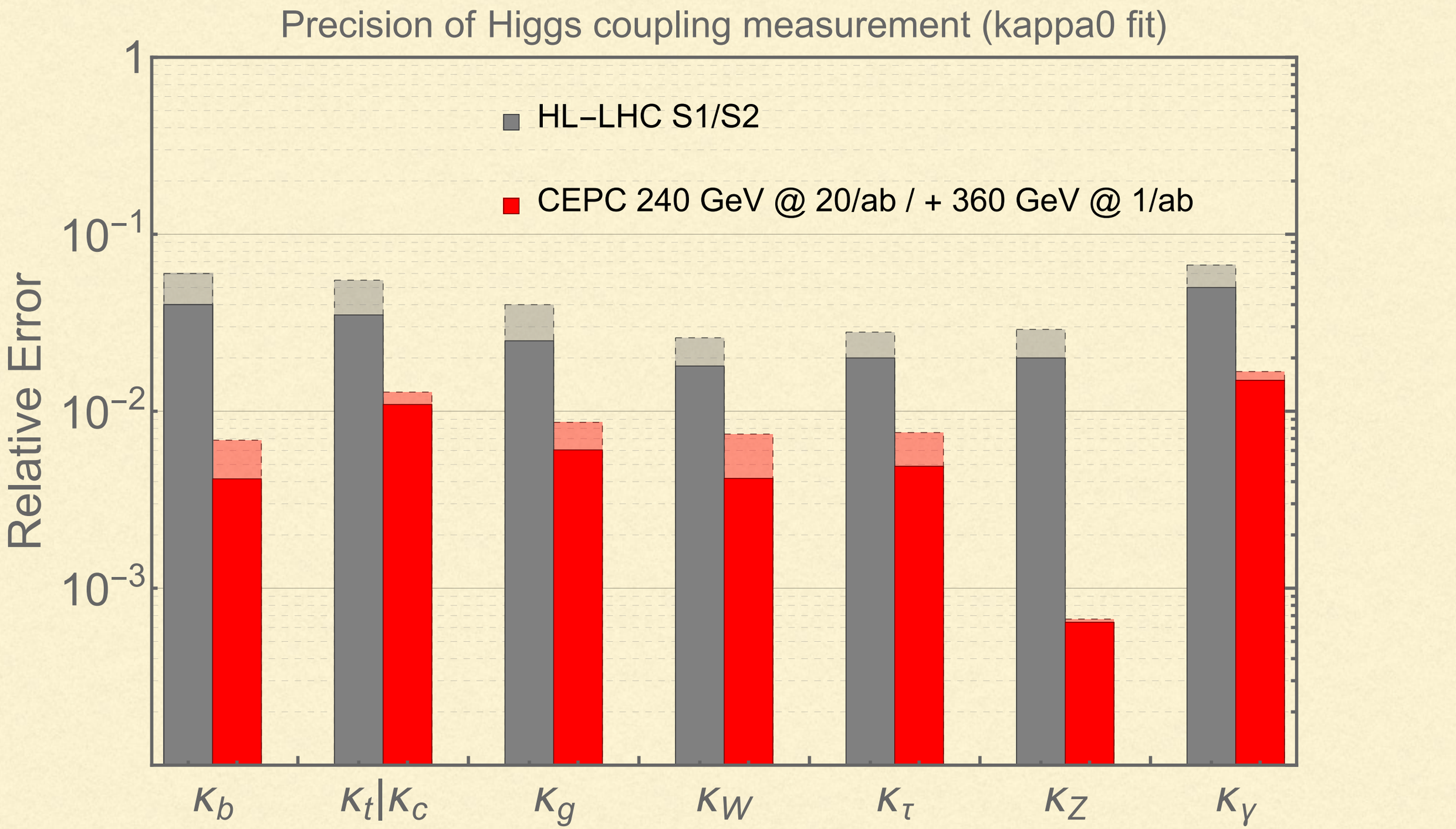
# PRECISE PREDICTION FOR HIGGS DECAY

Jian Wang
Shandong University

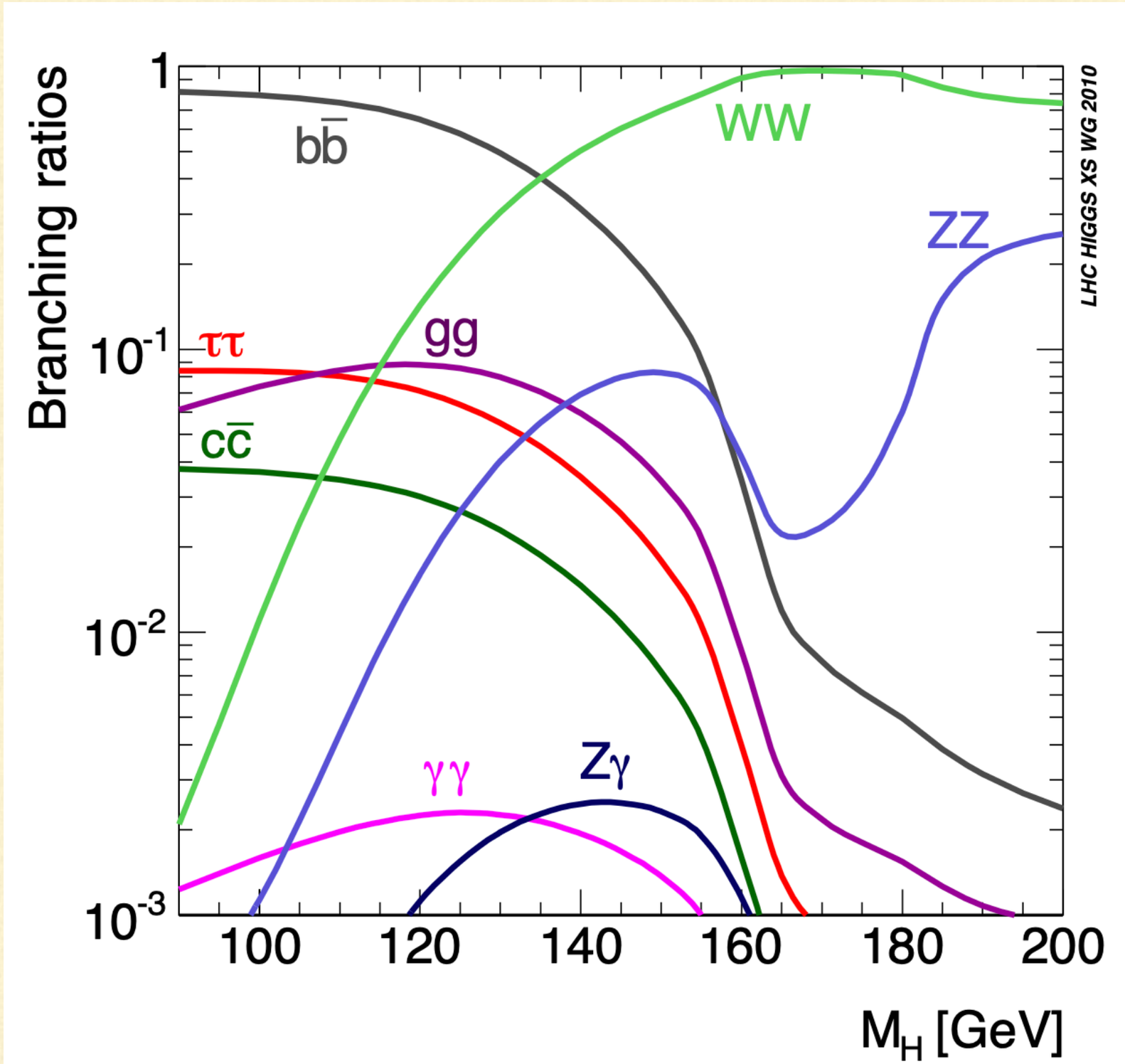
International Workshop on the High Energy Circular Electron Positron Collider
2026.10.22 Shanghai

| | 240 GeV, 20 $ab^{-1}$ | | 360 GeV, 1 $ab^{-1}$ | | |
|---|---|---|---|---|---|
| | ZH | **vvH** | ZH | **vvH** | **eeH** |
| inclusive | **0.26%** | | **1.40%** | \ | \ |
| H→bb | **0.14%** | **1.59%** | **0.90%** | **1.10%** | **4.30%** |
| H→cc | **2.02%** | | **8.80%** | **16%** | **20%** |
| H→gg | **0.81%** | | **3.40%** | **4.50%** | **12%** |
| H→WW | **0.53%** | | **2.80%** | **4.40%** | **6.50%** |
| H→ZZ | **4.17%** | | **20%** | **21%** | |
| $H \to \tau\tau$ | **0.42%** | | **2.10%** | **4.20%** | **7.50%** |
| $H \to \gamma\gamma$ | **3.02%** | | **11%** | **16%** | |
| $H \to \mu\mu$ | **6.36%** | | **41%** | **57%** | |
| $H \to Z\gamma$ | **8.50%** | | **35%** | | |
| $\mathrm{Br}_{upper}(H \to inv.)$ | **0.07%** | | | | |
| $\Gamma_H$ | **1.65%** | | **1.10%** | | |

"Physics potential of the CEPC", 2205.08553

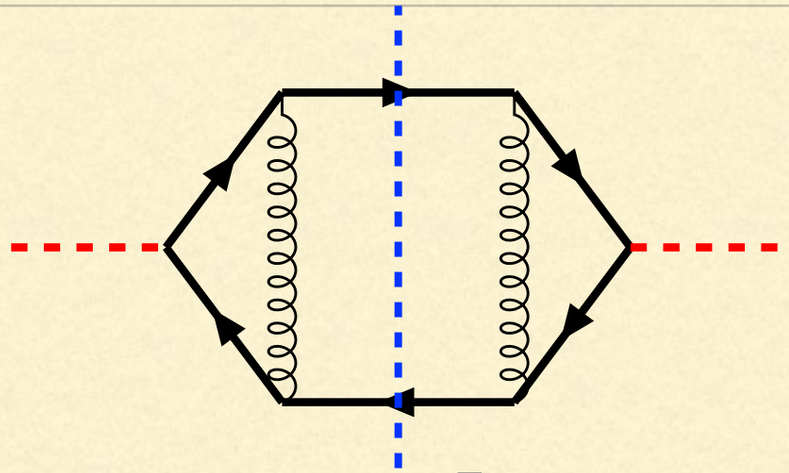



"Physics potential of the CEPC", 2205.08553




"Handbook of LHC Higgs cross sections", 1101.0593

# HIGGS DECAYS TO $b\bar{b} + X$



The decay rate is proportional to $y_b^2$.

