Tracing QCD Vacuum Fluctuations during Hadronization with Conserved Currents (守恒流探测强子化)

Wanchen Li (李婉辰)
wanchen1995@outlook.com

Based on: Weiyao Ke (柯伟尧), Hai Tao Li (李海涛), Wanchen Li, Xiaohui Liu (刘晓辉), Ding Yu Shao (邵鼎煜), arXiv: 2608.19708

2026 International Workshop on the High-Energy Circular Electron-Positron Collider (CEPC2026)
Shanghai, China, October 22–26, 2026



Outline

1. Charge flow.
2. Defects in n-collinear fragmentations.
   - No perturbative contribution,
   - Non-perturbative mechanism,
   - Jet Gell-Mann–Nishijima relation:

$$\langle Q_{\text{flow}} \rangle_f \simeq I_{3,f} + \frac{1}{2}\big(\langle B_{\text{flow}} \rangle_f + \langle S_{\text{flow}} \rangle_f\big).$$

3. Tests with Monte-Carlo simulations.
4. Summary and outlooks.

Charge Flow in Factorized Fragmentation

- The cross section factorization takes the form

$$d\sigma[Q_{\rm flow}] \sim f_a \otimes f_b \otimes \widehat{\sigma}_{ab\to f}\, J_f[Q_{\rm flow}].$$

- $J_f[Q_{\rm flow}]$ is the charge flow jet function measured on the detectors,

$$J_f[Q_{\rm flow}] \sim \langle 0|\xi_{n,f}\, W_n^\dagger\, \mathcal{M}_{Q,\,\rm flow}\, W_n\, \bar{\xi}_{n,f}|0\rangle.$$

- $\mathcal{M}_{Q,\,\rm flow}$ measures the total charge of the hadronic final state $|X\rangle$:

$$\mathcal{M}_{Q,\,\rm flow} = \sum_X \hat{Q}\, \delta\left(k^- - P_X^-\right) |X\rangle\langle X|,$$

where $\hat{Q}|X\rangle = Q_X|X\rangle$.

Hadronic charge and quark charge

Does the measured hadronic charge equal the initiating quark's charge?

- **Event by event.**
  Q_X is integer and Q_f is fractional, so $Q_X \neq Q_f$.

- **Ensemble average.**
  An integer-valued charge can have a fractional mean:

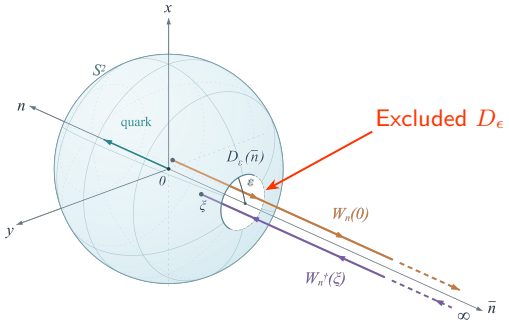
$$\langle Q_{\text{flow}} \rangle_f \stackrel{?}{=} Q_f.$$

  Charge conservation alone does not establish this equality for the measured sector.

Defects in the Fragmentation

The fragmentation contains Wilson lines that extend to infinity,

$$J_f[Q_{\text{flow}}] \sim \langle 0|\xi_{n,f}\, W_n^\dagger\, \mathcal{M}_{Q,\,\text{flow}}\, W_n\, \bar{\xi}_{n,f}|0\rangle.$$



- Celestial sphere S^2 covers the angular space of the factorized n-collinear sector.
- The Wilson line extends along $\bar{n}$; its endpoint is the excluded puncture $D_\epsilon(\bar{n})$.
- Charge bound to this endpoint is absent from the measured flow.

$$\widehat{Q}_{\text{flow}} = \lim_{\epsilon\to 0} \int_{S^2\setminus D_\epsilon(\bar{n})} d\Omega\, \widehat{Q}(\Omega).$$

Charge at the Wilson-line endpoint

Let x be the radiation's fraction of the jet's longitudinal momentum.

- Radiations with larger momentum contribute to common measured charge flow.
- **Zero momentum fraction, finite charge.** The disk contain only extreme soft radiations $x \to 0$. Its momentum weight vanishes, but the charge measurement has no such weight.
- The Wilson line itself is neutral under Q, B, S; dynamical quarks dressing it can carry these charges.

$$\underbrace{\langle \mathcal{Q}_{\text{flow}} \rangle_f}_{\text{measured hadrons}} + \underbrace{\langle \mathcal{Q}_D \rangle_f}_{\text{endpoint sector}} = \underbrace{\mathcal{Q}_f}_{\text{initiating quark}}, \qquad \mathcal{Q} = Q, B, S.$$

- A similar picture was proposed in different language by Collins and Rogers.

