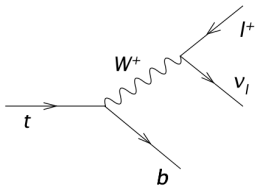
Complete angular analysis of polarized top-quark decay at NNLO in QCD

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$$t(\uparrow) \to W^+ X_b, \quad W^+ \to \ell^+ \nu_\ell$$

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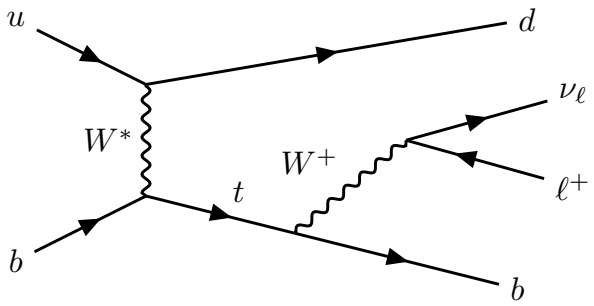
Shanghai, October 23, 2026

Outline

1. Motivation
2. Framework and calculation
3. Results
4. Summary

Why study polarized top decay?

Example: single-top production and decay



- Decay before hadronization preserves spin information.
- Single top: polarized tops.
- $t\bar{t}$: correlated spins, even without net polarization.

Polarization and spin correlations both require spin-dependent decay.

Bigi et al., Phys. Lett. B 181 (1986) 157; Mahlon and Parke, hep-ph/9912458; ATLAS, arXiv:1707.05393.
Spin-correlated $t\bar{t}$ production and decay: Czakon, Mitov and Poncelet, arXiv:2008.11133.

From the total width to the angular distribution

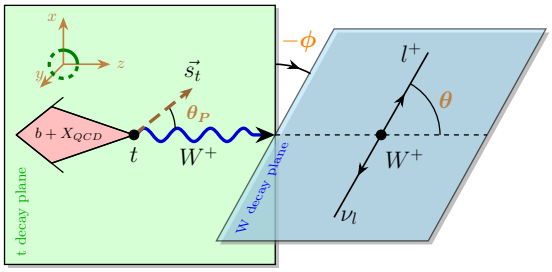
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Total decay width	NNLO (Gao, Li and Zhu, 2013) $\longrightarrow$ N^{3}LO for $m_b = 0$ (L. Chen et al., 2026). Finite-m_b dependence through NNLO (L. Chen et al., 2026).
Complete NLO angular analysis	All eight helicity structure functions, retaining the exact bottom-quark-mass dependence (Fischer et al., 2002).
Selected NNLO angular results	W helicity fractions (Czarnecki, Körner and Piclum, 2010) and W-direction spin-analyzing power (Czarnecki et al., 2018), for $m_b = 0$.

Fully differential NNLO calculations with spin information also exist for $m_b = 0$ (Brucherseifer, Caola and Melnikov, 2013).

This work: complete NNLO results for all eight functions, for both $m_b = 0$ and finite m_b.

Three angles, eight helicity structure functions



θ_P: spin versus W momentum
θ, ϕ: lepton direction in the W rest frame

Factor out the common coupling $gV_{tb}/\sqrt{2}$:

$$\mathcal{M}_X = \epsilon^*_\mu(q) M^\mu_X,$$

$$H^{\mu\nu}(q, s_t) = \sum_X M^\mu_X M^{\nu *}_X.$$

$\sum_X$: final-state spin and colour sums, and X_b phase space at fixed q.

Helicity projections

$$H_i(E_W) = \mathcal{P}^{\mu\nu}_i H^{\text{unpol.}}_{\mu\nu}, \quad i = U, L, F.$$

The corresponding spin projections define $H^P_i(E_W)$.

Spin independent

$H_U,\ H_L,\ H_F$

Spin dependent

$H^P_U,\ H^P_L,\ H^P_F$

Interference

$H^P_I,\ H^P_A$

Fourfold decay distribution

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}E_W\,\mathrm{d}\cos\theta_P\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \frac{\mathcal{N}(E_W)}{4\pi}\Big\{ \frac{3}{8}(H_U + P_t\cos\theta_P H_U^P)(1+\cos^2\theta) + \frac{3}{4}(H_L + P_t\cos\theta_P H_L^P)\sin^2\theta + \frac{3}{4}(H_F + P_t\cos\theta_P H_F^P)\cos\theta + \frac{3}{2\sqrt{2}}P_t\sin\theta_P H_I^P \sin 2\theta\cos\phi + \frac{3}{\sqrt{2}}P_t\sin\theta_P H_A^P\sin\theta\cos\phi\Big\}.$$

$$\Gamma_i = \int \mathrm{d}E_W\,\mathcal{N}(E_W)H_i(E_W), \qquad \Gamma_{UL} = \Gamma_U + \Gamma_L.$$

After integration over E_W, the same angular structure holds with $\mathcal{N}(E_W)H_i(E_W) \to \Gamma_i$.

