Two-loop mixed QCD-EW corrections to bottom pair production in electron-positron collisions

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Preliminary Result



National Natural Science Foundation of China

International Collaboration Fund for Creative Research Teams

Precision Calculations for Electron-Positron Colliders

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PCEPC Scientific Thrusts

Theory

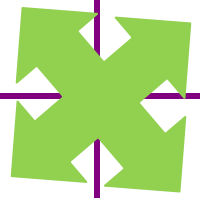
Theory

I. $e^+e^- \rightarrow f\,f$ @ NNLO

- MJRM, A. Vicini
- 2 post-docs
- 2 PhD students

II. $e^+e^- \rightarrow Zh$ @ NLO

- MJRM, XG He, S. Heinemeyer
- 2 Post-docs
- 2 PhD students



III. Theory-Expt Pipeline

- H. Yang, M. Ruan, J. Guo
- 2 post-docs
- 2 PhD students

IV. Novel EW Observables

- XG He, MJRM, M. Ruan
- 1 post-doc
- 2 PhD students

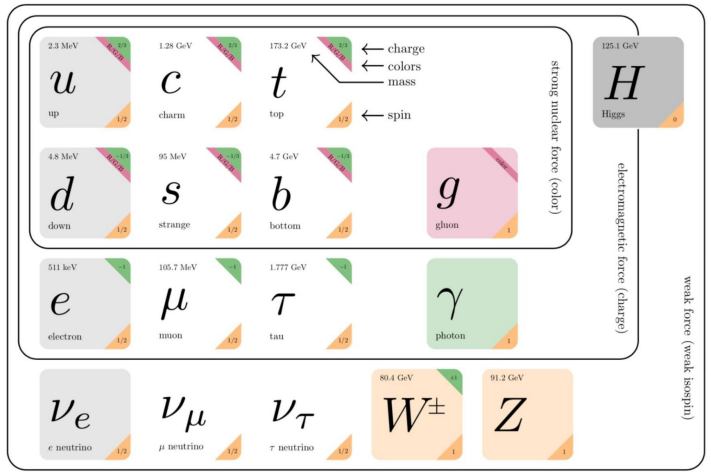
Experiment

Theory + Experiment

Total team: 22 = 7 Senior + 7 post-docs + 8 PhD students

Precision Standard Model Computations

- **Standard Model is Successful**
  - Three fundamental interaction
  - Test at quantum level

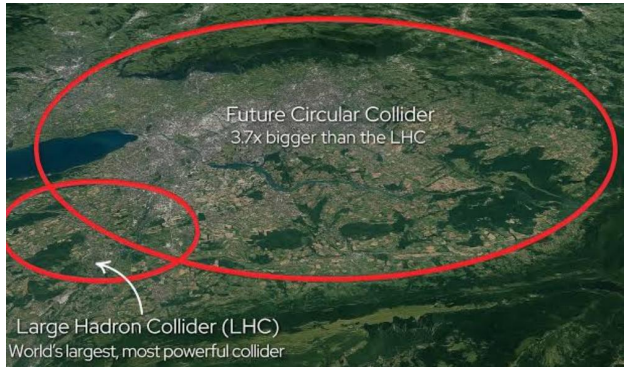


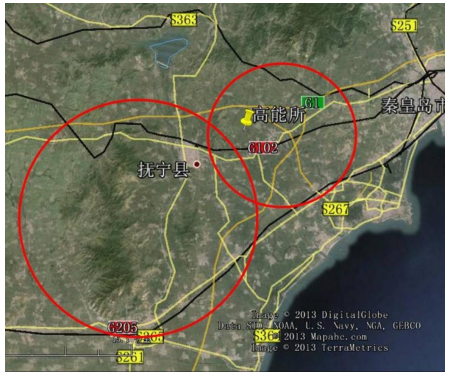
But we know it is not complete because serval question can not be explain

- Baryon asymmetry
- Neutrino mass
- Dark matter ...