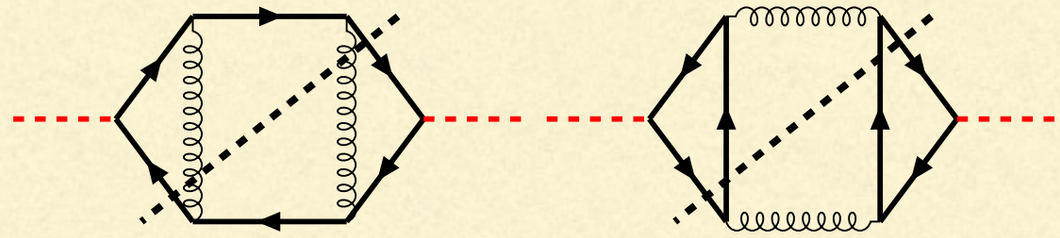
$$\Gamma(H \to b\bar{b}) = \Gamma_{\mathrm{LO}} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} X_1^{y_b y_b} + \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 (X_2^{y_b y_b} + X_2^{y_b y_t}) \right]$$

$$\Gamma_{\mathrm{LO}} = \frac{N_c}{16\pi} \bar{y}_b^2 m_H \left( 1 - \frac{4}{z} \right)^{3/2}, \quad z \equiv \frac{m_H^2}{m_b^2}$$

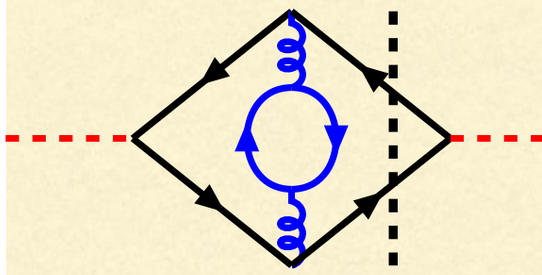
$$X_1^{y_b y_b} = 2 \log \left( \frac{\mu^2}{m_H^2} \right) + \frac{17}{3} + \frac{12 \log(z) + 10}{z} + \mathcal{O}(z^{-2})$$

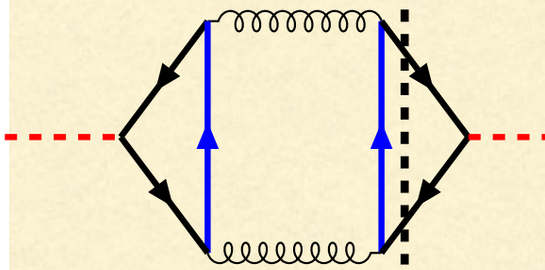
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

$$X_2^{y_b y_b} = \tilde{X}_{2,b\bar{b}}^{y_b y_b} + X_{2,b\bar{b}b\bar{b}}^{y_b y_b}$$



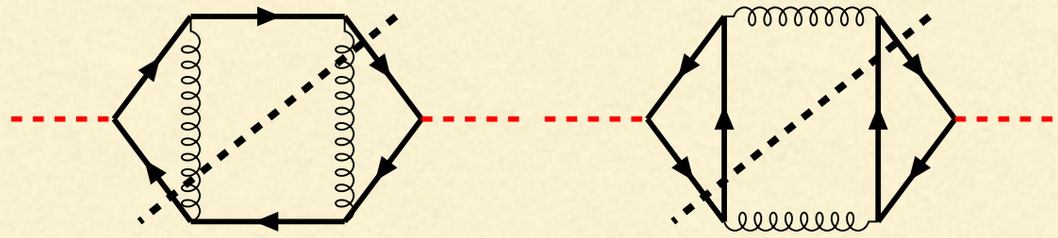
$$\begin{aligned}
\tilde{X}_{2,b\bar{b}}^{y_b y_b}|_{z\to\infty} = & -\frac{\log^3(z)}{27} + \frac{19\log^2(z)}{54} + \frac{2\zeta(3)\log(z)}{9} + \frac{\pi^2\log(z)}{54} \\
& - \frac{503\log(z)}{324} + \frac{55-2n_l}{12}\log^2\left(\frac{\mu^2}{m_H^2}\right) + \frac{307-11n_l}{9}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{2(3n_l-86)\zeta(3)}{9} \\
& + \frac{(3n_l-98)\pi^2}{54} - \frac{19\pi^4}{1620} + \frac{2\log(2)\pi^2}{9} + \frac{98681-3510n_l}{1296} \\
& + \frac{1}{z}\Bigg( - \frac{5\log^4(z)}{36} + \frac{\log^3(z)}{18} + \frac{\pi^2\log^2(z)}{18} - \frac{(2n_l-9)\log^2(z)}{2} - \frac{7\pi^2\log(z)}{54} \\
& - (2n_l-55)\log(z)\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{(117n_l-3625)\log(z)}{27} - 4\zeta(3)\log(z) \\
& - \frac{71\pi^4}{540} + \frac{4\pi^2\log(2)}{3} - \frac{5(2n_l-55)}{6}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{430\zeta(3)}{9} - \frac{(9n_l-175)\pi^2}{9} \\
& + \frac{36829-1548n_l}{324}\Bigg) + \mathcal{O}(z^{-2}),
\end{aligned} \tag{5.}$$



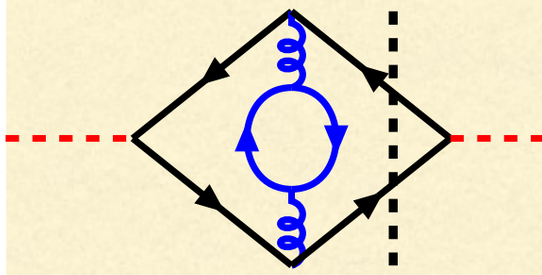


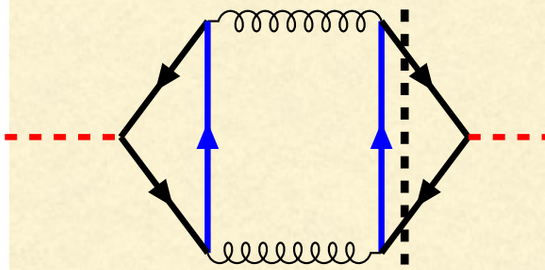
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

$$X_2^{y_b y_b} = \tilde{X}_{2,b\bar{b}}^{y_b y_b} + X_{2,b\bar{b}b\bar{b}}^{y_b y_b}$$



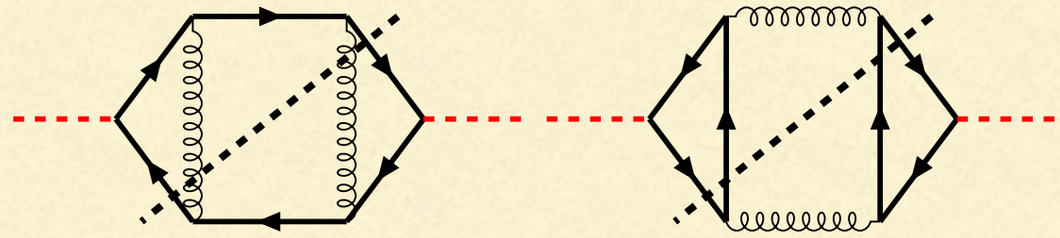
$$\tilde{X}_{2,b\bar{b}}^{y_b y_b}|_{z\to\infty} = -\frac{\log^3(z)}{27} + \frac{19\log^2(z)}{54} + \frac{2\zeta(3)\log(z)}{9} + \frac{\pi^2\log(z)}{54}$$
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$$+ \frac{1}{z}\Bigg( - \frac{5\log^4(z)}{36} + \frac{\log^3(z)}{18} + \frac{\pi^2\log^2(z)}{18} - \frac{(2n_l-9)\log^2(z)}{2} - \frac{7\pi^2\log(z)}{54}$$
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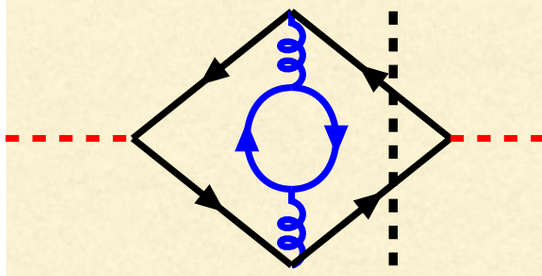


