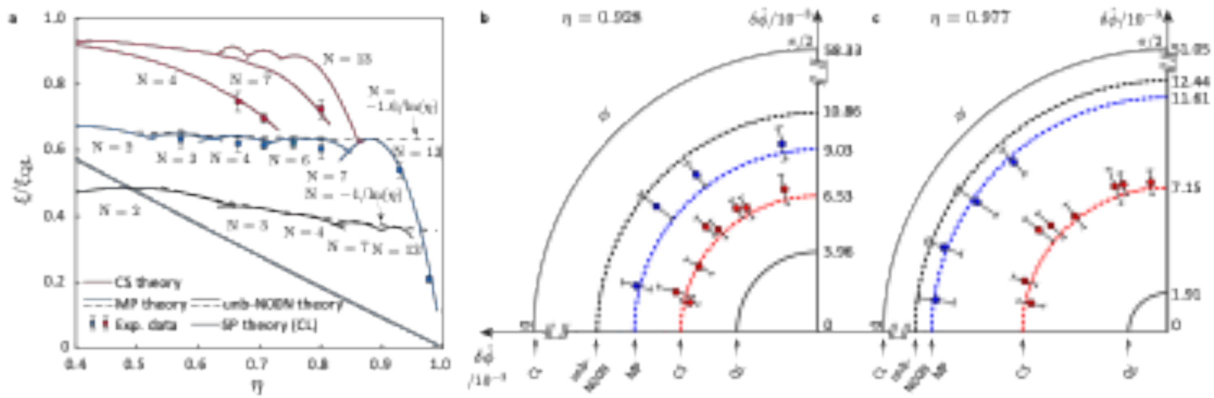


$$d = |\alpha|^2 \sum_{k=0}^{N} \eta^k$$

Control-enhanced sequential (CS) strategy



$$d = \sum_{k=0}^{N} |\alpha_k|^2$$



$$\xi_{CS} \leq \frac{4\eta}{1-\eta}(1-\eta^N)$$

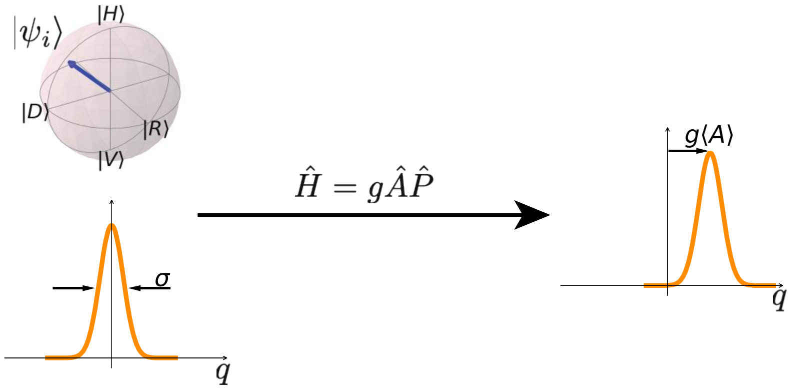
arXiv: 2203.10137
Q. Yu et al, Optica (accepted)

Outline

- Loss-tolerant quantum interferometry
- Precision metrology using weak measurements
- Multi-parameter quantum metrology

Weak Measurement and Weak Value

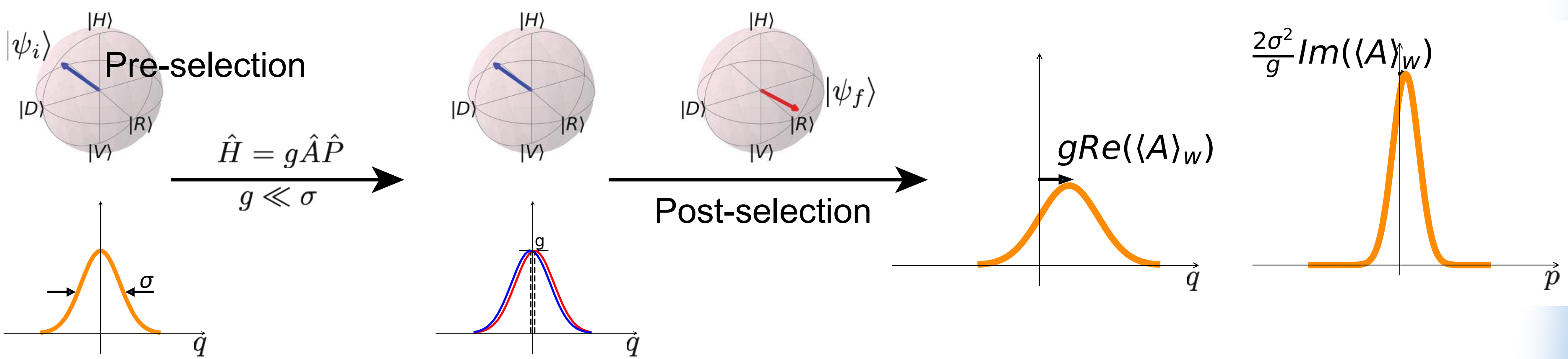
Von Neumann Measurement



$$\langle A \rangle = \langle \psi_i | A | \psi_i \rangle$$

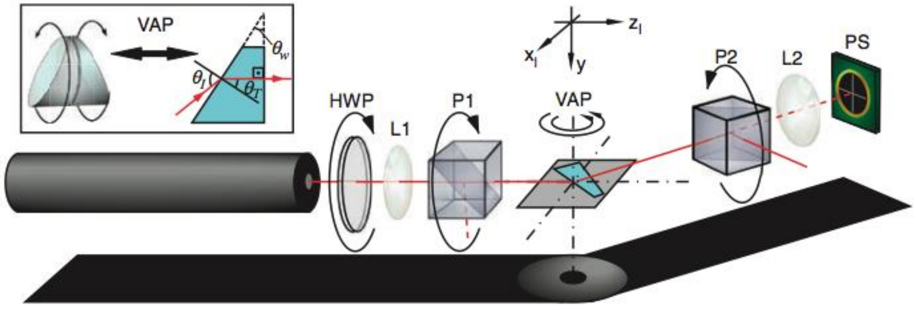
Weak measurement and weak value

$$\langle A \rangle_w = \frac{\langle \psi_f | A | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

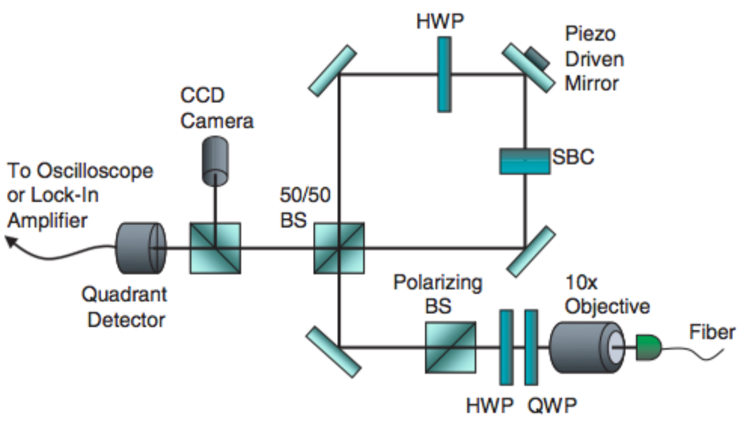


Weak-value amplification

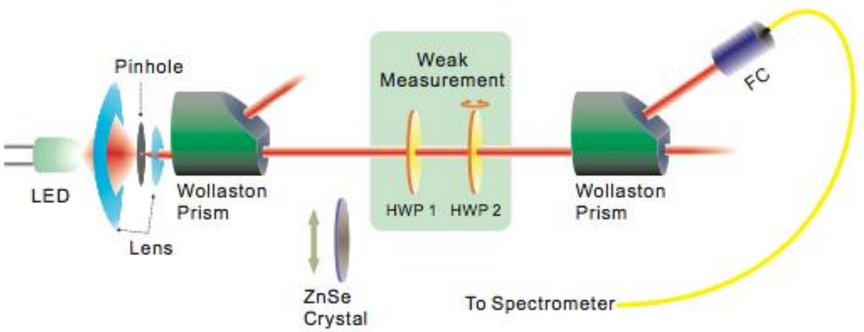
$$g \rightarrow gS_w \qquad S_w = \frac{\langle\psi_f|\hat{S}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$$



Science 319, 787 (2008)



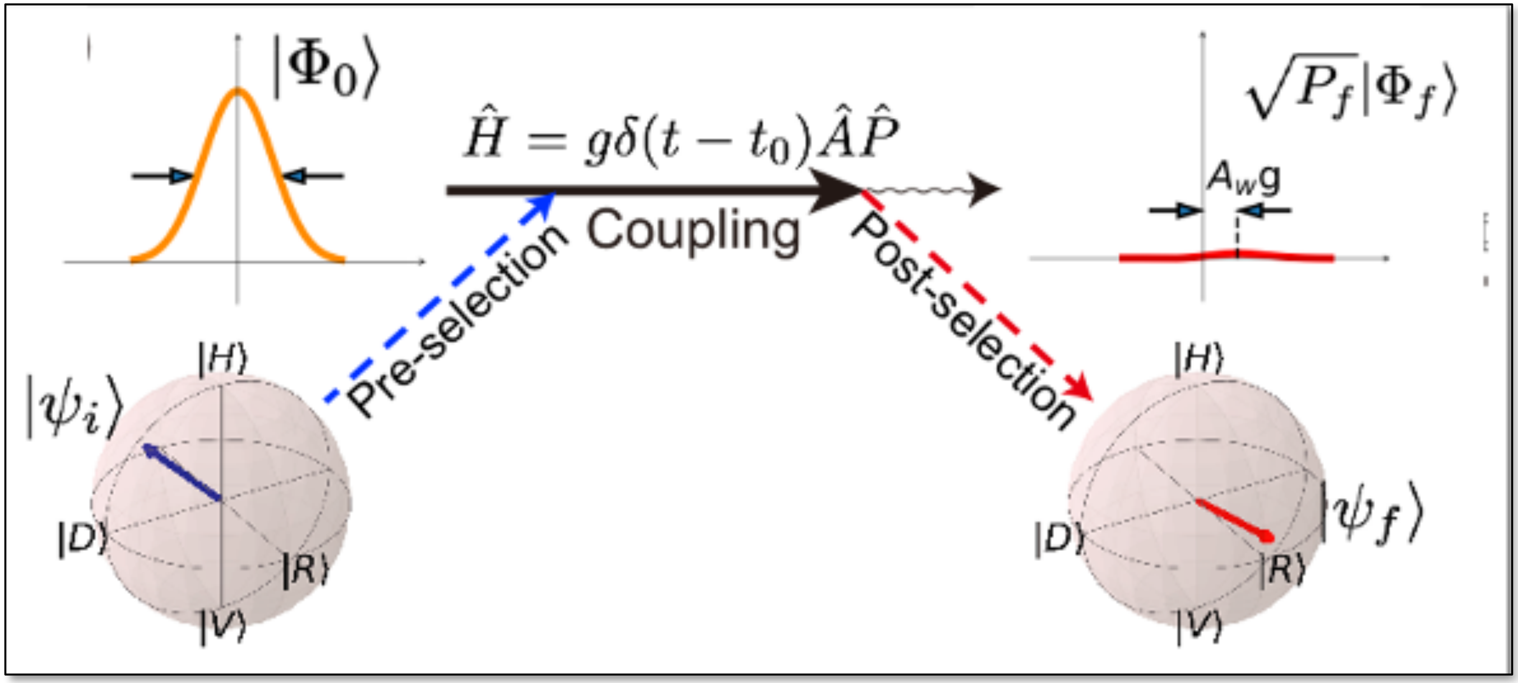
Phys. Rev. Lett. 102, 173601 (2009)



