Quantum information of photon pairs at lepton colliders

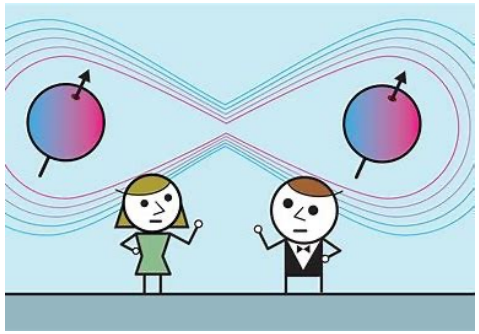
Bin Yan
Institute of High Energy Physics

CEPC 2026 @ Shanghai
Oct. 21- 26, 2026

K. Cheng, T. Han, G. Li and B. Yan, 2606.30759

Quantum information at collider

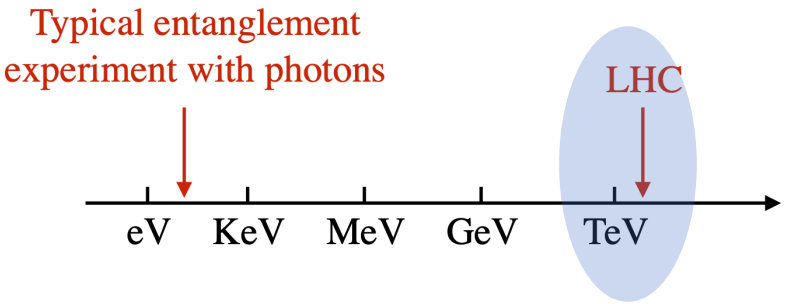
➢Quantum entanglement and non-locality are distinctive features of quantum systems



➢Collider physics: high energy and small size

Quantum information at collider

➢High energy: the scattering involves both the QCD and electroweak interactions

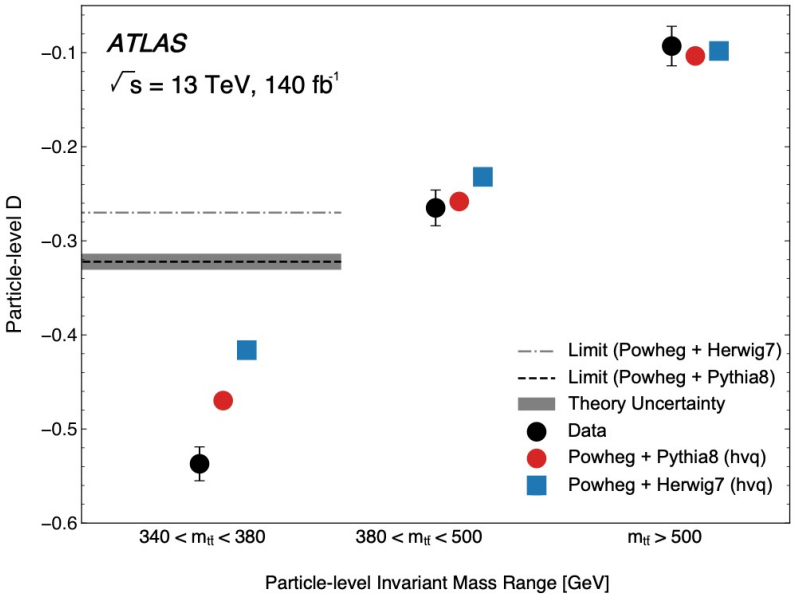


The results open up a new perspective on the complex world of quantum physics

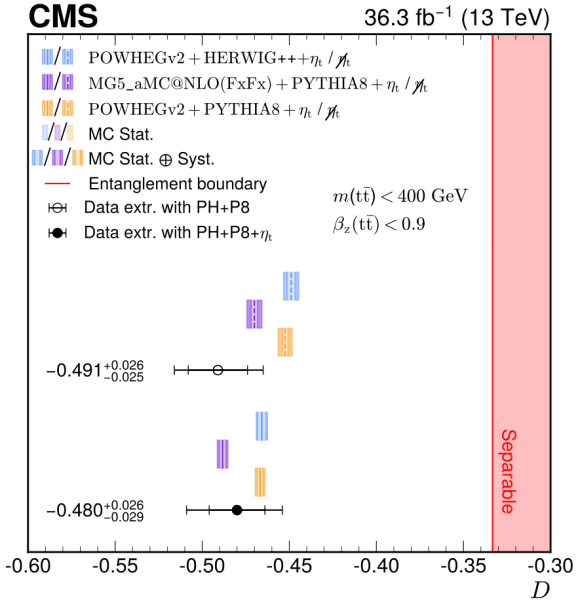


ATLAS and CMS has observed the spin entanglement of top quark pair

Nature 633, 542 (2024)

