Implementation of higher-order QED ISR corrections in ZFITTER

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Motivation

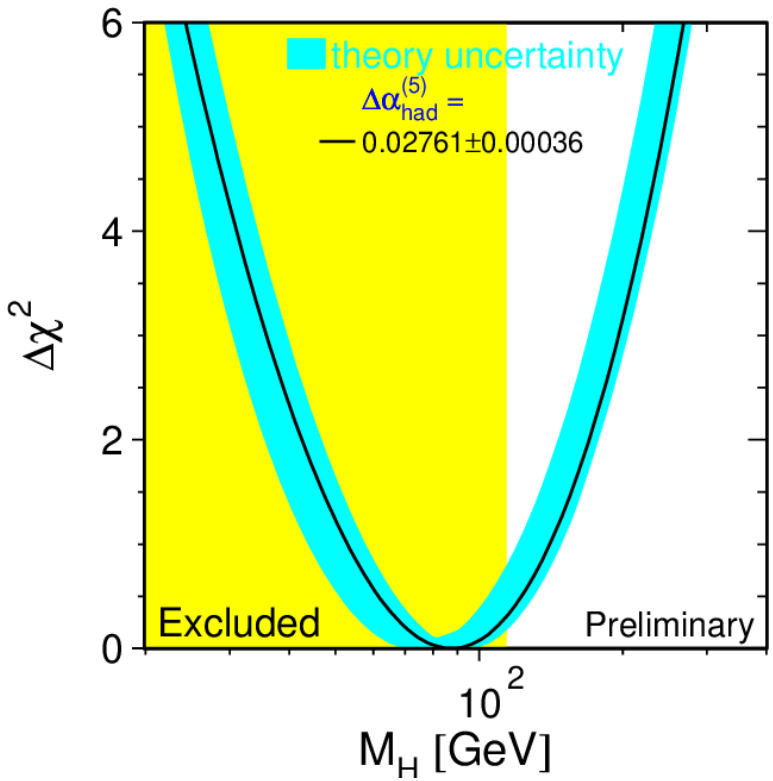
- **Future collider precision:** CEPC/FCC-ee aim at experimental precision of $\mathcal{O}(10^{-4})$ in luminosity measurement; the corresponding theoretical precision should reach $\mathcal{O}(10^{-5})$.
- **ZFITTER** is a legacy LEP/SLC analysis tool and remains a benchmark code. it will not be the mail code for future colliders, but it will be a good quick test for particular processes and MC
- **Exponentiation** is included in ZFITTER according to YFS approach. We plan to include photoinic and non-singlet pair corrections in Gribov-Lipatov approach

ZFITTER: History and Versions

- Fortran 77 semi-analytical program for fermion-pair production in e^+e^- annihilation and electroweak radiative corrections.
- Original authors: D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann.
- One of two main codes for analysis of EW data from LEP.

Calculates:

- M_W – the W boson mass;
- Γ_Z, Γ_W – Z and W decay widths;
- $\mathrm{d}\sigma/\mathrm{d}\cos\vartheta$ – differential cross-sections;
- σ_T – total cross-sections;
- A_{FB} – forward–backward asymmetries;
- A_{LR} – left–right asymmetries;
- A_{pol}, A_{pol}^{FB} – final-state polarisation effects for τ leptons.



For future colliders ZFITTER will not be the main program; it will be benchmark and quick test of particular small effects, Monte Carlo tests

From v6.21 to v6.42: Timeline and Additions

- **v6.21**: July 1999; described in Comput. Phys. Commun. **133** (2001) 229–395. Mainly intended for LEP1/SLC energies
- **v6.30**: LEP2 kinematics; final-state fermion-pair radiation; improved ISR QED with acceptance cuts; $|V_{tb}|$ option in DIZET.
- **v6.33+**: all known two- and three-loop corrections to M_W, $\sin^2\theta_{\rm eff}$, and Γ_Z gradually included.
- **v6.34** (26 Jan 2001): atomic parity violation; weak charge Q_W via subroutine `ZU_APV`.
- **v6.36** (21 Jul 2001): leading and next-to-leading Δr through heavy-top expansion in `m2tcor.f`.
- **v6.41** (15 Oct 2004): alleviated $b\bar{b}$ final-state mismatch in the two-loop electroweak treatment.
- **v6.42**: higher-order QED corrections; atomic parity violation; $\bar{\nu}_e\nu_e$ production; two-loop M_W and effective weak mixing angle; option `AMT4=6`.
- **v6.45** - current version, new version of DIZET
- **v6.46**β - in progress

