第一届强相互作用物理自旋研讨会，青岛，2026.07.26-31

北京航空航天大学
BEIHANG UNIVERSITY

相对论手征核力进展与展望

耿立升 北京航空航天大学

① Jun-Xu Lu, Chun-Xuan Wang, Yang Xiao, **LSG***, Jie Meng, and Peter Ring, PRL128 (2022)142002
② Wei-Jiang Zou, Jun-Xu Lu, Peng-Wei Zhao, **LSG***, Jie Meng, PLB 854 (2024)138732
③ Shihang Shen, Jun-Xu Lu, **LSG***, Jie Meng, Wei-Jiang Zou, 2507.01257, to appear in PRC
④ Ru-You Zheng, Zhi-Wei Liu, **LSG***, Jin-Niu Hu, Sibo Wang, PLB 864 (2025) 139416
⑤ Xuan Zeng, Ru-You Zheng, Zhi-Wei Liu, **LSG*,** Xian-Rong Zhou*, PRC 113 (2026) 014310
⑥ Qing-Yu Zhai, Dan-Yang Pang, Wen-Di Chen, O. A. Rubtsova, Rui-Rui Xu, Jun-Xu Lu*, Haozhao Liang,† and **LSG‡**, 2512.02475, to appear in PRC
⑦ Jun-Xu Lu and **LSG***, PRC 113 (2026) 054002
⑧ Qing-Yu Zhai, Jun-Xu Lu, and **LSG*,** in preparation

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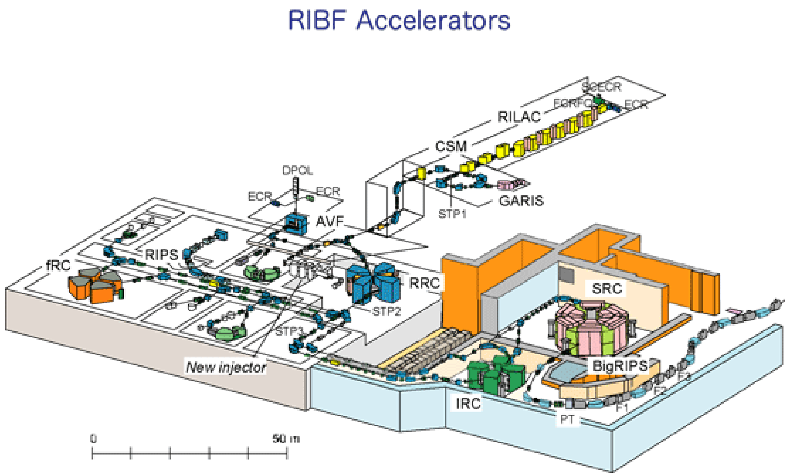
- ☞ **Why relativistic chiral nuclear force**
- ☞ **The first high-precision relativistic chiral nuclear force**
- ☞ **Recent developments and works in progress**
  - ➢ ***Progress in constructing higher-order NN and Ay puzzle***
  - ➢ ***Progress in studies of nuclear matter and finite nuclei***
- ☞ **Summary and outlook**

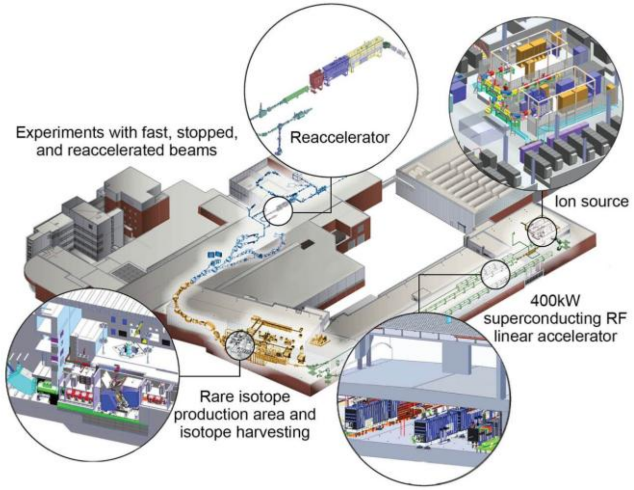
Nuclear force—always at the frontiers of nuclear physics

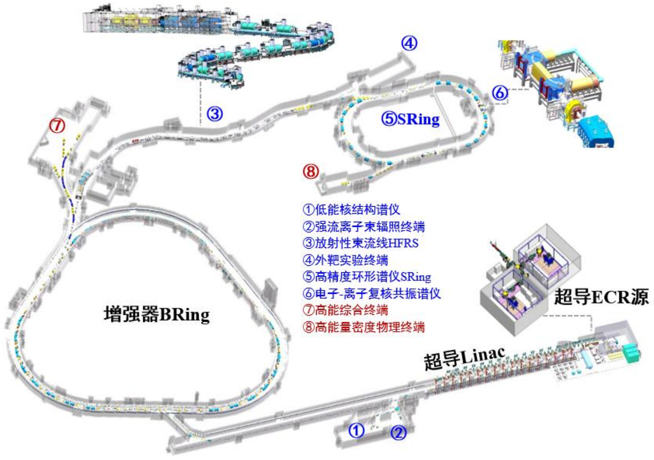
RIBF RIKEN NISHINA CENTER
RIKEN Nishina Center for Accelerator-Based Science
Introduction to RI Beam Factory and Users' Information

Facility for Rare Isotope Beams at Michigan State University

HIAF 强流重离子加速器装置
High Intensity heavy-ion Accelerator Facility







Three pillars

- **understanding of the nuclear force, especially the 3NF**
- understanding of the physics of the continuum
- understanding of the structure and dynamics of heavy element RIs,

Experimental nuclear physics

- map the nuclear landscape
- **understand the forces that bind nucleons into nuclei**
- answer questions about the astrophysical origin of nuclear matter
- address societal needs related to nuclear science and technology

HIAF scientific goals

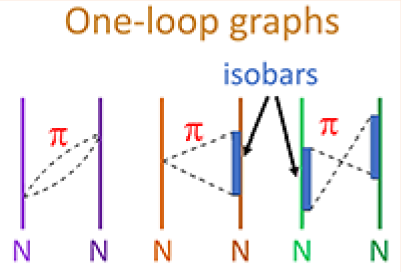
- **认识原子核内有效相互作用。**
- 探索宇宙中从铁到铀元素的来源
- 粒子辐照应用研究

Central to developments in the field of nuclear physics

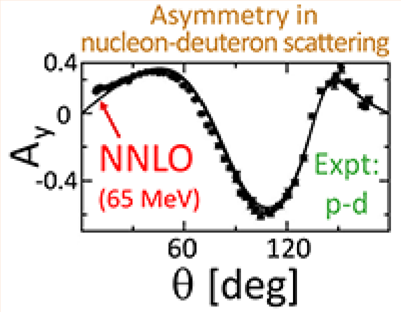
PHYSICAL REVIEW C

Remain central to developments in the field of nuclear physics
Announcing major discoveries/opening up new avenues of research

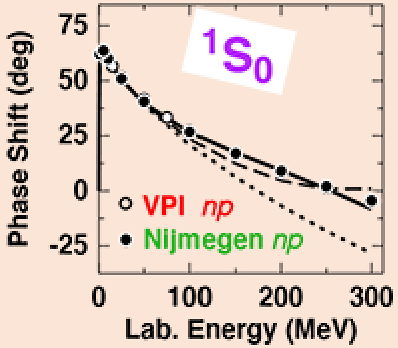
chiral EFT



Ordóñez, Ray, van Kolck
Phys. Rev. C **53**, 2086 (1996)

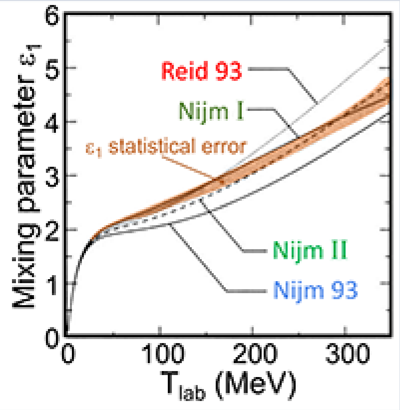


Epelbaum, Nogga, Glöckle, Kamada, Meißner, Witała
Phys. Rev. C **66**, 064001 (2002)

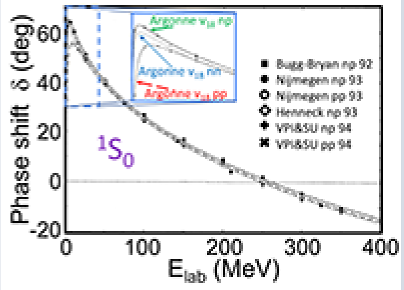


Entem, Machleidt
Phys. Rev. C **68**, 041001 (2003)

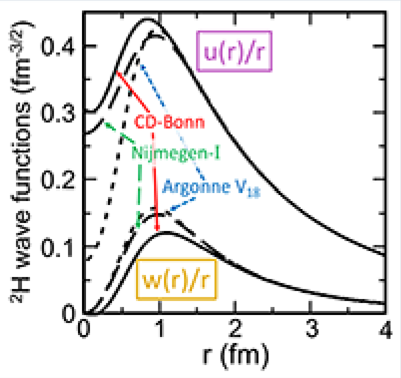
realistic



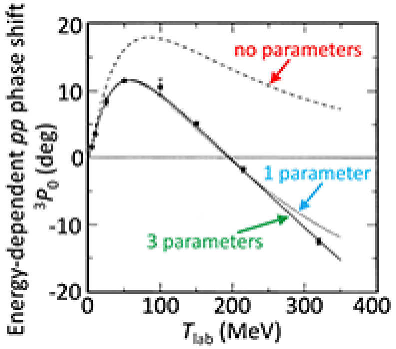
Stoks, Klomp, Terheggen, Swart
Phys. Rev. C **49**, 2950 (1994)



Wiringa, Stoks, Schiavilla
Phys. Rev. C **51**, 38 (1995)



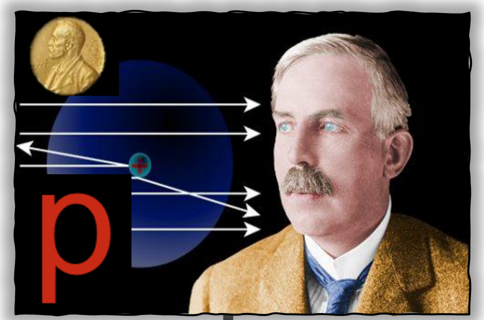
R. Machleidt
Phys. Rev. C **63**, 024001 (2001)



Stoks, Klomp, Rentmeester, Swart
Phys. Rev. C **48**, 792 (1993)

Milestones in nuclear force theories

Philos. Mag. 1919.



1918

Proton

Neutron

1932

Nature, 1932.

Proc. Phys. Math. Soc. Jap., 1935.

1935

Meson theory

1960s

Pheno. NF

AP, N.Y. 1968; NPA,1976; …

Phys. Rev. D, 1974.

1974

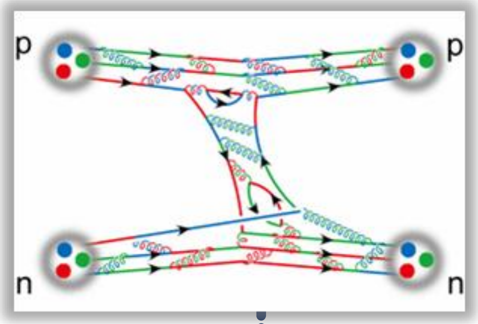
Lattice QCD

ChEFT

1990

PLB,1990; NPB,1991; PLB,1992.

