基于 QCD 的量子动理学理论

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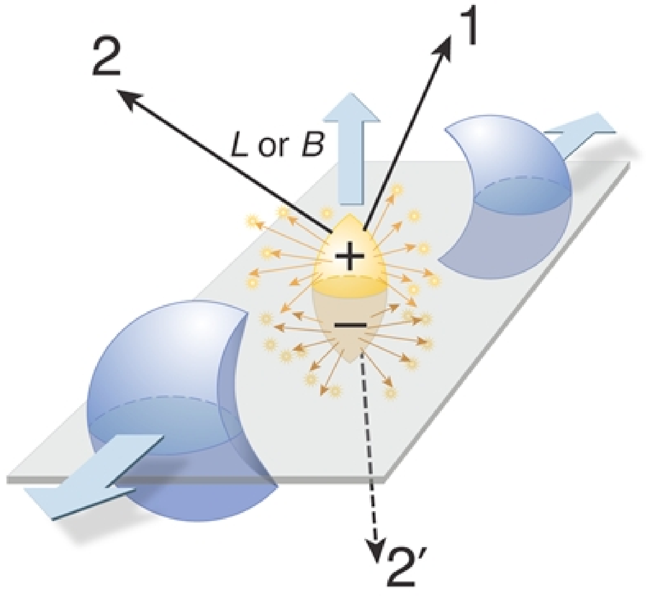
第一届强相互作用自旋物理会议 , 青岛 7.26-31, 2026

SL, 2603.04263

Outline

- Spin polarization in heavy ion collisions: theoretical uncertainties
- Quantum kinetic theory for QCD
- Applications to spin polarization: toward a complete description
- Summary and outlook

Beginning of spin phenomena in HIC



$$L_{ini} \sim 10^5\hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

spin-orbital coupling via parton scattering

quark polarization P_q

recombination

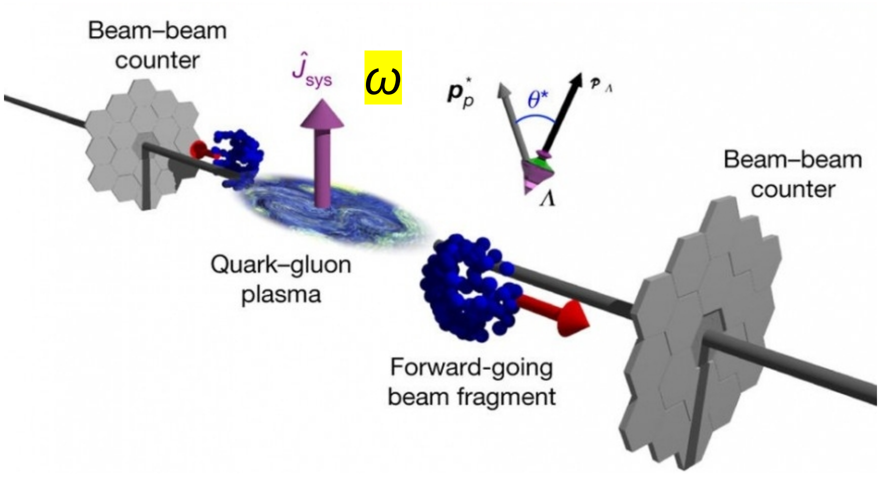
$qqq \longrightarrow B + X$ Baryon spin polarization

$q\bar{q} \longrightarrow V + X$ Vector meson spin alignment

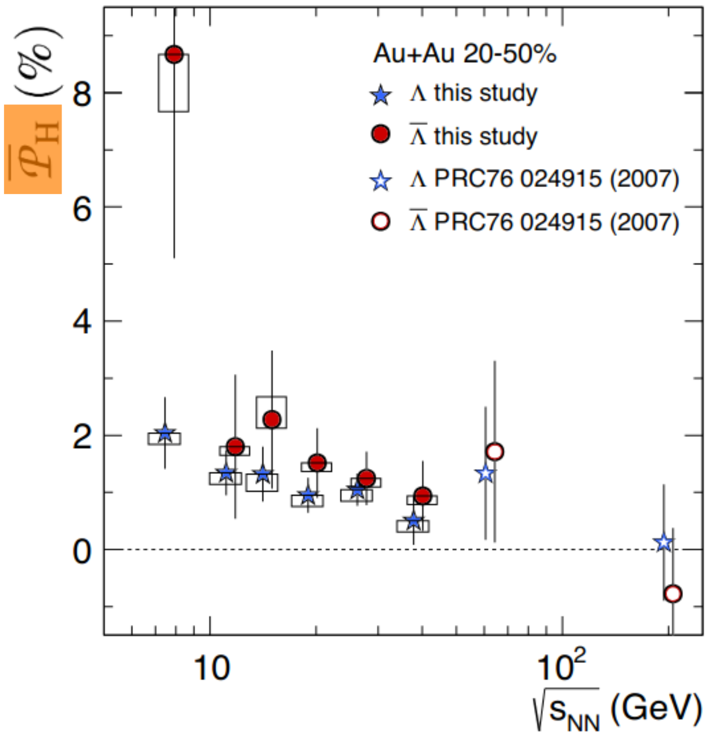
Global polarization of Λ: first exmaple

$$\Lambda \to p + \pi^-$$ weak decay

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}[1 + \alpha_H P_H \cos\theta^*]$$



Pu Shi's talk



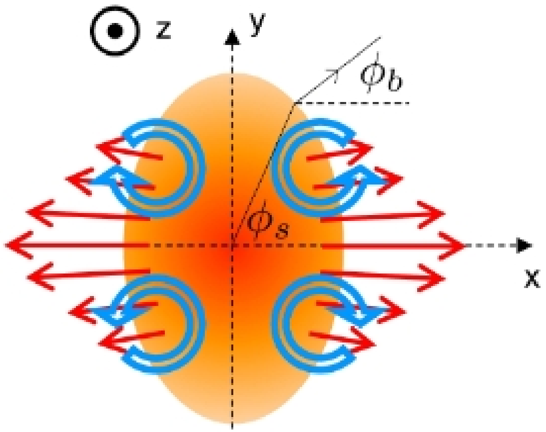
STAR, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

