Soft Contributions Stabilize NNLO QCD Corrections to Quarkonium Production and Decay

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Outline

- Introduction
  - Perturbative expansion
  - instabilities in quarkonium production and decay
- Soft contributions and the SIM scheme
- Phenomenological results
- Summary and outlook

Introduction to perturbative expansion

- Perturbative calculation in QCD at hadron colliders:

$$N(P_1) + N(P_2) \to \sum_{i=3}^{n} h_i(p_i, m_i) + X(p_X)$$

- QCD factorization theorem: John C. Collins, *et al*. (1989) , arXiv: hep-ph/0409313

$$p_1 = x_1 P_1, \quad p_2 = x_2 P_2$$

$$d\sigma = \sum_{ij} \int dx_1 dx_2 f_{i/N}(x_1) f_{j/N}(x_2) d\hat{\sigma}_{ij}(p_k, m_k)$$

$f(x)$: parton distribution function (PDF), non-perturbative

$\hat{\sigma}$: partonic cross section, perturbative

- Fixed-order expansion:

$$\hat{\sigma} = \hat{\sigma}^{(0)} \left(1 + \alpha_s \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \cdots\right)$$

Introduction to perturbative expansion

- Obtaining PDFs
  - PDF sets from LHAPDF, *e.g.*:

331100	NNPDF40_nnlo_as_01180	(tarball)	(info file)
331700	NNPDF40_nlo_as_01180	(tarball)	(info file)
331900	NNPDF40_lo_as_01180	(tarball)	(info file)

$$d\sigma = \sum_{ij} \int dx_1 dx_2 f_{i/N}(x_1) f_{j/N}(x_2) d\hat{\sigma}_{ij}(p_k, m_k)$$

σ: experimental measurement without any expansion in α_s

$$\hat{\sigma} = \hat{\sigma}^{(0)} \left(1 + \alpha_s \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \cdots\right)$$

$i = 0$: LO

$i = 1$: NLO

$i = 2$: NNLO

  - PDFs extraction: $\hat{\sigma}^{(i)} \leftrightarrow f^{(i)}(x)$

- Fixed order calculation:
  - The perturbative accuracy of the hard-scattering calculation should be consistent with that of the PDF set.

Introduction to perturbative expansion

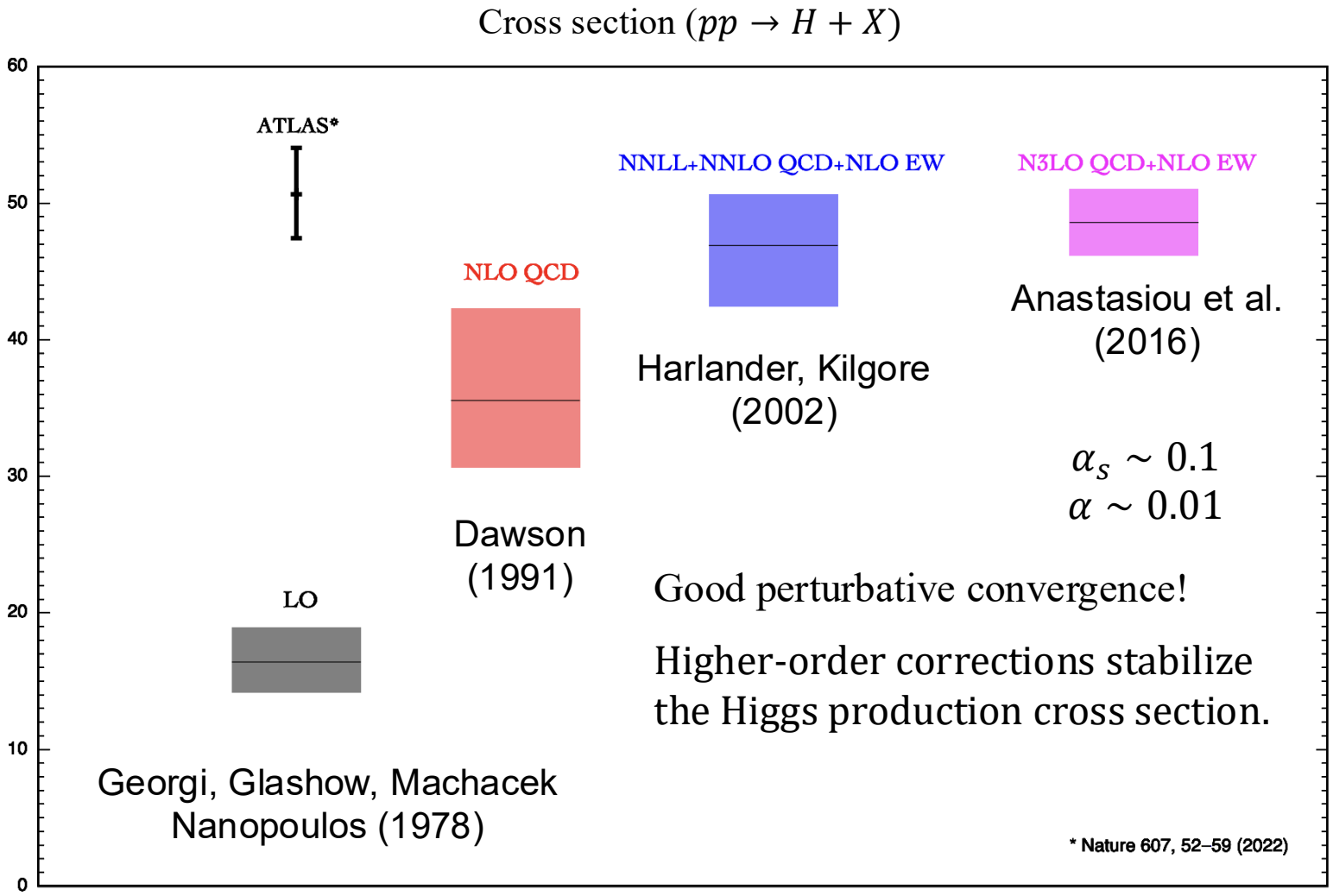


Figure from M. Grazzini's talk (2022).

