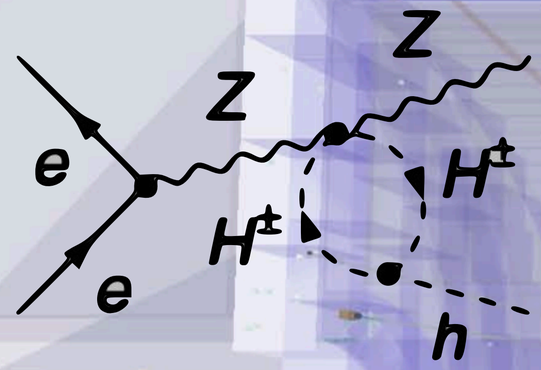
Probing the Real Triplet Higgs Model with NLO Higgsstrahlung at future e^+e^- colliders

CEPC detector event display arXiv:2510.05260

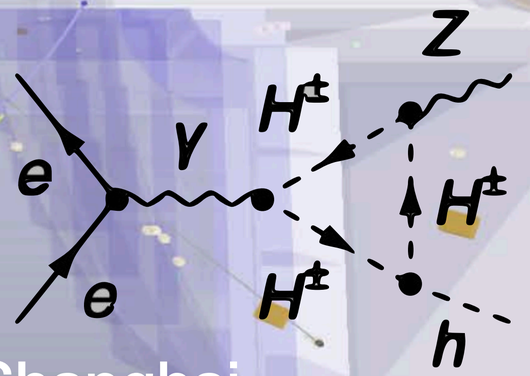


Cristian Sierra

Tsung-Dao Lee Institute, SJTU

On behalf of:
A.K. Barik, S. Heinemeyer (IFT, Madrid),
M. J. Ramsey-Musolf (TDLI),
Xiao-Gang He (TDLI), M. Y. Sassi
and Qin Yu

arXiv: 2611.XXXXX



CEPC2026 workshop, Shanghai

2026年10月23日

李政道研究所
TSUNG-DAO LEE INSTITUTE

Visualization of the CEPC detector and display of an $e+e- \to Zh$, $Z \to \mu+\mu-$, h $\to bb$ event

National Natural Science Foundation of China

International Collaboration Fund for Creative Research Teams

Precision Calculations for Electron-Positron Colliders

Michael Ramsey-Musolf [1]

Xiao-Gang He[1] Hai-Jun Yang[1] Jun Guo[1] Manqi Ruan[2] Sven Heinemeyer[3] Alessandro Vicini[4]

[1]Shanghai Jiao Tong University | *[2]Institute of High Energy Physics, CAS*

[3]Instituto de Fisica Teorica, Madrid | *[4]University of Milan*

January 2025-December 2027

李政道研究所
TSUNG-DAO LEE INSTITUTE

PCEPC Scientific Thrusts

Theory

Theory

I. $e^+e^- \rightarrow f\,f$ @ NNLO

- MJRM, A. Vicini
- 2 post-docs
- 2 PhD students

II. $e^+e^- \rightarrow Zh$ @ NLO

- MJRM, XG He, S. Heinemeyer
- 3 Post-docs
- 2 PhD students

III. Theory-Expt Pipeline

- H. Yang, ,M. Ruan, J. Guo
- 2 post-docs
- 2 PhD students

IV. Novel EW Observables

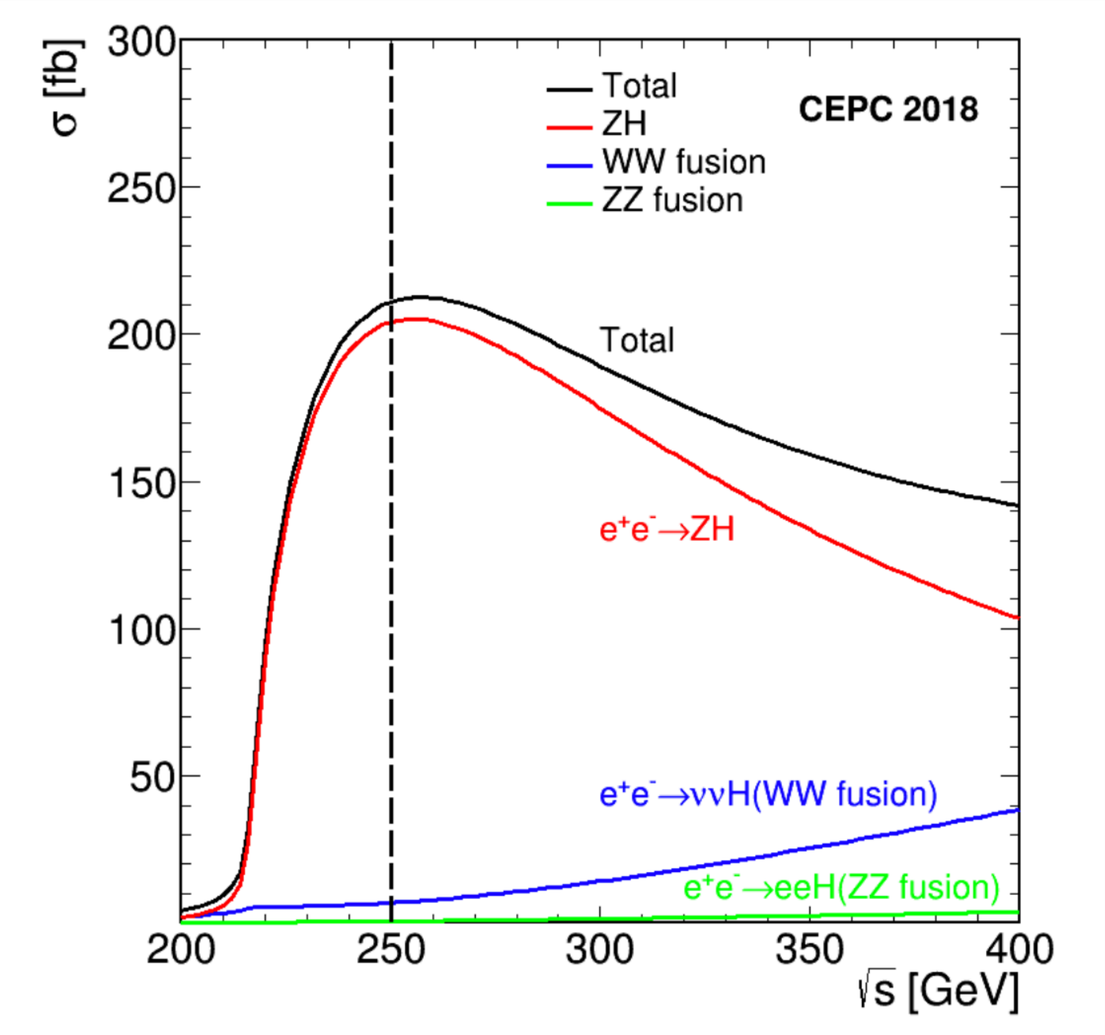
- XG He, MJRM, M. Ruan
- 2 post-doc
- 2 PhD students

Experiment

Theory+ Experiment

Total team: 22 = 7 Senior + 7 post-docs + 8 PhD students

Precision Higgsstrahlung at lepton colliders



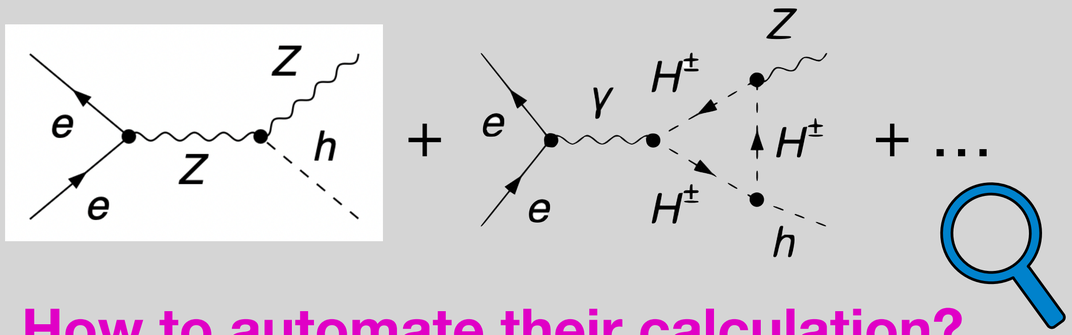
Plot: CEPC CDR, arXiv:1810.09037. Precision: 20 ab^{-1} at 240 GeV; arXiv:2205.08553, 2505.24810.

Why is $e^+e^- \to Zh$ important?

- CEPC, FCC-ee and linear colliders probe the Higgs sector near the Zh production maximum.
- Inclusive recoil measurements constrain the Higgs production rate, Higgs mass is simply (*See 9:00 am Bruce Mellado's talk*)

$$M^2_{\text{recoil}} = s + M_Z^2 - 2\sqrt{s}\,E_Z$$

- *Sensitivity to additional scalars entering self-energy and vertex corrections at NLO*



How to automate their calculation?

Focus of this talk!

Precision Higgsstrahlung at lepton colliders

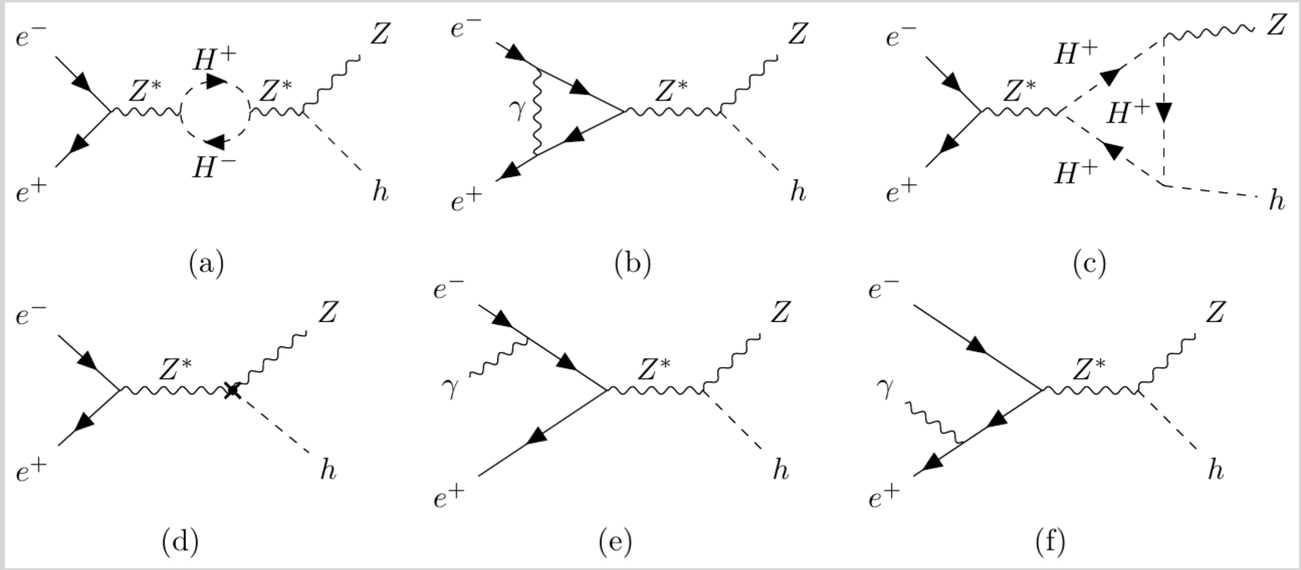
What do we want? **We want to compute the ratio of the (electroweak) NP cross-sections at NLO for Higgsstrahlung $e^+e^- \to Zh$ for *ANY MODEL*:**

$$R_{\rm NLO} = \frac{\sigma^{\rm NP}_{\rm NLO,EW}}{\sigma^{\rm SM}_{\rm NLO,EW}}, \qquad \delta\sigma_{Zh} = 100\,(R_{\rm NLO} - 1)\,\%$$

CEPC precision benchmarks

$$\delta\sigma_{Zh} = 0.26\,\% \quad (0.1\%)$$

arXiv:2505.24810



$$\sigma_{\rm NLO,EW} = \sigma_{\rm Born} + \frac{1}{2s}\left[\int d\Phi_2\, 2{\rm Re}(\overline{\mathscr{M}_0^* \mathscr{M}_1^{\rm ren}}) + \int d\Phi_3\, \overline{|\mathscr{M}_\gamma|^2}\right]$$

Precision Higgsstrahlung at lepton colliders

What do we want? **We want to compute the ratio of the (electroweak) NP cross-sections at NLO for Higgsstrahlung $e^+e^- \to Zh$ for *ANY MODEL*:**

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CEPC precision benchmarks

$$\delta\sigma_{Zh} = 0.26\,\% \quad (0.1\%)$$

arXiv:2505.24810

Specifically:

- To calculate the cross section we need all relevant vertices and counter-terms (CTs) given a **FeynRules/FeynArts.**
- Generate those CTs for *THE MODEL* -Real Triplet Higgs Model (RTHM) here.
- Send those to **FeynCalc/FormCalc** and check UV and IR finiteness.
- Pheno!: Run scans over parameter space constrained by physics.

