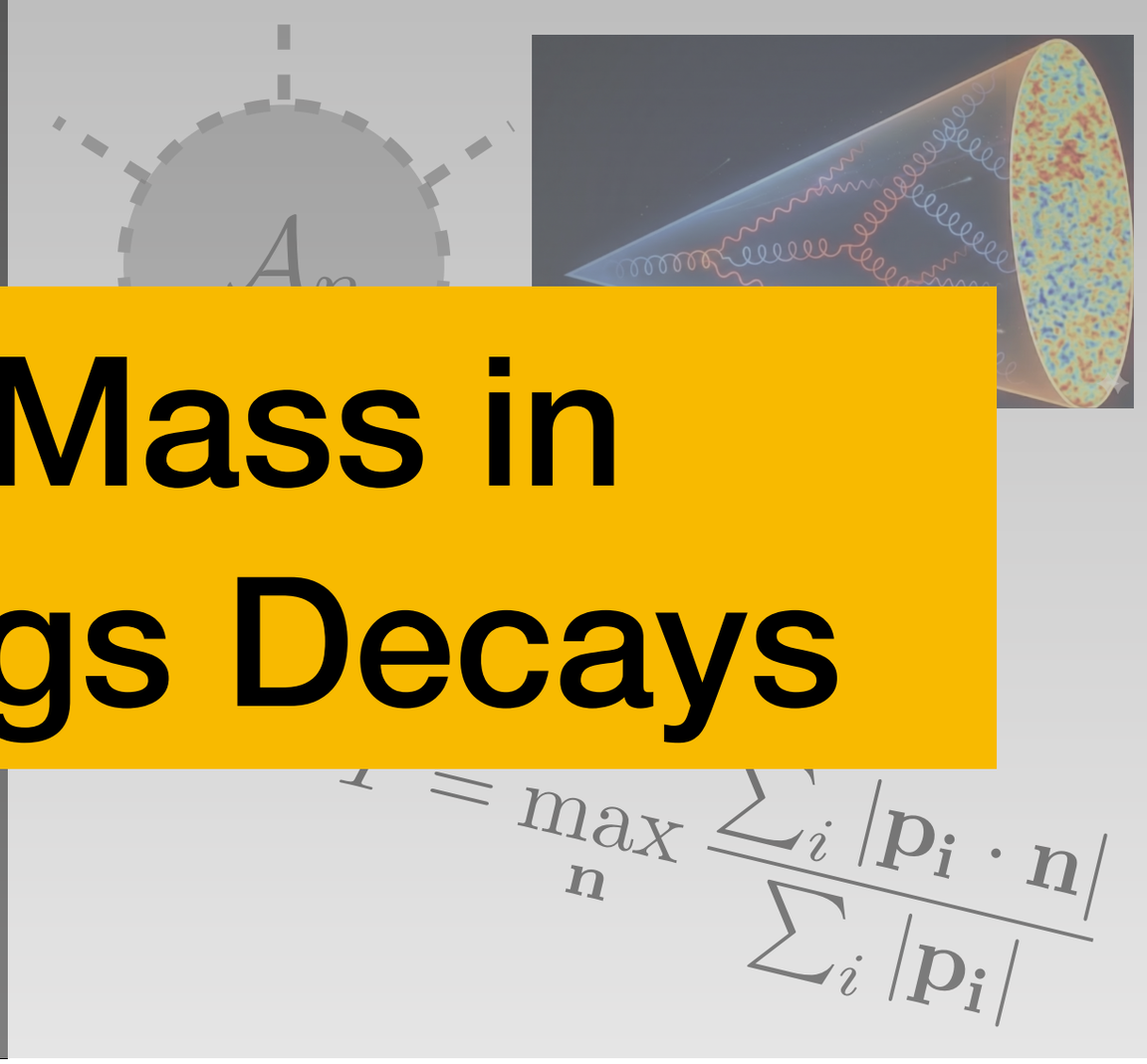




Heavy Jet Mass in Hadronic Higgs Decays

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CEPC 2026

Based on:

- [2502.12253]: with Miguel Benitez [Salamanca], Arindam Bhattacharya [DESY], André Hoang [Vienna], Vicent Mateu [Salamanca], Matthew D. Schwartz [Harvard] and Iain W. Stewart [MIT]
- [2607.25187]: with Luen Clingerman [Harvard]

Heavy jet mass (HJM)

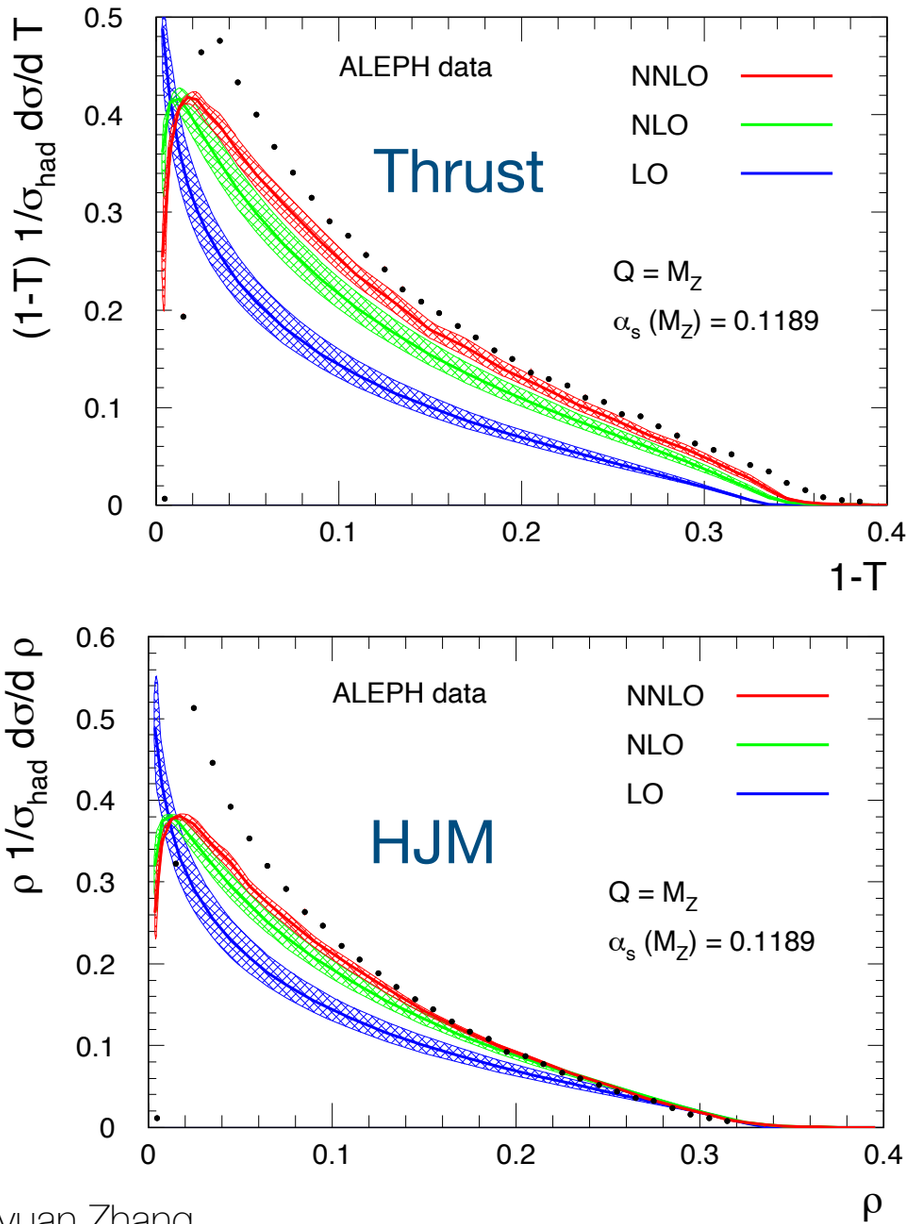
- Event shapes in e^+e^- collisions:

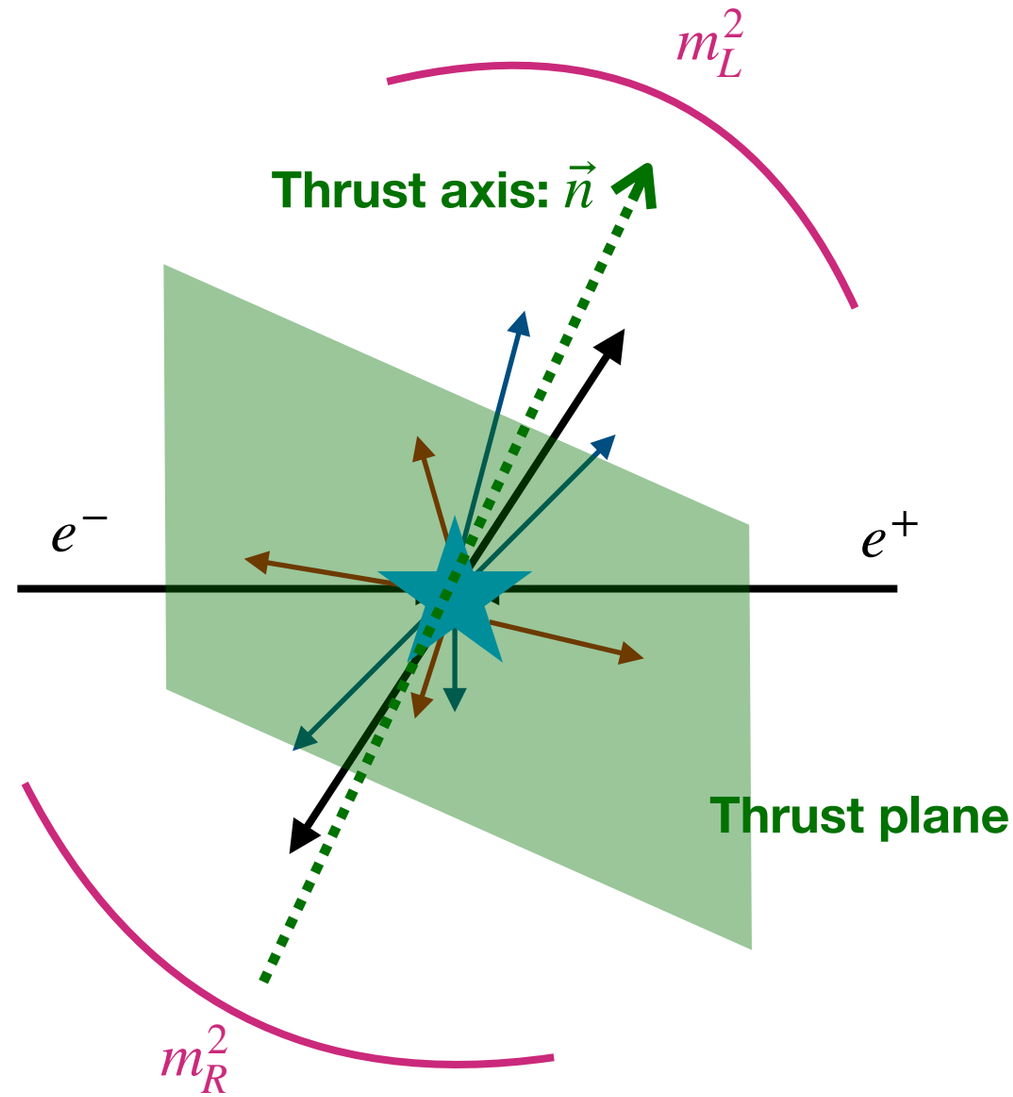
Thrust: $$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$$

HJM: $$\rho = \frac{1}{Q^2} \max\{m_L^2, m_R^2\}$$

$$\tau = 1 - T$$

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, 0711.4711]





Precision probes of QCD

- Event shapes provide a window to study QCD at different energy scales from collider data

1. The tensions of strong coupling constant α_s

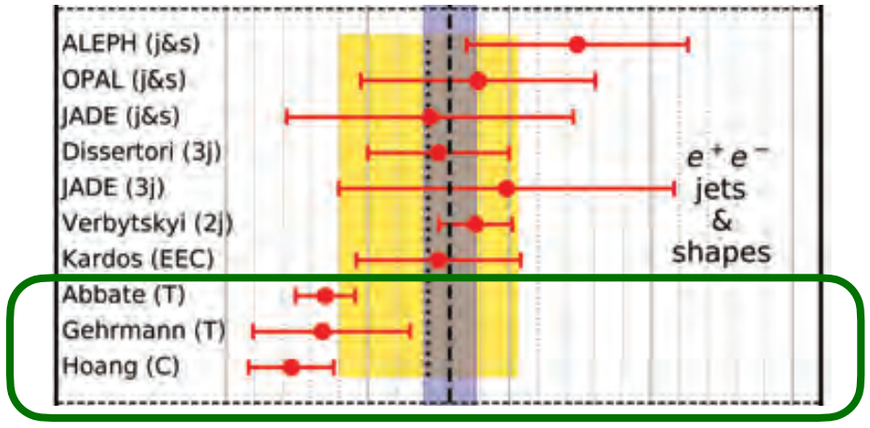
PDG 2025 average: $\alpha_s(m_Z) = 0.1180 \pm 0.0009$

e^+e^- 2025 average: $\alpha_s(m_Z) = 0.1189 \pm 0.0037$

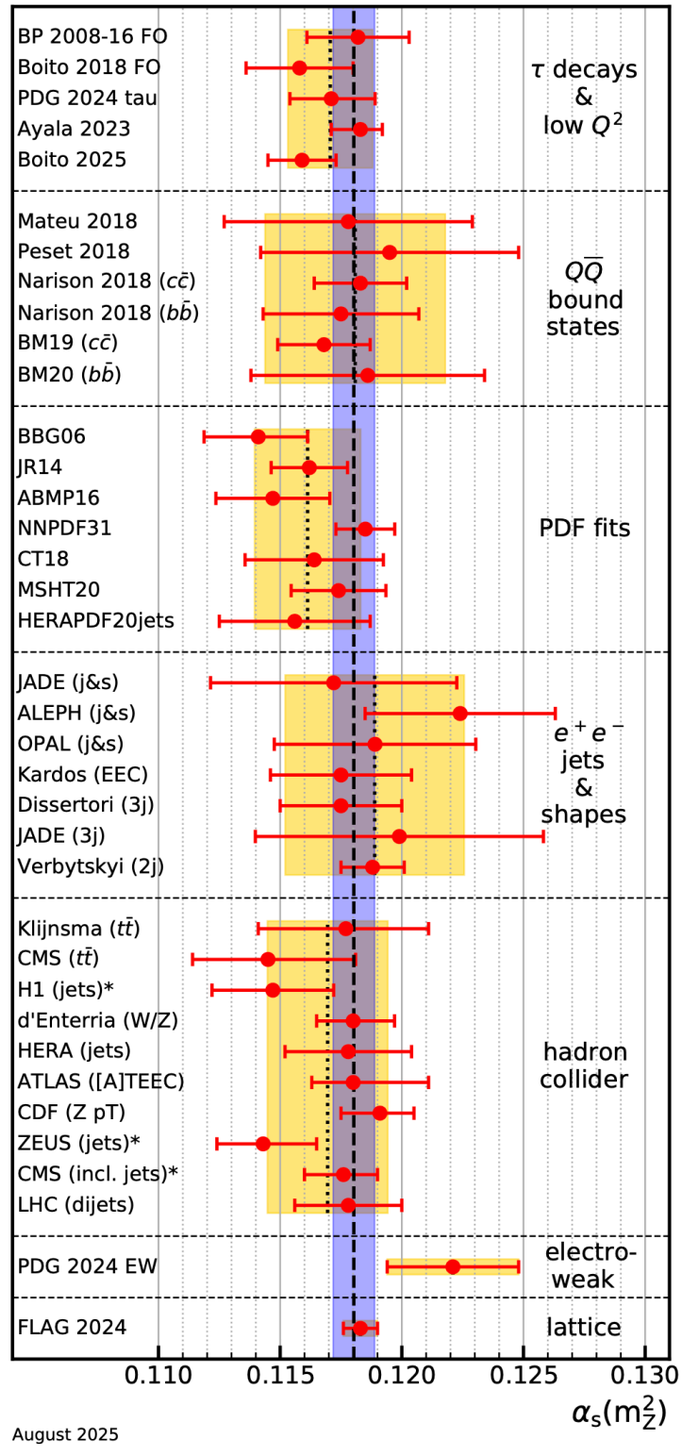
e^+e^- 2021 average: $\alpha_s(m_Z) = 0.1171 \pm 0.0031$

[Huston, Rabbertz, Zanderighi, PDG QCD review, 2312.14015]

Thrust/C parameter: N3LL+NNLO+ non-pert. corrections



[PDG QCD review 2025]



Latest SCET results

Thrust: 0.1136 ± 0.0012

HJM: $0.1148^{+0.0015}_{-0.0022}$

[Benitez, Hoang, Mateu, Stewart, Vita, 2412.15164]

[Benitez, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, XYZ, 2502.12253]

Precision probes of QCD

- Event shapes provide a window to study QCD at different energy scales from collider data

2. Extract non-perturbative QCD corrections Ω_1 from data

- Dijet configuration: do we have universality across different observables?

OPE analysis in SCET:

$$\Omega_1 = \frac{1}{N_c}\langle 0|\mathrm{tr}\,\bar{Y}_{\bar{n}}^T(0)Y_n(0)i\hat{\partial}Y_n^\dagger(0)\bar{Y}_{\bar{n}}^\star(0)|0\rangle$$

[Lee, Sterman, 0603066, 0611061]

In $\tau, \rho \to 0$ limit:

$$d\sigma^{\mathrm{pert}}(\rho) \Rightarrow d\sigma^{\mathrm{NP}}(\rho) = d\sigma^{\mathrm{pert}}\left(\rho - \frac{\Omega_1}{Q}\right)$$

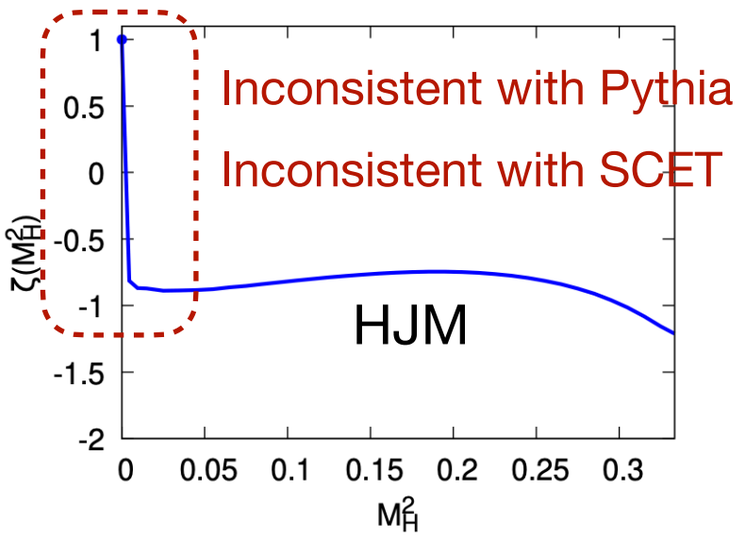
[Hoang, Mateu, Schwartz, Stewart, 2506.09130]

