海南师范大学
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Lattice QCD calculation of $\Xi^- \to \Lambda \ell^- \bar{\nu}_\ell$ semileptonic decay

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The 2026 International Workshop on the High Energy Circular Electron Positron Collider

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Outline

1. Motivation
2. Theorical framework
3. Numerical results
4. Summary

CKM unitarity

PDG2024:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

First row: 10^{-4} *accuracy*

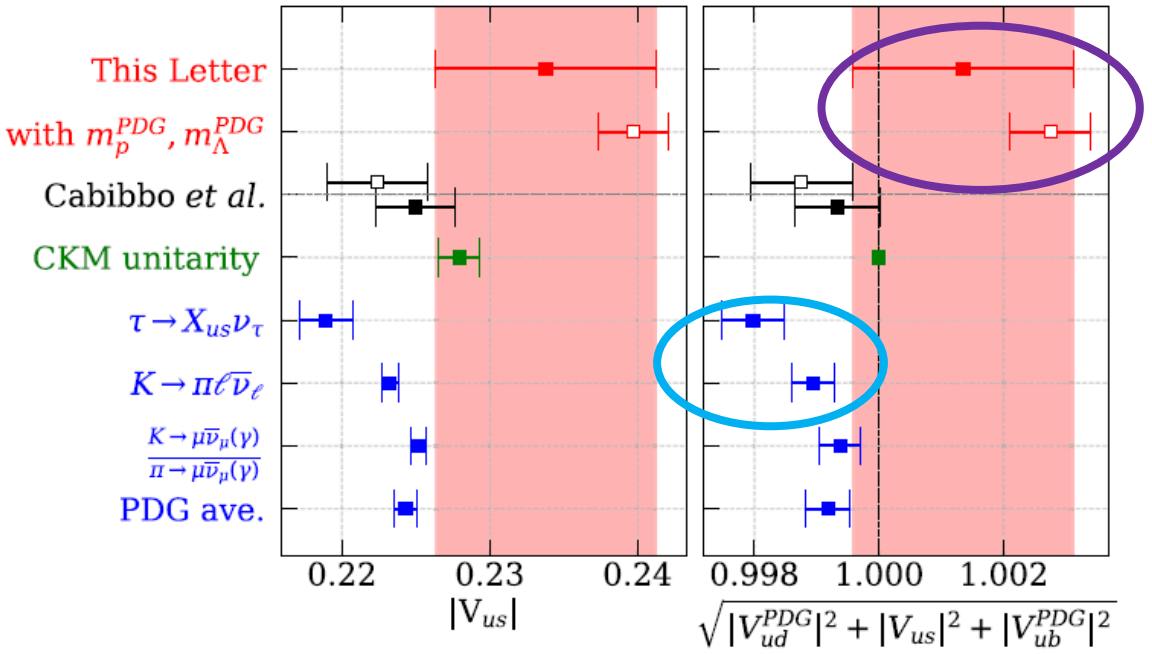
$|V_{ud}| = 0.97367 \pm 0.00032$ 0.03%

$|V_{us}| = 0.22431 \pm 0.00085$ 0.4%

$|V_{ub}| = 0.00382 \pm 0.00020$ 5%

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0007$$

$\Lambda \to p\ell^-\bar{\nu}_\ell$ from Lattice

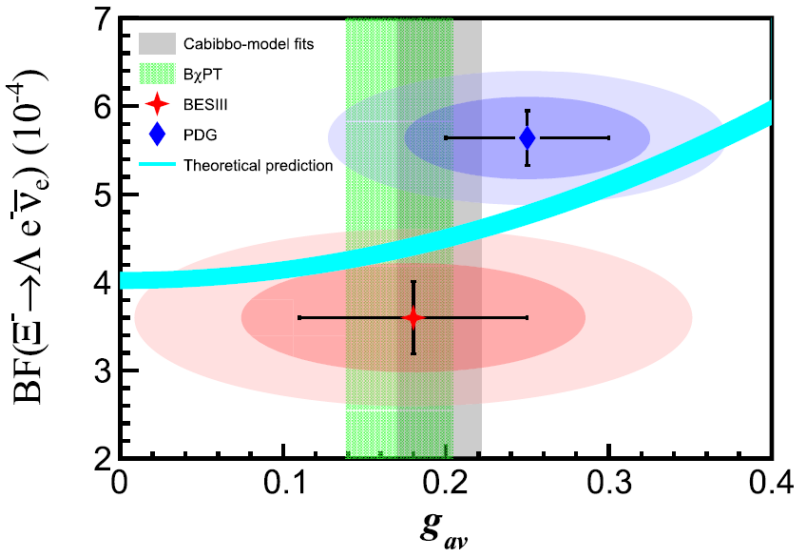


Phys.Rev.Lett. 135 (2025) 231901

$|V_{us}|$ of τ and K decay : differ by more than 2σ.

Hyperon decays provide an independent tests of CKM unitarity

Experiment measurement



BESIII: Phys.Rev.Lett. 137, 091801

Branching ratio of $\Xi^- \to \Lambda e\bar{\nu}_e$:

BESIII: $(3.60 \pm 0.41) \times 10^{-4}$

PDG2024: $(5.63 \pm 0.51) \times 10^{-4}$

3.9σ

Phenomenological calculation

SU(3) symmetry,

Quark model,

QCD sum rule …

Phys. Rev. D 51, 2262 (1995)
Phys. Rev. D 78, 094005 (2008)
Phys. Rev. D 100, 076008 (2019)
JHEP 06 (2024) 122
Phys. Rev. D 114 (2026) 074005

$Br \times 10^{-4}$	$\Xi^- \to \Lambda e\bar{\nu}_e$	$\Xi^- \to \Lambda \mu\bar{\nu}_\mu$
SU(3)	5.47(15)	1.58(4)
LFQM	4.85	1.31
CCQM	5.49	1.57
QCDSR	5.17(73)	1.54(23)
R3QM	4.41	1.24

First-principles calculations are necessary.

Beyond the standard model

The LO, NLO, NLO+SUSY and NLO+M331 results for the branching ratio of rare hyperon decays

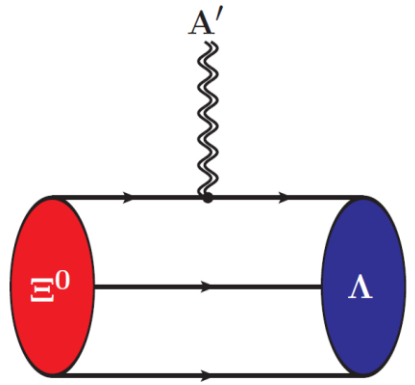
Branching ratio		$\mathcal{B}(\Lambda \to n\nu\bar{\nu})$	$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})$	$\mathcal{B}(\Xi^0 \to \Lambda\nu\bar{\nu})$
μ_c = 1 GeV	LO	2.85×10^{-12}	6.88×10^{-13}	1.06×10^{-12}
	NLO	1.98×10^{-12}	5.01×10^{-13}	7.35×10^{-13}
μ_t = 100 GeV	NLO+SUSY (Set.I)	8.14×10^{-12}	2.06×10^{-12}	3.02×10^{-12}
	NLO+SUSY (Set.II)	3.78×10^{-12}	9.55×10^{-13}	1.40×10^{-12}
	NLO+M331	1.24×10^{-11}	3.13×10^{-12}	4.59×10^{-12}
μ_c = 3 GeV	LO	1.10×10^{-12}	2.65×10^{-13}	4.10×10^{-13}
	NLO	1.20×10^{-12}	3.04×10^{-13}	4.46×10^{-13}
μ_t = 300 GeV	NLO+SUSY (Set.I)	5.85×10^{-12}	1.48×10^{-12}	2.17×10^{-12}
	NLO+SUSY (Set.II)	2.35×10^{-12}	5.94×10^{-13}	8.72×10^{-13}
	NLO+M331	1.02×10^{-11}	2.58×10^{-12}	3.80×10^{-12}
BESIII sensitivity [12]		3×10^{-7}	4×10^{-7}	8×10^{-7}

Chin.Phys.C 43 (2019) 093104

Eur. Phys. J. C 80 (2020) 824

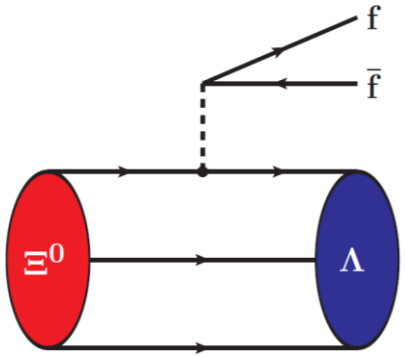
JHEP 04 (2019) 104

Rep. Prog. Phys. 86 016201



Dark photon radiative decay :

$\mathrm{Br}(\Xi^0 \to \Lambda A') < 2.4 \times 10^{-6}$



Semi-invisible decay :

$\mathrm{Br}(\Xi^0 \to \Lambda f \bar{f}) < 9.4 \times 10^{-7}$

New physics model need the form factors as input.