Introduction to quarkonium physics

- Quarkonium: bound state of a heavy quark-antiquark pair

$$\mathcal{Q} = Q\bar{Q}\,, \qquad Q = c,\, b$$

  - Heavy quark mass ⟹ relative velocity v is nonrelativistic

$$v_c^2 \sim 0.3\,,\ v_b^2 \sim 0.1$$

  - Non-relativistic QCD (NRQCD) factorization:

$$\mathcal{A} = \sum_n H[n](\mu_R, \mu_\Lambda) \times \langle \mathcal{O}[n] \rangle (\mu_s, \mu_\Lambda)$$

μ_R: renormalization scale
μ_Λ: NRQCD factorization scale
μ_s: soft scale

    - observable $\mathcal{A}$: cross section or decay width.
    - $H[n]$: the short-distance coefficient (SDC), perturbative
    - $\langle \mathcal{O}[n] \rangle$: the long-distance matrix element (LDME), non-perturbative

  - LDME extraction: $H[n]^{(i)} \leftrightarrow \langle \mathcal{O}[n] \rangle^{(i)}$

Introduction to quarkonium physics

- NRQCD exploits v as an additional expansion parameter:
  - For a Fock-state decomposition:

$$|J/\psi\rangle = \underbrace{\left|{}^3S_1^{[1]}\right\rangle}_{P\sim\mathcal{O}(v^0)} + \underbrace{\left|{}^3P_J^{[8]}g\right\rangle}_{P\sim\mathcal{O}(v^2)} + \underbrace{\left|{}^1S_0^{[8]}g\right\rangle}_{P\sim\mathcal{O}(v^3)} + \underbrace{\left|{}^3S_1^{[8]}gg\right\rangle}_{P\sim\mathcal{O}(v^4)} + \cdots .$$

Bodwin, Braaten, Lepage, PRD 51 (1994)

Spectroscopic notation: $n = {}^{2s+1}L_J^{[c]}$

S=spin, L= orbital angular mom., J= total angular mom., c=color (1 or 8)

- Negative rates:

$$\frac{\Gamma_{\rm NNLO}(\eta_c \to \gamma\gamma)}{\Gamma_{\rm LO}(\eta_c \to \gamma\gamma)} = 1 - 1.07\alpha_s - 11.16\alpha_s^2$$

Feng, Jia, Sang, PRL 115 (2015)
Abreu, Becchetti, Duhr, Ozcelik, JHEP 02 (2023)

$$\mu_R = m_Q$$

  - Large NNLO K factor: poor convergence
  - $\alpha_s(m_c) \sim 0.35$: negative NNLO decay width.

$$\frac{\Gamma_{\rm NNLO}(\eta_c \to \gamma\gamma)}{\Gamma_{\rm LO}(\eta_c \to \gamma\gamma)} = -0.74$$

→ Unstable and unphysical LDME

Introduction to quarkonium physics

- Negative rates (more examples):

Abreu, Becchetti, Duhr, Ozcelik, JHEP 02 (2023)
Feng, Jia, Mo, Pan, Sang, Zhang, arXiv: 2207.14259
Zeng, Gong, Wang, Niu, Huang, Li, arXiv: 2603.26199
Sang, Feng, Jia, Mo, Pan, Zhang, PRL 131 (2023)

Observable	**Data**	**LO**	**NLO**	**NNLO$_{\overline{\rm MS}}$**
$\Gamma(\eta_c \to \gamma\gamma)$ **[keV]**	6.4 ± 0.6	9.3	6.8	-0.5
$\Gamma(J/\psi \to e^+e^-)$ **[keV]**	5.5 ± 0.1	10.0	5.7	-0.5
$\Gamma(J/\psi \to \gamma\gamma\gamma)$ **[eV]**	1.1 ± 0.2	5.3	-0.1	-2.3
$\sigma(e^+e^- \to J/\psi J/\psi)$ **[fb]**	< 9.1	7.5	2.2	-3.6

Decay data from PDG2026
Upper limit for production from
Belle: PRD 70 (2004)

$\mu_R = 2m_Q, \sqrt{s}/2$

- Large (N)NLO K factor: poor convergence
- negative (N)NLO rates.

→ Unstable and unphysical LDME

- How to improve the perturbative calculation?

The SIM scheme

- The simpler NRQED:
  - Perturbative.
  - Large cancellations from LDME.

Czarnecki, Melnikov, Yelkhovsky, PRA 61 (2000)
Abreu, Becchetti, Duhr, Ozcelik, JHEP 09 (2022)

$$\frac{\Gamma_{\rm NNLO}}{\Gamma_{\rm LO}} = 1 + c_{\rm NLO}\alpha_s + \left(c_{\rm NNLO}^{\rm SDC} + c_{\rm NNLO}^{\rm soft}\right)\alpha_s^2$$

$c_{\rm NNLO}^{\rm SDC}$: NNLO contributions from SCD, large and negative

$c_{\rm NNLO}^{\rm soft}$: NNLO soft contributions to LDME, large and positive

$$c_{\rm NNLO} \sim \mathcal{O}(1)$$

- In the weak-coupling approach such a cancellation is also observed.

Beneke, Kiyo, Marquard, et.al., PRL 112 (2014)

  - Valid for many bottomonium processes, while limited for charmonium.

