Two-Loop Electroweak Renormalization in the Standard Model and Its Application in NNLO Corrections to $H \to b\bar{b}$ and Beyond

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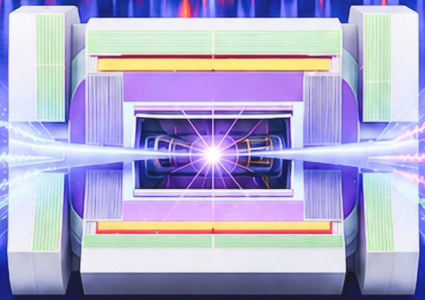
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Paper in preparation



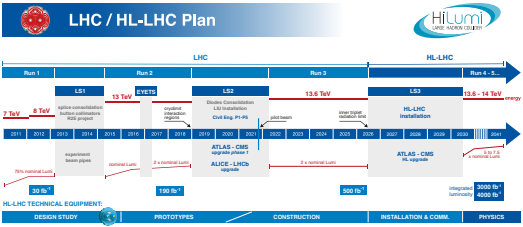
CEPC 2026

International Workshop on the High Energy Circular Electron Positron Collider

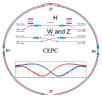
Shanghai, 25 October 2026

Background: CEPC2026 workshop poster

Precision Higgs Physics at Future Colliders



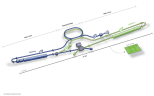
LHC / HL-LHC schedule (CERN, January 2025)



CEPC (CEPC Study Group)



FCC-ee (CERN/FCC)



ILC

LHC and HL-LHC

- Run 2 legacy: $\mu = 0.990^{+0.053}_{-0.051}$ [ATLAS '26], $1.014^{+0.055}_{-0.053}$ [CMS '26]; W, Z, t, b, τ couplings SM-like, $\pm$4–12%
- 14 TeV, 3000 fb^{-1} [Cepeda et al. '19]: at least 15 million Higgs bosons per year [CERN HiLumi]
- Large backgrounds, theoretical and systematic uncertainties: Higgs measurements typically limited to a few percent [CEPC '22]

Future e^+e^- Higgs factories

- **CEPC**: 240 GeV, 10 years, 2 IPs: 2.6–4.3 million Higgs bosons (30/50 MW) [CEPC '23]
- $\sigma(ZH) \times \mathrm{BR}(H \to b\bar{b})$ to 0.14% with 20 ab^{-1}: the most precise Higgs rate [CEPC '22]
- **FCC-ee**: almost 3 million Higgs bosons [FCC '25]
- Higgs couplings at the sub-percent level, $\sim$10$\times$ beyond HL-LHC [CEPC '23]

Take-home: sub-percent Higgs measurements ahead at CEPC — theory has to match experiment.

Perturbative QCD & EW: Where We Stand?

Perturbative QCD

- **NNLO QCD: the standard.** Nearly all LHC $2 \to 2$, fully differential (MATRIX, MCFM, **NNLOJET**); $2 \to 3$: $3j$ [Czakon et al. '21], $t\bar{t}H$ [Catani et al. '22], $t\bar{t}W$ [Buonocore et al. '23], $t\bar{t}j$ [Badger et al. '25]; e^+e^-: event shapes [Gehrmann-De Ridder et al. '07], four jets [Xuan Chen et al. '26; **talk: H. Sun, CEPC '26**], ZH, $H \to$ jets [Caletti et al. '25]
- **N^{3}LO QCD: the frontier.** Inclusive: ggF Higgs [Anastasiou et al. '16; Mistlberger '18], Drell–Yan [Duhr et al. '20], $e^+e^- \to t\bar{t}$ [Chen et al. '22], top decay [Chen et al. '23]; differential: ggF Higgs [Chen et al. '21], Drell–Yan [Camarda et al. '21], $H \to b\bar{b}$ [Mondini et al. '19], $e^+e^- \to jj$ [Chen et al. '25]
- **Amplitude frontier.** Two-loop six-point integrals [Henn et al. '24]; two-loop massive pentagons: $t\bar{t}j$, $t\bar{t}\gamma$, $t\bar{t}W$ [Badger et al. '24; talk: C. Brancaccio, HP^2 '26; Becchetti et al. '25]; planar three-loop $V+j$ [Gehrmann et al. '23]; three-loop five-point integrals [Liu et al. '24; Chicherin et al. '25]

Perturbative EW

- **NLO EW: automated.** On-shell [Sirlin '80; Aoki et al. '82] and complex-mass scheme [Denner et al. '05; Denner, Dittmaier '06]; public packages FeynArts/FormCalc, OpenLoops, RECOLA, MG5_aMC; up to $2 \to 8$ [Denner, Pellicciolli '21]
- **NNLO mixed QCD–EW $\mathcal{O}(\alpha\alpha_s)$.** Drell–Yan: on-shell Z [Bonciani, Buccioni, Rana, **Vicini** '20], $\ell^+\ell^-$, $\ell\nu$ [Bonciani et al. '21; Buonocore et al. '21; Armadillo et al. '24a,b]; $e^+e^- \to ZH$ [Gong et al. '16; Sun et al. '16], W^+W^- [**Z. Li** et al. '24], ZZ [**Z. Li** et al. '23]; $H \to b\bar{b}$ [Mihaila et al. '15]
- **NNLO EW $\mathcal{O}(\alpha^2)$: limited.** Electroweak precision observables complete [Awramik et al. '03; Dubovyk et al. '18, '19]; two-loop charge renormalization [Degrassi, **Vicini** '03]; $2 \to 2$ only with closed fermion loops: $e^+e^- \to ZH$ [Freitas, Song '22; Freitas et al. '23], $f\bar{f}$ [Freitas, Wallace '25], Drell–Yan [Armadillo, Devoto, Dradi, **Vicini** '25, '26]; unrenormalized two-loop ZH amplitude [X. Chen et al. '22]

Z. Li: the speaker. A selection only — apologies for any omissions.

Take-home: QCD: NNLO standard, N^{3}LO the frontier — EW: complete $\mathcal{O}(\alpha^2)$ for tree-level processes still limited.

Why Is NNLO EW Challenging?