Outline

1. Motivation

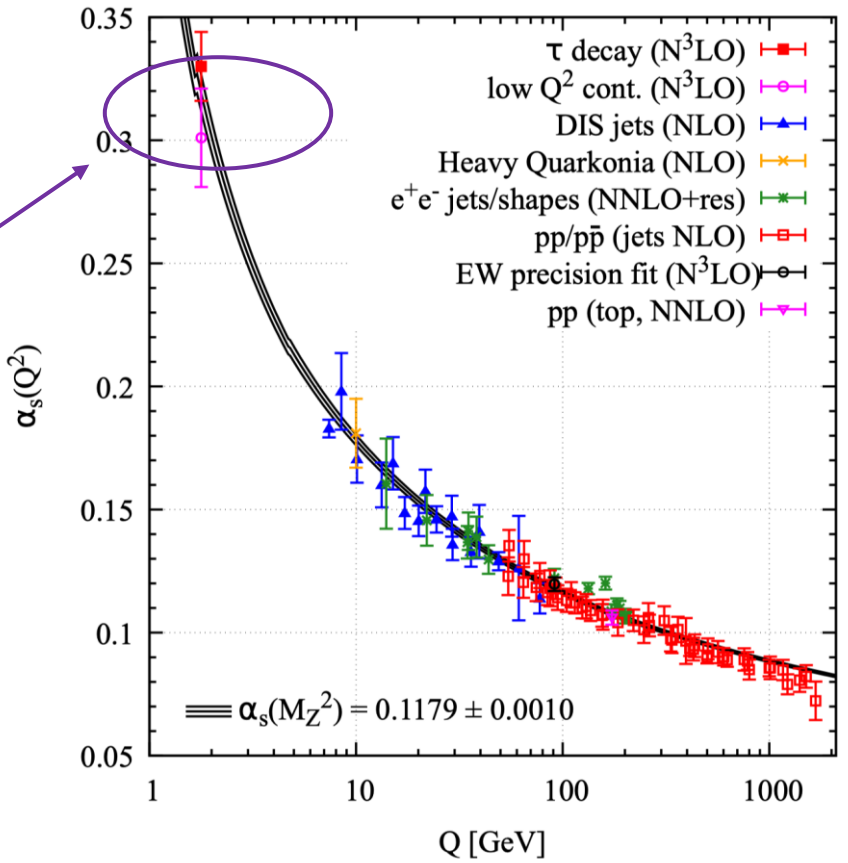
2. Theorical framework

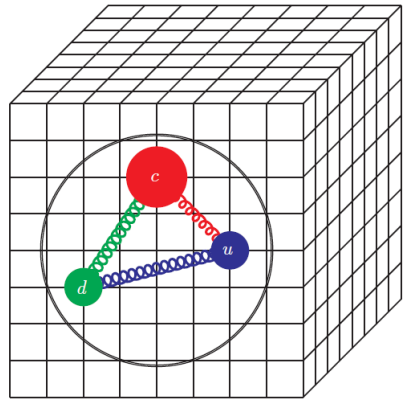
3. Numerical results

4. Summary and outlook

Hyperon decay: non-perturbative QCD

- Around the proton radius scale (~1 GeV), QCD cannot be solved analytically, and the theoretical calculation at this scale is one of the seven Millennium Prize Problems.
- Hyperon decay is just on the region where the transition from perturbative to nonperturbative occurs.



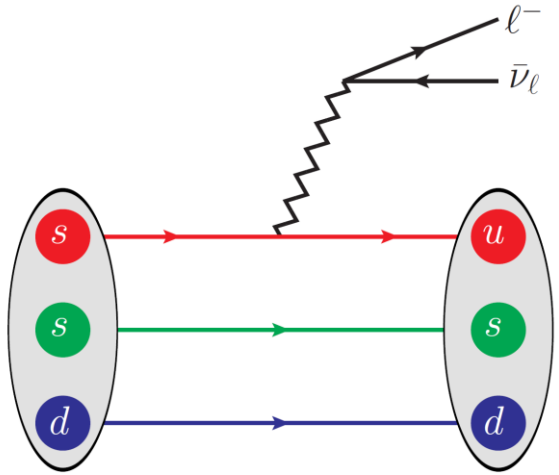




Siyuan @ TDLi

Hyperon decay is suitable for lattice QCD simulations.

Theorical framework



Differential decay width:

$$\frac{d\Gamma(B_1 \to B_2 l \nu_l)}{dq^2} = \frac{G_F^2 |V_{us}|^2 q^2 \sqrt{Q_+ Q_-}}{384\pi^3 M_\Xi^3} L^{\mu\nu} H_{\mu\nu}$$

Hadronic part

$$H_{\mu\nu} = M_\mu^{V-A} M_\nu^{V-A}$$

$$M_\mu^{V-A} = \langle \Lambda(p_f, s_f) | \bar{u}\gamma_\mu(1-\gamma_5)s | \Xi(p_i, s_i) \rangle$$

Matrix element

Weinberg form factors:

$$\langle B_f(p_f, s_f) | \bar{u}\gamma_\mu(1-\gamma_5)s | B_i(p_i, s_i) \rangle$$
$$= \bar{u}_{B_f}(p_f) \left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} q^\nu \frac{f_2(q^2)}{m_i} + q_\mu \frac{f_3(q^2)}{m_i} \right] u_{B_i}(p_i)$$
$$- \bar{u}_{B_f}(p_f) \left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} q^\nu \frac{g_2(q^2)}{m_i} + q_\mu \frac{g_3(q^2)}{m_i} \right] \gamma_5 u_{B_i}(p_i)$$

Axial charge g_A^H : $g_A^H = f_1^{SU(3)} * g_{av}$

Axial-vector coupling: $g_{av} = g_1(0)/f_1(0)$

Vector form factor in SU(3) limit: $f_1^{SU(3)} = -\sqrt{3/2}$

✅ Helicity form factor

$$
\begin{aligned}
&\langle \Lambda(p_f, s_f)|\bar{u}\gamma_\mu s|\Xi^-(p_i, s_i)\\
=&\bar{u}(p_f, s_f)\Bigg[(m_i - m_f)\frac{q^\mu}{q^2}f_0(q^2)\\
&+\frac{m_i + m_f}{s_+}\left(p_i^\mu + p_f^\mu - \frac{q^\mu}{q^2}(m_i^2 - m_f^2)\right)f_+(q^2)\\
&+\left(\gamma^\mu - \frac{2m_f}{s_+}p_i^\mu - \frac{2m_i}{s_+}p_f^\mu\right)f_\perp(q^2)\Bigg]u(p_i, s_i)
\end{aligned}
$$

$$
\begin{aligned}
&\langle \Lambda(p_f, s_f)|\bar{u}\gamma_\mu\gamma_5 s|\Xi^-(p_i, s_i)\\
=&-\bar{u}(p_f, s_f)\Bigg[(m_i + m_f)\frac{q^\mu}{q^2}g_0(q^2)\\
&+\frac{m_i - m_f}{s_-}\left(p_i^\mu + p_f^\mu - \frac{q^\mu}{q^2}(m_i^2 - m_f^2)\right)g_+(q^2)\\
&+\left(\gamma^\mu + \frac{2m_f}{s_-}p_i^\mu - \frac{2m_i}{s_-}p_f^\mu\right)g_\perp(q^2)\Bigg]u(p_i, s_i)
\end{aligned}
$$

Differential decay width：

$$
\begin{aligned}
\frac{d\Gamma}{dq^2} =& \frac{G_F^2|V_{us}|^2\sqrt{s_+s_-}}{384\pi^3 m_\Xi^3}\left(1 - \frac{m_\ell^2}{q^2}\right)^2\\
&\times\Bigg\{2(m_\ell^2 + 2q^2)(s_+g_\perp^2 + s_-f_\perp^2)\\
&+\frac{m_\ell^2 + 2q^2}{q^2}(s_+[(m_\Xi - m_\Lambda)g_+)]^2\\
&+s_-[(m_\Xi + m_\Lambda)f_+]^2)\\
&+\frac{3m_\ell^2}{q^2}(s_+[(m_\Xi - m_\Lambda)f_0]^2 + s_-[(m_\Xi + m_\Lambda)g_0]^2)\Bigg\}.
\end{aligned}
$$

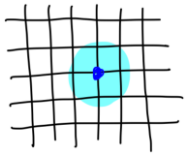
Differential decay width expressed as helicity form factor.

