Testing Bell inequalities and probing quantum entanglement at CEPC

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In collaboration with Youpeng Wu, Ruobing Jiang, Qiang Li et al. From PKU

Outline

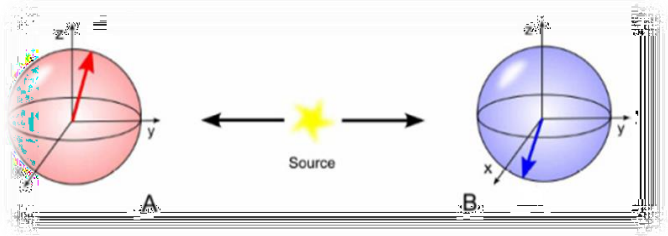
- **Quantum entanglement review**

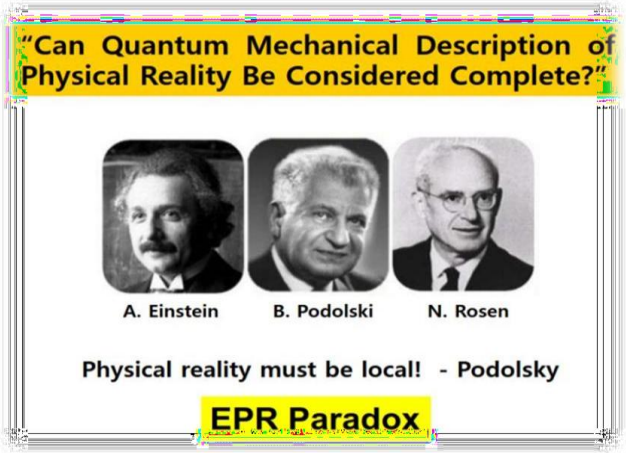
- **Quantum state tomography of ZZ pair form Higgs decay**
  - Definition of quantum state vectors of the particles produced at colliders
  - Spin density matrix
  - Quantum observables for entanglement detection

- **Simulation results for $H \to ZZ \to 4\ell$ at CEPC**

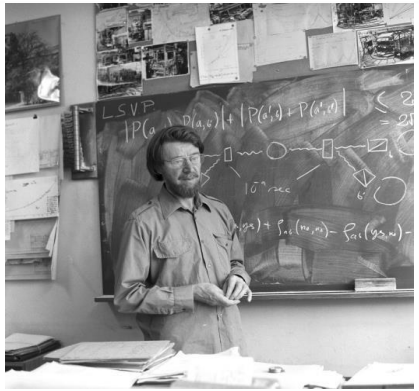
Entanglement origin: EPR paradox

➢ **Einstein-Podolsky-Rosen——EPR paper （1934），**





- Two entangled particles exhibit strong correlations, even when separated by large distances. EPR argued that quantum mechanics might therefore be incomplete.→ *missing local variable*



- Jhon Bell later showed that local hidden-variable theories must satisfy certain inequalities. Experiments violating these inequalities demonstrate that nature cannot be explained by local hidden-variable models.

