The 2026 International Workshop on the High Energy Circular Electron Positron Collider

Nonlinear Track Reconstruction Studies towards UKF-Based Tracking for CEPC

ACTS CKF · FreeToBoundCorrection · Unscented Kalman Filter

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Motivation: towards better nonlinear tracking

Can we improve track reconstruction by treating nonlinear uncertainty more effectively?

01 **Starting point**

ACTS CKF uses an EKF-like, Jacobian-based formulation.

02 **What we investigated**

Searching for better nonlinear treatment led us to FTBC in ACTS—and a missing CKF forwarding path.

03 **Where it led**

That investigation motivated our standalone UKF approach for CEPC track reconstruction.

Research direction | From FTBC sensitivity to sigma-point nonlinear estimation

Where FTBC sits in the CKF

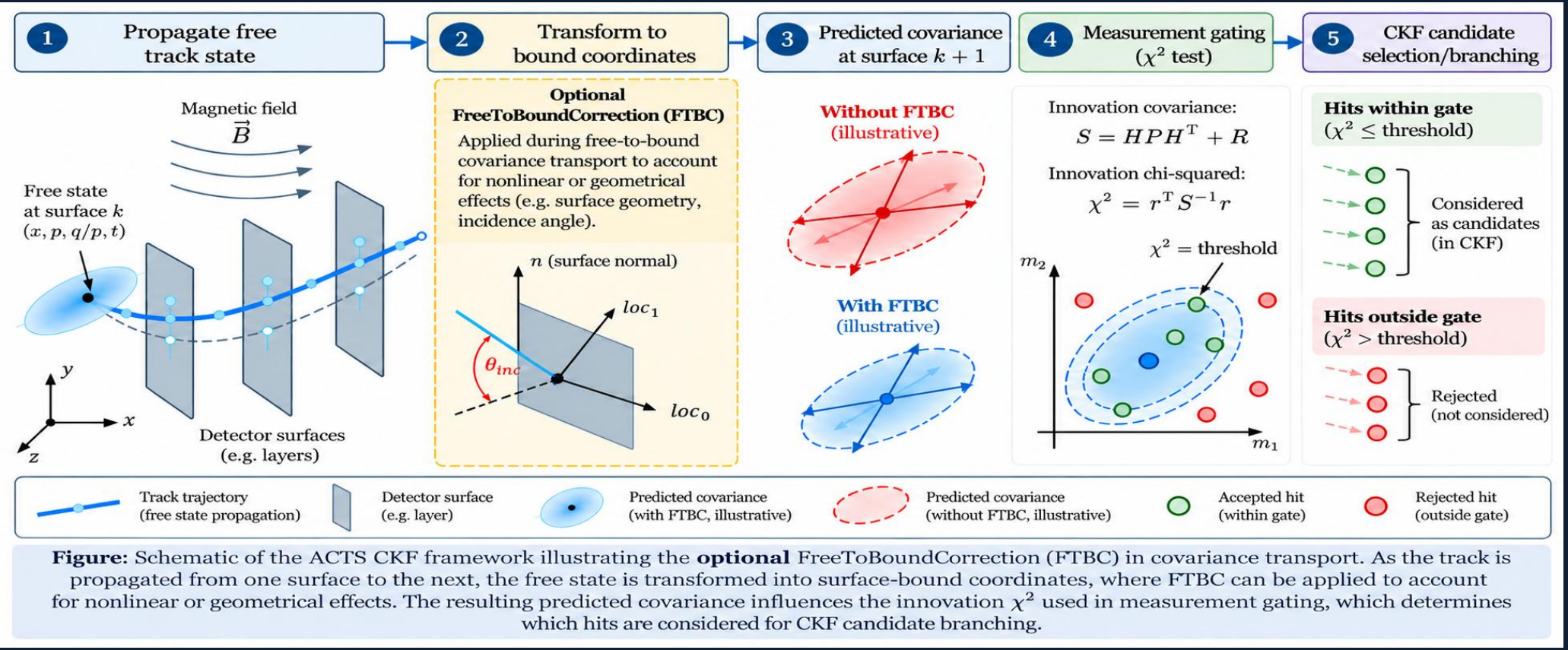


Figure: Schematic of the ACTS CKF framework illustrating the **optional** FreeToBoundCorrection (FTBC) in covariance transport. As the track is propagated from one surface to the next, the free state is transformed into surface-bound coordinates, where FTBC can be applied to account for nonlinear or geometrical effects. The resulting predicted covariance influences the innovation χ^2 used in measurement gating, which determines which hits are considered for CKF candidate branching.