Phys.Rev.Lett 2022



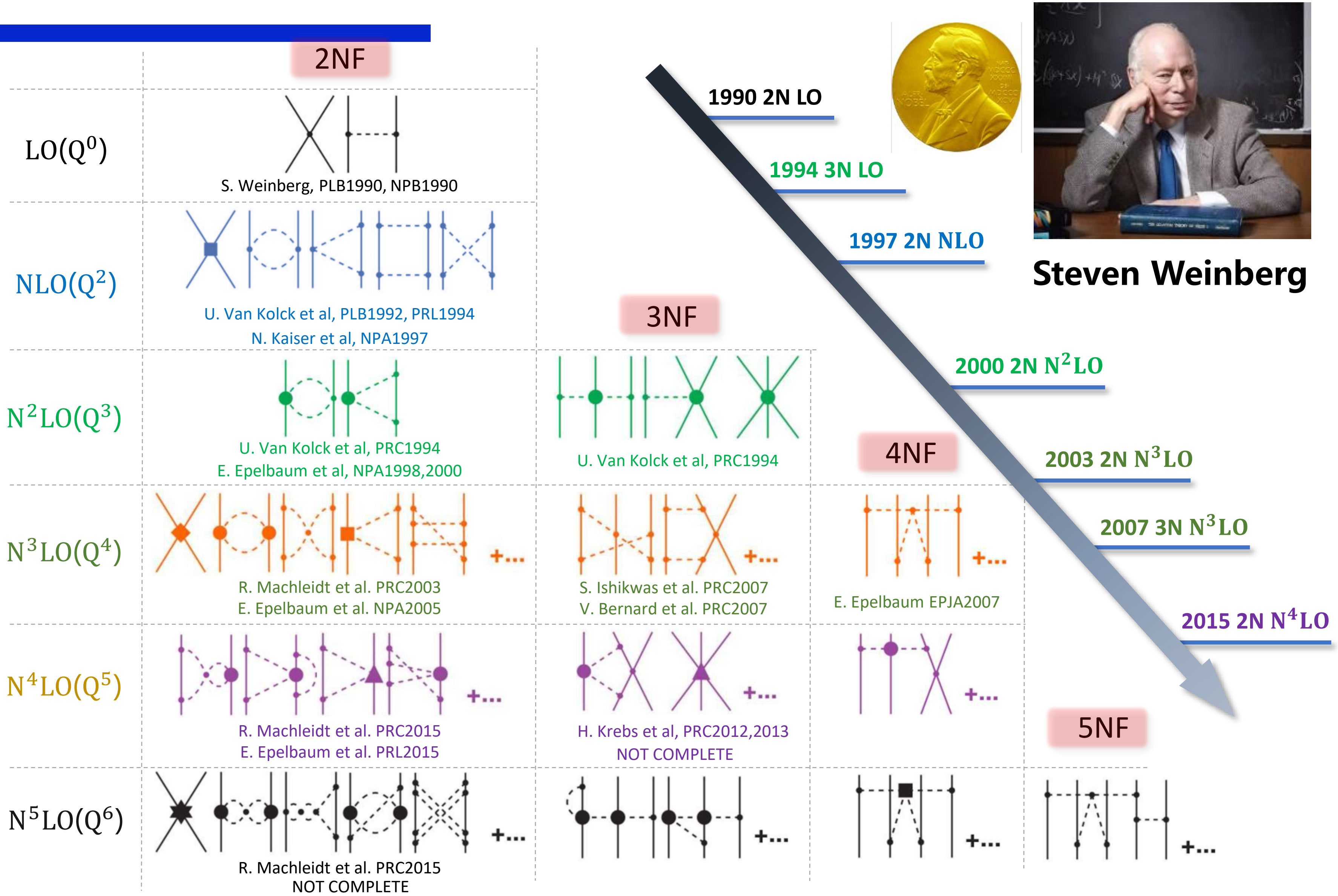
2022

rChEFT

Advantages of chiral forces—compared with conventional ones

1. Closer connection with QCD
2. Systematically improvable
3. Self-consistent treatment of many-body forces
4. Uncertainties quantifiable

Modern de facto standard: chiral nuclear forces



Adopted from Front.in Phys. 8 (2020) 57

Steven Weinberg

van Kolck

Kaiser

Epelbaum

Machleidt

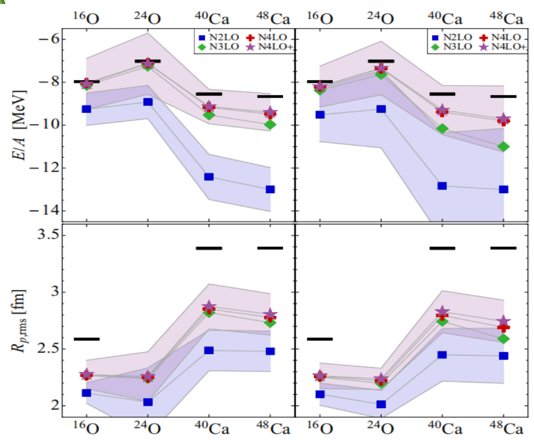
Philipps

Savage

NR chiral 2N and 3N forces still need to refined

Slow convergence

N^4LO?



EXP
N^4LO/ N^4LO+
N^3LO
N^2LO

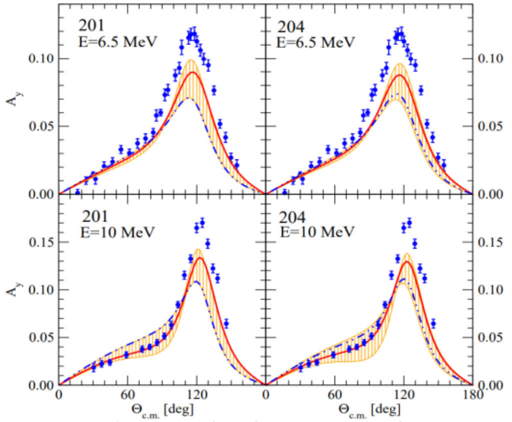
P. Maris et al. PRC2021

Ay puzzle

N-d polarization?

L. Girlanda et al. PRC2019

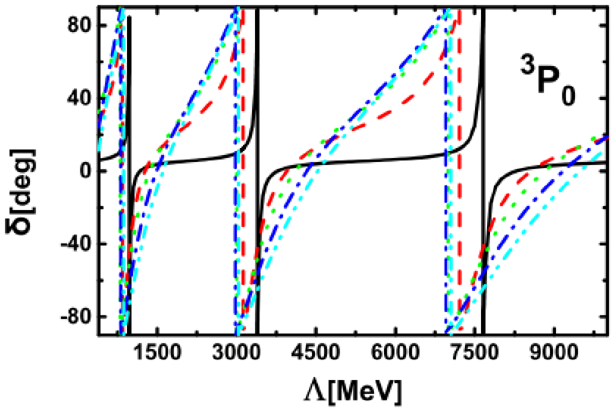
J. Golak et al. EPJA50,177



Red: N3LO chiral NN potential
Blue: N3LO chiral NN + full 3N potential

RG invariance

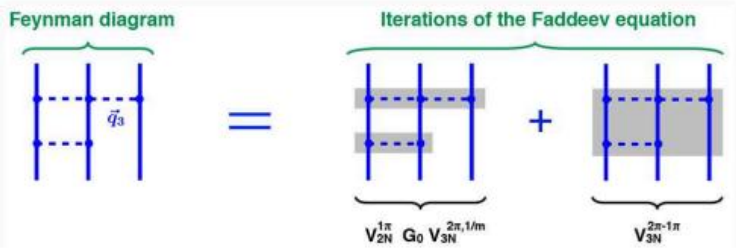
Cutoff dependent?



C.X. Wang et al. CPC2021

Self-consistency in 3N

E. Epelbaum et al. fphy2020

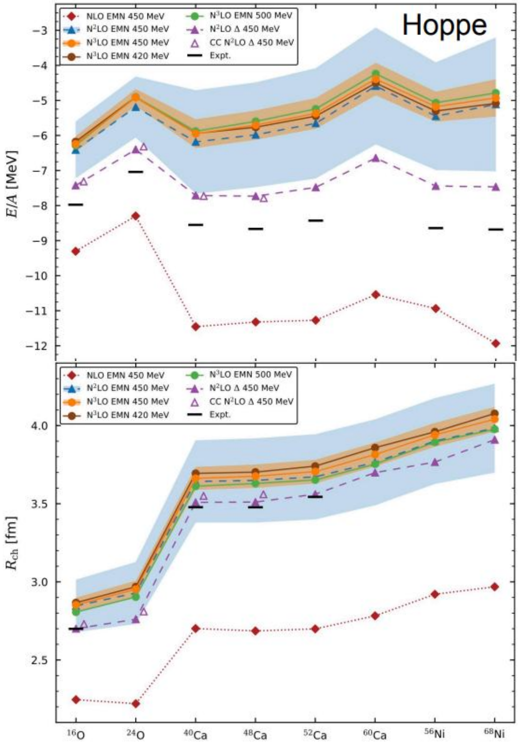


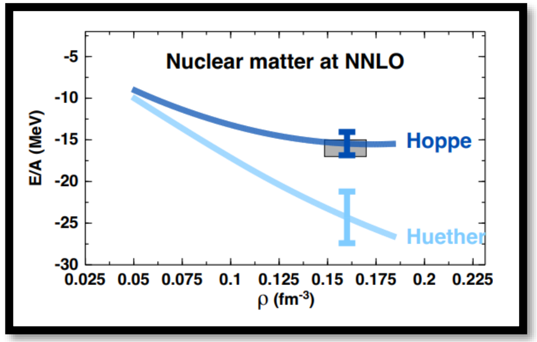
N^3LO 3N force breaks chiral symmetry

Many-body studies face self-consistency challenges

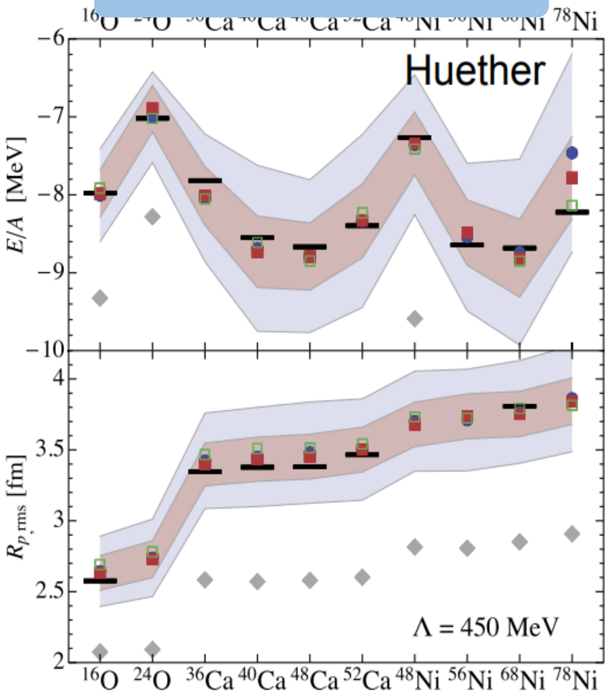
- Most are not truly ab initio， **with two exceptions, but they can not simultaneously describe medium mass nuclei and nuclear matter**

Hoppe： NM✓ BE×





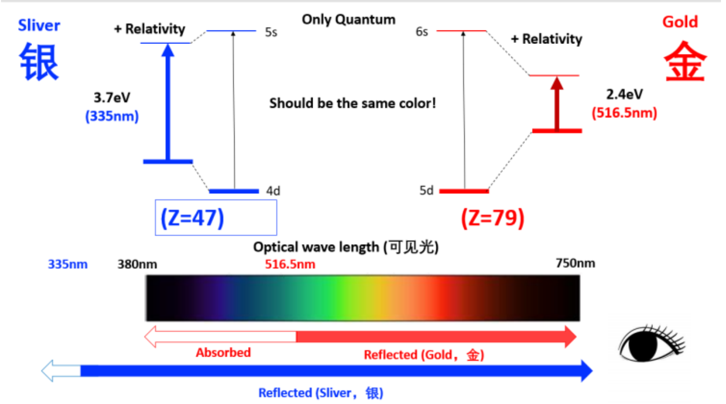
Huether： NM× BE✓



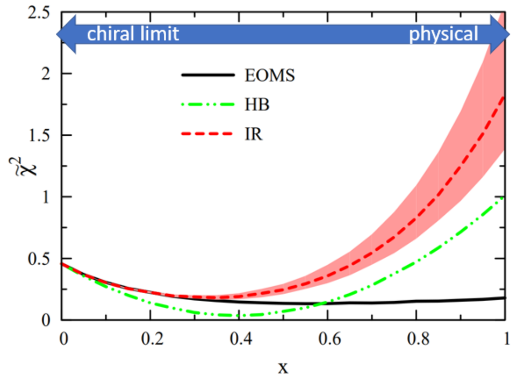
R. Machleidt and F. Sammarruca, Prog.Part.Nucl.Phys. 137 (2024) 104117