- In NRQCD, $\overline{\rm MS}$ scheme, only UV poles are subtracted: $\langle \mathcal{O}_{\rm bare}[n]\rangle = \langle \mathcal{O}_{\overline{\rm MS}}[n]\rangle Z_{\overline{\rm MS}}$
  - The finite soft contributions are not explicitly included in matching coefficients: $c_{\rm NNLO}^{\rm soft} = 0$

The SIM scheme

- LDME relates to the wave function at the origin: $\langle \mathcal{O}[n] \rangle \sim \left| \psi_{[n]}(0) \right|^2$
  - Non-Coulombic perturbative soft contribution to the $\overline{\text{MS}}$ renormalized wavefunction at the origin up to $\mathcal{O}(\alpha_s^2)$:

e.g. Kniehl, Penin, Smirnov, Steinhauser, PRL 90 (2003)

$$\delta\psi_{s,n}^{\text{Soft}}(0) = \psi_n^{\text{LO}}(0)\left\{ \frac{\alpha_s^2(\mu_S)}{2}\left[C_F^2\left(\frac{L_n}{2s+1} - \frac{15}{8n^2} + \frac{4}{(2s+1)n} + \frac{5}{3} + \frac{7}{3(2s+1)} - \frac{2S_1(n)}{2s+1} \right) \right.\right.$$
$$\left.\left. + C_A C_F\left(\frac{L_n}{2} + \frac{2}{n} + \frac{5}{4} - S_1(n) \right) \right] \right\}$$

$$L_n \equiv \log\left(\mu_\Lambda^2 a_n^2\right) = L_\Lambda + \tilde{L}_n$$

$$L_\Lambda = \log\left(\frac{\mu_\Lambda^2}{m_Q^2}\right), \quad \tilde{L}_n = \log\left(m_Q^2 a_n^2\right)$$

Bohr radius:

$$a_n = \frac{n}{(C_F \alpha_s(\mu_S) m_Q)}$$

- $\delta\psi_{s,n}^{\text{Soft}}(0)$ is divided into two parts:

$$\delta\psi_{s,n}^{\text{Soft}}(0) = \underbrace{\delta\psi_s^{\text{SD}}(0)}_{\text{state } (n) \text{ independent}} + \underbrace{\delta\psi_{s,n}^{\text{LD}}(0)}_{\text{state } (n) \text{ dependent}}$$

The SIM scheme

- The soft-improved matching (SIM) scheme:

$$\psi_{s,n}^{\overline{\rm MS}}(0)=\psi_{s,n}^{\rm SIM}(0)\left[1+\frac{\alpha_s^2(\mu_R)}{2}\delta Z_s^{\rm SIM}\right],\quad \delta Z_s^{\rm SIM}=C_F^2\left(\frac{L_\Lambda}{2s+1}+\frac{5}{3}+\frac{7}{3(2s+1)}\right)$$

$$\delta\psi_s^{\rm SD}(0) \Leftarrow +C_FC_A\left(\frac{L_\Lambda}{2}+\frac{5}{4}\right)$$

$$\delta\psi_{s,n}^{\rm Soft}(0)=\psi_n^{\rm LO}(0)\left\{\frac{\alpha_s^2(\mu_S)}{2}\left[C_F^2\left(\frac{L_n}{2s+1}-\frac{15}{8n^2}+\frac{4}{(2s+1)n}+\frac{5}{3}+\frac{7}{3(2s+1)}-\frac{2S_1(n)}{2s+1}\right)+C_AC_F\left(\frac{L_n}{2}+\frac{2}{n}+\frac{5}{4}-S_1(n)\right)\right]\right\}$$

  - $\delta\psi_{s,n}^{\rm LD}(0)$ is absorbed into the LD wavefunction at the origin, $\psi_{s,n}^{\rm SIM}(0)$.

$$L_n\equiv\log\left(\mu_\Lambda^2a_n^2\right)=L_\Lambda+\tilde{L}_n$$

- The bare LDME is scheme independent:

$$\langle\mathcal{O}_{\rm bare}\rangle=\langle\mathcal{O}_{\overline{\rm MS}}\rangle Z_s^{\overline{\rm MS}}=\langle\mathcal{O}_{\rm SIM}\rangle Z_s^{\rm SIM},\qquad Z_s^{\rm SIM}=Z_s^{\overline{\rm MS}}+\alpha_s^2(\mu_R)\delta Z_s^{\rm SIM}$$

$$L_\Lambda=\log\left(\frac{\mu_\Lambda^2}{m_Q^2}\right)$$

$$\langle\mathcal{O}[n]\rangle\sim\left|\psi_{[n]}(0)\right|^2$$

  - Combine the $\delta\psi_s^{\rm SD}(0)$ with the $\overline{\rm MS}$ SDC: $\quad {\rm SDC}^{\overline{\rm MS}}=H\times Z_s^{\overline{\rm MS}}$

$$\mathcal{A}^{\overline{\rm MS}}=H\times\langle\mathcal{O}_{\overline{\rm MS}}\rangle Z_s^{\overline{\rm MS}} \Rightarrow \mathcal{A}^{\rm SIM}=H\times\langle\mathcal{O}_{\rm SIM}\rangle Z_s^{\rm SIM}\qquad {\rm SDC}^{\rm SIM}=H\times Z_s^{\rm SIM}={\rm SDC}^{\overline{\rm MS}}+H\times\alpha_s^2\delta Z_s^{\rm SIM}$$

❖ *SIM is a finite renormalization on top of $\overline{\rm MS}$ scheme that reorganizes perturbative contributions between SDCs and LDMEs, without introducing new physical effects.*

The SIM scheme

- From the $\overline{\text{MS}}$ to the SIM scheme:

$$Z_s^{\text{SIM}} = Z_s^{\overline{\text{MS}}} + \alpha_s^2(\mu_R)\delta Z_s^{\text{SIM}}$$

$$\text{SDC}^{\text{SIM}} = H \times Z_s^{\text{SIM}} = \text{SDC}^{\overline{\text{MS}}} + H \times \alpha_s^2 \delta Z_s^{\text{SIM}}$$

$$Z_{\text{SIM}}^{s=0} = Z_{\overline{\text{MS}}} + \left[\left(C_F^2 + \frac{C_F C_A}{2}\right)\log\frac{\mu_\Lambda^2}{m_Q^2} + 12.11\right]\alpha_s^2(\mu_R), \quad \Rightarrow \text{Spin singlet}$$

$$Z_{\text{SIM}}^{s=1} = Z_{\overline{\text{MS}}} + \left[\left(\frac{C_F^2}{3} + \frac{C_F C_A}{2}\right)\log\frac{\mu_\Lambda^2}{m_Q^2} + 9.35\right]\alpha_s^2(\mu_R) \quad \Rightarrow \text{Spin triplet}$$