Precision Higgsstrahlung at lepton colliders

What do we want? **We want to compute the ratio of the (electroweak) NP cross-sections at NLO for Higgsstrahlung $e^+e^- \to Zh$ for *ANY MODEL*:**

$$R_{\rm NLO} = \frac{\sigma^{\rm NP}_{\rm NLO,EW}}{\sigma^{\rm SM}_{\rm NLO,EW}}, \qquad \delta\sigma_{Zh} = 100\,(R_{\rm NLO} - 1)\,\%$$

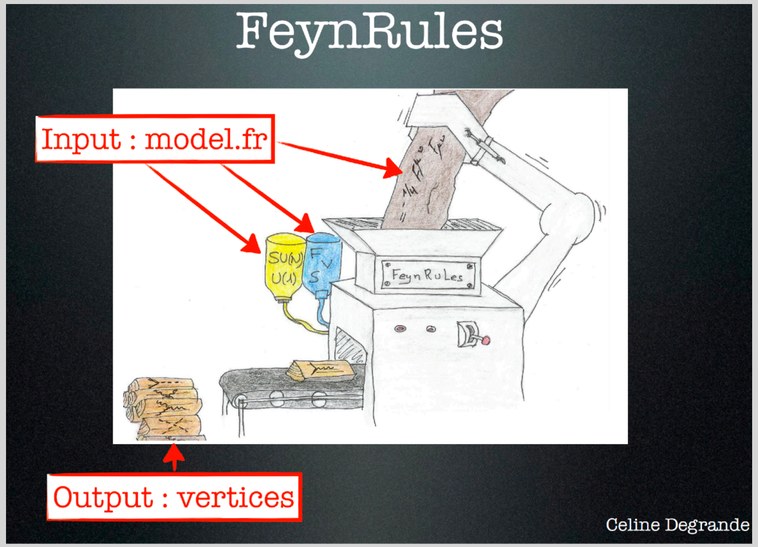
CEPC precision benchmarks

$$\delta\sigma_{Zh} = 0.26\,\% \quad (0.1\%)$$

arXiv:2505.24810

- **Two issues: 1. There's no automated framework for the full calculation**

FeynRules: Define Lagrangian



Calculation typically done case by case, timeframe of months-years!

→ ***Compute CTs from FeynRules*** → FeynCal → *Physics constraints*

Precision Higgsstrahlung at lepton colliders

What do we want? **We want to compute the ratio of the (electroweak) NP cross-sections at NLO for Higgsstrahlung $e^+e^- \to Zh$ for *ANY MODEL*:**

$$R_{\rm NLO} = \frac{\sigma^{\rm NP}_{\rm NLO,EW}}{\sigma^{\rm SM}_{\rm NLO,EW}}, \qquad \delta\sigma_{Zh} = 100\,(R_{\rm NLO} - 1)\,\%$$

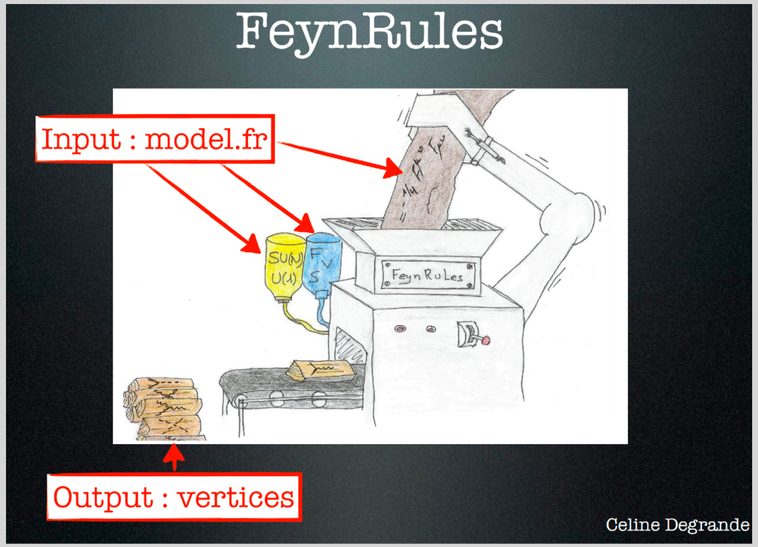
CEPC precision benchmarks

$$\delta\sigma_{Zh} = 0.26\,\% \quad (0.1\%)$$

arXiv:2505.24810

- **Two issues: 1. There's no automated framework for the full calculation**

FeynRules: Define Lagrangian



- **2. There's no tool doing this step automatically (hardest problem)**

→ ***Compute CTs from FeynRules*** → FeynCal → *Physics constraints*

FormCalc

Automated calculation workflow

Until now!
Higgsstrahlung-NLO
Package

FeynRules: Define Lagrangian

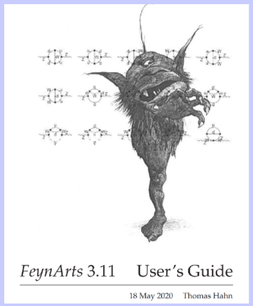
Compute CTs from FeynRules

New tool! FeynRCT, Mathematica package developed by Qin Yu

FormCalc

FORM
Abbreviations

Mathematica
FeynArts amplitudes
Analytical results

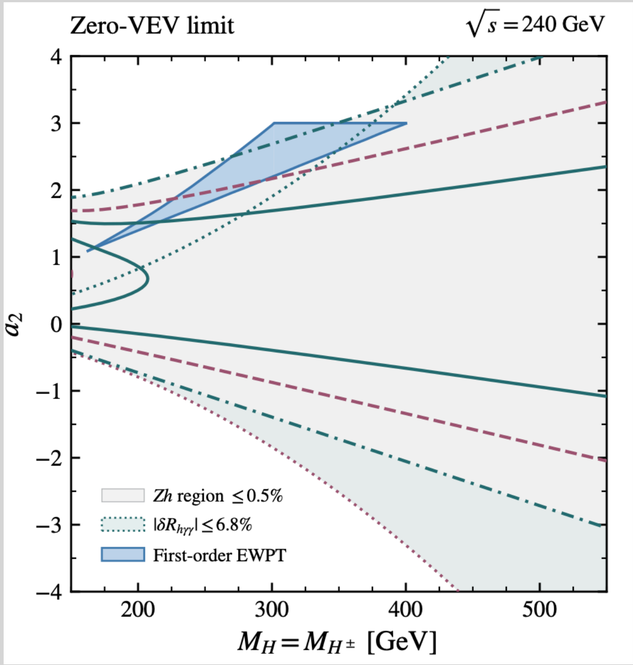


Generated Code
SquaredME
RenConst
Fortran

Driver programs

LoopTools: Evaluate integrals

Physics constraints



Adapted from https://feynarts.de/lectures/fafc.pdf by Thomas Hahn

Automated calculation workflow

Compute CTs automatically from FeynRules

New!

STEP 1 **FeynRCT**

Generate FeynArts model files with one-loop vertices as expressions of CT parameters

STEP 2 **Patch Files**

Patch model files for FormCalc usage such as input schemes, renormalization definitions which connect CT parameters with self-energies

STEP 3 **FormCalc**

Calculate the amplitude & **Integrate the phase space** to obtain the cross section

PRTS fixes the vertices

(Parameter renormalization tadpole scheme)

$$\delta t_i = -\widetilde{\mathrm{Re}}\, T_i, \qquad \delta M_i^2 = \widetilde{\mathrm{Re}}\, \Sigma_{ii}(M_i^2)$$

KOSY fixes the numerical renormalization definitions.

Kanemura et al, hep-ph/0408364

$$\widehat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) - \delta m_{ij}^2 + \frac{1}{2}\delta Z_{ij}(p^2 - m_i^2) + \frac{1}{2}\delta Z_{ji}(p^2 - m_j^2)$$

$$\delta Z_{ij} = \frac{2\,\widetilde{\mathrm{Re}}\,[\Sigma_{ij}(m_j^2) - \delta m_{ij}^2]}{m_i^2 - m_j^2}, \qquad \delta Z_{ji} = \frac{2\,\widetilde{\mathrm{Re}}\,[\Sigma_{ij}(m_i^2) - \delta m_{ij}^2]}{m_j^2 - m_i^2}$$