  J. Collins and T. C. Rogers, arXiv:2309.03346.

Perturbative cancellation at the endpoint

Can a finite mean charge survive in endpoint $0 < x < \delta$ as $\delta \to 0$?

- Pure virtual insertions carry no net Q, B, S.
- Singlet contribution is charge-even. Equal perturbative weights and opposite charges cancel in the mean for a soft $q\bar{q}$.
- The **charge-odd nonsinglet contribution** is integrable at $x = 0$: its charge in the shrinking interval vanishes.
  For example, splitting kernel $P_{NS}(x)$ only contain log divergent terms in $x \to 0$,

$$\int_0^\delta dx\ \ln x = \delta \ln \delta - \delta \longrightarrow 0.$$

Thus, at leading power perturbative contribution,

$$\boxed{\langle \mathcal{Q}_D \rangle_f^{\text{pert}} = 0, \qquad \langle \mathcal{Q}_{\text{flow}} \rangle_f^{\text{pert}} = \mathcal{Q}_f}.$$

Scale independence and flavor differences

- The μ dependence comes from ultraviolet renormalization, even when the matrix element has a nonperturbative contribution.
- Perturbatively, the bare (unrenormalized) charge sum rule gives $\langle Q_{\rm flow}\rangle_f^{\rm pert} = Q_f$,
  J. C. Collins and D. E. Soper, Nucl. Phys. B 194 (1982) 445.

$$\mu\frac{d}{d\mu}\langle Q_{\rm flow}\rangle_f = 0.$$

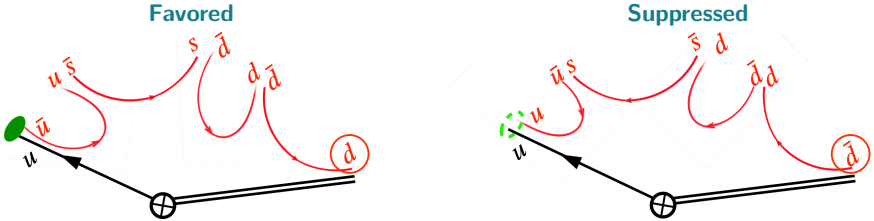
- Even if hadronization can generate a nonzero endpoint charge:

$$\langle Q_D\rangle_f = Q_f - \langle Q_{\rm flow}\rangle_f \quad\Longrightarrow\quad \mu\frac{d}{d\mu}\langle Q_D\rangle_f = 0.$$

- If we respect the light-flavor symmetry. The loss of initiating-flavor memory give

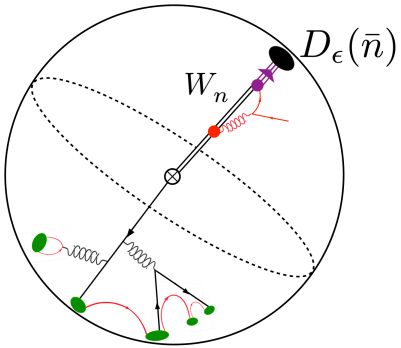
$$\boxed{\langle Q_D\rangle_u \simeq \langle Q_D\rangle_d \quad\Longrightarrow\quad \langle Q_{\rm flow}\rangle_u - \langle Q_{\rm flow}\rangle_d \simeq Q_u - Q_d = 1}.$$

Hadronization correlates the endpoints



- Consider a string breaking picture, the initiating u can bind to a neighboring $\bar{u}$. The reversed ordering with a neighboring u is suppressed.
- The two orderings leave a d or a $\bar{d}$ at the endpoint.
- Confinement prefer $u\bar{u}$ (left) over uu (right). Therefore, **unequal probabilities** give a nonzero net charge at the defect Q_D.

Color screening at the puncture



- Dynamical quarks and antiquarks combine with the colored Wilson-line endpoint to form a color singlet. These attached constituents form the screening cloud.
- Suppose attached quarks form the *screening cloud*: k quarks and ℓ antiquarks.
- For a q initiated fragmentation, color triality requires

$$\underbrace{1}_{\text{initiating}} \underbrace{-(k-\ell)}_{\text{cloud}} = 0 \pmod 3.$$

$$\boxed{B_D = \frac{k-\ell}{3} = \frac{1}{3} + m}, \quad m \in \mathbb{Z}.$$

- Triality fixes allowed quantum numbers; dynamics determines their probabilities.

Allowed screening clouds

			Leak through the edge		Measured	
Screening cloud	k	ℓ	B_D	Q_D	B_{flow}	Q_{flow}
q_a	1	0	$1/3$	Q_a	0	$Q_f - Q_a$
$(\bar{q}_a \bar{q}_b)_{\mathbf{3}}$	0	2	$-2/3$	$-(Q_a + Q_b)$	1	$Q_f + Q_a + Q_b$
Higher clouds	k	ℓ	$1/3 + m$	$\sum_a Q_a N_a$	$-m$	$Q_f - \sum_a Q_a N_a$

with $N_a = k_a - \ell_a$.