Why relativistic/covariant theories

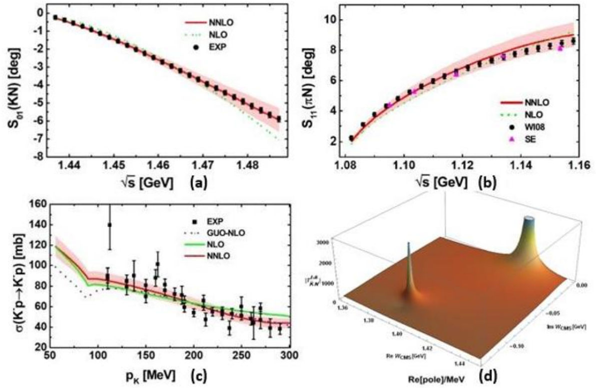
- **Lorentz invariance is one of the most essential symmetries in Nature,**
  where SPIN naturally emerges
- **Both kinematical and dynamical relativistic corrections self-consistently included**
- **Relativistic approaches have been successful in explaining many fine structures**
  - ✓ Atomic and molecular systems: why gold is yellow
  - ✓ Nuclear system：spin-orbit splitting, pseudospin symmetry
  - ✓ One-baryon sector：magnetic moments、masses、sigma terms
  - ✓ Meson-baryon scattering: πN、$\bar{K}N$、$\Lambda(1405)$



Why Gold is yellow, and Silver is white



LSG et al., PRL101 (2008) 222002



LSG et al, PRL130 (2023)071902*

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How to become relativistic/covariant

- **Dirac spinors and Clifford algebra** (instead of nonrelativistic wave functions and Pauli matrices)

$$u(\boldsymbol{p},s)=N_p\begin{pmatrix}1\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\epsilon_p}\end{pmatrix}\chi_s,\ N_p=\sqrt{\frac{\epsilon_p}{2M_N}}$$

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+
$\mathcal{O}$	0	1	0	0	0	−	0	1

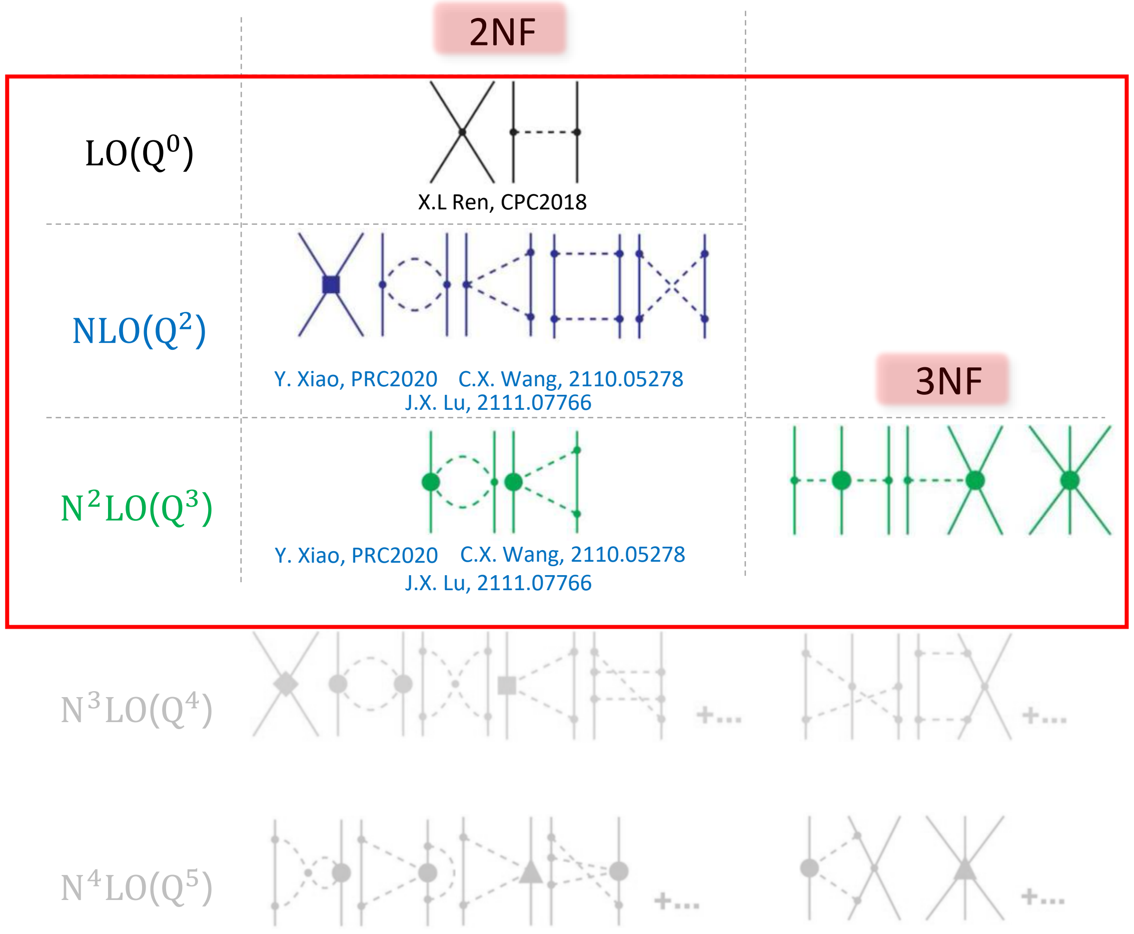
- **Covariant power counting** (vs. the Weinberg PC)

- **Covariant scattering equation** (instead of Lippmann-Schwinger eq.)

$$\mathcal{T}(p',p\mid W)=\mathcal{A}(p',p\mid W)+\int\frac{d^4k}{(2\pi^4)}\mathcal{A}(p',k\mid W)G(k\mid W)\mathcal{T}(k,p\mid W)$$

$$G(k\mid W)=\frac{i}{\left[\gamma^\mu(W+k)_\mu-m_N+i\epsilon\right]^{(1)}\left[\gamma^\mu(W-k)_\mu-m_N+i\epsilon\right]^{(2)}}$$

Accurate relativistic chiral NN interaction up to NNLO

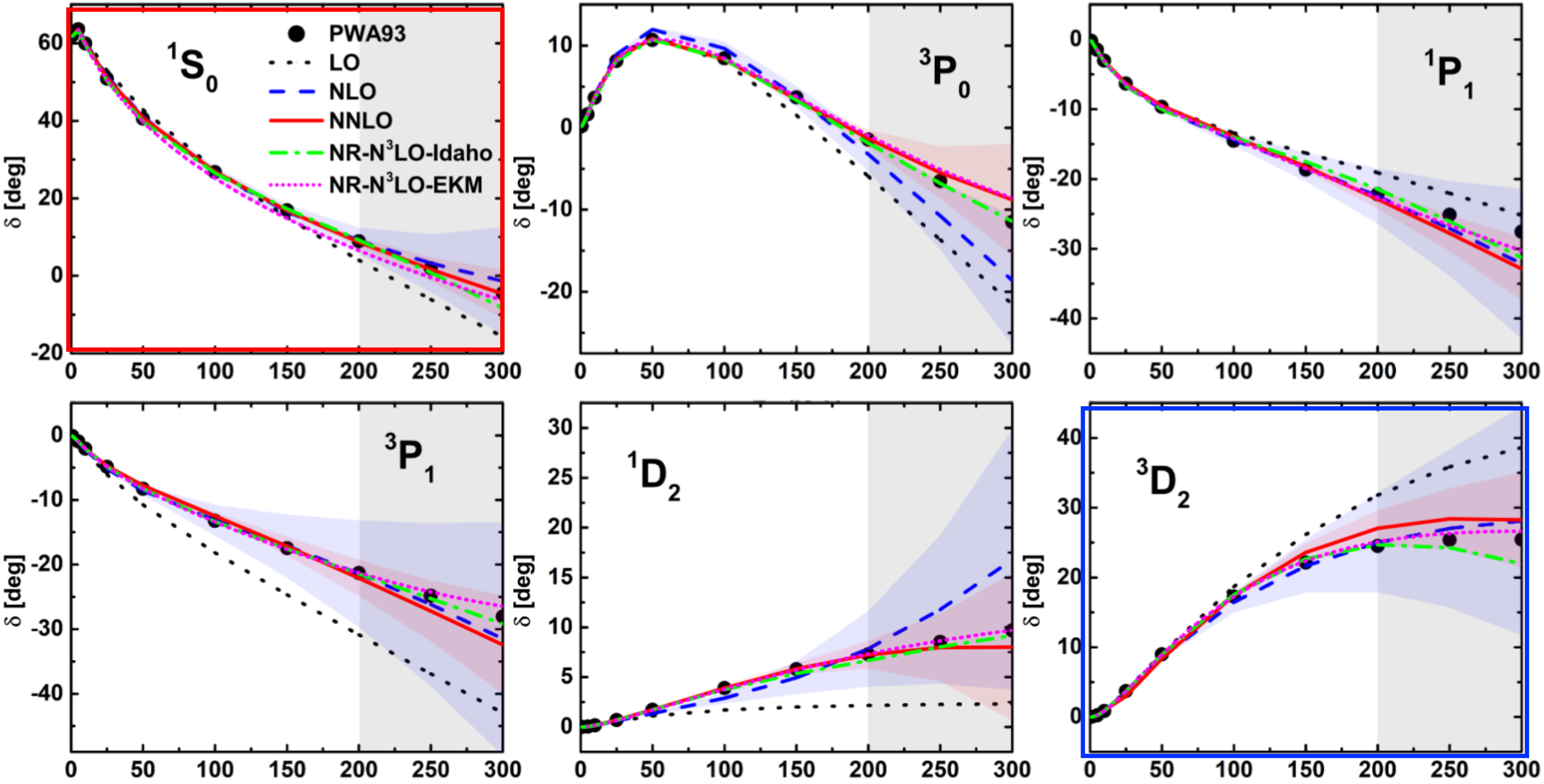


Three key inputs

1. **Contact Lagrangians:**
   - ✓ **PRC99(2019)024004**
2. **Meson-baryon vertices:**
   - ✓ **PRD99(2019)054024**
   - ✓ **PRL130(2023)071902**
3. **Two-meson exchanges**
   - ✓ **PRC102(2020)054001**
   - ✓ **PRC105(2022)014003**

Fit results for $J \leq 2$ partial waves

Jun-Xu Lu, Chun-Xuan Wang, Yang Xiao, LSG*, Jie Meng, and Peter Ring, PRL128 (2022)142002



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

Fit results for $J \leq 2$ partial waves

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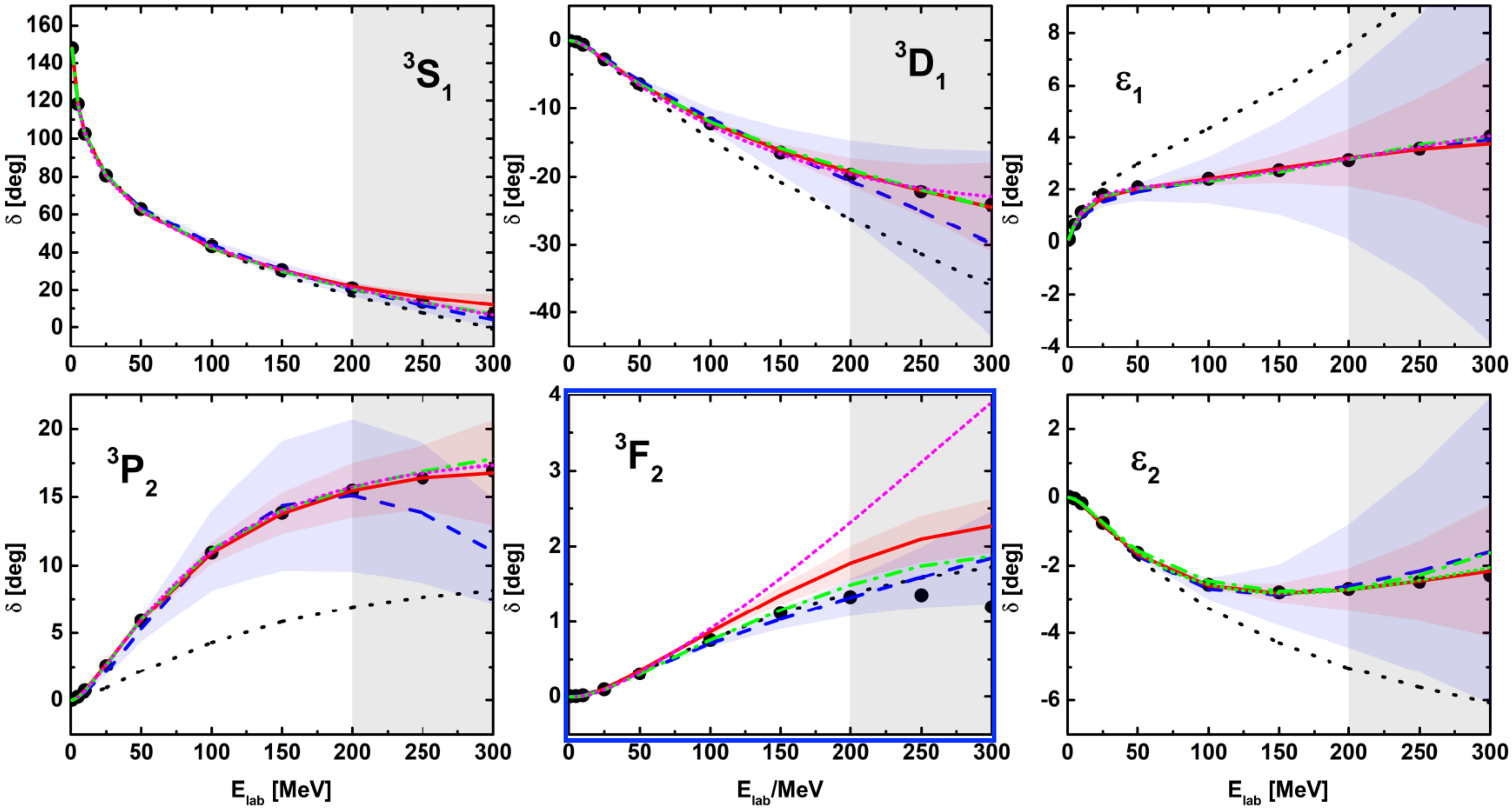
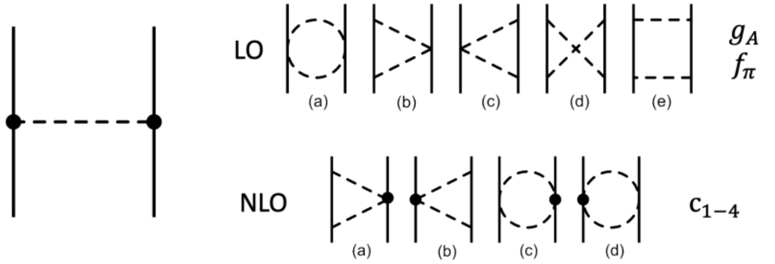