Phys. Rev. Lett. 111, 033604 (2013)

The amplification comes at a cost of signal intensity. The overall signal-to-noise ratio is not improved (PRA 84, 052111 (2011)).

Precision of weak measurement



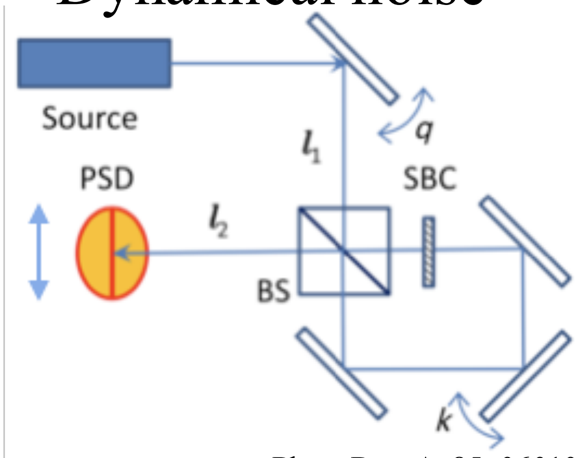
$$F_{WVA} \leq F_{tot} = p_d Q_d + (1 - p_d) Q_r + F_p \leq Q_j$$

In the ideal situation, weak measurement does not increase precision.

L. Zhang et al, Phys. Rev. Lett. 114, 210801 (2015)

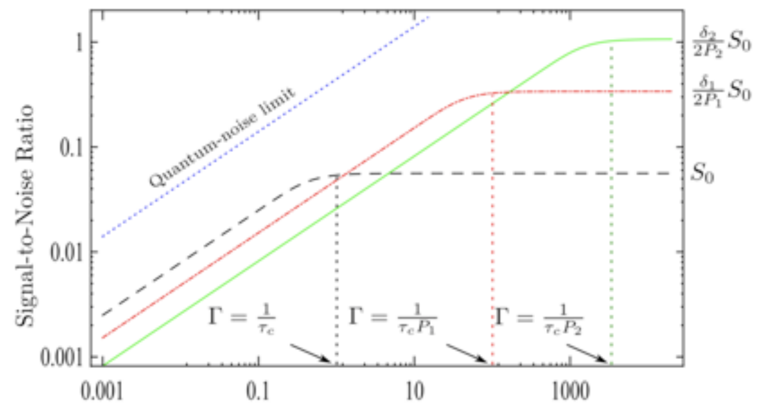
Practical systems are imperfect

Dynamical noise



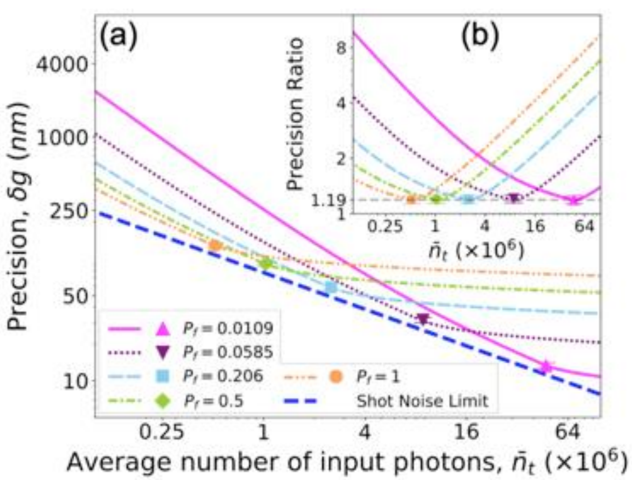
Phys. Rev. A 85, 060102(R) (2012)
Phys. Rev. X 4, 011031 (2014)

Correlated noise



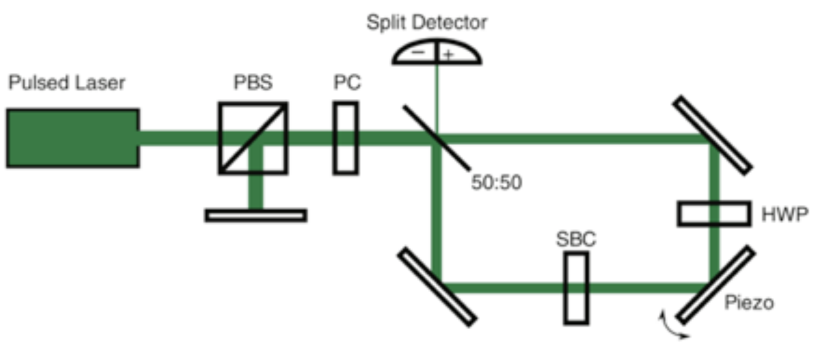
Phys. Rev. X 4, 011031 (2014)
Phys. Rev. Lett. 107, 133603 (2011)

Detector saturation



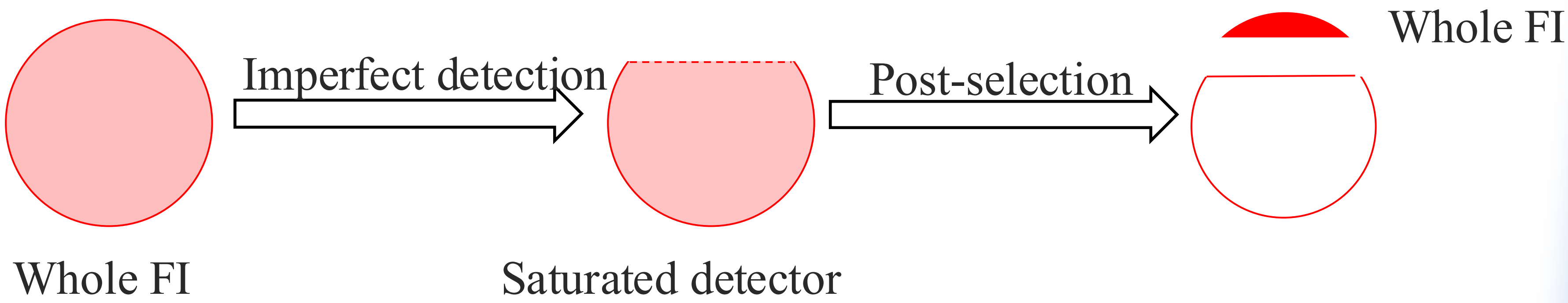
Phys. Rev. Lett 118, 070802 (2017)
Phys. Rev. Lett 125, 080501 (2020)

