

高能碰撞中的量子纠缠

Quantum Entanglement in High Energy Collisions

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The Einstein-Podolsky-Rosen (EPR) Paradox

Historical Development:

■ The Einstein-Bohr Debates (1920s-30s)

- Einstein: **"God does not play dice"**
- Bohr: **"Stop telling God what to do"**
- Fundamental clash: determinism vs probability

■ Schrödinger (1932)

- Coined **"Entanglement"** (Verschränkung)
- Identified non-local correlations as quintessential quantum feature

■ EPR Paper (1935) [Phys. Rev. 47, 777]

- "Can Quantum Description of Reality be Complete?"
- Formalized challenge via **local realism**
- Einstein: **"Spooky action at a distance"**

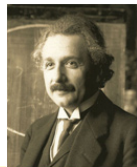
EPR "paradox" revealed the profound nature of entanglement



Bohr and Einstein



Schrödinger



EPR trio (1935)



Separable vs Entangled States: Two-Qubit Systems

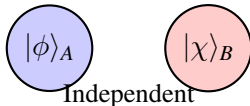
Separable States

- Can be written as: $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$
- No quantum correlations

Examples:

$$|00\rangle = |0\rangle_A \otimes |0\rangle_B, \quad |01\rangle = |0\rangle_A \otimes |1\rangle_B$$

$$\begin{aligned} |\psi_{\text{sep}}\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A \otimes |0\rangle_B \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \end{aligned}$$



Entangled States

- **Cannot** be written as product
- Genuine quantum correlations

Bell States (Maximally Entangled):

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



Bell's Theorem

■ Quantum Indeterminacy

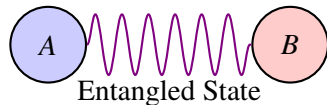
- **Realism:** Quantum indeterminacy reflects our ignorance of hidden variables; outcomes are determined but unknown.
- **Copenhagen:** Indeterminacy is fundamental; outcomes are truly probabilistic until measured.
- **Agnosticism:** The reality behind quantum events is unknowable; only predictive power of the theory matters.

■ Bell Nonlocality [Bell, 1964]

- Bell inequality: It makes an observable difference for Realism vs Copenhagen, and eliminates Agnostic view.
- Decisive evidence supporting QM (Copenhagen).

■ CHSH Inequality [Clauser et al., 1969]

- Generalized Bell inequality
- Foundation for quantum information theory



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

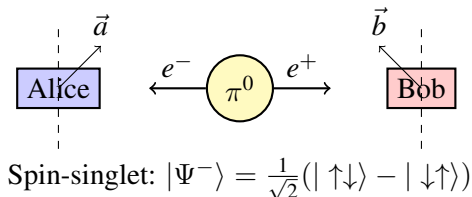


John Stewart Bell



EPRB Experiment: Testing Bell Nonlocality

Einstein-Podolsky-Rosen-Bohm Experiment



Correlation: $E(\vec{a}, \vec{b}) = \langle A(\vec{a}) \cdot B(\vec{b}) \rangle$

Bell CHSH Inequality:

$$\mathbb{B}_{HT} = |E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2$$

$$\mathbb{B}_{QM} = |\cos \theta_{ab} - \cos \theta_{ab'} + \cos \theta_{a'b} + \cos \theta_{a'b'}| \leq 2\sqrt{2}$$

QM violates Bell inequality $\Rightarrow$ Nature is nonlocal!

Local Hidden Variable Theory

- Pre-existing density $P(\lambda)$ for λ
- $A(\vec{a}, \lambda) = \pm 1$ predetermined
- $E(\vec{a}, \vec{b}) = \int P(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda$
- Local realism: $\mathbb{B}_{HT} \leq 2$

Quantum Mechanics

- No predetermined values
- $E(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} = -\cos \theta_{ab}$
- **Nonlocality:** $2 < \mathbb{B}_{QM} \leq 2\sqrt{2}$
Elementary proof with:
 $\alpha \cos \theta + \beta \sin \theta \leq \sqrt{\alpha^2 + \beta^2}$



Separable vs Entangled States

How to distinguish these two states:

Positive Partial Transpose (PPT)