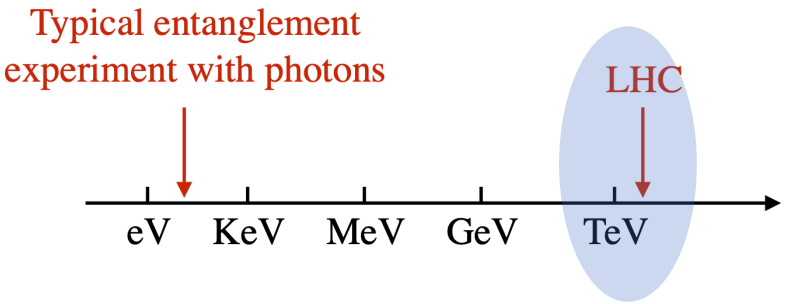


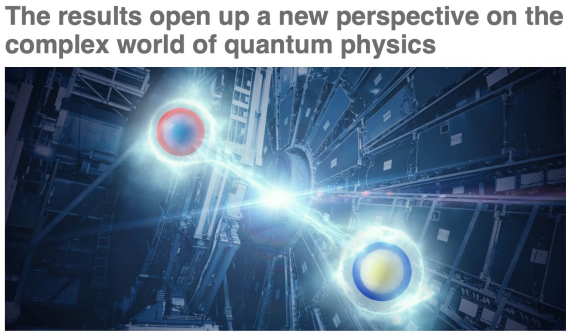
Rept. Prog. Phys. 87 (2024) 117801



Quantum information at collider

➢High energy: the scattering involves both the QCD and electroweak interactions





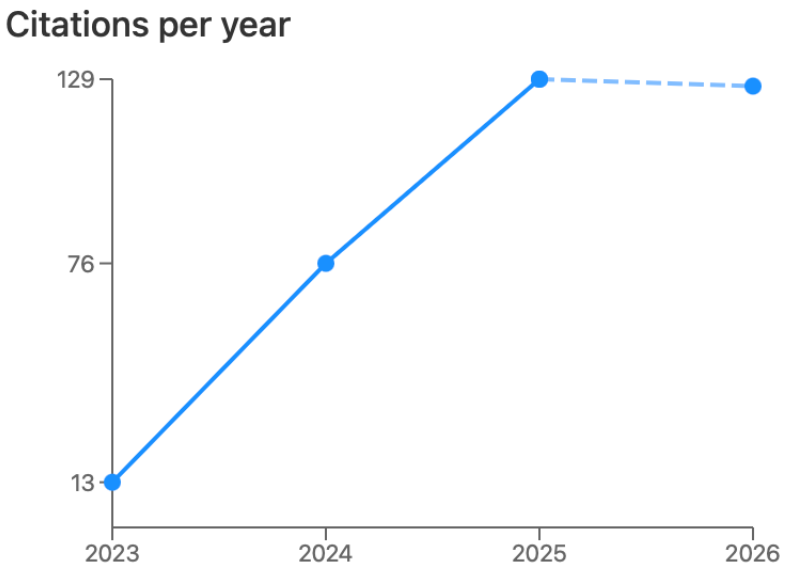
➢New features:

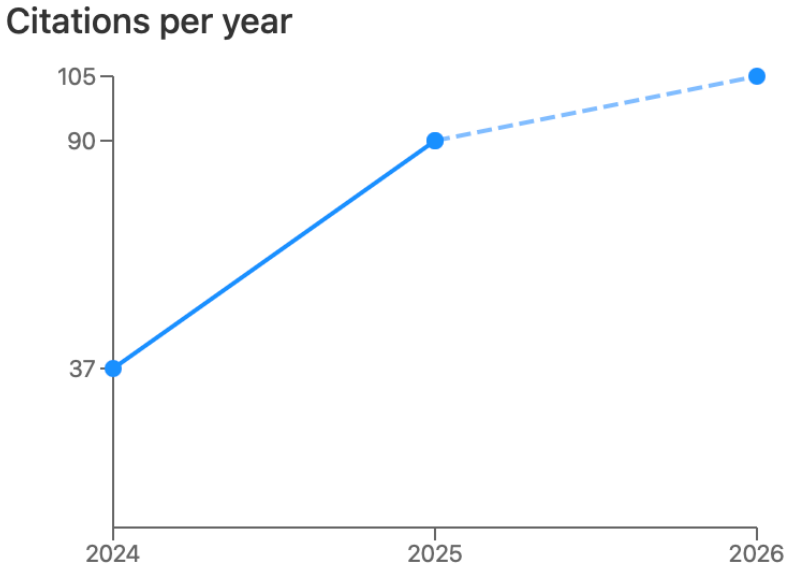
- ❑ Particle spin can not be measured directly
- ❑ New degree of freedom for spin-1 particle (Longitudinal mode)
- ❑ New features from interactions: parity violation, QCD confinement
- ❑ Entanglement beyond the spin space: flavor
- ❑ Quantum information and QCD confinement
- ❑ ……

Quantum information in high-energy physics is an emerging and rapidly growing field at the intersection of particle physics and quantum theory

Quantum information in collider

ATLAS and CMS has observed the spin entanglement of top quark pair





ATLAS: Nature 633 (2024) 8030, 542-547

CMS: Rept. Prog. Phys. 87 (2024) 117801

Quantum Entanglement

- Consider two subsystems A and B with a combined Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