Correlation function in Lattice QCD

Interpolator current of the baryons:

$$O_{\Xi^-} = \epsilon_{ijk}(d_i^T C\gamma_5 s_j)P_+ s_k,$$

$$O_\Lambda = \epsilon_{ijk}(u_i^T C\gamma_5 d_j)s_k,$$



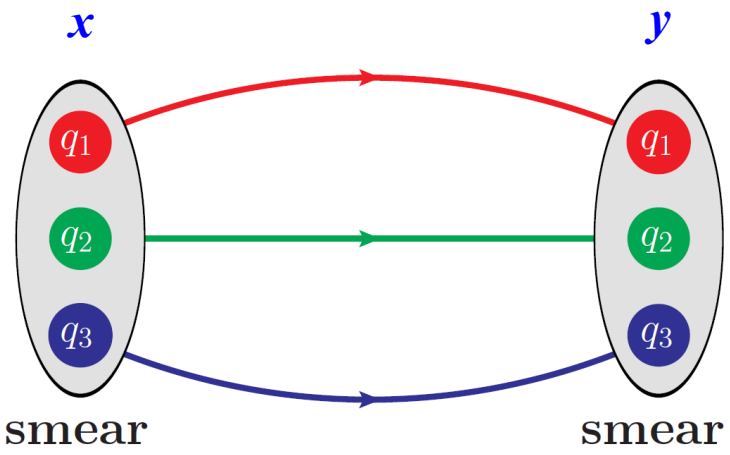
smear

Weak current:

$$J_\Gamma = \bar{u}\gamma_\mu(1-\gamma_5)s$$

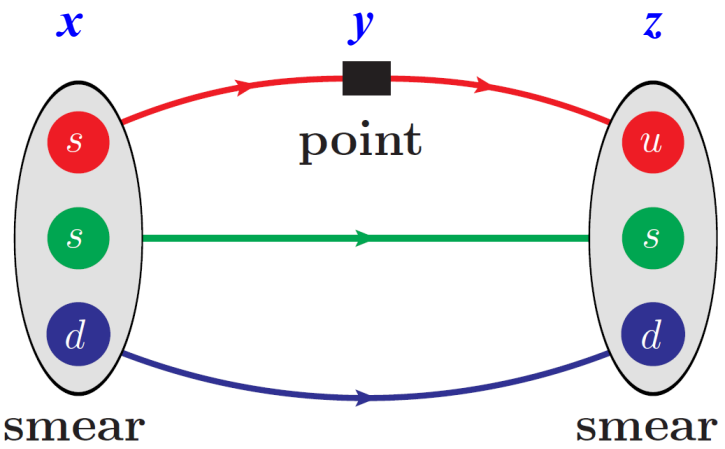
point

Two point correlation functions:



$$C_{2pt}(t) = \sum_x e^{-ip\cdot x}\,\mathrm{Tr}\{\langle 0|O_{\mathcal{B}}(x), \bar{O}_{\mathcal{B}}(0)|0\rangle T\}$$

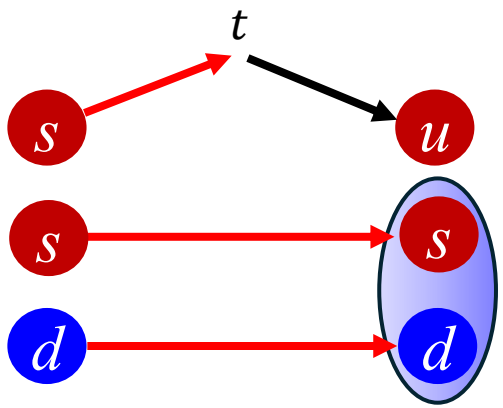
Three point correlation functions:



$$C^{3\mathrm{pt}}(t, t_{seq}, \vec{p}_f, \Gamma, T) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{-i\vec{p}_i\cdot\vec{y}}$$

$$\times\,\mathrm{Tr}\{\langle O_\Lambda(t_{seq}, \vec{y}), J_\Gamma(t, \vec{x}), O_\Xi(0)\rangle T\}$$

Sequential source method:



Sequential time: t_{seq}

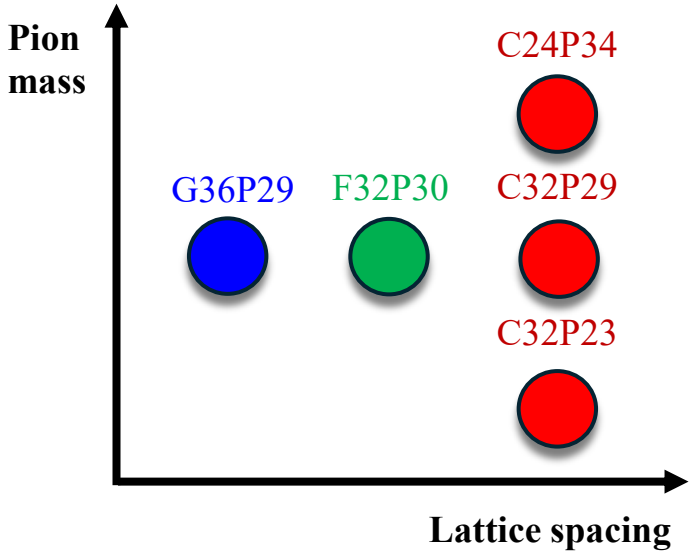
Lattice set-up

Ensembles from CLQCD Collaboration：

Lattice spacing： [0.069, 0.1052] fm

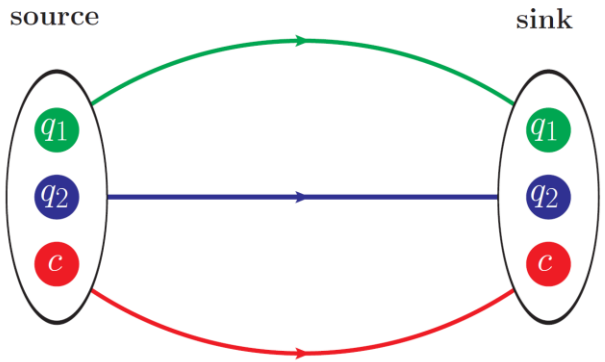
Pion mass： [227.9, 340.2] MeV

Ensemble	$L^3 \times T$	a (fm)	smear size	π (MeV)	Nconf	Nsrc
C24P34	$24^3 \times 64$	0.105	4 30	340	300	34
C32P29	$32^3 \times 64$	0.105	4 30	290	984	4
C32P23	$32^3 \times 64$	0.105	4 30	230	954	5
F32P30	$32^3 \times 96$	0.077	5 50	300	777	4
G36P29	$36^3 \times 108$	0.069	5 50	290	656	4

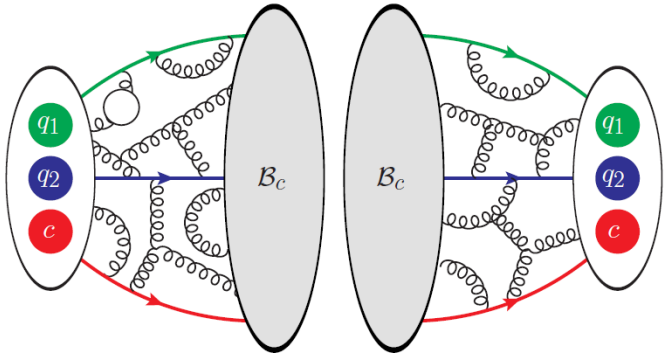


Extrapolation: Continuum limit and physical pion mass.

Two point correlation function



$$C_{2pt}(t) = \sum_x e^{-ip\cdot x}\,\mathrm{Tr}\{\langle 0|O_{\mathcal{B}}(x), \bar{O}_{\mathcal{B}}(0)|0\rangle T\}$$



$$C_{2pt}(t) = \boldsymbol{Z_B^{(0)2}} e^{-\boldsymbol{m_B} t}(1 + \Delta C \cdot e^{\Delta m \cdot t})$$