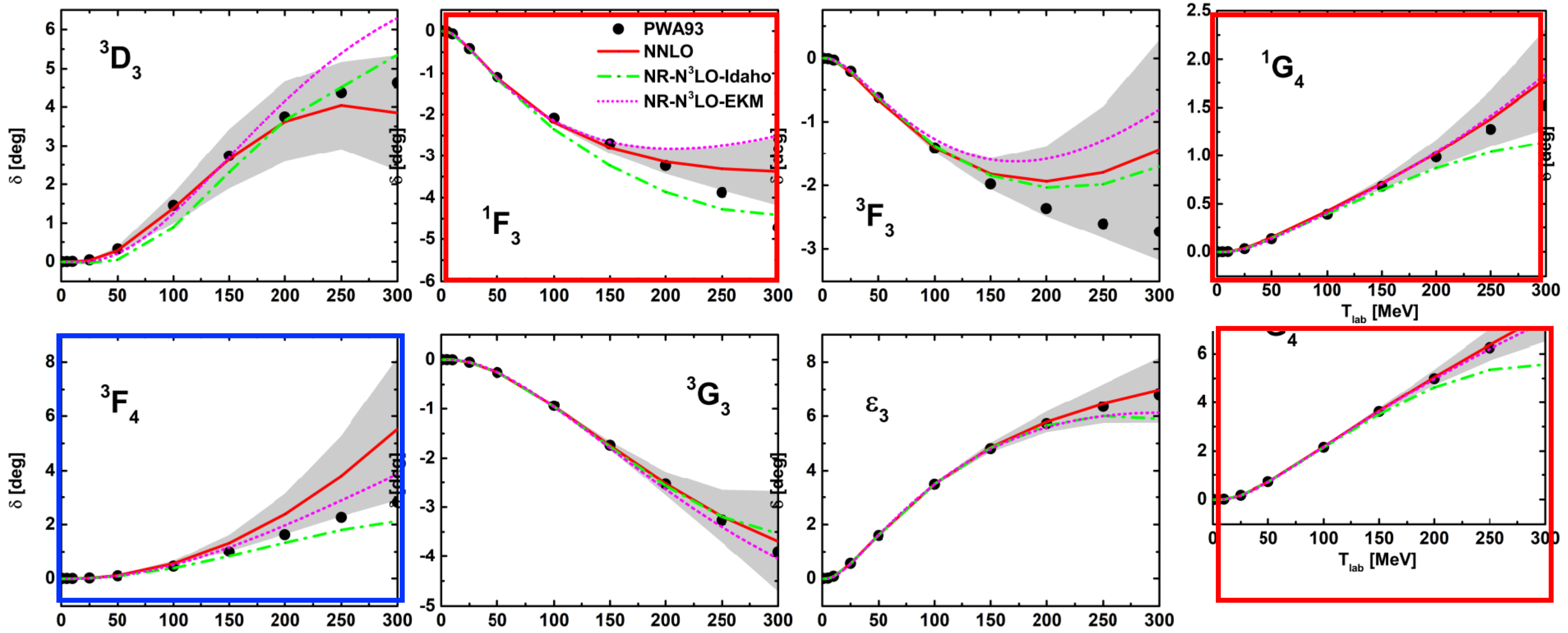
NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

Higher partial waves: chiral dyn. & rela.

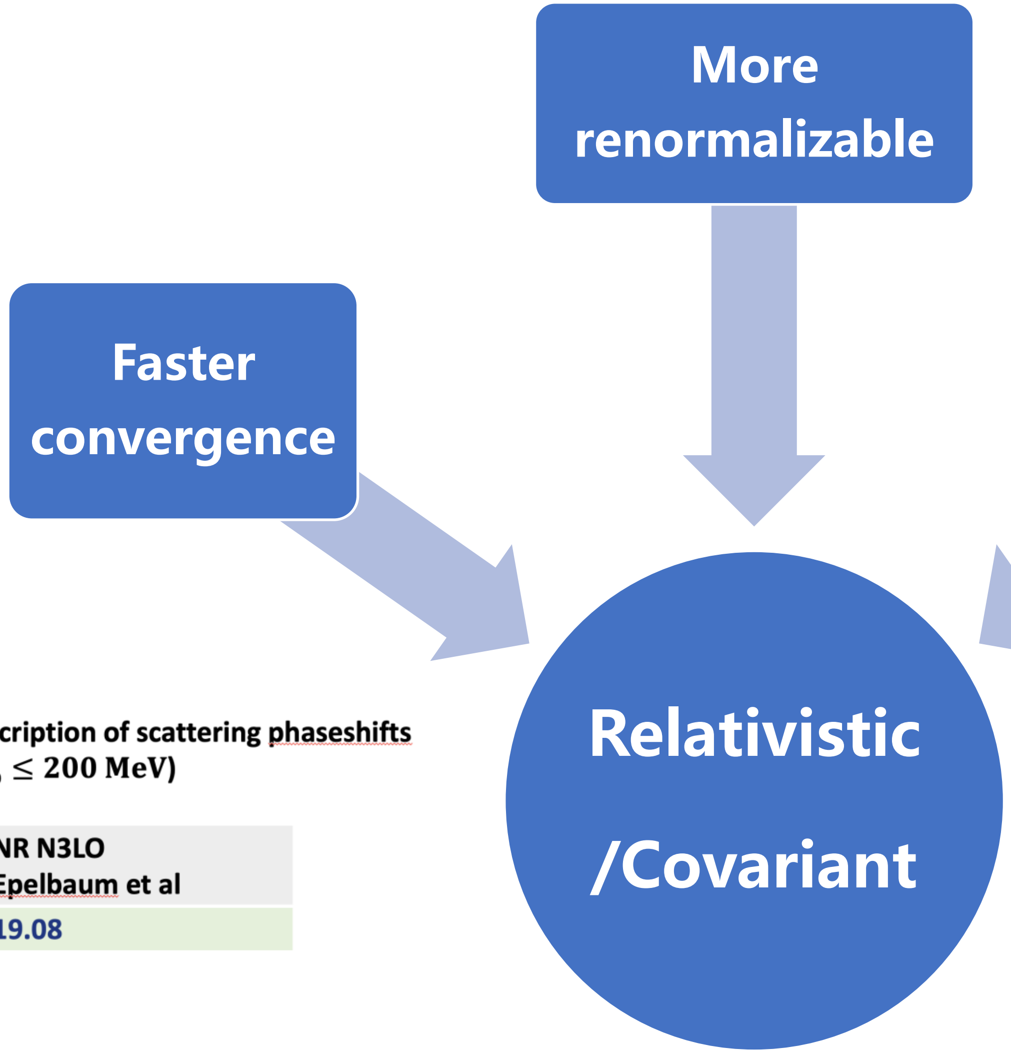


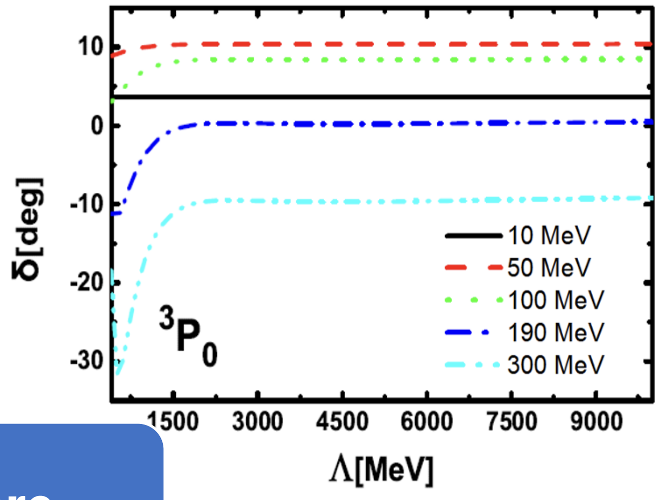
Test of relativistic treatment

	Total	$^3D_3$	$^1F_3$	$^3F_3$	$^3F_4$	$^3G_3$	ϵ_3	$^1G_4$	$^3G_4$
NNLO	0.98	0.03	0.03	0.21	0.70	0.00	0.01	0.00	0.00
NR-N^{3}LO-EKM	3.00	0.56	1.44	0.28	0.54	0.01	0.01	0.03	0.13



Three features of the relativistic force:

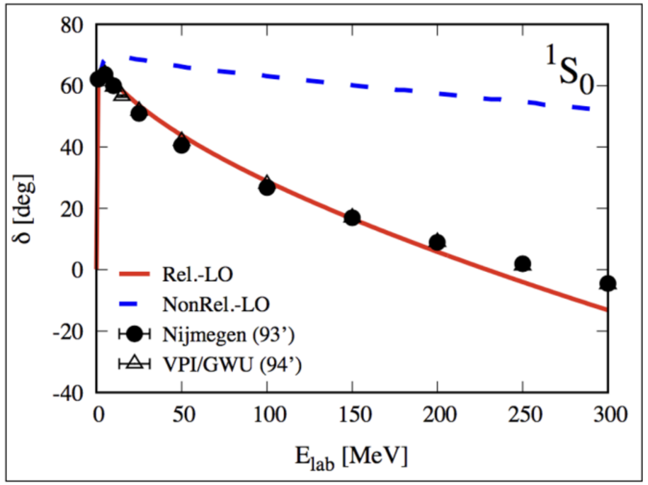




LSG* et al., CPC45,054101

Relativistic and non-relativistic description of scattering phaseshifts
$(0 \leq j \leq 4, T_{lab} \leq 200$ MeV$)$

Forces	Relativistic N2LO Geng et al.	NR N3LO Epelbaum et al
χ^2	17.57	19.08



LSG* et al., CPL38, 062101

Status quo

We have constructed the first high-precision relativistic chiral nuclear force

1990/91 Weinberg

1994, Van Kolck

2003, Machleidt

2015, Epelbaum

Beijing 2022

非相对论手征核力

相对论手征核力

LO NN, Weinberg, PLB251 (1990) 288

NN potential with Delta at **N2LO**, Ordonez, Ray, **U. van Kolck**, PRL72 (1994) 1982

Accurate NN potential at **N3LO,** Entem and **Machleidt**,PRC68(2003)041001

50 YEARS PHYSICAL REVIEW C

Precision NN potential at **N4LO**, **Epelbaum**, Krebs, Meißner, **PRL115 (2015)122301**

Relativistic NN potential at **N2LO**,Lu, Wang, Xiao, **Geng*,** Meng, Ring, **PRL128 (2022) 142002**