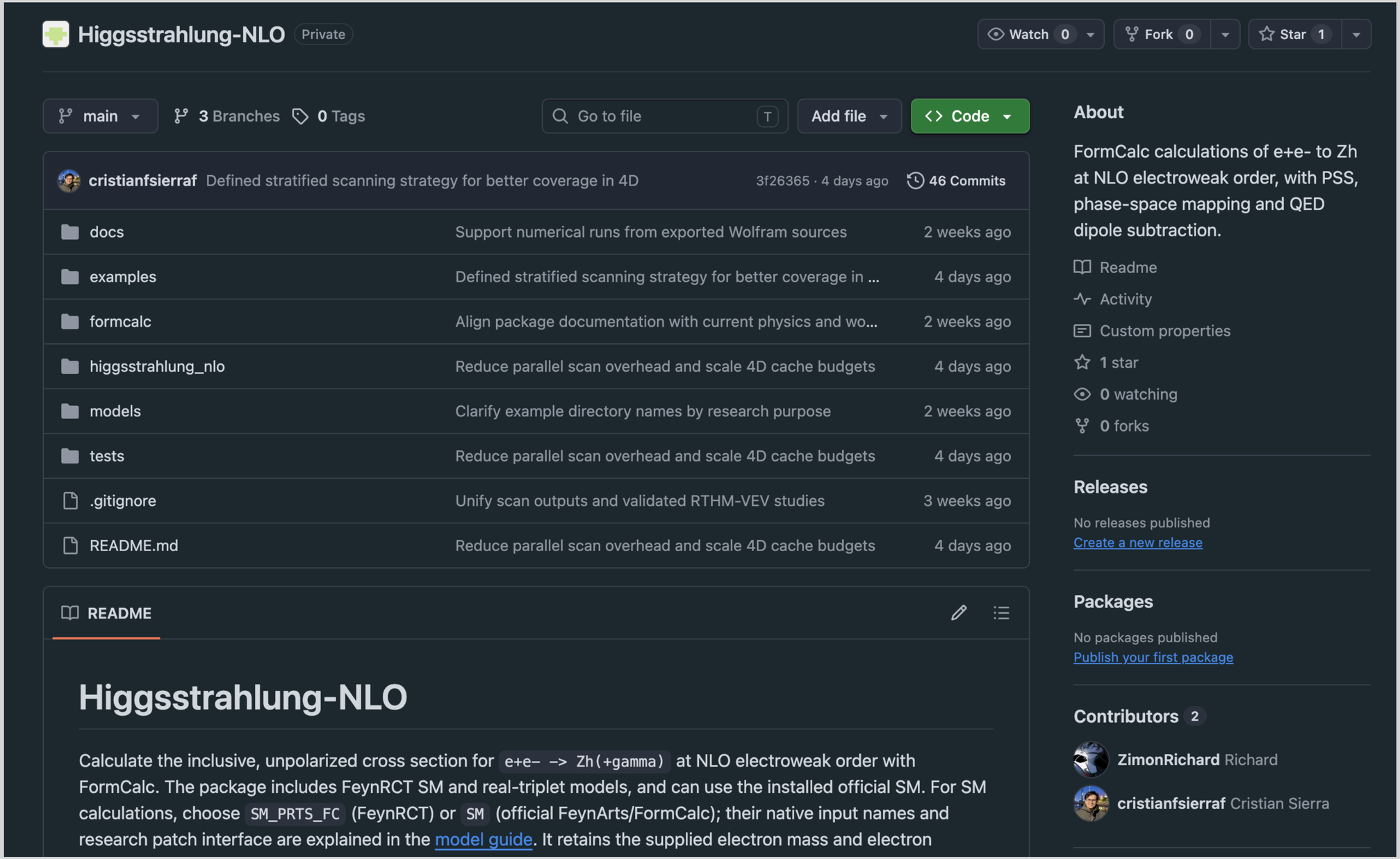
Born + Virtual: $e^+e^- \to Zh$

Born + Virtual + Soft + Hard: $e^+e^- \to Zh\gamma$

Workflow developed by Qin Yu. Analytical calculations, UV and IR checks done by Anjan Barik and Mohamed Sassi.

NLO contributions to Higgsstrahlung

To be released soon, stay tuned !



Higgsstrahlung-NLO Private

Watch 0 · Fork 0 · Star 1

main · 3 Branches · 0 Tags · Go to file T · Add file · Code

cristianfsierraf Defined stratified scanning strategy for better coverage in 4D — 3f26365 · 4 days ago — 46 Commits

docs	Support numerical runs from exported Wolfram sources	2 weeks ago
examples	Defined stratified scanning strategy for better coverage in ...	4 days ago
formcalc	Align package documentation with current physics and wo...	2 weeks ago
higgsstrahlung_nlo	Reduce parallel scan overhead and scale 4D cache budgets	4 days ago
models	Clarify example directory names by research purpose	2 weeks ago
tests	Reduce parallel scan overhead and scale 4D cache budgets	4 days ago
.gitignore	Unify scan outputs and validated RTHM-VEV studies	3 weeks ago
README.md	Reduce parallel scan overhead and scale 4D cache budgets	4 days ago

README

Higgsstrahlung-NLO

Calculate the inclusive, unpolarized cross section for `e+e- -> Zh(+gamma)` at NLO electroweak order with FormCalc. The package includes FeynRCT SM and real-triplet models, and can use the installed official SM. For SM calculations, choose `SM_PRTS_FC` (FeynRCT) or `SM` (official FeynArts/FormCalc); their native input names and research patch interface are explained in the model guide. It retains the supplied electron mass and electron

About

FormCalc calculations of e+e- to Zh at NLO electroweak order, with PSS, phase-space mapping and QED dipole subtraction.

Readme · Activity · Custom properties · 1 star · 0 watching · 0 forks

Releases

No releases published
Create a new release

Packages

No packages published
Publish your first package

Contributors 2

ZimonRichard Richard

cristianfsierraf Cristian Sierra

Example case: The Real Triplet Higgs Model (RTHM)

-Extends the SM by the triplet Σ with $Y_\Sigma = 0$ (*See Bruce Mellado's talk on the 152 GeV)*

$$\Phi = \begin{pmatrix} \phi^+ \\ (v_h + \phi + i\phi^0)/\sqrt{2} \end{pmatrix}, \qquad \Sigma = \frac{1}{2}\begin{pmatrix} v_\Sigma + \sigma^0 & \sqrt{2}\sigma^+ \\ \sqrt{2}\sigma^- & -(v_\Sigma + \sigma^0) \end{pmatrix}$$

-With Higgs potential given by

$$V = -\mu_h^2 \Phi^\dagger\Phi + \lambda_h(\Phi^\dagger\Phi)^2 - \mu_\Sigma^2 \mathrm{Tr}\Sigma^2 + b_4(\mathrm{Tr}\Sigma^2)^2$$
$$+ a_1 \Phi^\dagger\Sigma\Phi + a_2(\Phi^\dagger\Phi)\mathrm{Tr}\Sigma^2$$

Triplet self-coupling

Doublet-triplet quartic coupling

aka, portal coupling

-After SSB, we get two neutral scalars and a charged pair $H^\pm$

$$\begin{pmatrix} \phi \\ \sigma^0 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}\begin{pmatrix} h_{125} \\ H \end{pmatrix} \qquad c_\beta = \frac{v_h}{v}, \quad s_\beta = -\frac{2v_\Sigma}{v}, \quad v^2 = v_h^2 + 4v_\Sigma^2$$

$$\rho_{\mathrm{tree}} = \frac{M_W^2}{M_Z^2 c_W^2} = \frac{1}{c_\beta^2} \simeq 1 + \beta^2, \qquad c_W = c_\beta \frac{M_W}{M_Z}$$

ρ_{tree} parameter constraints β to be small

$$|\beta| \lesssim 0.03$$

Parameter scan and constraints

$$100 \leq M_{H^\pm}/\mathrm{GeV} \leq 2500, \quad \Delta m \equiv M_{H^\pm} - M_H$$
$$|\Delta m| \leq 2\ \mathrm{GeV}, \quad |\alpha| \leq 0.3, \quad -0.03 \leq \beta \leq -0.001$$

Theory constraints

- Boundedness from below and tree-level vacuum-depth tests.
- Unitarity and perturbativity: $|x_i| < 16\pi$; retain the stricter 8π subset.
- Vacuum stability.

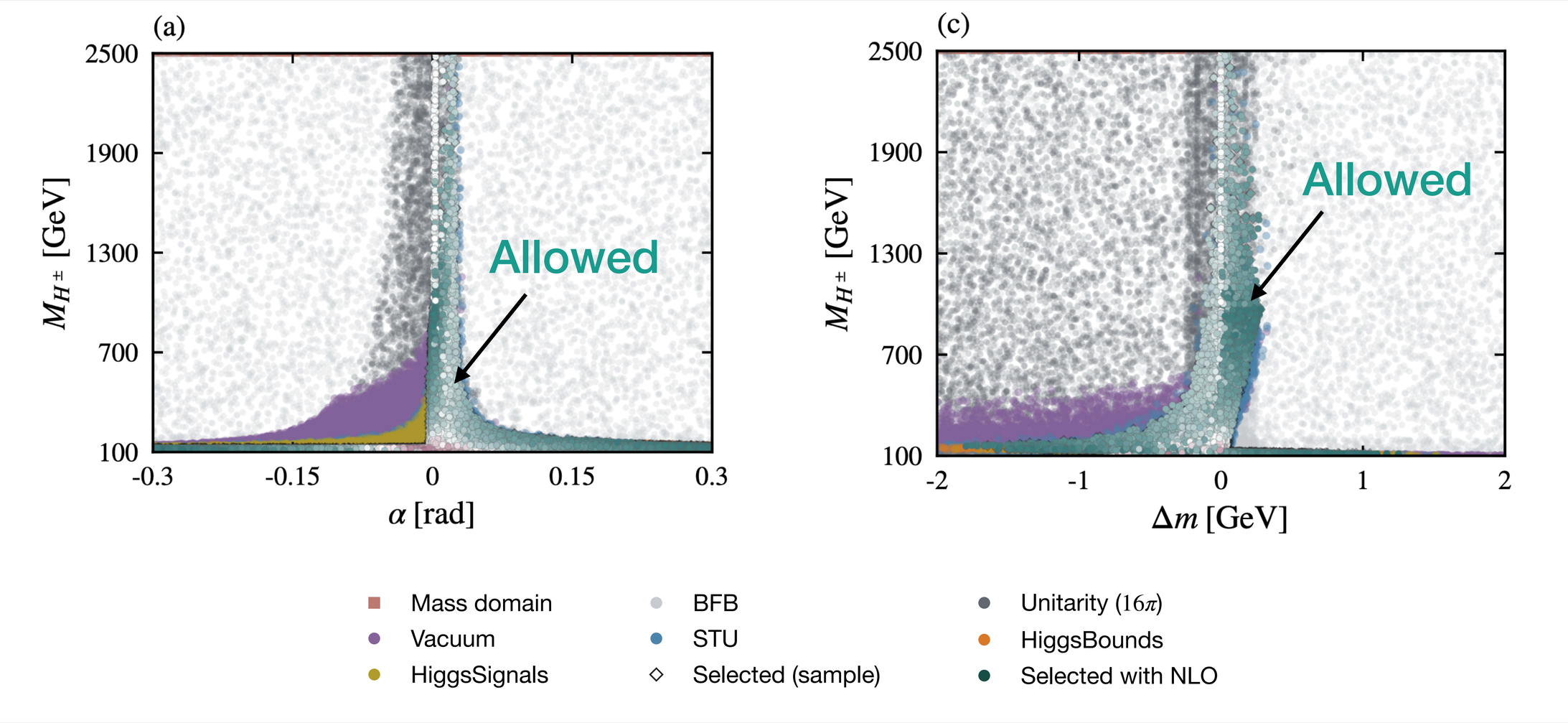
Experimental observables

- Oblique parameters STU.
- HiggsBounds, direct-search constraints.
- HiggsSignals, Higgs decays and rate fit: $\Delta\chi^2 \leq 9.5$ for the four-parameter scan.