Link: *https://sanc.jinr.ru/zfitter*

Improvements (preliminary results)

$\mathcal{O}(\alpha^2)$ order pair corrections:

- added corrections to the $\mathcal{O}(\alpha^2 L^0)$ contribution from [Blumlein et al., Phys.Lett.B 791 (2019)]

Difference with the previous version: 0.00361% with respect to Born cross section

$\mathcal{O}(\alpha^3 L^3)$ order pair corrections:

- replaced previous formula that reproduced $P_{ee}^{(0)\otimes 2}$ in $\mathcal{O}(\alpha^3 L^3)$ and part of $\mathcal{O}(\alpha^3 L^2)$ pair correction part with convolution I
- corrected coefficient at $P_{e\gamma}^{0} \otimes P_{\gamma e}^{(0)}$

Difference with the previous version: 0.06291% with respect to Born cross section

Flag IBLUM - switches between previous version and our additions
Flag INLO - will turn on $\mathcal{O}(\alpha^3 L^2)$ contribution

Problems:

- cross section in ZFITTER is not only integrated but partly exponentiated, difficult to reproduce and extract fixed order corrections
- not enough numerical integration precision, will switch to quadruple precision

ISR corrections, $\sqrt{s} = M_Z$, $z_{min} = 0.3$

Cross-section of e^+e^- annihilation:

$$\frac{d\sigma_{\bar{e}e}^{\mathrm{NNLO}}(s')}{ds'} = \frac{\sigma^{(0)}(s')}{s}\left\{\delta(1-z) + \sum_{\substack{k=1 \\ k\geq l\geq k-2}}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k L^l c_{kl}(z)\right\} + \mathcal{O}\left(\frac{m_e^2}{s}\right)$$

$$h_{ij} = \left(\frac{\alpha}{2\pi}\right)^k L^l \left(\int_{z_{min}}^{1} \sigma^{(0)}(sz) c_{kl}^{\theta}(z) dz \; + \sigma^{(0)}(1) c_{kl}^{\Delta}\right)$$

	h_{11}	h_{10}	h_{22}	h_{21}	h_{20}
γ	-32.74674	2.00210	4.88350	-0.59513	0.01521
Pairs	0	0	-0.30633	0.15852	-0.02151
Full	-32.74674	2.00210	4.57747	-0.43662	-0.00630

	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
γ	-0.37757	0.07102	0.00345	-0.00185	0.00315
Pairs	0.08757	-0.04604	-0.00909	0.00375	-0.00008
Full	-0.29002	0.02499	-0.00564	0.00190	0.00307

ISR corrections, $\sqrt{s} = 240\text{GeV}$, $z_{min} = 0.1$

	h_{11}	h_{10}	h_{22}	h_{21}	h_{20}
γ	324.14638	-11.76206	27.12979	-0.99088	-0.08026
Pairs	0	0	25.33367	-1.47717	-0.43752
Full	324.14638	-11.76206	52.46346	-2.46805	-0.51778

	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
γ	-0.62423	0.61404	-0.06641	0.01487	0.00247
Pairs	-0.02167	-0.03074	-0.01918	0.04599	0.00012
Full	-0.63690	0.58330	-0.08559	0.06086	0.00259

Corrections are large due to radiative return to resonance

Exponentiation

Future plans - implement exponentiation

Current state:

- Soft-photon effects exponentiation based on Yennie-Frautschi-Suura approach and exponentiation of the cross-section