All $H_i = H_i(E_W)$. $\mathcal{N}(E_W) = G_F|V_{tb}|^2 m_W^2\sqrt{E_W^2 - m_W^2}/(\sqrt{2}\pi)$.

P_t is the production polarization; the leptonic W branching fraction is factored out.

W helicity fractions

$$F_\pm = \frac{\Gamma_U \pm \Gamma_F}{2\Gamma_{UL}}, \qquad F_0 = \frac{\Gamma_L}{\Gamma_{UL}}.$$

Spin-independent combinations.

Spin and azimuthal modulations

$$\alpha_{\theta_P} = \frac{\Gamma^P_{UL}}{\Gamma_{UL}}, \qquad \gamma_\phi = \frac{3\pi^2}{8\sqrt{2}} \frac{\Gamma^P_A}{\Gamma_{UL}}.$$

$$\frac{1}{\Gamma_{UL}} \frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta_P} = \frac{1}{2}(1 + P_t \alpha_{\theta_P} \cos\theta_P),$$

$$\frac{1}{\Gamma_{UL}} \frac{\mathrm{d}\Gamma}{\mathrm{d}\phi} = \frac{1}{2\pi}(1 + P_t \gamma_\phi \cos\phi).$$

α_{θ_P} is the spin-analyzing power of the W direction.

A common framework for loop and phase-space integrals

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Fix $q^2 = m_W^2$ and E_W: project the semi-inclusive hadronic tensor

↓

Reverse unitarity: replace phase-space constraints by cut propagators

↓

IBP reduction $\longrightarrow$ master integrals [Blade / Kira]

↓

Differential equations in E_W/m_t + AMFlow boundary values

↓

Match local series; integrate over E_W at nonzero ϵ; reconstruct Γ_i

Numerical inputs and perturbative conventions

$$\Gamma_i = \Gamma_{\rm ref}\left[c_i^{(0)} + \alpha_s c_i^{(1)} + \alpha_s^2 c_i^{(2)}\right] + \mathcal{O}(\alpha_s^3).$$

$$m_t = 172.69\ \text{GeV}, \qquad \mu = m_t,$$
$$\alpha_s^{(5)}(m_t) = 0.1076, \qquad |V_{tb}| = 1,$$
$$\frac{m_W^2}{m_t^2} = \frac{112}{517}, \qquad \frac{m_b^2}{m_t^2} = \frac{1}{1305}.$$

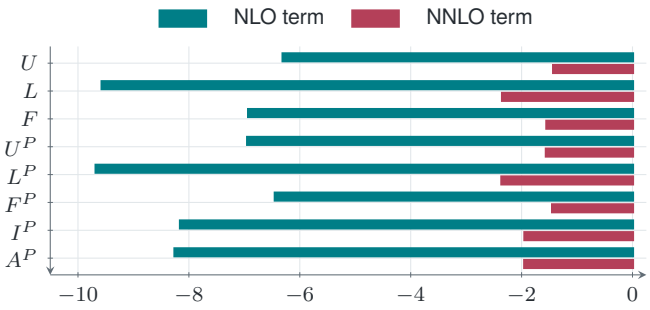
- Five-flavour $\overline{\rm MS}$ coupling; on-shell quark masses.
- Normalized observables use ratios of cumulative rates.

$m_b \simeq 4.7804$ GeV in the finite-mass calculation.

$\Gamma_{\rm ref} = G_F m_W^2 m_t |V_{tb}|^2/(12\sqrt{2})$, with nominal $m_W = 80.377$ GeV. Closed top loops are omitted for $m_b = 0$ but included for finite m_b; quoted NNLO mass effects include this difference. Two-mass counterterms: Fael et al., arXiv:2008.01102.

Massless benchmark: width and angular components

$$\Gamma_{UL}^{\mathrm{NNLO}} = \underbrace{1.4864}_{\text{Born}} \underbrace{- 0.1275}_{\text{NLO term}} \underbrace{- 0.0307}_{\text{NNLO term}} = 1.3282\ \mathrm{GeV}.$$



$$r_i^{(1)} = \frac{\Gamma_i^{\mathrm{NLO}} - \Gamma_i^{\mathrm{LO}}}{\Gamma_i^{\mathrm{LO}}},$$

$$r_i^{(2)} = \frac{\Gamma_i^{\mathrm{NNLO}} - \Gamma_i^{\mathrm{NLO}}}{\Gamma_i^{\mathrm{LO}}}.$$

Magnitude reductions

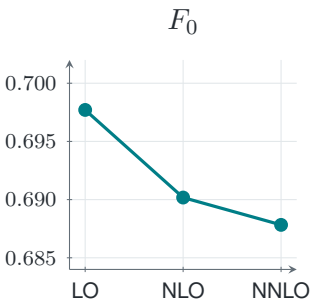
NLO term: 6.3–9.7%.