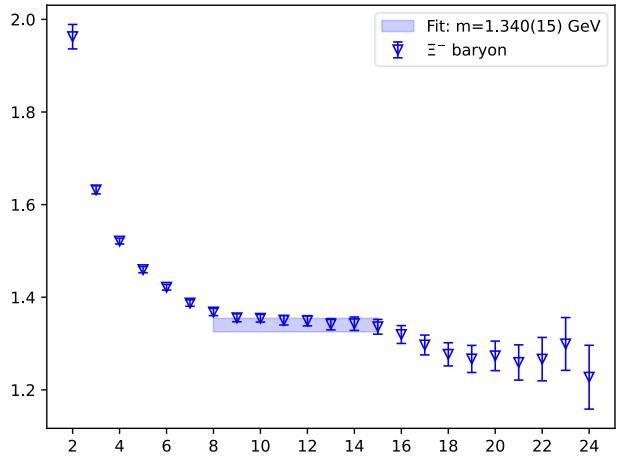
Decay constant Effective mass

$$\mathbb{1} = \frac{1}{V}\sum_n \frac{1}{2E_n}|n\rangle\langle n|$$

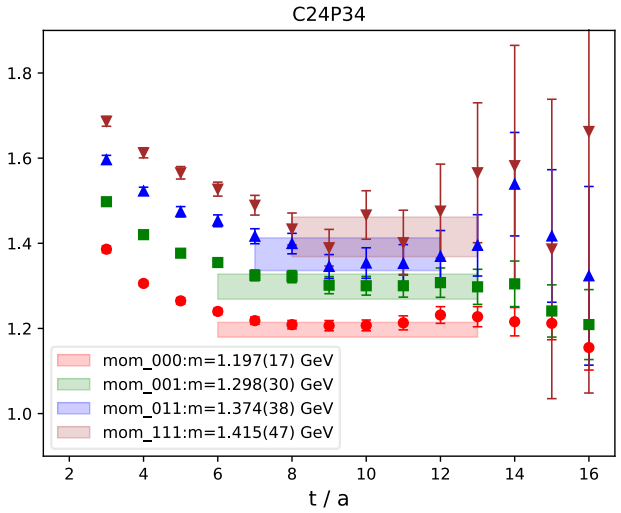
Complete set of hadronic states

Phys. Rev. D 113 054509

Ensembles: C24P34

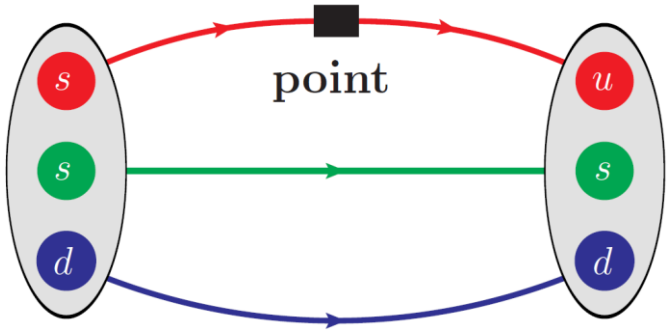


Ξ^- - baryon effective mass



Λ - baryon effective energy

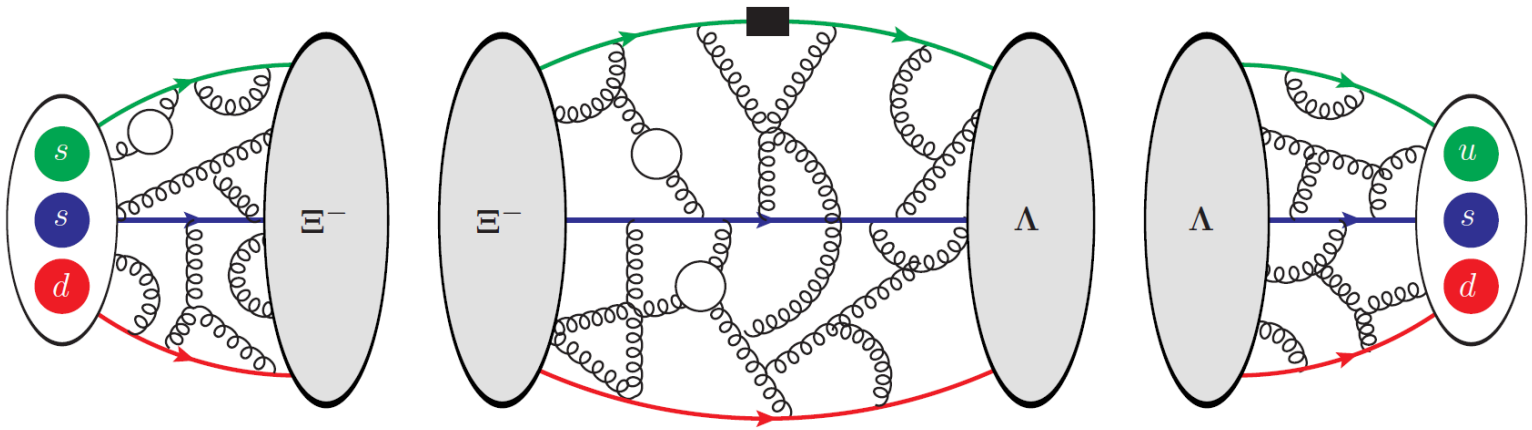
Three point correlation functions



$$C^{3\mathrm{pt}}(t, t_{seq}, \vec{p}_f, \Gamma, T) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_f \cdot \vec{x}} e^{-i\vec{p}_i \cdot \vec{y}} \mathrm{Tr}\{\langle O_\Lambda(t_{seq}, \vec{y}), J_\Gamma(t, \vec{x}), O_\Xi(0)\rangle T\}$$

$$\mathbb{1} = \frac{1}{V}\sum_n \frac{1}{2E_n}|n\rangle\langle n|$$

Complete set of hadronic states



Decay constant **Matrix element** **Decay constant**

Ratio of 3pt/2pt:

$$R(\Gamma, T, \vec{p}_f, t, t_{seq}) = \sqrt{\frac{C^{3\mathrm{pt}}(t, t_{seq}, \vec{p}_f, \Gamma, T) C^{3\mathrm{pt}}(t, t_{seq} - t, \vec{p}_f, \Gamma, T)}{C^{2\mathrm{pt}}_\Xi(t_{seq}, \vec{p}_i) C^{2\mathrm{pt}}_\Lambda(t_{seq}, \vec{p}_f)}}$$

Reduction formula

Reduction formula:

ground state + first excited state

$$R(\Gamma, T, \vec{p}_f, t, t_{seq}) = \frac{F(\Gamma, T, \vec{p}_f)}{8\sqrt{E_i E_f m_i m_f}} \sqrt{\frac{1 + D(e^{-\Delta E t} + e^{\Delta E (t - t_{seq})})}{1 + A_f e^{-\Delta E t_{seq}}}}$$

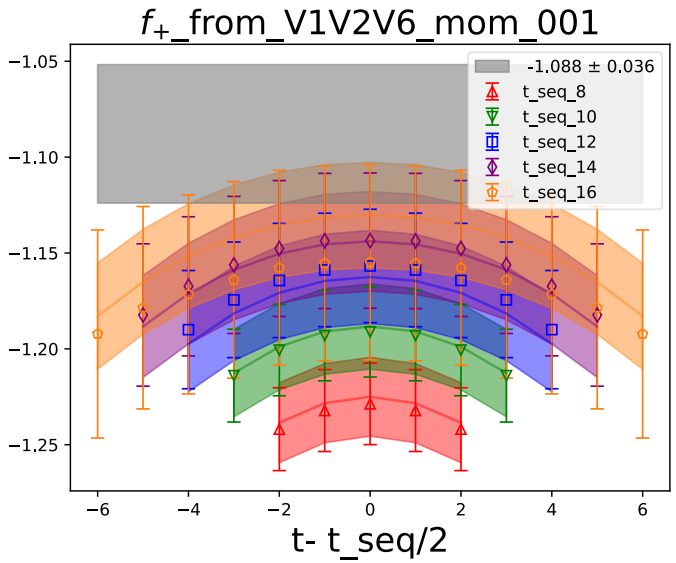
where $F(\Gamma, T, \vec{p}_f) = V_i \text{ or } A_i$

mom=(0,0,1)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$
$T = 1$	0	0	V_1	V_2
$T = \gamma^x$	V_3	0	0	0
$T = \gamma^y$	0	V_3	0	0
$T = \gamma^x\gamma^y$	0	0	0	0
$T = \gamma^z$	0	0	V_4	V_5
$T = \gamma^x\gamma^z$	V_6	0	0	0
$T = \gamma^y\gamma^z$	0	V_6	0	0
$T = \gamma^5\gamma^t$	0	0	0	0

Linear combination of helicity form factors

$$V_i = af_+(q^2) + bf_\perp(q^2) + cf_0(q^2), \qquad i = 1\sim6$$

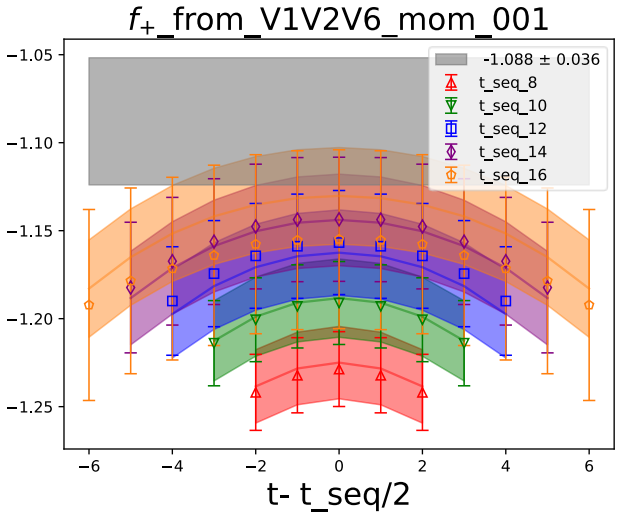
$$A_i = ag_+(q^2) + bg_\perp(q^2) + cg_0(q^2), \qquad i = 1\sim6$$



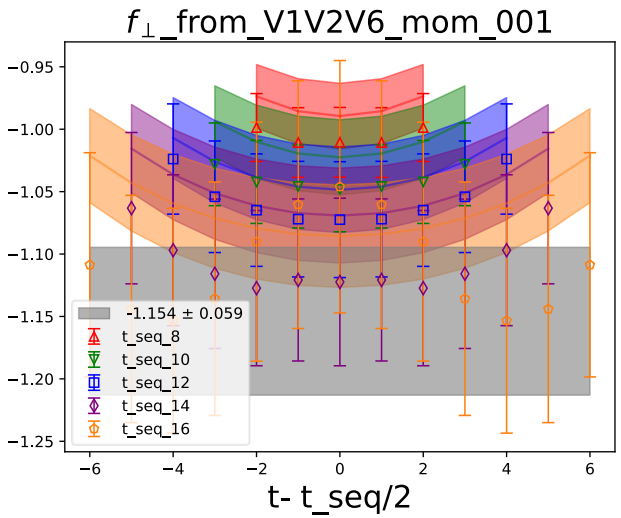
Extrapolation to sequential time

C32P23 : (0, 0, 1)