1. Separable (Unentangled) States:

$$\rho = \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes \rho_b^j$$

- $P(a_i, b_j) \geq 0$ are probabilities and $\sum_{i,j} P(a_i, b_j) = 1$
- ρ_a^i, ρ_b^j are local density matrices

2. Entangled States:

Cannot be written in separable form

$$\rho \neq \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes \rho_b^j$$

Peres Horodecki criterion:

- $\rho^{T_b} = \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes (\rho_b^j)^T$
- If ρ is separable, all eigenvalues of ρ^{T_b} must be non-negative;
- Otherwise, ρ is definitely entangled.
- For 2×2 and 2×3 , PPT is necessary and sufficient for separability.
- Higher *dim*, there are PPT entangled states. (PPT not sufficient)

Quantify entanglement [Wootters, 98]

- Use time reversal operation $\mathcal{T} = -i\sigma^y \mathcal{K}$ (Anti-Unitary) to flip spin.
- Define concurrence.



Time Reversal Operation and Kramers Degeneracy

Time Reversal for Spin-1/2

$$\boxed{\mathcal{T} = -i\sigma_y\mathcal{K}}$$

$$\mathcal{T}|\uparrow\rangle = |\downarrow\rangle \quad \mathcal{T}|\downarrow\rangle = -|\uparrow\rangle$$

- $\mathcal{K}$ is complex conjugation

$$\tilde{\chi} = \mathcal{T} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\beta^* \\ \alpha^* \end{pmatrix}$$

- Orthogonality: $\langle \tilde{\chi} | \chi \rangle = 0$

For arbitrary spin:

$$\mathcal{T} = e^{-i\pi S_y/\hbar} \mathcal{K}$$
$$\mathcal{T}^2 = -1 \text{ (for fermions)}$$

Kramers Degeneracy

**For half-integer spin systems with
time-reversal symmetry:
Every energy level is at least doubly
degenerate**



Concurrence: Measuring the Degree of Entanglement (Pure States)

Time Reversal Operation flips spins:

- $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
- $\mathcal{C}(|\psi\rangle) \equiv |\langle\tilde{\psi}|\psi\rangle|$, $|\tilde{\psi}\rangle = -\sigma_y \otimes \sigma_y |\psi^*\rangle$
- $|\tilde{\psi}\rangle = \delta^*|00\rangle - \gamma^*|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$, the spin-flipped complex conjugate.
- $\mathcal{C}(|\psi\rangle) = 2|\alpha\delta - \beta\gamma|$ measures overlap with time-reversed state.
- $\mathcal{C} = 0$: Separable (no entanglement)
- $0 < \mathcal{C} < 1$: Partially entangled
- $\mathcal{C} = 1$: Maximally entangled

$\mathcal{C}$ = invariance under time reversal

Separable State

$|\psi_1\rangle = |00\rangle$ and $\alpha = 1, \beta = \gamma = \delta = 0$

$$\mathcal{C} = 2|1 \cdot 0 - 0 \cdot 0| = 0$$

Bell State (Maximally Entangled)

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\mathcal{C} = 2|\frac{1}{2} - 0| = 1$$

Partially Entangled

$$|\psi_2\rangle = \frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$$

$$\mathcal{C} = 2|\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}}| = \frac{2\sqrt{2}}{3} \approx 0.94$$



Spin Density Matrix for Spin-1/2 Particles

Density Matrix Formalism

For a spin-1/2 particle, the density matrix is:

$$\rho = \frac{\mathbb{1}_2 + \vec{n} \cdot \vec{\sigma}}{2}$$

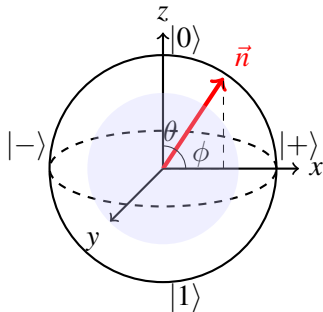
Bloch Vector: $n_i = \langle \sigma_i \rangle = \text{Tr}(\rho \sigma_i)$

- $|\vec{n}| = 1$: Pure state
- $|\vec{n}| < 1$: Mixed state
- $\vec{n} = 0$: Maximally mixed state