- η_c decay: $\eta_c \to \gamma\gamma$

$$\left.\frac{\Gamma_{\text{NNLO}}(\eta_c \to \gamma\gamma)}{\Gamma_{\text{LO}}(\eta_c \to \gamma\gamma)}\right|_{\overline{\text{MS}}} = 1 - 1.07\alpha_s + \underbrace{(-11.16)}_{\text{SDC}_{\text{NNLO}}^{\overline{\text{MS}}}}\alpha_s^2$$

$$\left.\frac{\Gamma_{\text{NNLO}}(\eta_c \to \gamma\gamma)}{\Gamma_{\text{LO}}(\eta_c \to \gamma\gamma)}\right|_{\text{SIM}} = 1 - 1.07\alpha_s + \underbrace{(-11.16+12.11)}_{\text{SDC}_{\text{NNLO}}^{\text{SIM}}=0.95\sim\mathcal{O}(1)}\alpha_s^2$$

- Features of the SIM scheme :
  - With our choice of the finite counterterm, the explicit μ_Λ dependence cancels through NNLO.
  - The essential improvement comes from the finite, non-logarithmic soft contributions.

Results in the SIM scheme

- $\eta_Q \to gg$:

$$\left.\frac{\Gamma_{\rm NNLO}(\eta_Q \to gg)}{\Gamma_{\rm LO}(\eta_Q \to gg)}\right|_{\mu_R=m_Q} = 1 + \begin{pmatrix} 1.54_{n_f=3} \\ 1.41_{n_f=4} \\ 1.27_{n_f=5} \end{pmatrix} \alpha_s + \left[\begin{pmatrix} -12.70_{n_f=3} \\ -13.37_{n_f=4} \\ -14.06_{n_f=5} \end{pmatrix} +12.11 \right] \alpha_s^2 .$$

- $V_Q \to \ell^+\ell^-$:

$$\left.\frac{\Gamma_{\rm NNLO}(V_Q \to \ell^+\ell^-)}{\Gamma_{\rm LO}(V_Q \to \ell^+\ell^-)}\right|_{\mu_R=m_Q} = 1 - 1.70\alpha_s + \left[\begin{pmatrix} -8.06_{n_f=3} \\ -7.98_{n_f=4} \\ -7.89_{n_f=5} \end{pmatrix} +9.35 \right] \alpha_s^2 .$$

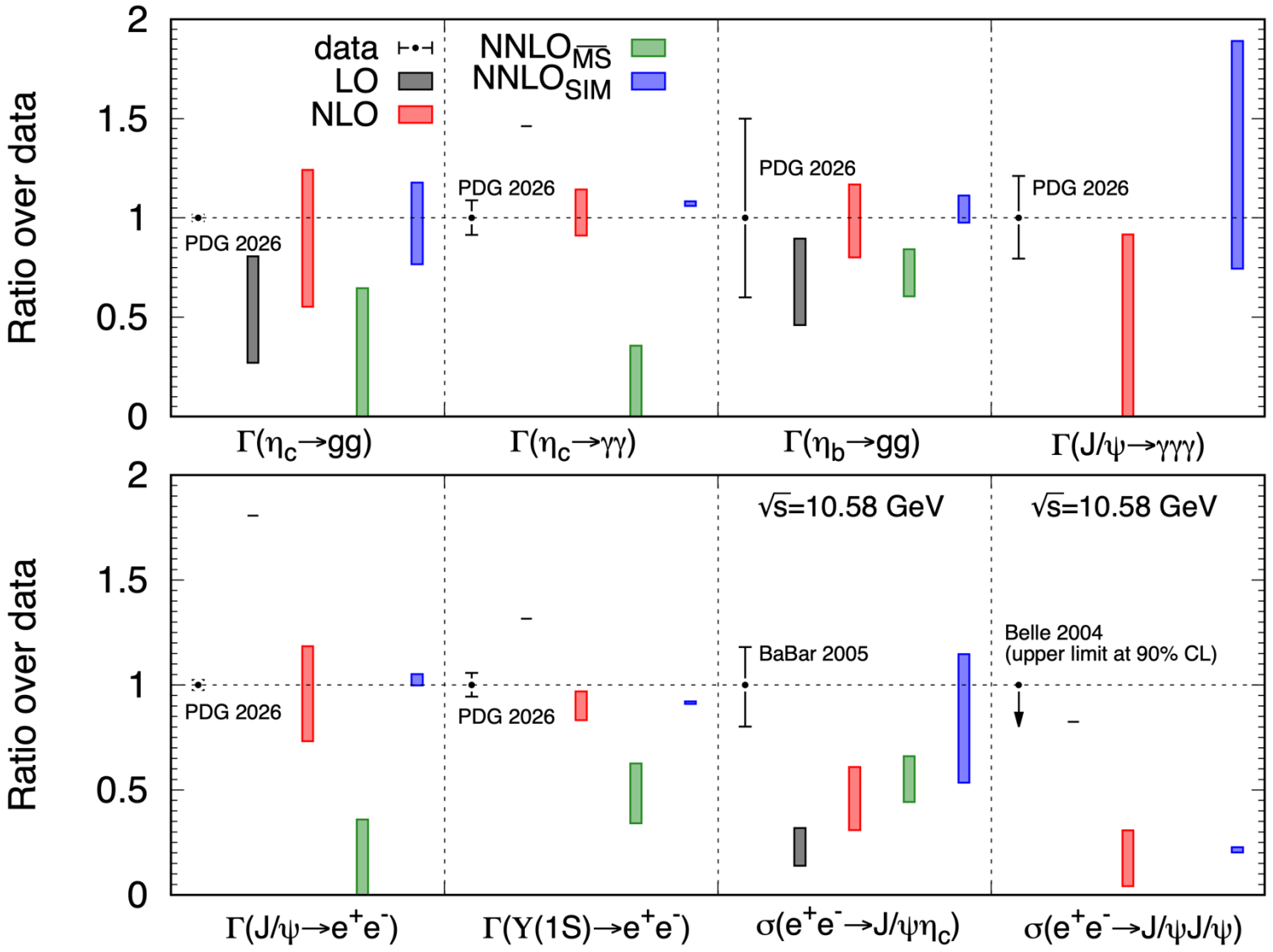
- $e^+e^- \to \eta_c\gamma$:

$$\left.\frac{\sigma_{\rm NNLO}(e^+e^- \to \eta_c\gamma)}{\sigma_{\rm LO}(e^+e^- \to \eta_c\gamma)}\right|_{\mu_R=\sqrt{s}/2,\sqrt{s}=3m_c} = 1 - 1.39\alpha_s + [-10.65+12.11]\,\alpha_s^2 .$$

$$\left.\frac{\sigma_{\rm NNLO}(e^+e^- \to \eta_c\gamma)}{\sigma_{\rm LO}(e^+e^- \to \eta_c\gamma)}\right|_{\mu_R=\sqrt{s}/2,\sqrt{s}=10m_c} = 1 - 0.55\alpha_s + [-10.83+12.11]\,\alpha_s^2 .$$

Maxia, Shao, Simon, *GW*, arXiv: 2606.28494

Results in the SIM scheme



Scales:

- $\mu_R = 2m_Q$ for decays
- $\mu_R = \sqrt{s}/2$ for productions
- $\mu_\Lambda = m_Q$

Charmonium LDMEs:

(tuned from data)

- $\langle \mathcal{O}_{\eta_c} \rangle = 0.318\ \text{GeV}^3$ from $\Gamma(\eta_c \to \gamma\gamma)$
- $\langle \mathcal{O}_{J/\psi} \rangle = 1.36\ \text{GeV}^3$ from $\Gamma(J/\psi \to e^+e^-)$

Bottomonium LDMEs:

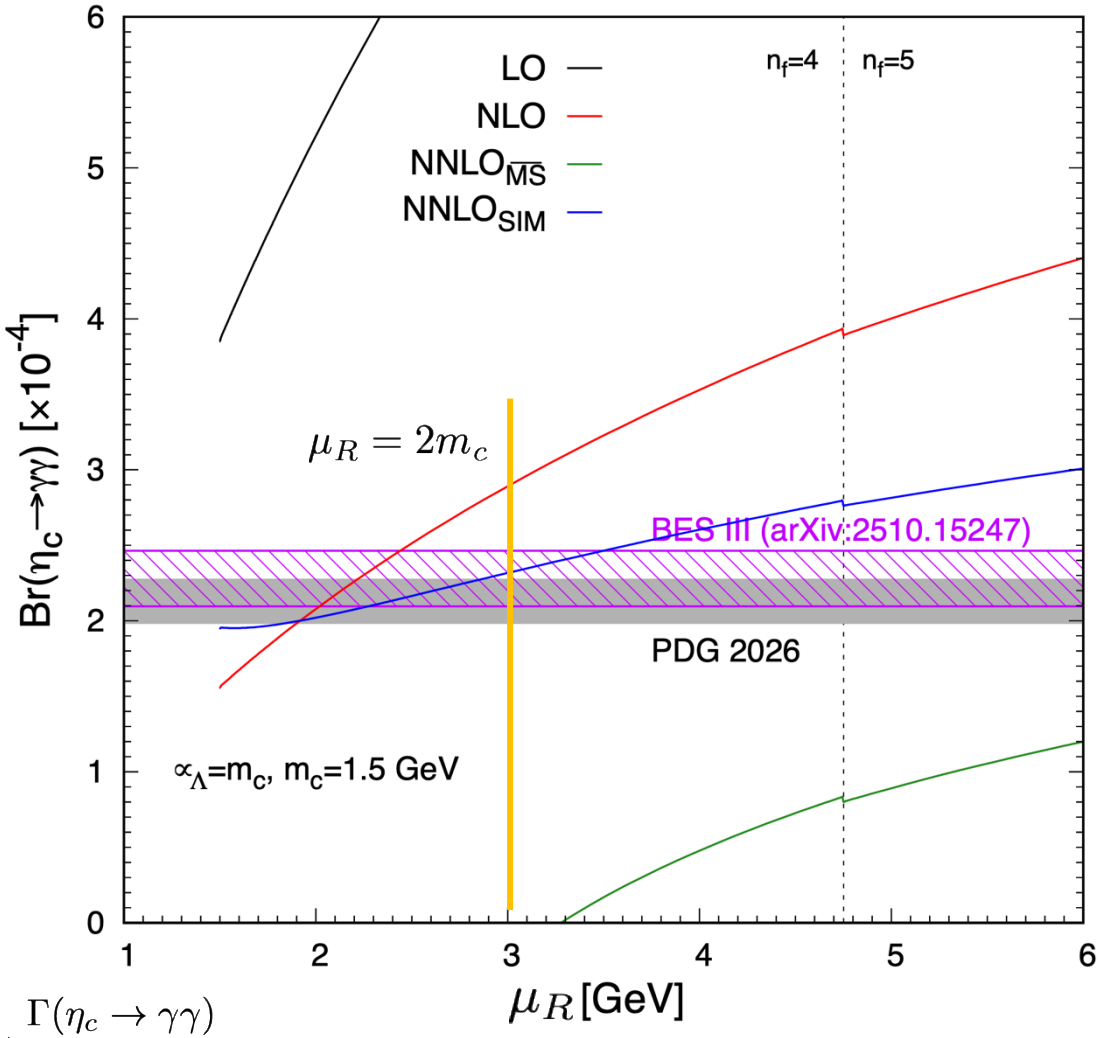
(from BT potential Eichten, Quigg, PRD 52 (1995))