Photon recycling

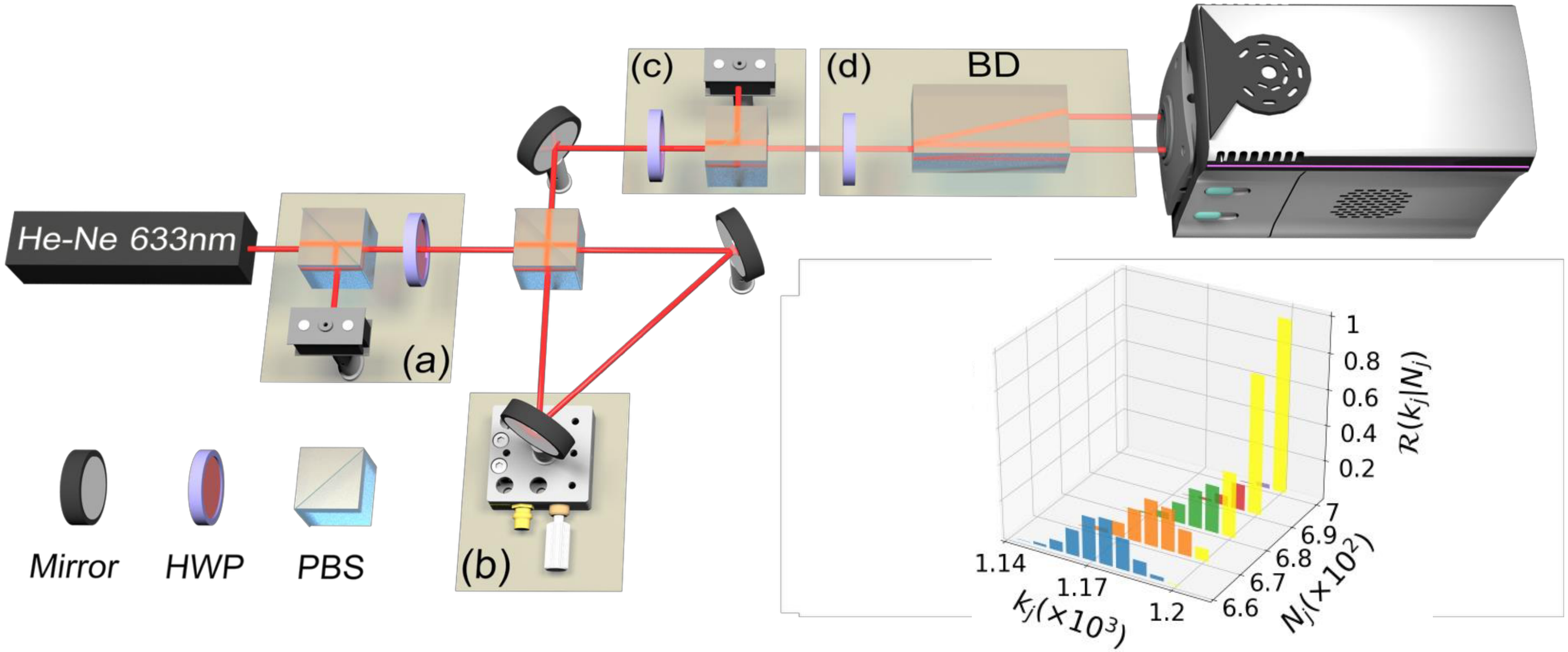


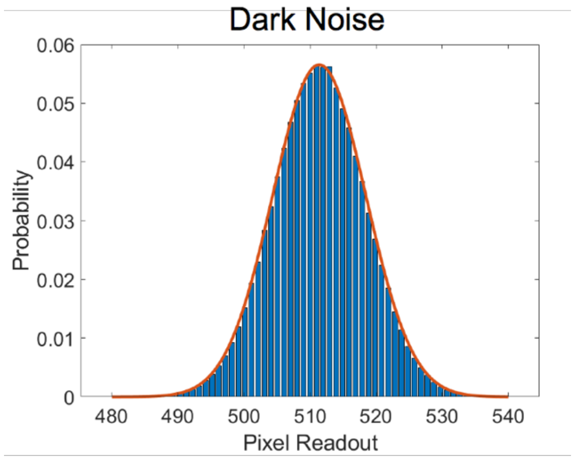
Phys. Rev. A 88, 023821 (2013)
Phys. Rev. X 4, 011031 (2014)
Phys. Rev. Lett.117, 230801 (2016)

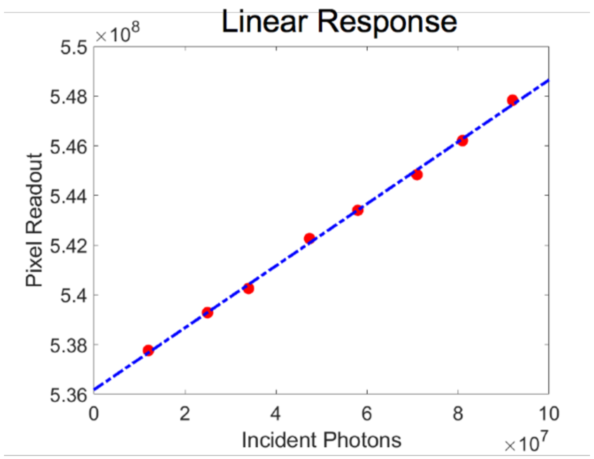
Effects with practical detectors

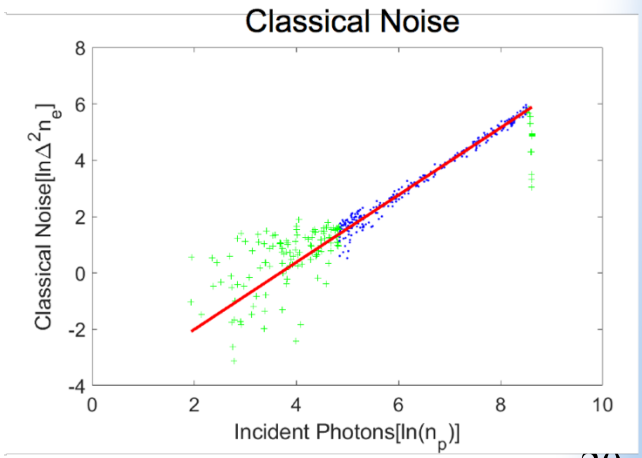


Experimental setup

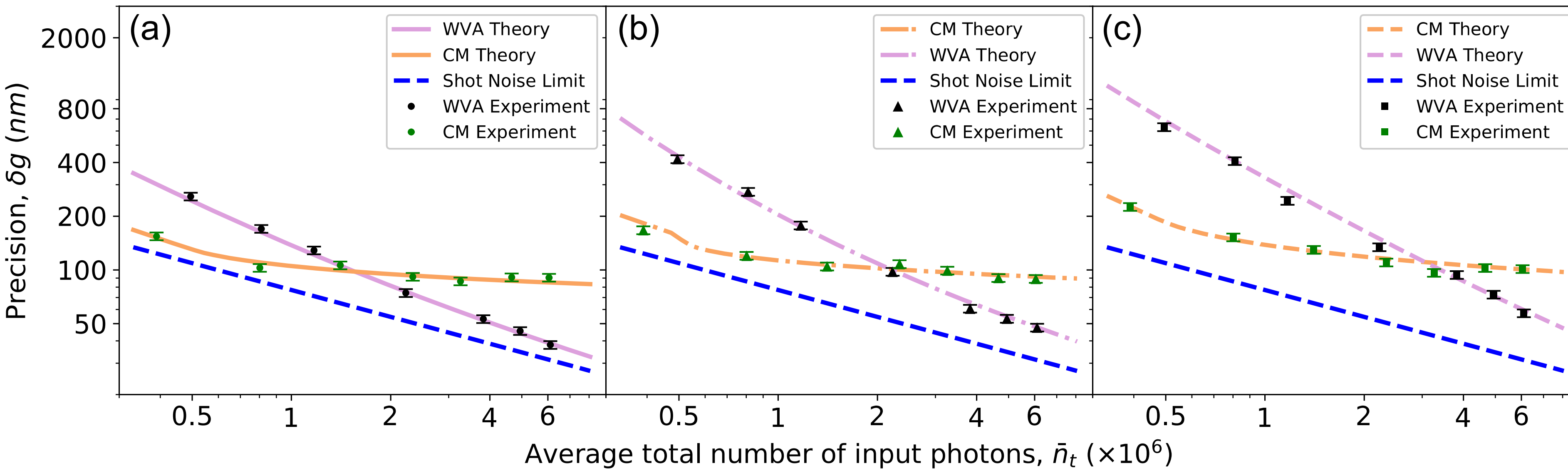








Precision of different estimators

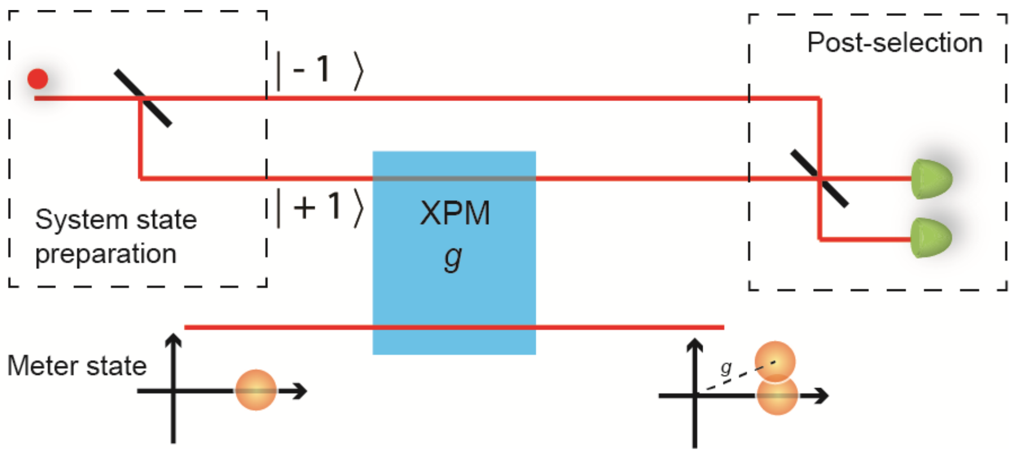


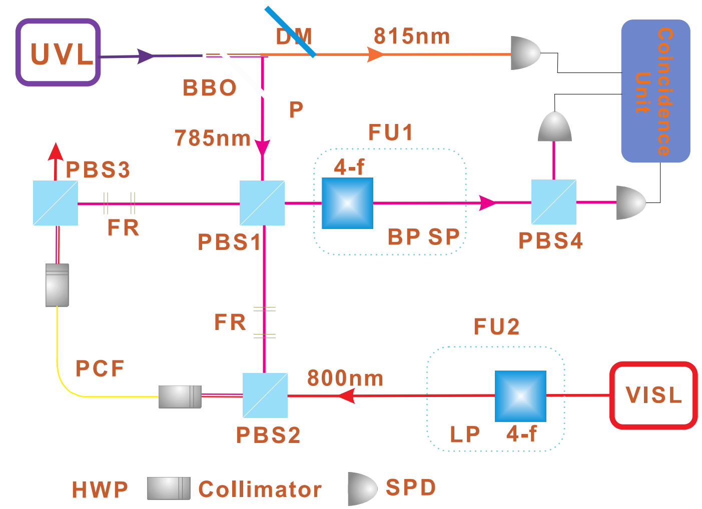
$$\frac{\partial \ln L(g)}{\partial g} = \sum_{l=1}^{n} \sum_{j=1}^{m} \frac{1}{p_l(k_j|g)} \frac{\partial p_l(k_j|g)}{\partial g} = 0 \qquad \hat{g}_{\mathrm{COM}} = \sum_{j=1}^{m} p_j x_j \qquad \hat{g}_{\mathrm{SD}} = \sqrt{\pi}\sigma(p_l - p_r)/\sqrt{2}$$

- ✓ Dynamic range increased by ~100
- ✓ Measurement precision increased by 6
- ✓ Approaching the quantum-limited precision

