Shower Shapes and Data-Driven Simulation Studies using a Highly Granular Crystal ECAL

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on behalf of the AGORA/CEPC Calorimetry Team

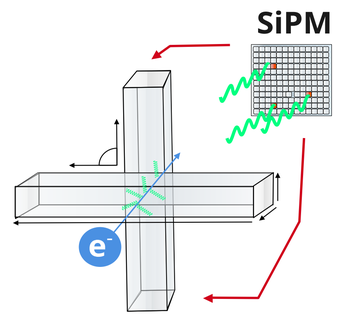
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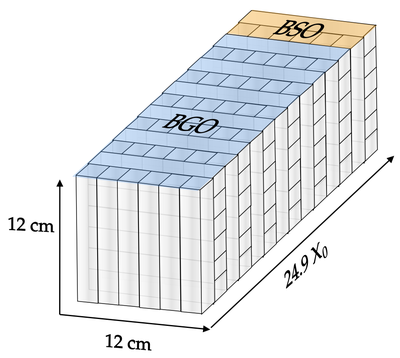
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Overview

Highly Granular Crystal ECAL

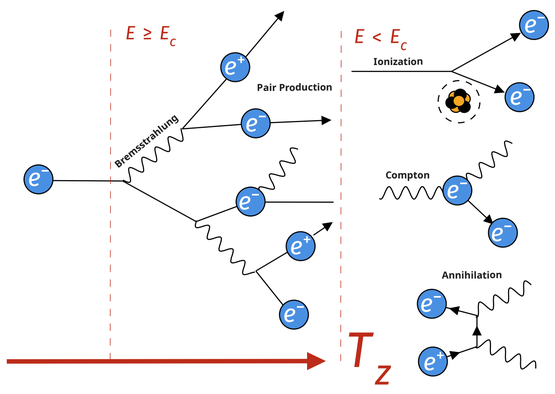
- **criss-crossing scintillating crystal bars [1]:**
  - CEPC detector volume: 1.5 × 1.5 × 40 cm³
  - prototype volume: 2 × 2 × 12 cm³
  - homogeneous;
  - crystal: BGO/BSO;
- **granularity:**
  - particle separation capability
  - energy density information;
- **excellent EM energy resolution: stochastic term ~1-2%**





Shower Shapes

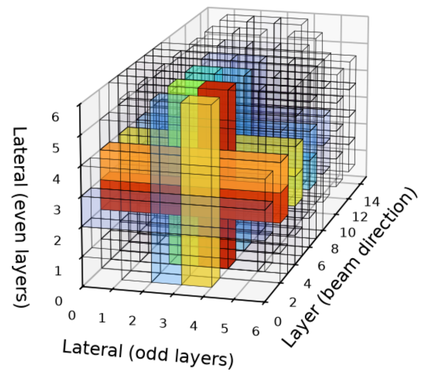
- **Shower shapes can be used for:**
  - studying shower fluctuations;
  - measuring absorber properties;
  - synthetic shower generation;
- **Gap in literature for shower shapes <1 GeV.**



Summary of Studies

2025 Testbeam Data

- Taken at Proton Synchrotron (PS) T9 Beamline;
- Electrons, 0.2-6 GeV
- ~2M electron events;
- PID used to select electrons;

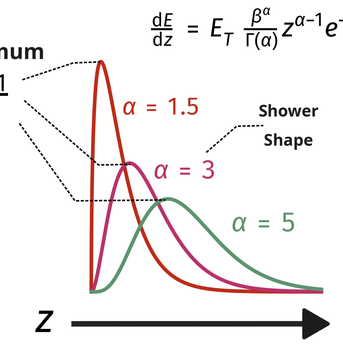


Fitted Shower Shape Models

- **Longitudinal Distribution: Gamma [2]**

$$\frac{dE}{dz} = E_T \frac{\beta^\alpha}{\Gamma(\alpha)} z^{\alpha-1} e^{-\beta z}$$

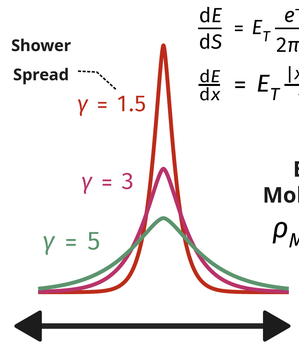
Shower Maximum $T_z = \frac{\alpha - 1}{\beta}$