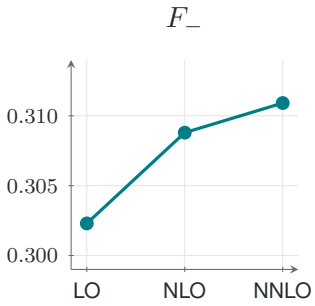
NNLO term: 1.4–2.4%.

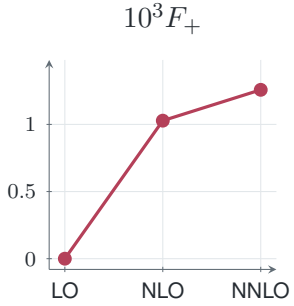
No common K factor for the angular components.

$m_b = 0$. Both increments are normalized to LO; negative percentages indicate reductions in $|\Gamma_i|$.

Helicity fractions: a small radiation-induced F_+







NNLO: $F_0 = 0.687829, \quad F_- = 0.310914, \quad F_+ = 0.001258.$

QCD radiation generates a nonzero right-handed W fraction.

$m_b = 0.$

Born-forbidden channels have two different origins

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Chirality + two-body kinematics

$\Gamma^{++}_{++},\ \Gamma^{--}_{00},\ \Gamma^{+-}_{+0}$

Require the opposite b helicity in the two-body decay. Finite m_b can populate them at Born level.

Two-body angular momentum

$\Gamma^{--}_{++},\ \Gamma^{++}_{--}$

Would require $\lambda_b = \pm 3/2$. They vanish for any m_b in a two-body final state.

$10^4\,\Gamma^{\lambda_t\lambda'_t}_{\lambda_W\lambda'_W}/\Gamma_{UL}$	NLO	NNLO
$\Gamma^{++}_{++}/\Gamma_{UL}$	9.260	11.457
$\Gamma^{--}_{00}/\Gamma_{UL}$	4.132	4.697
$\Gamma^{+-}_{+0}/\Gamma_{UL}$	−2.495	−2.638
$\Gamma^{--}_{++}/\Gamma_{UL}$	1.020	1.120
$\Gamma^{++}_{--}/\Gamma_{UL}$	1.308	1.573

Radiation populates all five; NNLO is their first subleading correction.

$\Gamma^{\lambda_t\lambda'_t}_{\lambda_W\lambda'_W}$: energy-integrated top–$W$ double-density entries. Table: $m_b = 0$, in units of 10^{-4}.

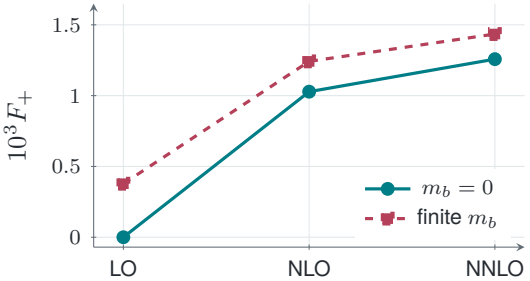
Spin and azimuthal correlations respond differently

Observable	LO	NLO	NNLO	NNLO/NLO -1
α_{θ_P}	0.395412	0.381641	0.377324	-1.13%
γ_ϕ	0.600948	0.603100	0.603968	$+0.14\%$

The NNLO corrections reduce the spin-analyzing power while slightly increasing the azimuthal modulation.

$m_b = 0.$

Finite m_b: a small width shift, a visible effect in F_+



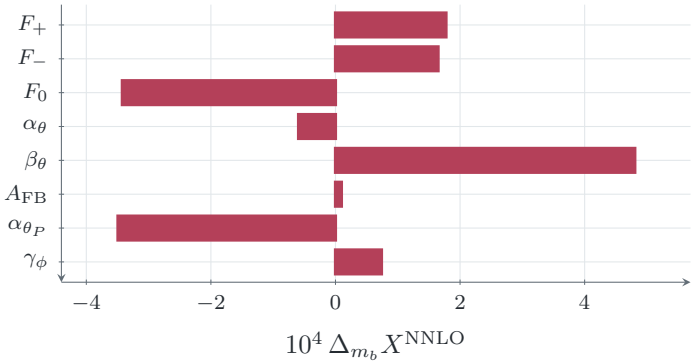
NNLO comparison

Observable	$m_b = 0$	Finite m_b	Relative shift
Γ_{UL} [GeV]	1.3282	1.3262	-0.154%
$10^3 F_+$	1.258	1.435	$+14.1\%$

$$\Delta_{m_b} F_+ = 1.77 \times 10^{-4}$$

NNLO differences follow the convention on slide 9, including the different treatment of closed top loops.