- Propagate a free track state through the field to the next surface.
- Transform the free state and covariance into surface-bound coordinates.
- FTBC modifies this covariance transport near nonlinear or geometrically sensitive transformations.
- The resulting predicted covariance enters innovation χ^2, gating and candidate selection.

FTBC investigation: implementation finding

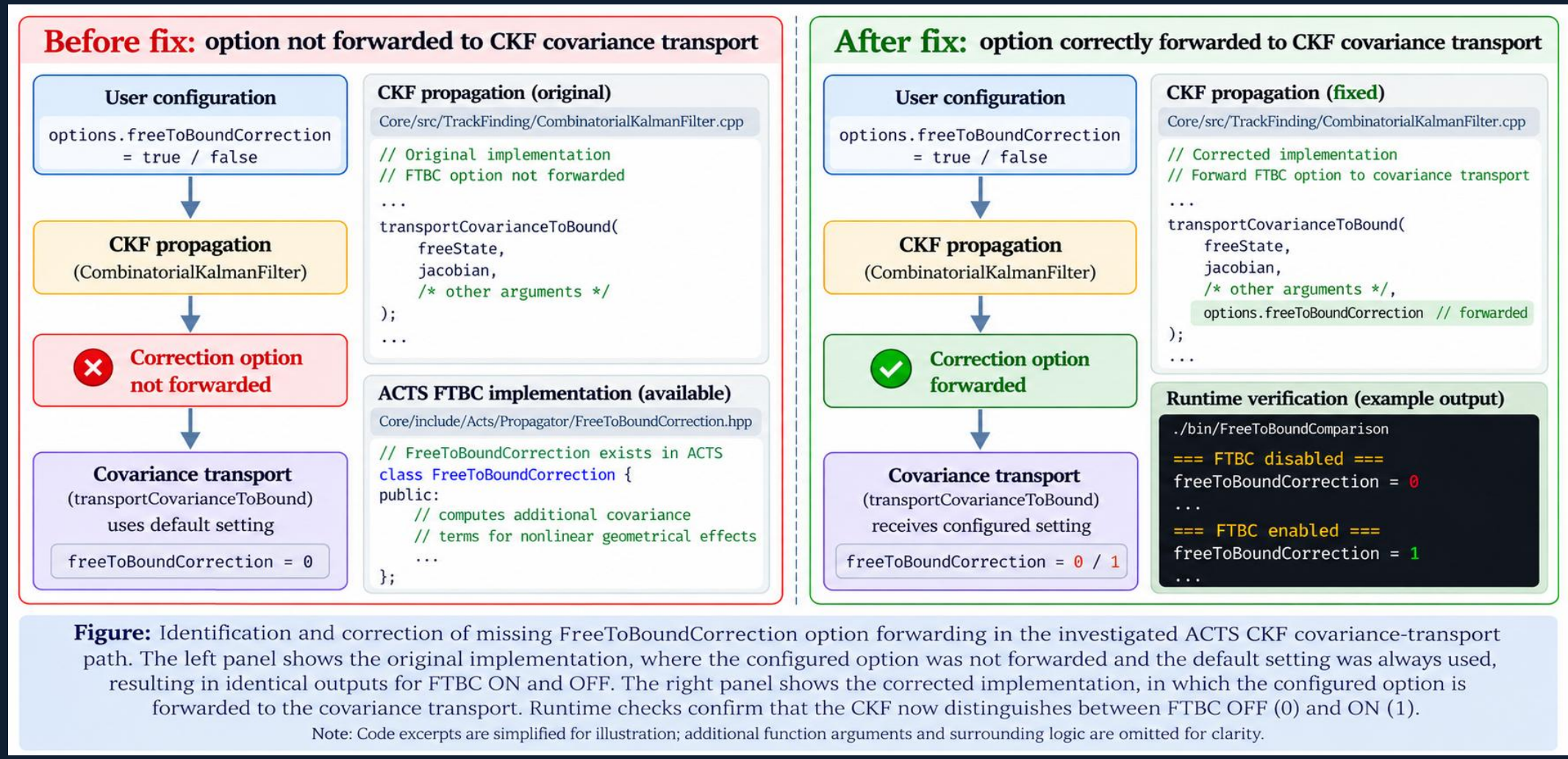


Figure: Identification and correction of missing FreeToBoundCorrection option forwarding in the investigated ACTS CKF covariance-transport path. The left panel shows the original implementation, where the configured option was not forwarded and the default setting was always used, resulting in identical outputs for FTBC ON and OFF. The right panel shows the corrected implementation, in which the configured option is forwarded to the covariance transport. Runtime checks confirm that the CKF now distinguishes between FTBC OFF (0) and ON (1).

Note: Code excerpts are simplified for illustration; additional function arguments and surrounding logic are omitted for clarity.

- FreeToBoundCorrection (FTBC) was already implemented in ACTS, but its configuration was not being forwarded to the CKF covariance-transport routine in the investigated code path. Correcting this enabled controlled FTBC ON/OFF comparisons.