15.17 million candidates → 270,665 selected → 25,483 NLO predictions

4D scan: masses and mixing

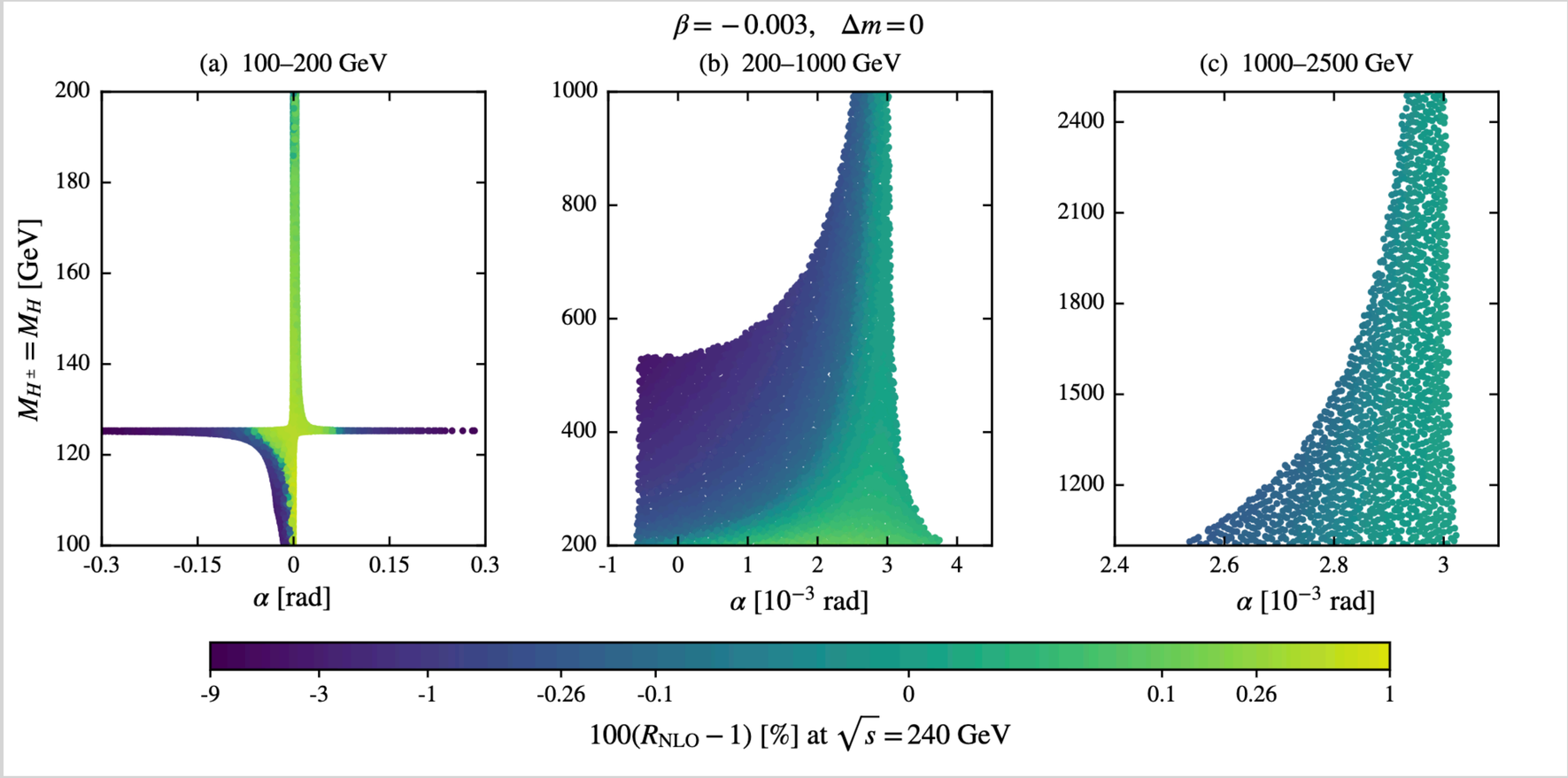
$$\sqrt{s} = 240 \text{ GeV}$$



- 15.17 million candidates → 270,665 selected → 25,483 NLO predictions.

At √s = 240 GeV, Mh = 125.13 GeV. Separate selections, not a combined 95% CL region. Point density is not a probability.

Pheno: 2D scan for mass–mixing slices



- For $\alpha = -\beta$, the mass-enhanced terms cancel exactly $a_2 = b_4 = 2M_h^2/v^2$ explaining the narrow band of small rate shifts near $\alpha = 0.003$ in the high-mass panel.
- This is an approximately SM-like regime, the hZZ and hWW couplings still differ from their SM values at finite β so it is not exact gauge-coupling alignment.

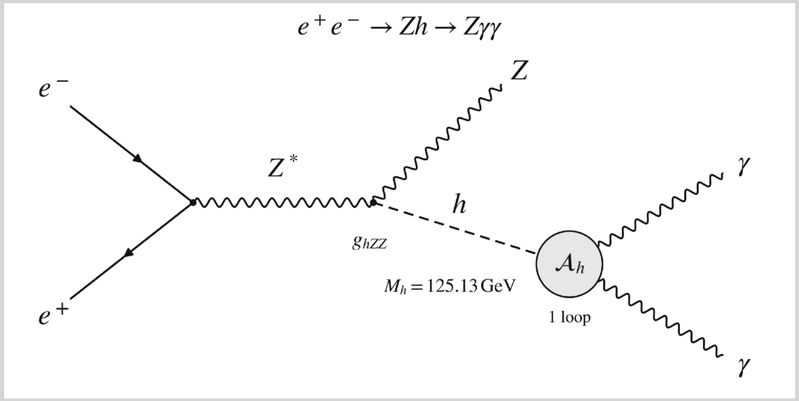
$$a_2 = \frac{2}{v^2}\left[M^2 + \frac{\sin 2\alpha}{\sin 2\beta}(M^2 - M_h^2)\right]$$

20,200 NLO predictions, work by Mohamed Sassi

Physics constraints: Diphoton and EWPT

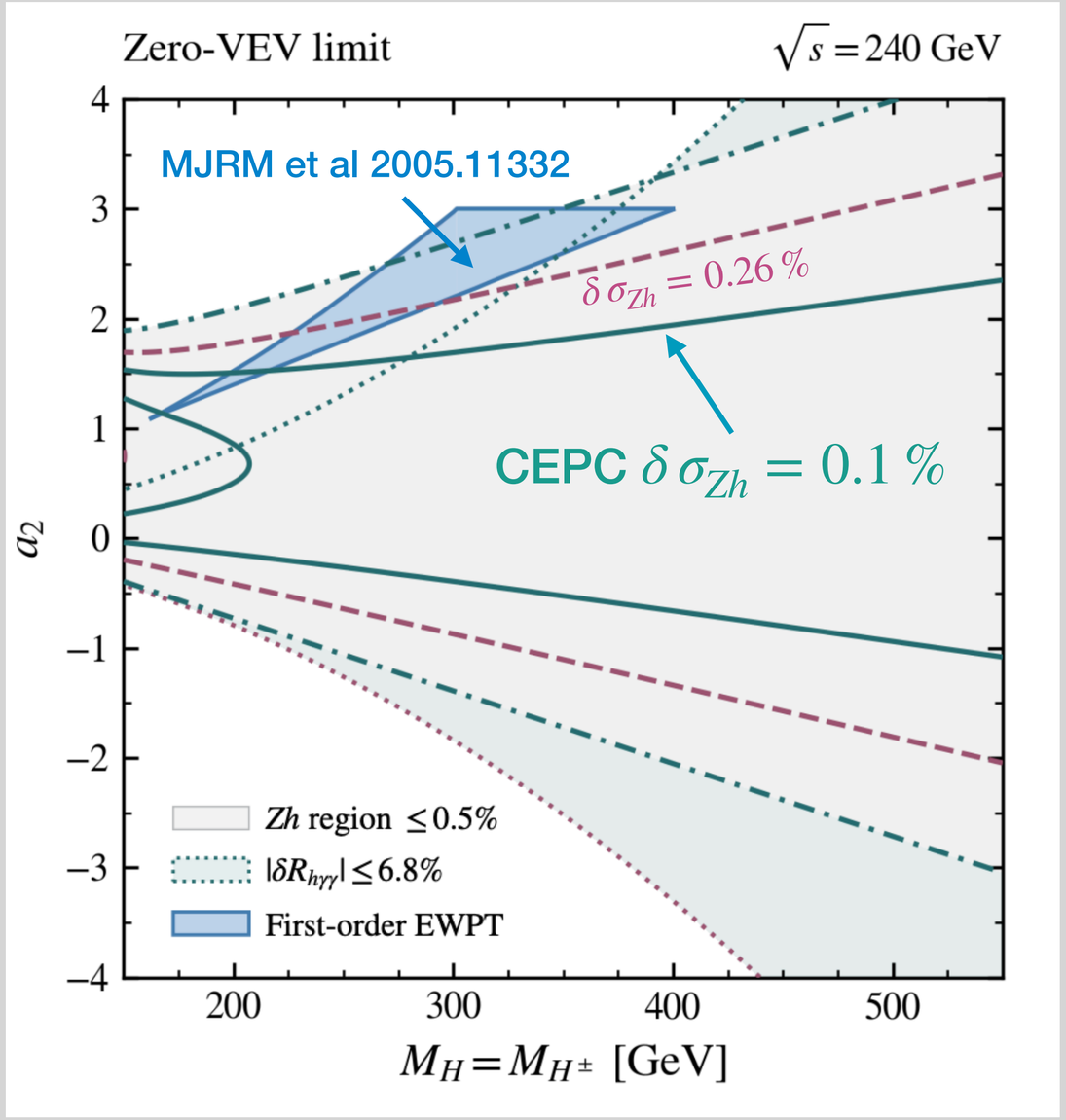
$b_4 = 0.25$

- SM Higgs-diphoton decay complementarity:



$$\delta R_{h\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma_{\text{SM}}(h \to \gamma\gamma)} - 1$$

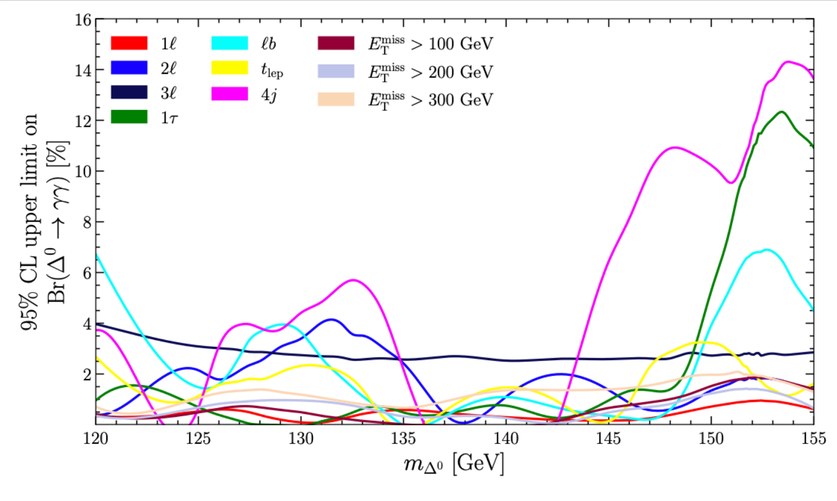
- SM reference: zero partial-width shift. This selected slice spans about –10.7% to 0%.
- Larger portal couplings retain sub-percent effects in Higgsstrahlung $\delta\,\sigma_{Zh}$ at larger masses.
- First-order EWPT parameter space can be probed with CEPC sensitivity (see Xiang Chen's talk for more on EWPTs and CEPC).

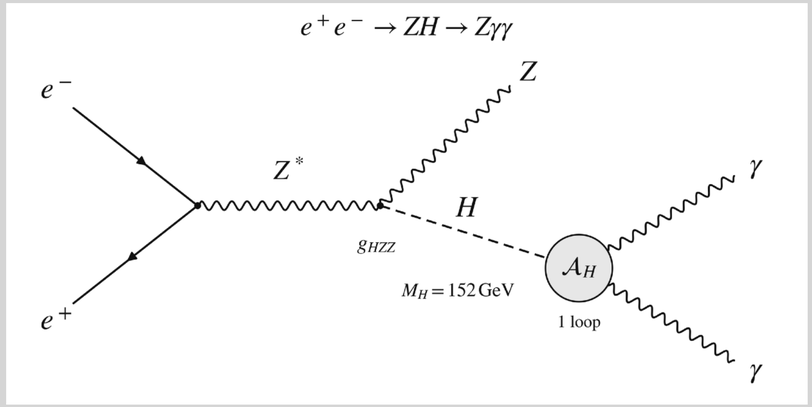


- ***Plot matches with MJRM et al 2104.10709.*** ✅

More physics: the 152 GeV interpretation

- Associated diphoton searches motivated a recast preferring Br(H → γγ) ≈ 0.7% at a local 3.9σ arXiv:2306.17209, 2411.18618, 2503.16245. (*See Bruce Mellado's talk on the 152 GeV*)