What we plan to include:

- Photonic and non-singlet pair corrections exponentiation (collinear effects) based on Gribov-Lipatov framework [A.Arbuzov, U.V., Asymmetry, 2026]
- Separate block for calculating completely without exponentiation to reproduce separate corrections

Notes on exponentiation

By construction $\sigma^{(exp)} \sim (\mathcal{D} \otimes \mathcal{D})^{(exp)} \otimes \sigma^{(0)}$ $\leftarrow$ note absence of $\bar{\sigma}^{(1)}$.
Part of NLO effects can be still covered (in μ_F and α)

Ad hoc exponentiation $\mathcal{D}^{(exp)} \otimes \mathcal{D}^{(exp)} \otimes \left(\sigma^{(0)} + \bar{\sigma}^{(1)}\right)$ in NLO fails

For $x \to 1$ we have $\mathcal{D}^{(exp)}(x,\beta) \otimes \mathcal{D}^{(exp)}(x,\beta) \equiv \mathcal{D}^{(exp)\otimes 2} \to \mathcal{D}^{(exp)}(x, 2\beta)$

Gribov-Lipatov formula:

$$\mathcal{D}_{ee}^{(exp,\gamma)}(z;\beta) = \frac{\beta}{2}\frac{(1-z)^{\frac{\beta}{2}-1}}{\Gamma(\frac{\beta}{2}+1)} \exp\left(\frac{\beta}{2}\left(\frac{3}{4} - \gamma_E\right)\right)$$

$$\gamma_E = 0.5772156649$$

Include the bulk of NS pair corrections

$$\beta = \frac{2\alpha}{\pi}(L-1) \to \widetilde{\beta} = \frac{2\alpha}{\pi}(L-1)\left[1 + \frac{\alpha}{6\pi}\left(L - \frac{7}{3}\right)\right]$$

so we suggest

$$\mathcal{D}_{ee}^{(exp,\gamma+NS)}(x,\widetilde{\beta}) = \frac{\widetilde{\beta}}{2}\,\frac{(1-x)^{\widetilde{\beta}/2-1}}{\Gamma(1+\widetilde{\beta}/2)} \exp\left\{\frac{\widetilde{\beta}}{2}\left(\frac{3}{4} - \gamma_E\right)\right\}$$

Matching of exponentiation to perturbative results

We suggest the improved approximation: include as much as possible perturbative results + higher-orders via exponentiation:

$$\sigma^{imp}(z) = \sigma^{pert}(z) - \mathcal{D}_{ser}^{(exp)\otimes 2}(z)\ \sigma^{(0)}(z) + D_{\infty}^{(exp)\otimes 2}(z)\ \sigma^{(0)}(z)$$

where

$$\sigma^{pert}(z) = \mathcal{D}_{ee} \otimes \mathcal{D}_{\bar{e}\bar{e}} \otimes \left(\sigma_{e\bar{e}}^{(0)}(z) + \frac{\alpha}{2\pi} \overline{\sigma}_{e\bar{e}}^{(1)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 \overline{\sigma}_{e\bar{e}}^{(2)}(z) \right)$$
$$+2\mathcal{D}_{\gamma e} \otimes \mathcal{D}_{\bar{e}\bar{e}} \otimes \left(\frac{\alpha}{2\pi} \sigma_{e\gamma}^{(0)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 \overline{\sigma}_{e\gamma}^{(1)}(z) \right)$$
$$+\mathcal{D}_{\bar{e}e} \otimes \mathcal{D}_{e\bar{e}} \otimes \left(\frac{\alpha}{2\pi} \sigma_{e\bar{e}}^{(0)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 \overline{\sigma}_{e\bar{e}}^{(1)}(z) \right)$$

and $\mathcal{D}_{ser}^{(exp)\otimes 2}(z)$ is perturbative expansion of $\mathcal{D}_{\infty}^{(exp)\otimes 2}(z)$ up to the order present in $\sigma^{pert}(z)$. So, we exclude double counting