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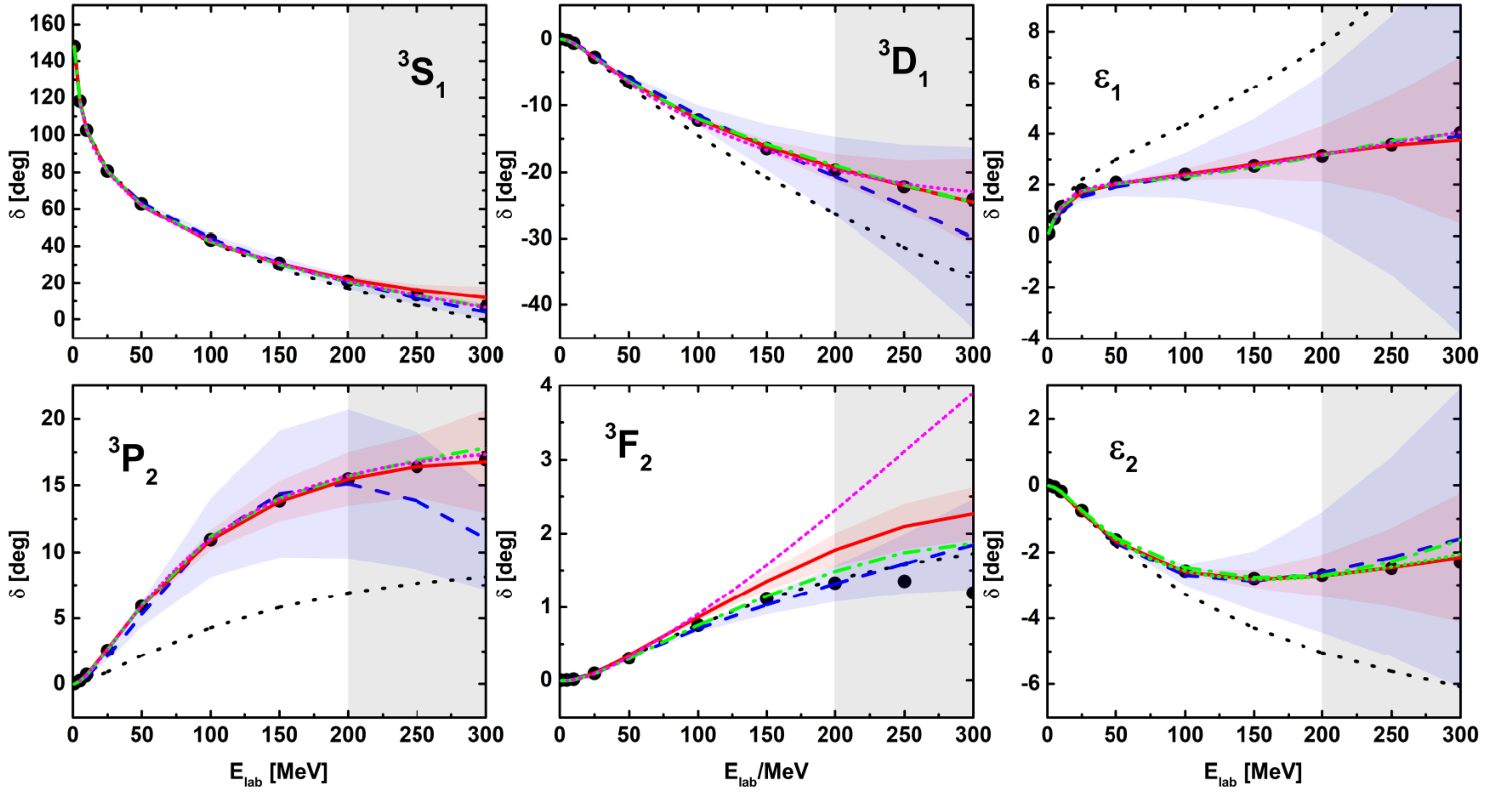
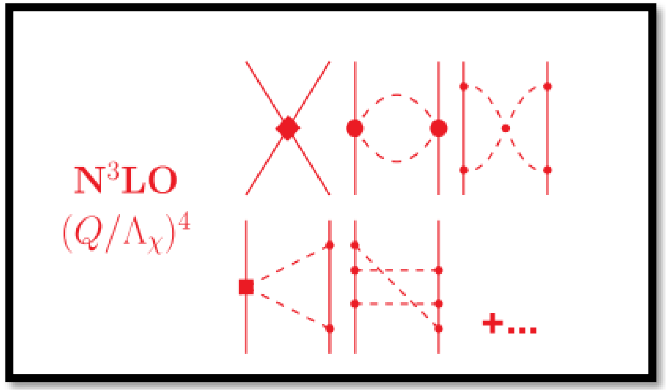
Why even higher orders

- NR-ChNF: N⁴LO is complete and partial N5LO already exists!
- R-ChNF: N3LO is needed to check the chiral convergence and describe better high-energy scattering data.



e.g., 3F2

Up to N³LO



- ✓ **Contact term** (23 terms)
- ✓ **One Pion** completely absorbed
- ✓ **Two Pion** One + Two loop
- ✓ **Three Pion** negligible

$$V_{1\pi} = V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \cdots$$

$$V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \cdots$$

$$V_{3\pi} = V_{3\pi}^{(4)} + \cdots,$$

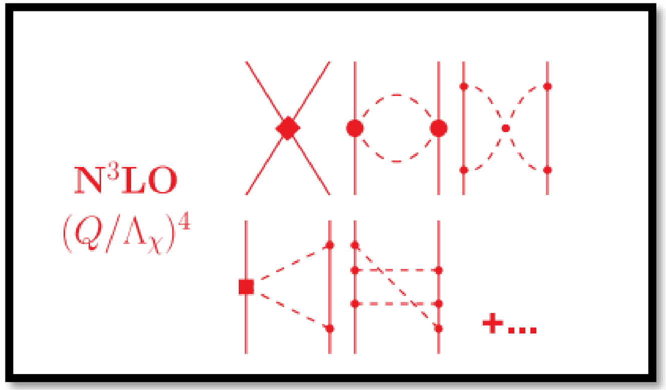
+all isospin breaking effects

➢ Contact terms (23 terms)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}\psi)\partial^2\partial_\alpha\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^\beta\partial_\nu\left(\bar\psi\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar\psi\psi)\partial^4(\bar\psi\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu\psi)\partial^4(\bar\psi\gamma_\mu\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^4(\bar\psi\gamma_5\gamma_\mu\psi)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}\psi)\partial^4(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_\alpha\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2(\bar\psi\gamma_\mu\psi)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu} i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^\alpha\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^2(\bar\psi\gamma_5\gamma_\mu\psi)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\psi)\partial^2(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\nu i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

Y. Xiao et al Phys.Rev.C 99,024004

Up to N³LO



- ✓ Contact term (23 terms)
- ✓ One Pion completely absorbed
- ✓ Two Pion One + Two loop
- ✓ Three Pion negligible

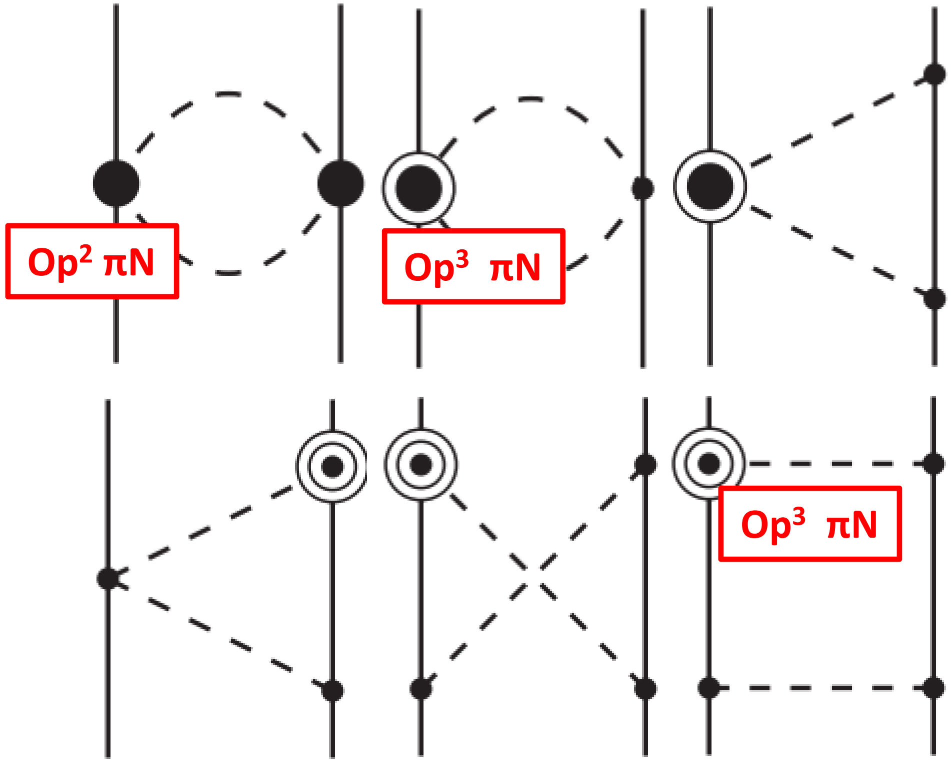
$$V_{1\pi} = V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \cdots$$

$$V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \cdots$$

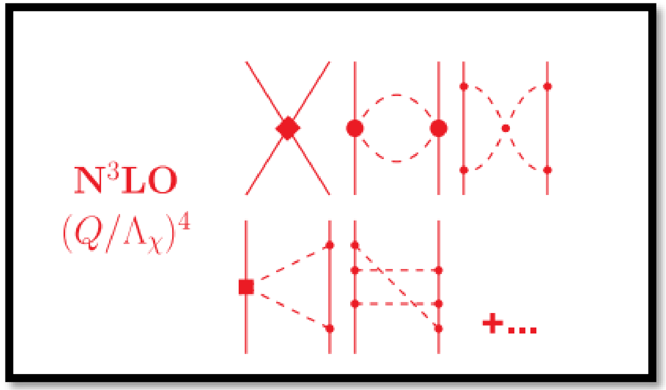
$$V_{3\pi} = V_{3\pi}^{(4)} + \cdots,$$

+all isospin breaking effects

- TPE (One-loop part)



Up to N^3LO



- ✓ Contact term (23 terms)
- ✓ One Pion completely absorbed
- ✓ Two Pion One + Two loop
- ✓ Three Pion negligible

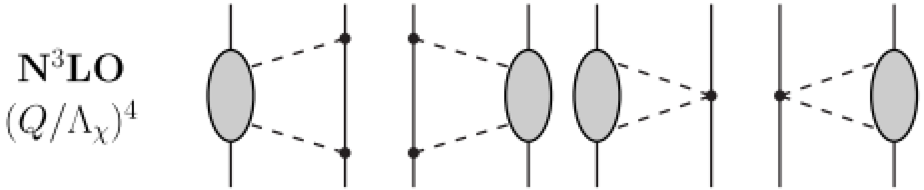
$$V_{1\pi} = V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \cdots$$

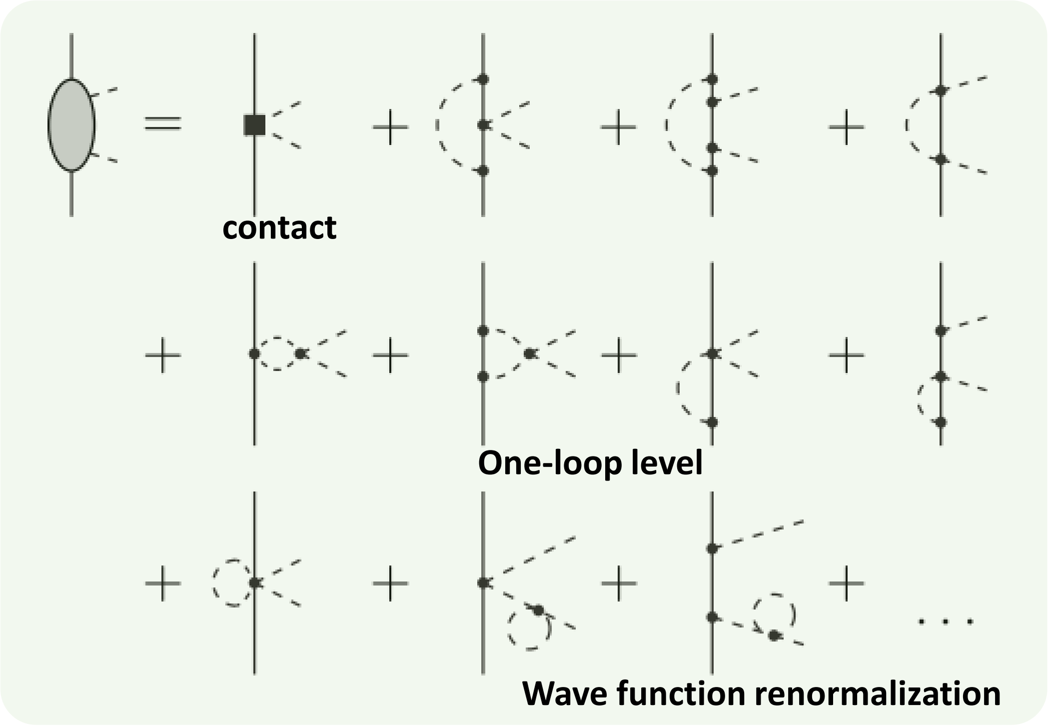
$$V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \cdots$$

$$V_{3\pi} = V_{3\pi}^{(4)} + \cdots,$$

+all isospin breaking effects

➢ TPE (two-loop part)