Computational obstacles

- **Size**: $\mathcal{O}(10^{3\text{–}4})$ two-loop diagrams, $\mathcal{O}(10^3)$ integral families
- **Multi-scale integrals**: M_W, M_Z, M_H, m_t, m_f at once; thresholds at the mass shells; elliptic sectors $\Rightarrow$ numerical, AMFlow [Liu, Ma '22; Huang, Liu, Ma '26]
- γ_5 with chiral couplings: anticommuting γ_5; traces with odd γ_5 fixed by g5anchor, anchor points of γ_5 [L. Chen '24], building on [Kreimer '90; Körner et al. '92]

Renormalization

- **No ready-to-use two-loop set**: standard at one loop [Denner, Dittmaier '19]; at two loops per observable [Freitas et al. '02; Chen, Freitas '20a]
- **Unstable H, W, Z, t**: the traditional real-axis mass is gauge dependent from two loops [Sirlin '91; Willenbrock, Valencia '91]

$$M^2_{\text{OS}} - \overline{M}^2 = \overline{M}\,\overline{\Gamma}_{(1)}\,\text{Im}\,\hat{\Sigma}'_{(1)}(\overline{M}^2) + \mathcal{O}(\alpha^3)$$

- **Complex-mass scheme** [Denner et al. '05, '06]: (1) master integrals analytically continued to the complex plane [Passarino et al. '10]; (2) complex masses and couplings; (3) unitarity violated beyond the working order [Denner, Lang '15]

Our choice: pole expansion

- Gauge-invariant complex pole $s_p = \overline{M}^2 - i\,\overline{M}\,\overline{\Gamma}$ (μ_t for the top) defines mass and width
- Real masses, mass counterterms, couplings: real master integrals, expanded about the real part order by order
- Field normalization is a convention: prescriptions (I), (II), (III)
- **Scope**: two-loop on-shell constants of H, W, Z, γ, all charged fermions incl. top; charge (on-shell, $\overline{\text{MS}}$); FT and Landau gauge; process independent, to be made public
- **First application**: $\Gamma(H \to b\bar{b})$ at $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$

A systematic two-loop EW renormalization framework for physical observables

Renormalization of Stable Particles

On-shell conditions [Sirlin '80; Denner '93]: (i) pole at the renormalized mass, (ii) unit residue, (iii) no on-shell γ–Z transitions

Photon and γ–Z mixing

(ii), (iii) fix δZ_{ZA}, δZ_{AZ}, δZ_{AA} through

$$\mathrm{Re}\,\hat{\Sigma}^T_{AZ}(0) = 0, \quad \mathrm{Re}\,\hat{\Sigma}^T_{AZ}(M_Z^2) = 0, \quad \mathrm{Re}\,\hat{\Sigma}^{T\prime}_{AA}(0) = 0$$

No photon mass counterterm: first condition + gauge invariance $\Rightarrow \hat{\Sigma}^T_{AA}(0) = 0$ [Dittmaier '21]

$$\begin{aligned}
\delta Z_{AA,(2)} &= -\mathrm{Re}\,\Sigma^{T\prime}_{AA,(2)}(0) - \tfrac{1}{4}\big(\delta Z_{ZA,(1)}\big)^2 \\
\delta Z_{ZA,(2)} &= \tfrac{2}{M_Z^2}\mathrm{Re}\,\Sigma^T_{AZ,(2)}(0) \\
&\quad - \big(\tfrac{\delta M^2_{Z,(1)}}{M_Z^2} + \tfrac{1}{2}\mathrm{Re}\,\delta Z_{ZZ,(1)}\big)\delta Z_{ZA,(1)} \\
\delta Z_{AZ,(2)} &= -\tfrac{2}{M_Z^2}\mathrm{Re}\,\Sigma^T_{AZ,(2)}(M_Z^2) \\
&\quad + \tfrac{\delta M^2_{Z,(1)}}{M_Z^2}\delta Z_{ZA,(1)} - \tfrac{1}{2}\delta Z_{AA,(1)}\delta Z_{AZ,(1)}
\end{aligned}$$

Mixing condition at the complex Z pole [Chen, Freitas '20a]: another field convention ($\delta Z_{AZ,(2)}$ shifted)

Stable fermions

Pole and unit residues of G_f at $p^2 = m_f^2$ (no absorptive parts), with $\Sigma_f^M \equiv \Sigma_f^L + \Sigma_f^R + 2\Sigma_f^S$:

$$\hat{\Sigma}^M_f(m_f^2) = 0, \qquad \hat{\Sigma}^{L/R}_f(m_f^2) + m_f^2\,\Sigma^{M\prime}_f(m_f^2) = 0$$

$$\begin{aligned}
\delta Z^{L/R}_{f,(2)} &= -\mathrm{Re}\left[\Sigma^{L/R}_{f,(2)} + m_f^2\,\Sigma^{M\prime}_{f,(2)}\right] \\
\delta m_{f,(2)} &= \tfrac{m_f}{2}\mathrm{Re}\,\Sigma^M_{f,(2)} + \tfrac{m_f}{8}\big(\delta Z^L_{f,(1)} - \delta Z^R_{f,(1)}\big)^2 \\
&\quad - \tfrac{1}{2}\delta m_{f,(1)}\big(\delta Z^L_{f,(1)} + \delta Z^R_{f,(1)}\big) \\
\delta Z^L_{\nu,(2)} &= -\mathrm{Re}\,\Sigma^L_{\nu,(2)}(0)
\end{aligned}$$

- Black terms, $(2) \to (1)$: one-loop constants [Denner '93]; $\Sigma_{(2)}$ at m_f^2, subloop-renormalized; blue: products of one-loop constants
- Landau gauge: if $\Sigma_f^{L/R}$ have absorptive parts from unphysical thresholds (massless would-be Goldstones), the top-quark prescriptions apply with $\Gamma_f = 0$ (B3)

Unstable Bosons: Pole and Field Prescriptions

Complex pole and residue

Pole on the unphysical sheet [Passarino et al. '10; Denner, Dittmaier '19]

$$\mathcal{D}(s) = \frac{1}{s - \overline{M}^2 + \hat{\Sigma}(s)} = \frac{R_p}{s - s_p} + \text{regular}$$

$$s_p \equiv \overline{M}^2 - i\,\overline{M}\,\overline{\Gamma}, \qquad R_p = \frac{1}{1 + \hat{\Sigma}'(s_p)}$$