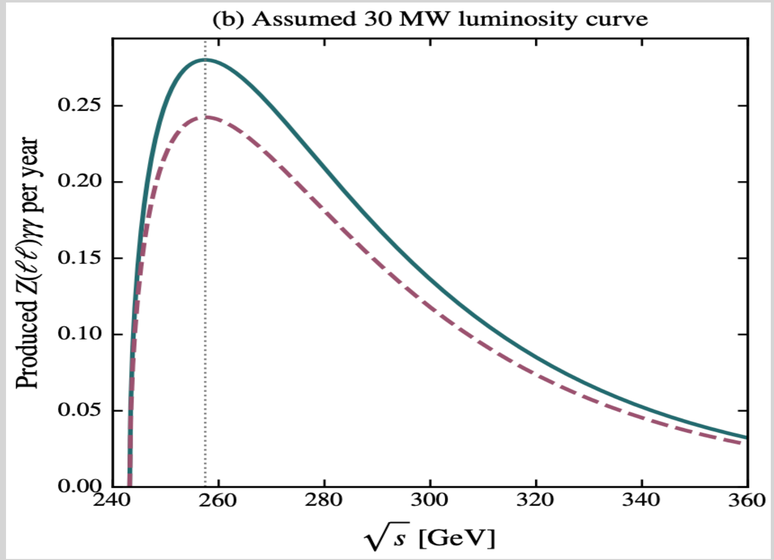
Not observed in ZZ decays:

Motivates triplet with $Y_\Sigma = 0$

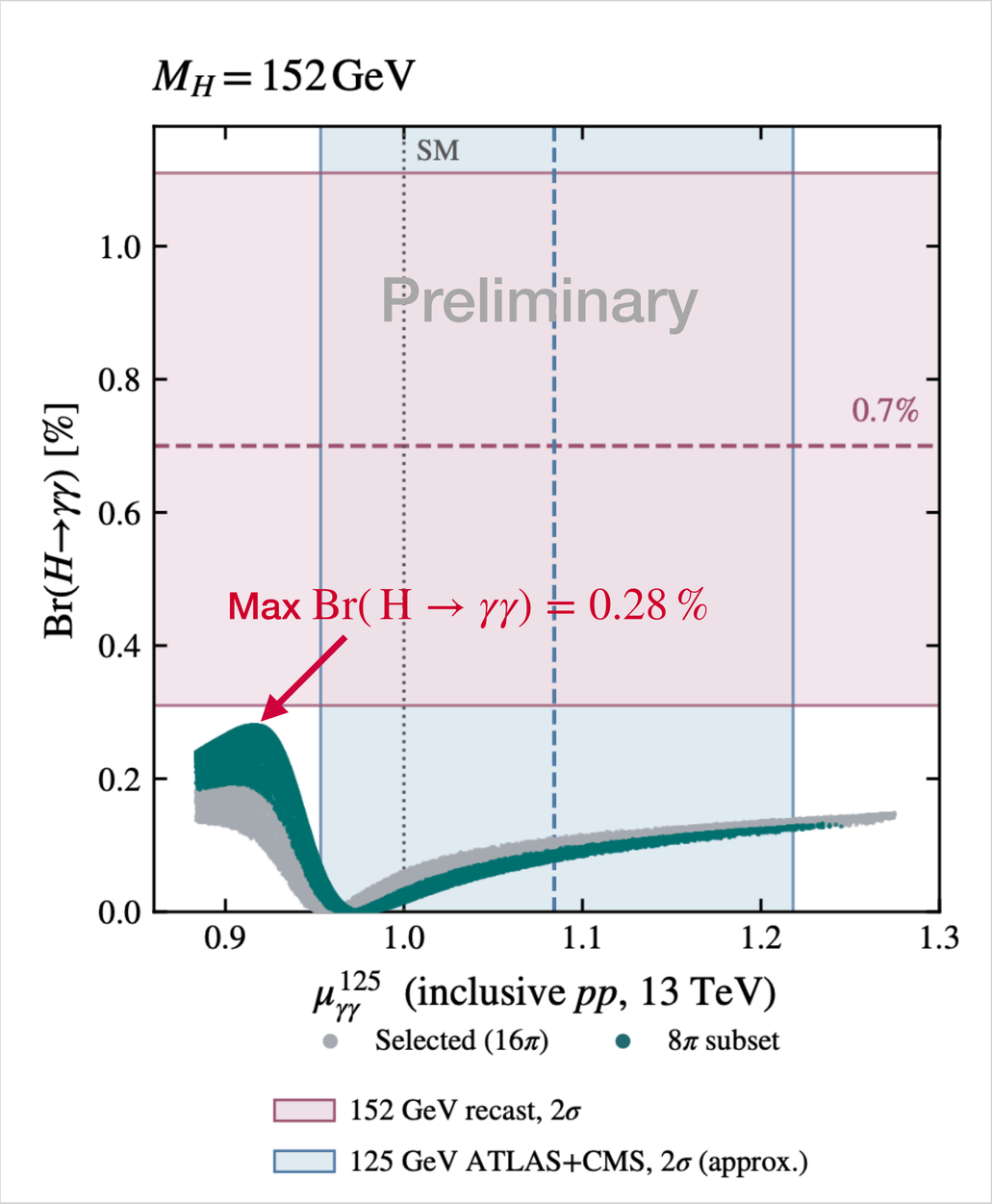
- On-shell threshold: ~243 GeV. Born max: ~280 GeV. Luminosity at CEPC shifts it to ~256 GeV:

$$N_{\ell\ell\gamma\gamma} = \mathscr{L}\,\sigma_{\rm ref}\,\mathscr{P}_\gamma\,\mathrm{Br}(Z \to \ell\ell)$$

$$\mathscr{P}_\gamma = s_\alpha^2 F_\beta\,\mathrm{Br}(H \to \gamma\gamma)$$



Diphotons connect the 125 and 152 GeV states



- The largest 152 GeV diphoton fraction found is 0.281%, below the recast's ~0.3–1.2% 95% C.L. interval.

- At that maximum, the 125 GeV diphoton signal strength is about ~0.92. ATLAS-CMS ~1.08 (Run-2 rates: ATLAS 2207.00348, CMS 2103.06956.)

$$\mu_{\gamma\gamma}^{125} = \frac{\sigma_h}{\sigma_h^{\text{SM}}} \frac{\Gamma_{\gamma\gamma}/\Gamma_{\gamma\gamma}^{\text{SM}}}{\Gamma_h/\Gamma_h^{\text{SM}}}$$

- We observe some tension with the RTHM interpretation. We are not fitting it.

- Establishing a search reach still needs detector-level analysis.

Conclusions

- We presented a new tool for the automated computation $e^+e^- \to Zh$ at NLO.
- A complete one-loop RTHM prediction connects scalar-sector parameters to precision Higgsstrahlung at future lepton colliders.
- After the adopted theoretical and experimental constraints, allowed rate shifts cover the 0.1–0.26% sensitive projections at the CEPC.
- Automated rich pheno: Higgsstrahlung, diphoton and EWPT observables provide complementary information on the scalar potential.
- The 152 GeV interpretation faces tension in the minimal model. Direct rates require correlated production and decay, and precision conclusions require further theory and collider studies.

谢谢！

Backup

Automated calculation workflow: FormCalc

$e^+e^- \to Zh$ | $e^+e^- \to Zh\gamma$

Born
Tree level

Virtual
One-Loop level
Weak + Virtual Photon (IR divergence)

Soft Photon (ΔE)
Tree level
Cancel the IR divergence; Introduce the energy cutoff ΔE

Hard Photon (ΔE)
Tree level
Cancel the energy cutoff ΔE

To cancel the IR divergence, FormCalc introduces photon mass square λ

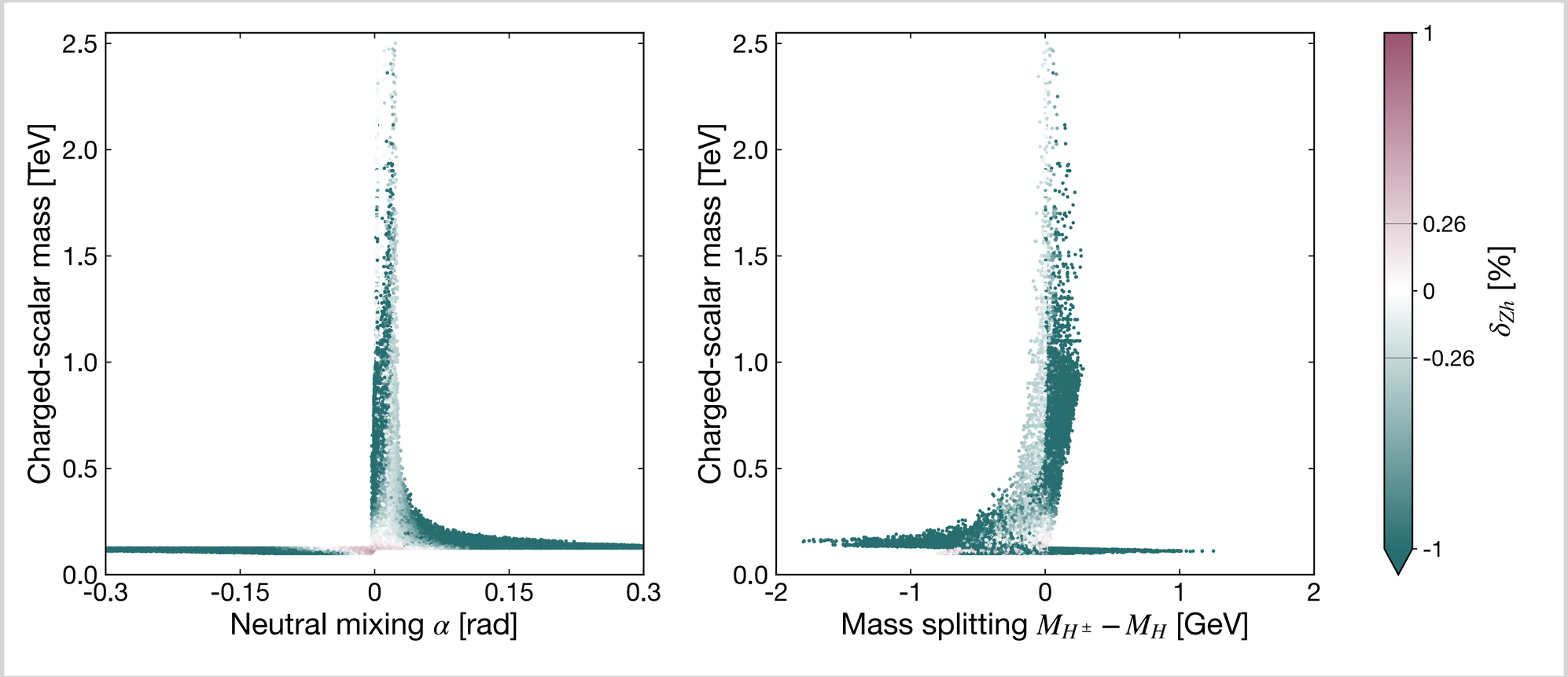
And replace the exact integration of **soft-slicing** $E_\gamma < \Delta E$ as analytic approximation. Thus the ΔE can not be exactly cancelled by hard photon part, with the error $O(\Delta E/E) = O(\delta_s)$, $E = \sqrt{s}/2$

This is the **soft-slicing** part idea of two-cutoff phase-space slicing (PSS)

Done automatically by FormCalc!!!

Slide by Qin Yu.