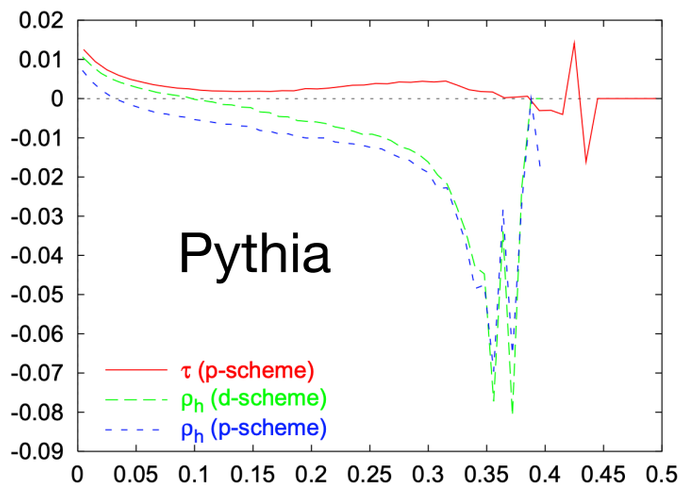
- 3-jet configuration:

[Luisoni, Monni, Salam, 2012.00622]

[Caola et al, 2108.08897 2204.02247]

[Nason, Zanderighi, 2301.03607]





[Salam, Wicke, hep-ph/0102343]

- Hadron mass effect: we can only measure 3-momentum (charged hadrons) or energy+direction (neutral hadrons) accurately

Hadronic Higgs decays at CEPC

- At CEPC, we can also study event shapes in hadronic Higgs decays

$$e^+e^- \to Z + H, \quad H \to q\bar{q}, gg$$

1. Individual channel

At $\sqrt{s} \approx m_Z + m_H = 216.2$ GeV, Higgs is nearly at rest.

At other energies, since ZH is clean, we can boost to Higgs rest frame.

- This provide additional data for extracting α_s and non-perturbative corrections.

  An example study for energy correlators: [Lin, Ruan, Xiao, Xu, 2406.10946]

- $H \to gg$ hadronization: opportunity to extract gluonic channel Ω_{1g}

2. Combining two channels

- The cross-section also depends on Yukawa coupling and Hgg effective coupling
- Combined with flavor information and the Higgs width measurements, this can constrain their values.

[Gao, 1608.01746]

HJM in hadronic Higgs decays

- Higgs effective field theory (HEFT)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\lambda(\mu)\mathcal{O}_g + \sum_q \frac{y_q(\mu)}{\sqrt{2}}\mathcal{O}_q, \qquad \mathcal{O}_g = H\,\text{Tr}(G^{\mu\nu}G_{\mu\nu}), \qquad \mathcal{O}_q = H\bar{\psi}_q\psi_q.$$

Massless quark limit: these two channels decouple.

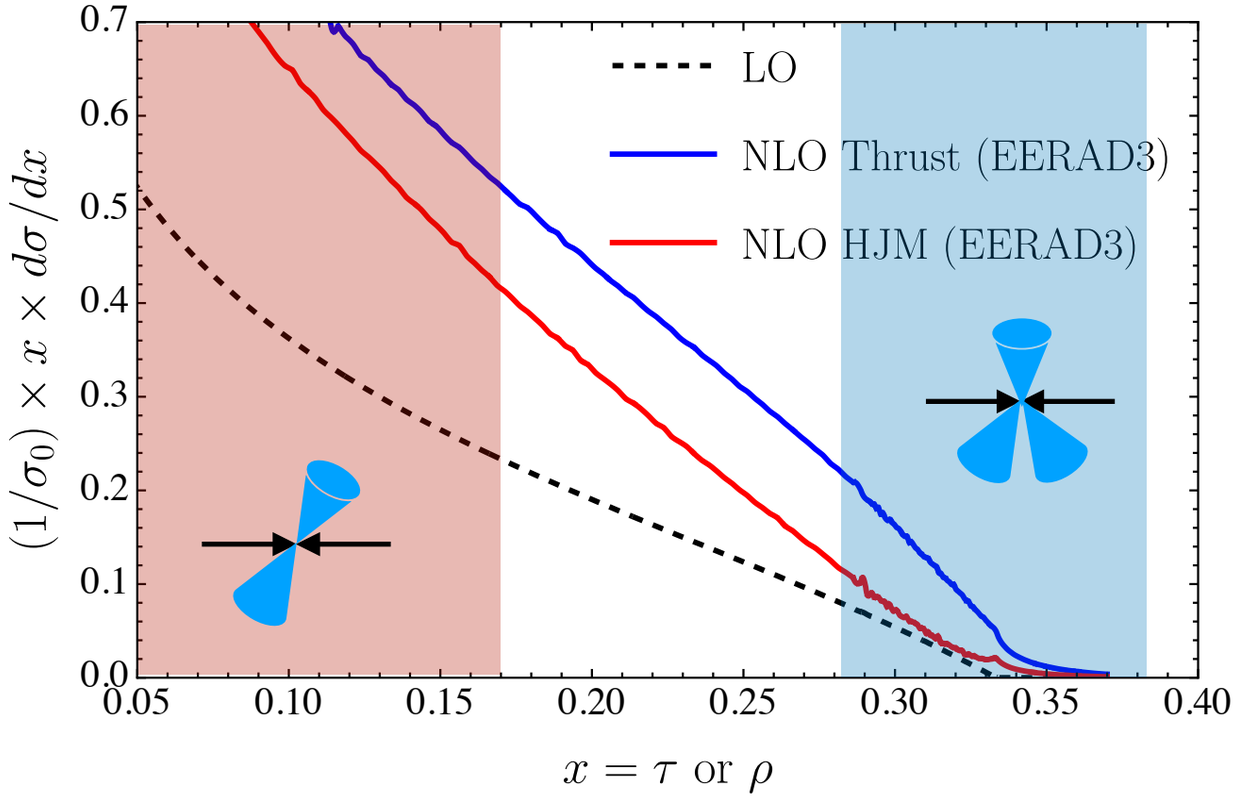
[Gao, Gong, Ju, Yang, 1901.02253]
[Ju, Xu, Yang, Zhou, 2301.04294]
[Fox et al., 2512.03263]

- How do we make precise predictions for HJM?

- Fixed-order prediction can be spoiled by the existence of large logarithms
  - Dijet logarithms: $\sum_{i=1}^{\infty}\sum_{j=0}^{2i-1}\alpha_s^i\, a_{ij}\,\frac{\ln^j\rho}{\rho}$
  - Shoulder logarithms: $r \equiv \frac{1}{3} - \rho$

$$\sum_{i=2}^{\infty}\sum_{j=0}^{2i-2}\alpha_s^i\left[r\theta(r)b_{ij}\ln^j r + (-r)\theta(-r)c_{ij}\ln^j(-r)\right]$$

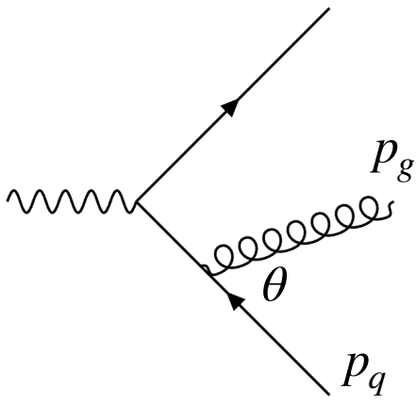
Sudakov shoulder:
[Catani, Webber, hep-ph/9710333]



Perturbative QCD: infrared divergence

- Perturbation theory receives large logarithmic enhancement:

$$\frac{1}{\sigma_0}\frac{d\sigma^{\rm LO}_{Hq\bar{q}}}{d\rho} = \delta(\rho) + \frac{\alpha_s}{2\pi}C_F\left[-4\frac{\ln\rho}{\rho} - \frac{3}{\rho} + 4 + 2\ln\rho + \cdots\right]$$



$$\frac{1}{(p_q+p_g)^2} = \frac{1}{2E_qE_g(1-\cos\theta)}$$

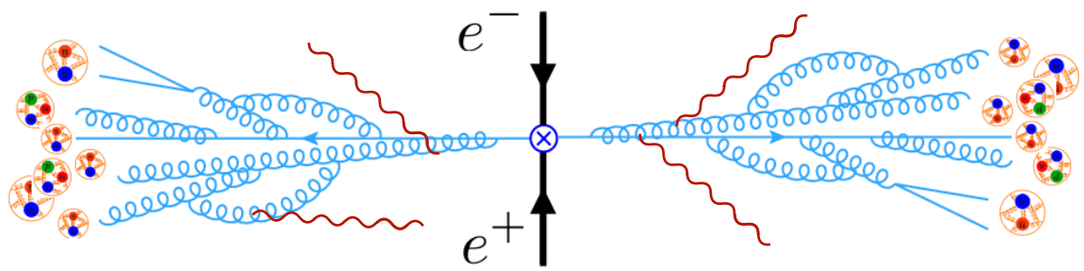
$E_g \to 0$: soft divergence

$\theta \to 0$: collinear divergence

Cumulant: $\Sigma(\rho) = \int_0^\rho \frac{d\sigma}{d\rho'}d\rho' \sim \alpha_s \frac{d\theta}{\theta}\frac{dE}{E} \quad \Rightarrow \quad \alpha_s \ln^2\rho$