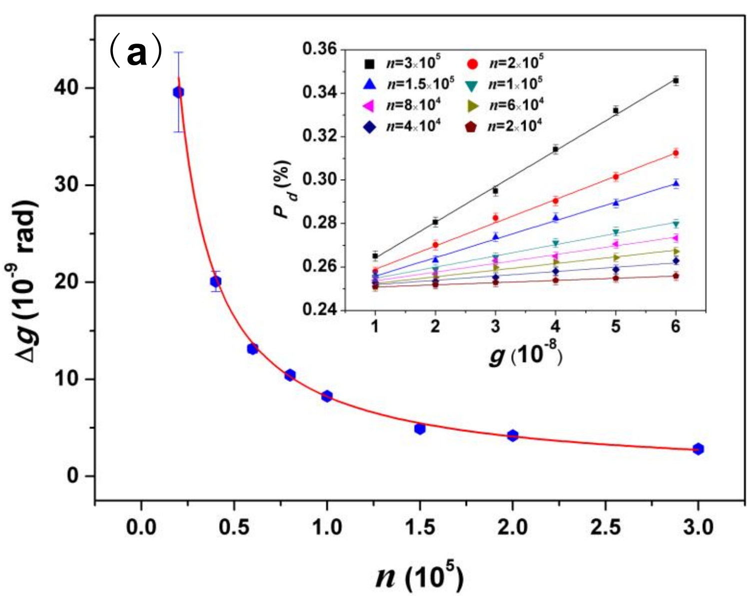
L. Xu et al, Phys. Rev. Lett 125, 080501 (2020)

Nonlinear weak measurement





Experiment by Prof. Chuan-Feng Li's group



- Achieving Heisenberg-scaling with classical resources
- Single photon carries the information from 10^5 photons

Scalable, robust

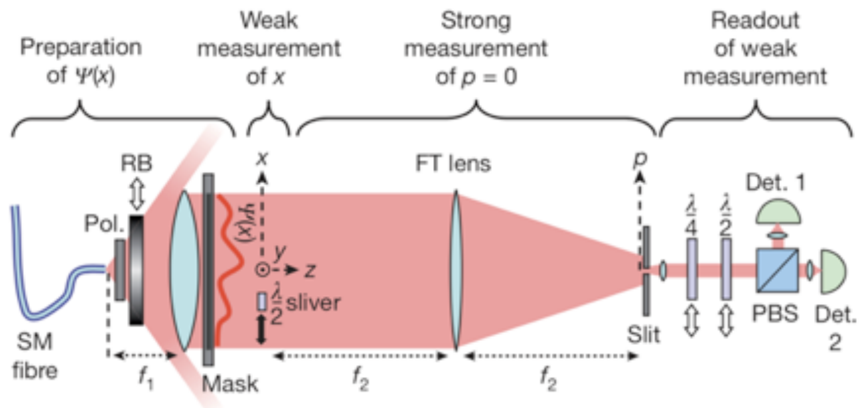
G. Chen et al, Phys. Rev. Lett. 121, 060506 (2018)

Measuring quantum systems using weak measurement

$$|\psi\rangle_s = \int dx\, \psi(x)|x\rangle$$

- Conventional measurement: $\hat{\pi}_x = |x\rangle\langle x|, |\langle x|\psi\rangle_x|^2 = |\psi(x)|^2$
- Weak measurement: $\hat{A} = \hat{\pi}_x = |x\rangle\langle x|, |\psi_f\rangle = |p_0\rangle$

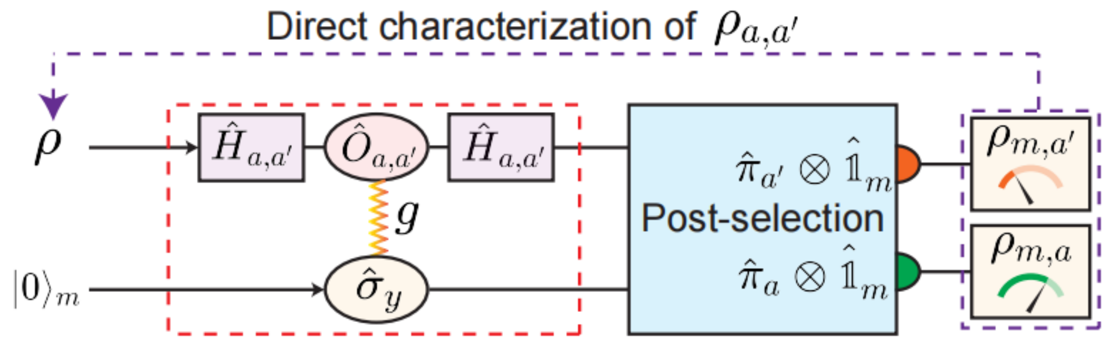
$$\langle \hat{\pi}_x \rangle_w = \frac{\langle p_0|x\rangle\langle x|\psi\rangle_s}{\langle p_0|\psi\rangle_s} \propto \langle x|\psi\rangle_s$$



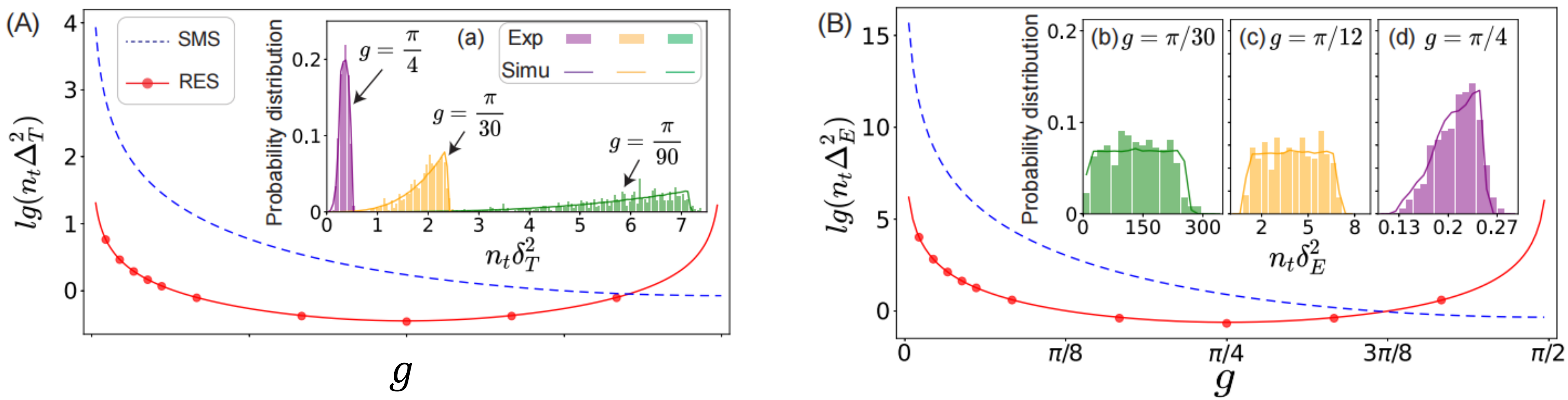
Nature 474, 188 (2011).

- Characterization of a general quantum state (mixed) requires **two** sequential weak measurement: high overhead and fluctuations
- Extension to quantum detector and channel

Resource-efficient direct tomography



- One weak measurement for arbitrary quantum states
- Optimization of measurement strength g



RES: resource-efficient scheme; SMS: sequential-measurement scheme

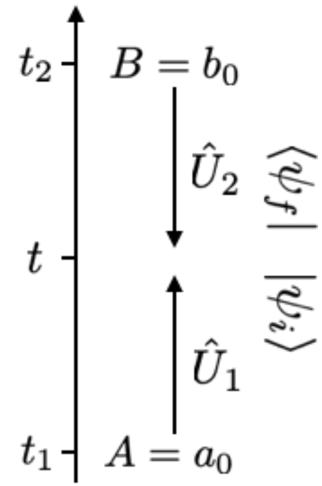
T: qutrit; E: two-entangled qubits

L. Xu et al, Phys. Rev. Lett. 132, 030201 (2024)

Direct tomography of quantum measurement

Time-symmetric description of quantum systems: two state vector formalism (TSVF)

$$\langle A\rangle_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$$



Direct state tomography

$$A_w = \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$$

unknown

Forward direction of TSVF

Direct detector tomography

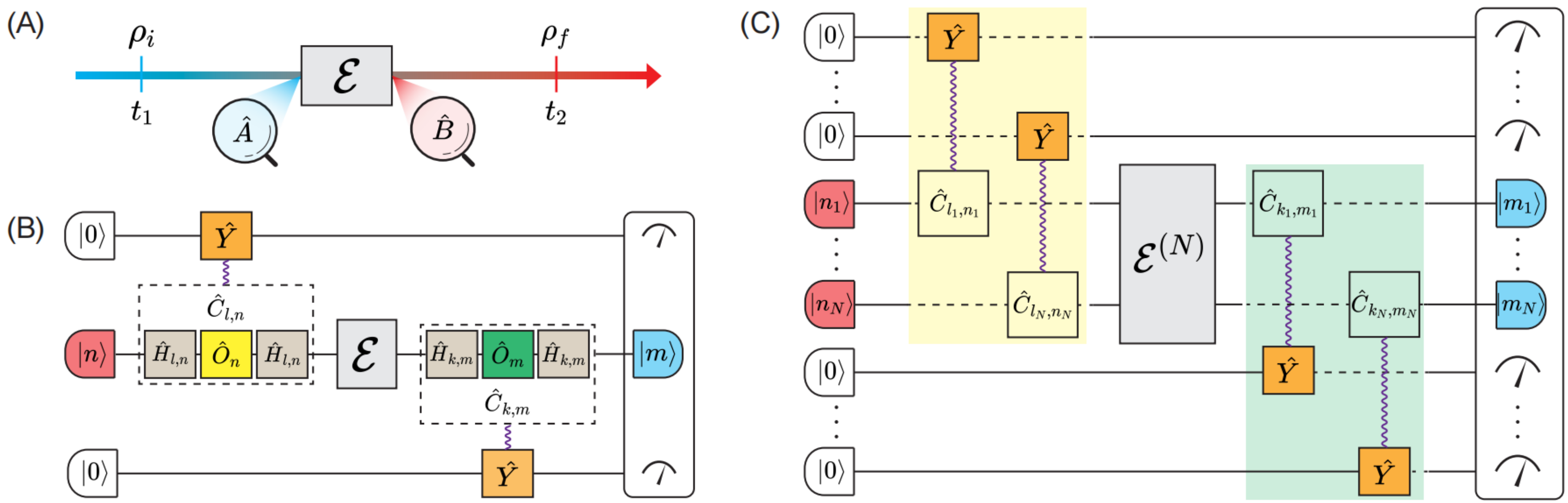
$$A_w = \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$$