$$|E(a,b) + E(a,b') + E(a',b) - E(a',b')| \leq 2$$

Entanglement detection at LHC as well as low energy case

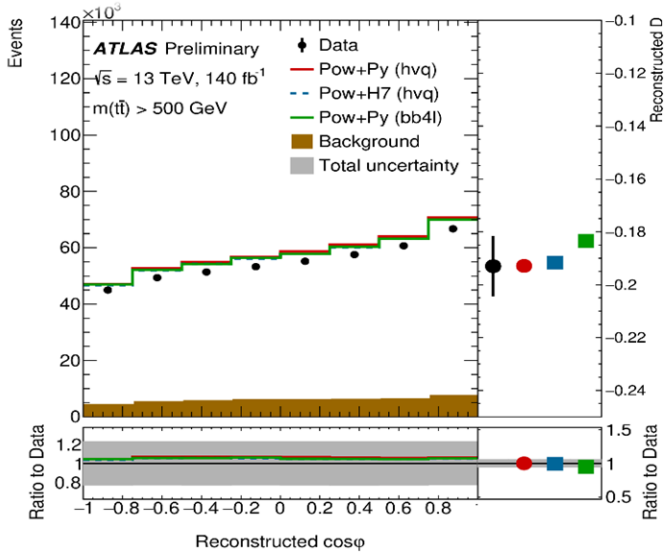
Entanglement marker

$$D = \frac{tr[C]}{3} = -3\,\langle cos\varphi \rangle$$

$$D \leq -\frac{1}{3}$$ Entanglement condition

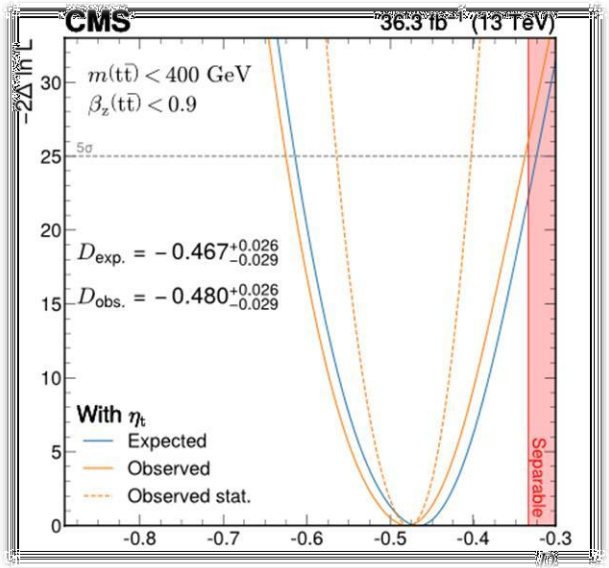
Y. Afik
Eur. Phys. J. Plus **136**, 907 (2021).

ATLAS collaboration



ATLAS Collaboration
Nature **633**, 542 (2024).

CMS collaboration



CMS collaboration.
Rep. Prog. Phys. **87**, 117801 (2024).

Bi-partite system

- **Quantum state of composite system** (i.e. bi-partite system)
  - Each spin-*s* particle has 2s+1 helicity states, i.e. s =1, $\lambda = +1,\ 0, -1$
  - A general pure state can be expressed in the helicity states as:

$$|\Psi\rangle = \sum_{\lambda_1,\lambda_2} c_{\lambda_1\lambda_2}|\lambda_1\rangle_A \otimes |\lambda_2\rangle_B, \qquad \sum_{\lambda_1,\lambda_2} |c_{\lambda_1\lambda_2}|^2 = 1.$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^{\mathrm{d}_a} \otimes \mathbb{C}^{\mathrm{d}_b}, \qquad \dim \mathcal{H} = d_a * d_b.$$

  - Statistical mixed states is described by Spin Density Matrix

$$\rho = \sum_k p_k |\Psi_k\rangle\langle\Psi_k|, \qquad p_k \geq 0, \quad \sum_k p_k = 1.$$

- **Quantum entanglement, Negativity, Bell inequality violation**

$$\rho_{\mathrm{sep}} = \sum_k p_k \rho_A^{(k)} \otimes \rho_B^{(k)}$$

$$\mathcal{N}(\rho) = \frac{\|\rho^{T_B}\|_1 - 1}{2} > 0 \ \Rightarrow \ \text{entangled.}$$

$I_3 > 2$: violation of local realism

Spin density matrix at a Collider

- In a collider process, the **spin density matrix** can be constructed from the production helicity amplitudes:

$$\rho_{\lambda_1\lambda_2;\lambda_1'\lambda_2'} = \frac{\sum_{\text{initial spins}} \mathcal{M}_{\lambda_1\lambda_2}\mathcal{M}^*_{\lambda_1'\lambda_2'}}{\sum_{\text{initial spins}}\sum_{\alpha,\beta}|\mathcal{M}_{\alpha\beta}|^2}.$$