- For example, in a u initiated jet,
  - Another u flows out from the defect, now

$$Q_{\text{flow}} = Q_u - Q_u = 0, \quad B_{\text{flow}} = B_u - B_u = 0,$$

  - $\bar{d}\bar{d}$ flows out from the defect, now

$$Q_{\text{flow}} = Q_u - 2Q_{\bar{d}} = 0, \quad B_{\text{flow}} = B_u - 2B_{\bar{d}} = 1.$$

Baryon flow and the channel probabilities

- **Endpoint probability.** Retain only the two minimal clouds. Then p_B is the fraction of endpoints screened by an anti-diquark, summed over flavors:

$$P[q] = 1 - p_B, \qquad P[(\bar{q}\bar{q})_{\mathbf{3}}] = p_B.$$

- The two clouds give $B_{\text{flow}} = 0$ and 1:

$$\langle B_{\text{flow}} \rangle_f \simeq \underbrace{(1 - p_B) \times 0}_{q \text{ screening}} + \underbrace{p_B \times 1}_{\bar{q}\bar{q} \text{ screening}} = p_B .$$

- **Vacuum production ratio.** r_{qq} is the effective vacuum diquark-to-quark production ratio. If screening weights are homogeneous, the endpoint samples the same ratio:

$$r_{qq} \simeq \frac{P[(\bar{q}\bar{q})_{\mathbf{3}}]}{P[q]} = \frac{p_B}{1 - p_B}, \qquad \langle B_{\text{flow}} \rangle_f \simeq \frac{r_{qq}}{1 + r_{qq}}.$$

Electric charge of the screening cloud

- **Flavor average.** Let the production rate $u : d : s = 1 : 1 : r_s$, thus the mean charge

$$\overline{Q}_q = \underbrace{\frac{2}{3}\cdot\frac{1}{1+1+r_s}}_{u} - \underbrace{\frac{1}{3}\cdot\frac{1}{1+1+r_s}}_{d} - \underbrace{\frac{1}{3}\cdot\frac{r_s}{1+1+r_s}}_{s} = \frac{1-r_s}{3(2+r_s)}.$$

- **Channel average.** Independent antiquark flavors give $\overline{Q}_{\bar{q}\bar{q}} = -2\overline{Q}_q$:

$$\langle Q_D\rangle_f \simeq \underbrace{(1-p_B)\overline{Q}_q}_{q \text{ cloud}} + \underbrace{p_B(\overline{Q}_{\bar{q}\bar{q}})}_{\bar{q}\bar{q} \text{ cloud}} = (1-3p_B)\overline{Q}_q.$$

- **Measured flow.** Subtract $\langle Q_D\rangle_f$ from Q_f and use $p_B \simeq \langle B_{\text{flow}}\rangle_f$:

$$\boxed{\langle Q_{\text{flow}}\rangle_f \simeq Q_f - \frac{1-r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\text{flow}}\rangle_f\right)}$$

Strangeness of the screening cloud

- **Flavor average.** Only the strange quark contributes, with $S_s = -1$:

$$\overline{S}_q = \underbrace{S_s}_{-1} \cdot \underbrace{\frac{r_s}{1+1+r_s}}_{\text{strange fraction}} = -\frac{r_s}{2+r_s}.$$

- **Channel average.** Use the same probabilities as for electric charge, with $\overline{S}_{\bar{q}\bar{q}} = -2\overline{S}_q$:

$$\langle S_D \rangle_f \simeq \underbrace{(1-p_B)\overline{S}_q}_{q \text{ cloud}} + \underbrace{p_B(\overline{S}_{\bar{q}\bar{q}})}_{\bar{q}\bar{q} \text{ cloud}} = (1-3p_B)\overline{S}_q.$$

- **Measured flow.** Subtract the endpoint strangeness from S_f:

$$\boxed{\langle S_{\text{flow}} \rangle_f \simeq S_f + \frac{3r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\text{flow}} \rangle_f\right).}$$

The jet Gell-Mann–Nishijima relation

- Combine the charge deficit with the strangeness excess to eliminate r_s:

$$2(Q_f - \langle Q_{\text{flow}} \rangle_f) + (\langle S_{\text{flow}} \rangle_f - S_f) \simeq \underbrace{\frac{2(1-r_s)+3r_s}{2+r_s}}_{=1} \left(\frac{1}{3} - \langle B_{\text{flow}} \rangle_f \right).$$