unknown

Backward direction of TSVF

L. Xu et al, Phys. Rev. Lett. 127, 180401 (2021), L. Xu et al, Adv. Photon. 3, 066001 (2021)

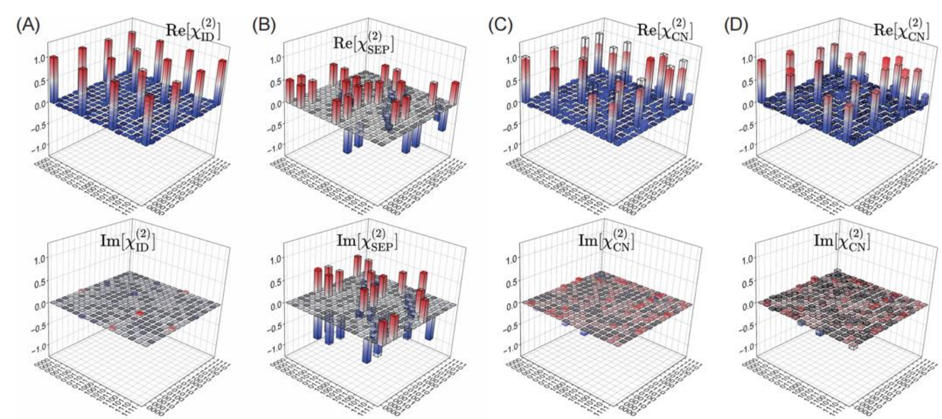
Direct tomography of quantum processes



Process weak value: $\langle \hat{B}\hat{A}\rangle_{if}^{\mathcal{E}} = \mathrm{Tr}[\rho_f \hat{B}\mathcal{E}(\hat{A}\rho_i)].$

$$\mathcal{E}(\rho) = \sum_{k,l,m,n} \chi_{klmn} |k\rangle\langle l|\rho|n\rangle\langle m|$$

$$\langle \hat{C}_{k,m}\hat{C}_{l,n}\rangle_{nm}^{\mathcal{E}} = \chi_{klmn}$$



L. Xu et al, submitted

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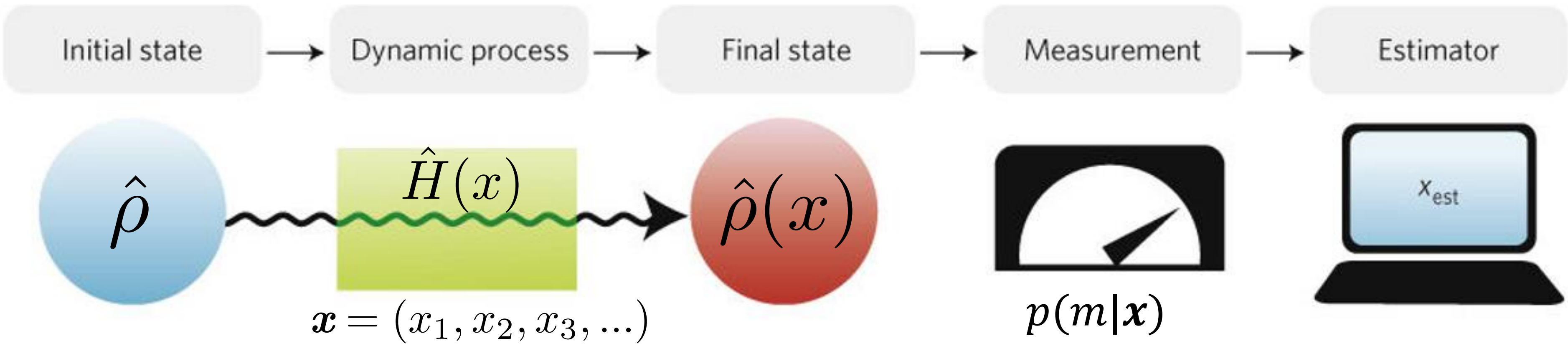
Multi-parameter quantum metrology



FI matrix $$F_{\mu\nu} = \int_m \frac{1}{p(m|\boldsymbol{x})} \frac{\partial p(m|\boldsymbol{x})}{\partial x_\mu} \frac{\partial p(m|\boldsymbol{x})}{\partial x_\nu}$$

QFI matrix $$[Q(\rho_{\boldsymbol{x}})]_{\mu\nu} = \frac{1}{2} Tr[\rho_{\boldsymbol{x}} \{L_\mu, L_\nu\}] \quad \partial_\kappa \rho_{\boldsymbol{x}} = \frac{L_\kappa \rho_{\boldsymbol{x}} + \rho_{\boldsymbol{x}} L_\kappa}{2}$$

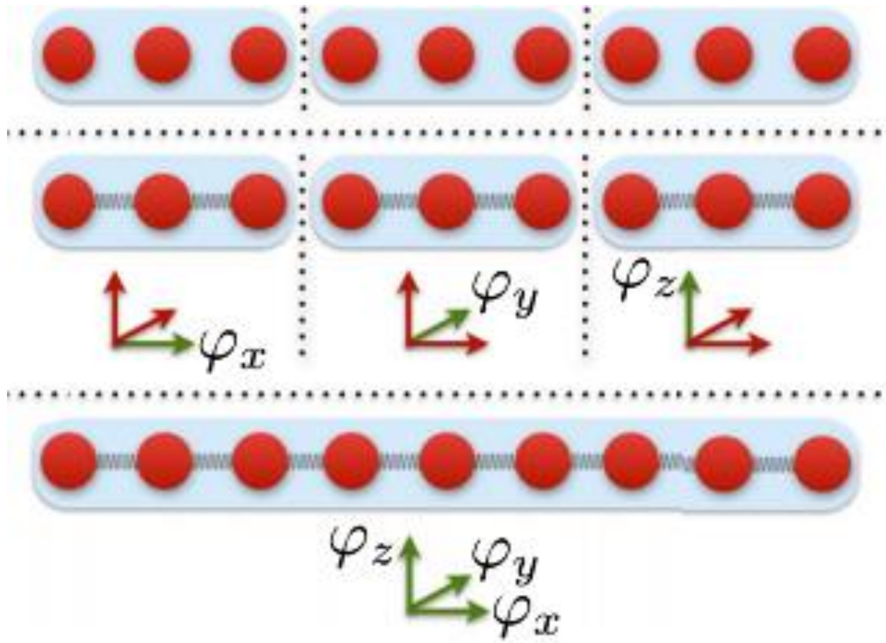
Cramer-Rao bound $$\Sigma \geq F^{-1} \geq Q^{-1}$$

Σ: Covariance matrix of the estimation

Advantage of the joint estimation of multiple parameters

Estimating each parameter separately

$\tilde{E}(\varphi_x)\quad \tilde{E}(\varphi_y)\quad \tilde{E}(\varphi_z)$

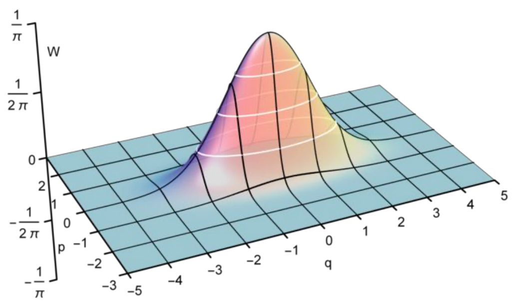


Estimating all parameters jointly

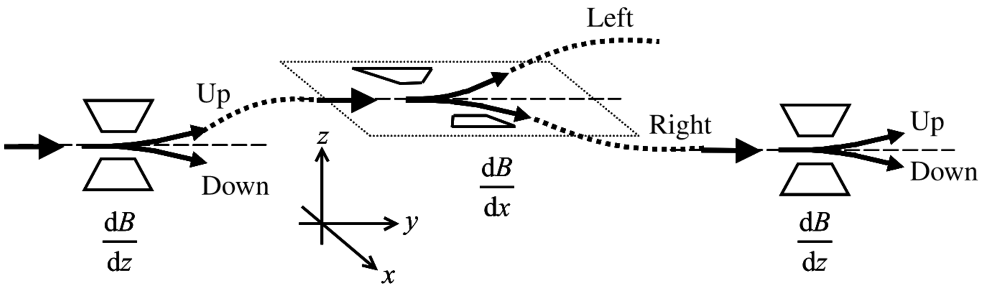
$\tilde{E}(\varphi_x, \varphi_y, \varphi_z)$

M parameters	Separate estimation	Joint estimation
Resources (particle, energy etc)	MN	N
Precision	N^{-1}	N^{-1}

Tradeoffs between different parameters



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



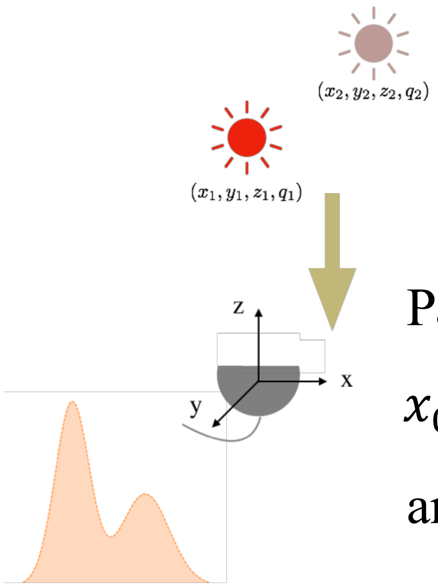
$$[S_i, S_j] = i\hbar\epsilon_{ijk}S_k$$

Due to the incompatibility in quantum mechanics, classical FI matrix F can be **strictly smaller** than QFI matrix Q, and the quantum Cramer-Rao bound (QCRB) $\Sigma = Q^{-1}$ may not be achieved.