Relativistic two-loop diagrams— the most tough task

☐ Dimensional regularization

- Convenient / straightforward
- **Full amplitude**—non-analytical part and polynomial part
- The loop momentum—pion momentum are integrated up to infinity
- Too complicated to be used for two-loop, especially in NF involving massive fermions

So far, the two-loop contributions to NF with the DR method remain untouched

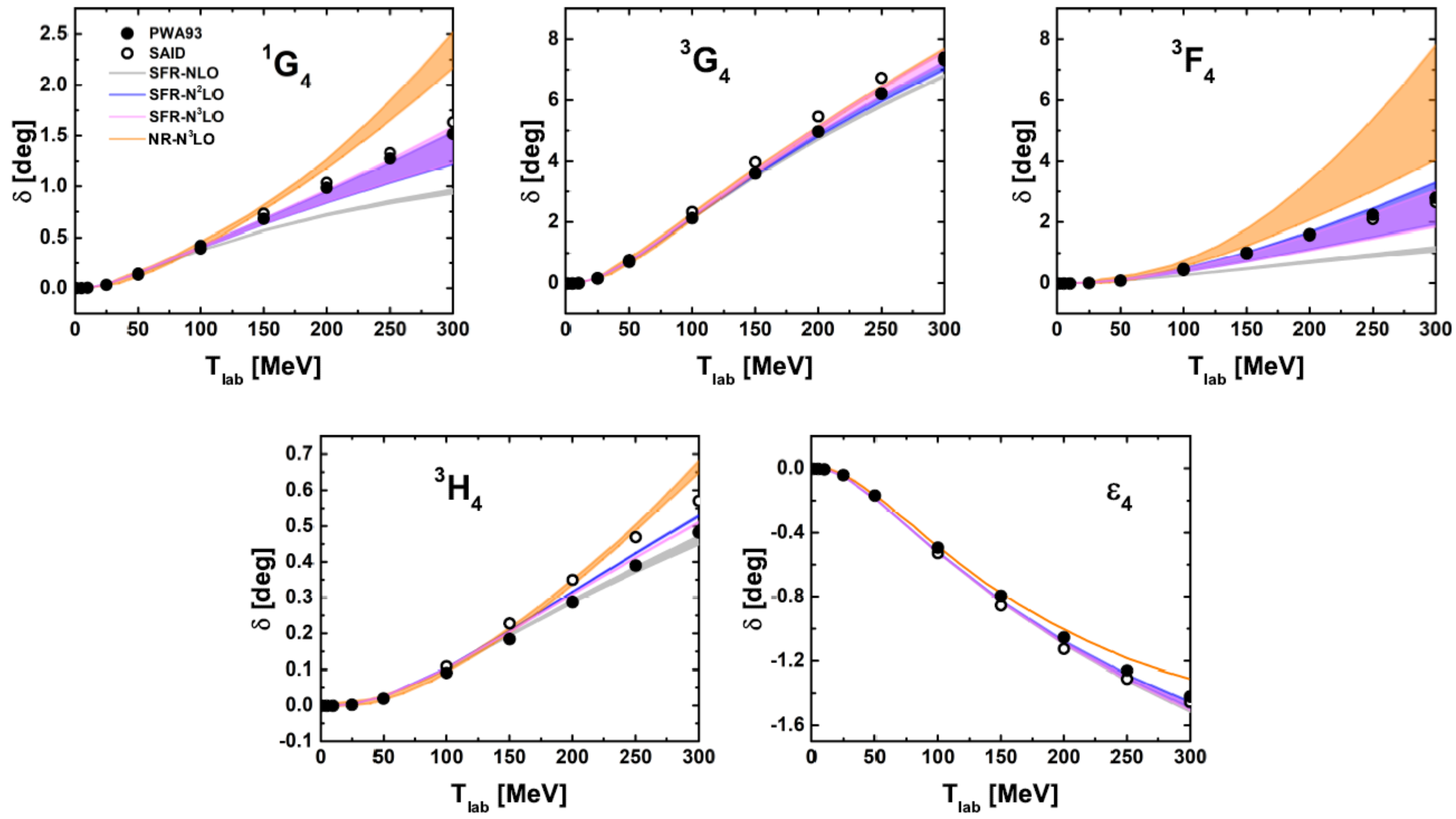
☐ Spectral representation

- **Dispersion relation** / Mandelstam representation
- Mostly **only** non-analytical parts due to subtraction
- Convenient to introduce cutoffs to the loop momentum
- From the experience of NR, it should be applicable for two-loops in the relativistic case as well.

It has become the common choice in the newly developed NR nuclear forces

- **Semi-local N^4LO NR-NF** Reinert et al. *Eur.Phys.J.A* 54, 86 (2018)
- **Local position-space N^3LO NR-NF** Saha et al. *Phys.Rev.C* 107, 034002 (2023)

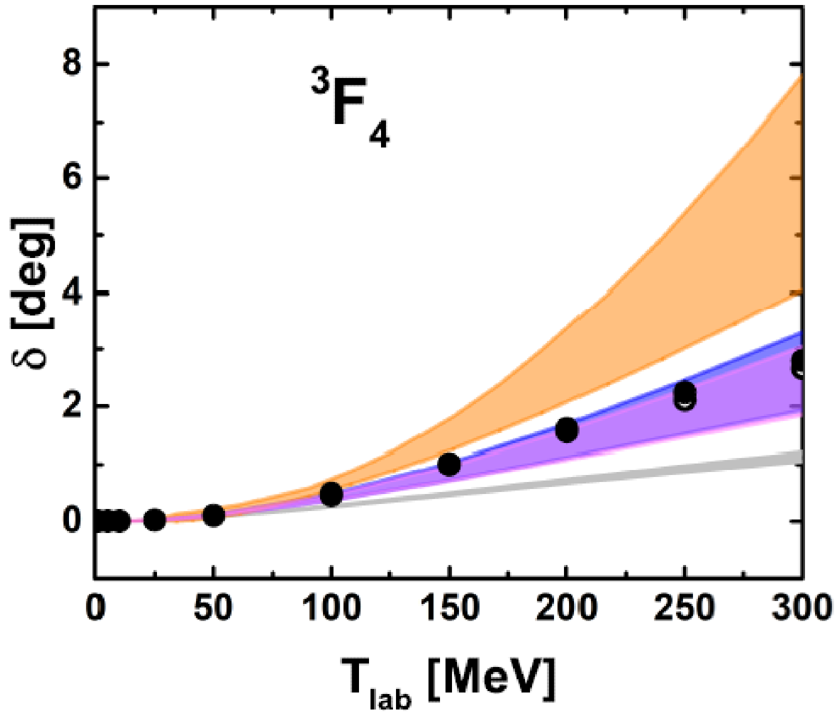
NLO&N2LO&N3LO SFR results—J=4

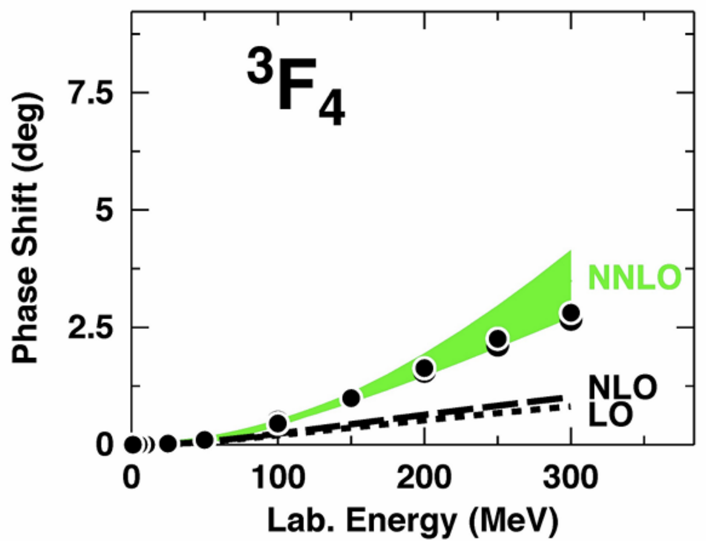


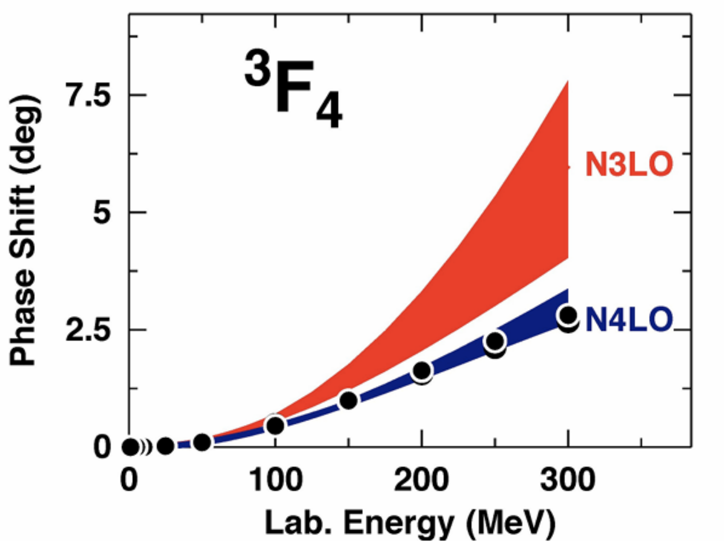
The convergence is very good! Better than NR

In comparison with the NR results

The Relativistic result







NR, PRC91, 014002

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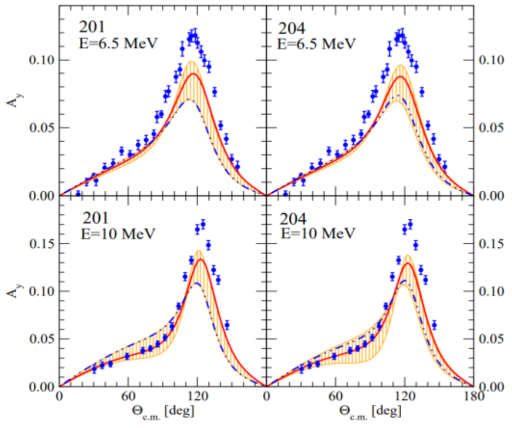
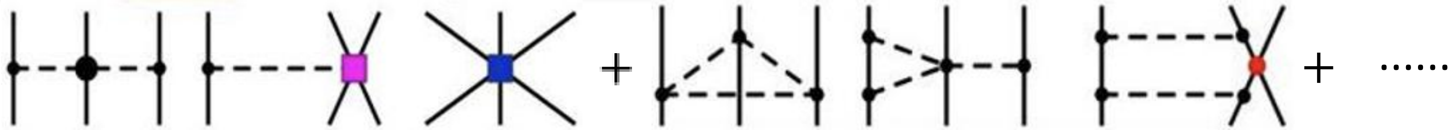
The A_y Puzzle in Nucleon–Deuteron Scattering