Connection between quantum mechanics and QFT through scattering amplitude

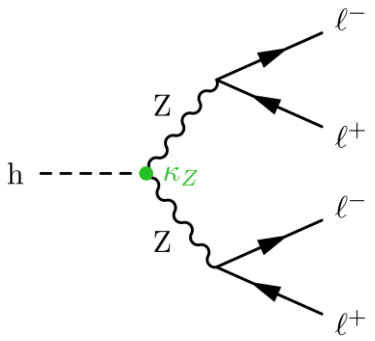
- Angular distributions of decay products probe spin polarization and entanglement

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \propto \sum_{\lambda_i,\lambda_i'} \rho_{\lambda_1\lambda_2;\lambda_1'\lambda_2'} D^{(1)}_{\lambda_1\lambda_1'} D^{(2)}_{\lambda_2\lambda_2'}$$

- The main quantum-information observables may include:

$$\rho \longrightarrow \begin{cases} \mathcal{N}(\rho) & \text{Entanglement,} \\ S(\rho_A) & \text{Pure-state entropy,} \\ I_3 & \text{Bell nonlocality.} \end{cases}$$

$H \to ZZ$ decay @ CECP



3-polarization vectors of Z boson

$$e^{\mu}_{\sigma}(m,\mathbf{k}) = \begin{pmatrix} 0 & k/m & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ \mathrm{i}/\sqrt{2} & 0 & \mathrm{i}/\sqrt{2} \\ 0 & -E/m & 0 \end{pmatrix}$$

- **H-ZZ is CP-even & $S_z = 0$:** Higgs produces an entangled ZZ* state

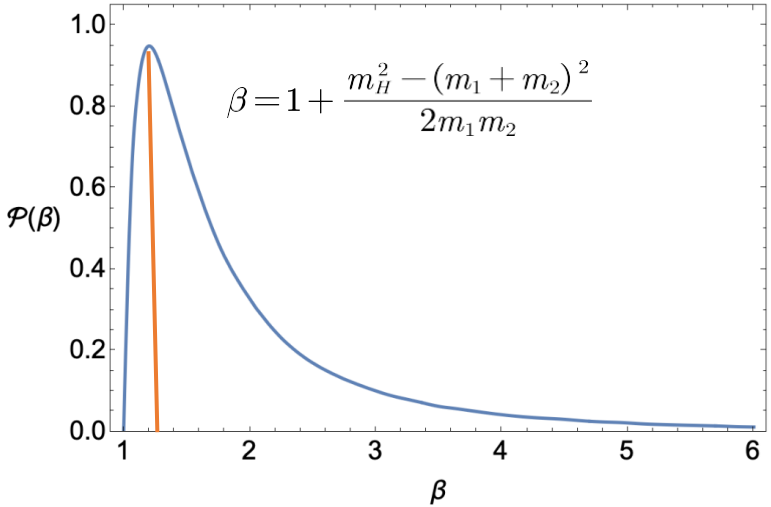
$$|u_1\rangle = |+,-\rangle, \qquad |u_2\rangle = |0,0\rangle, \qquad |u_3\rangle = |-,+\rangle.$$

$$|\psi_{ZZ}\rangle = \frac{|+-\rangle - \beta\,|00\rangle + |-+\rangle}{\sqrt{2+\beta^2}}$$

$$|\psi_{ZZ}\rangle \propto \sum_{\sigma,\lambda} g_{\mu\nu}\epsilon_1^{\mu*}(m_1,\mathbf{k};\sigma)\epsilon_2^{\nu*}(m_2,-\mathbf{k};\lambda)|\mathbf{k},\sigma\rangle|-\mathbf{k},\lambda\rangle.$$

$$\beta = 1 + \frac{m_H^2 - (m_{Z_1} + m_{Z_2})^2}{2m_{Z_1}m_{Z_2}}$$

Spin density matrix of ZZ^*



- Only 9 entries are non- zero! $\rho_{ZZ^*} = |\Psi\rangle\langle\Psi|$

$$\rho = \frac{1}{2+\beta^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & \beta^2 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

β = 1: threshold, exact spin singlet and maximal qutrit entanglement.
β > 1: stronger $|00\rangle$ component; entanglement is generally nonmaximal.