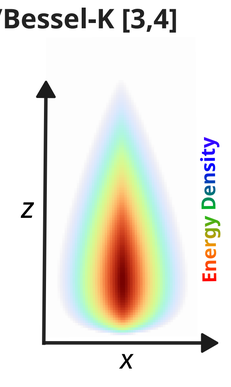
- **Lateral Distribution: Exponential/Bessel-K [3,4]**

$$\frac{dE}{dS} = E_T \frac{e^{-\frac{r}{\gamma}}}{2\pi\gamma^2}$$

$$\frac{dE}{dx} = E_T \frac{|x-c|}{\pi\gamma^2} K_1\left(\frac{|x-c|}{\gamma}\right)$$



Effective Molière Radius $\rho_M \simeq 3.89\gamma$

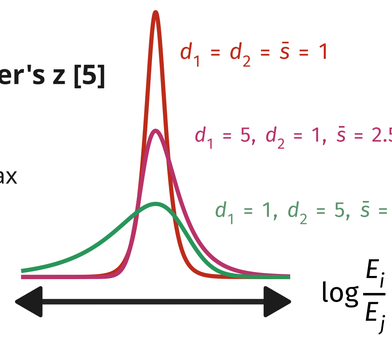


- **Log Energy Fraction Distribution: Fisher's z [5]**
  (Difference of Log Gamma Variates)

Log ratios related to energy fractions by Softmax

$[f_0, f_1 \cdots f_D] =$

$\text{Softmax}\left(\left[\log\frac{E_0}{E_D}, \log\frac{E_1}{E_D}, \cdots \log\frac{E_{D-1}}{E_D}, 0\right]\right)$

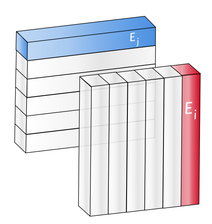


$Y = \mu + \bar{s} \log R$

$R = \frac{X_1/d_1}{X_2/d_2}$

$X_1 \sim \chi^2_{d_1}$

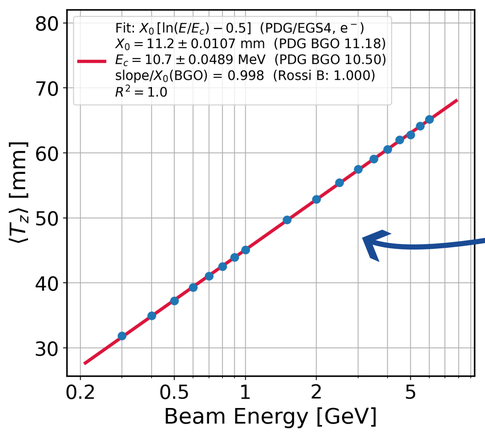
$X_2 \sim \chi^2_{d_2}$



Key Results

Physics Measurements of BGO and energy fractions

Mean T_z vs beam energy



Key Result:

$$\langle T_z \rangle = X_0 \left(\log\left(\frac{E}{E_c}\right) - 0.5 \right)$$

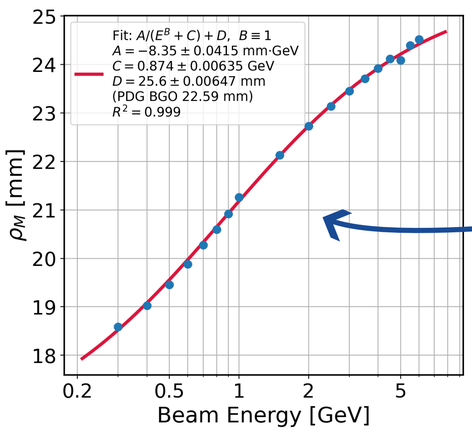
- **Radiation Length (X_0):**
  - Distance after which electron loses ~64% of its total energy;

PDG, BGO	Measurement
11.18 mm	11.2 ± 0.01 mm

- **Critical Energy (E_c):**
  - Below critical energy, EM shower extinction more probable than multiplication