We want to know whether there is new physics

As a result the lepton collider was proposed: CEPC, FCC_ee





300 times increasing

The lepton collider provide a cleaner background and push the experimental uncertainties to the theoretical limit

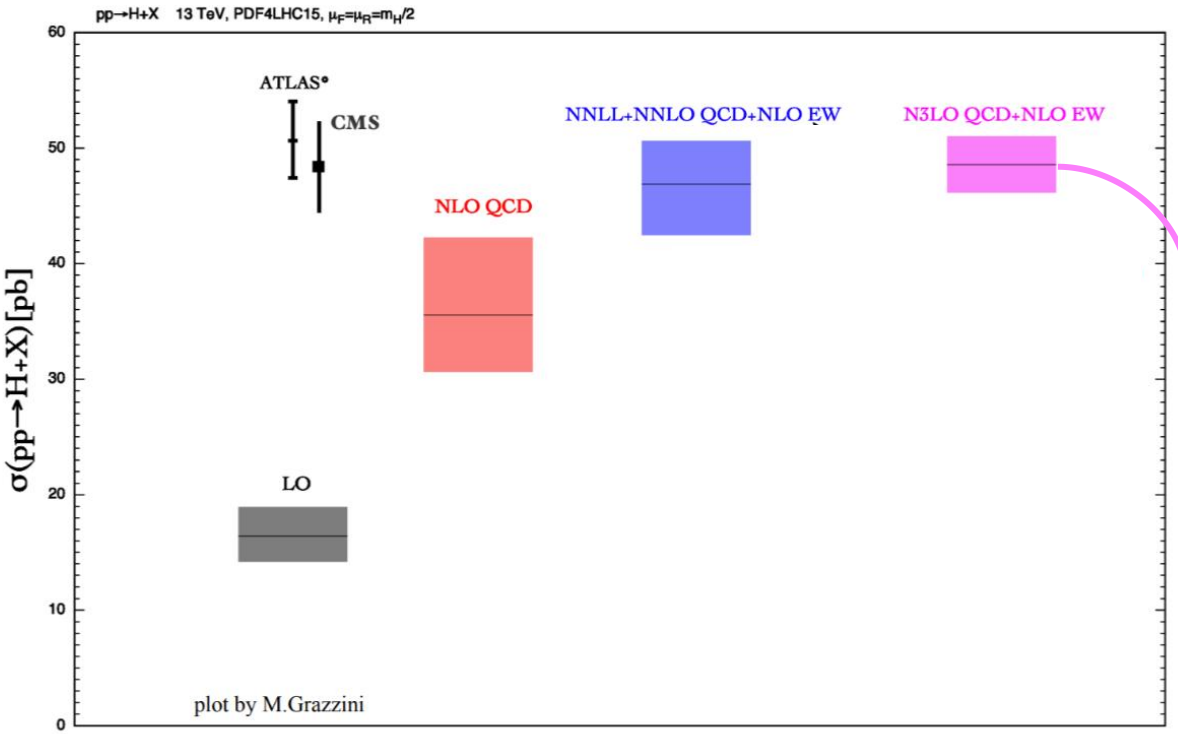
This development push theoritical computation into Standard model 2-loop region

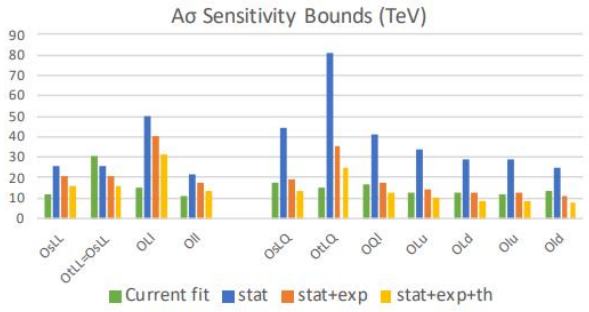
Precision Standard Model Computations

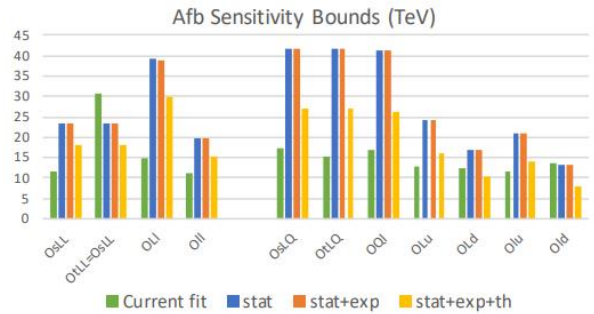
- **Why higher loop computation is important?**