- Use $B_f = 1/3$ and $I_{3,f} = Q_f - (B_f + S_f)/2$:

$$\boxed{\langle Q_{\text{flow}} \rangle_f \simeq I_{3,f} + \frac{1}{2}\big(\langle B_{\text{flow}} \rangle_f + \langle S_{\text{flow}} \rangle_f\big)}.$$

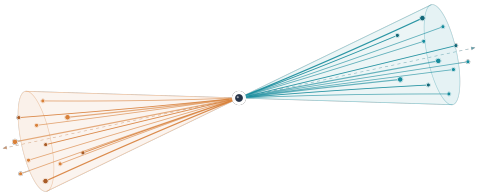
- Equal mean u, d screening gives $\langle I_{3,D} \rangle_f \simeq 0$: **the jet retains the initiating isospin.** Higher clouds can satisfy the same condition. GMN needs less than the two-channel probability extraction.

Monte Carlo tests of hemisphere flows

$e^+e^- \to q\bar{q}, \qquad q = u, d, s, \qquad T > 0.95$

Energy scan: $E_{\mathrm{CM}} = 50, 100, 200, 500, 1000$ GeV

Measurement: total Q, B, S in one hemisphere.
After QCD shower $\longrightarrow$ after hadronization.
QED and weak showers off; hadron decays disabled.



Hadronization parameters varied in the scans

$E_{\mathrm{CM}} = 100$ GeV

Generator	Strangeness	Baryon production
Pythia 8.312 Lund strings	`probStoUD` s-quark weight relative to u or d.	`probQQtoQ` Diquark weight relative to quarks.

Herwig 7.3
Clusters
	`PwtSquark`
	s-quark weight in cluster fission / decay.
	`SplitPwtSquark`
	Strange-quark weight in nonperturbative gluon splitting.
		`MesonToBaryonFactor`
		Relative competition between mesonic and baryonic reconnections.
		Joint scan: `BaryonicMesonic`.

The two Herwig strange-quark weights are varied together.

From Pythia parameters to the jet charge

Pythia input parameters

`probQQtoQ` $= 0.081$ `probStoUD` $= 0.217$

Two-channel prediction

$$\langle B_{\text{flow}} \rangle_f \simeq \frac{r_{qq}}{1 + r_{qq}} \simeq 0.075,$$

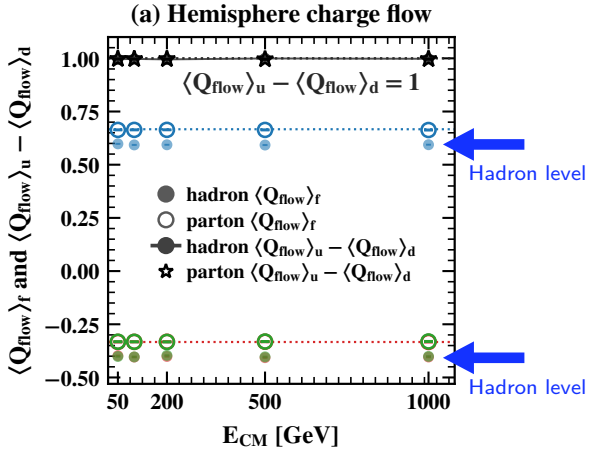
$$\langle Q_D \rangle_f \simeq \frac{1 - r_s}{2 + r_s} \left(\frac{1}{3} - \langle B_{\text{flow}} \rangle_f \right) \simeq 0.091.$$

Charge prediction

$$\langle Q_{\text{flow}} \rangle_f = Q_f - \langle Q_D \rangle_f$$

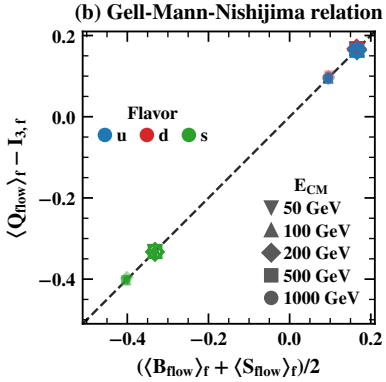
f	Input estimate	MC mean
u	0.575	0.594
d	-0.425	-0.403
s	-0.425	-0.401

MC: equal-weight mean over the five energies.



Monte Carlo test of the jet GMN relation

$$\langle Q_{\text{flow}}\rangle_f \simeq I_{3,f} + \frac{1}{2}\left(\langle B_{\text{flow}}\rangle_f + \langle S_{\text{flow}}\rangle_f\right).$$



Filled: hadron level; open: parton level.

Across flavors and energies

Parton and hadron points lie close to the same diagonal.

The initiating isospin is retained

$$\langle I_{3,\text{flow}}\rangle_f \simeq I_{3,f}.$$

Isospin-symmetric screening gives $\langle I_{3,D}\rangle_f \simeq 0$, while Q, B, and S each shift.