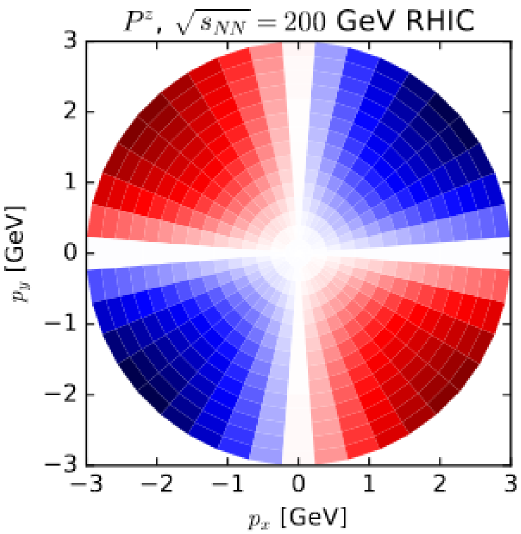
Becattini et al, PRC 2017

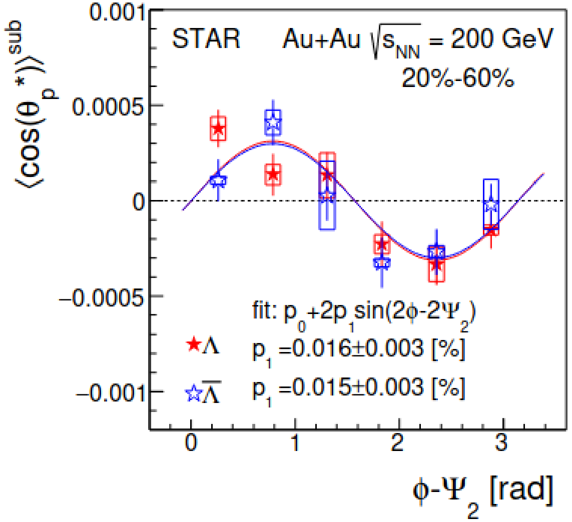
Spin couples to QGP vorticity

Local polarization of Λ: projection along beam



$P^z \propto \omega^z$



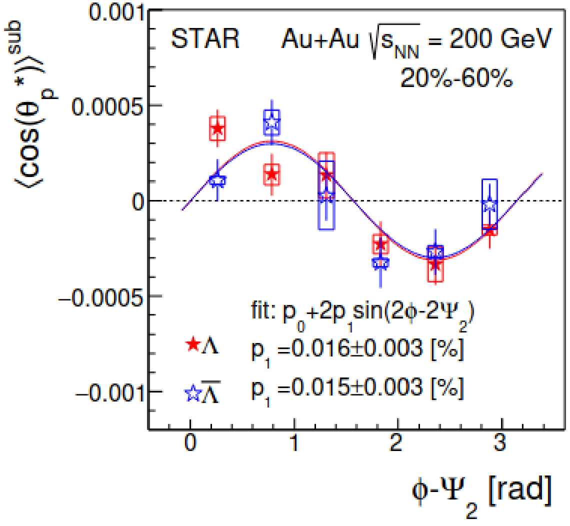


Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

STAR, PRL 2019

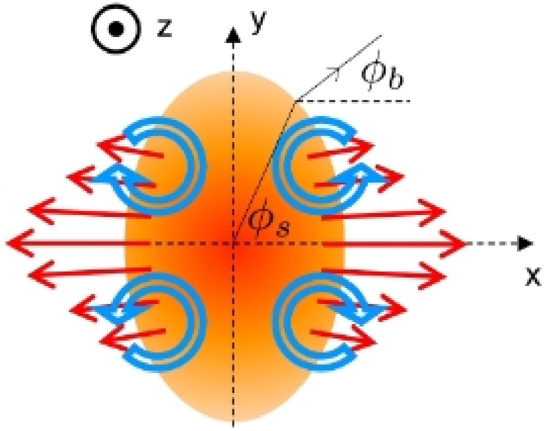
wrong sign

Local polarization of Λ: vorticity + shear



STAR, PRL 2019

vorticity



$$P^z \propto \omega^z$$

wrong sign

shear (free theory)

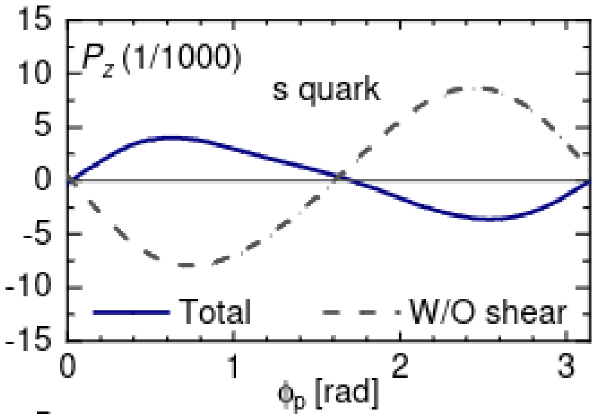
$$P^i \propto \epsilon^{ijk}\,\hat{p}_k\,\hat{p}_l\,\sigma_{jl}$$

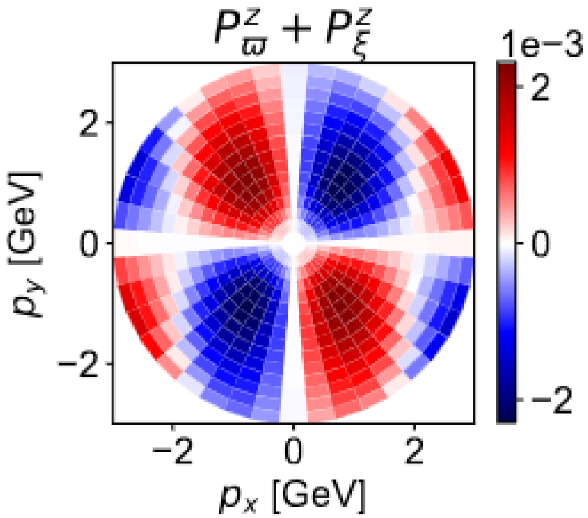
$$P^z \propto \left(\langle p_y{}^2\rangle - \langle p_x{}^2\rangle\right)\sigma_{xy}$$