Nielsen identity [Nielsen '75; Gambino, Grassi '99]

$$\partial_\xi \mathcal{D}^{-1}(s) = 2\Lambda_\xi(s)\,\mathcal{D}^{-1}(s)$$
$$\Longrightarrow \quad \partial_\xi s_p = 0, \qquad \partial_\xi \ln R_p = -2\Lambda_\xi(s_p)$$

- Pole gauge independent; residue gauge dependent, only that of a complete resonant amplitude is not [Grassi et al. '01]

Real-axis mass $\mathrm{Re}\,\mathcal{D}^{-1}(M_{\rm OS}^2) = 0$:

$$M_{\rm OS}^2 - \overline{M}^2 = \overline{M}\,\overline{\Gamma}_{(1)}\,\mathrm{Im}\,\hat{\Sigma}'_{(1)}(\overline{M}^2) + \mathcal{O}(\alpha^3 \overline{M}^2)$$

- Gauge-dependent thresholds $\Rightarrow$ $M_{\rm OS}$ gauge dependent from two loops [Sirlin '91]

Field normalization: three prescriptions

$\mathcal{D}^{-1}(s_p) = 0$ fixes δM^2 and $\overline{\Gamma}$; no asymptotic state, no LSZ condition $\Rightarrow$ field normalization is a convention:

(I):	$\hat{\Sigma}'(s_p) = 0,$	complex field constants
(II):	$\mathrm{Re}\,\hat{\Sigma}'(s_p) = 0,$	real field constants
(III):	$\mathrm{Im}\,\hat{\Sigma}(\overline{M}^2) = \overline{M}\,\overline{\Gamma},$	real field constants

- (I): $R_p = 1$, as in the complex-mass scheme [Denner et al. '05; Denner, Dittmaier '19], but with real $\overline{M}$, δM^2 and couplings
- (II): $\mathrm{Re}\,(1/R_p) = 1$, $R_p = 1/\left[1 + i\,\mathrm{Im}\,\hat{\Sigma}'(s_p)\right]$
- (III): width normalization [Kniehl et al. '00; Nekrasov '01; Freitas '14]

Relations

- Same δM^2, $\overline{\Gamma}$; δZ differ by finite terms, $Z^{(\rm I)} = Z^{(\rm II)}\,R_p^{(\rm II)}$
- $R_p^{1/2}$ per production/decay factor [Freitas, Song '22]; (I) $\equiv$ (II) with these explicit factors

Take-home: the pole s_p is gauge independent; the field normalization is a convention: (I)–(III) share δM^2, $\overline{\Gamma}$.

Unstable Bosons: Mass, Width and Fields

Mass and width: same for (I), (II), (III)

$$\mathrm{Re}\,\hat{\Sigma}(s_p) = 0, \qquad \overline{M}\,\overline{\Gamma} = \mathrm{Im}\,\hat{\Sigma}(s_p)$$

Count $\overline{\Gamma}/\overline{M} = \mathcal{O}(\alpha)$ (width from one loop): expand about $\overline{M}^2$ in loops and in $s_p - \overline{M}^2 = -i\overline{M}\overline{\Gamma}$ together, all at $\overline{M}^2$:

$$\hat{\Sigma}(s_p) = \hat{\Sigma}_{(1)} + \hat{\Sigma}_{(2)} - i\overline{M}\,\overline{\Gamma}_{(1)}\hat{\Sigma}'_{(1)} + \mathcal{O}(\alpha^3)$$

$$\begin{aligned}
\overline{M}\,\overline{\Gamma}_{(1)} &= \mathrm{Im}\,\Sigma_{(1)}, \qquad \delta M^2_{(1)} = \mathrm{Re}\,\Sigma_{(1)} \\
\overline{M}\,\overline{\Gamma}_{(2)} &= \mathrm{Im}\,\Sigma_{(2)} - \mathrm{Im}\,\Sigma_{(1)}\,\mathrm{Re}\,\hat{\Sigma}'_{(1)} \\
\delta M^2_{(2)} &= \mathrm{Re}\,\Sigma_{(2)} - \delta Z_{(1)}\,\delta M^2_{(1)} + \mathrm{Im}\,\Sigma_{(1)}\,\mathrm{Im}\,\Sigma'_{(1)}
\end{aligned}$$

(II, III): $\overline{M}\,\overline{\Gamma}_{(2)} = \mathrm{Im}\,\Sigma_{(2)}$; Im×Im: pole off axis [Freitas et al. '02]

Z: same on $\hat{\Sigma}^{\mathrm{eff}}_Z$; $\hat{\Sigma}^T_{ZZ} \ni \frac{s}{4}(\delta Z_{AZ})^2$; mixing term real at $\overline{M}^2_Z$

$$\begin{aligned}
\delta M^2_{Z,(2)} = \delta M^2_{(2)}\big|_{\Sigma\to\Sigma^T_{ZZ}} \\
+ \tfrac{1}{4}\overline{M}^2_Z\big(\delta Z_{AZ,(1)}\big)^2 + \big[\mathrm{Im}\,\Sigma^T_{AZ,(1)}\big]^2/\overline{M}^2_Z
\end{aligned}$$

Field constants and residue

$\delta Z^{(\mathrm{I})}_{(1)} = -\Sigma'_{(1)}$, $\delta Z^{(\mathrm{II,III})}_{(1)} = -\mathrm{Re}\,\Sigma'_{(1)}$; at two loops:

$$\begin{aligned}
\text{(I)}: &\quad \delta Z_{(2)} = -\Sigma'_{(2)} + i\,\mathrm{Im}\,\Sigma_{(1)}\,\Sigma''_{(1)} \\
\text{(II)}: &\quad \delta Z_{(2)} = -\mathrm{Re}\,\Sigma'_{(2)} - \mathrm{Im}\,\Sigma_{(1)}\,\mathrm{Im}\,\Sigma''_{(1)} \\
\text{(III)}: &\quad \delta Z_{(2)} = -\mathrm{Re}\,\Sigma'_{(2)} - \tfrac{1}{2}\,\mathrm{Im}\,\Sigma_{(1)}\,\mathrm{Im}\,\Sigma''_{(1)}
\end{aligned}$$

Same UV poles; finite differences from $\mathrm{Im}\,\Sigma_{(1)}$; Z: on $\hat{\Sigma}^{\mathrm{eff}}_Z$, extra $(\delta Z_{AZ,(1)})^2$ and mixing terms