- Dijet kinematics

$$\rho \sim \frac{\max\{p_L^2 + Qk_L,\, p_R^2 + Qk_R\}}{Q^2}$$



$$\sum_{i=1}^{\infty}\sum_{j=0}^{2i-1}\alpha_s^i\frac{a_{ij}}{\rho}\ln^j\rho$$

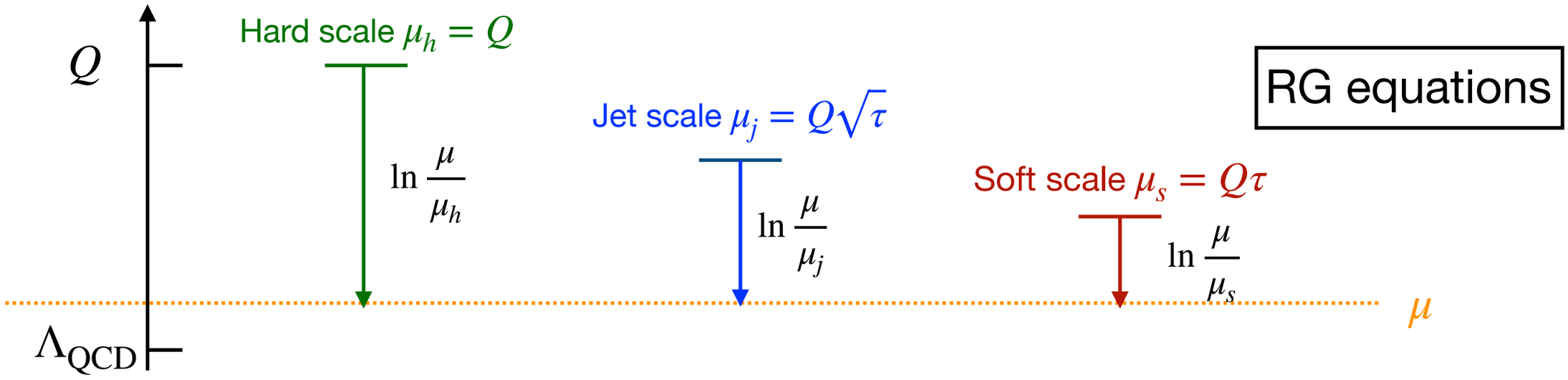
$\ln\frac{\mu}{Q} + \ln\frac{Q\sqrt{\rho}}{\mu}$ or $\ln\frac{\mu}{Q} + \ln\frac{Q\rho}{\mu}$

Hard scale — Collinear scale

Hard scale — Soft scale

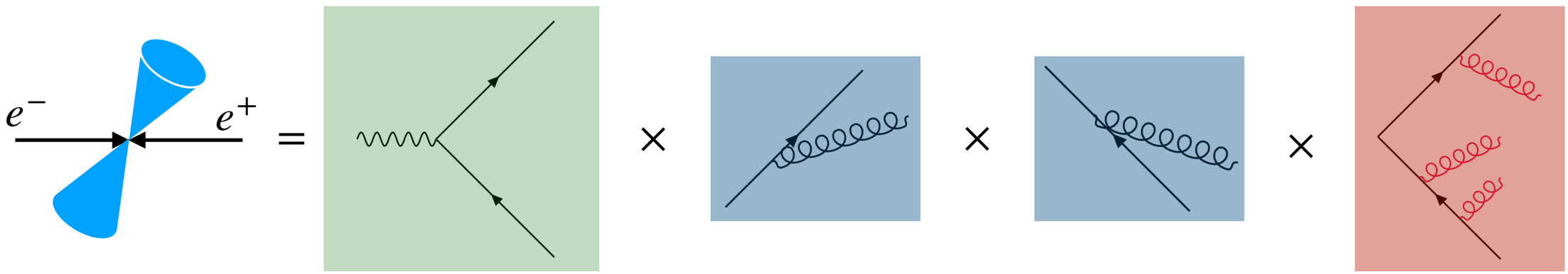
Effective field theory

- Soft-collinear effective theory (SCET): an EFT for QCD and collider physics



- SCET brings the factorization theorem

$$\frac{1}{\sigma_0}\frac{d^2\sigma_{\rm dij}}{dM_L^2\,dM_R^2} = H_i^{\rm dij}(Q^2,\mu)\int dk_L\,dk_R\, J_i(M_L^2 - Qk_L,\mu)\, J_i(M_R^2 - Qk_R,\mu)\, S_i^{\rm dij}(k_L,k_R,\mu)$$



Hard function | Jet function (n) | Jet function ($\bar{n}$) | Soft function

Effective field theory

- Dijet resummed cumulant:

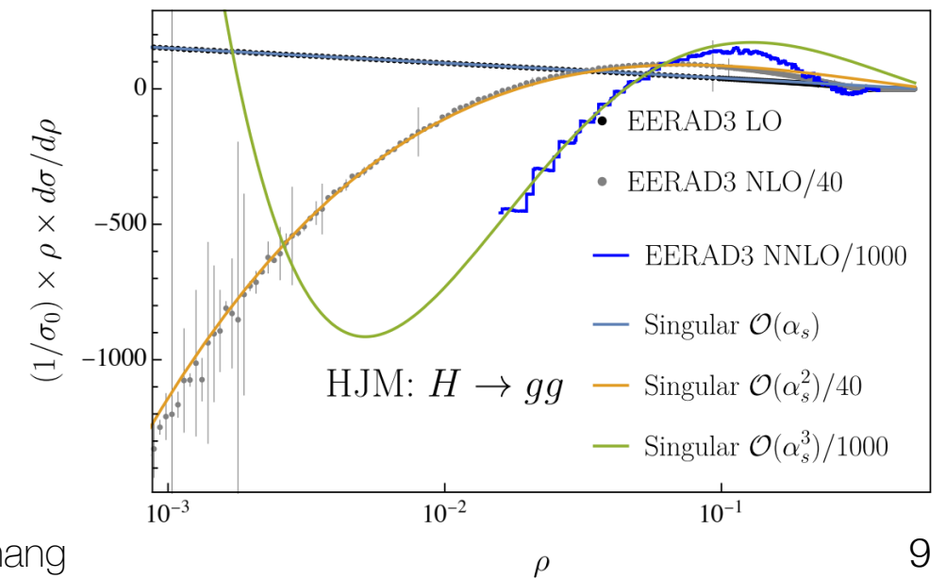
$$\Sigma_i^{\rho}(\rho) = W_i(\mu_h,\mu_j,\mu_s)\,\mathcal{N}(Q^2,\mu_h)\,H_i^{\rm dij}(Q^2,\mu_h)\widetilde{j}_i\left(\ln\frac{\mu_s Q}{\mu_j^2}+\partial_{\eta_1},\mu_j\right)\widetilde{j}_i\left(\ln\frac{\mu_s Q}{\mu_j^2}+\partial_{\eta_2},\mu_j\right)$$
$$\times\,\widetilde{s}^{\rm dij}_{\mu,i}(\partial_{\eta_1},\mu_s)\widetilde{s}^{\rm dij}_{\mu,i}(\partial_{\eta_2},\mu_s)\widetilde{S}_{{\rm NGL},i}(\partial_{\eta_1}-\partial_{\eta_2})\times\left(\frac{\rho Q}{\mu_s}\right)^{\eta_1+\eta_2}\frac{e^{-\gamma_E\eta_1}}{\Gamma(\eta_1+1)}\frac{e^{-\gamma_E\eta_2}}{\Gamma(\eta_2+1)}$$

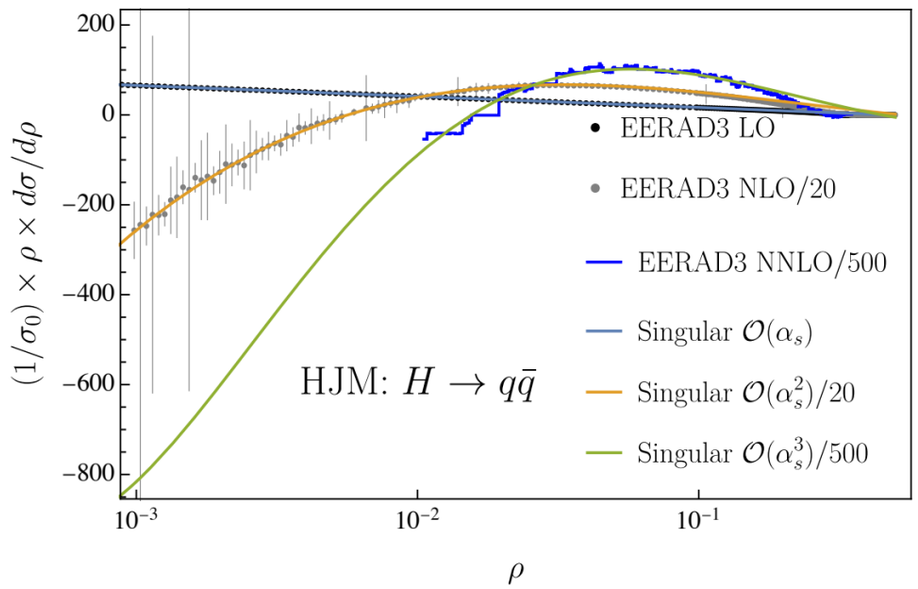
with evolution kernel

$$W_i(\mu_h,\mu_j,\mu_s) = \exp\left[4S_i(\mu_h,\mu_j)+4S_i(\mu_s,\mu_j)+2A^{\rm dij}_{S,i}(\mu_s,\mu_h)+4A_{J,i}(\mu_j,\mu_h)\right]\left(\frac{Q^2}{\mu_h^2}\right)^{-2A_{\Gamma,i}(\mu_h,\mu_j)}$$

$$S_i(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_i^{\rm cusp}(\alpha)}{\beta(\alpha)}\int_{\alpha_s(\nu)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')},\qquad A_{\Gamma,i}(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_i^{\rm cusp}(\alpha)}{\beta(\alpha)}$$