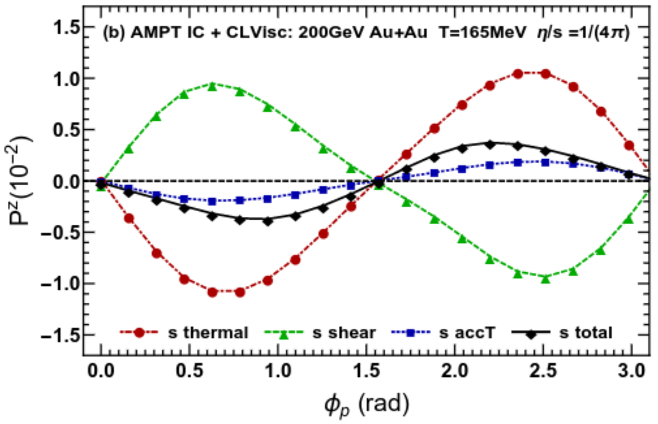
right sign

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Phenomenology: vorticity + shear







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

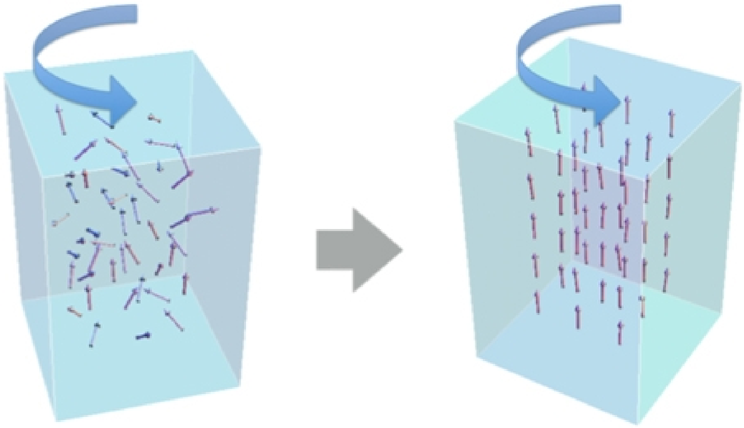
competition between shear and vorticity

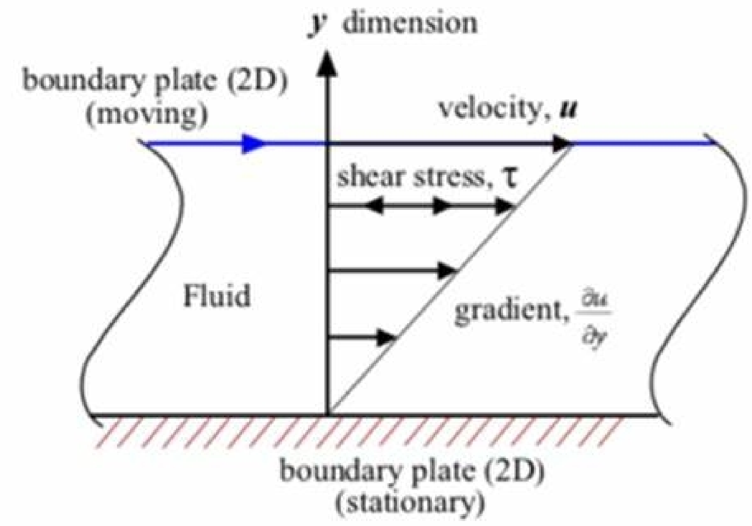
shear contribution may or not reverse the sign

Uncertainties in local polarization: collision effect

$P^z \propto \boxed{\omega^z}$ ~ B

$P^i \propto \epsilon^{ijk}\,\hat{p}_k\,\boxed{\hat{p}_l\,\sigma_{jl}}$ ~ E





Equilibrium: collision vanishes

time reversible

Steady state: collision nonvanishing

time irreversible

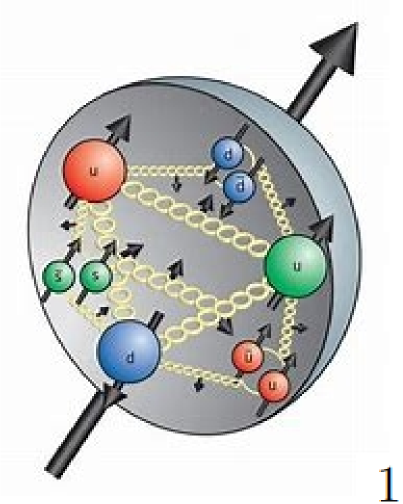
Uncertainties in local polarization: structure effect

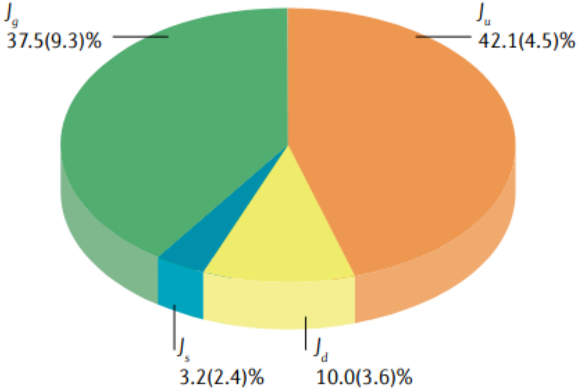
simplified structure

Λ: point particle

Λ: quark model

reality





$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

~40%

Alexandrou et al, PRD 2020

Hadron polarization: toward a complete description

hadron structure

quark polarization in QGP → hadron polarization

most phenomenology: collisionless
quantum kinetic theory (QKT)

Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

1. collisional effect included (QED)

Z. Wang, SL, JHEP 2022, PRDs 2025
Fang, Pu, Yang, PRD 2022, 2024, 2025

2. structure effect not included yet

Need QCD based QKT!