A necessary and sufficient condition to achieve QCRB

$$\mathrm{Tr}\left[\rho_{\boldsymbol{x}}[L_\mu, L_\nu]\right] = 0 \quad \text{Weak commutivity condition}$$

Imaging as a multi-parameter estimation problem



$$\rho = q\,|\Psi_1\rangle\langle\Psi_1| + (1-q)|\Psi_2\rangle\langle\Psi_2|$$

$$|\Psi_{1,2}\rangle = \exp\left(-i\hat{G}z_{1,2} - i\hat{p}_x x_{1,2}\right)|\Psi\rangle$$

Parameters to be estimated:

$x_0 = (x_1 + x_2)/2 \quad z_0 = (z_1 + z_2)/2 \quad s = x_1 - x_2 \quad t = z_1 - z_2$

and relative intensity q

QFI

$$Q = \begin{bmatrix} Q_{x_0x_0} & 2p^2(1-2q) & Q_{x_0z_0} & 0 & 4w\partial_s w \\ 2p^2(1-2q) & p^2 & 0 & 0 & 0 \\ Q_{x_0z_0} & 0 & Q_{z_0z_0} & 2\left(g^2-G^2\right)(-1+2q) & 4w\partial_t w \\ 0 & 0 & 2\left(g^2-G^2\right)(-1+2q) & g^2-G^2 & 0 \\ 4w\partial_s w & 0 & 4w\partial_t w & 0 & \frac{-1+w^2}{(-1+q)q} \end{bmatrix}$$

Weak commutivity

$$\Gamma = \begin{bmatrix} 0 & \Gamma_{x_0s} & \Gamma_{x_0z_0} & \Gamma_{x_0t} & 4\partial_s\phi(-1+2q)w^2 \\ -\Gamma_{x_0s} & 0 & \Gamma_{sz_0} & 0 & -2\partial_s\phi w^2 \\ -\Gamma_{x_0z_0} & -\Gamma_{sz_0} & 0 & \Gamma_{z_0t} & 4\left(\mathfrak{G}+\partial_t\phi\right)(-1+2q)w^2 \\ -\Gamma_{x_0t} & 0 & -\Gamma_{z_0t} & 0 & -2\left(\mathfrak{G}+\partial_t\phi\right)w^2 \\ -4\partial_s\phi(-1+2q)w^2 & 2\partial_s\phi w^2 & -4\left(\mathfrak{G}+\partial_t\phi\right)(-1+2q)w^2 & 2\left(\mathfrak{G}+\partial_t\phi\right)w^2 & 0 \end{bmatrix}$$

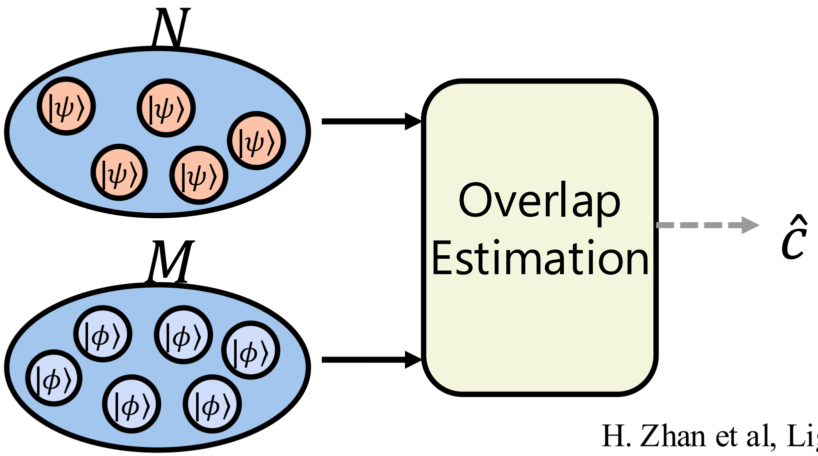
Phys. Rev. X 6, 031033 (2016), B. Wang et al, Photon. Res. 9. 1522 (2021), C. Hu et al, Quant. Front. 2, 20 (2023)

Quantum system characterization as a multi-parameter estimation problem

- Characterizing quantum state $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$ involves the estimation of θ, ϕ

QFI $Q = \begin{bmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{bmatrix}$ Weak commutivity $\Gamma = \frac{1}{2}\begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

- Evaluating function of quantum states generally involves the estimation of multiple parameters



H. Zhan et al, Light Science & Applications 14, 83 (2025)

How to overcome the precision tradeoff and achieve the optimal precision (quantum Cramér-Rao bound) of all the parameters?

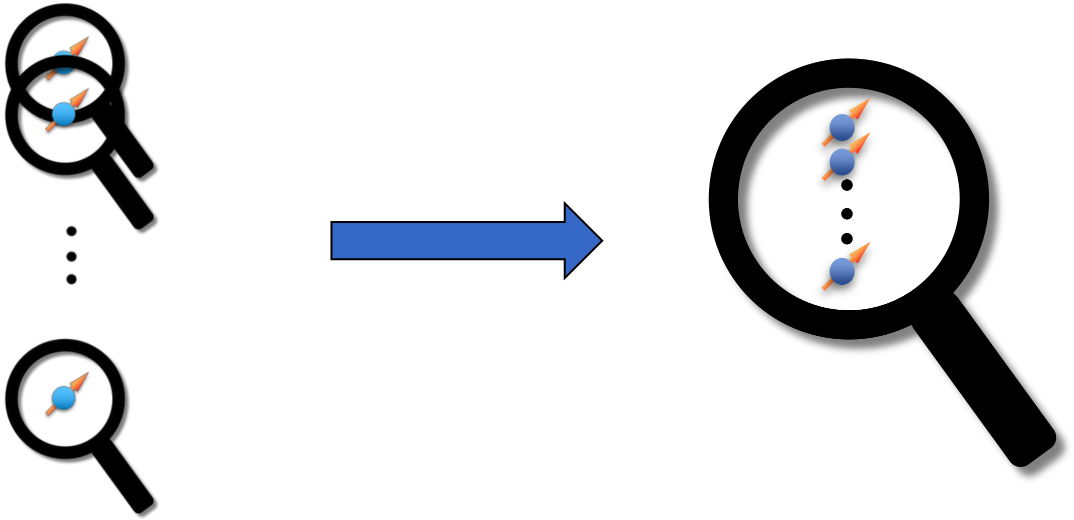
Mitigating tradeoff with collective measurement

- Pauli operators $\{\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y, \boldsymbol{\sigma}_z\}$ of a single qubit (spin) do not commute

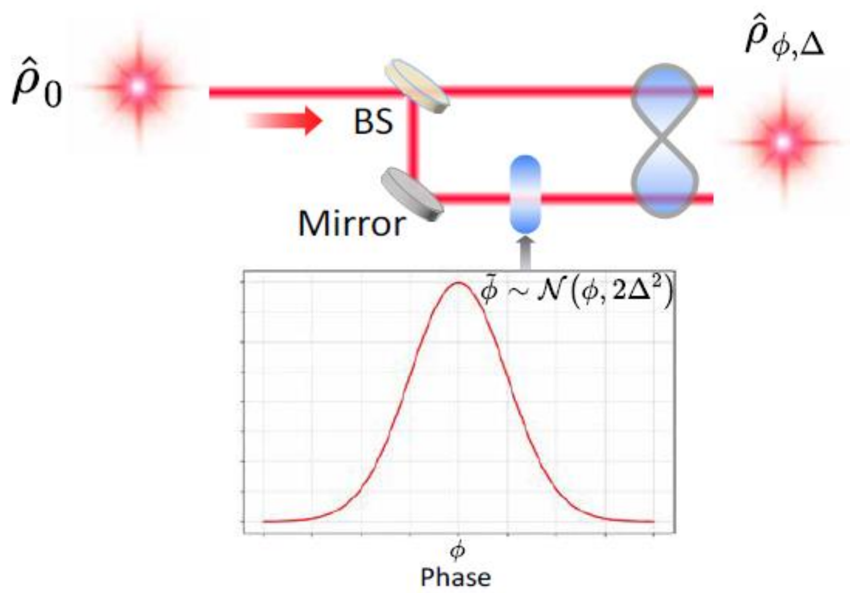
$$[\sigma_i, \sigma_j] = i\hbar\epsilon_{ijk}\sigma_k$$

- Tensor product of two Pauli operators $\{\boldsymbol{\sigma}_x\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y\boldsymbol{\sigma}_y, \boldsymbol{\sigma}_z\boldsymbol{\sigma}_z\}$ commute

$$[\sigma_i\sigma_i, \sigma_j\sigma_j] = 0$$



Simulatenous measurement of phase and phase diffusion



$$\hat{\rho}_{\phi,\Delta} = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\phi-\Delta^2} \\ e^{i\phi-\Delta^2} & 1 \end{pmatrix}$$

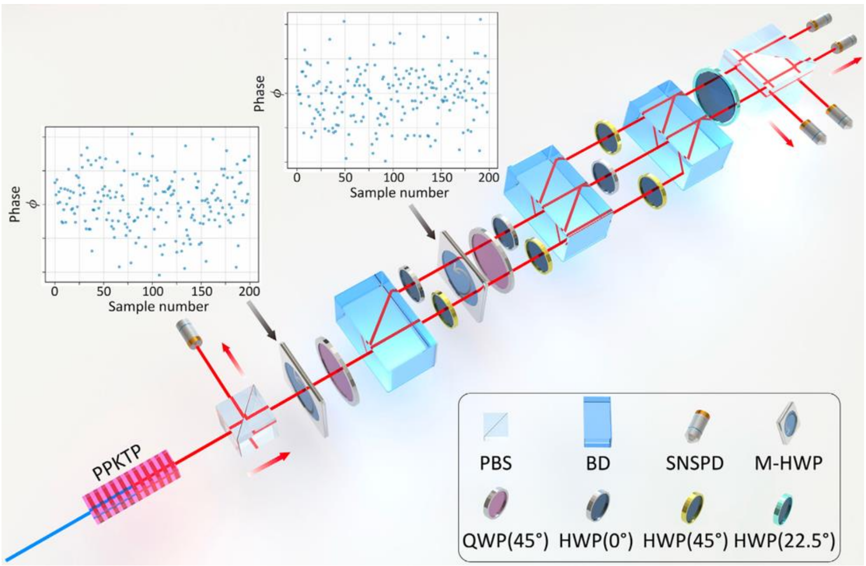
QFI matrix (θ=π/2)

$$\mathbf{Q} = \begin{bmatrix} e^{-2\Delta^2} & 0 \\ 0 & \frac{4\Delta^2}{e^{2\Delta^2}-1} \end{bmatrix}$$