Why covariance can change CKF decisions

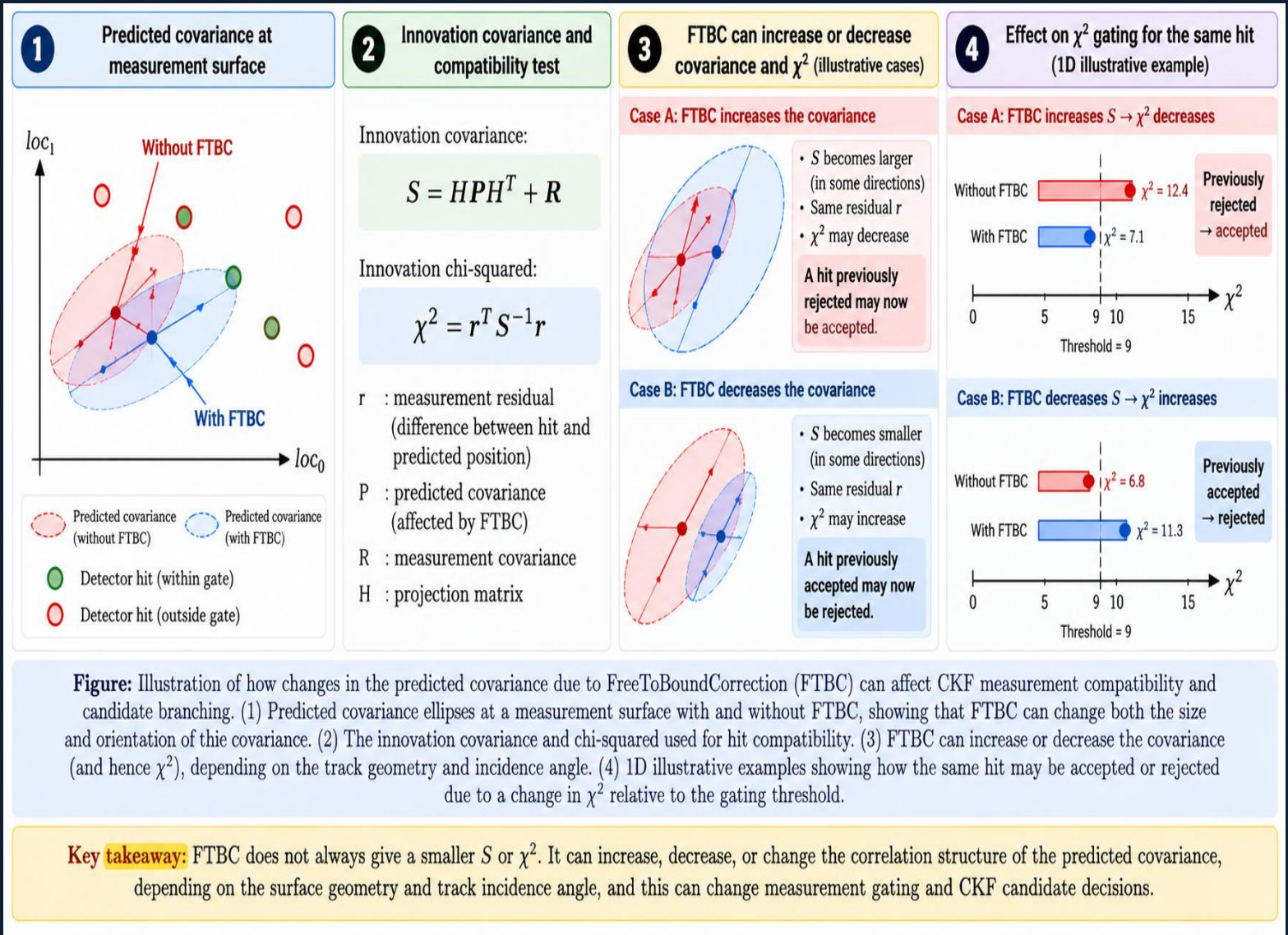


Figure: Illustration of how changes in the predicted covariance due to FreeToBoundCorrection (FTBC) can affect CKF measurement compatibility and candidate branching. (1) Predicted covariance ellipses at a measurement surface with and without FTBC, showing that FTBC can change both the size and orientation of thie covariance. (2) The innovation covariance and chi-squared used for hit compatibility. (3) FTBC can increase or decrease the covariance (and hence χ^2), depending on the track geometry and incidence angle. (4) 1D illustrative examples showing how the same hit may be accepted or rejected due to a change in χ^2 relative to the gating threshold.

Key takeaway: FTBC does not always give a smaller S or χ^2. It can increase, decrease, or change the correlation structure of the predicted covariance, depending on the surface geometry and track incidence angle, and this can change measurement gating and CKF candidate decisions.

- Innovation covariance: $S = H P H^T + R$.
- Candidate compatibility: $\chi^2 = r^T S^{-1} r$, with residual r.
- FTBC can change P, therefore S and χ^2, even if the nominal state barely changes.
- Near an acceptance threshold, a small χ^2 shift may change hit selection or branching.