For a heavy quark:

- **Production mechanism:** QCD processes determine initial Bloch vectors
- **Experimental access:** Weak decay measures spin projections $\langle \vec{n} \cdot \vec{\sigma} \rangle$

Bloch Sphere Representation



Geometry encodes quantum information

- $\vec{n} = (0, 0, 1)$: $\rho = |0\rangle\langle 0|$
- $\vec{n} = (1, 0, 0)$: $\rho = |+\rangle\langle +|$
- $\vec{n} = (0, 0, 0)$: $\rho = \frac{1}{2}\mathbb{1}_2$ (classical)



Density Matrix and Concurrence for Two-Qubit Systems

Extending to mixed states

Density Matrix Representation:

- Pure state: $\rho = |\psi\rangle\langle\psi|$
- Mixed state: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
- General form in computational basis:

$$\rho = \begin{pmatrix} \rho_{00,00} & \rho_{00,01} & \rho_{00,10} & \rho_{00,11} \\ \rho_{01,00} & \rho_{01,01} & \rho_{01,10} & \rho_{01,11} \\ \rho_{10,00} & \rho_{10,01} & \rho_{10,10} & \rho_{10,11} \\ \rho_{11,00} & \rho_{11,01} & \rho_{11,10} & \rho_{11,11} \end{pmatrix}$$

Properties:

- Hermitian: $\rho^\dagger = \rho$
- Trace $\text{Tr}(\rho) = 1$; Non-negative.

Concurrence in general:

[Hill, Wootters, 97; Wootters, 98]

- Define: $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$
- Compute: $\mathcal{R} = \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$
- Eigenvalues of $\mathcal{R}$: $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ (descending order)

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

Example - Werner State:

$$\rho_W = p |\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4} \mathbb{1}_4$$

- $p = 1$: Pure Bell state
- $\mathcal{C}(\rho_W) = \max\{0, \frac{3p-1}{2}\}$
- Entangled when $p > 1/3$



Spin Density Matrix: Physical Interpretation

The most general two-qubit density matrix:

$$\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_j^- \mathbb{1}_2 \otimes \sigma^j + C_{ij} \sigma^i \otimes \sigma^j \right)$$

Physical Quantities:

- $B_i^+ = \text{Tr } \rho(\sigma_i \otimes \mathbb{1}_2)$
- $B_j^- = \text{Tr } \rho(\mathbb{1}_2 \otimes \sigma_j)$
- $C_{ij} = \text{Tr } \rho(\sigma_i \otimes \sigma_j)$
Spin correlation /NB: Not $[C]$

Special Case:

For Bell states: $B_i^+ = B_j^- = 0$
(No individual spin polarization)

Bell States & Correlation Matrices:

State	Correlation Matrix
$ \Psi^-\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)$	$C_{ij} = \text{diag}(-1, -1, -1)$
$ \Psi^+\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)$	$C_{ij} = \text{diag}(1, 1, -1)$
$ \Phi^+\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle + \downarrow\downarrow\rangle)$	$C_{ij} = \text{diag}(1, -1, 1)$
$ \Phi^-\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle - \downarrow\downarrow\rangle)$	$C_{ij} = \text{diag}(-1, 1, 1)$

For singlet state:

$C_{ij} = -\delta_{ij}$ means spins are always anti-parallel.

Correlation matrix C_{ij} fully characterizes entanglement structure for Bell states

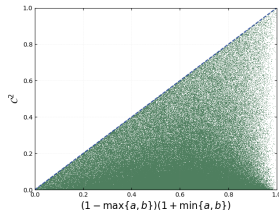


Upper Bound on Concurrence with Polarization

[Liu, Qi, He, Xiao] arXiv:2604.17756 [hep-ph] **Tighter Upper Bound:**

$$\mathcal{C}_{\max} = \sqrt{(1 - \max\{\|\vec{B}^+\|, \|\vec{B}^-\|\})(1 + \min\{\|\vec{B}^+\|, \|\vec{B}^-\|\})}$$

Numerical Verification:



2,000,000 random density matrices verify the bound

- **Depends on both** $\|\vec{B}^+\|$ and $\|\vec{B}^-\|$
- **Saturated by pure states** ($\vec{B}^+ = \vec{B}^-$)

Physical Interpretation:

- Polarization stores spin information **locally**
- Reduces room for **nonlocal** quantum correlation
- For a family of ρ with fixed B , the largest $\mathcal{C}$ achievable within that family.