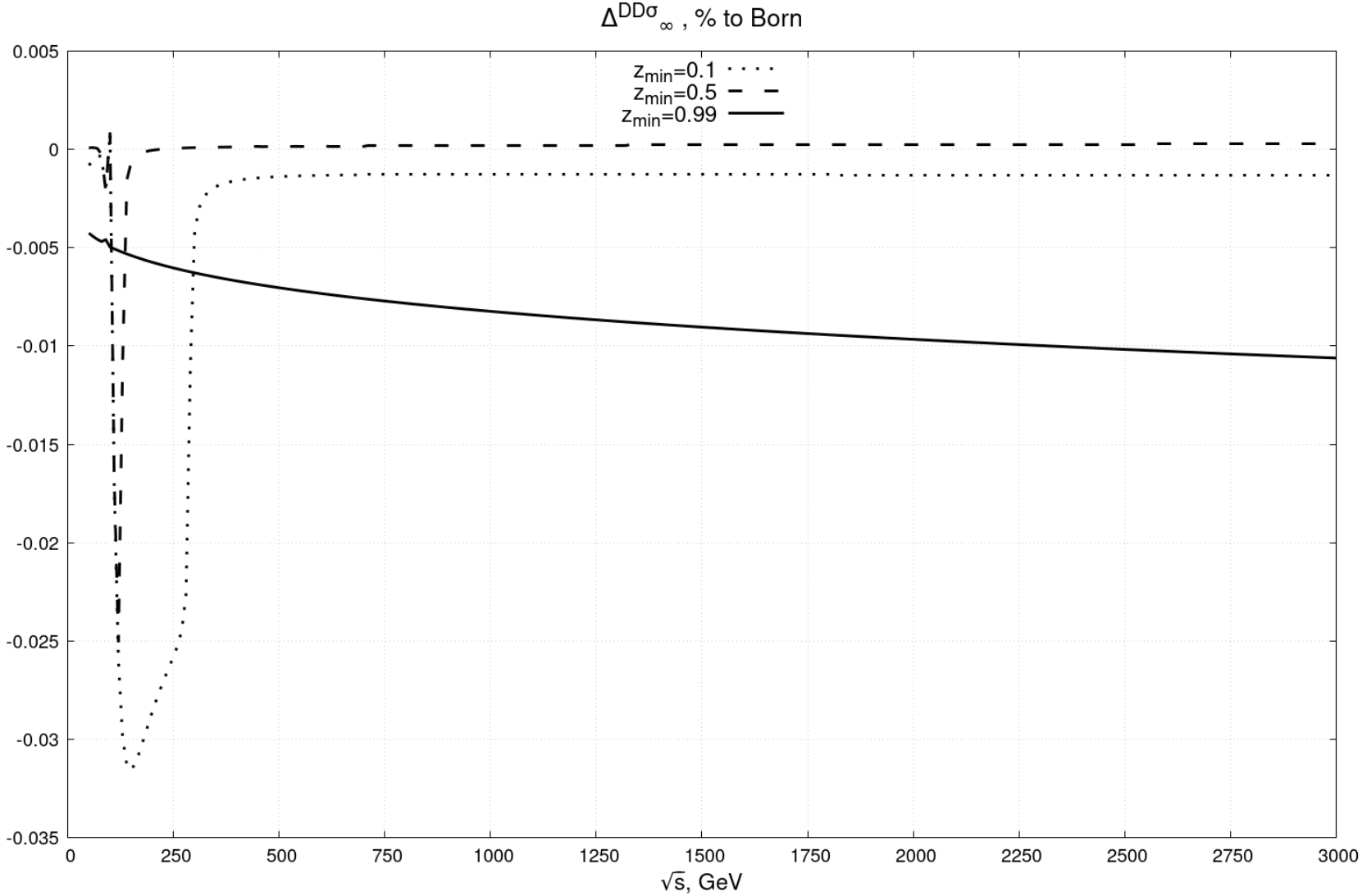
Numerical estimations of exponentiated corrections