Spin density matrix parameterization

➢ Gell-Mann matrices representation

$$\rho = \frac{1}{9} I_3 \otimes I_3 + \sum_{i=1}^{8} a_i \lambda_i \otimes I_3 + \sum_{j=1}^{8} b_j I_3 \otimes \lambda_j + \sum_{i,j=1}^{8} c_{ij} \lambda_i \otimes \lambda_j$$

$$a_i = \mathrm{Tr}[\rho(\lambda_i \otimes I_3)], \quad \rightarrow \text{Polarization of } Z_1$$

$$b_j = \mathrm{Tr}[\rho(I_3 \otimes \lambda_j)], \quad \rightarrow \text{Polarization of } Z_2$$

$$c_{ij} = \mathrm{Tr}[\rho(\lambda_i \otimes \lambda_j)]. \quad \rightarrow \text{Spin correlations of } Z_1 Z_2$$

80 parameters in total to represent polarizations and spin correlation!

Spin density matrix parameterization

➢ Irreducible tensor operators

$$T^{L_1}_{M_1}, T^{L_2}_{M_2} \in \{\mathbb{1}_3; T^1_1, T^1_0, T^1_{-1}; T^2_2, T^2_1, T^2_0, T^2_{-1}, T^2_{-2}\}$$

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\, T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\, \mathbb{1}_3 \otimes T^L_M + C_{L_1M_1L_2M_2}\, T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right];$$

➢ Connection between coefficients A^i_{LM}, C and spherical harmonics under a spatial rotation is easily obtain than previous parameterization form, with 9 entries are non-zero:

$$\rho = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

With the following relation:

$$\frac{1}{\sqrt{2}}A^1_{2,0} + 1 = C_{2,2,2,-2}$$

Reconstructing density matrix

- phase space is written in terms of the spherical coordinates with independent polar axis for the momenta of the final charged leptons in the respective rest frames of the decaying Z boson.

- $\Gamma_{1,2}$ are decay density matrices describing the polarization of the two Z boson into final leptons obtained from decay helicity amplitudes: $\Gamma \propto \mathcal{M}^{\dagger}\mathcal{M}$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left\{\rho\,(\Gamma_1 \otimes \Gamma_2)^T\right\}$$

$$\Gamma = \frac{1}{4}\begin{pmatrix} 1+\cos^2\theta - 2\eta_\ell\cos\theta & \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{i\varphi} & (1-\cos^2\theta)e^{2i\varphi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{-i\varphi} & 2\sin^2\theta & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{i\varphi} \\ (1-\cos^2\theta)e^{-2i\varphi} & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{-i\varphi} & 1+\cos^2\theta - 2\eta_\ell\cos\theta \end{pmatrix}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + A^1_{LM}B_L Y^M_L(\theta_1,\varphi_1) + A^2_{LM}B_L Y^M_L(\theta_2,\varphi_2) + C_{L_1M_1L_2M_2}B_{L_1}B_{L_2}Y^{M_1}_{L_1}(\theta_1,\varphi_1)Y^{M_2}_{L_2}(\theta_2,\varphi_2)\Big]$$

The average value of the spherical harmonic functions determine those coefficients!

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j)d\Omega_j = \frac{B_L}{4\pi}A^j_{LM}, \qquad j = 1,2.$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)Y^{M_2}_{L_2}(\Omega_2)d\Omega_1 d\Omega_2 = \frac{B_{L_1}B_{L_2}}{(4\pi)^2}C_{L_1M_1L_2M_2},$$

Testing Quantum entanglement

- Bell inequality of 2-qutrit system

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2$$

- Expectation value of I_3: $I_3 = \langle O_{\text{Bell}} \rangle = \text{Tr}\{\rho O_{\text{Bell}}\} \leq 2$

$$\mathcal{O}_{\text{Bell}} = \left[\frac{2}{3\sqrt{3}} (T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12} (T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2) + \frac{1}{2\sqrt{6}} (T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3} (T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4} T_0^2 \otimes T_0^2 \right] + \text{H.c.}$$