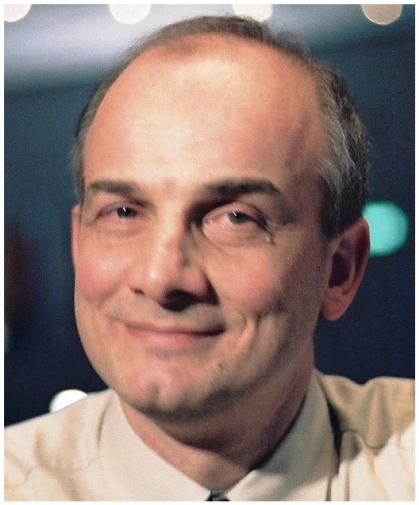
Separable sate (non-entangled) $\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$ From Hao Zhang's slide

"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "

$$\rho_{AB} \stackrel{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_i^N p_i = 1, \, p_i > 0 \right)$$

"Finally, we prove that the weak membership problem for the convex set of separable normalized bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits



L. Gurvits, *"Classical Deterministic Complexity of Edmonds' Problem and Quantum Entanglement"*, 2003

Quantum Entanglement

- **Peres-Horodecki criterion:** If partial transposition has any negative eigenvalue, then the state is entangled Peres, PRL 1996, 77: 1413, Horodecki, PLA, 1997, 232]

  Partial transposition $\rho_{AB}^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$

  $$\text{Negativity} = \sum_i \frac{|\lambda_i| - \lambda_i}{2}$$ λ_i: eigenvalues of matrix: $\rho_{AB}^{T_B}$

- **Concurrence** W. K. Wootters, PRL 80 (1998) 2245

  $$\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \qquad \tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$$

  λ_i: eigenvalues of matrix: $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

  $$0 \leq \mathcal{C}(\rho) \leq 1$$

  Seperable — Maximal entangled

For a 2×2 systems negativity is equivalent to concurrence

Bell inequality

- Bell (CHSH) inequality: $\hat{A}_i = \pm 1,\ \hat{B}_i = \pm 1$

$$\left|\langle \hat{A}_1\hat{B}_1\rangle + \langle \hat{A}_1\hat{B}_2\rangle + \langle \hat{A}_2\hat{B}_1\rangle - \langle \hat{A}_2\hat{B}_2\rangle\right| \le 2$$

[Clauser et al, *PRL* 23, 880 (1969)]

$$\hat{A} = \vec{a}\cdot\hat{\sigma},\quad \hat{B} = \vec{a}\cdot\hat{\sigma},\quad \langle \hat{A}\hat{B}\rangle = \vec{a}\cdot C\cdot\vec{b}$$

- Bell inequality is violated iff we can find four directions $\vec{a}_{1,2}, \vec{b}_{1,2}$ so that

$$\left|\vec{a}_1\cdot C\cdot(\vec{b}_1-\vec{b}_2) + \vec{a}_2\cdot C\cdot(\vec{b}_1+\vec{b}_2)\right| > 2$$

Fix some direction $\vec{a}_{1,2}, \vec{b}_{1,2}$

e.g. $\sqrt{2}|C_{xx} \pm C_{yy}| > 2$

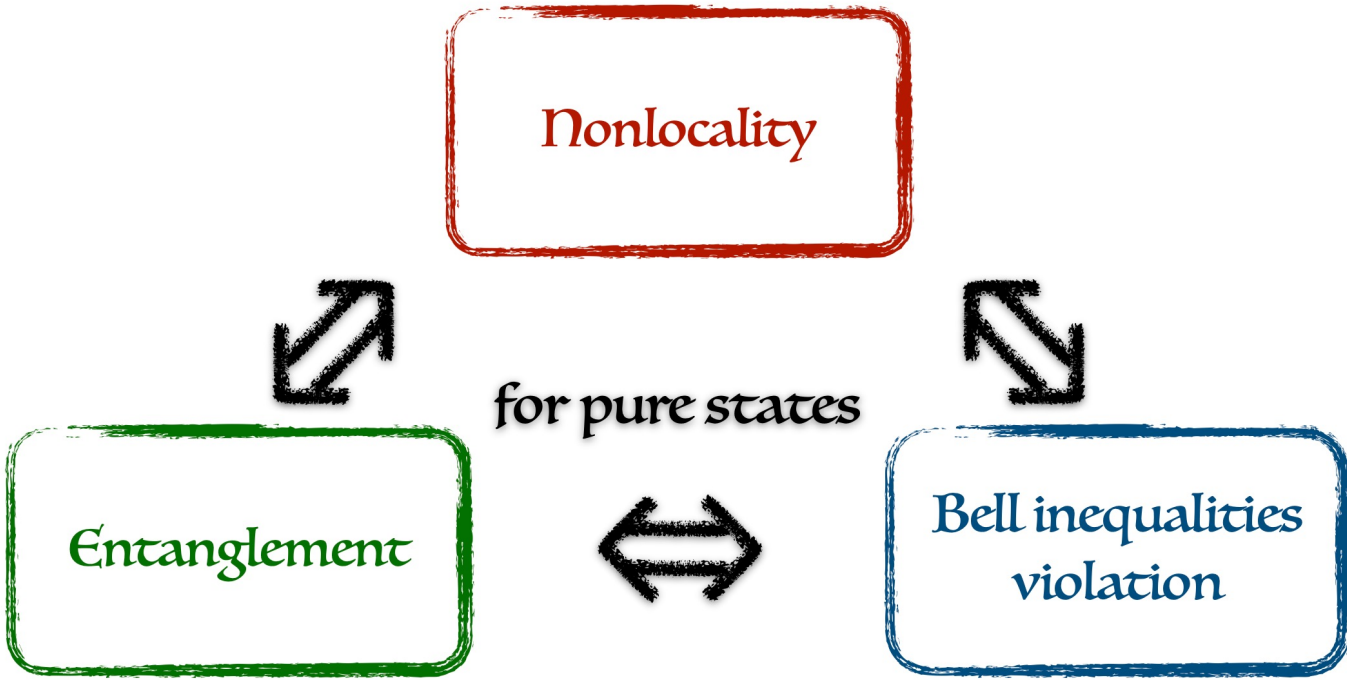
Scan $\vec{a}_{1,2}, \vec{b}_{1,2}$ **to maximize**