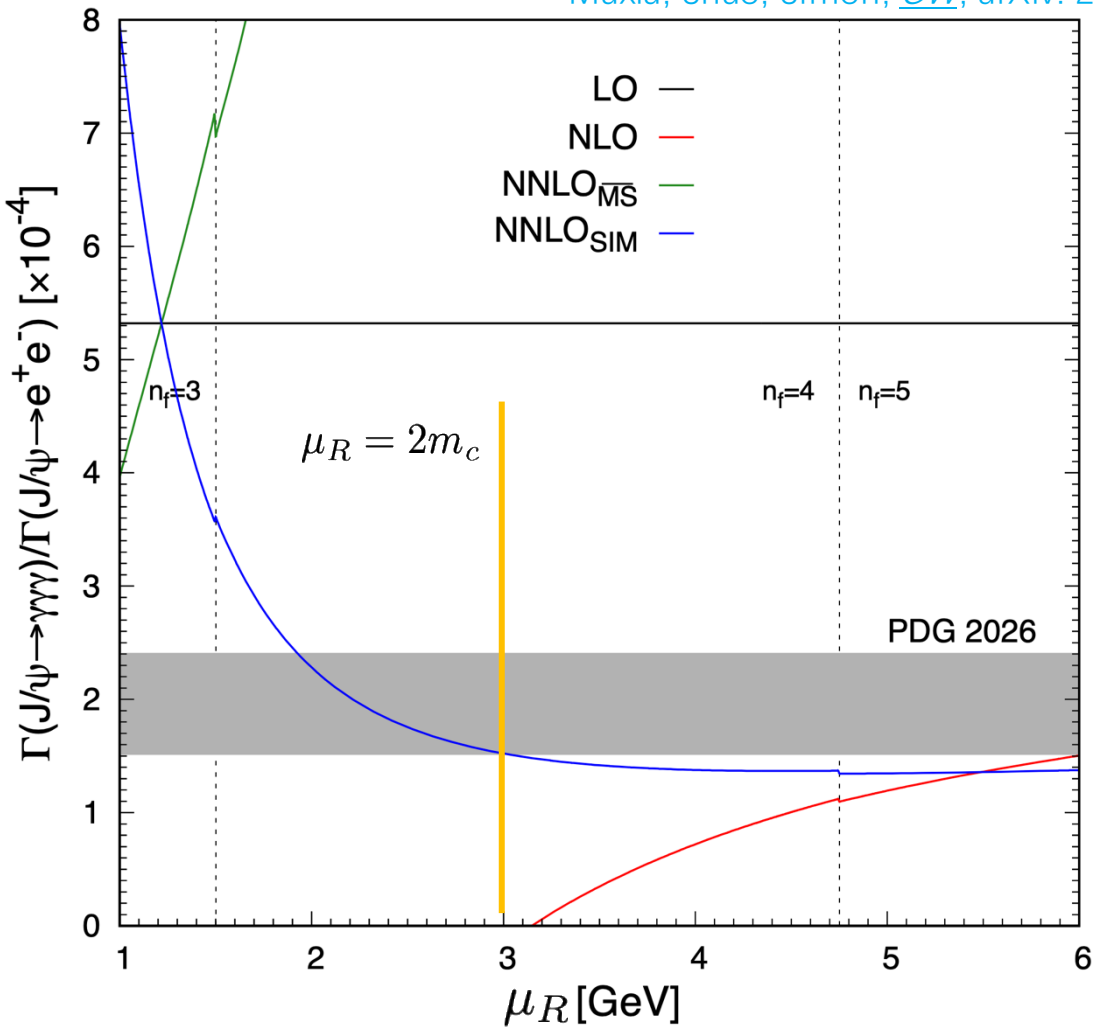
- $\langle \mathcal{O}_{\eta_b} \rangle = 3.093\ \text{GeV}^3$
- $\langle \mathcal{O}_{\Upsilon} \rangle = 9.28\ \text{GeV}^3$

Maxia, Shao, Simon, *GW*, arXiv: 2606.28494

Results in the SIM scheme

Maxia, Shao, Simon, *GW*, arXiv: 2606.28494





$$\sim \frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(\eta_c \to gg)}$$

The ratios are not dependent on the choice of the LDMEs.

Summary and outlook

- We propose a soft-improved matching scheme (SIM) to Stabilize NNLO QCD Corrections to Quarkonium Production and Decay.
  - A matching scheme on top of the $\overline{\text{MS}}$ scheme and explicit μ_Λ dependence cancels through NNLO.
  - Applicable to both exclusive and inclusive processes involving S-wave color-singlet quarkonium at leading power in the NRQCD velocity expansion.
- Our SIM scheme is applied to the S-wave color-singlet quarkonium processes with principal quantum number n=1 in this work.
- We achieve substantially improved perturbative convergence and agreement with experimental data.

- Relativistic corrections should also be taken into account.
- Extend the analysis to color-octet channels.
- Multi-scale quarkonium processes might benefit from resummation of large logarithms, e.g., $\log(s/m_c^2)$ in $e^+e^- \to \eta_c J/\psi$.