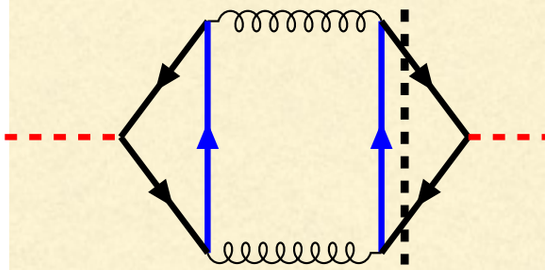
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

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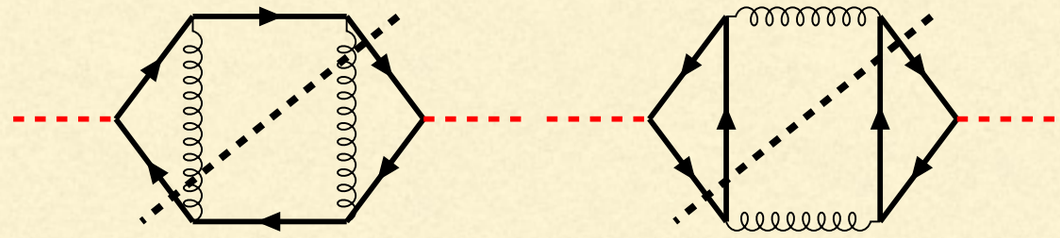




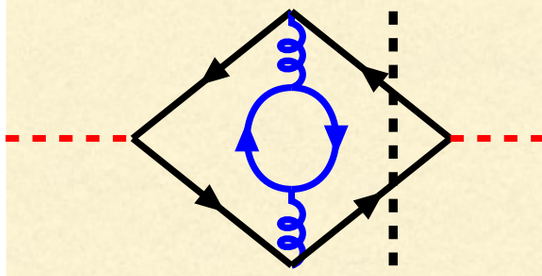
It is divergent in the massless limit.

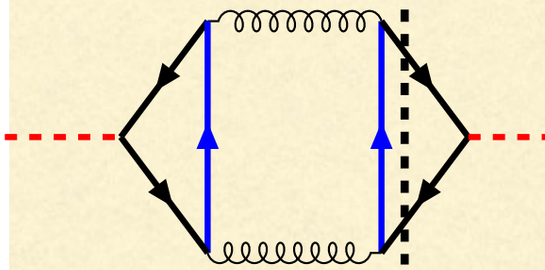
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

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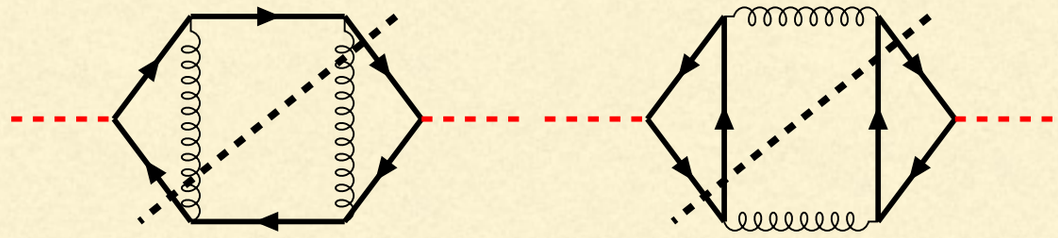




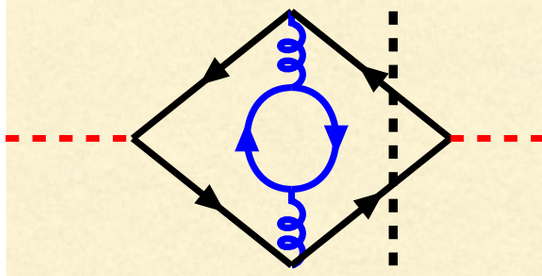
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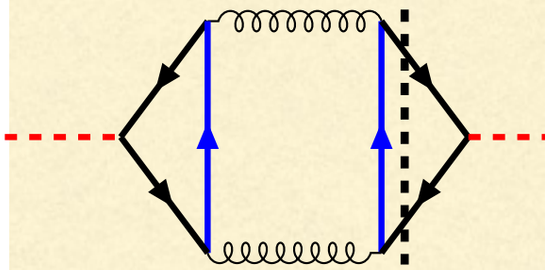
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

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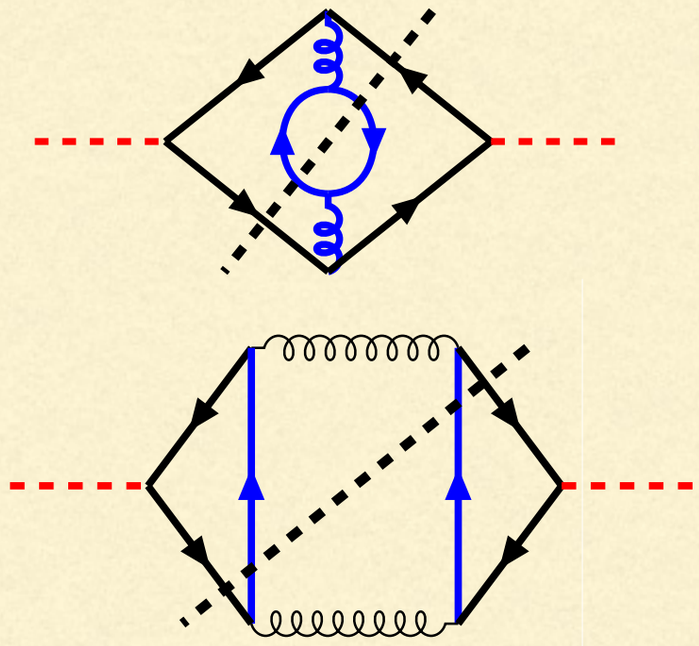


It is divergent in the massless limit.

Large log induced by soft quarks.

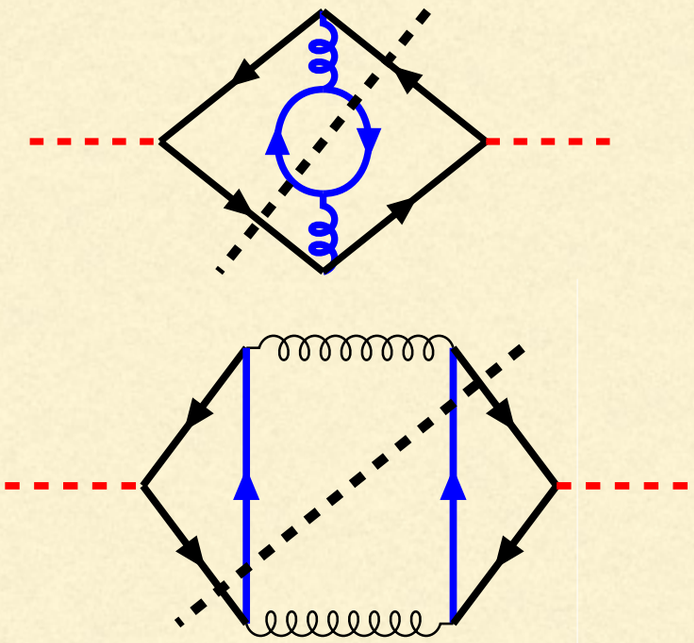
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

$$
\begin{aligned}
X^{y_b y_b}_{2,b\bar{b}b\bar{b}}|_{z\to\infty} &= \frac{\log^3(z)}{27} - \frac{19\log^2(z)}{54} - \frac{2\zeta(3)\log(z)}{9} - \frac{\pi^2\log(z)}{54} + \frac{503\log(z)}{324} \\
&+ \frac{19\pi^4}{1620} - \frac{2\log(2)\pi^2}{9} + \frac{5\zeta(3)}{18} + \frac{31\pi^2}{108} - \frac{2491}{648} \\
&+ \frac{1}{z}\left(\frac{\log^4(z)}{18} - \frac{\log^3(z)}{18} - \frac{2\pi^2\log^2(z)}{9} + \frac{7\pi^2\log(z)}{54} - \frac{\log^2(z)}{3} + 4\zeta(3)\log(z)\right. \\
&\left. + \frac{211\log(z)}{54} + \frac{13\pi^4}{270} - \frac{7\zeta(3)}{9} + \frac{19\pi^2}{18} - \frac{8353}{324}\right) + \mathcal{O}(z^{-2})\,.
\end{aligned}
$$



JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

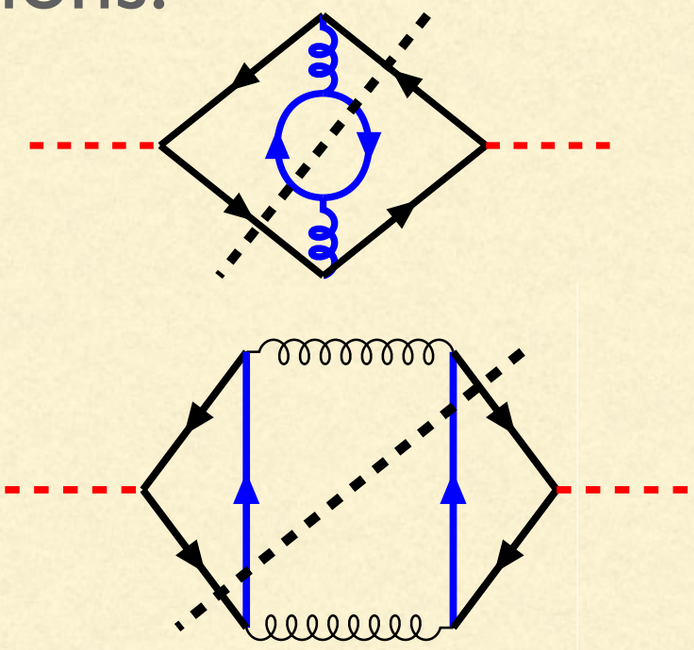
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JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

# Cancel with that in double virtual corrections.

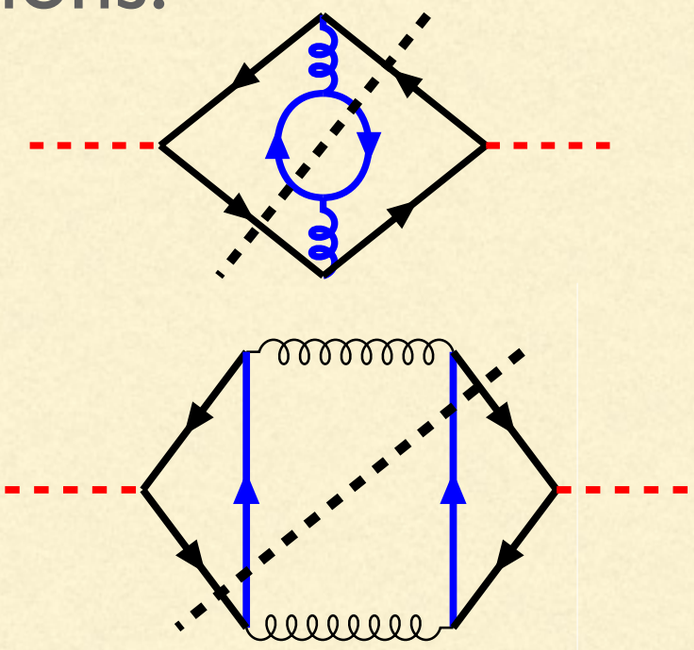
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JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

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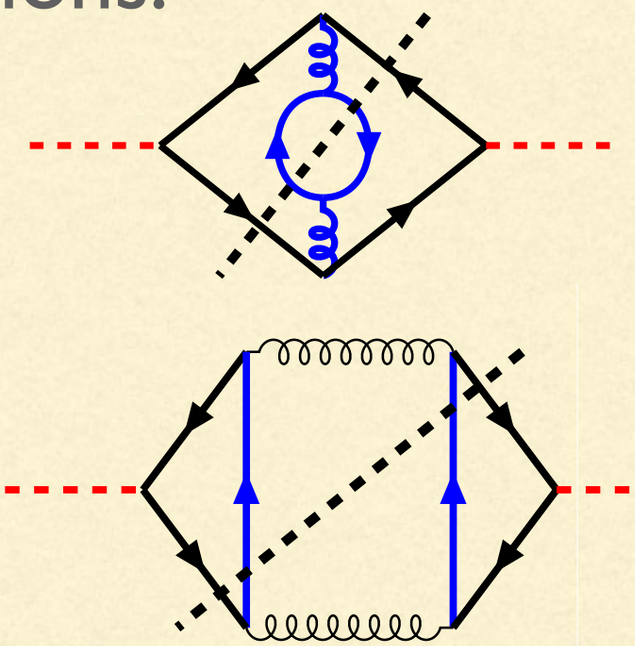
$$X^{y_b y_b}_{2,b\bar{b}b\bar{b}}|_{z\to\infty} = \frac{\log^3(z)}{27} - \frac{19\log^2(z)}{54} - \frac{2\zeta(3)\log(z)}{9} - \frac{\pi^2\log(z)}{54} + \frac{503\log(z)}{324}$$
$$+ \frac{19\pi^4}{1620} - \frac{2\log(2)\pi^2}{9} + \frac{5\zeta(3)}{18} + \frac{31\pi^2}{108} - \frac{2491}{648}$$
$$+ \frac{1}{z}\left(\frac{\log^4(z)}{18} - \frac{\log^3(z)}{18} - \frac{2\pi^2\log^2(z)}{9} + \frac{7\pi^2\log(z)}{54} - \frac{\log^2(z)}{3} + 4\zeta(3)\log(z)\right.$$
$$\left. + \frac{211\log(z)}{54} + \frac{13\pi^4}{270} - \frac{7\zeta(3)}{9} + \frac{19\pi^2}{18} - \frac{8353}{324}\right) + \mathcal{O}(z^{-2})\,.$$



JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

# Cancel with that in double virtual corrections.

$$
\begin{aligned}
X^{y_b y_b}_{2,b\bar{b}b\bar{b}}|_{z\to\infty} = & \frac{\log^3(z)}{27} - \frac{19\log^2(z)}{54} - \frac{2\zeta(3)\log(z)}{9} - \frac{\pi^2\log(z)}{54} + \frac{503\log(z)}{324} \\
& + \frac{19\pi^4}{1620} - \frac{2\log(2)\pi^2}{9} + \frac{5\zeta(3)}{18} + \frac{31\pi^2}{108} - \frac{2491}{648} \\
& + \frac{1}{z}\left(\frac{\log^4(z)}{18} - \frac{\log^3(z)}{18} - \frac{2\pi^2\log^2(z)}{9} + \frac{7\pi^2\log(z)}{54} - \frac{\log^2(z)}{3} + 4\zeta(3)\log(z)\right. \\
& \left. + \frac{211\log(z)}{54} + \frac{13\pi^4}{270} - \frac{7\zeta(3)}{9} + \frac{19\pi^2}{18} - \frac{8353}{324}\right) + \mathcal{O}(z^{-2})\,.
\end{aligned}
$$

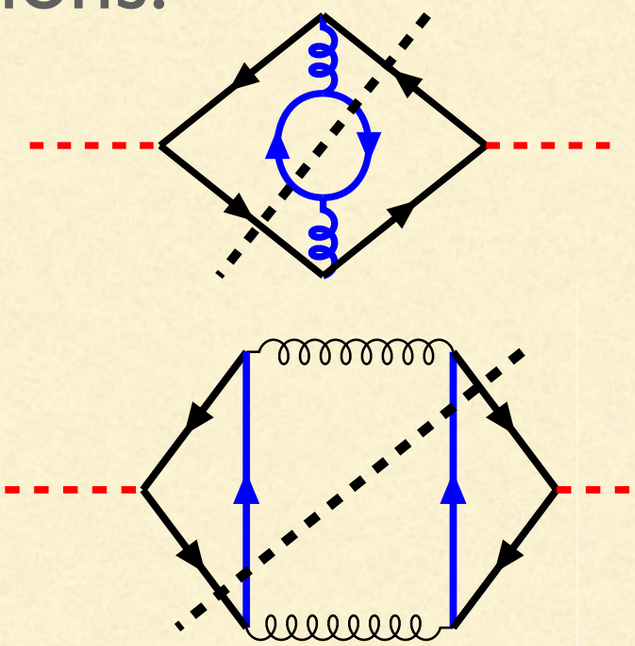


Does not cancel.

JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

# Cancel with that in double virtual corrections.

$$X^{y_b y_b}_{2,b\bar{b}b\bar{b}}|_{z\to\infty} = \frac{\log^3(z)}{27} - \frac{19\log^2(z)}{54} - \frac{2\zeta(3)\log(z)}{9} - \frac{\pi^2\log(z)}{54} + \frac{503\log(z)}{324}$$
$$+ \frac{19\pi^4}{1620} - \frac{2\log(2)\pi^2}{9} + \frac{5\zeta(3)}{18} + \frac{31\pi^2}{108} - \frac{2491}{648}$$
$$+ \frac{1}{z}\left(\frac{\log^4(z)}{18} - \frac{\log^3(z)}{18} - \frac{2\pi^2\log^2(z)}{9} + \frac{7\pi^2\log(z)}{54} - \frac{\log^2(z)}{3} + 4\zeta(3)\log(z)\right.$$
$$\left. + \frac{211\log(z)}{54} + \frac{13\pi^4}{270} - \frac{7\zeta(3)}{9} + \frac{19\pi^2}{18} - \frac{8353}{324}\right) + \mathcal{O}(z^{-2})\,.$$



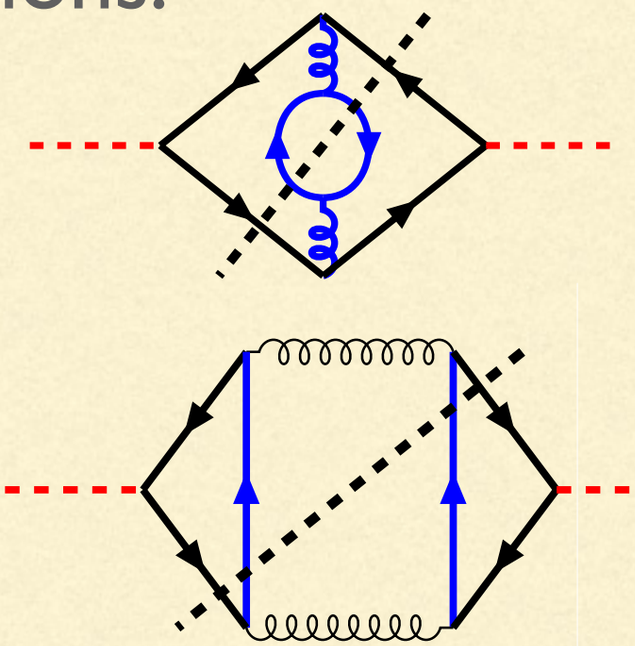
Does not cancel.