$$\mathcal{B}[\rho] = 2\sqrt{c_1^2 + c_2^2} > 2$$

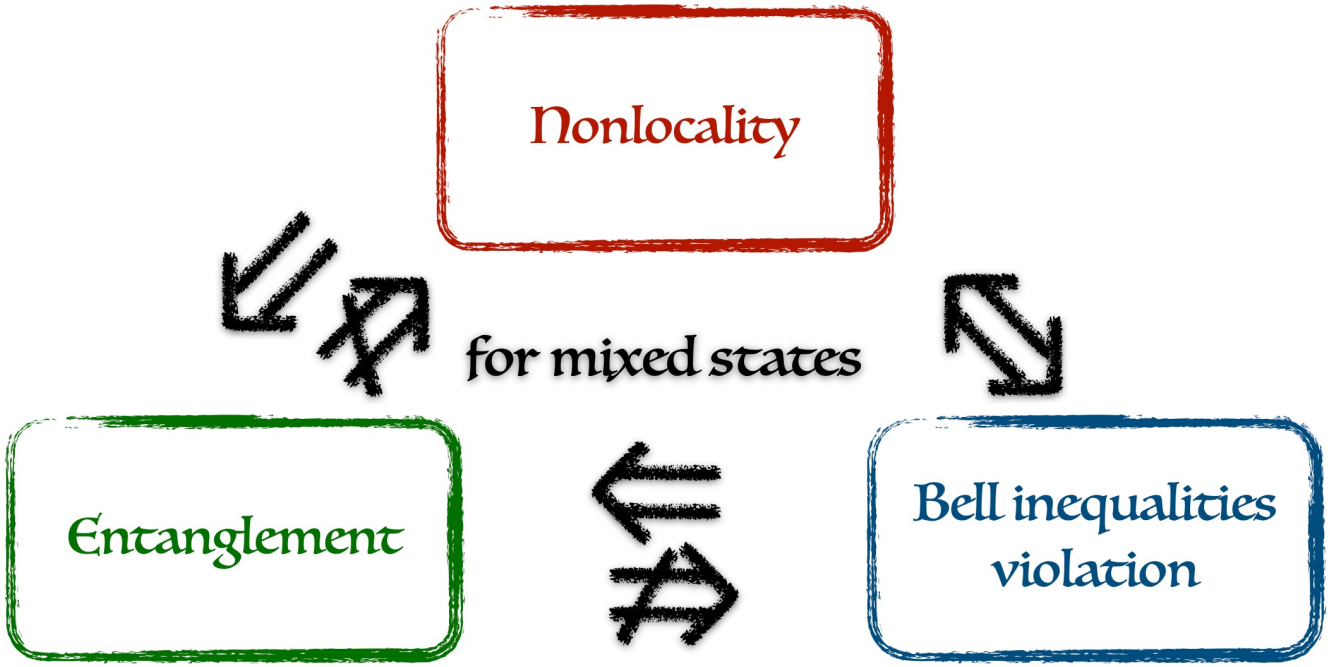
c_1^2, c_2^2 **are the largest two eigenvalue of** C^TC.

Quantum entanglement and nonlocality

- Gisin Theorem: For any bipartite pure state, entanglement is equivalent to Bell nonlocality



Quantum entanglement and nonlocality



e.g. Werner states in quantum information

$$\rho_{\text{Werner}}(\alpha) = \alpha|\Psi_0\rangle\langle\Psi_0| + \frac{1-\alpha}{4}\hat{I}_4$$