The lepton collider provide a cleaner background and push the experimental uncertainties to the theoretical limit

This development push theoretical computation into Standard model 2-loop region







S.F Ge, Z.N Qian, Michael, J. Zhou, : 2410.17605

The 2-loop effect is quite big and can not be ignored if we want to study the new physics

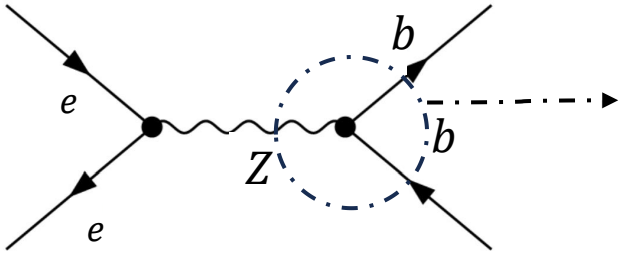
So, it is necessary to push our computation into the next level

On which target?

Precision Standard Model Computations

- **What process we want to study?**

  Z-boson resonance & fermion pair:



$$\frac{g}{4\cos\theta_W}\left[1-(1-\frac{4}{3}\sin^2\theta_W)\gamma^5\right]\gamma^\mu$$

Which process we want to consider? and How to extract the physics parameter from the experiment?

1. New physics is coupled to heavy field.
2. How do we find experimental evidence study Z coupling to bb in week mixing angle.
3. $\sin\theta_W$ measurement from AFB at LEP and ALR in SLAC are problematic.

Motivation of the work both in LEP and CEPC

- **Precision Standard Model Computations: $e^+e^- \to \bar{b}b$ EW-QCD mixed contribution at NNLO**

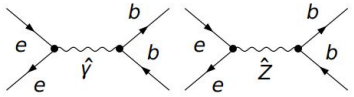
$$e^+e^- \to \bar{b}bX \longrightarrow e^+e^- \to \bar{b}b$$

But 2-loop computation is very difficult

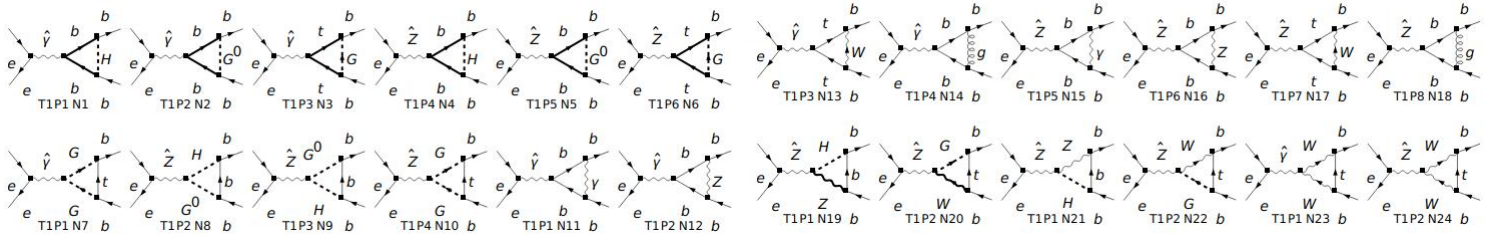
Precision Standard Model Computations

- **Demonstration: 1-loop vs 2-loop only in Vertex diagram**

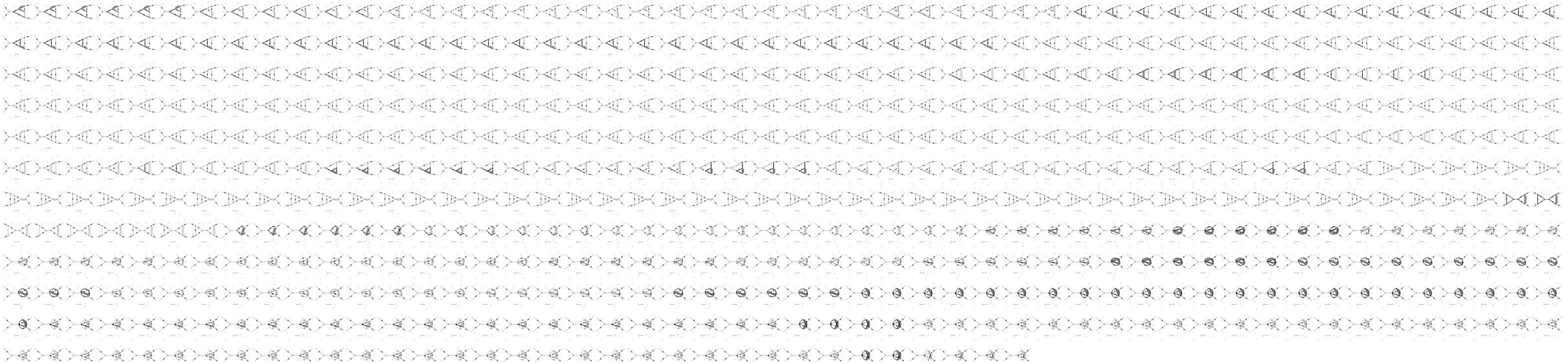
Born diagram 2:



One-loop diagram 24:



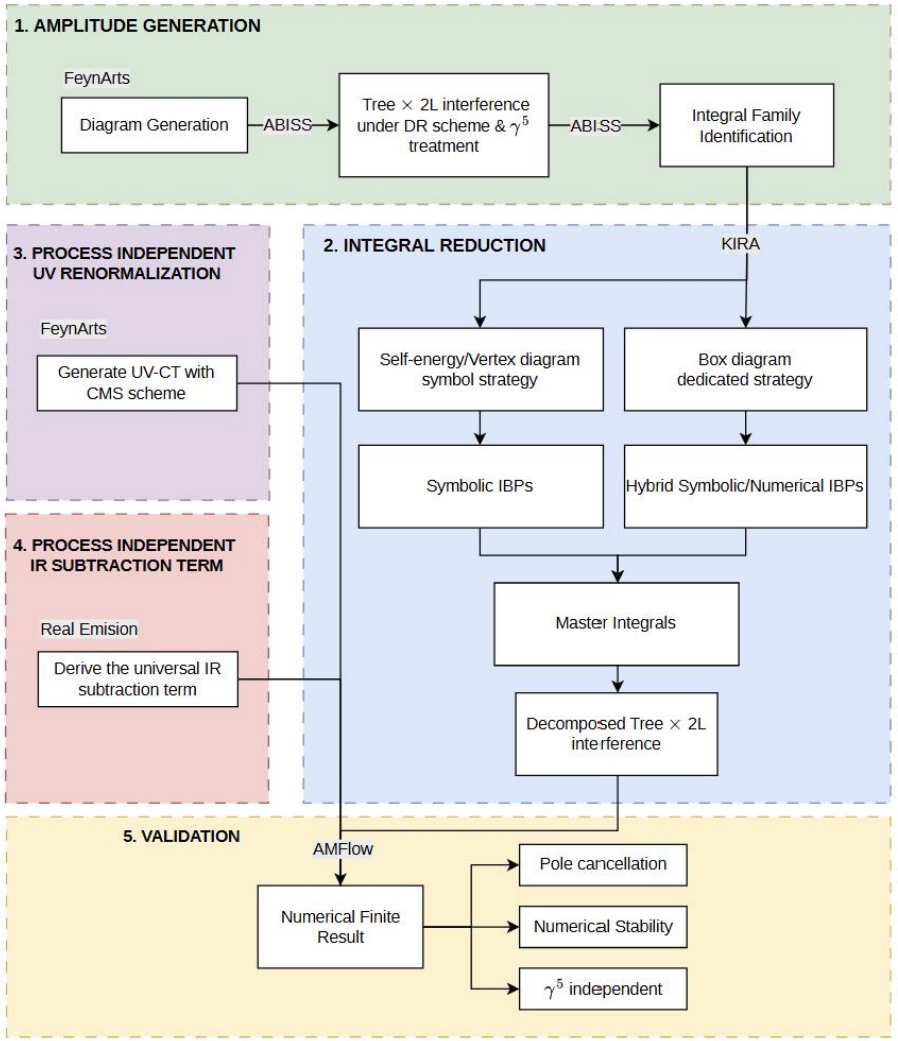
Two-loop diagram 500+:



Not just numbers: $\langle \mathcal{M}_0^{(0,0)} | \mathcal{M}_0^{(1,1)} \rangle = \sum_i c_i(s,t,\{m_i\}) SI_i(s,t,\{m_i\})$