From microscopic theory to kinetic theory

N-body distribution $F_N(z_1, \cdots, z_N, t)$ $z_i = (x_i, v_i)$

reduced distribution $f_s(z_1, \cdots, z_s, t) = \frac{N!}{(N-s)!} \int dz_{s+1} \cdots dz_N F_N(z_1, \cdots, z_N, t)$

BBGKY

$$(\partial_t + v_1 \cdot \nabla_{x_1}) f_1(z_1) = \int dz_2 \, K_{12} f_2(z_1, z_2)$$

$$(\partial_t + v_1 \cdot \nabla_{x_1} + v_2 \cdot \nabla_{x_2} + K_{12}) f_2(z_1, z_2)$$
$$= \int dz_3 \, (K_{13} + K_{23}) f_3(z_1, z_2, z_3).$$

$f_1 \to f_2 \to f_3 \to \cdots$

correlation suppressed
(molecular chaos)

$f_2 = f_1 f_1 + \cancel{g_2}$

assumed by physicists, proven by mathematicians for hard sphere model (Fields medal 2026)

Difficulties with QCD based QKT

- Elastic/inelastic collisions, IR divergences
- On-shell and off-shell DOF
- Gauge dependencies
- Generalization to spin DOF (QKT)

$$\left(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}}\right) f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f].$$

Arnold, Moore, Yaffe, early 00s

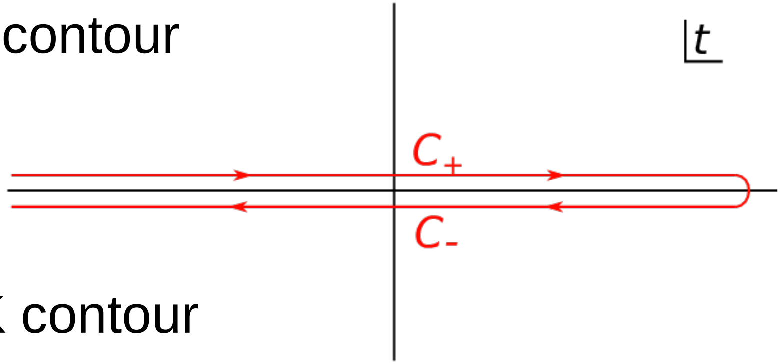
Boltzmann equation (spin-averaged kinetic theory) written down based on physical considerations, generalization to QKT difficult

Derivation of the QKT

Dyson-Schwinger equation (BBGKY) on the SK contour

truncation to
two-point function

Kadanoff-Baym equation (Boltzmann) on the SK contour



Chou, Su, Hao, Yu, Phys. Rept. 1985
Blaizot, Iancu, Phys. Rept. 2002

$$(i\partial\!\!\!/_x - m)\, S^<(x,y) = \int_z \left(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\right)$$

$$\tilde{S}^<(X = \frac{x+y}{2}, P) = \int d^4(x-y)e^{iP\cdot(x-y)}\langle S^<(x,y)\rangle \propto f(X,P)$$

- higher point functions fixed by two-point function — molecular chaos
- off-equilibrium state
- preserve conservation laws

K-B equation for quark

$$\frac{i}{2}\not{\partial}S^{<} + (\not{P} - m)S^{<} = \mathrm{Re}[\Sigma_R]S^{<} + \Sigma^{<}\mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)$$

thermal mass/width

collision

real part: quasi-particle dispersion

imag part: kinetic equation

diag $\rightarrow$ $$\mathrm{tr}\left(\not{\partial}S^{<}\right) = \mathrm{tr}\left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)$$

Boltzmann

off-diag $\rightarrow$ spin kinetic equation

Gradient expansion

$$\frac{i}{2}\partial\!\!\!/ S^< + (P\!\!\!/ - m)S^< = \mathrm{Re}[\Sigma_R]S^< + \Sigma^<\mathrm{Re}[S_R] + \frac{i}{2}(\Sigma^> S^< - \Sigma^> S^<)$$

$$S^< = S^{<(0)} + S^{<(1)} + \cdots .$$

$$S^<(X,P) = S^{<(0)}(X,P) + S^{<(1)}_{\mathrm{un}}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)\,(P\!\!\!/ + m)\,(-f_q(X,P))$$

$$+ S^{<(1)}_{\mathrm{pol}}(X,P) = \gamma^5\gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

$$f_q = f_q^{\mathrm{leq}} + \delta f_q.$$

$$\partial\!\!\!/ S^{<(0)} = (\Sigma^> S^< - \Sigma^> S^<)^{(1)}$$

$$f_q^{\mathrm{leq}}(X,P) = (\exp((P\cdot u(X) - \mu(X)/T(X)) + 1)^{-1}$$

hydrodynamic gradient: shear, vorticity etc

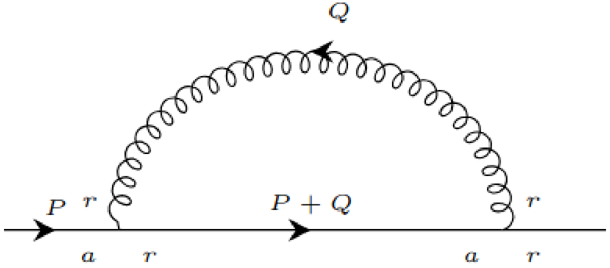
Power counting in coupling

$$\frac{i}{2}\not{\partial} S^< + (\not{P} - m)S^< = \mathrm{Re}[\Sigma_R]S^< + \Sigma^< \mathrm{Re}[S_R] + \frac{i}{2}(\Sigma^> S^< - \Sigma^> S^<)$$

thermal mass/width

collision

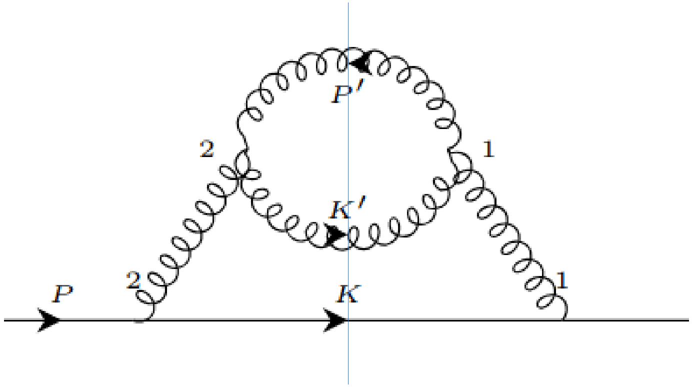
dispsersion correction



$\sim O(g^2)$ parametrically suppressed

$$\not{\partial} S^{<(0)} = (\Sigma^> S^< - \Sigma^> S^<)^{(1)} \quad \sim O(\partial\, g^0)$$