Four-dimensional scan: masses and mixing

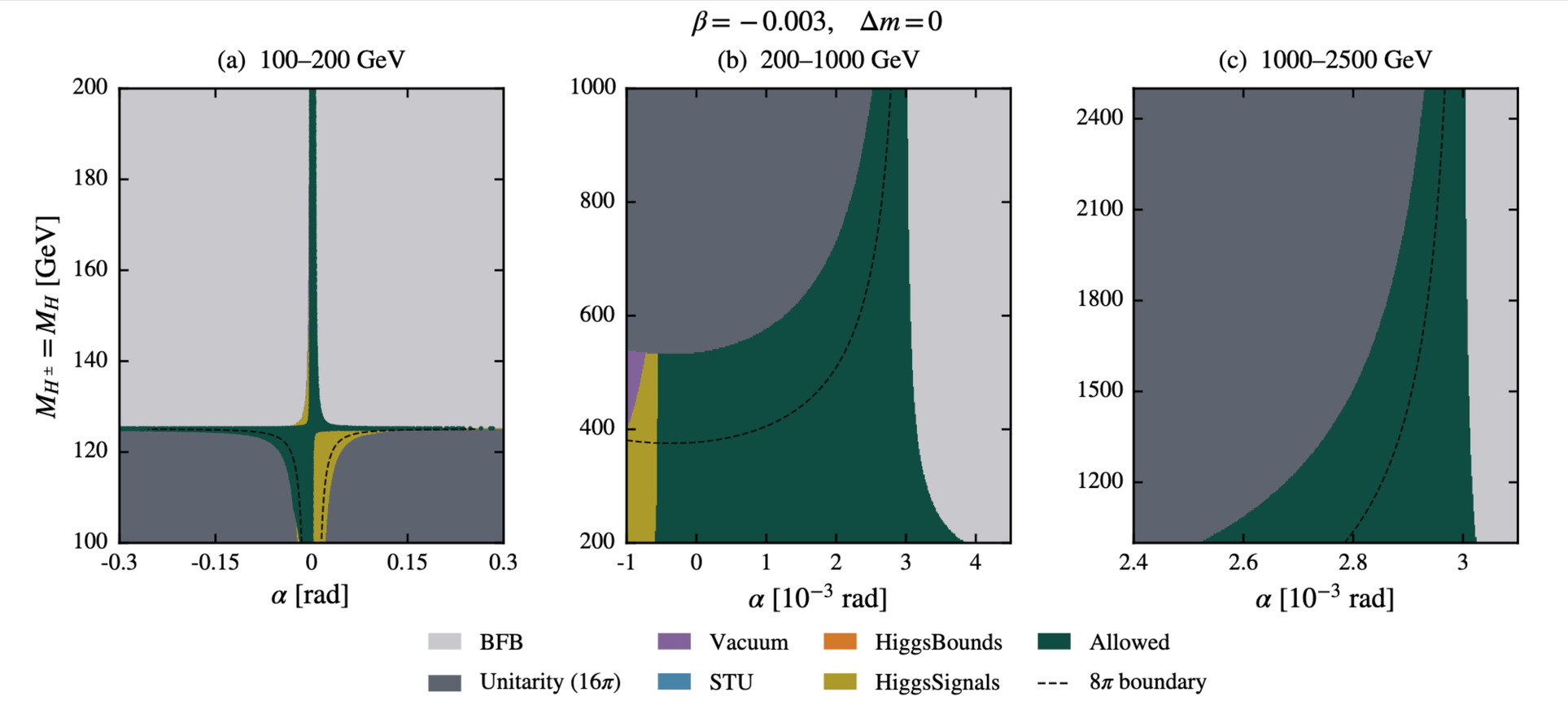


- Large masses restrict neutral mixing through the reconstructed quartics and unitarity.
- The spectrum is nearly degenerate and hidden parameters remain free in each projection.

Colour: δZh at 240 GeV. Displayed range –1% to +1%; lower overflow retains larger negative shifts.

Fixed mass–mixing slices

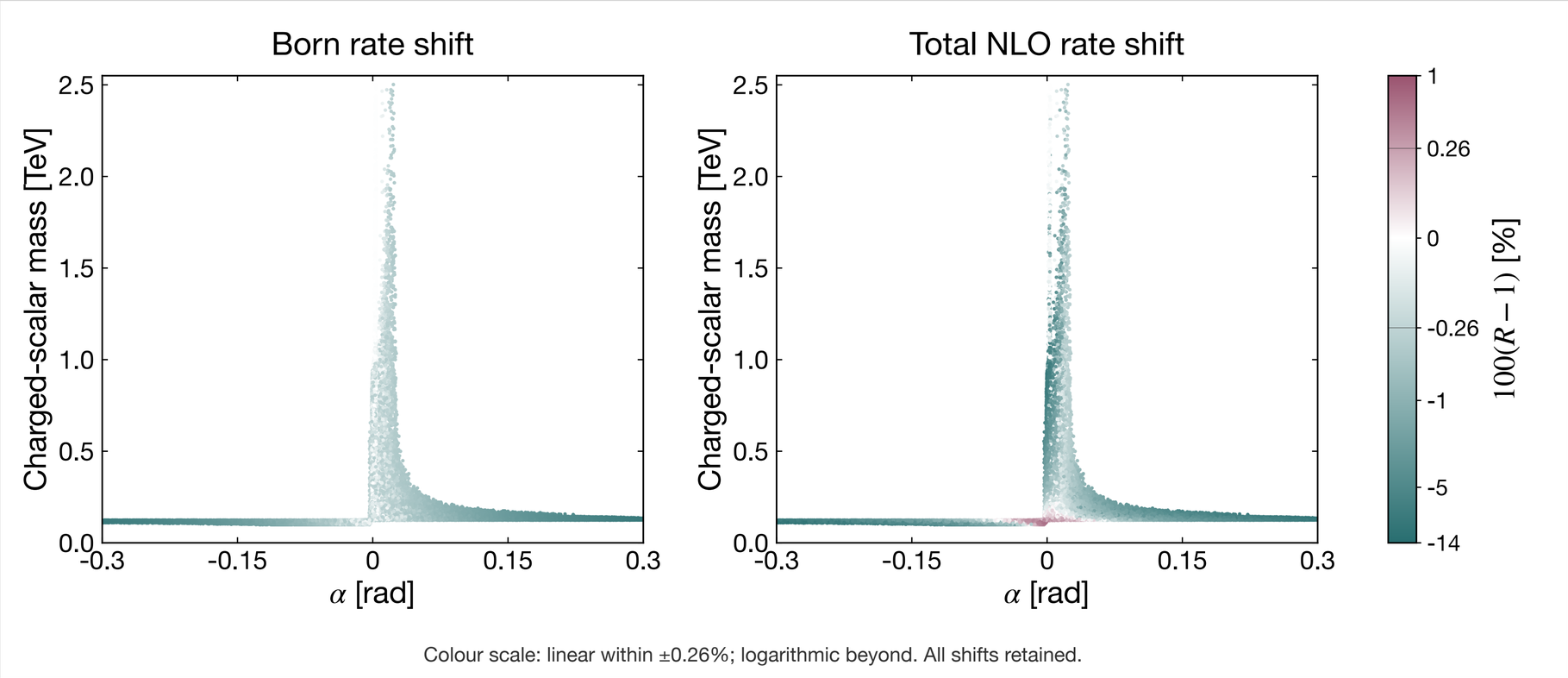
$$\beta = -0.003, \qquad M_H = M_{H^\pm}, \qquad \sqrt{s} = 240\ \text{GeV}$$



- Fixing the hidden parameters resolves the ambiguity of projected scan points.
- The allowed mixing interval narrows at high mass; all three panels use the same precision scale.

20,200 supplied NLO predictions after the current selection, with independent checks in each mass range.

Born mixing and radiative effects



Colour scale: linear within ±0.26%; logarithmic beyond. All shifts retained.

$$R_{\rm Born} \simeq c_\alpha^2 F_\beta, \qquad R_{\rm NLO} = R_{\rm Born}\,\frac{1+\Delta_{\rm EW}^{\rm RTHM}}{1+\Delta_{\rm EW}^{\rm SM}}$$

- The most negative total NLO point is –13.12%, with an approximately –9% Born shift.
- The NLO ratio is a model prediction; it must not be interpreted as a pure new-scalar loop correction.

ΔEW = (σNLO – σBorn)/σBorn within each model. Born approximation neglects electron-Yukawa-suppressed branches.

Cross section computation at NLO

In order to do that we need to compute the ratio of the differential cross-sections

$$\delta\sigma_{Zh} = \frac{\sigma^{1-\text{loop}}_{\text{BSM}}}{\sigma^{\text{LO}}_{\text{SM}}}$$

Where $\sigma^{\text{1-loop}}_{\text{BMS}}$ is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}\left|\mathrm{M}^{\text{NLO}}\right|^2.$$

With

$$\begin{aligned} i\mathbf{M}^{\text{NLO}}_{e^+e^-\to Zh} &= i\mathbf{M}^{\text{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\text{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\text{vert}}_{e^+e^-\to Zh}, \\ &= -i\frac{\hat{e}^2 M_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\,\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\text{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\text{vert}}_{\gamma^*\to Zh}, \end{aligned}$$

Cross section computation at NLO

$$i\mathbf{M}^{\mathrm{NLO}}_{e^+e^-\to Zh} = i\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{e^+e^-\to Zh},$$

$$= -i\frac{\hat{e}^2\hat{M}_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\,\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh},$$

Cross section computation at NLO

$$
\begin{aligned}
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\end{aligned}
$$

Cross section computation at NLO

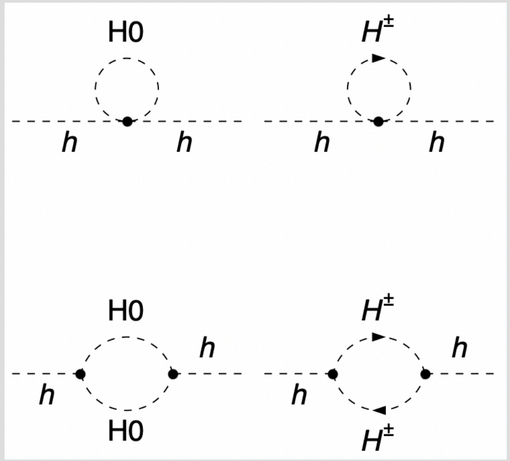
$$i\mathbf{M}^{\mathrm{NLO}}_{e^+e^-\to Zh} = i\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{e^+e^-\to Zh},$$

$$= -i\frac{\hat{e}^2\hat{M}_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh},$$

$$\hat{\rho}_{NC}(s) = \frac{1}{s - \hat{M}_Z^2 + \hat{\Sigma}_T^{ZZ}(s)}\left(1 + \frac{1}{2}\delta\hat{Z}_{ZZ} + \frac{1}{2}\delta\hat{Z}_h\right),$$

$$= \frac{1}{s - M_Z^2 + i\Gamma_Z M_Z}\left(1 + \frac{\mathrm{Re}\hat{\Sigma}_T^{ZZ}(M_Z^2) - \mathrm{Re}\hat{\Sigma}_T^{ZZ}(s)}{s - M_Z^2} + \frac{1}{2}\delta\hat{Z}_{ZZ} + \frac{1}{2}\delta\hat{Z}_h\right)$$

Higgs self-energy



Cross section computation at NLO

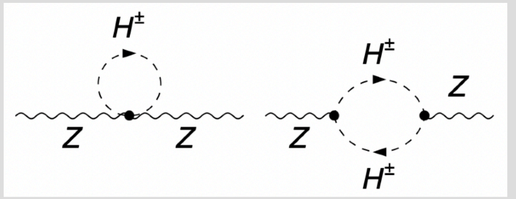
$$i\mathbf{M}^{\mathrm{NLO}}_{e^+e^-\to Zh} = i\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{e^+e^-\to Zh},$$

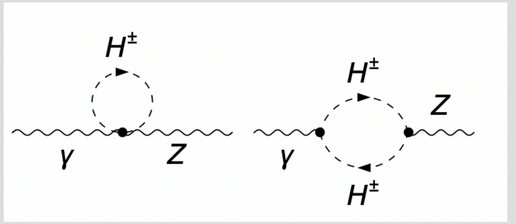
$$= -i\frac{\hat{e}^2\hat{M}_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\,\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh},$$