Application to $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$:

- **Parity violation** generates polarization
- **Up:** $\mathcal{C}_{\max} \simeq 0.744$; **Down:** $\mathcal{C}_{\max} \simeq 0.353$
- Much smaller than unpolarized case ($\mathcal{C}_{\max} = 1$)



Entanglement and Bell Nonlocality Conditions

Starting from the spin density matrix: $\rho_{\alpha\alpha',\beta\beta'} = \frac{1}{4} \left(\mathbb{1}_{\alpha\alpha',\beta\beta'} + C_{ii} \sigma_{\alpha\beta}^i \otimes \sigma_{\alpha'\beta'}^i \right)$

- Anti-correlated spins: $C_{xx}, C_{yy}, C_{zz} < 0$
- Def: $D = (C_{xx} + C_{yy} + C_{zz})/3 = \text{tr}C/3$
- $D = -1$: Perfect anti-correlation

Four eigen values of $\mathcal{R} = \rho$ (since $\tilde{\rho} = \rho$)

$$\begin{aligned}\lambda_1 &= \frac{1}{4}(1 - C_{xx} - C_{yy} - C_{zz}), \\ \lambda_2 &= \frac{1}{4}(1 + C_{xx} + C_{yy} - C_{zz}), \\ \lambda_3 &= \frac{1}{4}(1 + C_{xx} - C_{yy} + C_{zz}), \\ \lambda_4 &= \frac{1}{4}(1 - C_{xx} + C_{yy} + C_{zz}).\end{aligned}$$

Entanglement Condition

- Concurrence $\mathcal{C}[\rho] = \frac{1}{2}(-3D - 1) > 0$:

$$D < -\frac{1}{3}$$

Bell Nonlocality Condition

- For CHSH violation $\mathbb{B} > 2$:
[Horodecki, et al, 95]

$$D < -\frac{1}{\sqrt{2}} \approx -0.707$$

Hierarchy: Bell Nonlocality $\subset$ Entanglement $\subset$ All Quantum States



Top Quark Weak Decay and Spin Transfer

Top Quark Decay: Choose its rest frame

$$t \rightarrow W^+ b \rightarrow \ell^+ \nu_\ell b, \bar{t} \rightarrow W^- \bar{b} \rightarrow \ell^- \bar{\nu}_\ell \bar{b}$$

Decay Spin Density Matrix:

$$\Gamma_\pm = \frac{\mathbb{1}_2 + \kappa_\pm \vec{\sigma}_t \cdot \hat{l}_\pm}{2}$$

Parity Violating Angular Distribution:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \kappa_\pm \cos\theta$$

- Weak decay **Spin-momentum correlation** reveals spin directions.
- $\kappa_\pm = \pm 1$ ($t\bar{t}$) **spin analyzing power**
- $\sigma_{l_+ l_-} \propto \text{tr}[\Gamma_+ \otimes \Gamma_- \rho]$ NB $\text{tr}[\sigma^i \sigma^j] = 2\delta^{ij}$

Correlation between di-leptons

$$\frac{d^2\sigma}{\sigma d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} \left[1 - \hat{l}_+ \cdot C \cdot \hat{l}_- \right]$$

Entanglement Signature

$$\langle \cos\varphi \rangle = -\frac{1}{3}D = -\frac{1}{9}\text{Tr}(C)$$

Experimental Reach:

- Extract $D = \text{Tr}(C)/3$ parameter directly
- **Quantum Tomography**: all elements of ρ can be measured. [Bernreuther, Heisler, Si, 15; ATLAS, 1612.07004; CMS, 1907.03729]



First Observation of Quark Entanglement at the LHC

[ATLAS (Nature 2024):] First observation of entanglement in quarks at the highest-energy.