Residue in (II): only $\mathrm{Re}\,\hat{\Sigma}'(s_p) = 0$:

$$\begin{aligned}
R_p &= 1/\big[1 + i\,\mathrm{Im}\,\hat{\Sigma}'(s_p)\big], \\
|R_p| &= 1 - \tfrac{1}{2}\big[\mathrm{Im}\,\Sigma'_{(1)}(\overline{M}^2)\big]^2 + \mathcal{O}(\alpha^3)
\end{aligned}$$

- $R_p^{1/2}$ per production/decay factor [Freitas, Song '22]; (I) absorbs it: $Z^{(\mathrm{I})} = Z^{(\mathrm{II})} R_p^{(\mathrm{II})}$
- (I): $-\delta M^2_{(1)}\delta Z_{(1)}$ complex ⇒ use the hatted equations

W: $W\gamma$ threshold exactly at $\overline{M}^2_W$ [Kniehl, Sirlin '01] ⇒ same regulator ϵ for IR and UV, in both virtual and real corrections [Freitas et al. '02; Denner '93]

Charge Renormalization at Two Loops

Thomson limit: on-shell charge to all orders

$$\bar{u}(p)\,\hat{\Gamma}^{ee\gamma}_{\mu}(p,p)\,u(p)\Big|_{p^2=m_e^2} = ie\,\bar{u}(p)\gamma_\mu u(p)$$

Defines $\alpha(0) = e^2/(4\pi)$; S-matrix element: gauge independent, universal [Aoki et al. '82; Denner et al. '94]; QED: $Z_e = Z_{AA}^{-1/2}$. Isosinglet with charge $\to 0$, coupling to $A_0 + \frac{s_{W,0}}{c_{W,0}} Z_0$ [Dittmaier '21]; $e_0 = Z_e\, e$:

$$\boldsymbol{Z_e\left[\sqrt{1+\delta Z_{AA}} + \frac{s_W+\delta s_W}{c_W+\delta c_W}\,\frac{\delta Z_{ZA}}{2}\right] = 1}$$

$$\begin{aligned}
\delta Z_{e,(1)} &= -\tfrac{1}{2}\delta Z_{AA,(1)} - \tfrac{s_W}{2c_W}\delta Z_{ZA,(1)} \\
\delta Z_{e,(2)} &= -\tfrac{1}{2}\delta Z_{AA,(2)} - \tfrac{s_W}{2c_W}\delta Z_{ZA,(2)} + (\delta Z_{e,(1)})^2 \\
&\quad + \tfrac{1}{8}(\delta Z_{AA,(1)})^2 - \tfrac{1}{2c_W^3}\,\delta Z_{ZA,(1)}\,\delta s_{W,(1)}
\end{aligned}$$

- **Two-point functions only**, no two-loop vertex; agrees with [Freitas et al. '02; Awramik et al. '02; Dittmaier '21]
- R_ξ gauge: $\Sigma^T_{AA}(0) \neq 0$ beyond one loop $\Rightarrow$ use $\Sigma^{T\prime}_{AA}(0)$

Light-fermion vacuum polarization

$$\Delta\alpha(M_Z^2) = \Pi_{\text{light}}(0) - \text{Re}\,\Pi_{\text{light}}(M_Z^2)$$

- $\Delta\alpha^{(5)}_{\text{had}}$: free quarks 0.0357, data $0.02760(10)$ [Davier et al. '19] $\Rightarrow$ from data:
  $\Pi_{\text{light}}(0) \;\to\; \text{Re}\,\Pi_{\text{light}}(M_Z^2) + \Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$
- In Z_e [Degrassi, Vicini '03]: changes the representation of the Thomson charge, not its definition
- $\Delta\alpha(M_Z^2)$ counted as one loop:

$$\begin{aligned}
\delta Z_{e,(1)} &= \tfrac{1}{2}\big[\Delta\alpha(M_Z^2) + \text{Re}\,\Pi_{\text{light},(1)}(M_Z^2) \\
&\quad + \Pi_{\text{heavy},(1)}(0)\big] - \tfrac{s_W}{2c_W}\,\delta Z_{ZA,(1)} \\
\delta\tilde{Z}_{AA,(2)} &= -\text{Re}\,\Sigma^{T\prime}_{AA,(2)}(0) + \Pi_{\text{light},(2)}(0) \\
&\quad - \text{Re}\,\Pi_{\text{light},(2)}(M_Z^2) - \tfrac{1}{4}\big(\delta Z_{ZA,(1)}\big)^2
\end{aligned}$$

- $\delta\tilde{Z}_{AA}$ everywhere in $\delta Z_{e,(2)}$ (products, external photon lines): no separate $\Delta\alpha_{(2)}$

EW Renormalization: What We Got

FeynRules counterterm vertices → **FeynArts** FT and Landau gauge → **CalcLoop** g5anchor γ_5 → **Kira + Blade** IBP reduction → **DE series + AMFlow** master integrals

FeynRules [Alloul et al. '13]; FeynArts [Hahn '00]; CalcLoop [Y.-Q. Ma]; g5anchor [L. Chen '24]; Kira [Klappert et al. '20]; Blade [Guan et al. '19, '24]; AMFlow [Liu, Ma '22; Huang, Liu, Ma '26]

Ingredients

- Higgs tadpole, Σ_H, $\Sigma^T_{W_2 ZZ, AZ, AA}$, chiral fermion form factors: at the mass shells and at $p^2 = 0$
- Derivatives before the on-shell limit: first $\Rightarrow$ field constants; second, of one-loop functions $\Rightarrow$ complex-pole expansion

$$\delta X^g_{(2)} = \sum\nolimits_a c^g_a(D, \{m\})\, I^g_a, \qquad g = \mathrm{FT}, \mathrm{LG} \quad \text{(schematic)}$$

Masters include elliptic sectors, evaluated numerically to high precision

Two sets, to be made public

(i) massless light fermions, massive top; FT; in A_0, B_0, TVID 2 functions [Bauberger et al. '19]

(ii) all quark and charged-lepton masses; FT and Landau; in our master integrals

Massless neutrinos in both. **Also** (GitHub): master-integral definitions, one-loop constants through $\mathcal{O}(\epsilon)$, normalization of the loop measure