Experimental observations: FTBC ON vs OFF

0 T toy study • incidence-dependent χ^2 response

Cases with lower χ^2 (FTBC ON)



14 cases

Cases with higher χ^2 (FTBC ON)



22 cases

Mean $\Delta\chi^2$ (ON – OFF) ≈ $+7.93 \times 10^{-5}$

FTBC did not consistently reduce χ^2; the response depends on track geometry and incidence angle.

Reported toy sample • χ^2 changes do not by themselves establish hit-association switching

Can UKF be better?

- FTBC revealed sensitivity of CKF covariance transport to nonlinear surface geometry and incidence angle.
- FTBC addresses a local free-to-bound transformation; Jacobian-based transport remains a first-order approximation.
- UKF propagates sigma points through actual nonlinear dynamics to reconstruct the predicted mean and covariance.
- Objective: test whether sigma-point propagation improves momentum estimation and uncertainty consistency in CEPC.

Research question | Can UKF better capture nonlinear uncertainty in CEPC tracking?

Standalone CEPC UKF implementation

- Six-dimensional bound state: loc0, loc1, ϕ, θ, q/p and time– n=6.
- 13 sigma points (2n+1); α = 0.7, β = 2, κ = 0 (scaling parameters).
- ACTS propagation for each sigma point for weighted mean and covariance.
- Circular handling of ϕ, 2D local-position updates and covariance validation.