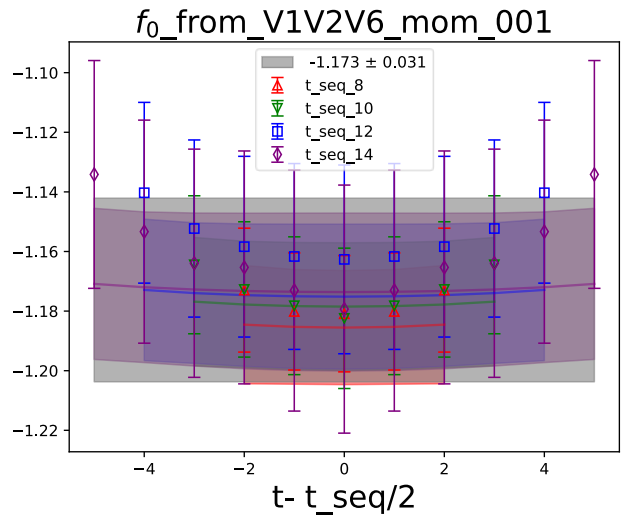
f_+



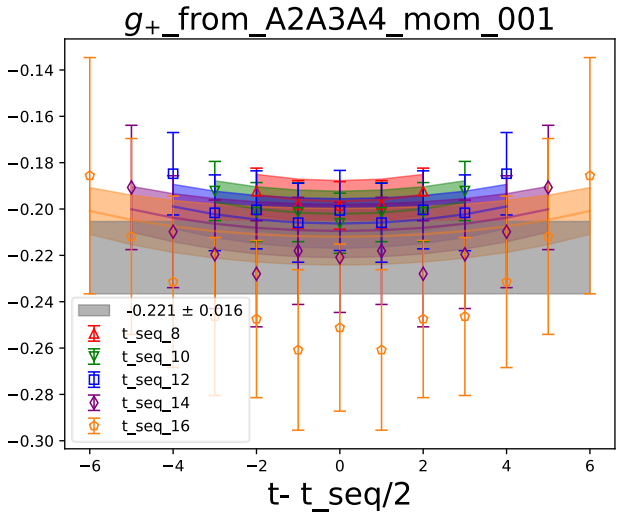
$f_\perp$



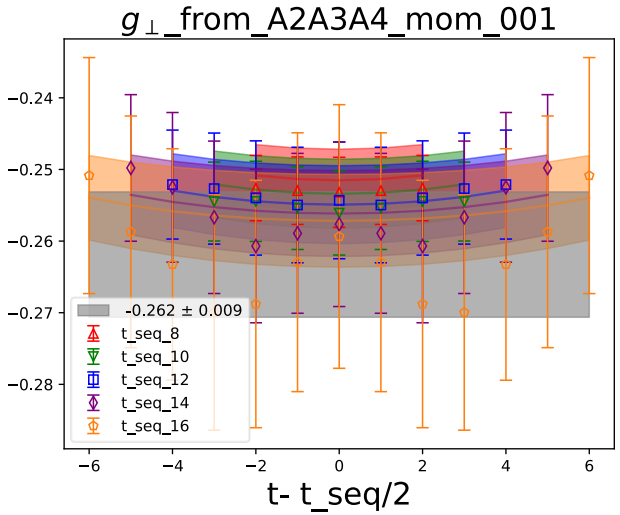
f_0



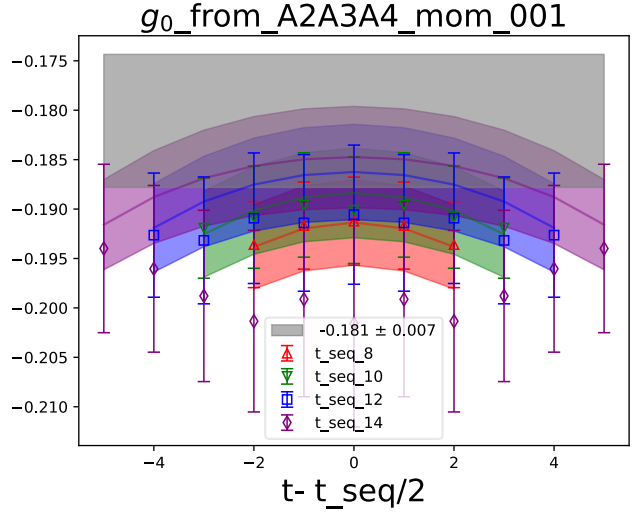
g_+



$g_\perp$



g_0



Renormalization

Bare lattice currents

$$V_\mu^{\rm lat} = \bar{u}\gamma_\mu s, \quad A_\mu^{\rm lat} = \bar{u}\gamma_\mu\gamma_5 s$$

$$V_\mu^R = Z_V^{us} V_\mu^{\rm lat}, A_\mu^R = Z_A^{us} A_\mu^{\rm lat}$$

Renormalization of form factors

$$f_i^R(q^2) = Z_V^{us} f_i^{\rm bare}(q^2), g_i^R(q^2) = Z_A^{us} g_i^{\rm bare}(q^2)$$

For the flavor-changing current

$J = \bar{u}\Gamma_{V/A}$s, we use

$$Z_V^{us} = \sqrt{Z_V Z_V^s}, Z_A^{us} = \sqrt{Z_V Z_V^s}\,\frac{Z_A}{Z_V}$$

Phys. Rev. D 111, 054504 (2025)

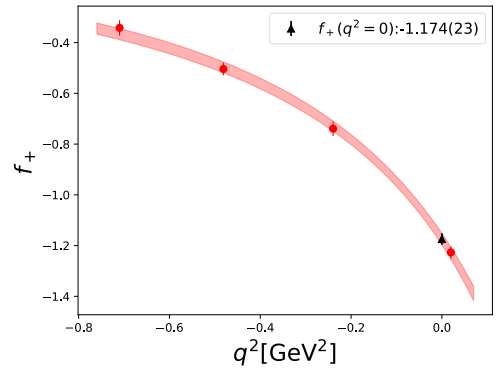
TABLE VI. The renormalization constants on each ensemble.

a (fm)	Symbol	Z_V	Z_V^s (optional)	Z_V^c	Z_A/Z_V
0.10524(05)(62)	C24P34	0.79676(32)	0.85034(10)	1.57130(20)	1.07677(72)
	C24P29	0.79814(23)	0.85184(06)	1.57353(18)	1.07244(70)
	C32P29	0.79810(13)	0.85167(04)	1.57163(14)	1.07648(63)
	C32P23	0.79957(13)	0.85350(04)	1.57644(12)	1.07375(40)
	C48P23	0.79954(05)	0.85339(02)	1.57326(08)	1.07317(70)
	C48P14	0.79957(06)	0.85359(02)	1.57415(08)	1.07320(68)
0.08973(20)(53)	E28P35	0.81768(04)	0.85877(02)	1.40361(05)	1.06029(53)
0.07753(03)(45)	F32P30	0.83548(12)	0.86900(03)	1.30566(07)	1.05549(54)
	F48P30	0.83511(04)	0.86880(02)	1.30451(06)	1.05687(66)
	F32P21	0.83579(09)	0.87031(03)	1.30782(08)	1.05416(50)
	F48P21	0.83567(05)	0.86880(02)	1.30673(04)	1.05434(88)
0.06887(12)(41)	G36P29	0.84636(09)	0.87473(05)	1.23990(13)	1.04500(22)
0.05199(08)(31)	H48P32	0.86855(04)	0.88780(01)	1.12882(11)	1.03802(28)

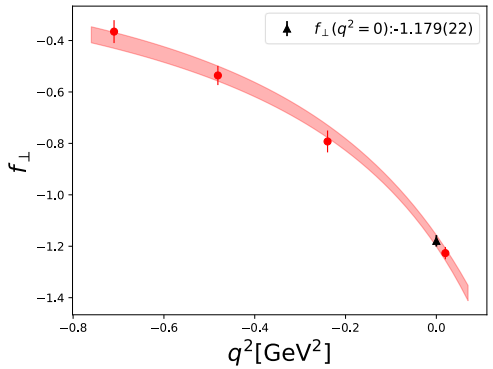
q^2 dependenc : z-series expansion

C24P34

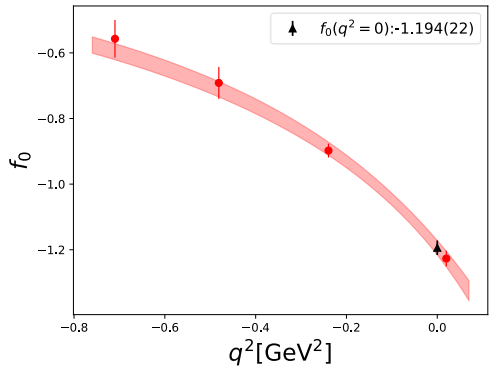
f_+ $m_{pole} = m_{K^*} = 0.892$ GeV



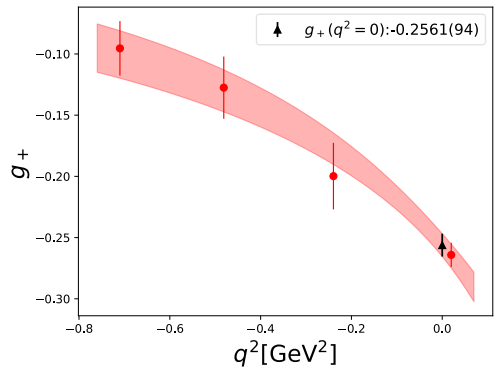
$f_\perp$ $m_{pole} = m_{K^*} = 0.892$ GeV