Blue: N3LO chiral NN + full 3N



Red: N3LO chiral NN

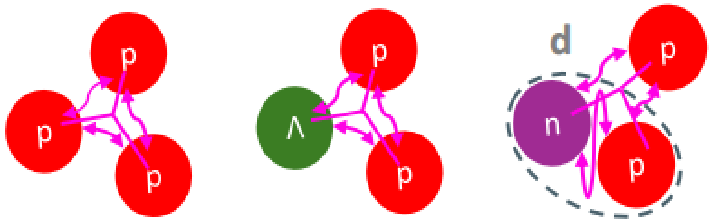
- Emergence of the A_y puzzle: failure of high-precision NN forces — A. Kievsky et al, *PRC*52 (**1995**) R15
- Benchmark calculations with realistic NN interactions — W. Glöckle et al, *Physics Reports* 274 (1996) 107; A. Kievsky et al, *PRC* 58 (1998) 3085
- Identification of the origin: ^3P_J problem — D. Hüber et al, *PRC* 58, 674 (1998)
- Attempts with conventional three-nucleon forces — C.R. Brune et al, *PLB* 428 (1998) 13-17
  - Tucson–Melbourne (TM/TM99) • Urbana IX • Brazil force
- Chiral three-nucleon forces and remaining A_y discrepancy — L. Girlanda et al, *PRC* 99 (2019) 5, 054003

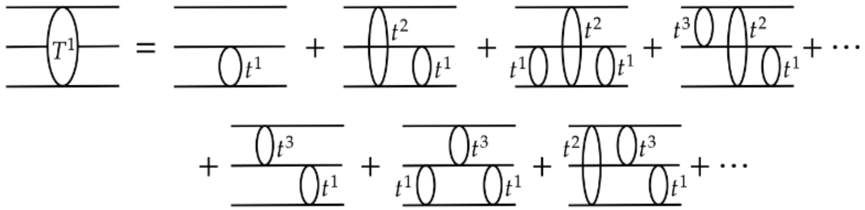


- Beyond conventional 3NF: new mechanisms
  - Modified NN *P*-waves — D.R. Entem et al, *PRC* 65 (2002) 064005
  - Relativistic effects — H. Witała et al, *PRC* 77 (2008) 034004
  - Higher-order chiral three-nucleon forces — E. Epelbaum et al, *EPJA* 56 (2020) 92

3B theoretical framework – the nonrelativistic Faddeev equation

3N interactions + **covariant three-body scattering eq.**





$$t_i = v_i + v_i G_0 t_i$$

$$\langle \vec{p}_i \vec{q}_i | t_i(E) | \vec{p}_i{}' \vec{q}_i{}' \rangle = \delta(\vec{q}_i - \vec{q}_i{}') \langle \vec{p}_i | t_i(E - q_i^2/2\tilde{\mu}_i) | \vec{p}_i{}' \rangle$$

$$\begin{pmatrix} T^{(i)} \\ T^{(j)} \\ T^{(k)} \end{pmatrix} = \begin{pmatrix} t_i \\ t_j \\ t_k \end{pmatrix} + \begin{pmatrix} 0 & t_i & t_i \\ t_j & 0 & t_j \\ t_k & t_k & 0 \end{pmatrix} G_0 \begin{pmatrix} T^{(i)} \\ T^{(j)} \\ T^{(k)} \end{pmatrix}$$

L.D. Faddeev
23 Mar. 1934 – 26 Feb. 2017

Faddeev L D. *Journal of Experimental and Theoretical Physics*, 1961, 12(5): 1014-1019

First relativistic three-body scattering equation

Lippmann-Schwinger equation (LSE)

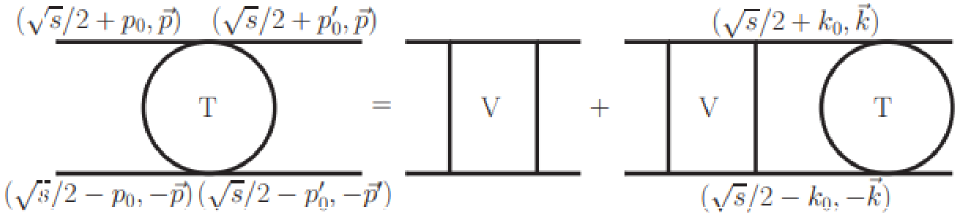
$$T = V + VG_0T$$

$G_0 = \frac{1}{E - H_0 + i\epsilon}$: the non-relativistic propagator

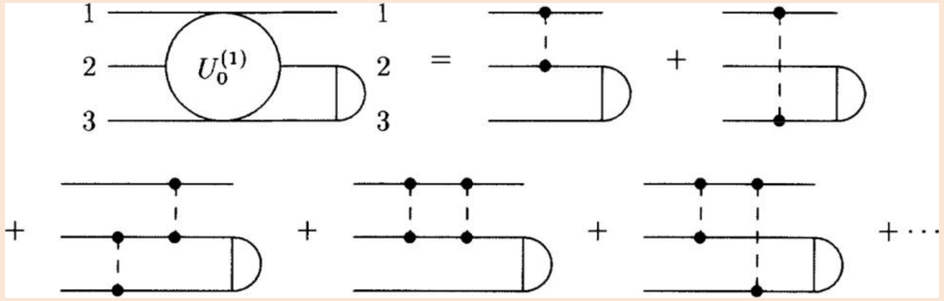
Bethe-Salpeter equation (BSE)

$$T = \mathcal{V} + \mathcal{V}\mathfrak{g}T$$

$\mathcal{V}$: all irreducible connected Feynman diagrams
$\mathfrak{g}$: relativistic propagator of two nucleons



R.M WOLOSHYN & A D. JACKSON *Nuclear Physics B* 64 (1973) 269-288



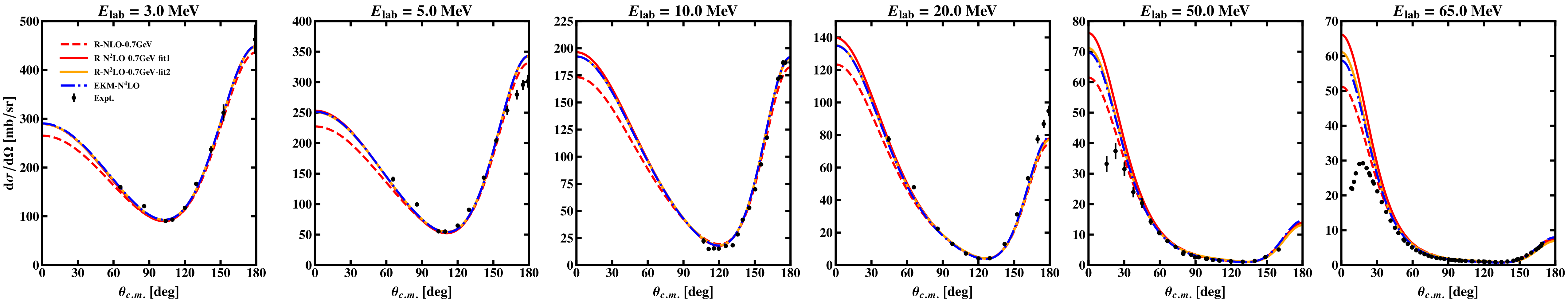
$$\boldsymbol{U = Pv + PtG_0U}$$

P: permutation operators in the covariant framework
v: all irreducible connected Feynman diagrams
(two-body or three-body interactions)
G_0 : relativistic propagators of three-nucleons

At the end, the equation has the same form as the Faddeev equation, similar to the relationship between the BS and LS equations. Thus, we call it **the relativistic scattering equation.**

First study of the nD scattering in the relativistic framework (preliminary)

☐ The differential cross section —only two-body force up to J=2



✓ Results from R-NNLO and EKM-N4LO almost overlap. Differences emerge at extremely small angles.

☐ Further comparison for tritium binding energy

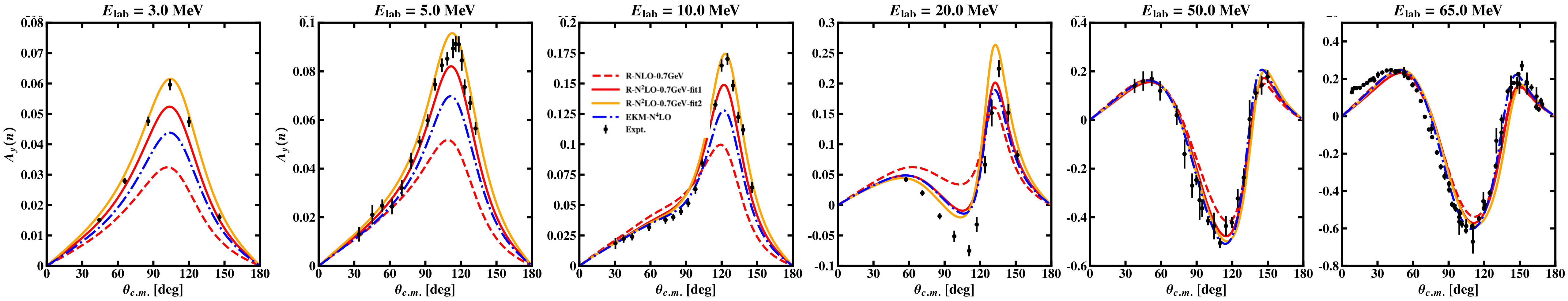
$EKM - N^3LO$	**isospin breaking**	**7.37145 MeV**
	isospin symmetry	**7.53719 MeV**
$CChNF - N^2LO$	**isospin symmetry**	**8.30648 MeV**

Exp. 8.4818

EKM-N⁴LO: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

First study of the nD scattering in the relativistic framework (preliminary)

- The polarization Ay for n-d scattering—only two-body force up to J=2



✓ The description of Ay is significantly improved up to NNLO.
✓ Sensitive to the P-wave NN force

Covariance may indeed play a crucial role in the description of polarization Ay and related observables of few-body system.

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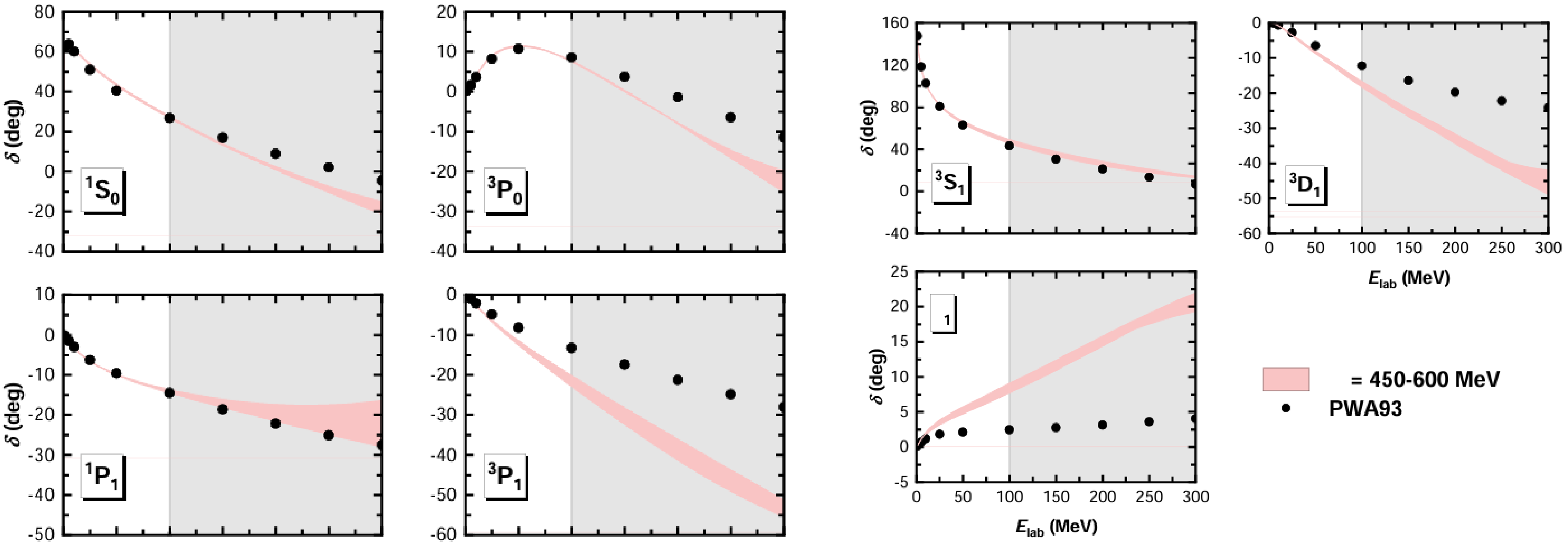
SNM in the DBHF theory (vs. BHF)