CEPC benchmark configuration

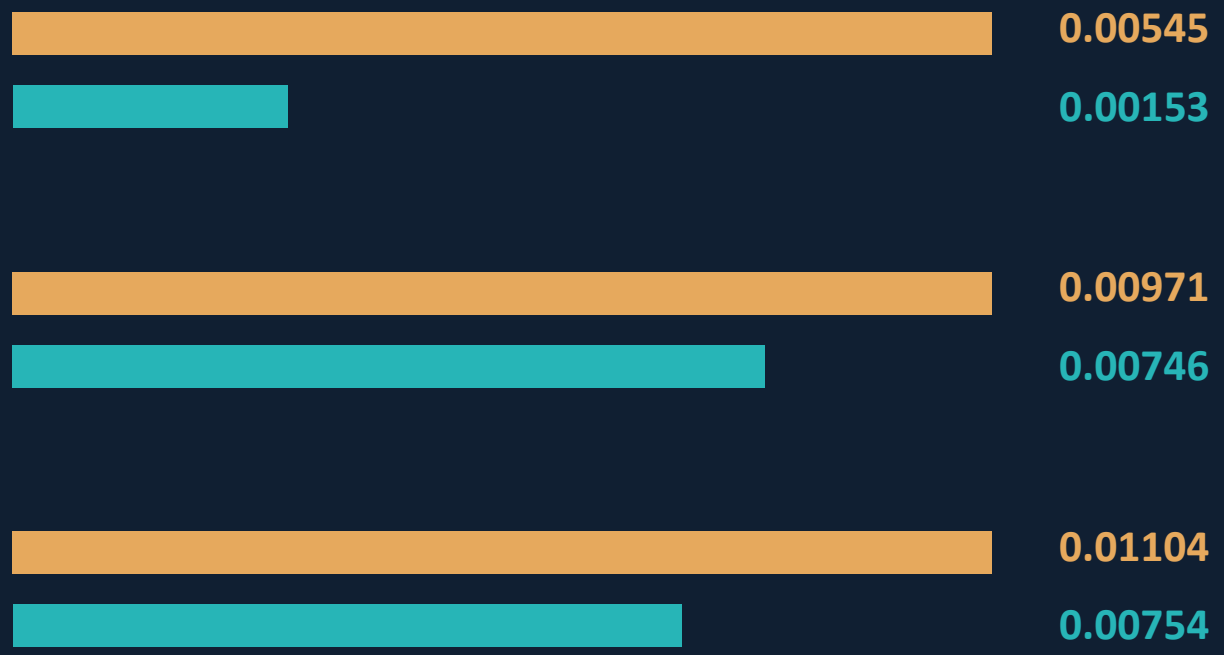
- ACTS v35.2.0 with CEPC TGeo geometry (~2,090 detector elements).
- Uniform axial magnetic field Bz = 3 T.
- Nominal muon truth momentum 10 GeV/c; initial seed 9 GeV/c.
- 234 noisy 2D local measurements; σ = 0.1 mm.

Study design: 50 independent simulated events

- 50 independent measurement-noise realisations of one nominal 10 GeV/c particle trajectory.
- Same detector geometry and 3 T magnetic field; random seeds 421262–421311.
- Each CKF–UKF pair uses the same simulated measurements (234 per run).
- Both filters produced momentum estimates; both processed all 234 measurements in each run.

Momentum reconstruction: observed results

Error metrics across 50 paired simulations



Mean momentum: CKF 9.994555 | UKF 9.998472 GeV/c (truth: 10 GeV/c)

Conclusions and next steps: towards UKF-based track finding

WHAT WE LEARNED

- FTBC changes local covariance transport; χ^2 responses depend on surface geometry and incidence angle.
- Near a gating threshold, covariance changes may alter hit acceptance and CKF branching.
- Standalone UKF showed promising momentum metrics in a preliminary, paired CEPC benchmark.

NEXT: TEST ASSOCIATION SWITCHING DIRECTLY

- Test UKF-based prediction and covariance within combinatorial hit association—not only fixed-hit fitting.
- Compare association stability, efficiency, fake-hit rate, accuracy and runtime across CEPC conditions.