- Remember only nine-entries are non-zero, this constrains the form the I_3:

$$I_3 = \frac{1}{36} \left[18 + 16\sqrt{3} - \sqrt{2}(9 - 8\sqrt{3}) A_{2,0}^1 - 8(3 + 2\sqrt{3}) C_{2,1,2,-1} + 6 C_{2,2,2,-2} \right].$$

- Entanglement condition:

Peres–Horodecki criterion

Partial transpose: $\left(\rho^{T_B}\right)_{ij,kl} = \rho_{il,kj}.$

$\lambda_{\min}(\rho^{T_B}) < 0 \implies$ Entangled

Entangled $\iff C_{2,1,2,-1} \neq 0$ or $C_{2,2,2,-2} \neq 0$

Reference system

(i) A unit vector is defined in the incident beam direction $\hat{p} = (0, 0, 1)$ in the lab frame.

(ii) Then we set the direction of the three-momentum of the on-shell Z boson as $\hat{z}$ axis in the Higgs rest frame. The angle Θ is then between these two axes.

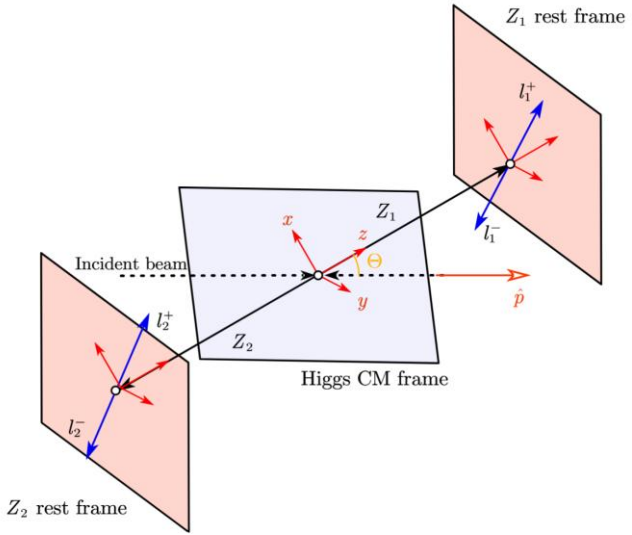
(iii) $\hat{x}$ axis is defined as

$$\hat{x} = \frac{\text{sign}(\cos\Theta)\,(\hat{p} - \hat{z}\cos\Theta)}{\sin\Theta},$$

so that it lies on the production plane.

(iv) $\hat{y}$ axis is perpendicular to the production plane and is given as

$$\hat{y} = \hat{z} \times \hat{x}.$$



➢ Event reconstruction and selection

1. Select four charged leptons; pair opposite-sign leptons into Z candidates.

2. Closest dilepton mass to m_Z identifies Z_1; the other pair forms off-shell Z_2*.

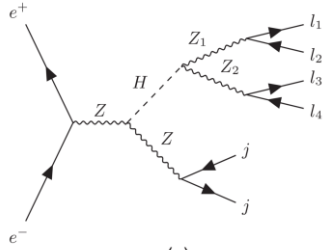
3. Boost to H and each Z rest frame; extract Ω_1, Ω_2; form moment estimators.

Simulation and results

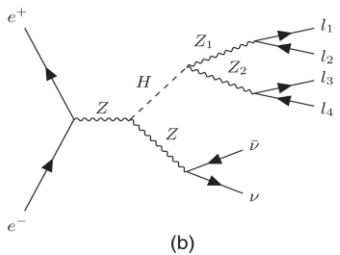
Signal

I. $e^+e^- \to HZ, Z \to jj$

II. $e^+e^- \to HZ, Z \to \overline{\nu}\nu$



(a)