PDG, BGO	Measurement
10.5 MeV	10.7 ± 0.05 MeV

Mean ρ_M vs beam energy



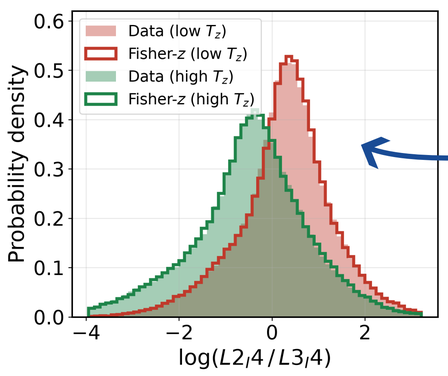
Key Result:

$$\langle \rho_M \rangle = \frac{A}{(E^B + C)} + D$$

- **Effective Molière Radius (D):**
  - Radius containing 90% of energy;
  - Larger than theory due to granularity/energy threshold

PDG, BGO	Measurement
22.59 mm	25.6 ± 0.06 mm

Log energy ratios conditioned on shower shape (electrons, E = 1 GeV)

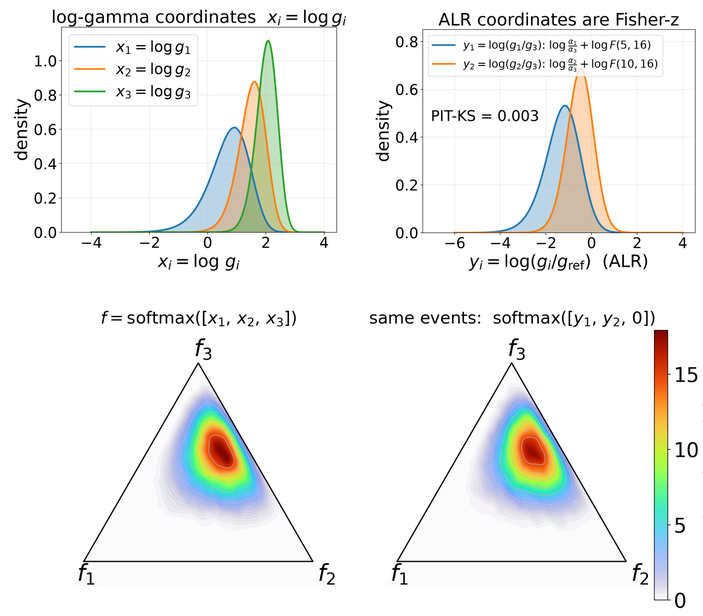


Key Result:

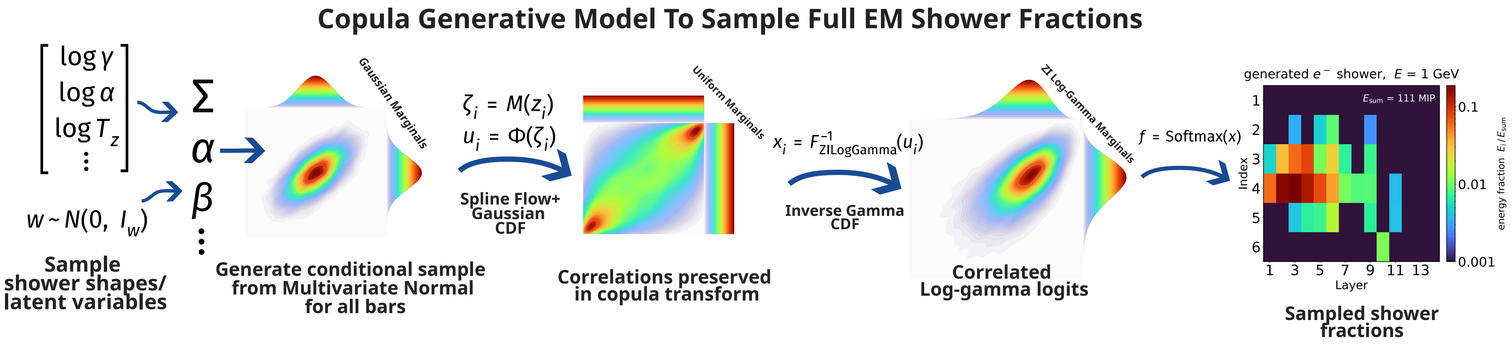
- Log fraction distribution fitted by Fisher's z distribution;
- Log fraction fully conditioned on shower shape
- Scale indicates correlation between cells;