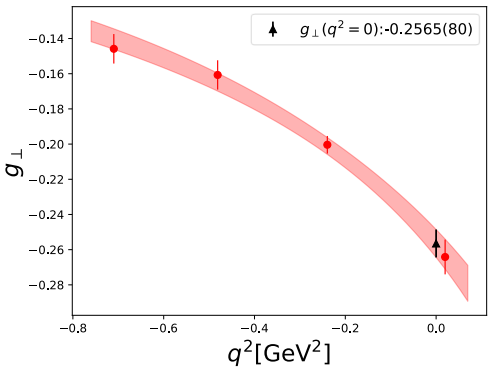
f_0 $m_{pole} = m_{K_0^*} = 0.838$ GeV



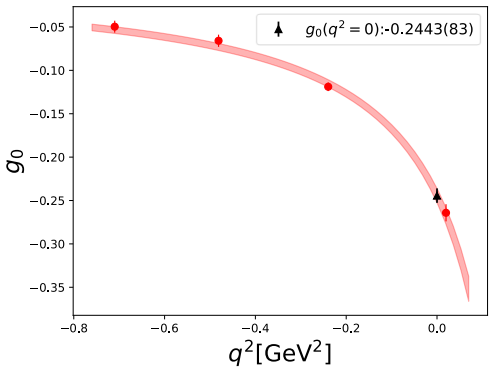
g_+ $m_{pole} = m_{K_1} = 1.27$ GeV



$g_\perp$ $m_{pole} = m_{K_1} = 1.27$ GeV



g_0 $m_{pole} = m_{K^\pm} = 0.494$ GeV

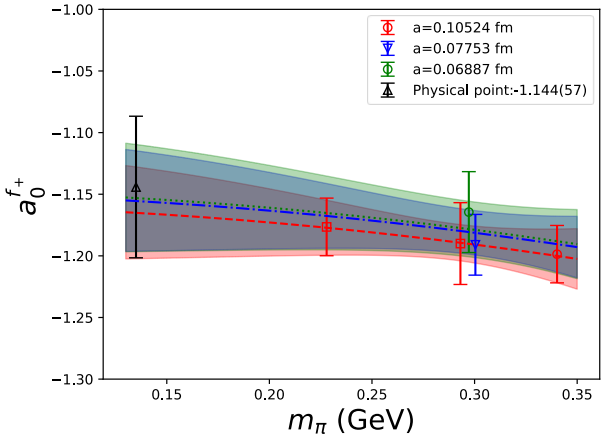


z-series expansion: $$f(q^2) = \frac{1}{1 - q^2/(m_{\rm pole})^2} \sum_{n=0}^{n_{\rm max}} a_n^f [z(q^2)]^n$$

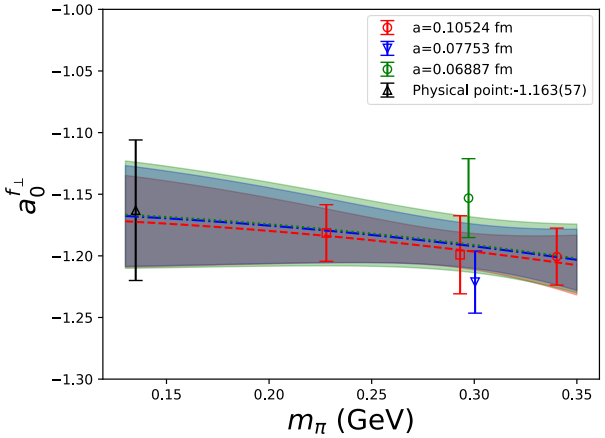
Phys. Rev. D 103, 015033 (2021)

Chiral and continuum limit

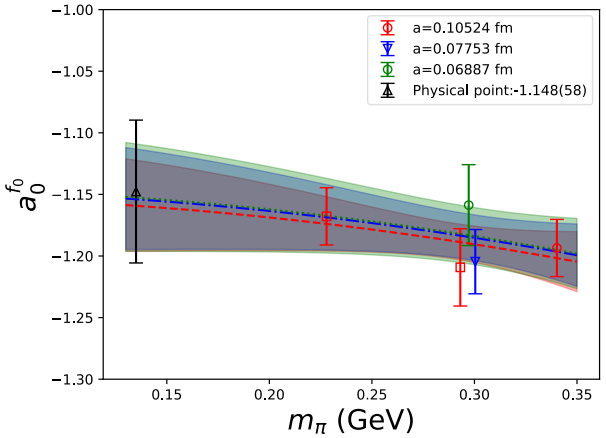
$f_+ : a_0$



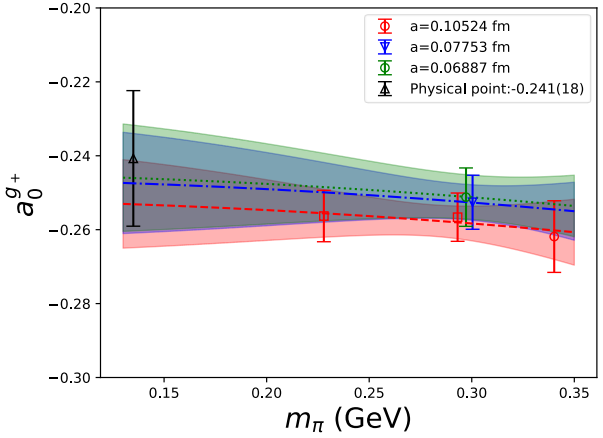
$f_\perp : a_0$



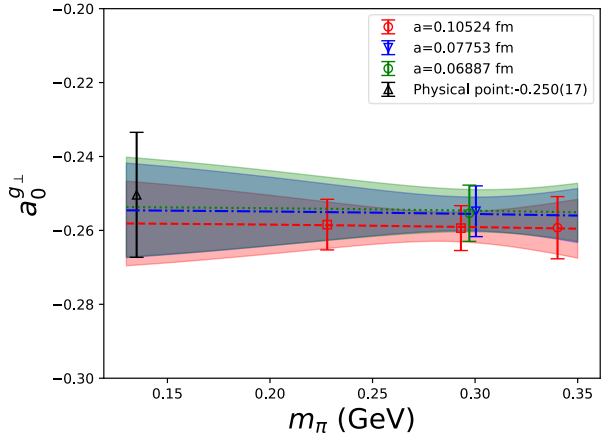
$f_0 : a_0$



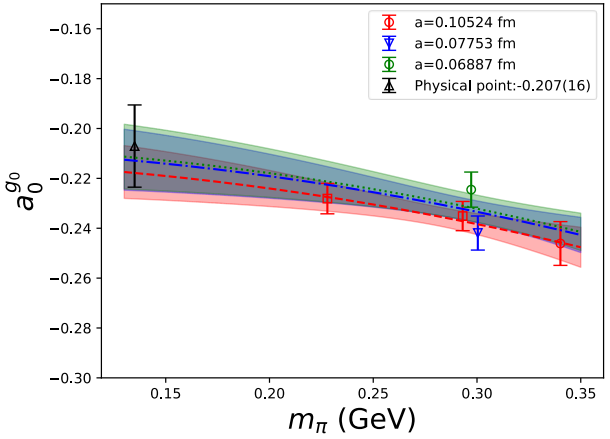
$g_+ : a_0$



$g_\perp : a_0$

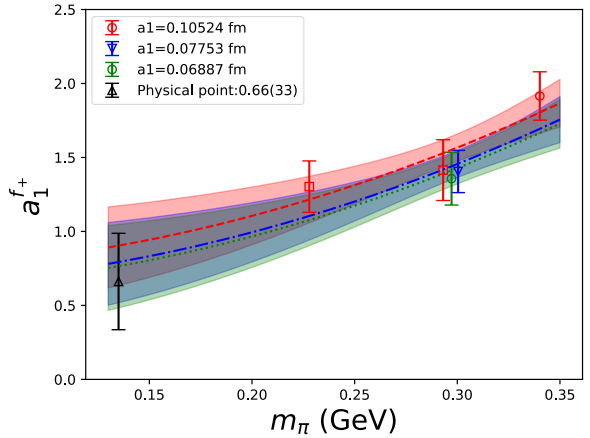


$g_0 : a_0$

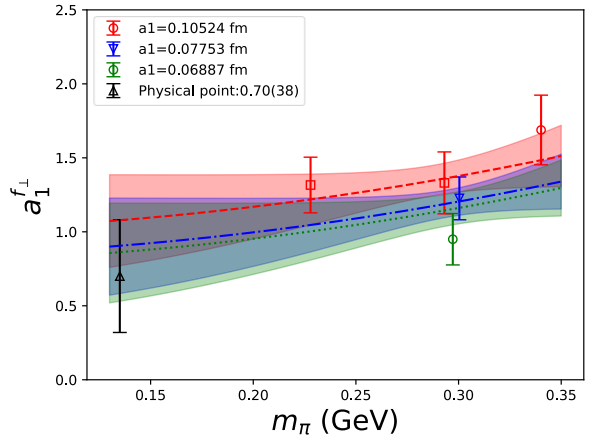


Extrapolation formula $a_i = a_{i,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2, \quad i = 0, 1$