Applications

- Set (ii), full massive model: $H \to b\bar{b}$ at $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ (this talk); $W \to \ell\nu$, $W \to u\bar{d}$; $Z \to f\bar{f}$
- Set (i), massless light fermions: $Z \to f\bar{f}$

$\Gamma(H \to b\bar{b})$: Background and State of the Art

$$\Gamma(H \to b\bar{b}) = \Gamma_{\rm LO}\left[1 + \sum_{k\geq 1} \delta_{\rm QCD}^{\rm N^kLO} + \delta_{\rm EW}^{\rm NLO} + \delta_{\rm QCD\text{-}EW}^{\rm NNLO} + \delta_{\rm EW}^{\rm NNLO} + \cdots\right]$$

The dominant Higgs decay

- BR $\simeq 58\%$: normalizes all branching ratios [LHC HXSWG '16]; direct access to y_b; observed in 2018 [ATLAS '18; CMS '18]
- CEPC: $\sigma(ZH) \times \mathrm{BR}(b\bar{b})$ to 0.14% $(20\,\mathrm{ab}^{-1})$ [CEPC '22]
- Intrinsic uncertainty $< 0.4\%$ today; projected 0.2% with the complete $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$, EW part then $< 0.1\%$ [Freitas et al. '19; de Blas et al. '26]

Known corrections

- **QCD:** NLO [Braaten, Leveille '80; Sakai '80]; massless b to $\mathcal{O}(\alpha_s^4)$ [Gorishnii et al. '90; Chetyrkin '96; Baikov et al. '05; Herzog et al. '17]: beyond NNLO $< 0.2\%$
- Mass effects at NNLO [Chetyrkin, Kwiatkowski '95; Harlander, Steinhauser '97], exact m_b [Wang et al. '23]; differential [Bernreuther et al. '18; Behring, Bizoń '19; Somogyi, Tramontano '20]
- y_t terms: $\mathcal{O}(\alpha_s^{2,3})$ [Primo et al. '18; Mondini et al. '20; Wang et al. '24], y_t^2 at $\mathcal{O}(\alpha_s^4)$ [Wang et al. '26]
- Massive b to $\mathcal{O}(\alpha_s^4)$: new $y_b y_t$ term $+0.26\%$ [Chen et al., HP^2 '26, prelim.]
- **EW:** $\mathcal{O}(\alpha)$ [Bardin et al. '91; Dabelstein, Hollik '92; Kniehl '92], accidentally small [Freitas et al. '19]; $\mathcal{O}(\alpha\alpha_s)$ [Kataev '97; Mihaila et al. '15]
- $\mathcal{O}(\alpha^2)$ only partial: $\mathcal{O}(G_F^2 m_t^4)$ [Butenschoen et al. '06, '07], $\mathcal{O}(G_F^2 M_H^4)$ [Durand et al. '94; Ghinculov '94]

Recall J. Wang's talk (23 Oct): $\Gamma_{\overline{\rm MS}}^{\rm N^3LO\ QCD} = 2.410\,\mathrm{MeV}$, +NLO EW: $2.382\,\mathrm{MeV}$ (-1.2%) [Wang et al. '24] $\Rightarrow$ NNLO EW needed
M. Niggetiedt, HP^2 '26 (prelim.): massive b, $\mathcal{O}(\alpha_s^4)$: $\Gamma_{\overline{\rm MS}} = 2.4326\,\mathrm{MeV}$, $y_b y_t$: $+0.26\%$, scale 0.25% [Chen, Chen, Guan, Niggetiedt '26]

Take-home: $\Gamma(H \to b\bar{b})$: **intrinsic theory error** $< 0.4\%$ **today; projected** 0.2% **needs complete** $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$.

$H \to b\bar{b}$ at NNLO: Observable and Computation

$$\delta\Gamma^{(2)}_{\text{EW}} = \frac{1}{2M_H}\Big\{ \int d\Phi_{b\bar{b}} \left[2\text{Re}\left(\mathcal{M}^{(0,0)*}\mathcal{M}^{(2,0)}\right) + |\mathcal{M}^{(1,0)}|^2\right] + \int d\Phi_{b\bar{b}\gamma}\, 2\text{Re}\left(\mathcal{M}^{(0,0)*}_{b\bar{b}\gamma}\mathcal{M}^{(1,0)}_{b\bar{b}\gamma}\right) + \int d\Phi_{b\bar{b}\gamma\gamma}\, |\mathcal{M}^{(0,0)}_{b\bar{b}\gamma\gamma}|^2 + \sum_f \int d\Phi_{b\bar{b}f\bar{f}}\, |\mathcal{M}^{(0,0)}_{b\bar{b}f\bar{f}}|^2 \Big\}$$

$$\delta\Gamma^{(2)}_{\text{QCD-EW}} = \frac{1}{2M_H}\Big\{ \int d\Phi_{b\bar{b}}\, 2\text{Re}\left(\mathcal{M}^{(0,0)*}\mathcal{M}^{(1,1)} + \mathcal{M}^{(1,0)*}\mathcal{M}^{(0,1)}\right) + \int d\Phi_{b\bar{b}g}\, 2\text{Re}\left(\mathcal{M}^{(0,0)*}_{b\bar{b}g}\mathcal{M}^{(1,0)}_{b\bar{b}g}\right) + \int d\Phi_{b\bar{b}\gamma}\, 2\text{Re}\left(\mathcal{M}^{(0,0)*}_{b\bar{b}\gamma}\mathcal{M}^{(0,1)}_{b\bar{b}\gamma}\right) + \int d\Phi_{b\bar{b}g\gamma}\, |\mathcal{M}^{(0,0)}_{b\bar{b}g\gamma}|^2 \Big\}$$

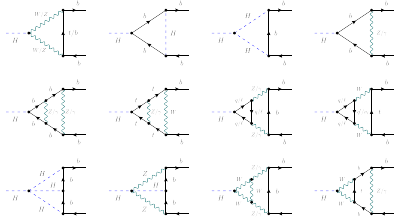
Width from $|\mathcal{M}|^2$: phase space $\xrightarrow{\text{reverse unitarity}}$ cut loop integrals $\xrightarrow{\text{IBP + masters}}$ $\delta\Gamma$ [Anastasiou, Melnikov '02]; Kira + Blade, DE + AMFlow