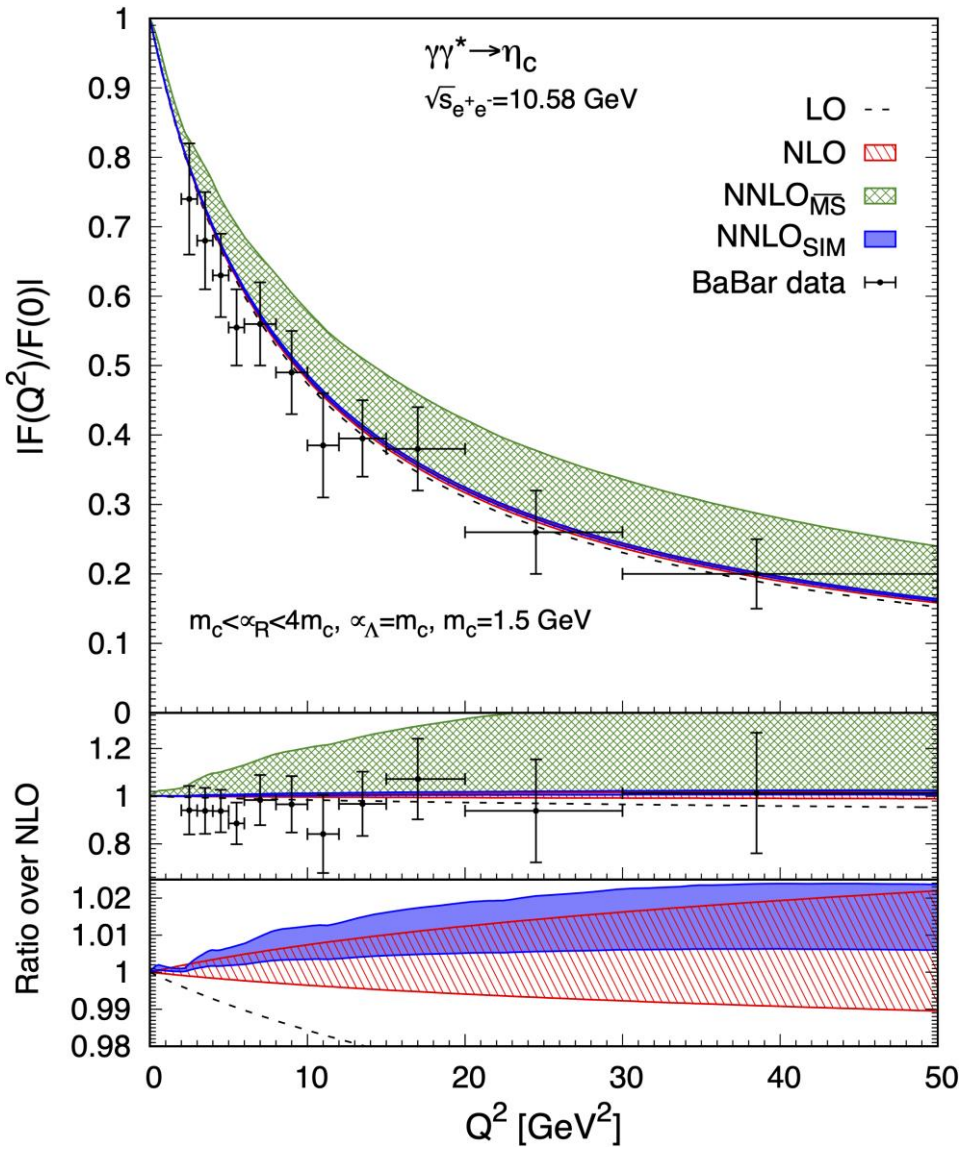
Thank you for your attention!

Back up

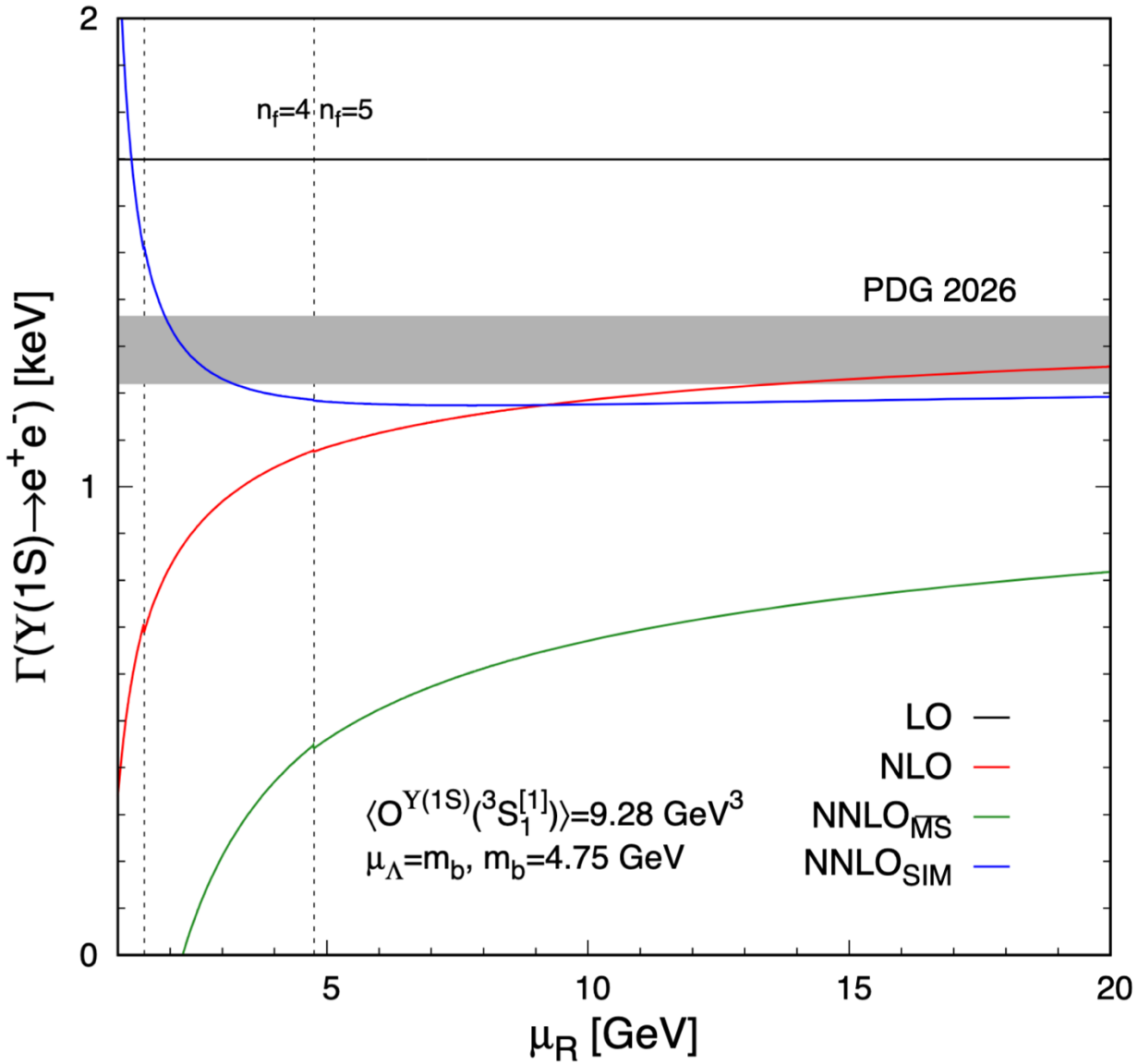
$$\frac{\partial f_i(x,Q^2)}{\partial \ln Q^2} = \sum_j P_{ij}(x,\alpha_s(Q^2)) \otimes f_j(x,Q^2)$$