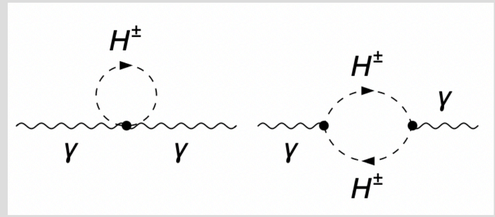
$$\hat{\rho}_{NC}(s) = \frac{1}{s - \hat{M}_Z^2 + \hat{\Sigma}_T^{ZZ}(s)}\left(1 + \frac{1}{2}\delta\hat{Z}_{ZZ} + \frac{1}{2}\delta\hat{Z}_h\right),$$

$$= \frac{1}{s - M_Z^2 + i\Gamma_Z M_Z}\left(1 + \frac{\mathrm{Re}\hat{\Sigma}_T^{ZZ}(M_Z^2) - \mathrm{Re}\hat{\Sigma}_T^{ZZ}(s)}{s - M_Z^2} + \frac{1}{2}\delta\hat{Z}_{ZZ} + \frac{1}{2}\delta\hat{Z}_h\right)$$

Vector boson self energies



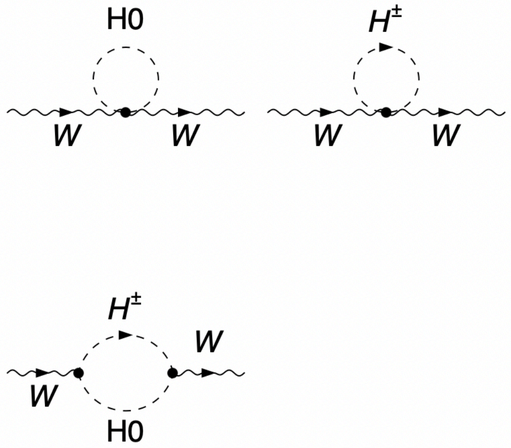




Cross section computation at NLO

$$i\mathbf{M}^{\mathrm{NLO}}_{e^+e^-\to Zh} = i\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{e^+e^-\to Zh},$$

$$= -i\frac{\hat{e}^2\hat{M}_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\,\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh},$$



$$\hat{M}_V^2 = M_V^2 + \mathrm{Re}\hat{\Sigma}_T^{VV}\left(M_V^2\right),$$

$$\hat{c}^2 \equiv 1 - \hat{s}^2 = \frac{\hat{M}_W^2}{\hat{M}_Z^2},$$

$$\hat{e} \equiv e\left(1 - \frac{1}{2}\delta\hat{Z}_{\gamma\gamma} - \frac{1}{2}\frac{\hat{s}}{\hat{c}}\delta\hat{Z}_{Z\gamma}\right)$$

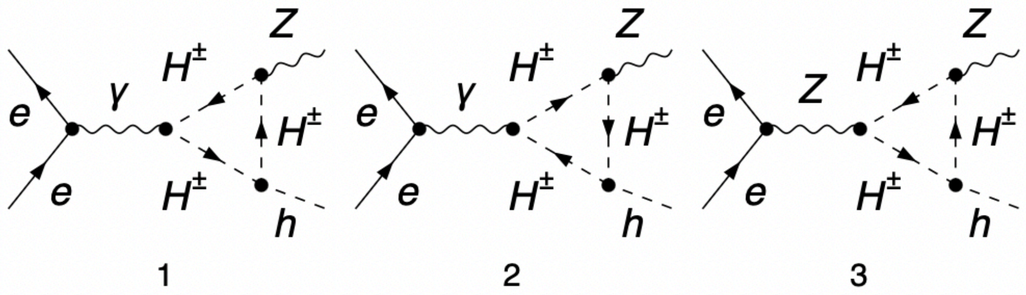
W boson self energy

Cross section computation at NLO

$$i\mathbf{M}^{\mathrm{NLO}}_{e^+e^-\to Zh} = i\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{self}}_{e^+e^-\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{e^+e^-\to Zh},$$

$$= -i\frac{\hat{e}^2\hat{M}_Z}{\hat{s}\hat{c}}\hat{\rho}_{NC}(s)\,\bar{v}(p_2)\,\gamma^\mu\left(g_v^{eff} - g_a^{eff}\gamma_5\right)u(p_1)\,\epsilon_\mu(k_1) + i\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh} + i\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh},$$

Vertex contributions



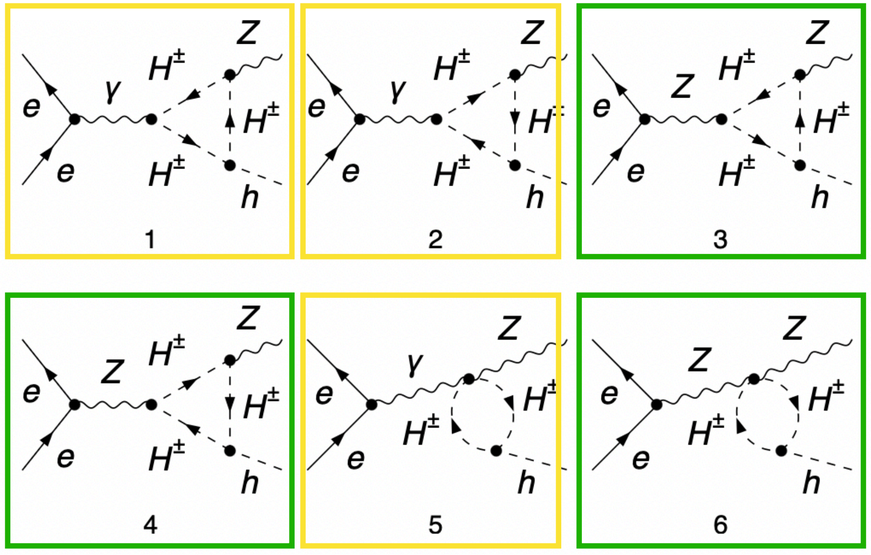
Cross section computation at NLO

$$2\ \mathrm{Re}\left[\left(\mathbf{M}^{\mathrm{vert}}_{Z^*\to Zh}+\mathbf{M}^{\mathrm{vert}}_{\gamma^*\to Zh}\right)\cdot\left(\mathbf{M}^{\mathrm{tree}}_{e^+e^-\to Zh}\right)^*\right]$$

$$=-\frac{1}{16\pi^2}\frac{4\ e^3}{M_Z s_W c_W}\sum_{s,(M_1,M_2)}\left(\frac{g_{ssh}g^2_{ssZ}\left(g_v^2+g_a^2\right)}{\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}+\frac{g_{ssh}g_{ss\gamma}g_{ssZ}\ g_v\left(s-M_Z^2\right)}{s\left[\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right]}\right)\times$$

$$2\mathrm{Re}\Big\{\left(tu+2sM_Z^2-M_h^2M_Z^2\right)\left(2C_{24s}\left(M_1,M_2\right)-\frac{1}{2}B_0\left(k_2,M_2,M_2\right)\right)$$

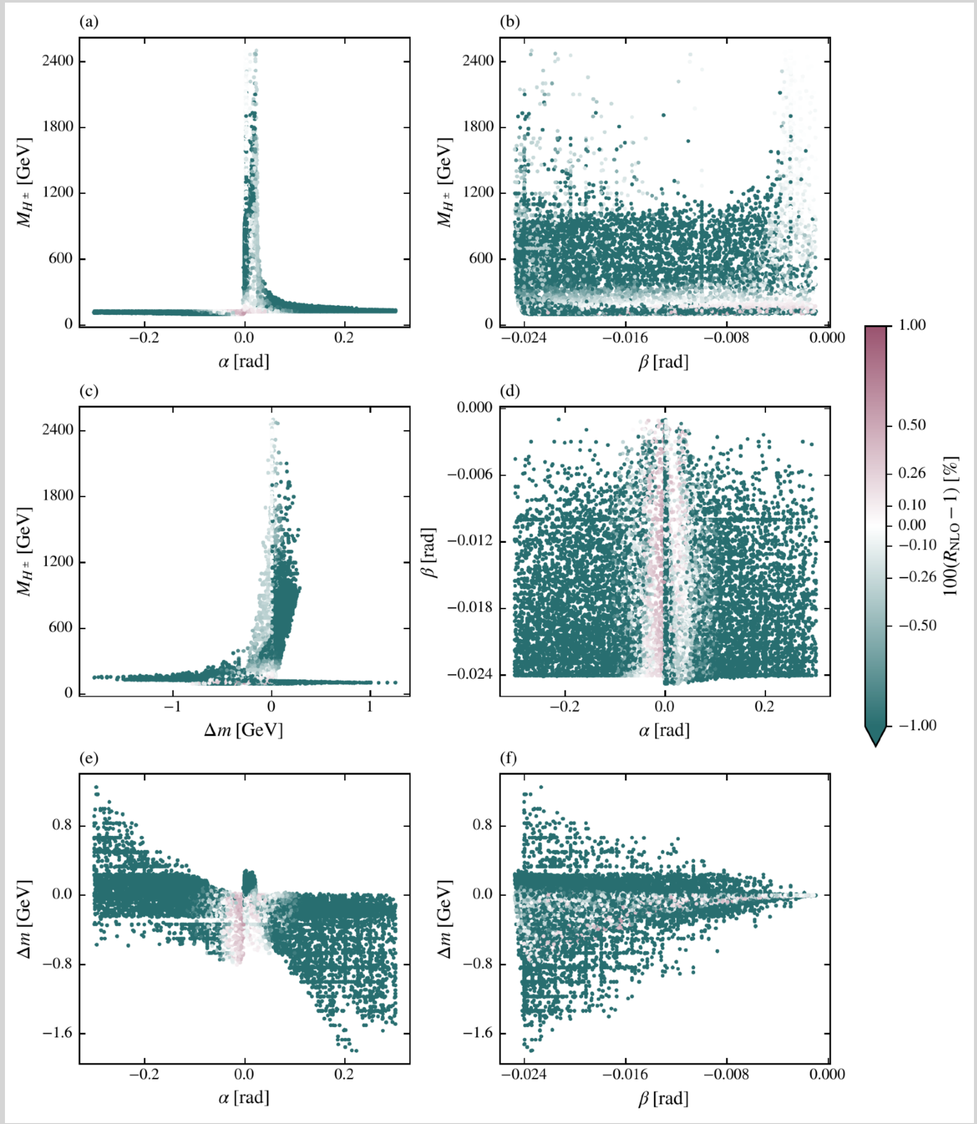
$$+\left(s+M_Z^2-M_h^2\right)\left(M_h^2M_Z^2-tu\right)\left(C_{22s}\left(M_1,M_2\right)-C_{23s}\left(M_1,M_2\right)\right)\Big\},$$