Double-virtual $2\text{Re}\left(\mathcal{M}^{(0,0)*}\mathcal{M}^{(2,0)}\right) + |\mathcal{M}^{(1,0)}|^2$

one loop

two loops

two loops



Real–virtual, double-real, $4f$

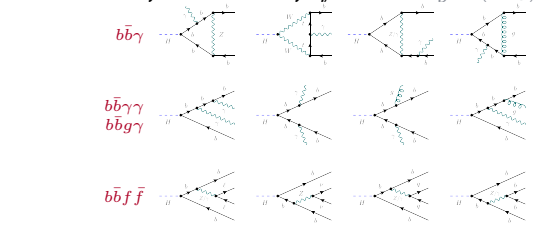
with g: $\mathcal{O}(\alpha\alpha_s)$

$b\bar{b}\gamma$

$b\bar{b}\gamma\gamma$

$b\bar{b}g\gamma$

$b\bar{b}f\bar{f}$



$\mathcal{M}^{(i,j)}_F$: renormalized amplitude for final state F at relative order $\alpha^i\alpha_s^j$ (incl. counterterms); first term of $\delta\Gamma^{(2)}_{\text{EW}}$: double-virtual

$H \to b\bar{b}$: Numerical Results

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	$\alpha(0)$ scheme		$\alpha(M_Z^2)$ scheme	
Contribution	Γ [MeV]	K [%]	Γ [MeV]	K [%]
LO	5.399		5.738	
NLO QCD	3.477	-35.591	3.696	-35.591
NLO EW	3.585	$+1.982$	3.457	-4.164
NNLO QCD–EW	3.572	-0.241	3.570	$+1.976$
NNLO EW (FT)	**3.5826**	**+0.20242**	**3.5907**	**+0.35759**
NNLO EW (LG)	**3.5826**	**+0.20245**	**3.5907**	**+0.35763**

- Γ cumulative, $K = \delta\Gamma/\Gamma^{(0)}$; FT and LG rows: alternatives
- EW benchmark, not a complete width prediction: on-shell m_b, no pure QCD beyond $\mathcal{O}(\alpha_s)$, no $4f$
- $\alpha(0)$ with $\Delta\alpha$ from data; $\alpha(M_Z^2)$ by finite conversion

Inputs [GeV]: pole masses $M_W = 80.334$, $M_Z = 91.1538$, $M_H = 125.13$; $m_t = 172.6$, $m_b^{\rm OS} = 4.788$, $1/\alpha(M_Z^2) = 128.951$, $\alpha_s(M_H) = 0.112677$

What the numbers say

- NLO QCD dominates (-35.6%): on-shell Yukawa, $\ln(M_H^2/m_b^2)$ not absorbed
- $\alpha(M_Z^2)$ absorbs light-fermion running into Born ($+6.3\%$): NLO EW, mixed K flip sign
- Mixed QCD–EW and pure NNLO EW are both sub-percent in $\alpha(0)$: -0.24% and $+0.20\%$
- They partly cancel in $\alpha(0) \Rightarrow$ both are needed at the 0.2% level
- As large as $\mathcal{O}(\alpha_s^3)$, $\mathcal{O}(\alpha_s^4)$ QCD [Baikov et al. '05]: $+0.20\%$, -0.15% ($\overline{\rm MS}$ y_b, $M_H = 120\,\text{GeV}$)

Higher-order light-current terms overlapping with $\Delta\alpha^{(5)}_{\rm had}$: subtraction still to be established

Take-home: NNLO EW, no $4f$: $+0.20\%$ $[\alpha(0)]$, $+0.36\%$ $[\alpha(M_Z^2)]$ — as large as the projected 0.2% theory error

$H \to b\bar{b}$: Four-Fermion States and Gauge Test

PRELIMINARY · L. Chen, X. Chen, A. Freitas, H.-Y. Han, **Z. Li**, Y.-Q. Ma, A. Tan, in preparation

Four-fermion final states

$\delta\Gamma^{(2)}_{4f}$ [10^{-3} MeV]	$\alpha(0)$	$\alpha(M_Z^2)$
$b\bar{b}\,\ell^+\ell^-,\ \ell = e,\mu,\tau$	1.7346	2.0817
$b\bar{b}\,\nu_\ell\bar{\nu}_\ell$	1.0254	1.2306
$b\bar{b}\,q\bar{q},\ q = u,d,c,s$	3.9608	4.7535
$b\bar{b}\,b\bar{b}$, incl. exchange	0.49443	0.59339
sum	7.21523	8.65919
$100\ K^{(2)}_{4f}$ [%]	**0.13364**	**0.15091**

$$\delta\Gamma^{(2)}_{\rm EW} = \delta\Gamma^{(2)}_{\rm EW,\,no\,4f} + \delta\Gamma^{(2)}_{4f}$$

- Tree level: $f\bar{f}$ from γ^*/Z^* off the b quarks; $H \to ZZ^*, Z \to b\bar{b}$: different channel, not included
- $\approx \frac{2}{3}\,[\alpha(0)]$ and $\frac{2}{5}\,[\alpha(M_Z^2)]$ of the rest of $\mathcal{O}(\alpha^2)$
- $b\bar{b}\,q\bar{q}$ dominates: 55% of $\delta\Gamma^{(2)}_{4f}$ in both schemes

Gauge test: FT vs. Landau gauge (LG)

$$\Delta_\xi K^{(2)} = \frac{\delta\Gamma^{(2)}_{\rm EW,\,no\,4f}\big|_{\rm LG} - \delta\Gamma^{(2)}_{\rm EW,\,no\,4f}\big|_{\rm FT}}{\Gamma^{(0)}}$$

[10^{-3} MeV]	$\alpha(0)$		$\alpha(M_Z^2)$	
	FT	LG	FT	LG
2b	3.9776	3.9776	52.864	52.864
2f	15.860	15.860	19.035	19.035
fb	−8.9087	−8.9070	−51.380	−51.378

- 2b, fb, 2f (0,1,2 closed fermion loops): each must be ξ independent
- $\Delta_\xi K^{(2)} = 3 \times 10^{-7}\ [\alpha(0)],\ 4 \times 10^{-7}\ [\alpha(M_Z^2)]$, from fb
- No external-Higgs residue: LG−FT $= 1.1, 1.2 \times 10^{-4}$ of $\Gamma^{(0)}$ $= 1 - |R_H| \Rightarrow$ residue needed
- **Potential source:** Γ at real $p^2 = \overline{M}_H^2$, not at s_p: $\frac{1}{2}(\Gamma^{(0)}/\overline{M}_H)|X_u| = 3.2, 3.6 \times 10^{-7}$, size and sign of LG−FT (X_u: LG unphysical cut in $\mathrm{Im}\,\hat{\Sigma}'_H$; fb, $\propto m_b^2$); to be tested