Precision Standard Model Computations

- **The Workflow and What it can Do?**



Explain them in detailed in next

1 Generate amplitudes — **FeynArts / ABISS**
DR, γ^5 treatment and integral families

2 Reduce integrals — **KIRA**
Symbolic and hybrid IBP strategies

3 Renormalize UV divergences
Through complex-mass counterterms

4 Subtract universal IR singularities
Include 2-loop and 1-loop and subtraction term

5 Evaluate and validate — **AMFlow**
Poles, γ^5 independence and stability

Hight Precision Finite Result: Monte Carlo simulations

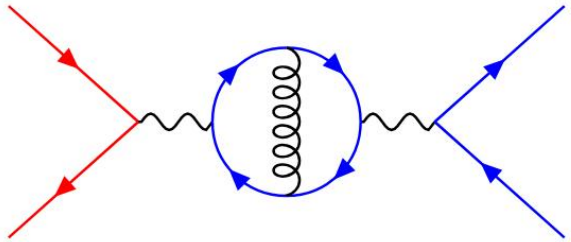
Precision Standard Model Computations

- **How to compute the Feynman integral generated from the 2-loop diagram?**

We find in total **13069** different 2-loop Feynman-integral
With **up to 5 massive lines / 3 mass values.**

$s,\ m_q$

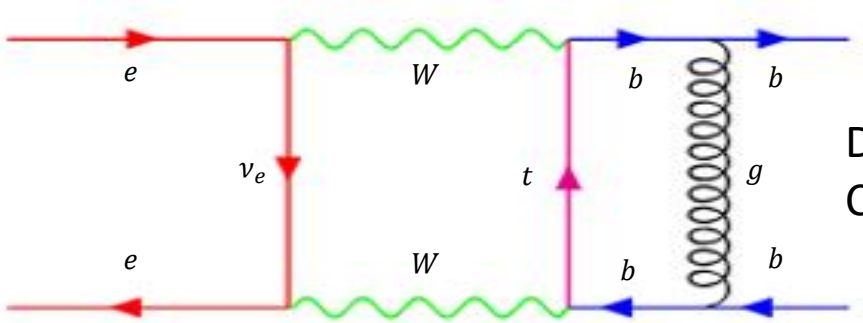
Complexity: 1 adimensional ratio



10 mins

$s, t,\ \mu_W,\ m_t,\ m_b$

10 adimensional ratio



DAYS TO COMPUTE

But those Feynman-integrals are not all independent

1. Classify **13069** integrals into 35 integral families by topology.(***ABISS***)
2. Identify the independent master-integral(MIs) for each family through IBP(***KIRA***)
3. Express the **13069** integrals as linear combination of MIs $SI^a = \sum_i c_i^a I_i$ in each family.(**ABISS**)

13,069 scalar integrals → IBP identities → 1,210 masters

Precision Standard Model Computations

- **How to generate IBPs?**

  The fully symbolic reduction of the Box topology is very demanding: RAM

Self-energy / vertex: symbolic

- KIRA with FireFly reconstruction
- Preserve symbolic dependence
- Search for effective simplifications
- Flexible reuse of reduction rules

Massive boxes: hybrid

- Numerical masses except for W/Z
- Symbolic dependence on kinematics
- Generate only the needed sectors and numerator powers

- **The finial structure of the Result**

$$\langle \mathcal{M}_0^{(0,0)} | \mathcal{M}_0^{(1,1)} \rangle = \sum_{k=1}^{1210} c_k(s,t,\{m_f\},\{\mu_B\},\epsilon) I_k(s,t,\{m_f\},\{\mu_B\},\epsilon).$$

MIs can be evaluated through ***AMFlows***

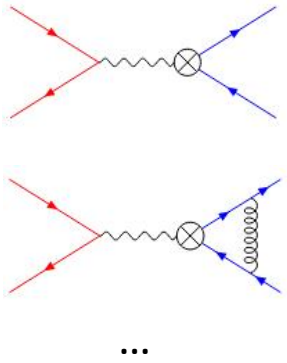
Precision Standard Model Computations

- **UV and IR Poles and Finite Result**

UV renormalization in DR:

- Renormalize parameters and fields

$$|\mathcal{M}\rangle = Z_{e^-}^{\frac{1}{2}} Z_{e^+}^{\frac{1}{2}} Z_b^{\frac{1}{2}} Z_{\bar{b}}^{\frac{1}{2}} \times 16\pi^2 \sum_{j,k=0}^{\infty} \left(\frac{g+\delta g_s}{4\pi}\right)^{2j} \left(\frac{e+\delta e}{4\pi}\right)^{2k+2} \times |\mathcal{M}_0^{(j,k)}(s,t,\{m_f+\delta m_f\},\{\mu_B^2+\delta\mu_B^2\})\rangle$$



...

- On-shell complex-mass counterterms
- Counterterms remain complex.

$$\mu_V^2 = M_V^2 - iM_V\Gamma_V, \quad V = W, Z.$$

- Verified Ward identities

$$\Sigma_{\mathrm{L}}^{\hat{A}\hat{A}}(k^2) = 0,$$
$$\Sigma_{\mathrm{L}}^{\hat{A}\hat{Z}}(k^2) = 0,$$
...

Guarantee the gauge invariance

IR subtraction:

- Hard process + Real emission = ***IR free result***
- In Soft-Collinear limit the the amplitude factorizes as **universal** IR terms $\mathcal{I}$ times lower order amplitude.

Finite Result:

$$\langle \mathcal{M}^{(0,0)}|\mathcal{M}^{(1,1),fin}\rangle = \langle \mathcal{M}^{(0,0)}|\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}\langle \mathcal{M}^{(0,0)}|\mathcal{M}^{(0,0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}\langle \mathcal{M}^{(0,0)}|\mathcal{M}^{(1,0),fin}\rangle - \mathcal{I}^{(1,0)}\langle \mathcal{M}^{(0,0)}|\mathcal{M}^{(0,1),fin}\rangle$$

Precision Standard Model Computations

- **γ^5 Problem in DR Scheme**

  γ^5 definition is a problem in DR scheme, in 4-d theory

$$\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 \,,$$

  This would yield properties of γ^5 matrix

$$\{\gamma_\mu, \gamma_5\} = 0 \,, \qquad Tr(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = 4i\epsilon_{\mu\nu\rho\sigma} \,, \qquad Tr(ABCD) = Tr(DABC)$$

  However, the three preparties would contradict with each others, in $\boldsymbol{d}$-dimension we have to sacrifice one of those conditions to achieve Dirac trace in $\boldsymbol{d}$-dimension

Property	Larin	HVBM	KKS (Kreimer)
Anticommuting γ^5 in D dimensions	✗	✗	✓
Trace cyclicity	✓	✓	✗
Consistent γ^5 treatment	✓*	✓	✓*
Preserves gauge symmetry without extra finite counterterms	△	✗	✓
Suitable for automated two-loop calculations	✓*	△	✓

Convenient for higher loop computation

Precision Standard Model Computations

- ***How can we Trust our Result?***
- ***How can we Checks our Calculations?***

Precision Standard Model Computations

- **Summarize of the Check and problem we should deal with**

Drawing all the relative Feynman diagrams of $ee \to \bar{b}b$

⇩

Numerically evaluate all the loop integrals

⇩

Evaluate all the CTs to obtained the UV free result

⇩

Evaluate real emissions to get the IR free result

⇩

Combine all the result with high numerical precision

The two-loop result must be checked by:

Gauge consistency	Verified Background-field Ward identities.
γ^5 independent	The result is not dependent how we write the Dirac trace.
Numerical stability	Large cancellation under control thanks to arbitrary precision.
UV + IR pole free	Numerical cancellation of the UV and IR poles.

"Pole-free" means below the precision threshold, not an exact symbolic zero.