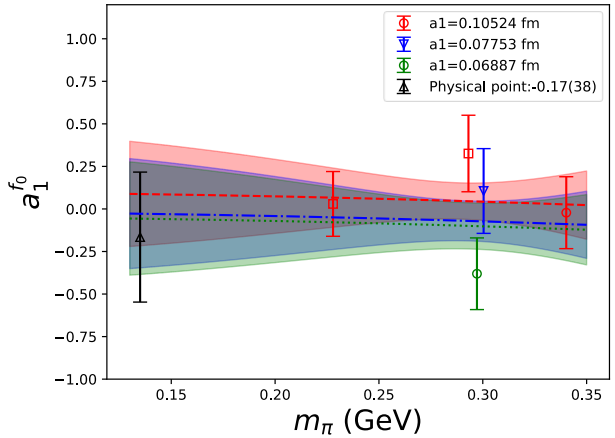
$f_+ : a_1$



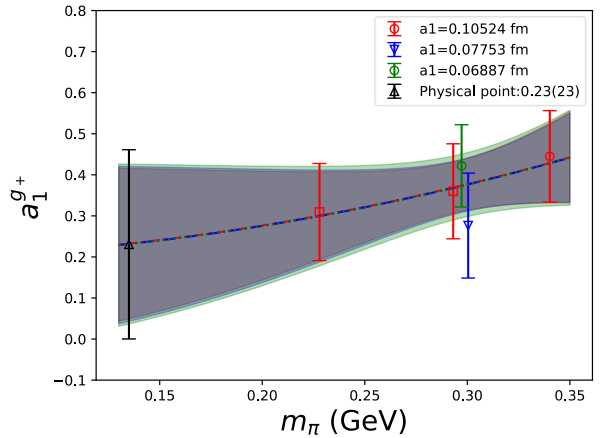
$f_\perp : a_1$



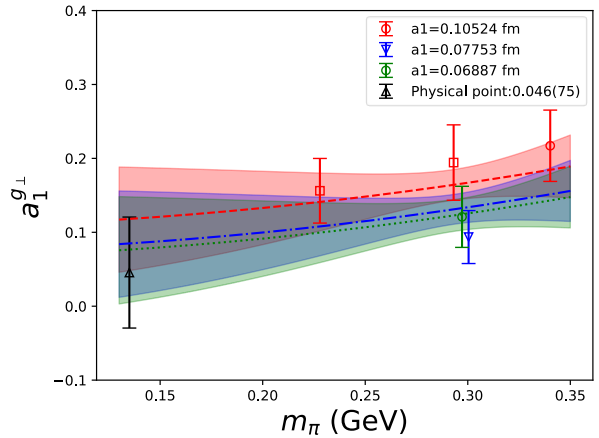
$f_0 : a_1$



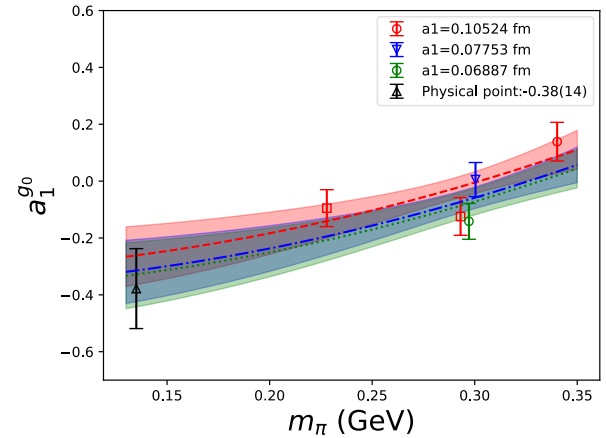
$g_+ : a_1$



$g_\perp : a_1$



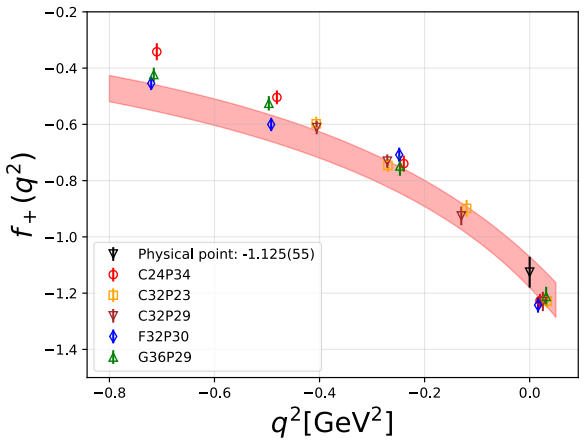
$g_0 : a_1$



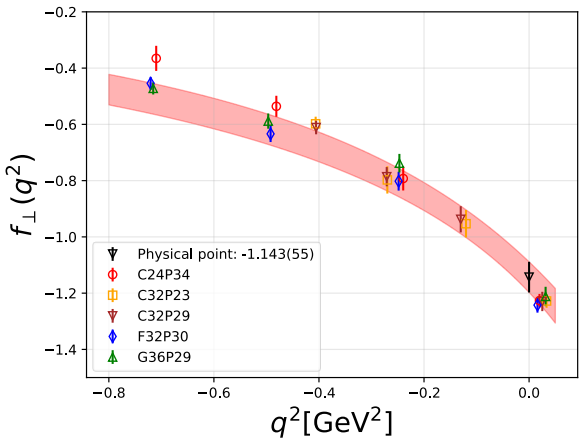
Extrapolation formula $a_i = a_{i,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2, \quad i = 0, 1$