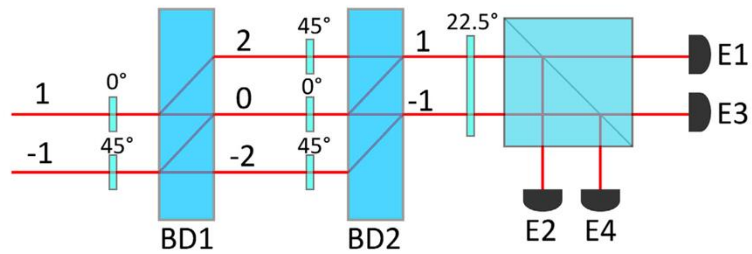
- Without parameter tradeoffs, one expect a Fisher information matrix $F = Q$, or $\mathrm{Tr}(Q^{-1}F) = 2$
- Single-copy measurement only achieves $\mathrm{Tr}(Q^{-1}F) \leq 1$
- Bell measurement on two copies can achieve $\mathrm{Tr}(Q_2^{-1}F_2) \leq 1.5$

Nat. Comm. 5, 3532 (2014)

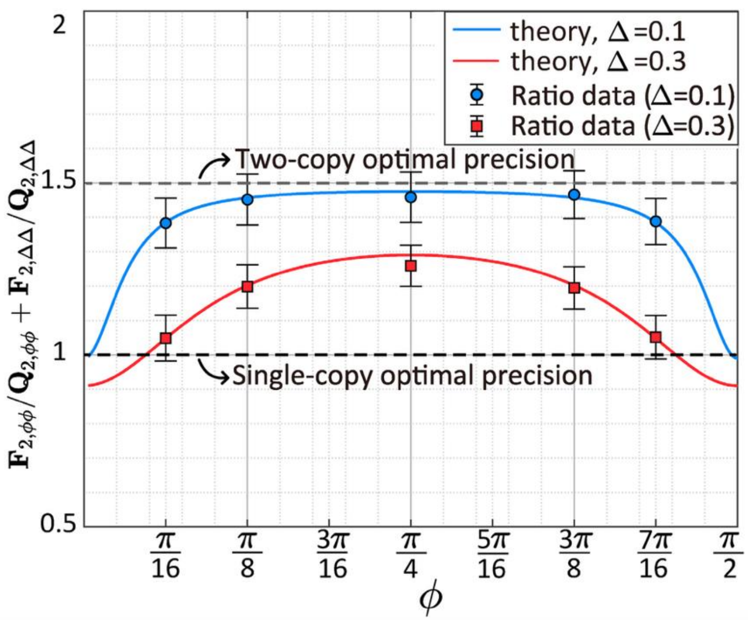
Experimental demonstration



Use path and polarization to encode two qubits



Collective measurement



Collective measurement can reduce the parameter tradeoffs, but may not be able to completely remove it.

B. Wang, et al. Adv. Sci. e18375 (2025)

Removing tradeoff with antiunitary symmetry

If there exists an antiunitary operator (e.g. conjugation) such that

$$\Theta\rho_x\Theta^\dagger = \rho_x$$

for all parameter points, the state is said to have **global antiunitary symmetry (GAS)**.

- GAS ensures weak commutivity condition

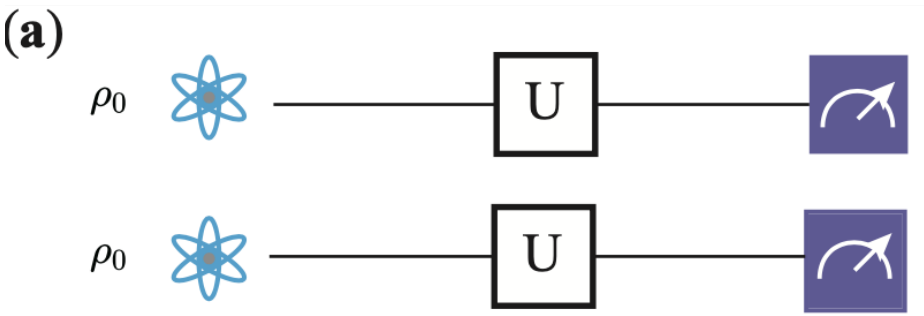
$$\Theta\rho_x\Theta^\dagger = \rho_x \quad \Longrightarrow \quad \mathrm{Tr}(\rho_x[\hat{L}_i, \hat{L}_j]) = 0, \forall i, j$$

- The optimal measurement is parameter-independent

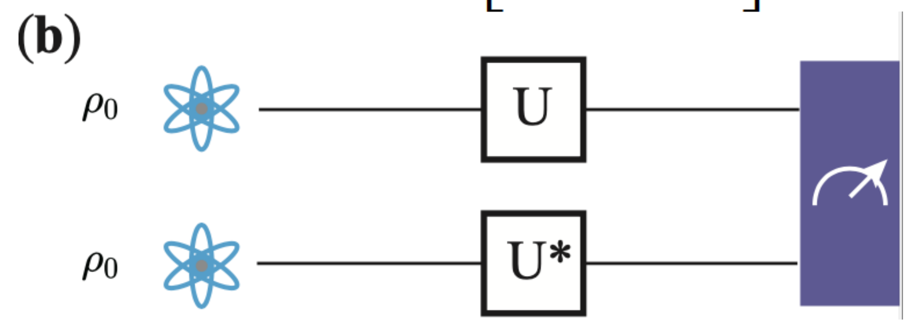
Quantum 6, 665 (2022).

Comparison between GAS and standard stratigies

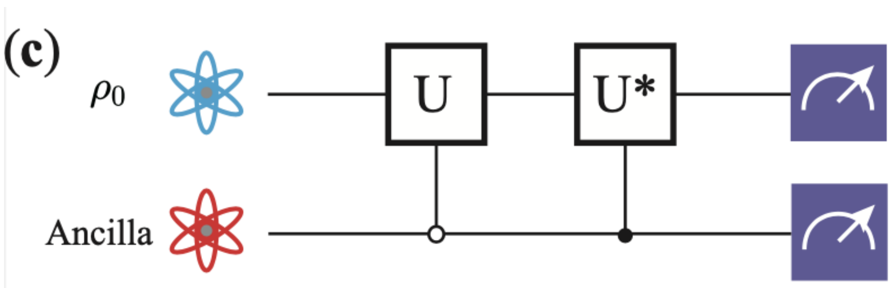
$$|\psi_x\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \qquad \Gamma = \frac{1}{2}\begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$



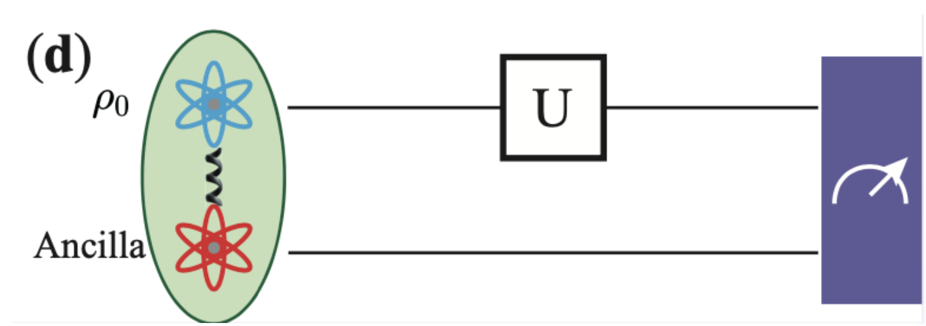
Parallel model $\quad |\psi_x\rangle \otimes |\psi_x\rangle$



Mutually conjugate model (MCM) $\quad |\psi_x\rangle \otimes |\psi_x^*\rangle$



Ancilla-assisted mutually conjugate model (AAMCM) $\quad \dfrac{|\psi_x\rangle|0\rangle + |\psi_x^*\rangle|1\rangle}{\sqrt{2}}$

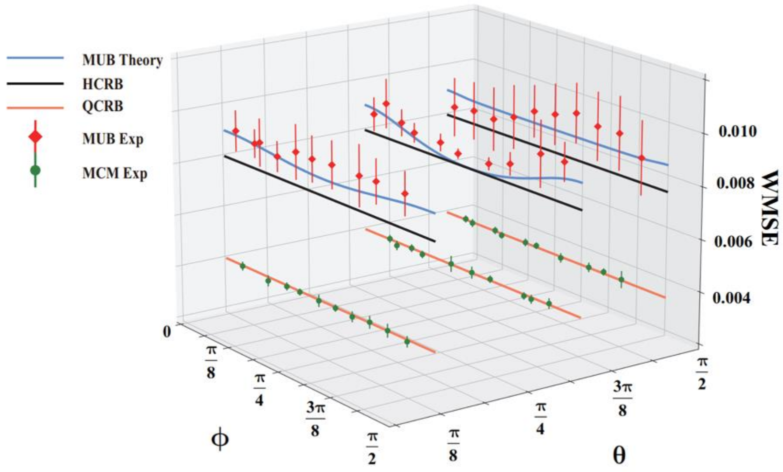


Maximal entanglement model (MEM) $\quad (U_x \otimes \mathbb{I})\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$

B. Wang et al, Phys. Rev. Lett. 133, 210801 (2024)

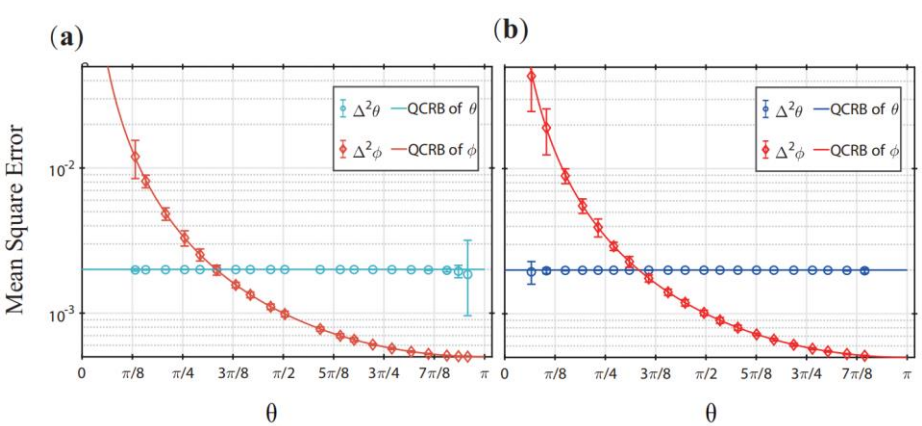
Experimental results

MCM $|\psi_x\rangle \otimes |\psi_x^*\rangle$ vs Parallel model $|\psi_x\rangle \otimes |\psi_x\rangle$