Bell singlet

$\frac{1}{3} < \alpha < 1$ Entanglement

$\frac{1}{\sqrt{2}} < \alpha < 1$ Bell inequalities violation

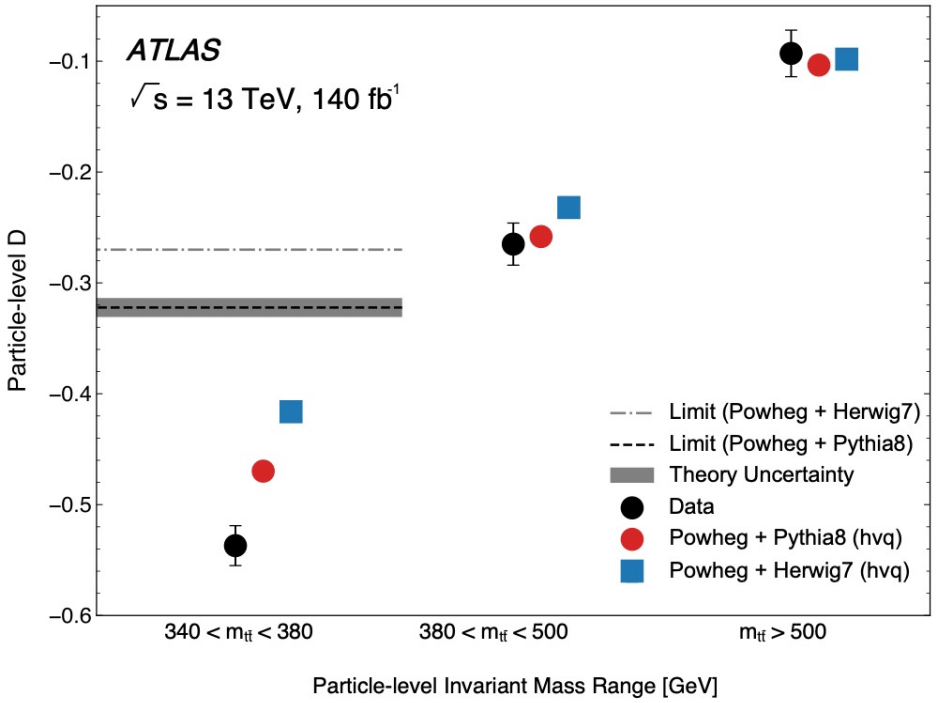
Testing Bell inequalities violation is much more difficult than entanglement

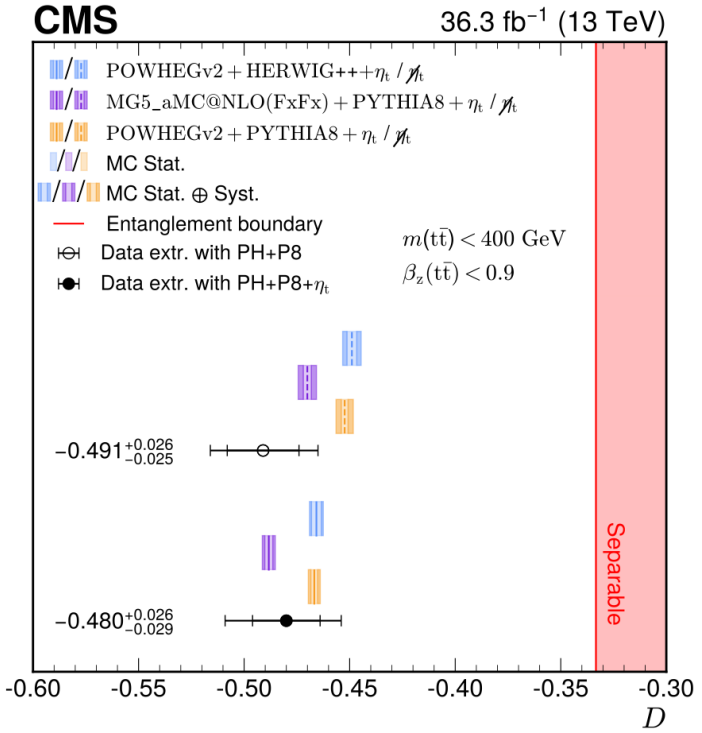
Quantum spin entanglement

Top quark pair: $\mathcal{C} = \dfrac{-1-3D}{2}$ $\qquad \dfrac{1}{\sigma}\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\varphi} = \dfrac{1}{2}(1 - D\cos\varphi)$

$$D = \min\{\frac{\mathrm{tr}(C)}{3}, \frac{c_1 - c_2 - c_3}{3}, \frac{c_2 - c_1 - c_3}{3}, \frac{c_3 - c_1 - c_2}{3}\}, \qquad c_i = \mathrm{eig}(C)$$

Y. Afik, J. de Nova, EPJC 136 (2021) 907





Entangled : $D < -1/3$, $\quad$ separable : $D > -1/3$

Quantum entanglement at colliders

- Top quark pair

Y. Afik, J. R. M. n. de Nova Eur. Phys. J. Plus 136, 907 (2021)
M. Fabbrichesi, R. Floreanini, G. Panizzo, PRL 127, 161801 (2021)
C. Severi. C. D. E. Boschi, F. Maltoni, and M. Sioli, EPJC 82, 285 (2022)
T. Han, M. Low, T. A. Wu, JHEP 07, 192 (2024)
T. Han, M. Low, N. McGinnis, and S. Su, 2412.21158
K. Cheng, T. Han and M. Low, 2410.08303,
…

- Tau lepton pair

M. M. Altakach et al, PRD 107, 093002 (2023)
K. Ehataht et al, PRD 109, 032005 (2024)
Y. Du, X.-G. He, C.-W. Liu and J.-P. Ma, 2409.15418
Y. Zhang et al, 2504.01496
T. Han, M. Low, Y. Su, 2501.04801
…

- Gauge boson pair

A. J. Barr et al, Quantum 7, 1070 (2023)
Q. Bi, Q.-H. Cao, K. Cheng, H. Zhang, PRD 109, 036022 (2024)
R. Ding et al, 2504.09832
…

- Flavor

K. Chen, Z. Xing, R. Zhu, 2407.19242
H. Feng. H. Tang, W. Guo Q. Qin, 2504.15798
….

- Entanglement & NP

R. Aoude et al, PRD 106 (2022) 055007
M. Fabbrichesi et al, EPJC 83 (2023) 162, JHEP 09 (2023) 195
A. Bernal et al, EPJC 83 (2023) 11, 1050
…

The spin correlation between particles can be measured from its decay products



How about the stable particles, e.g. light quarks and photons?