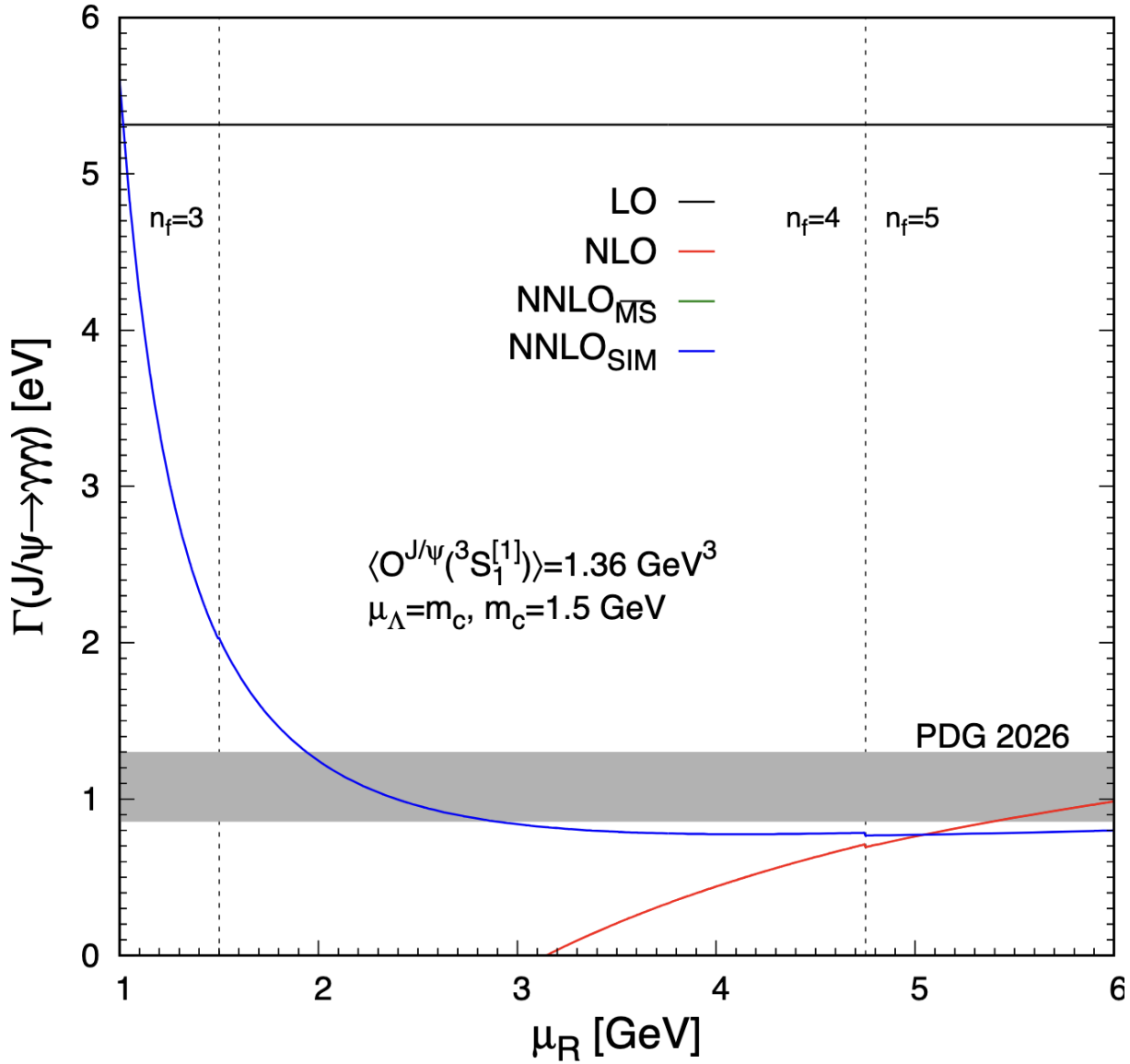


perturbative



The absolute transition form-factor ratio $|F(Q^2)/F(0)|$ for $\gamma\gamma^* \to \eta_c$ as a function of the photon virtuality Q^2 is shown at LO (dashed), NLO (red hatched), $\mathrm{NNLO}_{\overline{\mathrm{MS}}}$ (green hatched), and $\mathrm{NNLO}_{\mathrm{SIM}}$ (blue band). Experimental data from the **BaBar** experiment are taken from ref. [46]. The lower panels show ratios normalized to the central NLO value. In the bottom panel, both $\mathrm{NNLO}_{\overline{\mathrm{MS}}}$ and the **BaBar** data are not shown.


2
1
0
5
10
15
20
n_f=4 n_f=5
PDG 2026
$\Gamma(\Upsilon(1S)\to e^+e^-)$ [keV]
μ_R [GeV]
$\langle O^{\Upsilon(1S)}(^3S_1^{[1]})\rangle$=9.28 GeV^3
μ_Λ=m_b, m_b=4.75 GeV
LO
NLO
NNLO$_{\overline{MS}}$
NNLO$_{SIM}$


LO
NLO
NNLO$_{\overline{MS}}$
NNLO$_{SIM}$
n_f=3
n_f=4
n_f=5
$\langle O^{J/\psi}(^3S_1^{[1]})\rangle$=1.36 GeV^3
μ_Λ=m_c, m_c=1.5 GeV
PDG 2026
$\Gamma(J/\psi\to\gamma\gamma)$ [eV]
μ_R [GeV]
0
1
2
3
4
5
6