Take-home: $4f$ **final states add** 0.13–0.15%; **FT and LG differ by** 3–4×10^{-7} **of the Born width (fb), origin open.**

Summary and Outlook

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Summary

Two-loop on-shell EW renormalization

- Reusable, process-independent two-loop constants: masses, fields, charge; FT and Landau gauge
- Unstable H, W, Z, t: gauge-invariant complex pole, expanded about its real part; field normalization by prescriptions
- Charge (Thomson limit) from two-point functions; $\Delta\alpha^{(5)}_{\text{had}}$ from data

$\Gamma(H \to b\bar{b})$ **at** $\mathcal{O}(\alpha^2)$ **and** $\mathcal{O}(\alpha\alpha_s)$

- NNLO EW: $+0.20\%$ $[\alpha(0)]$, $+0.36\%$ $[\alpha(M_Z^2)]$ of $\Gamma^{(0)}$; FT and LG differ by 3–4 $\times$ 10^{-7} of $\Gamma^{(0)}$
- Four-fermion final states: $+0.13\%$ $[\alpha(0)]$, $+0.15\%$ $[\alpha(M_Z^2)]$

Outlook

- W decays and top-quark decay at NNLO EW and mixed QCD-EW
- Public release of the two-loop constants shortly; update to a compact form
- $2 \to 2$ processes at future e^+e^- and hadron colliders

Thank you for your attention!

W-Boson Decays at NNLO EW (Preliminary)

PRELIMINARY · L. Chen, X. Chen, A. Freitas, H.-Y. Han, **Z. Li**, Y.-Q. Ma, A. Tan, in preparation

	$W^+ \to e^+\nu_e$		$W^+ \to u\bar{d}$	
Contribution	Γ [MeV]	K [%]	Γ [MeV]	K [%]
LO	218.767		656.302	
NLO QCD	218.767	0	681.435	+3.829
NLO EW	226.435	+3.505	704.136	+3.459
NNLO QCD–EW	227.753	+0.602	708.507	+0.666
NNLO EW (FT)	**228.0104**	**+0.11773**	**710.0716**	**+0.23844**
NNLO EW (LG)	**228.0146**	**+0.11967**	709.6983†	+0.18155†

$$\Gamma = \frac{|R_W|}{2\overline{M}_W}\,\frac{1}{3}\sum_{\lambda,X}\int d\Phi\,|\mathcal{M}_\lambda|^2, \qquad |R_W| = 1 - \tfrac{1}{2}x_W^2$$

$$x_W = \mathrm{Im}\,\Sigma^{T\prime}_{W,(1)}(\overline{M}^2_W) = 3.36\,\alpha\ (\mathrm{FT}),\ 3.14\,\alpha\ (\mathrm{LG})$$

- $\alpha(0)$ scheme, real field constants (II); Γ cumulative, K vs. same-channel Born; no $4f$, no QCD beyond NLO
- †LG hadronic: uncancelled simple pole (2.4×10^{-5} of Born), ϵ^0 coefficient only

What the numbers say

- Poles cancel: NLO > 50 digits, NNLO QCD–EW exactly, NNLO EW $< 2 \times 10^{-13}$ (FT); IR poles of $\delta Z_{W,(2)}$ cancel against virtual + real (p. 7)
- Residue indispensable: $100(|R_W| - 1) = -0.030$ (FT), -0.026 (LG); lowers $K^{(2)}_{\mathrm{EW}}$ by 20% ($e\nu$), 11% ($u\bar{d}$)
- NNLO EW (FT): +0.118% ($e\nu$), +0.238% ($u\bar{d}$); $|K^{(2)}_{\mathrm{EW}}/K^{(1)}_{\mathrm{EW}}| = 0.034, 0.069$ vs. 0.134 for $H \to b\bar{b}$
- Leptonic QCD–EW +0.602%: only from $\mathcal{O}(\alpha\alpha_s)$ parts of δZ_W, δs_{W}, δZ_e (50, 41, 10%)
- Gauge: leptonic LG−FT $\Delta_\xi K^{(2)} = +1.9 \times 10^{-5}$ (1.6% of the coefficient, $\sim 60 \times H \to b\bar{b}$): origin under study

On-Shell Setup: Parameters and Fields

Parameters and fields

Input parameters: α, M_W, M_Z, M_H, m_f
(charged ℓ, q); massless ν, unit CKM;
$c_{\rm w}^2 = M_W^2/M_Z^2$ [Sirlin '80]

$$\alpha_0 = \frac{e_0^2}{4\pi} = \frac{g_0^2 g_0'^2}{4\pi\left(g_0^2+g_0'^2\right)}, \qquad M_{H,0}^2 = \frac{\lambda_0 v_0^2}{2},$$

$$M_{W,0}^2 = \frac{g_0^2 v_0^2}{4}, \qquad M_{Z,0}^2 = \frac{\left(g_0^2+g_0'^2\right)v_0^2}{4}$$

Renormalized parameters: $i = H, W, Z$

$$M_{i,0}^2 = M_i^2 + \delta M_i^2, \qquad m_{f,0} = m_f + \delta m_f,$$

$$e_0 = (1+\delta Z_e)\, e$$

Renormalized fields:

$$H_0 = Z_H^{1/2} H, \qquad W_0^{\pm} = Z_W^{1/2} W^{\pm},$$

$$f_0^{L/R} = \left(Z_f^{L/R}\right)^{1/2} f^{L/R}$$

γ–Z mixing under renormalization: off-diagonal δZ_{ZA}, δZ_{AZ}, linear by definition [Denner, Dittmaier '19]

$$\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{1+\delta Z_{ZZ}} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & \sqrt{1+\delta Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