Pythia 8.312, $T > 0.95$.
Herwig gives the same pattern (backup).

Strangeness controls the charge–baryon slope

$E_{\mathrm{CM}} = 100$ GeV, $\quad T > 0.95$

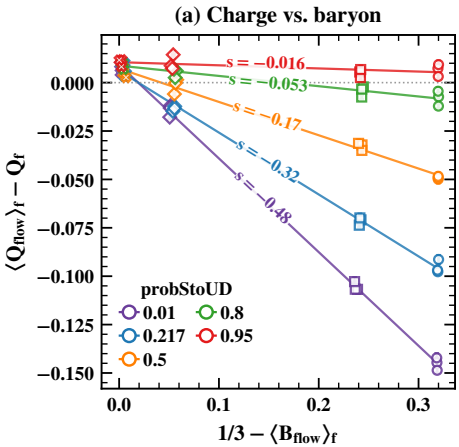
Read the colors: vary strange production

Use $r_s \simeq$ `probStoUD` to predict the slope:

$$\langle Q_{\mathrm{flow}} \rangle_f - Q_f \simeq -\tfrac{1-r_s}{2+r_s} \left(\tfrac{1}{3} - \langle B_{\mathrm{flow}} \rangle_f \right).$$

`probStoUD`	Input estimate	MC slope
0.010	−0.493	−0.484
0.217	−0.353	−0.320
0.500	−0.200	−0.172
0.800	−0.071	−0.053
0.950	−0.017	−0.016

Larger r_s gives flatter slopes.



Baryon production controls the jet baryon number

Read the markers: vary diquark production

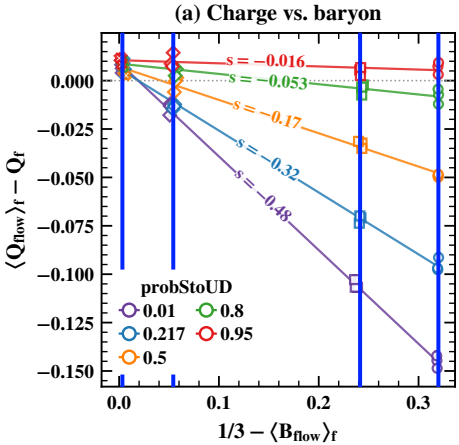
Use $r_{qq} \simeq$ `probQQtoQ` as the input estimate:

$$\langle B_{\mathrm{flow}} \rangle_f \simeq \frac{r_{qq}}{1 + r_{qq}}.$$

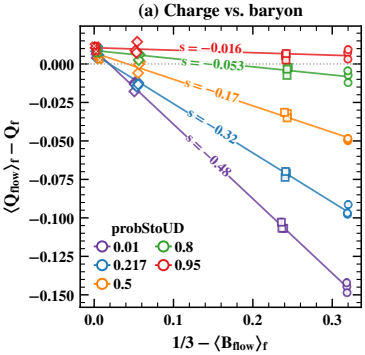
	`probQQtoQ`	Input estimate	MC $\langle B_{\mathrm{flow}} \rangle_f$
◯	0.010	0.0099	0.0139
□	0.081	0.0749	0.0922
◇	0.500	0.3333	0.2790
×	0.800	0.4444	0.3298

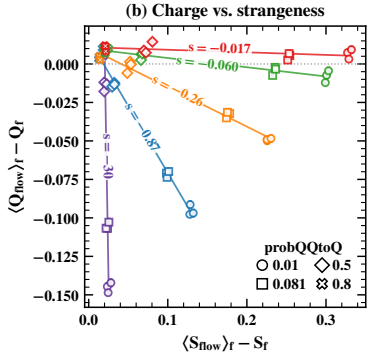
MC: equal-weight mean over five colors and three flavors.

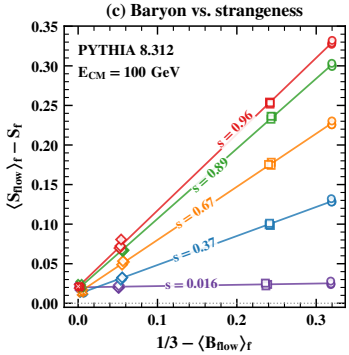
Fixed marker: $\langle B_{\mathrm{flow}} \rangle_f$ barely changes with strangeness.