$$h^{exp}_{55,\gamma} = \left(\int_{z_{min}}^{1} dz D^{(exp,\gamma)\otimes 2}_{ser,5}(z)\ \sigma^{(0)}(sz) - \int_{z_{min}}^{1} dz D^{(exp,\gamma)\otimes 2}_{ser,4}(z)\ \sigma^{(0)}(sz) \right) \Big/ \sigma^{(0)}(s) \cdot 100\%$$

$$\Delta^{DD\sigma}_{\infty,\gamma} = \left(\int_{z_{min}}^{1} dz D^{(exp,\gamma)\otimes 2}_{\infty}(z)\ \sigma^{(0)}(sz) - \int_{z_{min}}^{1} dz D^{(exp,\gamma)\otimes 2}_{ser,5}(z)\ \sigma^{(0)}(sz) \right) \Big/ \sigma^{(0)}(s) \cdot 100\%$$

	$\sqrt{s} = 160$ GeV			$\sqrt{s} = 240$ GeV			$\sqrt{s} = M_Z$		
	h_{55}	h^{exp}_{55}	$\Delta^{DD\sigma}_{\infty}$	h_{55}	h^{exp}_{55}	$\Delta^{DD\sigma}_{\infty}$	h_{55}	h^{exp}_{55}	$\Delta^{DD\sigma}_{\infty}$
$Z_{min} = 0.1$									
γ	0.00562	0.01695	-0.00369	0.00247	0.02316	-0.00389	0.00315	0.00341	-0.00013
Pairs	-0.00164	-	-	0.00012	-	-	-0.00008	-	-
Full	0,00397	0.02057	-0.00343	0.00259	0.01950	-0.00219	0.00307	0.00344	-0.00086
$Z_{min} = 0.5$									
γ	-0.00022	-0.00097	0.00034	-0.00020	-0.00089	0.00035	0.00315	0.00341	-0.00013
Pairs	0.0002	-	-	0.00013	-	-	-0.00008	-	-
Full	-0.00002	-0.00018	0.00009	-0,00007	-0.00005	0.00014	0.00307	0.00344	-0.00086
$Z_{min} = 0.9$									
γ	0.00143	0.00156	-0.00017	0.00161	0.00179	-0.00019	0.00319	0.00345	-0.00013
Pairs	-0.00083	-	-	-0.00092	-	-	-0.00009	-	-
Full	0.00058	0.00060	0.00004	0.00069	0.00073	0.00004	0.00310	0.00347	-0.00087

Exponentiation of pair corrections



$\Delta^{DD\sigma}_{\infty,\gamma+NS}$ in percent vs. center-of-mass energy for three different values of z_{min}.

Theoretical uncertainty

We estimated theoretical uncertainty at Z-peak

$$\Delta_{LL,exp} = \frac{h_{55}}{h_{44}}(h_{55}^{(exp)} - h_{55})$$

$$\Delta_{NLL} = \frac{h_{43}}{h_{32}} h_{43}$$

$$\Delta_{NNLL} = \frac{h_{20}}{h_{21}} h_{32}$$

Таблица: Theoretical uncertainty in % at $\sqrt{s} = M_Z$.

z_{min}	$\Delta_{LL,exp}$	Δ_{NLL}	Δ_{NNLL}
0.1	0.00019	0.00014	0.00036
0.5	0.00019	0.00014	0.00036
0.9	0.00021	0.00014	0.00036

Conclusions

- **Higher-order QED ISR corrections implemented in ZFITTER:**
  - $\mathcal{O}(\alpha^2)$ corrections added (Blumlein et al.)
  - $\mathcal{O}(\alpha^3 L^3)$ coefficient for pair corrections corrected
- **Plans:**
  - Separate block in ZFITTER to reproduce fixed order corrections
  - Implement exponentiation according to Gribov-Lipatov approach
  - Improve integration precision
- **Exponentiation of photonic and NS pair corrections:** estimated h_{55} correction and the residual "tail" for different energies and z_{min}, calculated theoretical uncertainty