$$X_2^{y_b y_b}|_{z\to\infty\,(m_b^2\to 0)} = \frac{55-2n_l}{12}\log^2\left(\frac{\mu^2}{m_H^2}\right) + \frac{307-11n_l}{9}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{(4n_l-113)\zeta(3)}{6}$$
$$+ \frac{(2n_l-55)\pi^2}{36} + \frac{10411-390n_l}{144} + \frac{1}{z}\left(-\frac{\log^4(z)}{12} - \frac{\pi^2\log^2(z)}{6} - \frac{(6n_l-25)\log^2(z)}{6}\right.$$
$$- \frac{(26n_l-829)\log(z)}{6} - (2n_l-55)\log(z)\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{5(2n_l-55)}{6}\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{\pi^4}{12}$$
$$\left. + \frac{4\pi^2\log(2)}{3} + 47\zeta(3) - \frac{(2n_l-41)\pi^2}{2} + \frac{791-43n_l}{9}\right) + \mathcal{O}(z^{-2})\,. \qquad (5.12)$$

JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

# Cancel with that in double virtual corrections.

$$X^{y_b y_b}_{2,b\bar{b}b\bar{b}}|_{z\to\infty} = \frac{\log^3(z)}{27} - \frac{19\log^2(z)}{54} - \frac{2\zeta(3)\log(z)}{9} - \frac{\pi^2\log(z)}{54} + \frac{503\log(z)}{324}$$
$$+ \frac{19\pi^4}{1620} - \frac{2\log(2)\pi^2}{9} + \frac{5\zeta(3)}{18} + \frac{31\pi^2}{108} - \frac{2491}{648}$$
$$+ \frac{1}{z}\left(\frac{\log^4(z)}{18} - \frac{\log^3(z)}{18} - \frac{2\pi^2\log^2(z)}{9} + \frac{7\pi^2\log(z)}{54} - \frac{\log^2(z)}{3} + 4\zeta(3)\log(z)\right.$$
$$\left. + \frac{211\log(z)}{54} + \frac{13\pi^4}{270} - \frac{7\zeta(3)}{9} + \frac{19\pi^2}{18} - \frac{8353}{324}\right) + \mathcal{O}(z^{-2})\,.$$



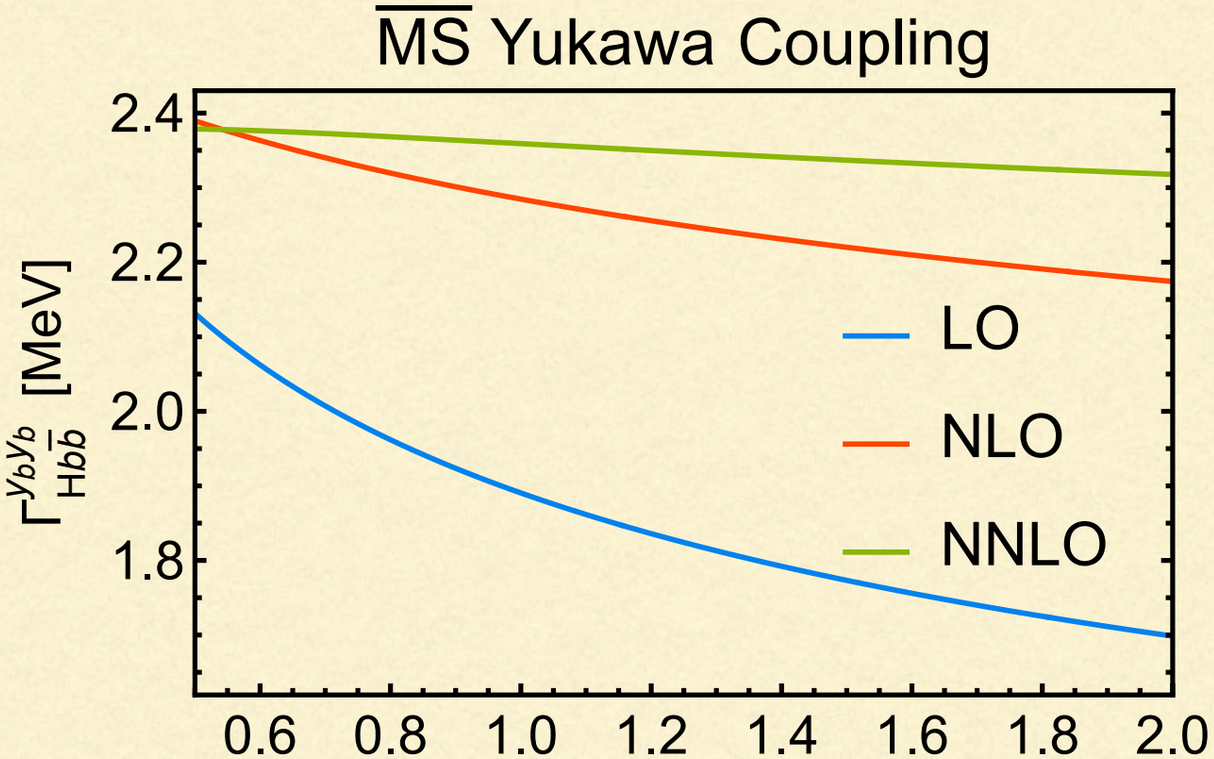
Does not cancel.

$$X_2^{y_b y_b}|_{z\to\infty\,(m_b^2\to 0)} = \frac{55-2n_l}{12}\log^2\left(\frac{\mu^2}{m_H^2}\right) + \frac{307-11n_l}{9}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{(4n_l-113)\zeta(3)}{6}$$
$$+ \frac{(2n_l-55)\pi^2}{36} + \frac{10411-390n_l}{144} + \frac{1}{z}\left(-\frac{\log^4(z)}{12} - \frac{\pi^2\log^2(z)}{6} - \frac{(6n_l-25)\log^2(z)}{6}\right.$$
$$- \frac{(26n_l-829)\log(z)}{6} - (2n_l-55)\log(z)\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{5(2n_l-55)}{6}\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{\pi^4}{12}$$
$$\left. + \frac{4\pi^2\log(2)}{3} + 47\zeta(3) - \frac{(2n_l-41)\pi^2}{2} + \frac{791-43n_l}{9}\right) + \mathcal{O}(z^{-2})\,. \qquad (5.12)$$

# The large logarithms at subleading power: $\log^4(z) = 1815$

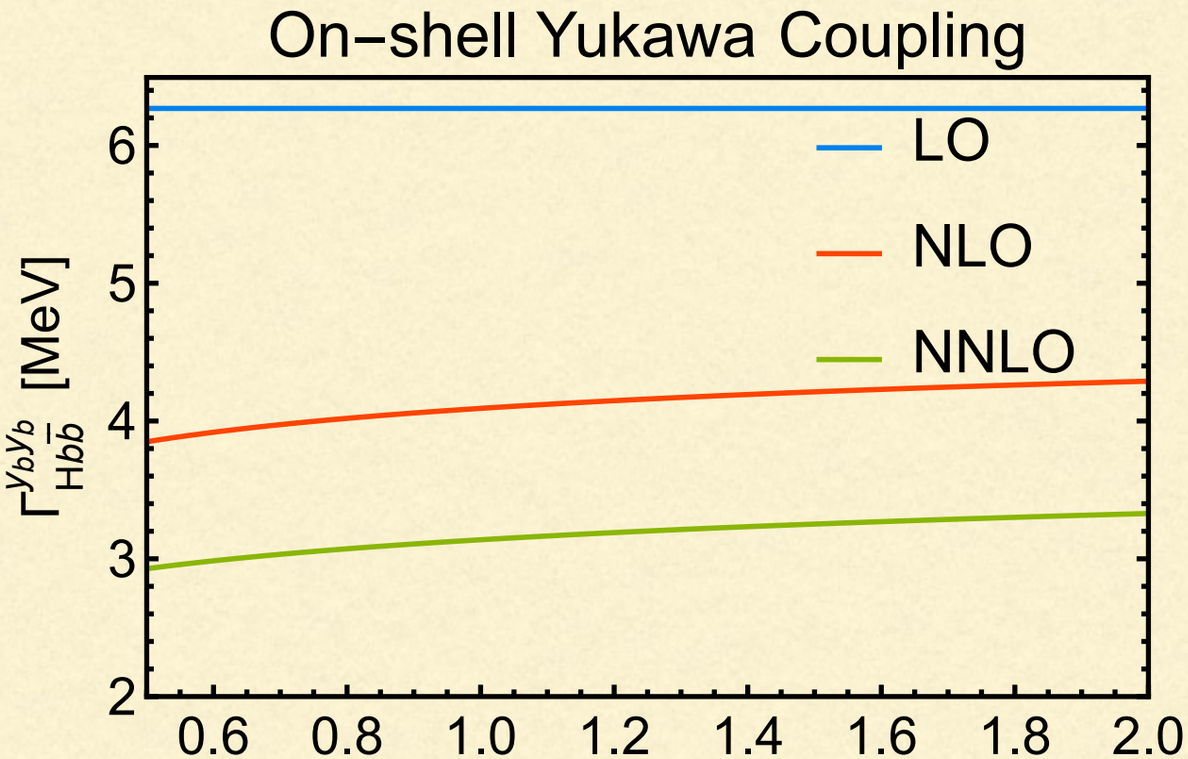
JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