Pythia: joint Q, B, S correlations







$$\langle Q_{\rm flow}\rangle_f - Q_f \simeq -\frac{1-r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\rm flow}\rangle_f\right)$$

$$\langle Q_{\rm flow}\rangle_f - Q_f \simeq -\frac{1-r_s}{3r_s}\left(\langle S_{\rm flow}\rangle_f - S_f\right)$$

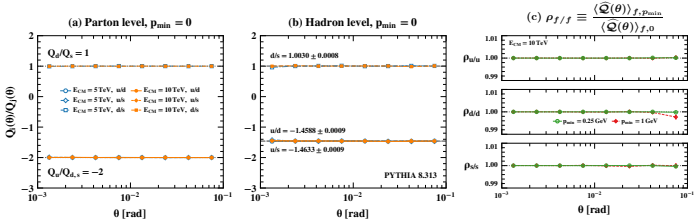
$$\langle S_{\rm flow}\rangle_f - S_f \simeq \frac{3r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\rm flow}\rangle_f\right)$$

Colors: `probStoUD`; markers: `probQQtoQ`. $E_{\rm CM} = 100$ GeV, $T > 0.95$; free-intercept fits.

Charge correlators

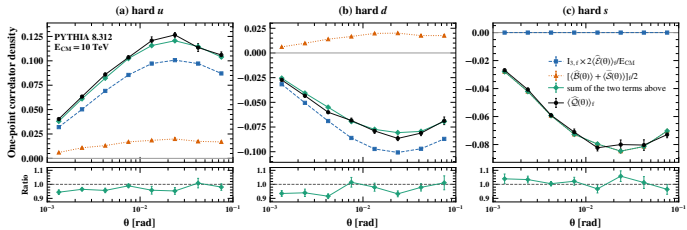
See arXiv:2608.19708 for more results: Herwig comparisons and conserved-charge correlators.

Flavor ratios & momentum cuts



Insensitive to momentum cutoffs of order Λ_{QCD}.

Differential GMN



Summary

- **Conserved currents provide a perturbative baseline.**
  At leading power, charge-flow shifts measure nonperturbative endpoint screening.
- **Joint Q, B, S flows resolve the screening content.**
  With minimal clouds and common flavor weights, B fixes r_{qq} and correlated Q, S shifts fix r_s. Pythia and Herwig support these relations.

Jet GMN follows from isospin-symmetric screening

$$\boxed{\langle Q_{\text{flow}}\rangle_f \simeq I_{3,f} + \frac{1}{2}\big(\langle B_{\text{flow}}\rangle_f + \langle S_{\text{flow}}\rangle_f\big)} \qquad \big(\langle I_{3,D}\rangle_f \simeq 0\big).$$

A further test with heavy-flavor jets Net charm and bottomness survive hadronization. With the same light-flavor screening, before weak decays,

$$\langle \mathcal{Q}_{\text{flow}}\rangle_c \simeq \langle \mathcal{Q}_{\text{flow}}\rangle_u, \qquad \langle \mathcal{Q}_{\text{flow}}\rangle_b \simeq \langle \mathcal{Q}_{\text{flow}}\rangle_d, \qquad \mathcal{Q} = Q, B, S.$$

Thank you!

Extensions and Experimental Availability

Additional screening mechanisms

- **Color reconnection and recombination**
  Allow screening by surrounding partons.
- **Beam remnants and baryon junctions**
  Modify B transport and Q–B correlations.

Christiansen–Skands (2015); Fries et al. (2003); Kharzeev (1996); STAR, 2408.15441.

What joint Q, B, S flows can test

- Track strange and baryonic screening versus multiplicity in pp, pA and AA.
- Strangeness and baryon-to-meson enhancements motivate testing a possible thermodynamic limit.

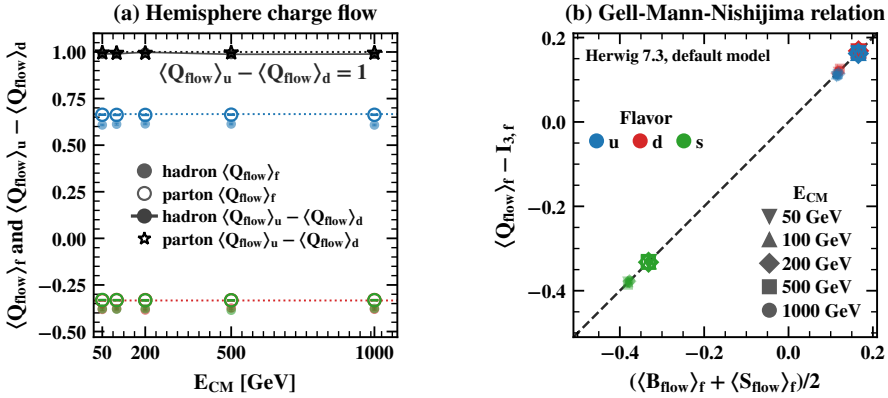
ALICE, Nature Phys. 13 (2017) 535; PRC 99 (2019) 024906; JHEP 09 (2024) 102.

Experimental starting points

Identified-hadron measurements at LEP, RHIC and the LHC provide a basis for joint-flow studies.
DELPHI, EPJC 17 (2000) 207; SLD, PRD 59 (1999) 052001; ALICE, JHEP 09 (2024) 102.