Light quarks: K. Cheng, B. Yan, PRL (2025)

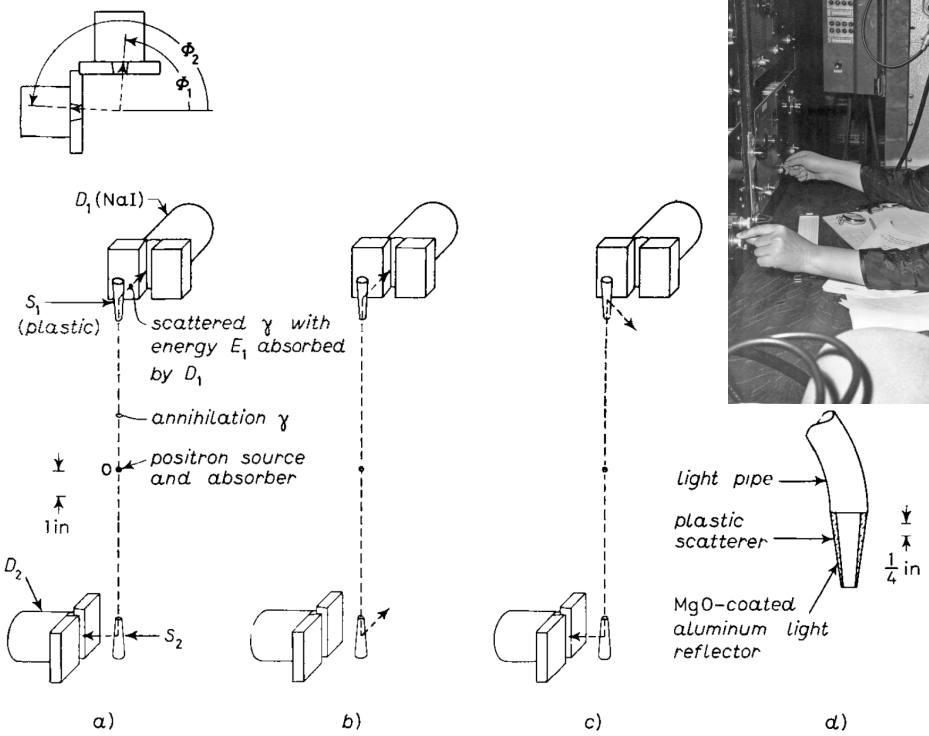
Quantum information of photon pair

- The photon pair state is linearly polarized in lepton collider

$$e^+e^- \to \gamma\gamma$$

$$\psi = \frac{|XY\rangle - |YX\rangle}{\sqrt{2}}$$

Compton Scattering with electrons in detector



$$P(E_1E_2\Phi_1\Phi_2) = \frac{1}{4\pi^2}F(E_1)F(E_2)[1 - m(E_1)m(E_2)\cos 2(\Phi_2 - \Phi_1)]$$

L. R. Kasday, J. D. Ullman, and C.-S. Wu, Il Nuovo Cimento B (1971-1996) 25, 633 (1975).

Quantum information of photon pair

- Virtual-photon splitting as a polarimeter
- The virtual photon should be described by a qutrit state

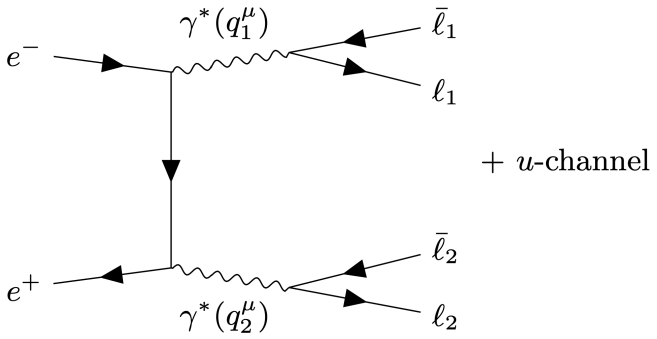
$$\epsilon_0^\mu(q_i) = \frac{q_i^\mu}{\sqrt{q_i^2}} + \mathcal{O}(\delta_i)$$

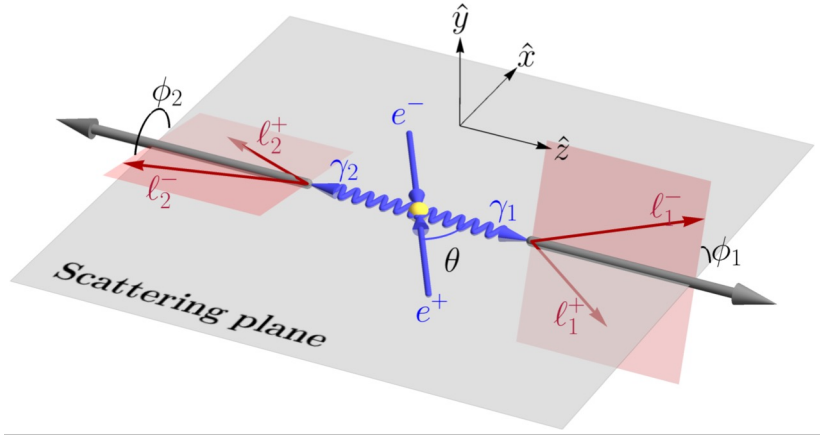
- For nearly on-shell photons, the longitudinal contributions are suppressed, we can use an effective two-qubit description
- The splitting process