Dyson-resummed propagators

$$G_H = \frac{i}{k^2 - M_H^2 + \hat{\Sigma}_H(k^2)}, \qquad G_f = \frac{i}{\not{p} - m_f + \hat{\Sigma}_f} \equiv i\,\frac{N_f}{D_f}$$

$$G_W^{\mu\nu}(k) = \frac{-i\,P_T^{\mu\nu}(k)}{k^2 - M_W^2 + \hat{\Sigma}_W^T(k^2)} + P_L^{\mu\nu}(k)\, G_W^L(k^2)$$

$$G_{VV'}^{\mu\nu}(k) = -i\,P_T^{\mu\nu}(k)\left[\mathcal{K}^{-1}(k^2)\right]_{VV'} + P_L^{\mu\nu}(k)\, G_{VV'}^L(k^2)$$

- $\mathcal{K}(s) = \begin{pmatrix} s - M_Z^2 + \hat{\Sigma}_{ZZ}^T & \hat{\Sigma}_{AZ}^T \\ \hat{\Sigma}_{AZ}^T & s + \hat{\Sigma}_{AA}^T \end{pmatrix}$, $s = k^2$, basis (Z, A)
- Z: photon eliminated from the coupled equations,
  $\left[\mathcal{K}^{-1}\right]_{ZZ} = 1/\left[s - M_Z^2 + \hat{\Sigma}_Z^{\rm eff}\right]$,
  $\hat{\Sigma}_Z^{\rm eff} = \hat{\Sigma}_{ZZ}^T - \left[\hat{\Sigma}_{AZ}^T\right]^2/\left(s + \hat{\Sigma}_{AA}^T\right)$: photon exchange between two γ–Z transitions, from two loops; algebraic, not absolute, square;
  Z pole and residue conditions on $\hat{\Sigma}_Z^{\rm eff}$ [Freitas et al. '02; Chen, Freitas '20a]
- Fermions: $\hat{\Sigma}_f = \not{p} P_L \hat{\Sigma}_f^L + \not{p} P_R \hat{\Sigma}_f^R + m_f \hat{\Sigma}_f^S$ (left, right, scalar)
- Transverse parts enter the on-shell conditions; $\hat{\Sigma}_{(2)} = \Sigma_{(2)}$ (subloop-renormalized) + local counterterms

Top Quark: Pole, Mass and Field Prescriptions

Complex pole and mass counterterm

$$D_t(\mu_t^2) = 0, \qquad \mu_t = m_t - \tfrac{i}{2}\Gamma_t$$

- $m_t \equiv \mathrm{Re}\,\mu_t$, gauge independent [Kniehl, Sirlin '08]; $\mathrm{Re}\,D_t = \mathrm{Im}\,D_t = 0$ fix $\delta m_t, \Gamma_t$
- Expand at $p^2 = m_t^2$, $X = L, R, S$: $\Sigma_t^X(\mu_t^2) = \Sigma_{(1)}^X + \Sigma_{(2)}^X - i m_t \Gamma_{t,(1)} \Sigma_{(1)}^{X\prime}$

$$\begin{aligned}
\delta m_{t,(1)} &= \tfrac{m_t}{2}\mathrm{Re}\,\Sigma_{(1)}^M, \qquad \Gamma_{t,(1)} = m_t\,\mathrm{Im}\,\Sigma_{(1)}^M \\
\delta m_{t,(2)} &= \tfrac{m_t}{2}\mathrm{Re}\,\Sigma_{(2)}^M - \tfrac{1}{2}\delta m_{t,(1)}\mathrm{Re}\left(\delta Z_{(1)}^L + \delta Z_{(1)}^R\right) \\
&+ \tfrac{m_t}{8}\mathrm{Re}\left[(\delta Z_{(1)}^L - \delta Z_{(1)}^R)^2\right] - \tfrac{m_t}{8}\mathrm{Re}\left[(\hat\Sigma_{(1)}^L - \hat\Sigma_{(1)}^R)^2\right] \\
&+ \tfrac{m_t}{4}\mathrm{Im}\,\Sigma_{(1)}^M\,\mathrm{Im}\left[\hat\Sigma_{(1)}^L + \hat\Sigma_{(1)}^R + 2m_t^2\,\Sigma_{(1)}^{M\prime}\right]
\end{aligned}$$

- $\Sigma^M \equiv \Sigma^L + \Sigma^R + 2\Sigma^S$; $\mathrm{Im}\times\mathrm{Im}$ (last term) only in $\delta m_{t,(2)}$, not in $\Gamma_{t,(2)}$ (cf. p. 7)
- No absorptive parts $\Rightarrow$ stable-fermion $\delta m_{f,(2)}$ (p. 5)

Residue and field prescriptions

$$\left.\frac{N_t}{D_t'}\right|_{p^2=\mu_t^2} = \not{p} + \mu_t \;\Rightarrow\; r_L = r_R = 1$$

$r_{L/R} \equiv (1 + \hat\Sigma_t^{R/L})/D_t'$; $\mathrm{Im}\,(\Sigma_t^L - \Sigma_t^R) \neq 0 \Rightarrow$ **real** δZ_t **cannot give both** [Kniehl, Sirlin '08]

I, complex: $\delta Z_{t,(1)}^{L/R} = -\Sigma_{t,(1)}^{L/R} - m_t^2 \Sigma_{t,(1)}^{M\prime}$: $r_L = r_R = 1$, as in the complex-mass scheme

II, real: only $\mathrm{Re}\,r_{L/R} = 1$; $\Gamma_{t,(2)} = m_t\,\mathrm{Im}\,\Sigma_{t,(2)}^M$ as for bosons; residue $r_{L/R} = 1 - i q_{L/R} - i t_{L/R}$, $q_{L/R} = \mathrm{Im}\left[\Sigma_{t,(1)}^{L/R} + m_t^2 \Sigma_{t,(1)}^{M\prime}\right]$

$$G_t\big|_{\mathrm{pole}} = i\,F\,\frac{\not{p} + \mu_t}{p^2 - \mu_t^2}\,\bar F, \qquad F = P_L\sqrt{r_L} + P_R\sqrt{r_R}$$

External top: $u \to Fu$, $\bar u \to \bar u \bar F$: the role of $R_p^{1/2}$ (p. 7)

Relation: $Z_t^{(\mathrm{I})} = Z_t^{(\mathrm{II})}\, r^{(\mathrm{II})}$, $Z_t^{(\mathrm{II})} = \mathrm{Re}\,Z_t^{(\mathrm{I})}$ to all orders