Generative Model using energy fraction distribution

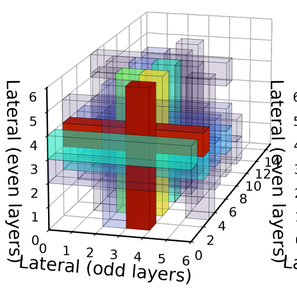
- A fixed shower shape means log energy ratios drawn from Fisher's z distribution;
- Softmax maps log ratios to energy fractions;
- Equivalent to Softmax of zero-inflated log-Gamma variates;
- **Fast shower simulation method:**
  - *Generate correlated log-Gamma variates;*
  - *Apply Softmax operation;*
  - *Obtain energy fractions for each bar;*
- **Correlations from Gaussian Copula method;**
- **Fit of marginal/correlation matrix with censored log-likelihood for 'dark' bars;**



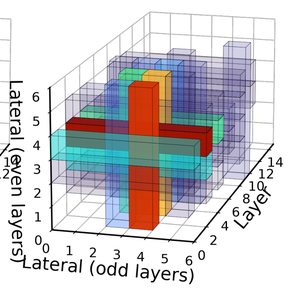
Copula Generative Model To Sample Full EM Shower Fractions

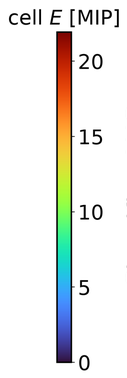


Data



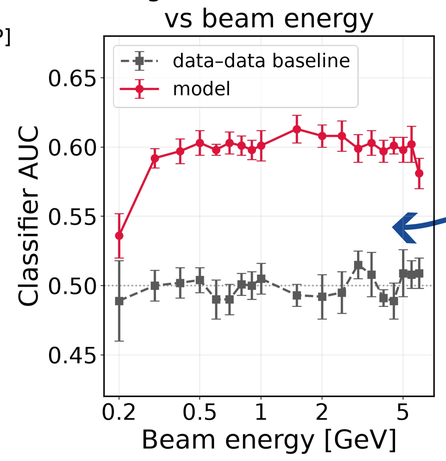
Model





$E = 1$ GeV, $E_{\text{sum}} = 111$ MIP

Data–generated classifier AUC vs beam energy



Key Result:

- DNN Classifier can only distinguish generated vs. real up to AUC=0.6;
- AUC within CaloChallenge 2022 review [6] top-5 range for electrons
- Global shower variables reconstructed with excellent agreement;
- Average generation time on GPU: 33 µs

[1] The CEPC Study Group, CEPC Technical Design Report -- Reference Detector, arXiv:2510.05260 [hep-ex] (2025), https://arxiv.org/abs/2510.05260.
[2] Grindhammer, G. & Peters, S., The Parameterized Simulation of Electromagnetic Showers in Homogeneous and Sampling Calorimeters, arXiv:hep-ex/0001020 (2000), DOI: 10.48550/arXiv.hep-ex/0001020
[3] O.L. Pinto, Shower Shapes in a Highly Granular SiPM-on-Tile Analog Hadron Calorimeter, PhD thesis, Universität Hamburg (2022), https://ediss.sub.uni-hamburg.de/handle/ediss/9855
[4] Bonamente, M., Properties and Approximations of a Bessel Distribution for Data Science Applications, Int. J. Stat. Distrib. Appl. 11(3) (2025) 85–97, DOI: 10.11648/j.ijsda.20251103.11
[5] Fisher, R.A., On a distribution yielding the error functions of several well known statistics, Proc. Int. Congr. Math., Toronto, vol. 2, pp. 805–813 (1924)
[6] Krause, C. et al., CaloChallenge 2022: a community challenge for fast calorimeter simulation, Rep. Prog. Phys. 88 (2025) 116201, DOI: 10.1088/1361-6633/ae1304