$$\frac{1}{16q^2}\Gamma_{\lambda\lambda'} = D(z,q^2)\delta_{\lambda\lambda'} + H(z,q^2)\left[\cos(2\phi)(\sigma_x)_{\lambda\lambda'} + \sin(2\phi)(\sigma_y)_{\lambda\lambda'}\right]$$

Unpolarized linearly polarized

$$z = \frac{E_\ell + p_\ell^z}{E_\gamma + p_\gamma^z}$$ light-cone momentum fraction

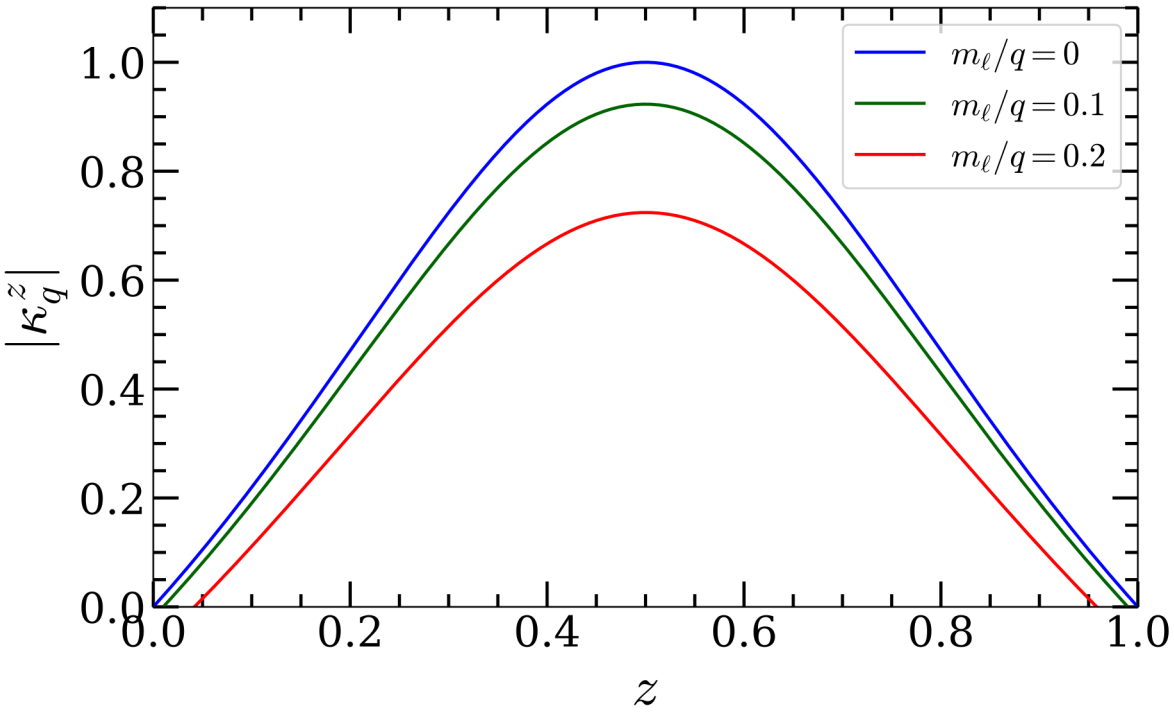




Spin analyzing power

- Spin analyzing power from splitting

$$D(z,q^2)=\frac{1}{2}+\frac{m_\ell^2}{q^2}-z(1-z) \qquad H(z,q^2)=\frac{m_\ell^2}{q^2}-z(1-z) \qquad \kappa_q^z=\frac{H(z,q^2)}{D(z,q^2)}$$



- The electron has a larger spin analyzing power

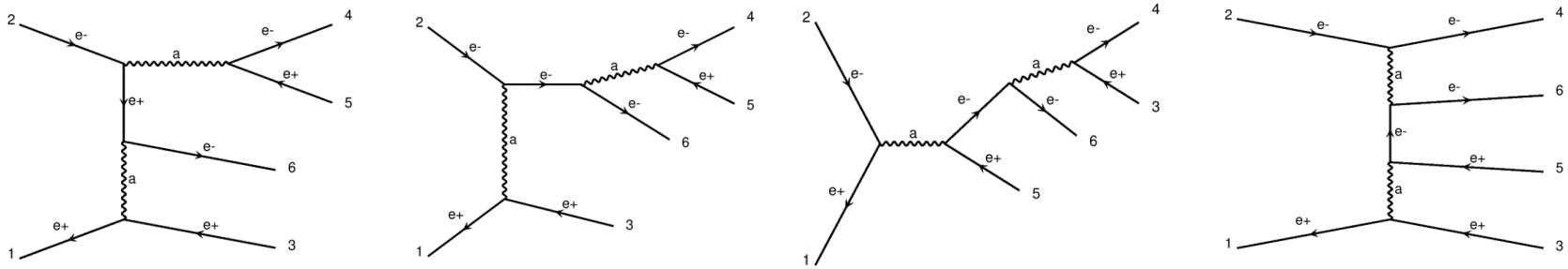
Quantum information of photon pair

- Bell variable:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1\mathrm{d}\Omega_2} \propto \frac{D(z_1,q_1^2)D(z_2,q_2^2)}{q_1^2 q_2^2}\sigma_{\mathrm{hard}}\left[1+\alpha_{q_1,q_2}^{z_1,z_2}\times\left(\mathcal{B}_-\cos 2(\phi_1+\phi_2)+\mathcal{B}_+\cos 2(\phi_1-\phi_2)\right)\right].$$