collision

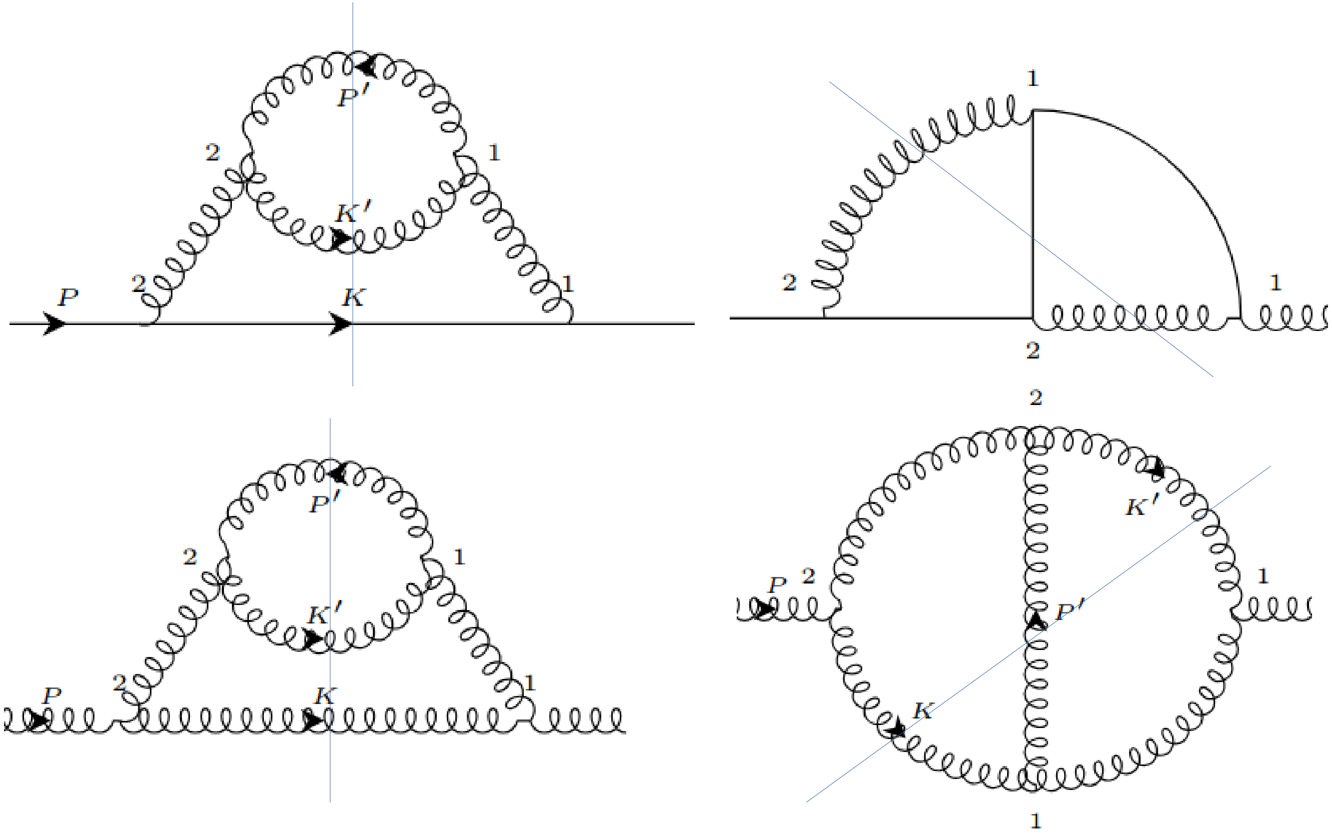


naively $\sim O(g^4)$

enhanced by $\delta f \sim \dfrac{\partial_X f_{\mathrm{leq}}}{g^4} \sim \tau_R \partial_X f_{\mathrm{leq}}$