MCM doubles the precision with the elimination of parameter tradeoffs

AAMCM $\frac{U(\theta,\phi)|0\rangle|0\rangle + U^*(\theta,\phi)|0\rangle|1\rangle}{\sqrt{2}}$ vs MEM $\left(U(\theta,\phi)\otimes\mathbb{I}\right)\frac{|0\rangle|0\rangle+|1\rangle|1\rangle}{\sqrt{2}}$

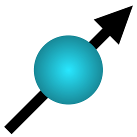


Both models achieve the comparable precision, but AAMCM only requires local measurement

B. Wang et al, Phys. Rev. Lett. 133, 210801 (2024)

Strategy using classical correlations

Anti-parallel spins



$$|\boldsymbol{n}\rangle = cos\frac{\theta}{2}|0\rangle + e^{-i\phi}sin\frac{\theta}{2}|1\rangle = U(\theta,\phi)|0\rangle \qquad U(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$|\mathbf{n}, -\mathbf{n}\rangle = U(\theta,\phi)|0\rangle \otimes U(\theta,\phi)|1\rangle$$

$$\langle\partial_\theta(\mathbf{n}, -\mathbf{n})|\partial_\phi(\mathbf{n}, -\mathbf{n})\rangle = 0$$

Weak-commutativity condition is satisfied

QFI $\mathbf{Q}(|\mathbf{n}, -\mathbf{n}\rangle) = \begin{pmatrix} 2 & 0 \\ 0 & 2\sin^2\theta \end{pmatrix} = 2 \cdot \mathbf{Q}(|\mathbf{n}\rangle)$

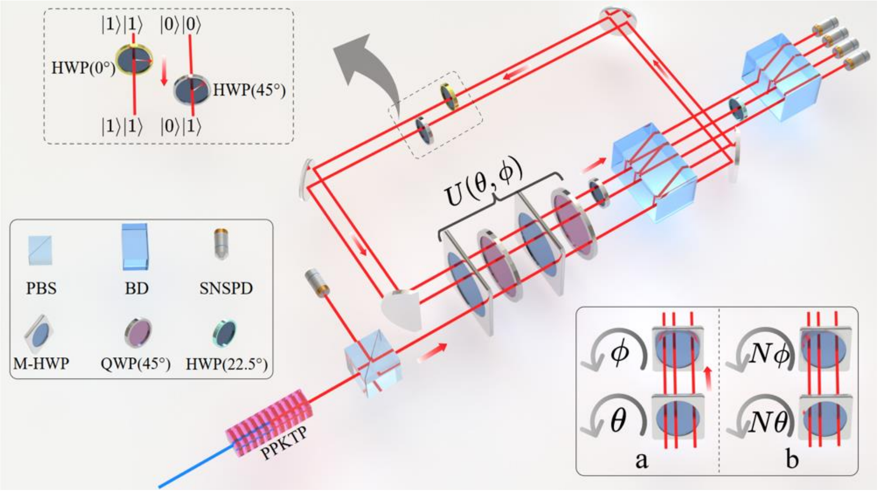
Optimal measurement: $\left\{|01\rangle, |10\rangle, \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \frac{|00\rangle - |11\rangle}{\sqrt{2}}\right\}$

Classical Correlations can improve the precision of multi-parameter quantum metrology!

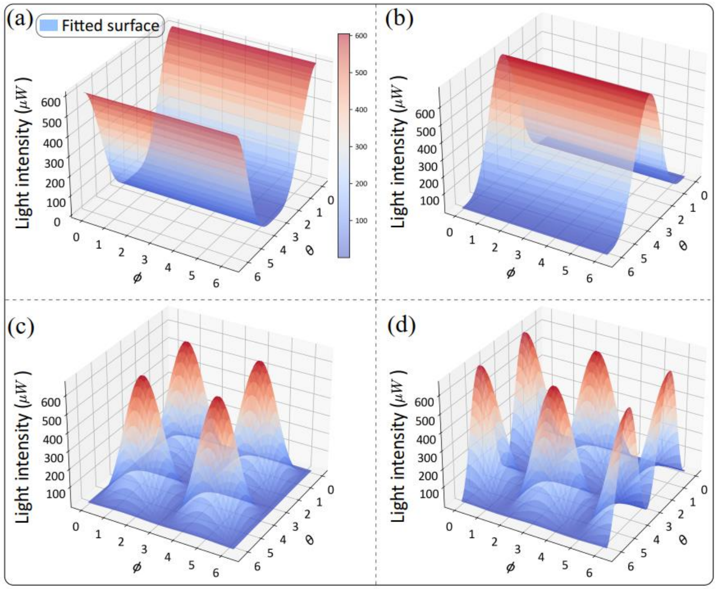
M. Mi et al, Phys. Rev. Lett. 135, 110804 (2025)

Experimental demonstration

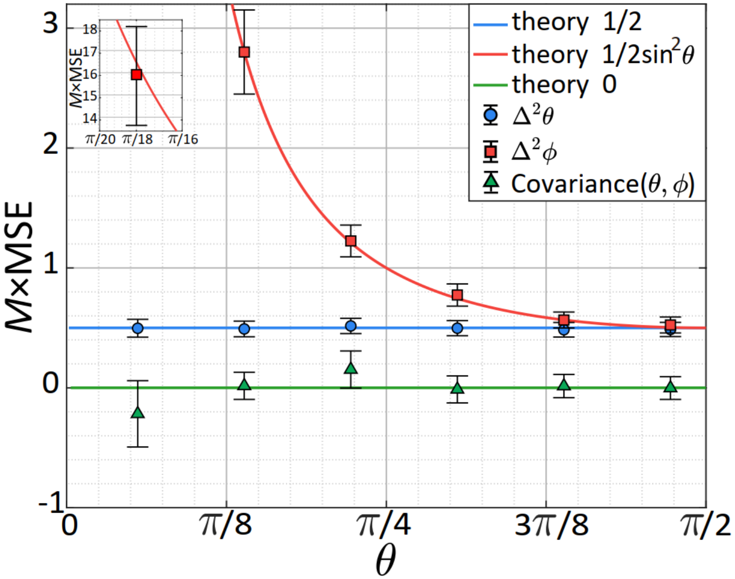
Setup



Likelihood



Precision

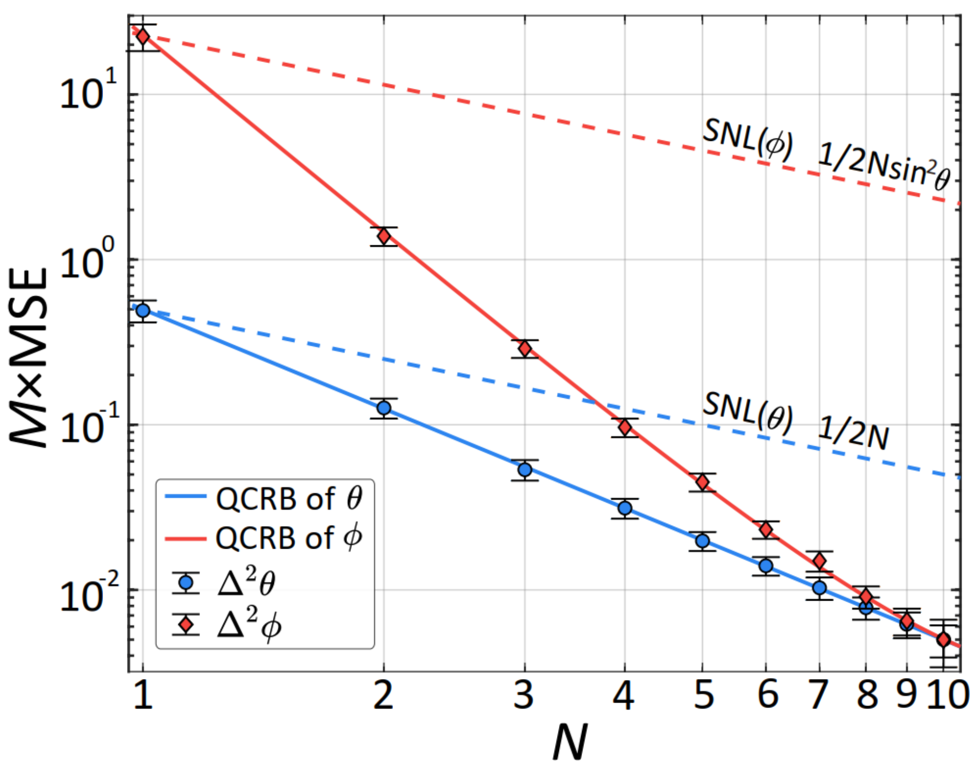


M. Mi et al, Phys. Rev. Lett. 135, 110804 (2025)

Heisenberg-limited scaling with sequential measurement

Applying $U(\theta,\phi)$ N times together with control achieve $U(N\theta,N\phi)$

$$\mathbf{Q}(U(N\theta,N\phi)|0\rangle \otimes U(N\theta,N\phi)|1\rangle) = \begin{pmatrix} 2N^2 & 0 \\ 0 & 2N^2\sin^2(N\theta) \end{pmatrix}$$



M. Mi et al, Phys. Rev. Lett. 135, 110804 (2025)

Summary

- Quantum metrology promises the precision beyond the classical limit, but to achieve it requires to overcome **practical challenges**: noise, loss, complexity, tradeoff…
- Loss-tolerant quantum interferometry with entangled photons and control-enhanced sequential strategy
- Practical advantage of weak measurement in precision metrology and characterization of quantum systems
- Overcoming the parameter tradeoffs in multi-parameter quantum metrology with collective measurement, antiunitary symmetry and classical correlations

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