$\overline{m_b}(m_H) = 2.78$ GeV




LO unc. ~23%
NLO unc.~9%
NNLO unc.~3%
NLO/LO=1.12-1.28
NNLO/NLO=0-1.07

$m_b = 5.07$ GeV




NLO unc.~11%
NNLO unc.~13%
NLO/LO=0.65
NNLO/NLO=0.78

JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

| width | LO | NLO | NNLO($y_b^2$) | NNLO($y_b y_t$) | NNNLO($y_b^2$) | NNNNLO($y_b^2$) |
|---|---|---|---|---|---|---|
| $\overline{\text{MS}}$ | $1.891^{+0.241}_{-0.192}$ | $2.285^{+0.105}_{-0.110}$ | $2.359^{+0.020}_{-0.041}$ | $2.376^{+0.026}_{-0.046}$ | $2.379^{+0.005}_{-0.015}$ | $2.377^{+0.006}_{-0.006}$ |
| on-shell | 6.269 | $4.092^{+0.197}_{-0.242}$ | $3.138^{+0.191}_{-0.210}$ | $3.193^{+0.181}_{-0.197}$ | $2.804^{+0.112}_{-0.096}$ | $2.649^{+0.065}_{-0.049}$ |

Larin et al, hep-ph/9506465
Chetyrkin hep-ph/9608318,
Baikov et al, hep-ph/0511063,
Herzog et al, 1707.01044

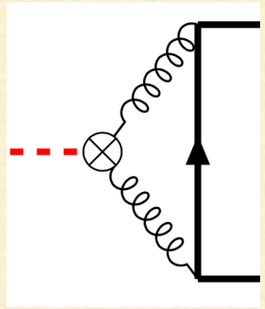
The results in MS scheme become stable quickly.
Scale unc. ~0.3%.
The higher-order beyond NNLO is much less than 1%.

In onshell scheme, N3LO/NNLO=0.88, N4LO/N3LO=0.95.
Scale unc.~2%.

JW, Yefan Wang, Da-Jiang Zhang, JHEP03(2024)068

$$\mathscr{L}_{\text{eff}} = -\frac{H}{v}\left(C_1\mathcal{O}_1^R + C_2\mathcal{O}_2^R\right) + \mathscr{L}_{\text{QCD}}$$

$$\mathcal{O}_1 = (G^0_{a,\mu\nu})^2, \quad \mathcal{O}_2 = m_b^0 \bar{b}^0 b^0$$



$$C_1 = -\left(\frac{\alpha_s}{\pi}\right)\frac{1}{12} - \left(\frac{\alpha_s}{\pi}\right)^2\frac{11}{48} - \left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{2777}{3456} + \frac{19}{192}L_t - n_f\left(\frac{67}{1152} - \frac{1}{36}L_t\right)\right] + \mathcal{O}(\alpha_s^4),$$

$$C_2 = 1 + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{5}{18} - \frac{1}{3}L_t\right] + \left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{841}{1296} + \frac{5}{3}\zeta(3) - \frac{79}{36}L_t - \frac{11}{12}L_t^2 + n_f\left(\frac{53}{216} + \frac{1}{18}L_t^2\right)\right] + \mathcal{O}(\alpha_s^4),$$

Chetyrkin, Kniehl, Steinhauser,
hep-ph/9701277, 9708255

The relative error of using this EFT is below 1%.
The correction to the total decay width is below 0.01%.