Form factors at the physical point

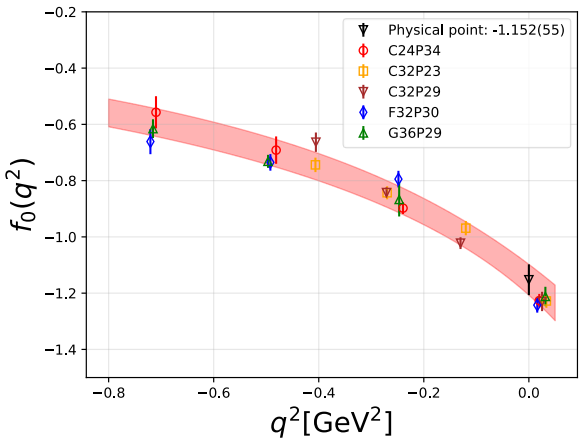
f_+



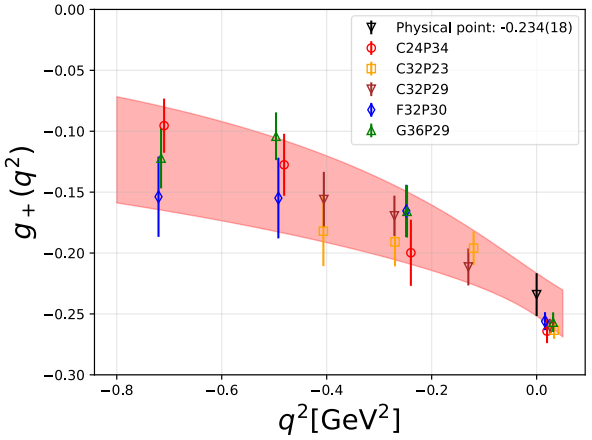
$f_\perp$



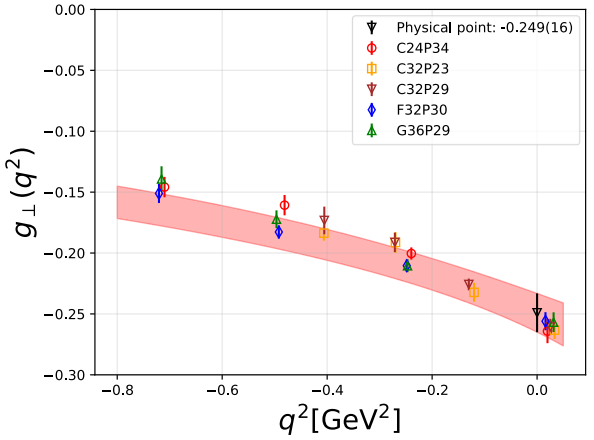
f_0



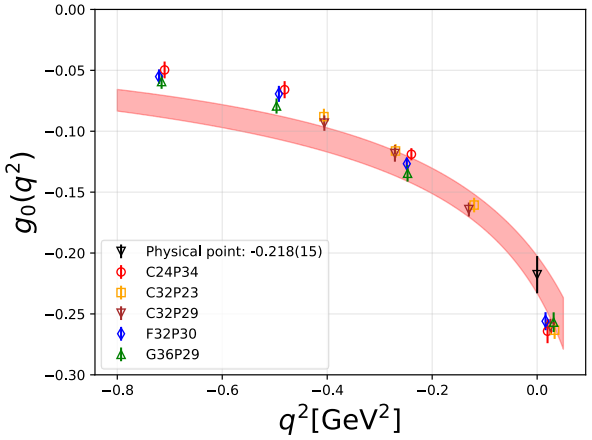
g_+



$g_\perp$



g_0

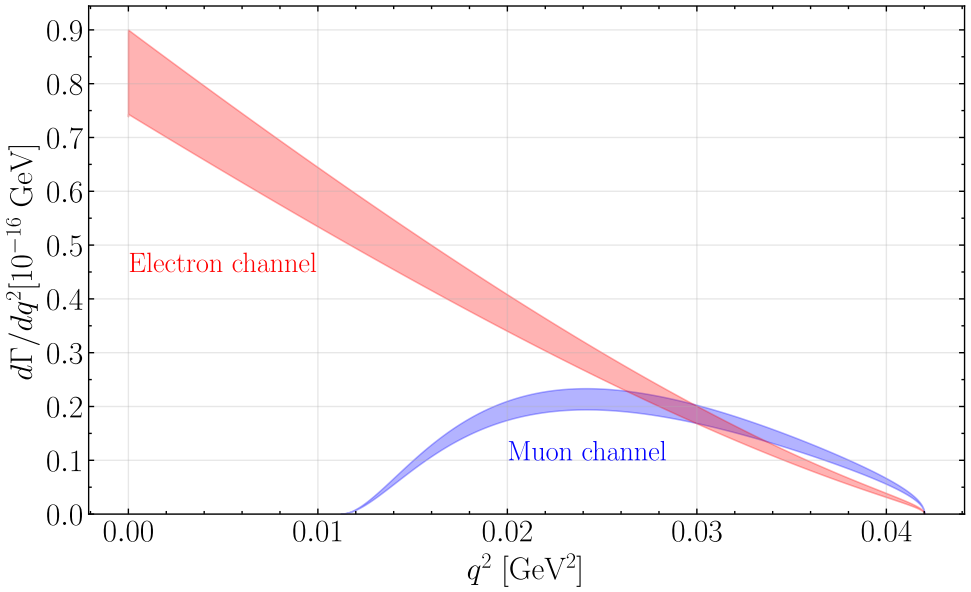


Outline

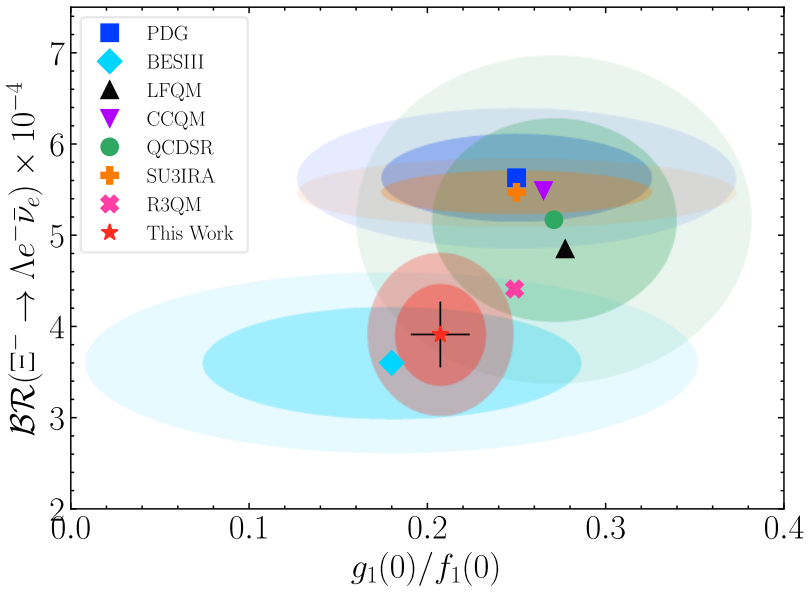
1. Motivation
2. Theorical framework
3. Numerical results
4. Summary

Numerical results

Differential decay width



Branching ratio

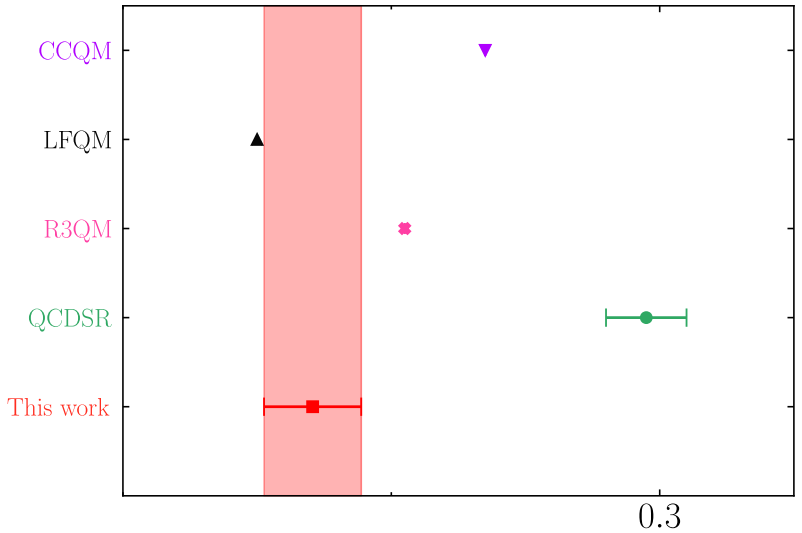


	This work	BESIII	PDG
$\mathbf{Br}(\Xi \to \Lambda e^- \bar{\nu}_e) \times 10^4$	$\mathbf{3.91(36)_{stat}}$	$\mathbf{3.60(40)_{stat}(10)_{syst}}$	$\mathbf{5.63\ (31)_{total}}$

Electron-channel branching fraction: within 1σ agree with BESIII, but more than 2σ with PDG.

Numerical results

Lepton-flavor ratio

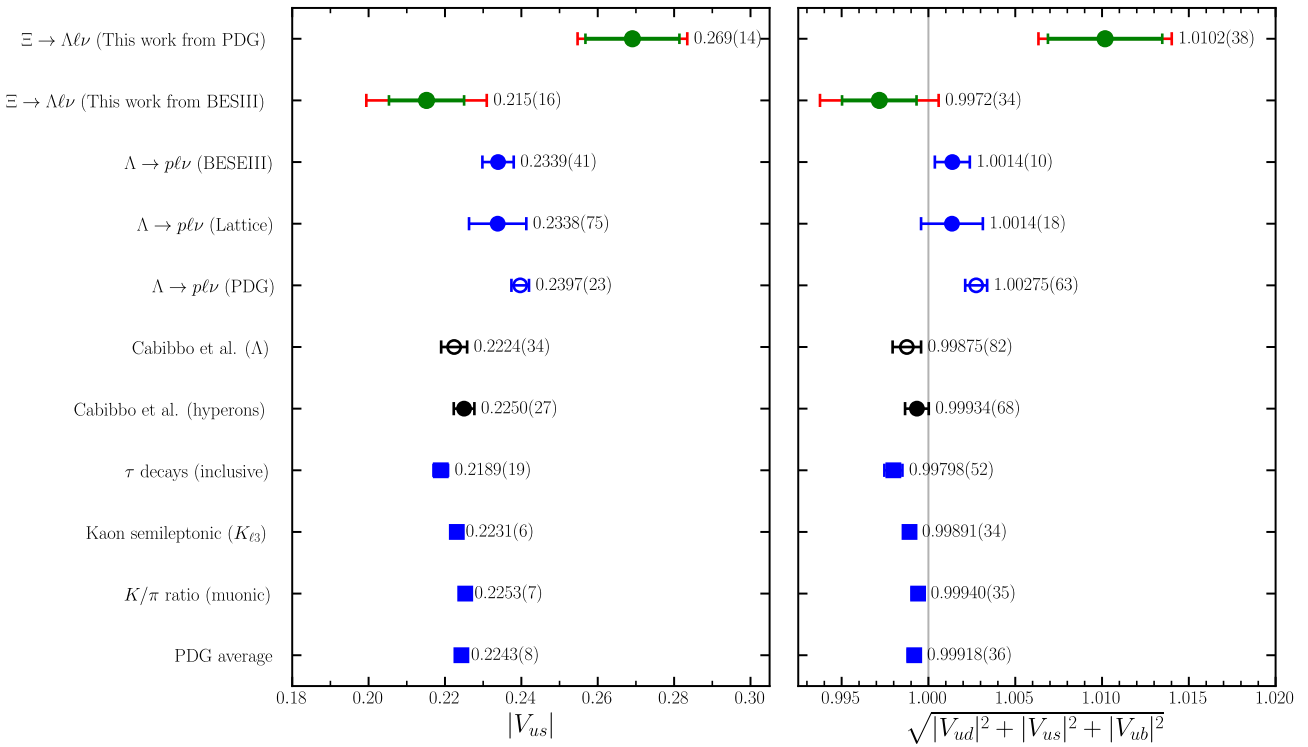


$$\mathcal{R}^{\mu e} = \mathcal{B}_{\Xi\to\Lambda\mu\nu_\mu}/\mathcal{B}_{\Xi\to\Lambda e\nu_e}$$

$$\mathcal{R} = \frac{\mathrm{Br}(\Xi^- \to \Lambda\mu^-\bar{\nu}_\mu)}{\mathrm{Br}(\Xi^- \to \Lambda e^-\bar{\nu}_e)} = 0.2741(36)$$

CKM matrix element |Vus|

Unitarity



CKM matrix unitarity: within 1σ of BESIII, but more than 2σ from PDG.

Summary

1. We calculated helicity form factors of $\Xi \to \Lambda \ell \bar{\nu}_\ell$ in Lattice QCD
2. Performed chiral and continuum limit extrapolations
3. Obtained the preliminary results of branching ratio and g_{av}

Outlook

- Other processes like $\Xi_c \to \Xi \ell \bar{\nu}_\ell \ldots$
- Apply 3pt to other baryon observables

Thanks for your attention!