Both factors factorize from PaVe functions

Full four-parameter projections

17,326 NLO predictions



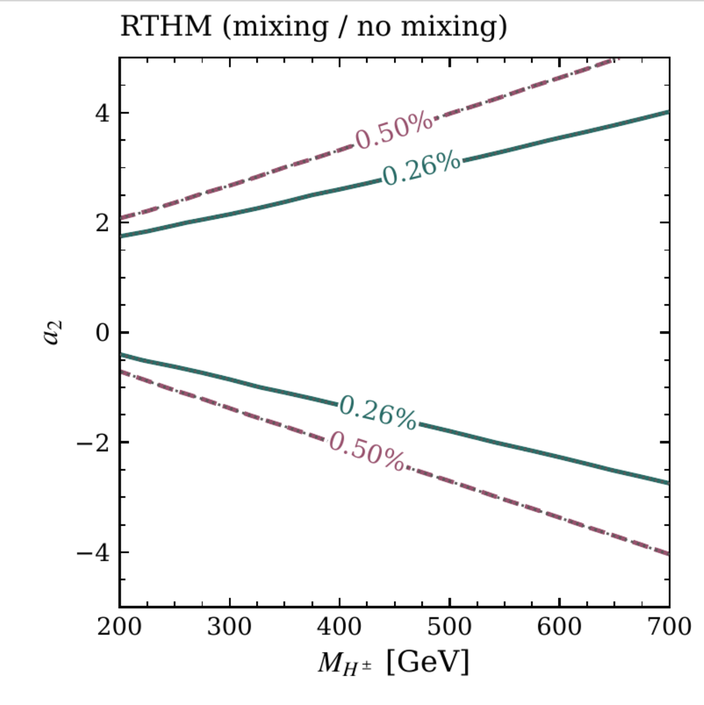
Full range
−13.12% to
+0.662%

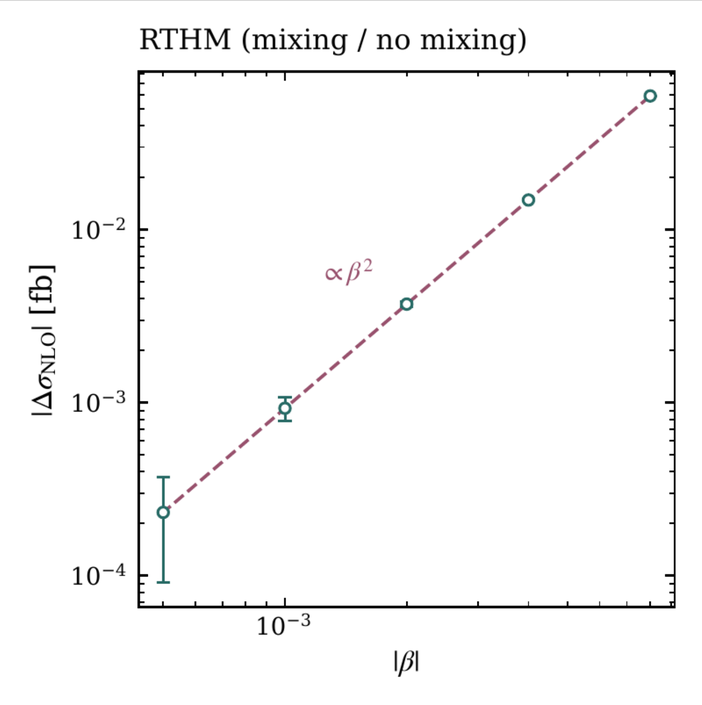
Colour scale
−1% to +1%

Overlapping points can have different hidden parameters. Neither density nor projected support is a confidence region.

The inert limit as an internal check

$$a_1 = v_\Sigma = 0, \qquad \alpha = \beta = 0, \qquad \Sigma \to -\Sigma$$





- Matched inputs and radiation settings; the difference decreases along a controlled small-VEV path.
- An external comparison must also match the SM LO versus NLO normalization.

The displayed internal comparison plane has no phenomenological selection. Literature normalization: arXiv:2104.10709, Eq. (19).

Numerical stability and theory precision

- Radiation-cutoff changes below 5.1×10^{-5} fb at the two reported benchmark points.
- Slicing versus dipole subtraction: differences below 7.9×10^{-5} fb.
- Cuhre versus Divonne: 26 comparisons agree within integration errors.
- Targeted 0.1% contour checks agree within 0.002 percentage points, including integration errors.
- Remaining theory work: physical angle matching, finite-mixing oblique effects and omitted higher orders.

Correlated 152 GeV benchmarks

Each row is a single selected parameter point

Point	M(H±) [GeV]	α	β	Brγγ [%]	Pγ
Largest Brγγ	152.00006	0.000731	-0.001006	0.28086	1.50×10^{-9}
Largest rate, 16π	151.14402	0.162818	–0.014776	0.14852	3.90×10^{-5}
Largest rate, 8π	151.25813	0.160461	-0.016169	0.13238	3.37×10^{-5}

- The largest branching ratio is accompanied by strong production suppression.
- Rate maxima occur near the adopted unitarity boundaries.

Mh = 125.13 GeV, MH = 152 GeV. Rounded coordinates identify saved points and should not be used to retest boundary cuts.

Finite-VEV input matching

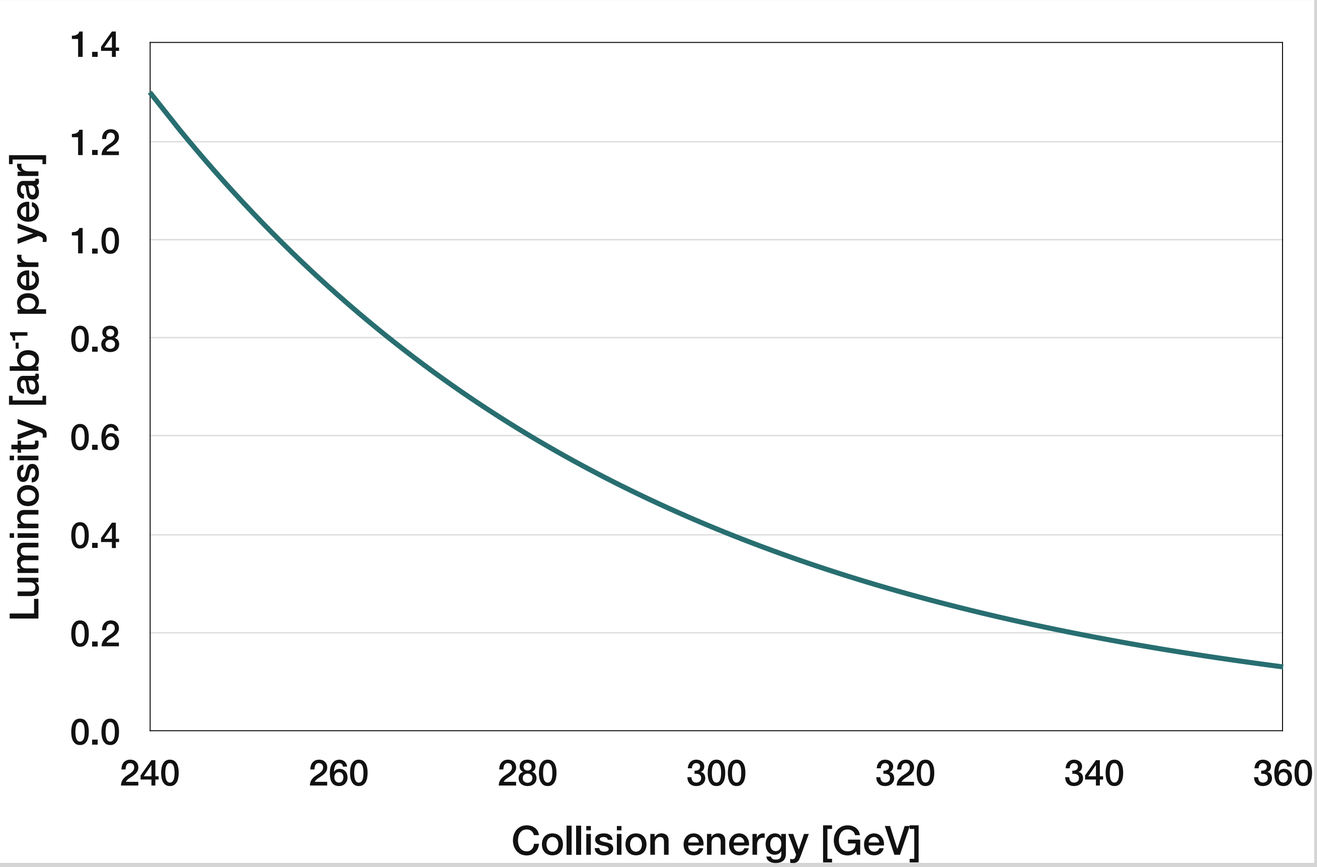
$$c_W = c_\beta \frac{M_W}{M_Z}, \quad v = \frac{2 M_W s_W}{e}, \quad v_h = v c_\beta, \quad v_\Sigma = -\frac{v}{2} s_\beta$$

$$F_\beta = \left(\frac{s_W^{\rm SM}}{c_\beta s_W} \right)^4 \frac{(-1 + 4 s_W^2)^2 + 1}{[-1 + 4 (s_W^{\rm SM})^2]^2 + 1}$$

$$b_4 > 0 \quad \Longrightarrow \quad \Delta m < \sqrt{\frac{c_\alpha^2 M_H^2 + s_\alpha^2 M_h^2}{c_\beta^2}} - M_H$$

- The SM weak angle follows from the same W and Z input masses.
- The inert radiative splitting must not be added to independently supplied masses without matching.

Luminosity assumption and inclusive recoil



$$\mathscr{L}_{\mathrm{yr}}(E) = 1.3\ \mathrm{ab}^{-1}\, 10^{-(E-240\,\mathrm{GeV})/(120\,\mathrm{GeV})}$$

- 30 MW scenario, summed over two interaction points.
- Intermediate energies are an interpolation, not published operating points.
- Inclusive recoil removes the diphoton branching penalty; backgrounds and efficiencies set the reach.

CEPC accelerator TDR, arXiv:2312.14363. The 50 MW upgrade is a separate scenario.

Scalar mixing counterterms

$$\delta\alpha = \frac{\widetilde{\mathrm{Re}}\,\Sigma_{hH}(M_H^2) + \widetilde{\mathrm{Re}}\,\Sigma_{hH}(M_h^2)}{2(M_h^2 - M_H^2)}$$

$$\delta\beta = -\frac{\widetilde{\mathrm{Re}}\,\Sigma_{GH}(M_{H^\pm}^2) + \widetilde{\mathrm{Re}}\,\Sigma_{GH}(0) - 2\Delta_{GH}}{2M_{H^\pm}^2}$$

$$\Delta_{GH} = \frac{2(v_h\delta t_\sigma - v_\Sigma \delta t_\phi)}{v_h^2 + 4v_\Sigma^2}, \qquad \delta t_i = -\widetilde{\mathrm{Re}}\,T_i$$

- Charged subtraction at $p^2 = 0$ does not change the MW mass of internal Goldstone propagators.
- PRTS / KOSY in Feynman gauge; no gauge-independent angle matching is implied.

Current manuscript, Appendix B. Nondegenerate neutral masses and finite triplet VEV assumed.

Statistical interpretation of the selection

$$\chi^2_{\mathrm{HS}}(p) - \chi^2_{\mathrm{HS,min}} \leq 9.4877, \qquad \chi^2_{\mathrm{HS,min}} = 150.02155$$

- The HiggsSignals threshold refers to the four-parameter fit, including when drawing a fixed slice.
- HiggsBounds, HiggsSignals, STU and theory conditions are applied as separate selections.
- A precision contour compares a predicted shift with an experimental uncertainty. It is not a confidence boundary.
- Numerical cancellation and integration checks do not establish the missing-higher-order uncertainty.

Why PCEPC?

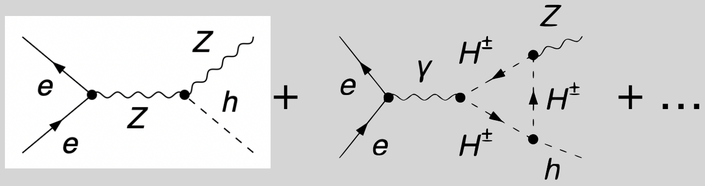
PCEPC: Precision Calculations for EP colliders

1. Scientific Question

- How do NLO corrections from NP impact $e^+e^- \to Zh$ predictions at e^+e^- colliders?

2. Importance

- Critical for sub-1% Higgs measurements (couplings, width).
- Sensitive to new physics in deviations from SM predictions.



3. What's New?

- First full NLO EW tool specific for EPC energies ($\sqrt{s} = 240$ GeV).

4. Implications

- Reduces theoretical uncertainties to match experimental precision.
- Benchmark calculation for various future e^+e^- colliders (CEPC, FCC-ee, ILC/LCF).