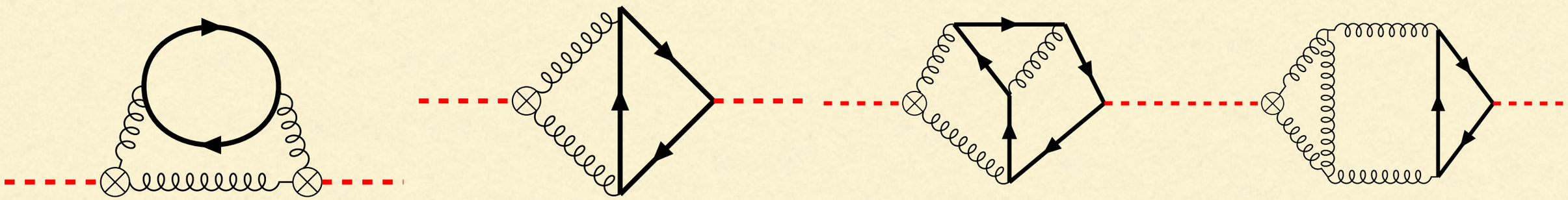
Primo et al, 1812.07811

$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}$$

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

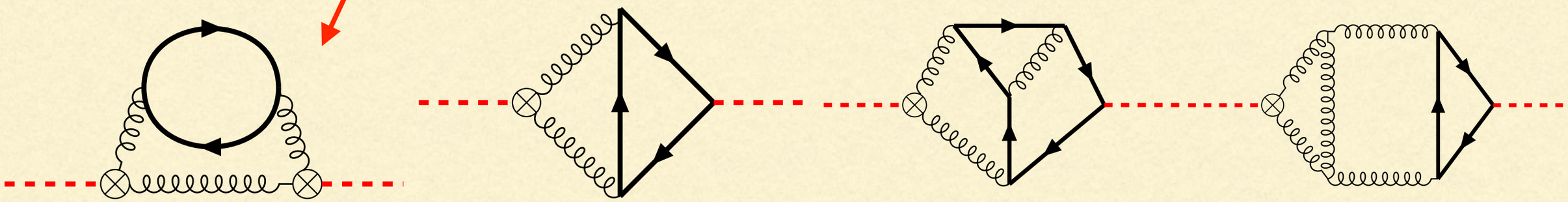


$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}$$

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

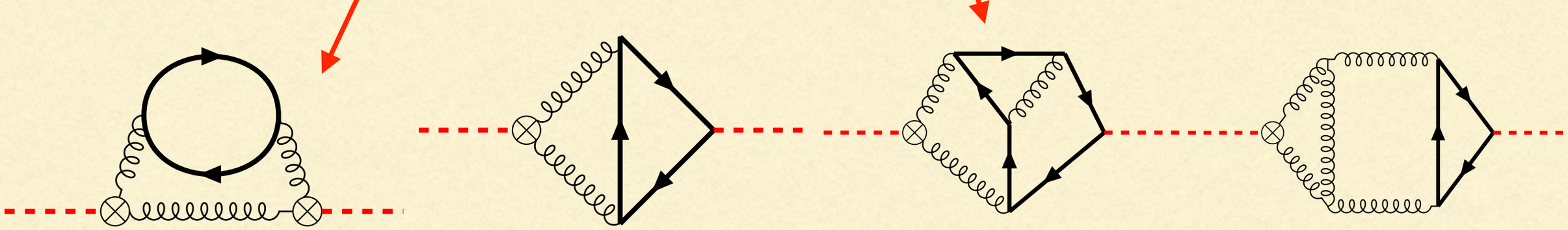


$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}$$

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

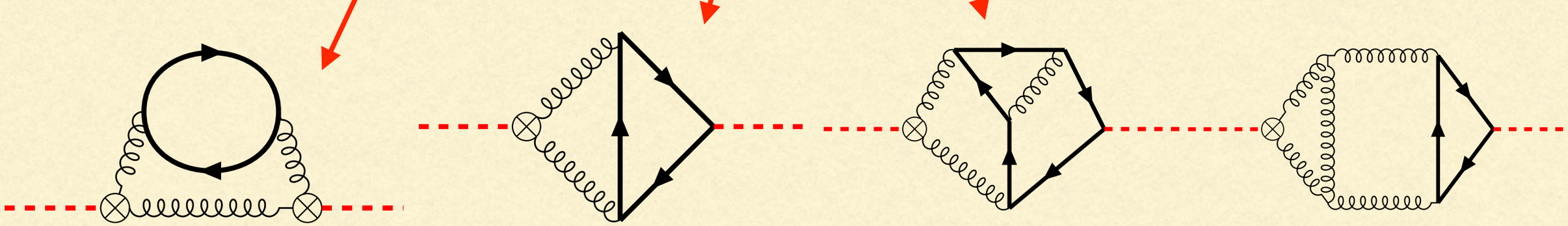


$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}$$

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

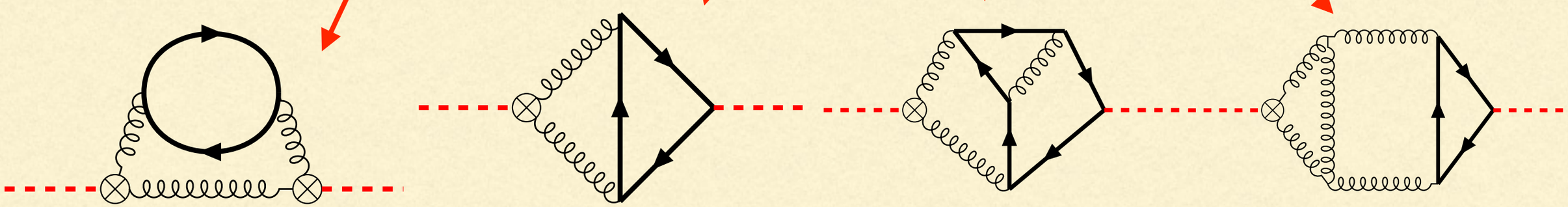


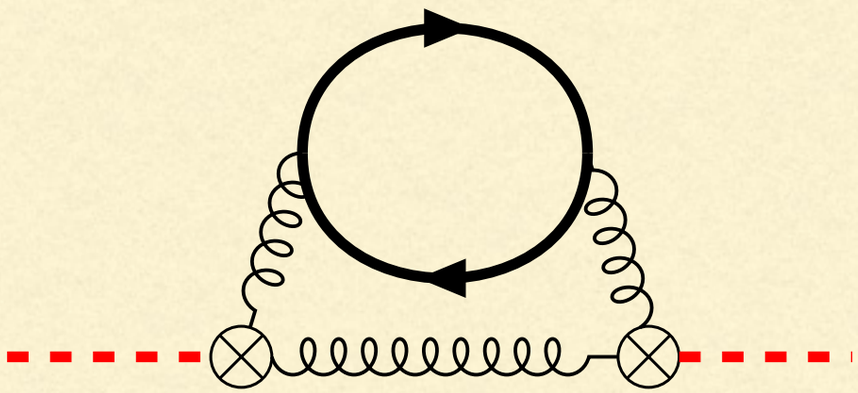
$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}$$

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

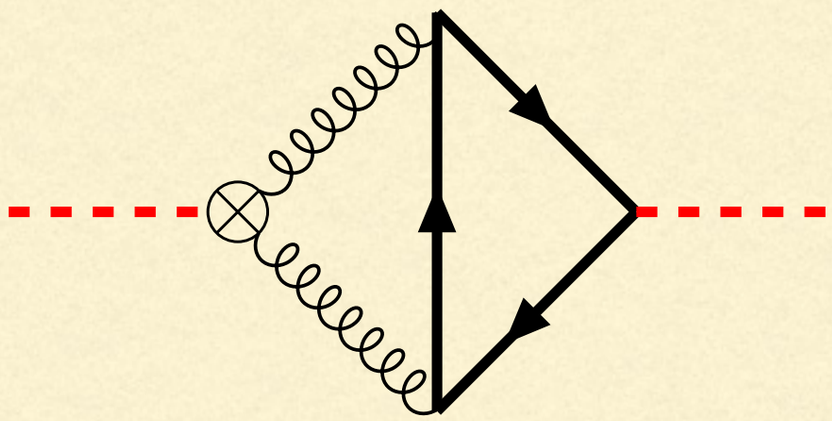




$$\Delta^{C_1C_1}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H^3}{\pi v^2} C_A C_F \left[\frac{1}{6}\log(z) - \frac{7}{12} + \frac{3}{z}\right] + \mathcal{O}(z^{-2})$$

It is power enhanced and divergent in $m_b \to 0$ limit.
The extraction of yb is highly non-trivial after considering such contributions.

JW, Xing Wang, Yefan Wang, JHEP03(2025)163



$$\Delta^{C_1C_2}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H m_b \overline{m_b}(\mu)}{\pi v^2} C_A C_F \left[ -\frac{1}{8}\log^2(z) - \frac{3}{4}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{\pi^2}{8} - \frac{19}{8} \right.$$
$$\left. + \frac{1}{2}\frac{\log^2(z)}{z} + 2\frac{\log(z)}{z} + \frac{9}{2z}\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{\pi^2}{2z} + \frac{15}{2z} \right] + \mathcal{O}(z^{-2}),$$

The double logarithm dominates.
It is induced by soft massive quarks, different from traditional Sudakov double logarithms.

$$-\frac{m_H m_b \overline{m_b}(\mu)}{8\pi v^2} C_A C_F \log^2(z) \times \frac{1}{24}\left(C_A - C_F\right)\log^2(z)$$

JW, Xing Wang, Yefan Wang, JHEP03(2025)163

$$
\begin{aligned}
\Gamma_{H\to b\bar{b}} = \frac{3m_H\overline{m_b}^2}{8v^2\pi}\Bigg\{1 &+ \left(\frac{\alpha_s}{\pi}\right)\frac{17}{3} \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{1}{9}\log^2(\bar{z}) - \frac{2}{3}\log(x) - \frac{97\zeta(3)}{6} - \frac{17\pi^2}{12} + \frac{9235}{144}\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^3\Bigg[\frac{5}{648}\log^4(\bar{z}) + \frac{59}{324}\log^3(\bar{z}) - \frac{31\pi^2}{324}\log^2(\bar{z}) + \frac{989}{648}\log^2(\bar{z}) \\
&+ \frac{32\zeta(3)}{27}\log(\bar{z}) - \frac{41\pi^2}{324}\log(\bar{z}) + \frac{137}{216}\log(\bar{z}) - \frac{23}{18}\log^2(x) - \frac{49}{6}\log(x) \\
&+ \frac{1945\zeta(5)}{36} - \frac{13\pi^4}{3240} - \frac{81239\zeta(3)}{216} - \frac{81239\zeta(3)}{216} - \frac{22291\pi^2}{648} + \frac{37434709}{46656}\Bigg]\Bigg\} \\
&+ \frac{m_H^3}{v^2\pi}\left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{\log(\bar{z})}{216} - \frac{7}{432}\right] + \mathcal{O}(\bar{z}^{-1}) + \mathcal{O}(x) + \mathcal{O}(\alpha_s^4)
\end{aligned}
$$

JW, Xing Wang, Yefan Wang, JHEP03(2025)163

| | [MeV] | $\mu = \frac{1}{2}m_H$ | $\mu = m_H$ | $\mu = 2m_H$ |
|---|---|---|---|---|
| $\mathcal{O}(\alpha_s^0)$ | $\Gamma^{C_2C_2}_{Hb\bar{b}}$ | 2.1314 | 1.8905 | 1.6985 |
| $\mathcal{O}(\alpha_s^1)$ | $\Gamma^{C_2C_2}_{Hb\bar{b}}$ | 0.25813 | 0.39409 | 0.47563 |
| $\mathcal{O}(\alpha_s^2)$ | $\Gamma^{C_2C_2}_{Hb\bar{b}}$ | -0.0043084 | 0.076819 | 0.143670 |
| | $\Gamma^{C_1C_2}_{Hb\bar{b}}$ | 0.027078 | 0.024746 | 0.022608 |
| $\mathcal{O}(\alpha_s^3)$ | $\Gamma^{C_2C_2}_{Hb\bar{b}}$ | -0.015360 | 0.0048336 | 0.038198 |
| | $\Gamma^{C_1C_2}_{Hb\bar{b}}$ | 0.010314 | 0.013585 | 0.015170 |
| | $\Gamma^{C_1C_1}_{Hb\bar{b}}$ | 0.0088676 | 0.0064617 | 0.0048595 |

MS scheme

NLO/LO=20%

NNLO/NLO=4%

1% correction!

For C1C2, NNLO/NLO=55%

Fortunately, the C1C1 conbribution is not large.

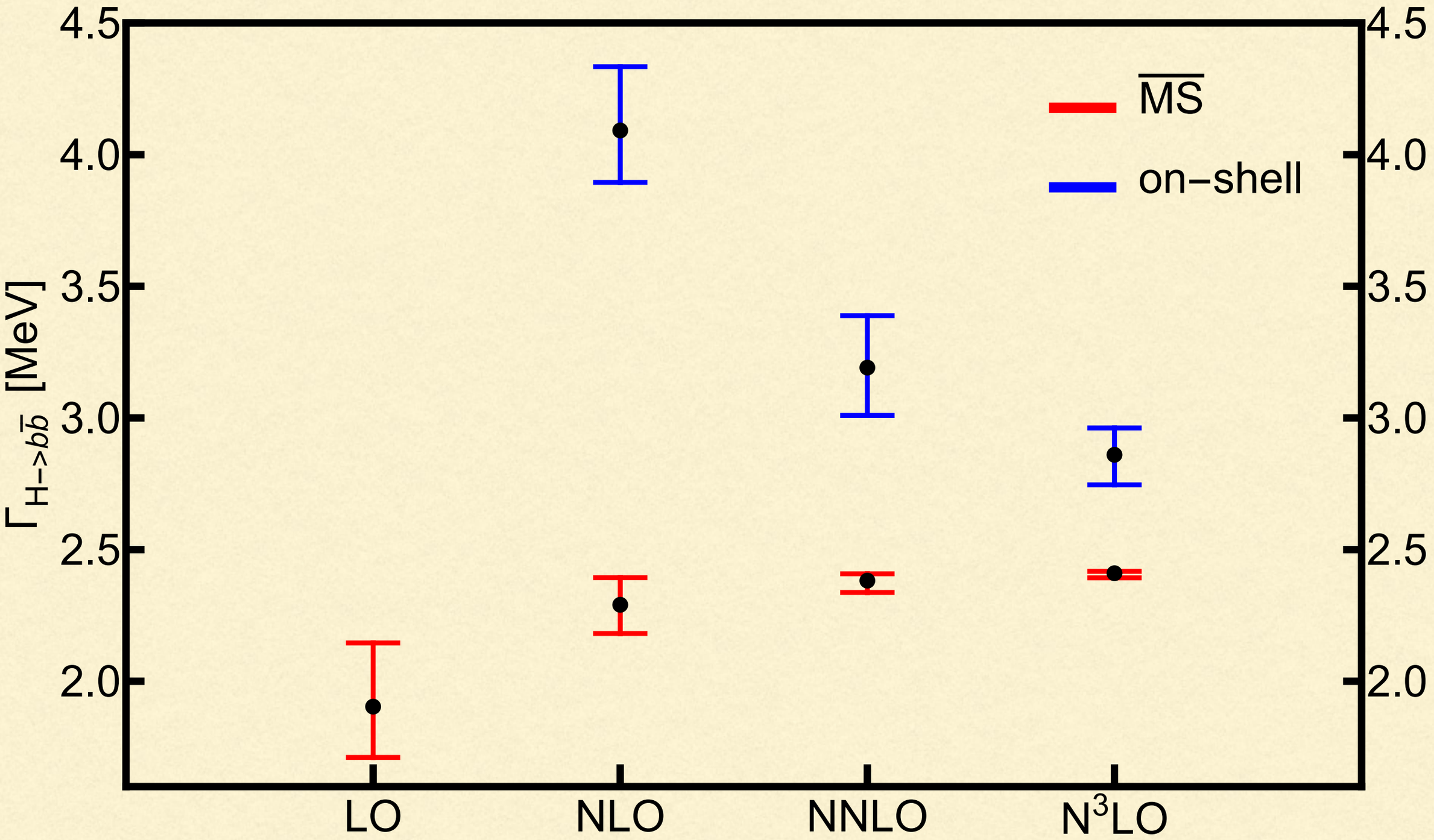
JW, Xing Wang, Yefan Wang, JHEP03(2025)163