Recent work measures electric-charge flow at DELPHI and proposes heavy-flavor flow measurements.
Zhang, Lee, Moult et al., 2608.14779, 2608.15357, 2608.15367; Lee–Moult, 2609.13366.

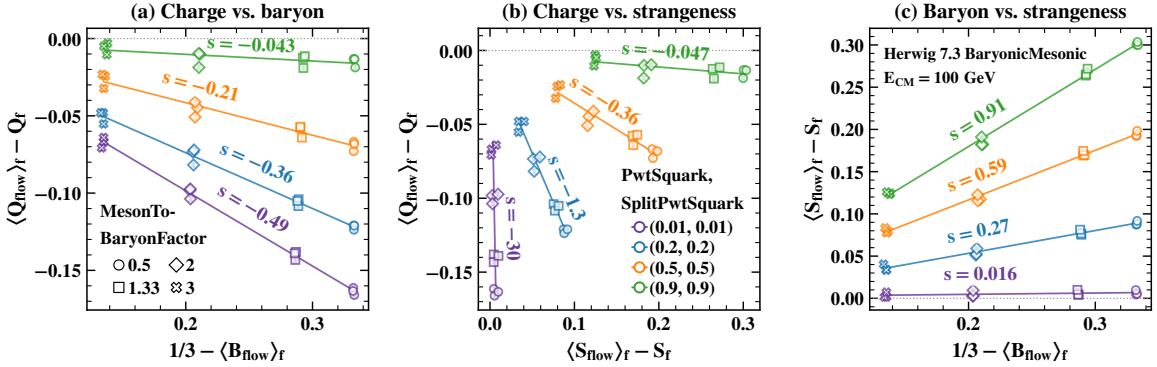
Herwig: charge shifts and jet GMN



Hadron level: $(\langle Q_{\text{flow}}\rangle_u, \langle Q_{\text{flow}}\rangle_d, \langle Q_{\text{flow}}\rangle_s) \simeq (0.611, -0.379, -0.379)$.

Equal-weight mean over $E_{\text{CM}} = 50, 100, 200, 500, 1000$ GeV.

Herwig: joint Q, B, S correlations



$$\langle Q_{\rm flow}\rangle_f - Q_f \simeq -\frac{1-r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\rm flow}\rangle_f\right)$$

$$\langle Q_{\rm flow}\rangle_f - Q_f \simeq -\frac{1-r_s}{3r_s}\left(\langle S_{\rm flow}\rangle_f - S_f\right)$$

$$\langle S_{\rm flow}\rangle_f - S_f \simeq \frac{3r_s}{2+r_s}\left(\frac{1}{3} - \langle B_{\rm flow}\rangle_f\right)$$

`BaryonicMesonic`; matched strange weights; varying `MesonToBaryonFactor`. $E_{\rm CM} = 100$ GeV, $T > 0.95$.

Charge correlators: the OPE connection

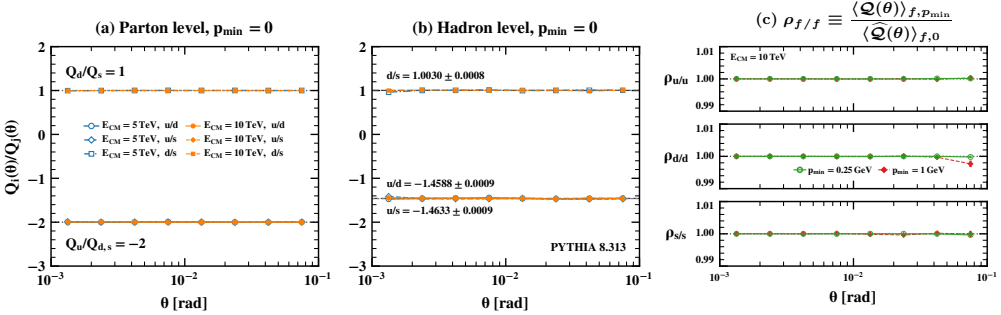
The OPE in the perturbative small-angle region gives

$$\langle \widehat{\mathcal{Q}}(\theta) \rangle_f \simeq \underbrace{\mathcal{K}_q(\theta)}_{\text{common angular shape}} \underbrace{\langle \mathcal{Q}_{\text{flow}} \rangle_f}_{\text{measured normalization}}, \qquad \mathcal{Q} = Q, B, S.$$

- $\mathcal{K}_q(\theta)$ is the common perturbative angular function decided by OPE and Sudakov evolutions, see W. Li, X. Liu and D. Y. Shao, arXiv:2607.16392.
- Hadronization modifies total measured charge flow.
- The common factor cancels in flavor ratios. The integrated Q, B, S relations therefore also govern the angular distributions.