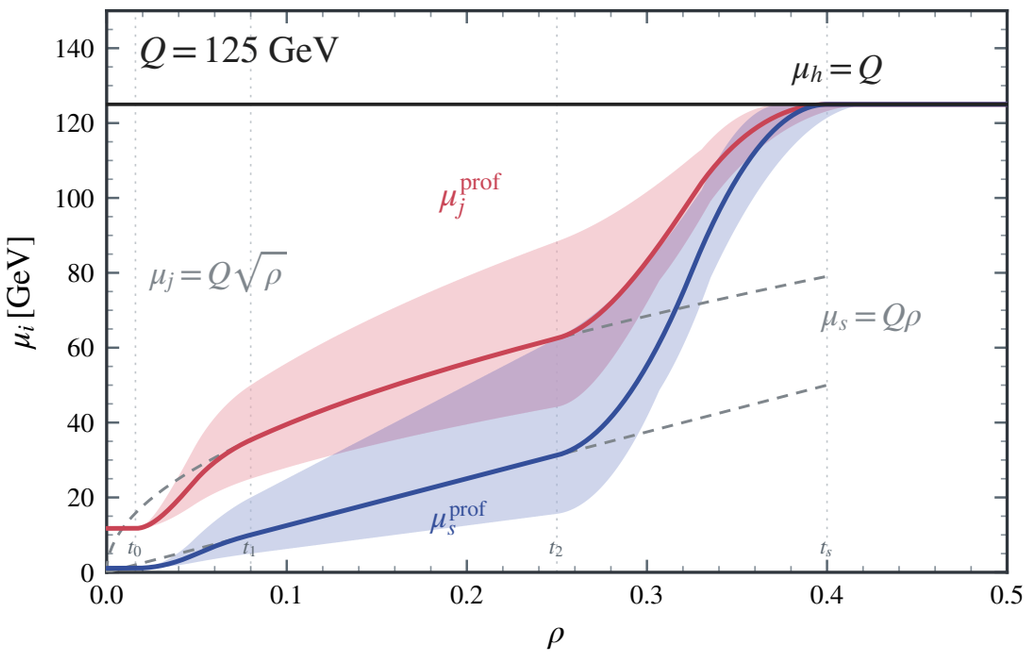
- Check against EERAD3 v2 program: [Campillo et al., 2503.20610]

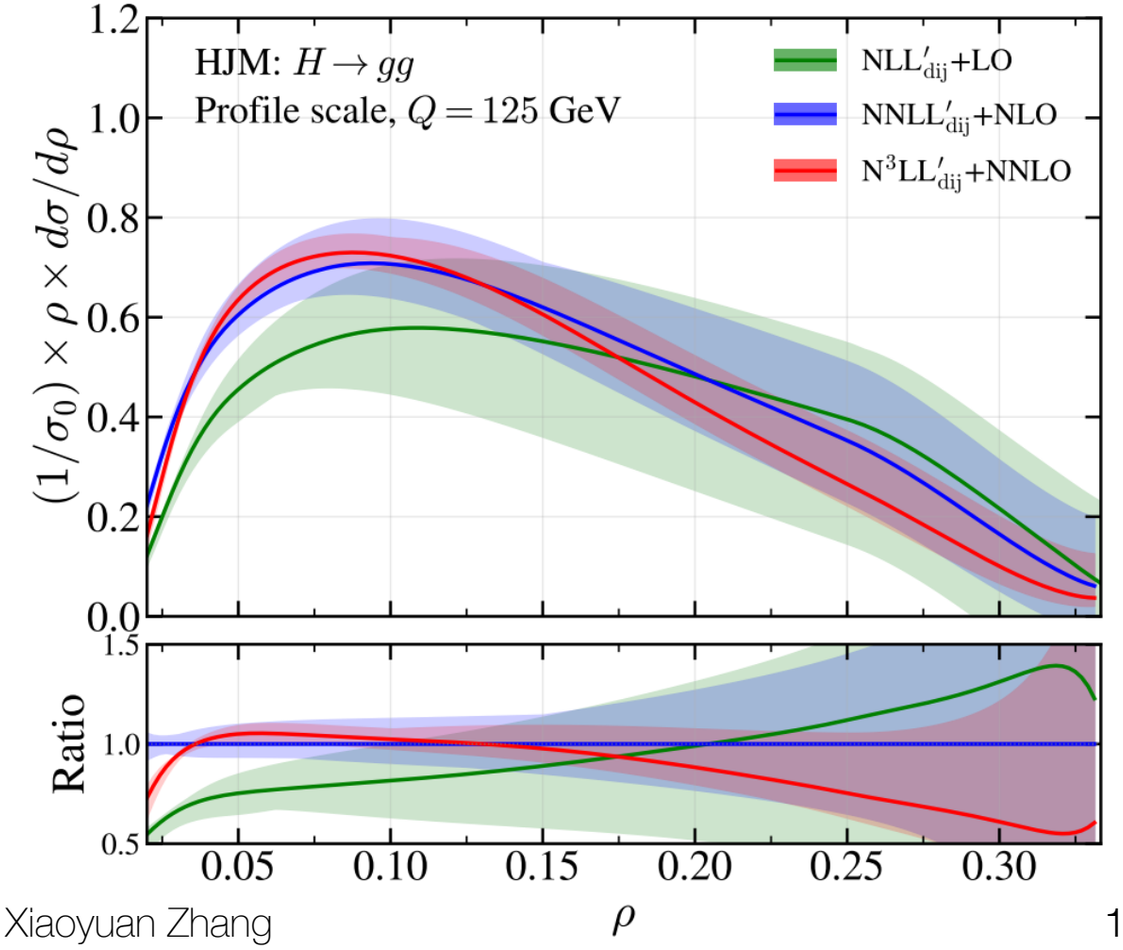


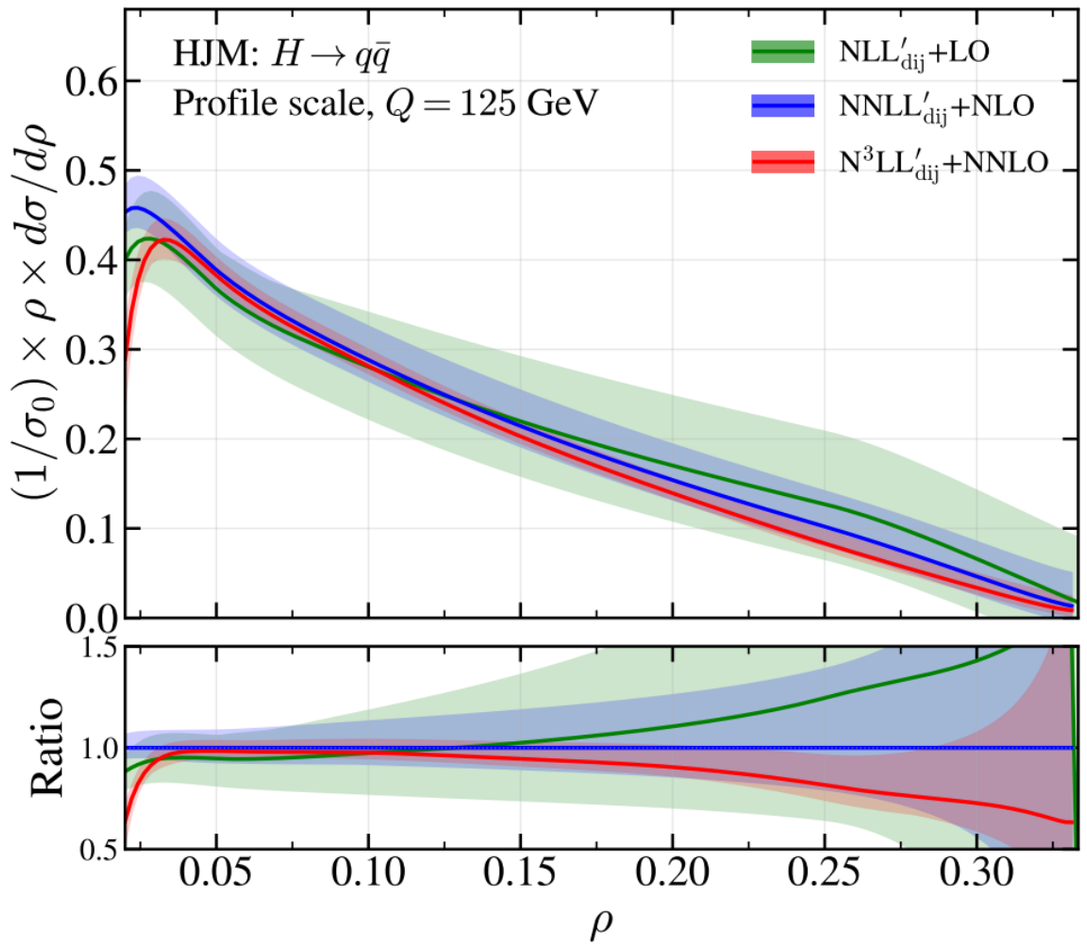


Dijet resummation in Higgs decays

- Fixed-order matching with profile scales
- Profile scales are designed to turn off the resummation (i.e. use fixed-order) when the large logarithms are no longer large.
- Numerical results:

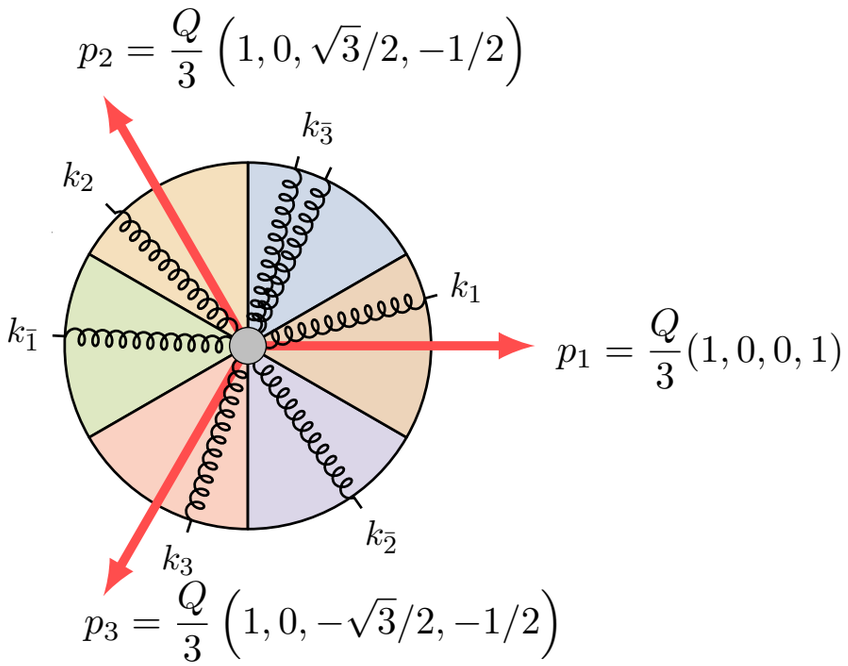






Sudakov shoulders

- Sudakov shoulder arises from trijet configurations



The maximal value of τ, ρ depend on the final-state multiplicity

3 partons $\quad \tau, \rho \leq \frac{1}{3}$

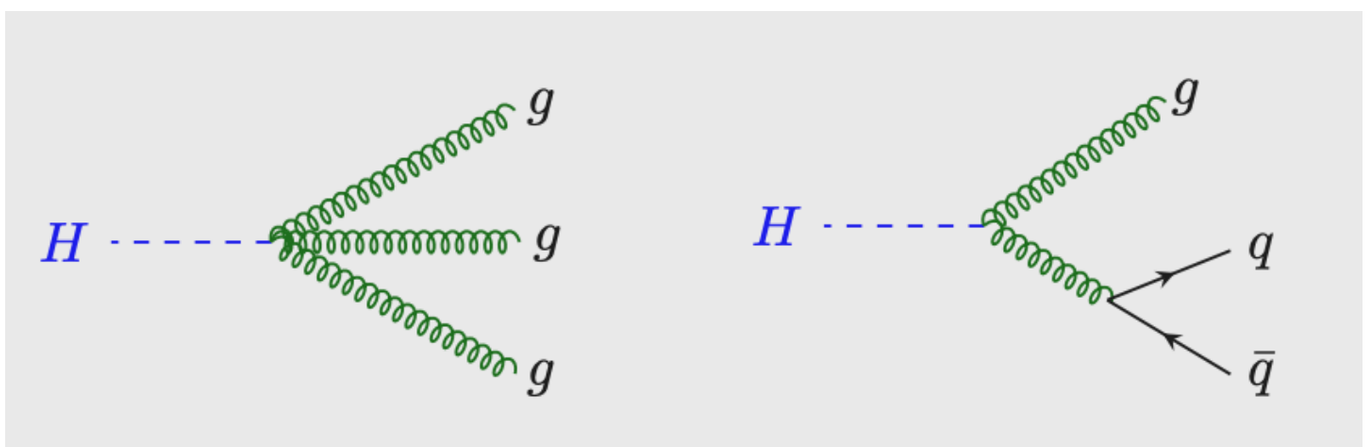
4 partons $\quad \tau, \rho \leq 0.42$

Incomplete cancellation of infrared divergence

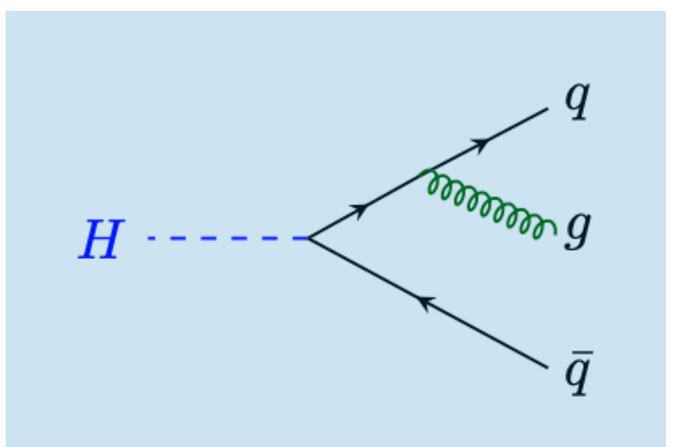
$\Rightarrow$ divergence, kinks, etc.

$\Rightarrow$ large logarithms

- Subprocesses near trijet limit $\tau, \rho \to 1/3$



$H \to gg$



$H \to q\bar{q}$

Sudakov shoulders

- 1. NLO Parameterization: $H(Q) \to i(p_1) + j(p_2) + k(p_3) + l(p_4)$

$$p_3 = z\omega n + \frac{q_\perp^2}{4z\omega}\bar{n} + q_\perp, \qquad p_4 = (1-z)\omega n + \frac{q_\perp^2}{4(1-z)\omega}\bar{n} - q_\perp,$$

$$p_1 = z_1 n + \frac{p_\perp^2}{4z_1}\bar{n} + p_\perp, \qquad p_2 = z_2 n + \frac{p_\perp^2}{4z_2}\bar{n} - p_\perp.$$

$$\Rightarrow \quad \{s_{234},\, s_{34},\, z,\, \omega,\, \phi\}, \qquad \text{with} \quad s_{234} \equiv (p_2+p_3+p_4)^2, \quad s_{34} \equiv (p_3+p_4)^2$$

- 2. Trijet expansion: $x \equiv \omega - \frac{1}{3}, \quad y \equiv s_{234} - \frac{1}{3}$
  - Generate the $H \to 4$ partons integrand using QGRAF+FORM
  - Power expansion: $s_{34} \sim \lambda,\, x \sim \lambda,\, y \sim \lambda,\, z \sim \lambda^0,\, \phi \sim \lambda^0$
  - Simplify thrust constraints also in the trijet limit. For example,

$$T_1^2 = T_{234}^2 = (1-s_{234})^2, \quad T_2^2 = T_{134}^2 = \left[\frac{2\omega(1+s_{234}-2\omega)-s_{34}}{2\omega}\right]^2,$$

$$T_{12}^2 = T_{34}^2 = \left(\frac{4\omega^2 - s_{34}}{2\omega}\right)^2, \quad T_{14}^2 = T_{23}^2 = \left[\frac{s_{34}z - 2\omega\,(1+s_{234}-2\omega(1-z))}{2\omega}\right]^2 - 4s_{23}$$