| | [MeV] | $\mu = \frac{1}{2}m_H$ | $\mu = m_H$ | $\mu = 2m_H$ |
|---|---|---|---|---|
| $\Gamma_{H\to b\bar{b}}\,(\overline{\rm MS})$ | $\mathcal{O}(\alpha_s^0)$ | 2.1454 | 1.9036 | 1.7108 |
| | $\mathcal{O}(\alpha_s^1)$ | 0.24806 | 0.38682 | 0.47051 |
| | $\mathcal{O}(\alpha_s^2)$ | 0.014742 | 0.091773 | 0.15580 |
| | $\mathcal{O}(\alpha_s^3)$ | 0.0092203 | 0.028117 | 0.055816 |
| $\Gamma_{H\to b\bar{b}}\,(\rm OS)$ | $\mathcal{O}(\alpha_s^0)$ | 6.2687 | 6.2687 | 6.2687 |
| | $\mathcal{O}(\alpha_s^1)$ | -2.4192 | -2.1770 | -1.9797 |
| | $\mathcal{O}(\alpha_s^2)$ | -0.85590 | -0.90061 | -0.91641 |
| | $\mathcal{O}(\alpha_s^3)$ | -0.23550 | -0.33107 | -0.39831 |

$$\Gamma^{\rm N^3LO\ QCD}_{H\to b\bar{b}}\left(\overline{\rm MS}\right) = 2.410^{+0.007}_{-0.017}\ {\rm MeV}$$

$$\Gamma^{\rm N^3LO\ QCD+NLO\ EW}_{H\to b\bar{b}}\left(\overline{\rm MS}\right) = 2.382^{+0.007}_{-0.017}\ {\rm MeV}$$

JW, Xing Wang, Yefan Wang, JHEP03(2025)163




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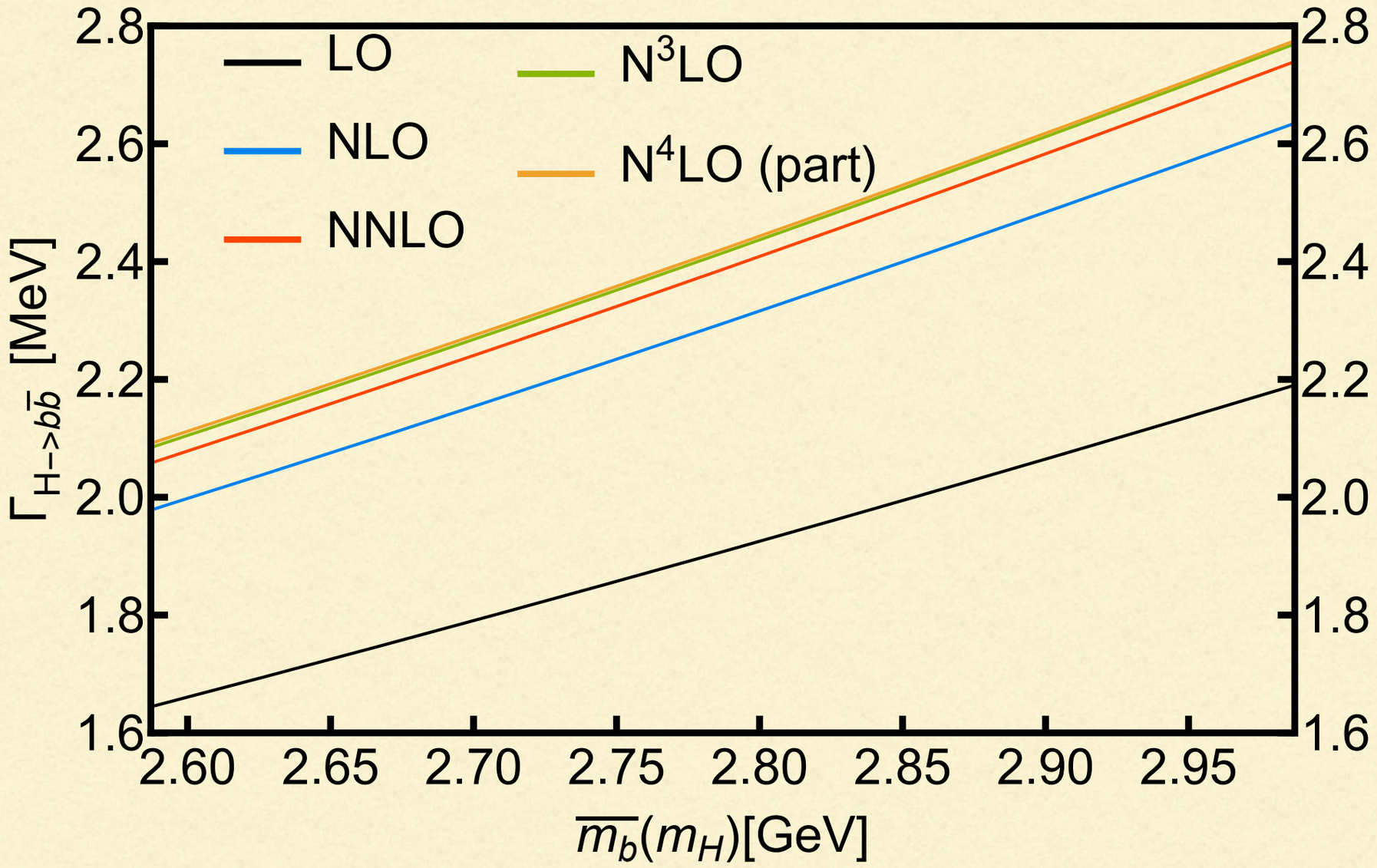
| $\mu = m_H$ | [MeV] | $\Gamma_{H b\bar{b}}^{C_2 C_2}$ | $\Gamma_{H b\bar{b}}^{C_1 C_2}$ | $\Gamma_{H b\bar{b}}^{C_1 C_1}$ |
|---|---|---|---|---|
| $\Gamma_{H\to b\bar{b}}\left(\overline{\text{MS}}\right)$ | $\mathcal{O}(\alpha_s^0)$ | 1.9076 | - | - |
| | $\mathcal{O}(\alpha_s^1)$ | 0.3873 | - | - |
| | $\mathcal{O}(\alpha_s^2)$ | 0.0735 | 0.0183 | - |
| | $\mathcal{O}(\alpha_s^3)$ | 0.0048 | 0.0142 | 0.0090 |
| | $\mathcal{O}(\alpha_s^4)$ | -0.0025 | * | 0.0087 |

-0.1%

+0.4%

$$\Gamma_{H\to b\bar{b}}^{\text{N}^4\text{LO QCD}}\left(\overline{\text{MS}}\right) = 2.421^{+0.008}_{-0.010}(\text{scl.})^{+0.005}_{-0.005}(\alpha_s)\ \text{MeV}$$

JW, Xing Wang, Yefan Wang, JHEP07(2026)139




If the decay width is measured with an uncertainty of 0.21%, the bottom quark mass can be derived with a precision around 0.36%.

JW, Xing Wang, Yefan Wang, JHEP07(2026)139

# CONCLUSIONS

- Higgs decays to $b\bar{b}$ can be measured with an accuracy better than 1%.
- Theoretical predictions are investigated with full dependence on the bottom quark mass.
- Large corrections are found due to the logarithms $\log(m_H^2/m_b^2)$.
- Extraction of the bottom quark Yukawa coupling is nontrivial after considering higher order corrections.

# Thank you!