long relaxation time & large deviation from leq

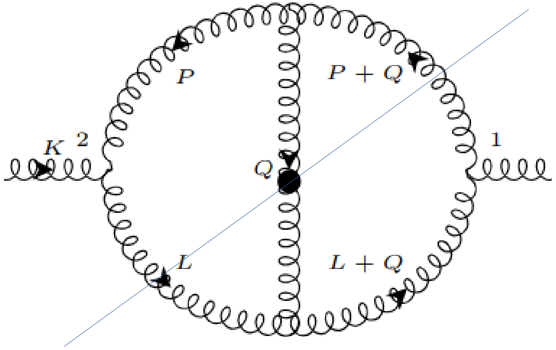
Self-energies: elastic collisions

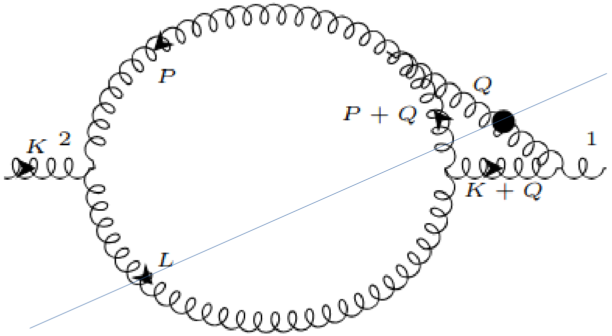


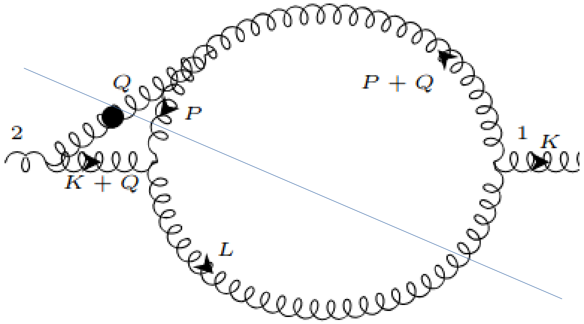
+ many other 2-loop diagrams

binary collisions

Self-energies: inelastic collisions

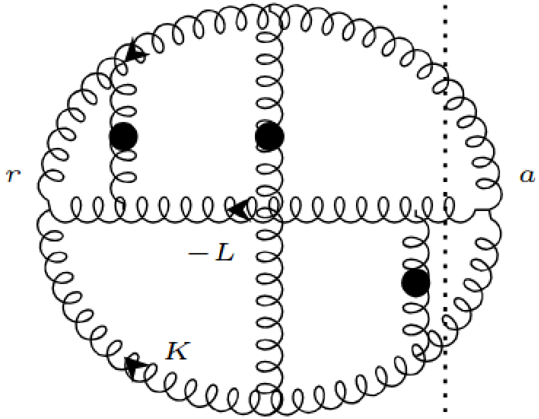






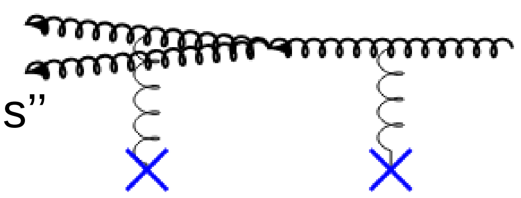
medium-induced
collinear radiation

multiple interaction
with medium



3-body
“bound state”

3g-vertex in spin basis



$$\epsilon^s(L) = \frac{1}{\sqrt{2}}\left(1, is, -\frac{l^s}{l_\parallel}\right) \qquad s = \pm 1$$

$$l^s \equiv l_1 + isl_2$$

quantization axis:
collinear direction z

$$l_\parallel \sim O(1) \quad l_\perp \sim O(g)$$

$$\epsilon_i^s(P)\epsilon_j^{-s'*}(L)\epsilon_k^{s''}(K)\left[\delta_{ik}(-p+k)_j + \delta_{kj}(-k-l)_i + \delta_{ji}(l+p)_k\right]$$

$$= \delta_{s,-s''}\sqrt{2}\left[\frac{p_\parallel k_\perp - k_\parallel p_\perp}{l_\parallel}\right]^{s'} + \delta_{-s',s''}\sqrt{2}\left[\frac{k_\parallel l_\perp - l_\parallel k_\perp}{p_\parallel}\right]^{s} + \delta_{s,-s'}\sqrt{2}\left[\frac{l_\parallel p_\perp - p_\parallel l_\perp}{k_\parallel}\right]^{s''}$$

$$\Delta S = -s' \qquad \Delta L = s'$$

partial wave

$$q_1 + is'q_2 \sim Y^*_{1,-s'}$$

orbit-spin conversion
with local collisions

qqg-vertex in spin basis

quantization axis:
collinear direction z

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_s = \left(\frac{2l_{||}}{p_{||}}\right)^{1/2} \frac{k_{||}p_s - p_{||}k_s}{k_{||}}, \quad \Delta S = -s,\ \Delta L = s$$

$$Y_{1,s}(\hat{p}) \propto p_s$$

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_{-s} = \left(\frac{2l_{||}}{p_{||}}\right)^{1/2} \frac{k_{||}p_{-s} - p_{||}k_{-s}}{k_{||}}, \quad \Delta S = s,\ \Delta L = -s$$

$$\bar{u}_s(L)\gamma^i u_{-s}(P)\epsilon^i_s = \left(\frac{1}{2l_{||}p_{||}}\right)^{1/2} 2mk_{||}s. \quad \Delta S = \Delta L = 0. \qquad Y_{1,0}(\hat{p}) \propto p_{||},$$

quark mass induces quark spin flip,
but conserve total spin

orbit-spin conversion
with local collisions

Application to spin polarization

$$\frac{i}{2}\not{\partial} S^{<(0)} + \boxed{(\not{P} - m) S^{<(1)}} = \frac{i}{2}\left(\Sigma^{>} S^{<} - \Sigma^{<} S^{>}\right)^{(1)}$$ constraint equation

$$\boxed{\frac{i}{2}\not{\partial} S^{<(1)}} + (\not{P} - m) S^{<(2)} = \frac{i}{2}\left(\Sigma^{>} S^{<} - \Sigma^{<} S^{>}\right)^{(2)}$$ kinetic equation

Non-dynamical part from constraint equation

free collisional

$$S^{<(1)}_{\rm pol}(X,P) = \gamma^5 \gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu, \gamma_\nu]}{4} \mathcal{S}^{\mu\nu}$$

$$\mathcal{D}_\nu = \boxed{\partial_\nu} - \boxed{\Sigma^{>}_\nu - \Sigma^{<}_\nu \frac{1-f_q}{f_q}} \quad g^4 \delta f \sim O(\partial_X)$$

$$\mathcal{A}^\mu = -2\pi \epsilon(P\cdot u) \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u + m)} \delta(P^2 - m^2)$$

Early studies with QED:
SL, Wang, JHEP 2022, PRDs 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

Dynamical part from kinetic equation

$$\mathcal{A}^a_\mu = 2\pi \delta(P^2 - m^2) a_\mu f_A \quad \sim O(m)$$

SL, Wang, PRD 2025

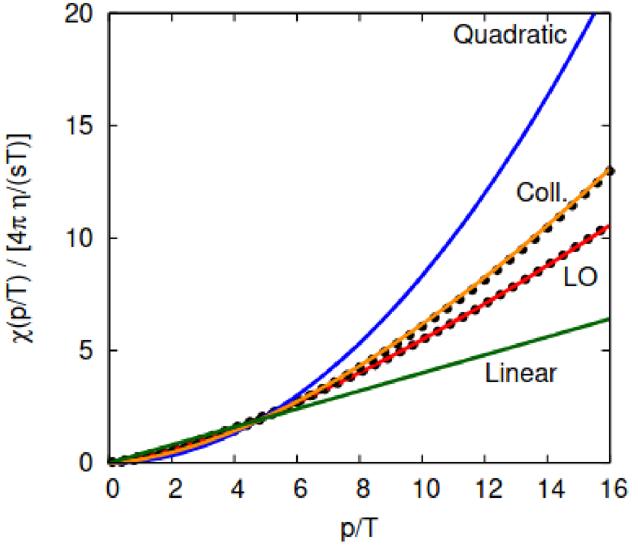
Expectation from QCD based collision

$$S^{<(1)}_{\rm pol}(X,P) = \gamma^5\gamma_\mu\mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

free collisional

$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2)$$

$$\mathcal{D}_\nu = \partial_\nu - \Sigma^>_\nu - \Sigma^<_\nu\frac{1-f_q}{f_q}$$