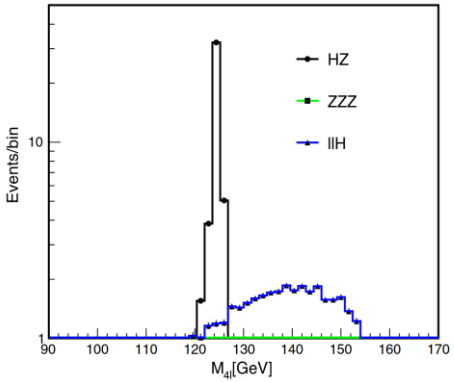
(b)

Background

(i) $e^+e^- \to ZZ.$

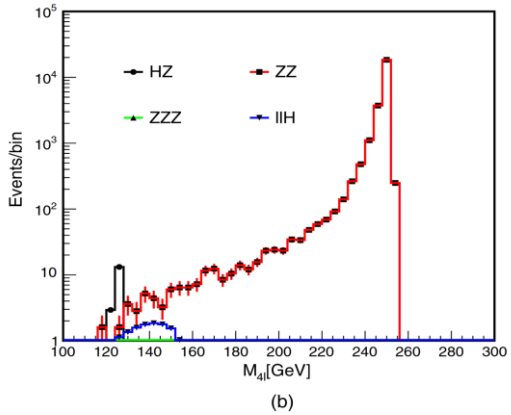
(ii) $e^+e^- \to ZZZ.$

(iii) $e^+e^- \to \ell^+\ell^- H.$



(a)

Semi-leptonic channel



(b)

leptonic channel

Simulation and results

TABLE I. The numerical result of the observable $\mathcal{I}_3$ for the signal processes. (Semileptonic) $\mathcal{L} = 50\ \text{ab}^{-1}$.

M_z^* (GeV)	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.823 \pm 0.640(1.29\sigma)$	$-1.080 \pm 0.420(2.57\sigma)$	$0.637 \pm 0.559(1.14\sigma)$
10	$2.913 \pm 0.692(1.32\sigma)$	$-1.126 \pm 0.451(2.50\sigma)$	$0.677 \pm 0.598(1.13\sigma)$
20	$3.092 \pm 0.800(1.37\sigma)$	$-1.225 \pm 0.514(2.38\sigma)$	$0.761 \pm 0.734(1.04\sigma)$
30	$3.048 \pm 1.816(0.58\sigma)$	$-1.160 \pm 1.192(0.97\sigma)$	$0.875 \pm 1.338(0.65\sigma)$

TABLE II. The Numerical result of the observable $\mathcal{I}_3$ for the signal processes. (Leptonic) $\mathcal{L} = 50\ \text{ab}^{-1}$.

M_z^* (GeV)	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.713 \pm 1.167(0.61\sigma)$	$-1.008 \pm 0.745(1.35\sigma)$	$0.608 \pm 0.931(0.65\sigma)$
10	$2.780 \pm 1.328(0.59\sigma)$	$-1.044 \pm 0.849(1.23\sigma)$	$0.644 \pm 1.038(0.62\sigma)$
20	$2.936 \pm 1.455(0.64\sigma)$	$-1.119 \pm 0.940(1.19\sigma)$	$0.754 \pm 1.083(0.70\sigma)$
30	$3.016 \pm 2.465(0.41\sigma)$	$-1.129 \pm 1.616(0.70\sigma)$	$0.905 \pm 1.617(0.56\sigma)$

Summary

- ATLAS and CMS experiments already observed that quantum entanglement is truly present particle colliders, proving that Quantum Mechanics is a non-local theory.
- Because of the spin-zero property of the Higgs boson, the ZZ system arising from its decay is entangled state.
- Simulations of the process $H \rightarrow ZZ \rightarrow 4\ell$ at a CEPC with $\sqrt{s} = 250$ GeV and $L = 50$ ab^{-1} shows that quantum entanglement is detectable.
- Higgs decay at CEPC can also provide means of testing Bell inequalities and probing quantum entanglement of a qutrit-qutrit, qubit-qutrit, qubit-qubit system.


谢谢
THANK YOU