Normalized mass shifts remain small in absolute terms



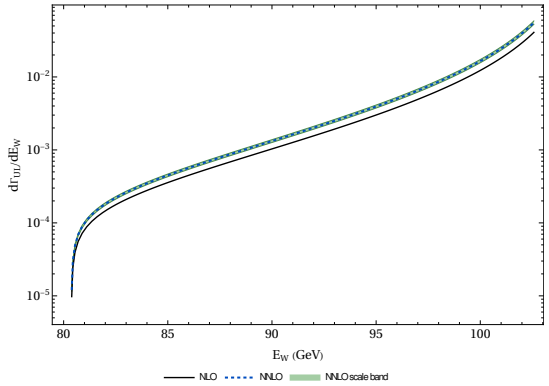
$$\Delta_{m_b} X = X(m_b) - X(0), \qquad \delta_{m_b} X = \frac{X(m_b) - X(0)}{X(0)}.$$

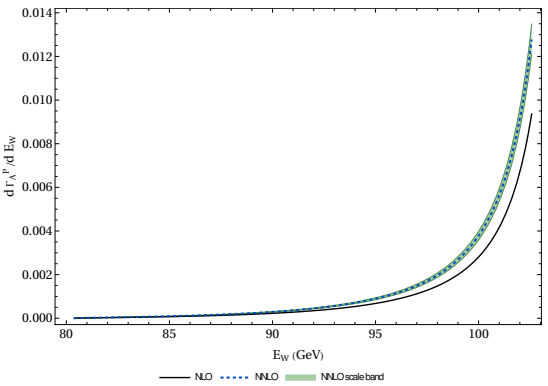
Relative shift (%)

X	$\delta_{m_b} X$
F_+	$+14.111$
F_-	$+0.053$
F_0	-0.050
α_θ	$+0.016$
β_θ	-0.076
$A_{\rm FB}$	-0.004
α_{θ_P}	-0.093
γ_ϕ	$+0.012$

NNLO cumulative predictions. Relative shifts use the signed massless value $X(0)$.

Massless W-energy spectra





$$\frac{\mathrm{d}\Gamma_{UL}}{\mathrm{d}E_W} = \mathcal{N}(E_W) H_{UL}(E_W)$$

$$\frac{\mathrm{d}\Gamma_A^P}{\mathrm{d}E_W} = \mathcal{N}(E_W) H_A^P(E_W)$$

Continuous radiation region only; endpoint terms are not shown. Central scale: $\mu = m_t$; NNLO band: $m_t/2 \le \mu \le 2m_t$. The right-hand panel is a signed angular-coefficient density.

Connection to measured angular moments

ATLAS measures the three-angle distribution in single-top production.

$$\frac{1}{\Gamma_{UL}}\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_P\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sum_{k=0}^{1}\sum_{l=0}^{2}\sum_{|m|\le\min(k,l)} a_{k,l,m}\,M_{k,l}^{m},$$

$$M_{k,l}^{m} = \sqrt{2\pi}\,Y_k^m(\theta_P,0)Y_l^m(\theta,\phi).$$

Two examples connect the polarized decay functions to angular moments:

$$a_{1,0,0} = \frac{P_t}{2\sqrt{6\pi}}\frac{\Gamma_{UL}^{P}}{\Gamma_{UL}}, \qquad \mathrm{Re}\,a_{1,1,1} = \frac{P_t}{2\sqrt{\pi}}\frac{\Gamma_{A}^{P}}{\Gamma_{UL}}.$$

Measured angular moments → Background and detector corrections → NNLO coefficients + production polarization

NNLO decay coefficients complement the production polarization input.

ATLAS triple-differential analysis: arXiv:1707.05393; quadruple-differential analysis: arXiv:2510.23372. For the 2017 transverse-axis convention, $\phi_{\mathrm{ATLAS}} = \phi + \pi$ and $a_{k,l,m}^{\mathrm{ATLAS}} = (-1)^m a_{k,l,m}$.

Summary

1. **Eight functions through NNLO.**
   Independent massless and finite-bottom-mass calculations.

2. **Different angular components receive different corrections.**
   The angular shape cannot be inferred from a total-width K factor.

3. **Suppressed channels expose small effects.**
   The NNLO F_+ shift is 1.77×10^{-4}, about 14.1% relative to the massless result.

Thank you!