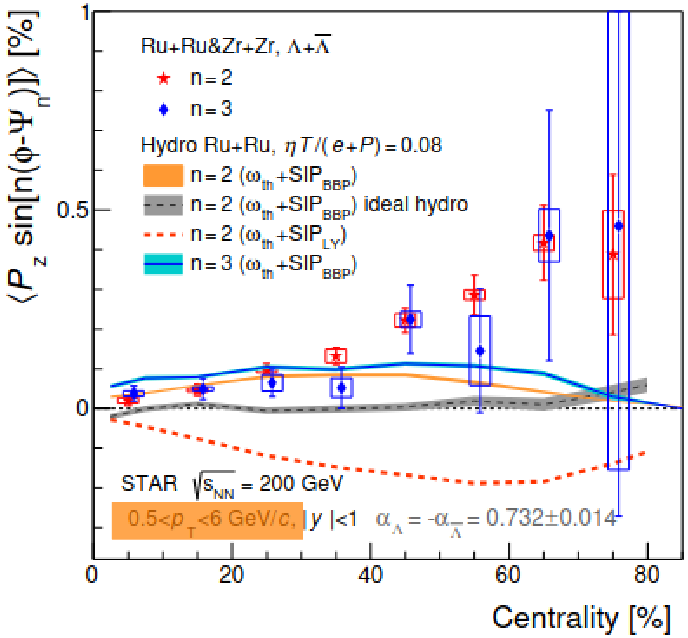


$$\delta f(p) = -n_p\chi(\tilde{p})\hat{p}^i\hat{p}^j\langle\partial_i u_j\rangle$$

Dusling, Moore, Teaney, PRC 2010

Coll.=elastic
LO=elastic+inelastic

expect large correction from inelastic collision



Gauge dependencies

QKT derived in Coulomb gauge, other gauges?

Boltzmann equation (diagonal) gauge independent

$$S_{\rm un}^{<(1)}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)\left(\not{P}+m\right)\left(-\delta f_q(X,P)\right),$$

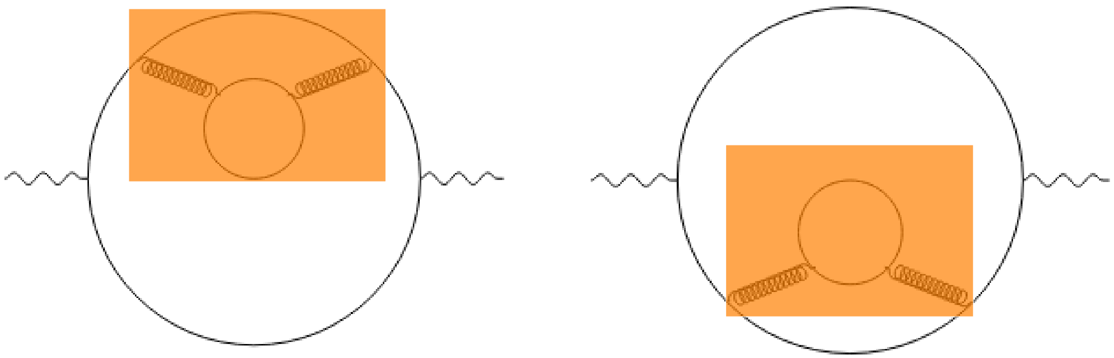
Spin polarization (off-diagonal) gauge dependent

$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2) \qquad \mathcal{D}_\nu = \partial_\nu - \boxed{\Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q}}$$

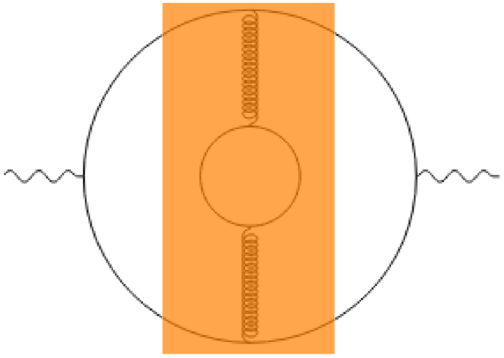
gauge dependencies expected to cancel
between self-energy and vertex corrections

SL, Z.y. Wang, in preparation

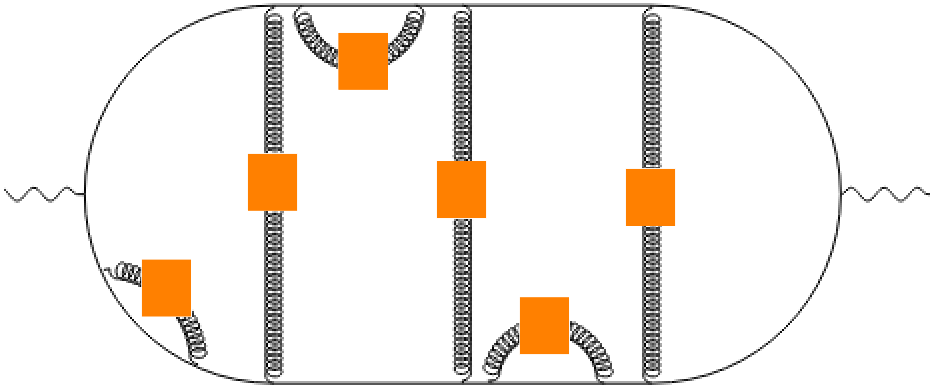
From quark to hadron polarization



shear induced Q/Qbar polarization



shear correction to potential



gauge dependencies cancel in Bethe-Salpeter amplitude

SL, Z.y. Wang, in preparation

Summary and outlook

- Derived QKT for QCD. 0^{th} order reproduces Boltzmann equation, 1^{st} order gives spin kinetic equation
- Inelastic collision as a mechanism for orbit-spin conversion
- From quark spin polarization to hadron polarization with cancelled gauge dependencies

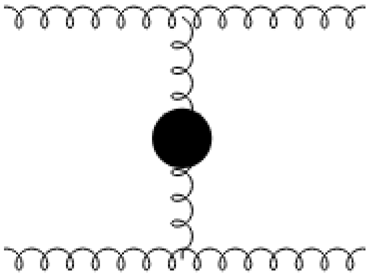
Thank you!