For us "0" mean 10^{-23}, which is also the boundary of our result

Precision Standard Model Computations

- **Numerical checks**

  γ^5 scheme independent

  As we mentioned before KKS scheme sacrificed the trace cyclicity. So we need to make sure different choice of trace rules didn't change the physical result

Ordering I: $\mathrm{Tr}\left[(\not{p}_1+m_1)A(\not{p}_2-m_2)B\right]\mathrm{Tr}\left[(\not{p}_3+m_3)C(\not{p}_4-m_4)D\right]$

Ordering II: $\mathrm{Tr}\left[(\not{p}_2-m_2)B(\not{p}_1+m_1)A\right]\mathrm{Tr}\left[(\not{p}_3+m_3)C(\not{p}_4-m_4)D\right]$

Powers of ϵ	Spinor ordering I	Spinor ordering II
ϵ^{-2}	$-111.87867444499615+71.39575849824868\,i$	$-111.87867444499615+71.39575849824868\,i$
	$111.87867444499615-71.39575849824868\,i$	$111.87867444499615-71.39575849824868\,i$
	0	0
ϵ^{-1}	$414.2026774143816-521.8700903387290\,i$	$414.2026774143816-521.8700903387290\,i$
	$-1697.7165848553345+1083.4036414186723\,i$	$-1697.7165848553345+1083.4036414186723\,i$
	$1283.5139074409529-561.5335510799433\,i$	$1283.5139074409529-561.5335510799433\,i$
ϵ^{0}	$-1082.578129337419+1953.168032413456\,i$	$-912.016346039799+2026.456264089261\,i$
	$12754.493508174951-8139.323627085281\,i$	$12754.493508174951-8139.323627085281\,i$
	$-15232.585522220781+6825.157089559418\,i$	$-15403.147305518401+6751.868857883613\,i$
$\langle\mathcal{M}^{(0,0)}\vert\mathcal{M}^{(1,1),fin}_{\mathrm{box}}\rangle$	$-3560.670143383249+639.001494887593\,i$	$-3560.670143383249+639.001494887593\,i$

Precision Standard Model Computations

- **Numerical checks**

Pole cancellation

Powers of ϵ	Individual coefficients	Net coefficient
ϵ^{-3}	$-96.037444831343384754117143226765 9 + 61.286623674992446894055395208109 4\ i$	$10^{-31} + 10^{-31}\ i$
	$96.037444831343384754117143226765 9 - 61.286623674992446894055395208109 4\ i$	
	0	
ϵ^{-2}	$549.64698096126642874719163 5 - 1042.909866793504348258865005\ i$	$10^{-24} + 10^{-24}\ i$
	$-1039.998715375774381297790937 + 1135.433925663155411965509782\ i$	
	$490.35173441450795255059930 2 - 92.524058869651063706644777\ i$	
ϵ^{-1}	$-7368.37063893291373338072136 + 5351.64606797110399891415785\ i$ $+(603.42106230336687452888419 - 385.07521340135713448914441\ i)\Delta\alpha^{(5)}_{\rm had}(M^2_{Z,\rm OS})/\alpha$	$10^{-23} + 10^{-23}\ i$
	$4536.68622225092270337759730 - 10053.80847407197270156725540\ i$	
	$2831.68441668199103000312405 + 4702.16240610086870265309755\ i$ $-(603.42106230336687452888419 - 385.07521340135713448914441\ i)\Delta\alpha^{(5)}_{\rm had}(M^2_{Z,\rm OS})/\alpha$	

Finite remanent

$$\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(1,1),fin} \rangle$$
$$= -21678.171896726779812600 4003 - 21293.882901188480274024 4403\ i$$
$$+(3080.970813023263155538 2841 - 581.345807250410661461 7399\ i)\Delta\alpha^{(5)}_{\rm had}(M^2_{Z,\rm OS})/\alpha.$$

Perturbative controlled

Light-quark non-perturbative nature

Precision Standard Model Computations

- **Summary**

1. Workflow which can deal with IR+UV pole and γ^5 in DR scheme simultaneously and consistently.
2. $e^+e^- \to b\bar{b}$ two-loop amplitude with massive bottom quarks at $\mathcal{O}(\alpha^2\alpha_S)$.
3. Obtaining the UV-renormalized and IR-finite matrix element.

- **Outlook**

1. Scan the phase space relevant to future e^+e^- colliders with ***Seasyde*+*AMFlow*** → Numerical Grid.
2. Phenomenology: interpolation of the Grid, e.g. In Monte Carlo simulations.
3. Evaluation of Z-boson form factors at $\mathcal{O}(\alpha^2\alpha_S)$ and prediction of the effective weak-mixing angle $\sin\theta_w$.
4. Precision top-quark pairs production can be studied by the same method (already important at LHC).