$$\alpha_{q_1,q_2}^{z_1,z_2}=\frac{\kappa_{q_1}^{z_1}\kappa_{q_2}^{z_2}}{2} \qquad \mathcal{B}_\pm \equiv C_{xx}\pm C_{yy} \qquad B_+=0, \quad B_-=\frac{2\sin^2\theta}{1+\cos^2\theta}$$

- Backgrounds:



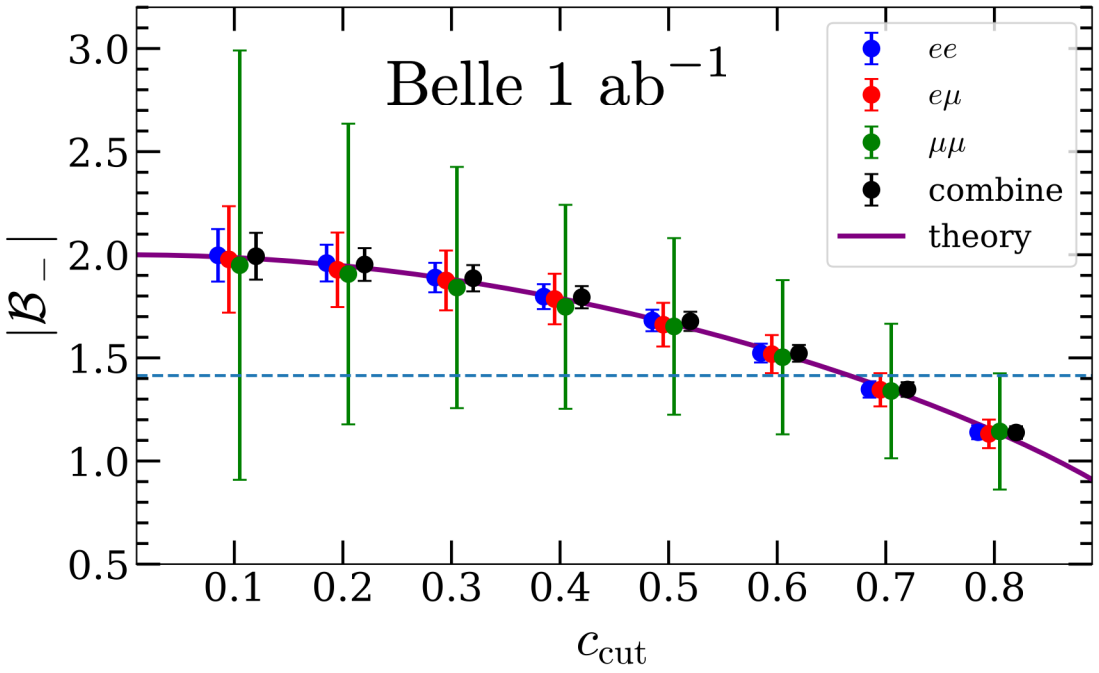
- Cuts: Lepton angle, energy, Photon virtuality, scattering angle and ΔR

$$|\cos\theta_\ell|<0.9, E_\ell>0.1\ \mathrm{GeV}, m_{ee}<0.5\ \mathrm{GeV}, m_{\mu\mu}<1\ \mathrm{GeV}, \Delta R>10^{-3}$$

Quantum information of photon pair

- Bell variable:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1 \mathrm{d}\Omega_2} \propto \frac{D(z_1, q_1^2) D(z_2, q_2^2)}{q_1^2 q_2^2} \sigma_{\mathrm{hard}} \left[1 + \alpha_{q_1,q_2}^{z_1,z_2} \times \left(\mathcal{B}_- \cos 2(\phi_1 + \phi_2) + \mathcal{B}_+ \cos 2(\phi_1 - \phi_2) \right) \right].$$



$$B_- = \frac{2\sin^2\theta}{1+\cos^2\theta}$$

$$|\cos\theta| < c_{\mathrm{cut}}$$

- Low virtuality sample is well described by the effective two-qubit approximation
- Belle variable will be enhanced in the central scattering region
- CHSH type Bell inequality violation @ Belle: 7.3 σ for $c_{\mathrm{cut}} = 0.3$

Summary

- Quantum information studies at colliders have only just begun
- Prior work limited to massive particles; massless photons unexplored in colliders
- Exploiting the photon splitting process for nearly on-shell photons, we formulate an effective two-qubit description of photon pair in lepton colliders
- Bell-inequality violation for photons could be observed with $> 5\sigma$ significance