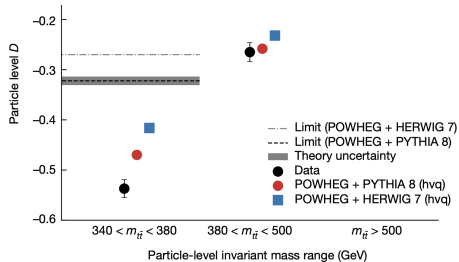
Entanglement Measure:

$$D = \text{tr}[C]/3 = -3\langle \cos \phi \rangle$$

where ϕ is the angle between charged leptons in their parent top/antitop rest frames

Key Features:

- Spin transferred to decay products
- Measured near $t\bar{t}$ threshold
- From atomic physics to high-energy collisions: **A new frontier!**
- **CMS, STAR, BES-III** more to come.



$\sqrt{s} = 13 \text{ TeV}, 140 \text{ fb}^{-1}$ data (2015-2018)

Measured: $D < -1/3$ (Entanglement criterion)
 $D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.)

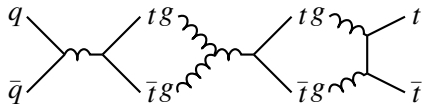
- Observed: $> 5\sigma$ from no entanglement
- **Yet, Bell Nonlocality:** $D < -1/\sqrt{2}$



Theory vs Experiment: Top Quark Entanglement

Quantum State Tomography $\rho_{\alpha\alpha',\beta\beta'} = R_{\alpha\alpha',\beta\beta'} / \text{tr} R$ [Afik, de Nova, 2022]

Top quark pair production



$$R_{\alpha\alpha',\beta\beta'} = \frac{1}{N} \sum \mathcal{M}_{t_\alpha \bar{t}_{\alpha'}}^* \mathcal{M}_{t_\beta \bar{t}_{\beta'}}$$

- **Measured $D \approx -0.54$** near threshold
- **Gluon fusion dominance** at LHC
- **Angular momentum conservation** determines spin correlations
- **Statistical mixture** of $q\bar{q}$ and gg

"Observation of Entanglement but not Bell Nonlocality due to Quark channel mixture"

Near Threshold ($\beta \rightarrow 0$):

- $q\bar{q}$: **Separable state** ($\mathcal{C} = 0$), since $t\bar{t}$ spin (± 1) is equally mixed along beam.
- gg : **Maximally entangled** singlet Ψ^-

High Energy ($\beta \rightarrow 1$) with $\theta = \pi/2$:

- Both channels: Maximally entangled triplet Ψ^+ along $\hat{n}$ with nonzero OAM.

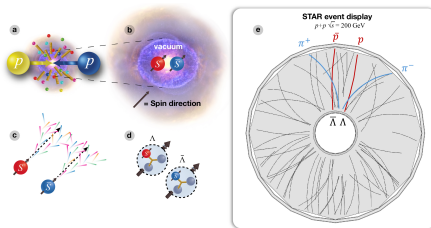
Mixed State at LHC

$$\rho = w_{q\bar{q}} \rho_{q\bar{q}} + w_{gg} \rho_{gg}$$



First evidence of spin correlation in $\Lambda\bar{\Lambda}$ hyperon pairs

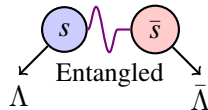
STAR Collaboration [Nature 2026; arXiv:2506.05499] $p + p$ collisions at $\sqrt{s} = 200$ GeV



- Relative polarization (same as D): $P_{\Lambda\bar{\Lambda}} = (18 \pm 4)\%$
- Parallel: $1/3$; Antiparallel: -1 ; no spin correlation 0 .
- Short-range pairs show maximal entanglement
- Long-range pairs: correlation vanishes (decoherence)
- Evidence for quantum entanglement in QCD vacuum

"Entanglement: A new paradigm for exploring QCD phenomena"

Entanglement as a Tool



- QCD Confinement
- Chiral Symmetry
- Spin Dynamics
- Decoherence
- Bell Nonlocality
- Nuclear Medium !?