- 3. Decompose the phase space into collinear, soft, and soft-collinear

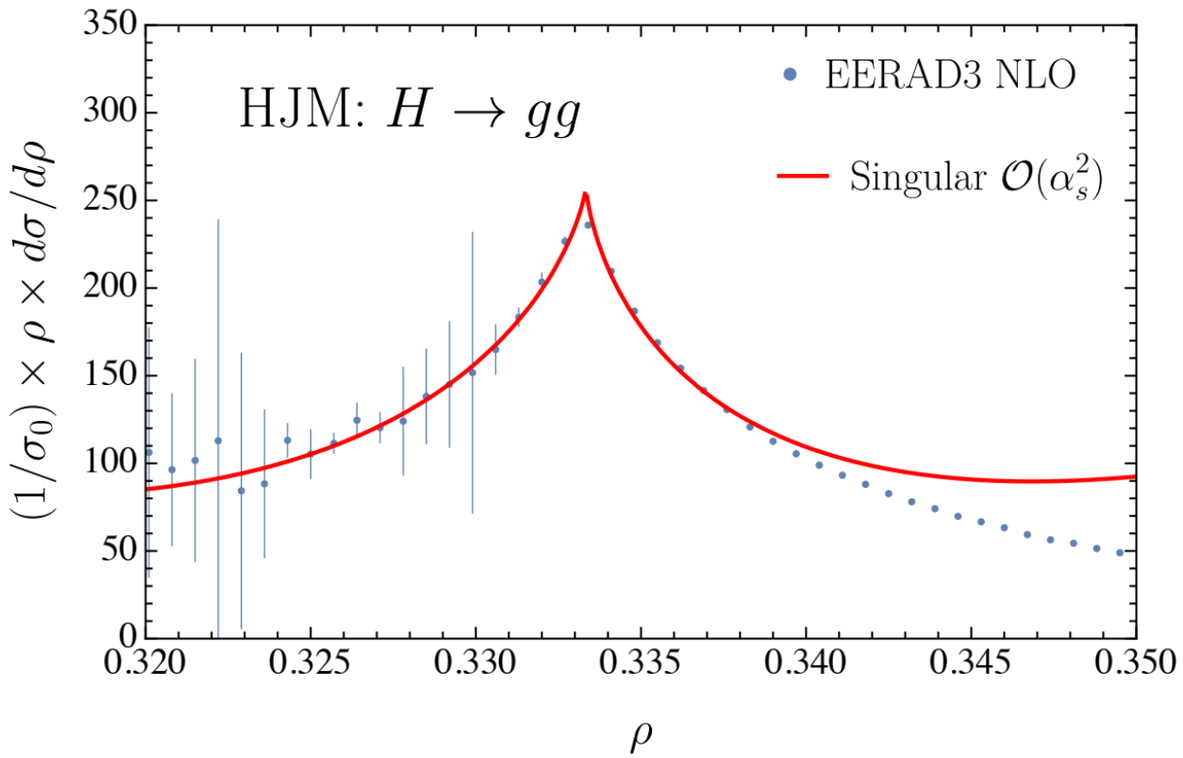
Sudakov shoulders

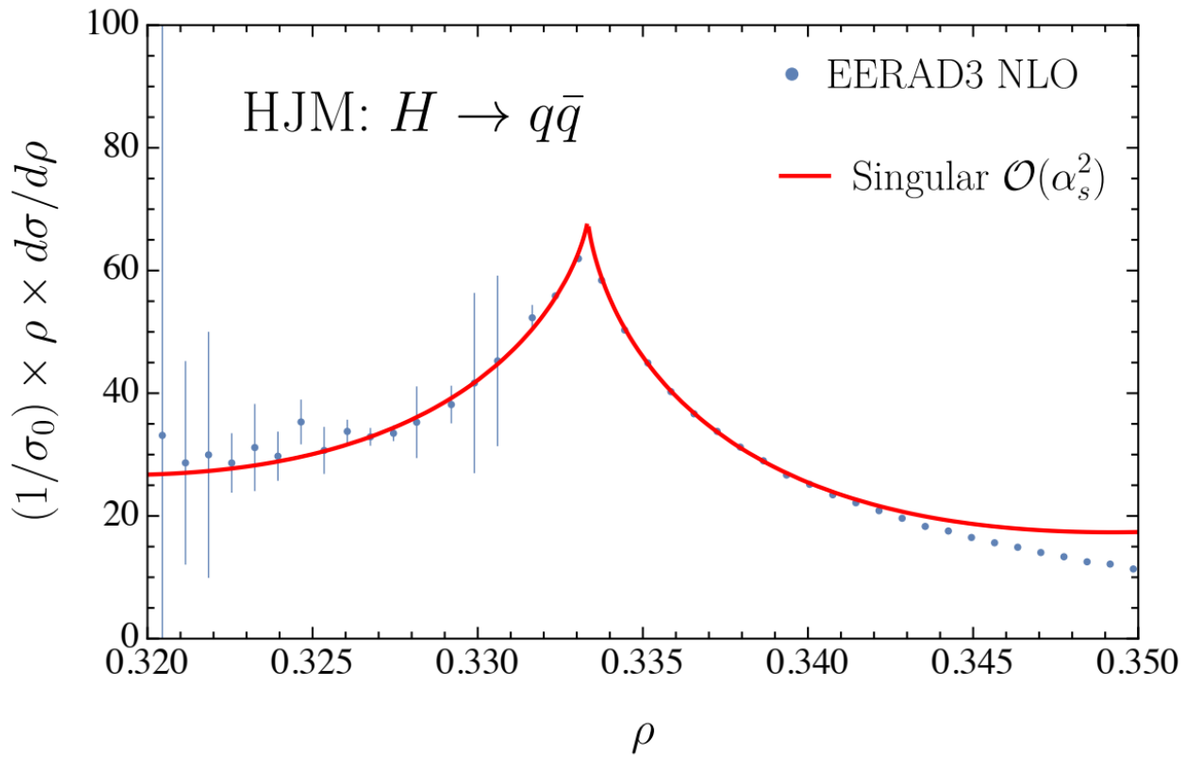
- Adding all regions gives the singular expansions. For example, left shoulder:

$$\left.\frac{1}{\sigma_0}\frac{d\sigma_{Hq\bar{q}}}{d\rho}\right|_{\rho\to\frac{1}{3}^-} = \left(\frac{\alpha_s}{4\pi}\right)^2 r\Big\{C_F^2\left[-240\ln^2 r + (120 + 960\ln 2 - 480\ln 3)\ln r\right]$$

$$r \equiv \frac{1}{3} - \rho \qquad + C_A C_F\left[-120\ln^2 r + (20 + 480\ln 2 - 240\ln 3)\ln r\right] + 40 C_F n_f \ln r\Big\}$$

- Check against EERAD3 v2 program (+ fitted constant and linear terms)





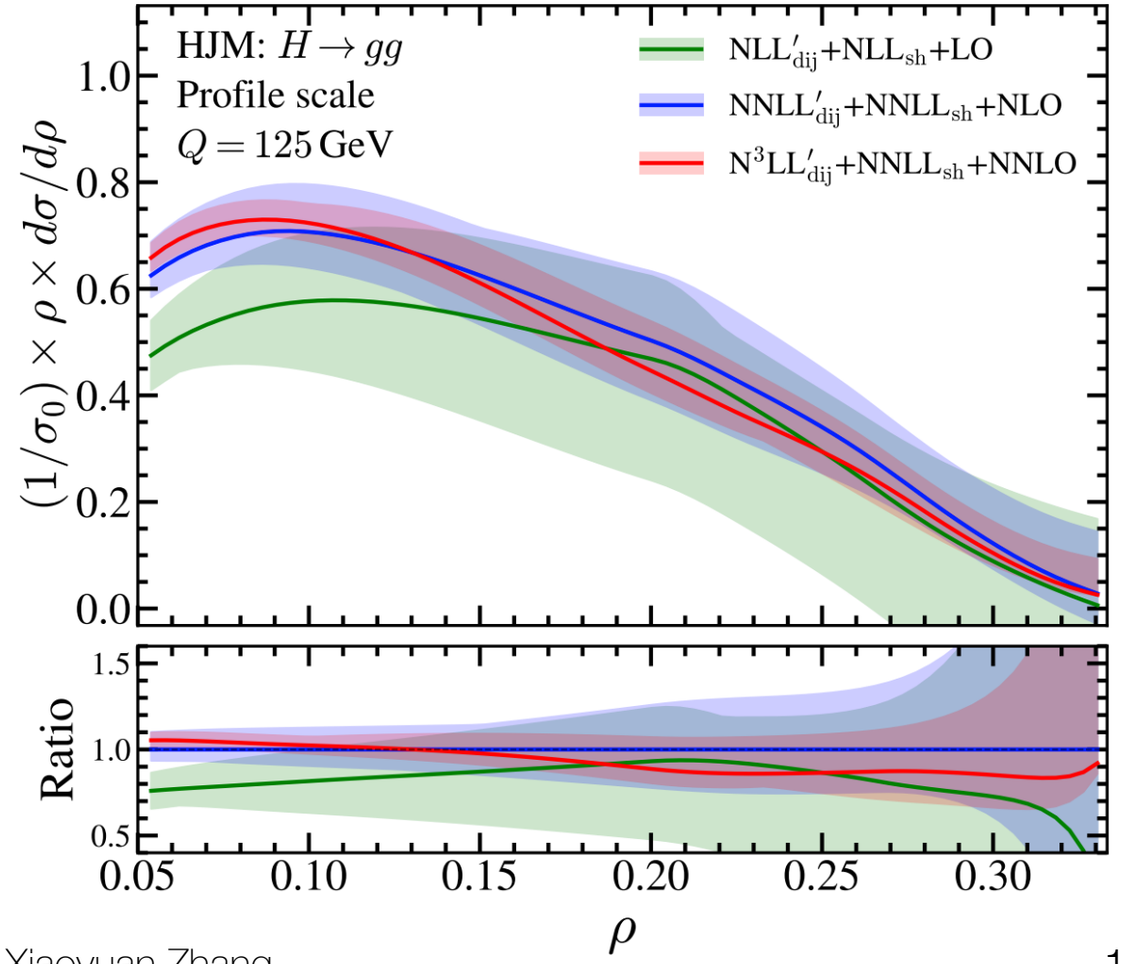
Joint resummation

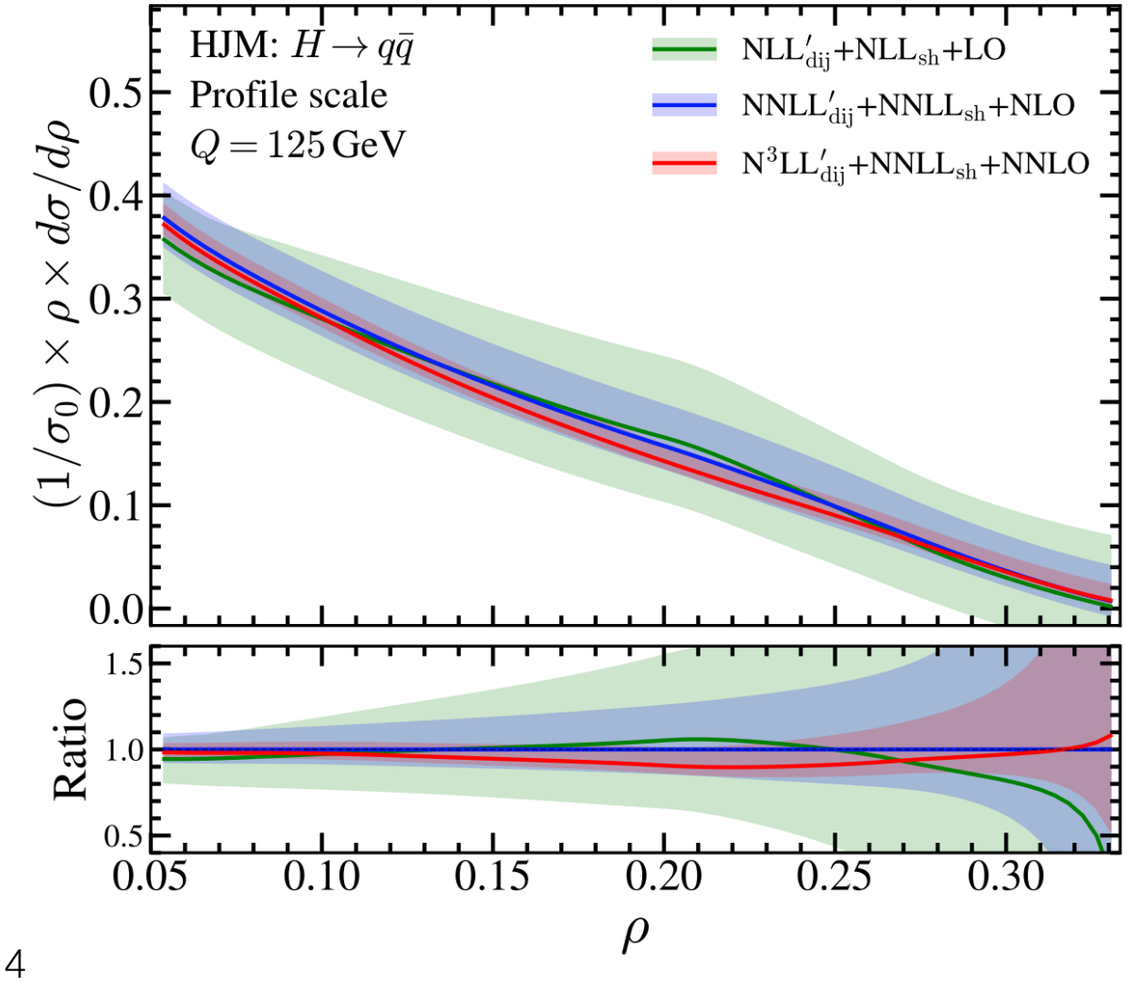
- Shoulder resummation, in Fourier space:

$$\widetilde{\sigma}_{\rm sh}(z) = \Pi(\mu_h, \mu_{j\ell}, \mu_{jh}, \mu_{s\ell}, \mu_{sh}) \cdot \widetilde{sj} \cdot \left(-iz\frac{e^{\gamma_E}\mu_{s\ell}}{Q}\right)^{-a} \left(iz\frac{e^{\gamma_E}\mu_{sh}}{Q}\right)^{-b}$$

$$\widetilde{sj} = \widetilde{j}_{h_1}\left(\ln\frac{Q^2e^{-\gamma_E}}{iz\mu_{jh}^2}\right)\widetilde{j}_{h_2}\left(\ln\frac{Q^2e^{-\gamma_E}}{iz\mu_{jh}^2}\right)\widetilde{j}_{\ell}\left(\ln\frac{Q^2e^{-\gamma_E}}{-iz\mu_{j\ell}^2}\right)\widetilde{s}_{\ell;h_1h_2}\left(\ln\frac{Qe^{-\gamma_E}}{-iz\mu_{s\ell}}, \ln\frac{Qe^{-\gamma_E}}{iz\mu_{sh}}\right)$$

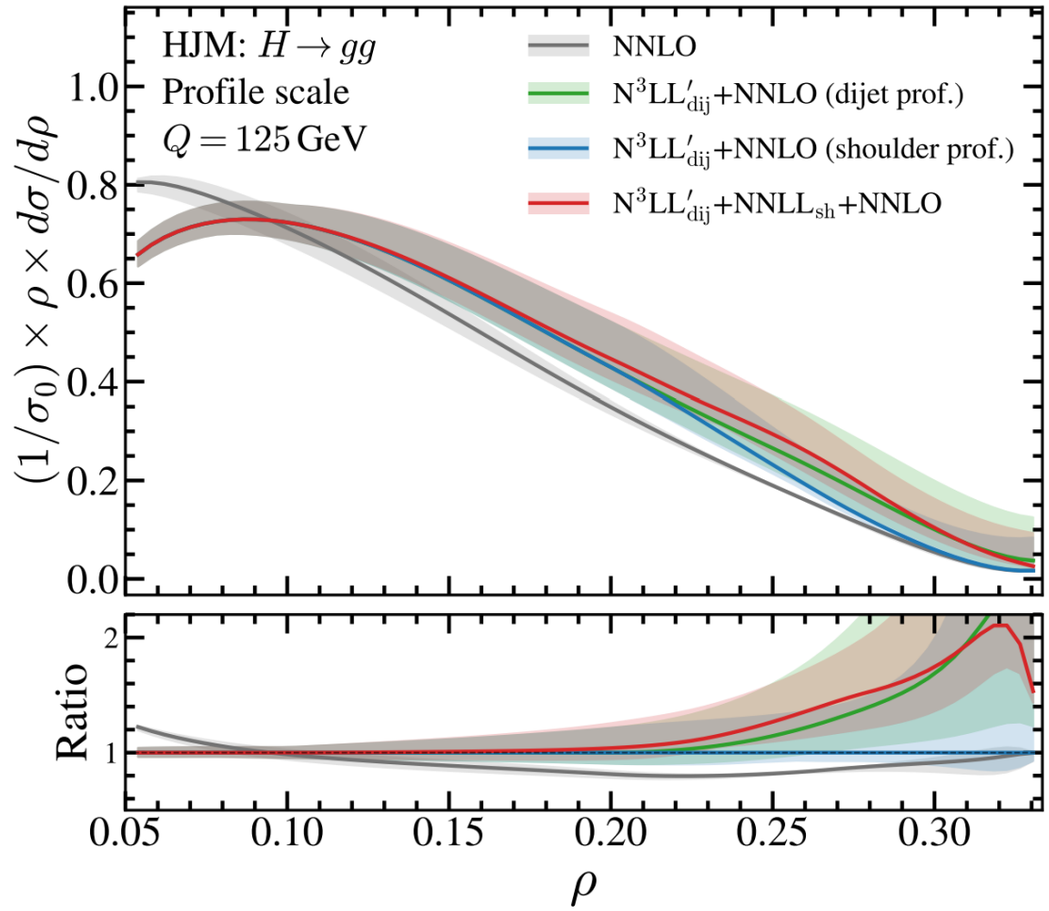
- Joint resummation with profile scales:

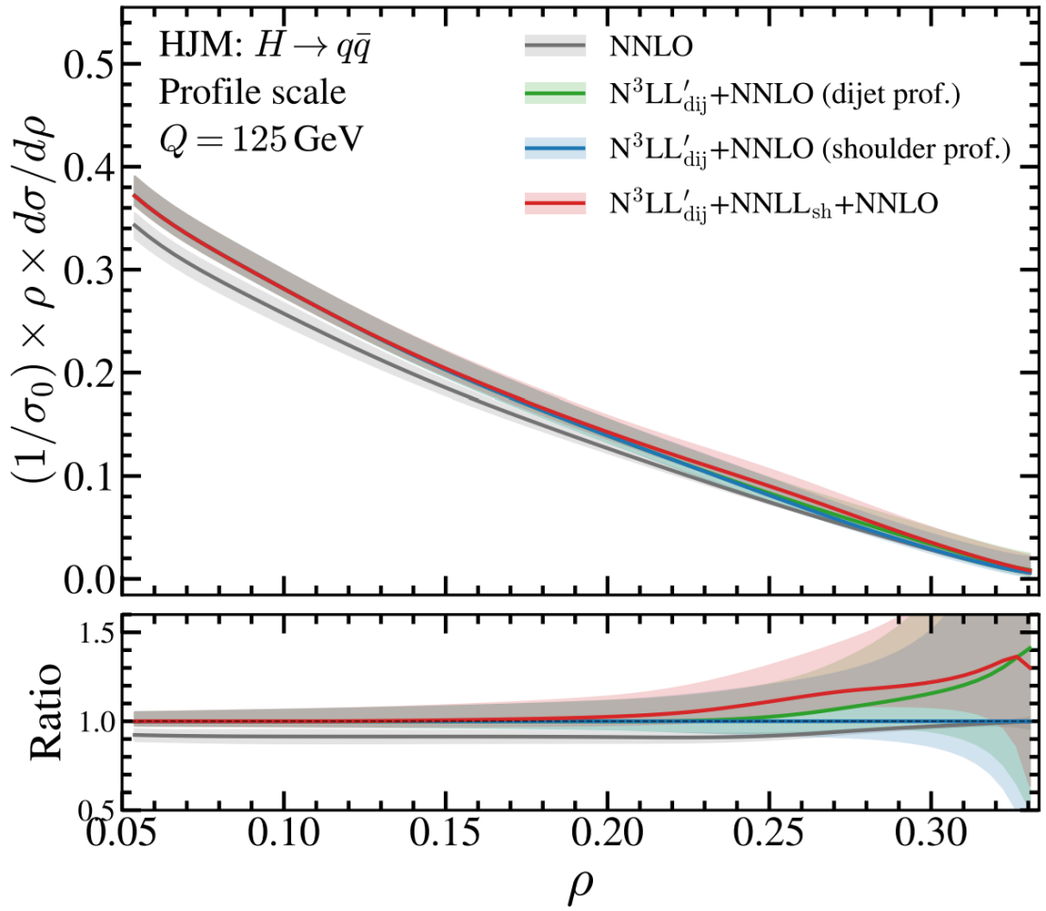




Joint resummation

- Compare different settings: ratio to dijet resum.+NNLO (modified profile)





- Compare with NNLO: both dijet and shoulder resummation are crucial
- Dijet+shoulder resummation+NNLO provides SOTA pert. prediction for HJM
- The gluon channel has a different shape and provides an opportunity to determine the hadronization parameter Ω_{1g}

Conclusion

- We extend the HJM factorization formula in both dijet and shoulder regions to hadronic Higgs decays: $H \to gg + X$ and $H \to q\bar{q} + X$
- Combining N^{3}LL$'$ dijet resummation, NNLL shoulder resummation, and NNLO fixed order, we obtain a precision perturbative prediction for HJM in the Higgs rest frame.
- HJM can be measured at future e^+e^- collider (CEPC) and used to extract fundamental parameters (like α_s) and non-perturbative parameters (like Ω_1).
- Future: add renormalon subtraction in both dijet and trijet limits; include the leading non-pert. power corrections; include EW corrections and QED ISR effects; restore quark masses in the calculation.