Charge correlators and momentum acceptance

$$\frac{\langle \widehat{Q}(\theta) \rangle_{f_1}}{\langle \widehat{Q}(\theta) \rangle_{f_2}} \simeq \frac{\langle Q_{\text{flow}} \rangle_{f_1}}{\langle Q_{\text{flow}} \rangle_{f_2}}, \qquad \theta \ll 1$$



(a, b) Flavor ratios

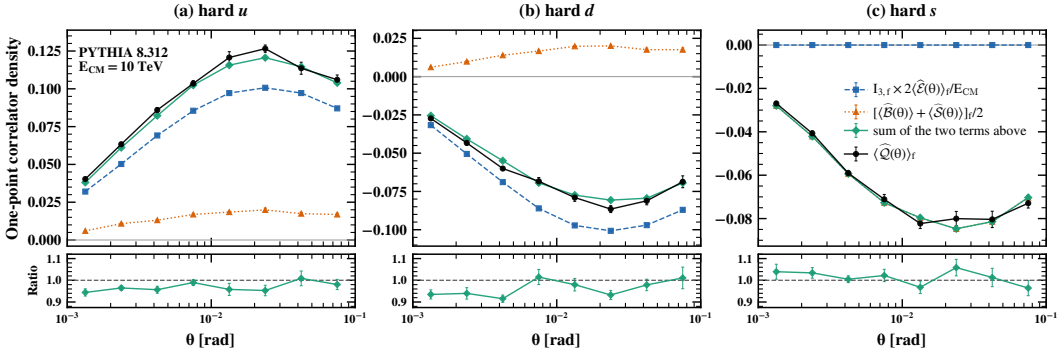
$u/d,\ u/s:\ -2 \to \simeq -1.46; \quad d/s \simeq 1.$

(c) Hadron acceptance momentum

$p_{\text{min}} = 0,\ 0.25,\ 1$ GeV.

A differential test of the GMN relation

$$\langle \widehat{Q}(\theta) \rangle_f \simeq I_{3,f} \frac{\langle \widehat{\mathcal{E}}(\theta) \rangle_f}{E_{\mathrm{CM}}/2} + \frac{\langle \widehat{B}(\theta) \rangle_f + \langle \widehat{S}(\theta) \rangle_f}{2}$$



Energy-correlator normalization at LL accuracy. $E_{\mathrm{CM}} = 10$ TeV; no thrust cut.

Backup: Pythia strange-screening ratios

Input parameter	Extracted from correlations		
probStoUD	$r_s^{Q\text{-}B}$	$r_s^{Q\text{-}S}$	$r_s^{S\text{-}B}$
0.010	0.0217(59)	0.01105(61)	0.01080(69)
0.217	0.273(13)	0.2766(61)	0.2785(25)
0.500	0.560(15)	0.566(11)	0.5771(31)
0.800	0.8489(64)	0.8477(61)	0.8381(57)
0.950	0.952(17)	0.952(16)	0.948(11)

- The three determinations show broadly consistent trends.
- The effective ratio tracks the input, with visible model-dependent shifts.

Parentheses: propagated one-standard-error uncertainties.

Backup: Herwig strange-screening ratios

Input parameters	Extracted from correlations		
Matched strange weights	$r_s^{Q\text{-}B}$	$r_s^{Q\text{-}S}$	$r_s^{S\text{-}B}$
0.01	0.0177(65)	0.0110(16)	0.0104(14)
0.2	0.2047(33)	0.1996(23)	0.1975(38)
0.5	0.479(27)	0.483(23)	0.486(16)
0.9	0.876(35)	0.878(34)	0.876(34)

- Matched weights: `PwtSquark` = `SplitPwtSquark`.
- The correlations test a common screening description across two generators.

Parentheses: propagated one-standard-error uncertainties.

Backup: Pythia baryonic screening and topology

$$r_{qq} \simeq r_B \equiv \frac{\langle B_{\text{flow}} \rangle_f}{1 - \langle B_{\text{flow}} \rangle_f}, \qquad r_{\text{topology}} = \frac{P(B_{\text{flow}} = 1)}{P(B_{\text{flow}} = 0)}$$

Input parameter | Measured flow | Screening ratios

`probQQtoQ`	$\langle B_{\text{flow}} \rangle_f$	r_B	r_{topology}
0.010	0.0139	0.01410(9)	0.01526(9)
0.081	0.0922	0.1015(3)	0.1102(3)
0.500	0.2790	0.3869(7)	0.4215(6)
0.800	0.3298	0.4921(8)	0.5341(8)

- `probQQtoQ` weights eligible local steps; r_B includes the full fragmentation history.
- r_B and r_{topology} agree within 9%, supporting the two-channel approximation.