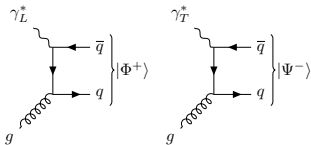
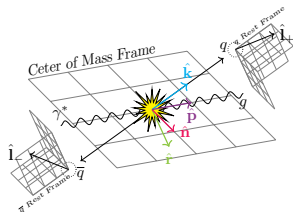
Quark Pair Production in Photon-Gluon Fusion: Longitudinal case

[Qi, Guo, Xiao] ▶ arXiv:2506.12889v1 [hep-ph]

Photon-Gluon Fusion Process

$$\gamma_{\lambda=\pm,0}^* + g \rightarrow q + \bar{q}$$

$$\rho_L = \frac{1}{4} (\mathbb{1}_4 + C_{ij} \sigma^i \otimes \sigma^j)$$



For $q\bar{q}$ with $\beta \rightarrow 0$ and $\theta = \frac{\pi}{2}$

Longitudinal photons contribution:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\chi_1 & -\chi_2 \\ 0 & -\chi_2 & \chi_1 \end{pmatrix}$$

with
$$\chi_1 = \frac{1 - 2z^2 + z^2\beta^2}{1 - z^2\beta^2}, \quad \chi_2 = \sqrt{1 - \chi_1^2}.$$

■ ρ_L is given by a **pure state** $= |\Psi\rangle \langle\Psi|$, with

$$|\Psi\rangle = \frac{1}{2} (\sqrt{1 + \chi_1}, i\sqrt{1 - \chi_1}, i\sqrt{1 - \chi_1}, \sqrt{1 + \chi_1}).$$

■ **Near Threshold** ($\beta \rightarrow 0$) with $\theta = \frac{\pi}{2}$: $|\Phi^+\rangle$.

■ **High Energy** ($\beta \rightarrow 1$): $|\Phi^+\rangle$.

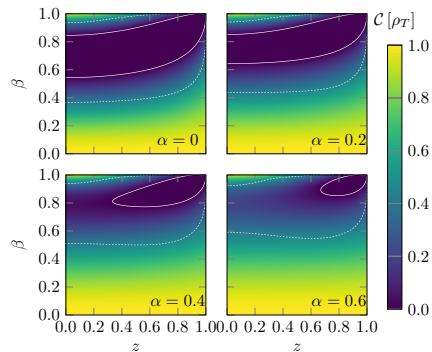
■ $q\bar{q}$ has spin 1 with nonzero OAM and $\mathcal{C}[\rho_L] \equiv 1!$

Always Maximally Entangled! Very Special!



Quark Pair Production in Photon-Gluon Fusion: Transverse case

[Qi, Guo, Xiao] [arXiv:2506.12889v1 \[hep-ph\]](#) **Transverse photons:** similar to $gg \rightarrow q\bar{q}$ channel.

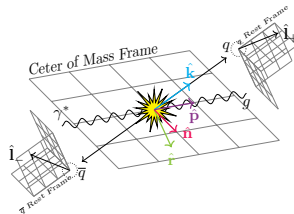


- **Density plots of the concurrence** for transverse photon as functions of β and $z = \cos \theta$ at given $\alpha \equiv Q^2/\hat{s}$.
- Solid lines (entanglement ($C[\rho_T] = 0$)) and dashed lines (Bell nonlocality).
- **Near Threshold ($\beta \rightarrow 0$):**
Maximally entangled singlet Ψ^-
- **High Energy ($\beta \rightarrow 1$) with $\theta = \pi/2$:**
Maximally entangled triplet Φ^- .

- **Low background and Maximal signal.** Better to have LT separation! (Also UPC)
- Possible measurements: $b\bar{b}$ or $c\bar{c}$ or hyperon $\Lambda\bar{\Lambda}$.
- Diffractive DIS [Fucilla and Hatta, 2509.05267] L : Maximal Entanglement; T : entanglement is necessary and sufficient for Bell-nonlocality.
- QI at EIC [Cheng, Han, Trifinopoulos, 2510.23773] (Polarized ep collisions)



Summary and Outlook



- **Entanglement** and **Bell Nonlocality** are measurable at high energy collisions.
- **EIC** offers a unique and clean experimental environment for measuring **maximal** entanglement and Bell Nonlocality.
- Using entanglement as a tool to probe **nuclear environment** and other QCD effects.
- **New opportunities** to explore the interplay of quantum information phenomena and high energy and hadronic physics in the years to come.