The 10th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter, Zhuhai, Nov 16-20, 2026

https://indico.ihep.ac.cn/event/29668/

Where dispersion correction not ignored

- screening IR divergence (elastic collision)
- regulate collinear divergence (inelastic collision)



K-B equation for gluon

$$\left[-P^2\eta^{\mu\nu}+\frac{i}{2}\left(-2P\cdot\partial\eta^{\mu\nu}+\partial^\nu P^\mu-\frac{1}{\xi}P^{\mu\alpha}P^{\nu\beta}\partial_\beta P_\alpha\right)\right]D^{<(0)}_{\nu\rho}=\mathrm{Re}[\Pi^{T(0)}_R]P^{\mu\nu}_T D^{<(0)}_{\nu\rho}+\Pi^{<(0)}_T P^{\mu\nu}_T\mathrm{Re}[D^{R(0)}_{\nu\rho}]$$

$$+\frac{i}{2}\left(\Pi^{\mu\nu>(0)}D^{<(0)}_{\nu\rho}-\Pi^{\mu\nu<(0)}D^{>(0)}_{\nu\rho}\right)$$

thermal mass/width

collision

$$D^{<(0)}_{\mu\nu,ab}(X,P)=2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\mu\nu}f^{\mathrm{leq}}_g(X,P)\delta_{ab}$$

0^{th} order: local equilibrium

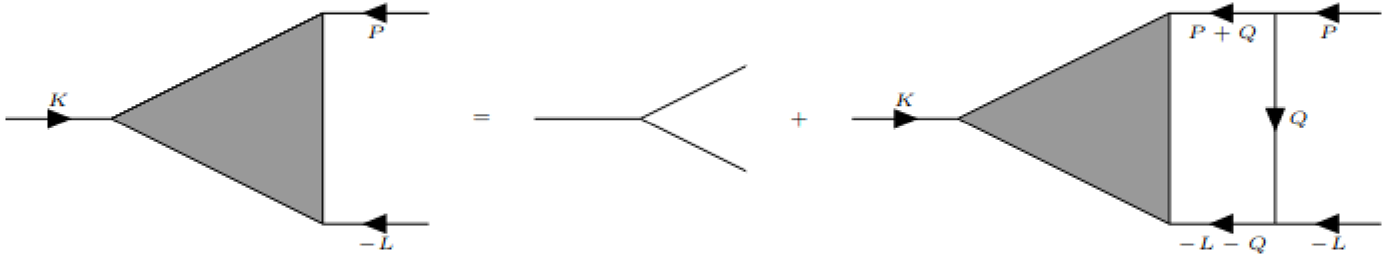
transverse

spin average $2P\cdot\partial P^{\nu\rho}_T D^{<(0)}_{\nu\rho}=\left(\Pi^{\mu\nu>}D^{<}_{\nu\mu}-\Pi^{\mu\nu<}D^{>}_{\nu\mu}\right)^{(1)}$

→ 1^{st} order: unpolarized part

$$D^{<(1)}_{\mu\nu,\mathrm{un}}(X,P)=2\pi\epsilon(p\cdot u)\delta(P^2)P^T_{\mu\nu}\delta f_g(X,P),$$

3-g bound state interpretation



$$i\delta E F(P,-L,K) = I_0(P,-L,K) + g^2\frac{C_A}{2}\int_Q 2\pi\delta(q_0-q_{\parallel})D^*_{rr,\mu\nu}(Q)\hat{e}^\mu\hat{e}^\nu\Big([F(P+Q,-L-Q,K)-F(P,-L,K)]+$$
$$[F(P+Q,-L,K-Q)-F(P,-L,K)]+[F(P,-L-Q,K+Q)-F(P,-L,K)]\Big)$$

kinetic

$$\delta E = \frac{p_\perp^2+m_g^2}{2p_\parallel}+\frac{l_\perp^2+m_g^2}{-2l_\parallel}+\frac{k_\perp^2+m_g^2}{2k_\parallel}$$

3-gluon 2D "bound state"

pairwise potential

mass: $(p_\parallel, -l_\parallel, k_\parallel)$

momenta: $(\mathbf{p}_\perp, -\mathbf{l}_\perp, \mathbf{k}_\perp)$

Inelastic collision rate ~ spectral density of bound state

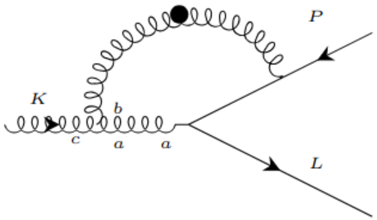
qqg bound state interpretation

kinetic

$$i\delta E F(P,-L,Q) = I_0(P,-L,K) + g^2 \int_Q 2\pi\delta(q_0 - q_{\|}) D^*_{rr,\mu\nu}(Q)\hat{e}^\mu \hat{e}^\nu \Big[$$

$$\left(C_F - \frac{1}{2}C_A\right) F(P+Q,-L-Q,K) + \frac{C_A}{2} F(P+Q,-L,K-Q) + \frac{C_A}{2} F(P,-L-Q,K+Q)\Big].$$

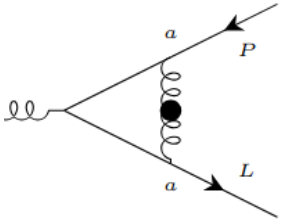
weighted pairwise potential



qqg 2D “bound state”

mass: $(p_{\|}, -l_{\|}, k_{\|})$

momenta: $(\mathbf{p}_\perp, -\mathbf{l}_\perp, \mathbf{k}_\perp)$



Inelastic collision rate ~ spectral density of bound state