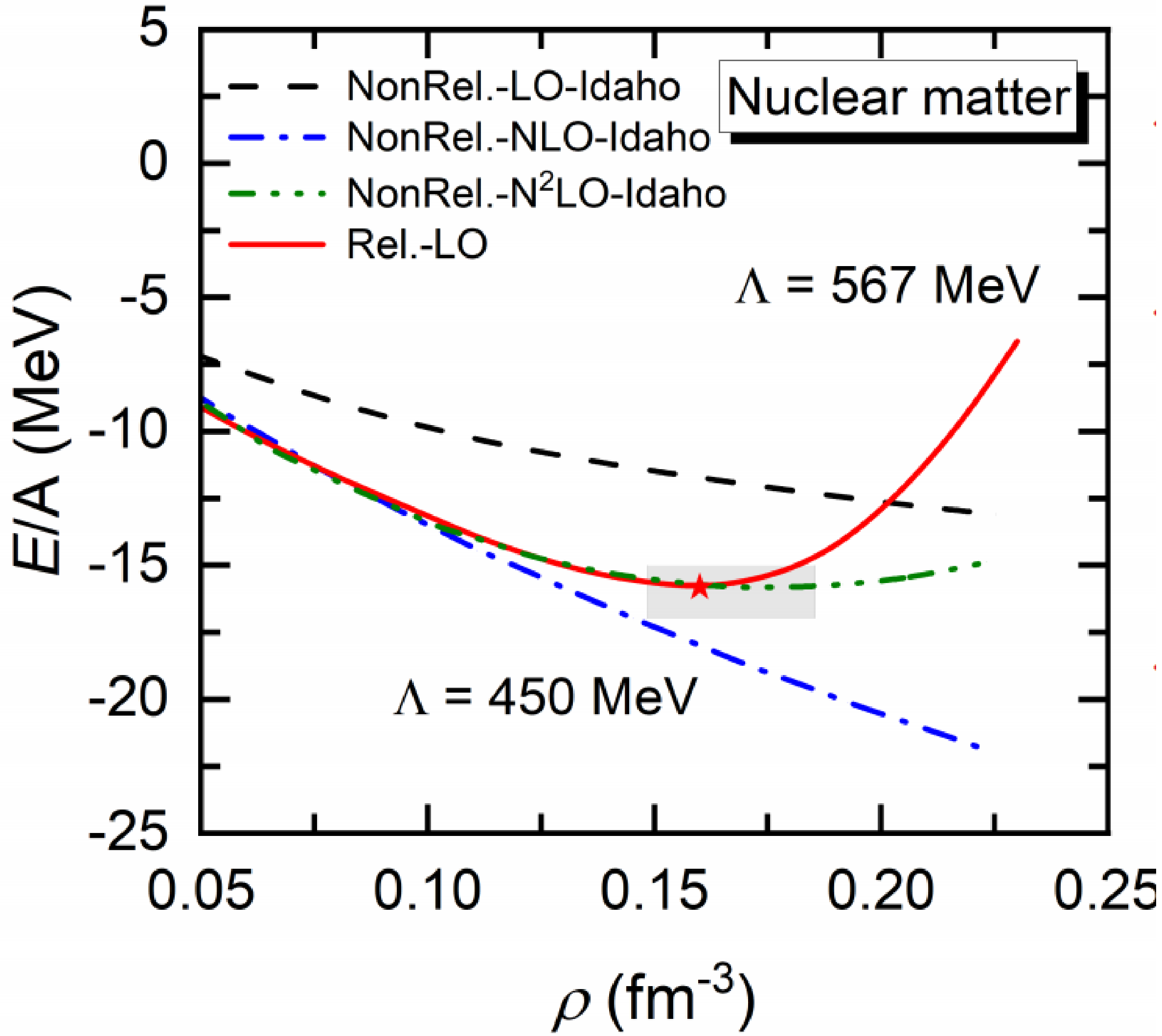
Wei-Jiang Zou, Jun-Xu Lu, Peng-Wei Zhao, LSG*, Jie Meng, Phys.Lett.B 854 (2024)138732



Only four LECs

SNM in the DBHF theory (vs. BHF)

Wei-Jiang Zou, Jun-Xu Lu, Peng-Wei Zhao, LSG*, Jie Meng, Phys.Lett.B 854 (2024)138732



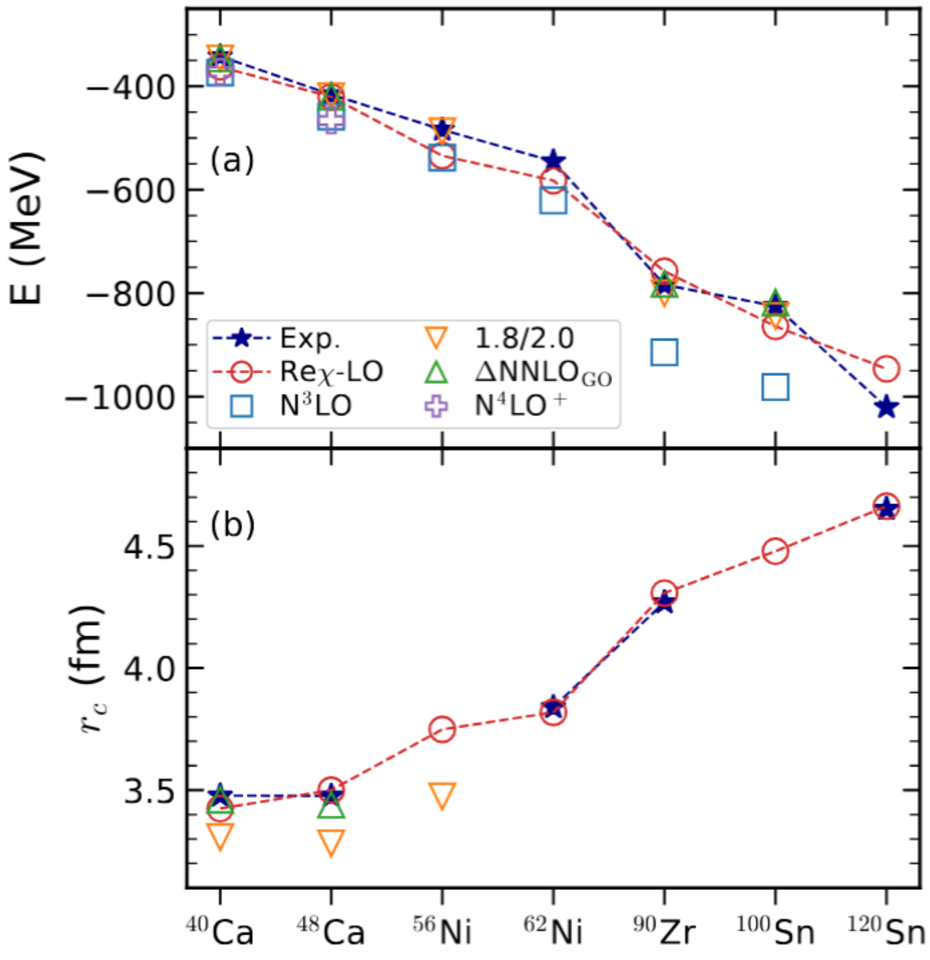
- No saturation in the BHF up to NLO.
- Only at N^2LO and with 3NF considered, saturation can be achieved in the BHF.
- Below the saturation point, results are similar

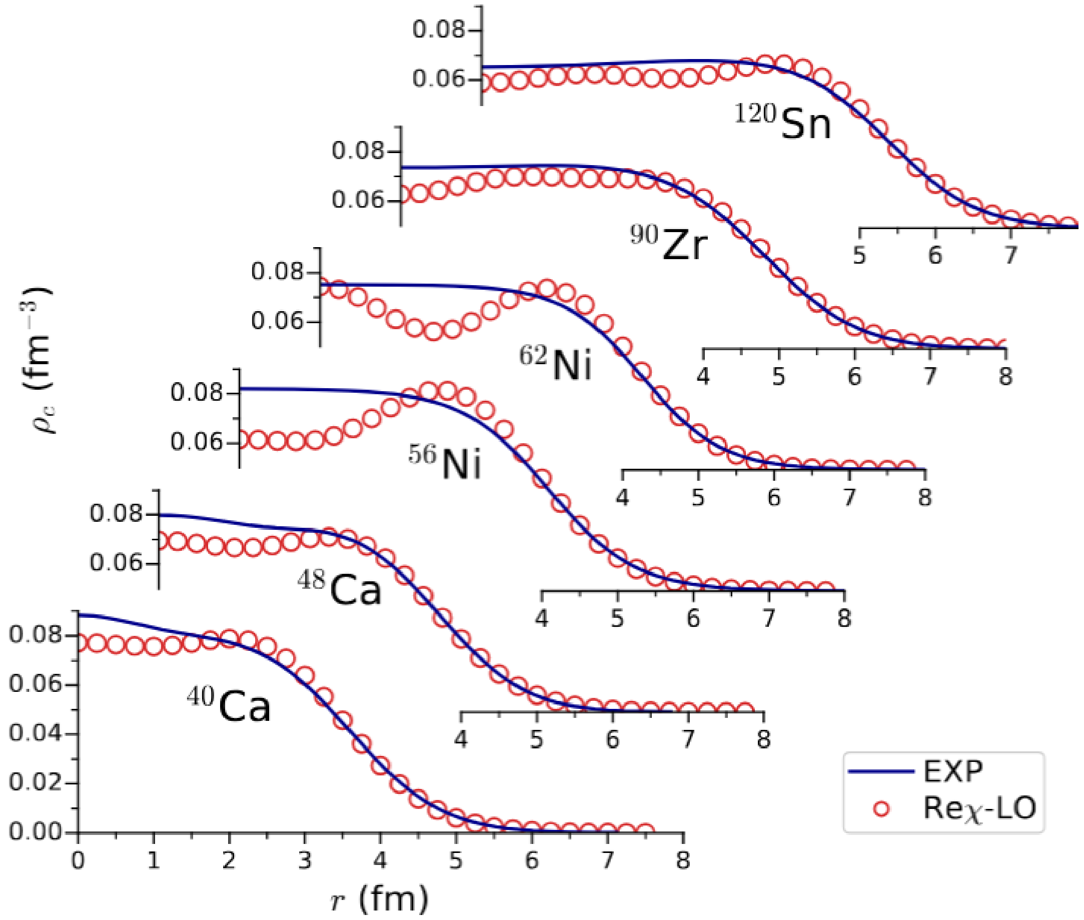
F. Sammarruca et al, PRC104, 064312

Only four LECs

Finite nuclei in the DBHF theory

Shi-Hang Shen, Jun-Xu Lu, LSG*, Jie Meng, Wei-Jiang Zou, 2507.01257, to appear in PRC

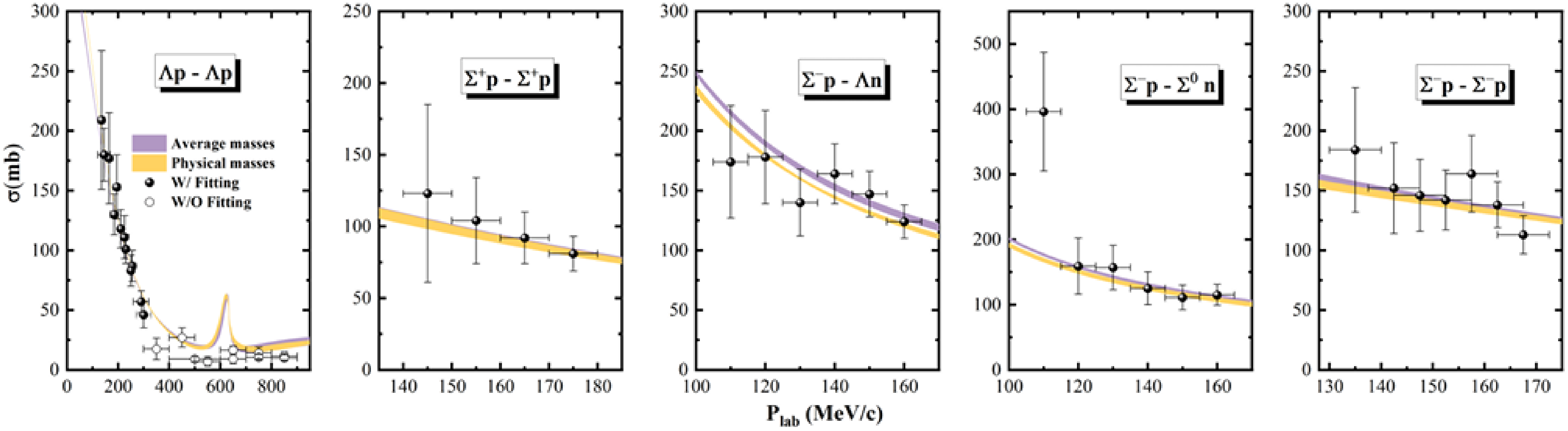




Only four LECs

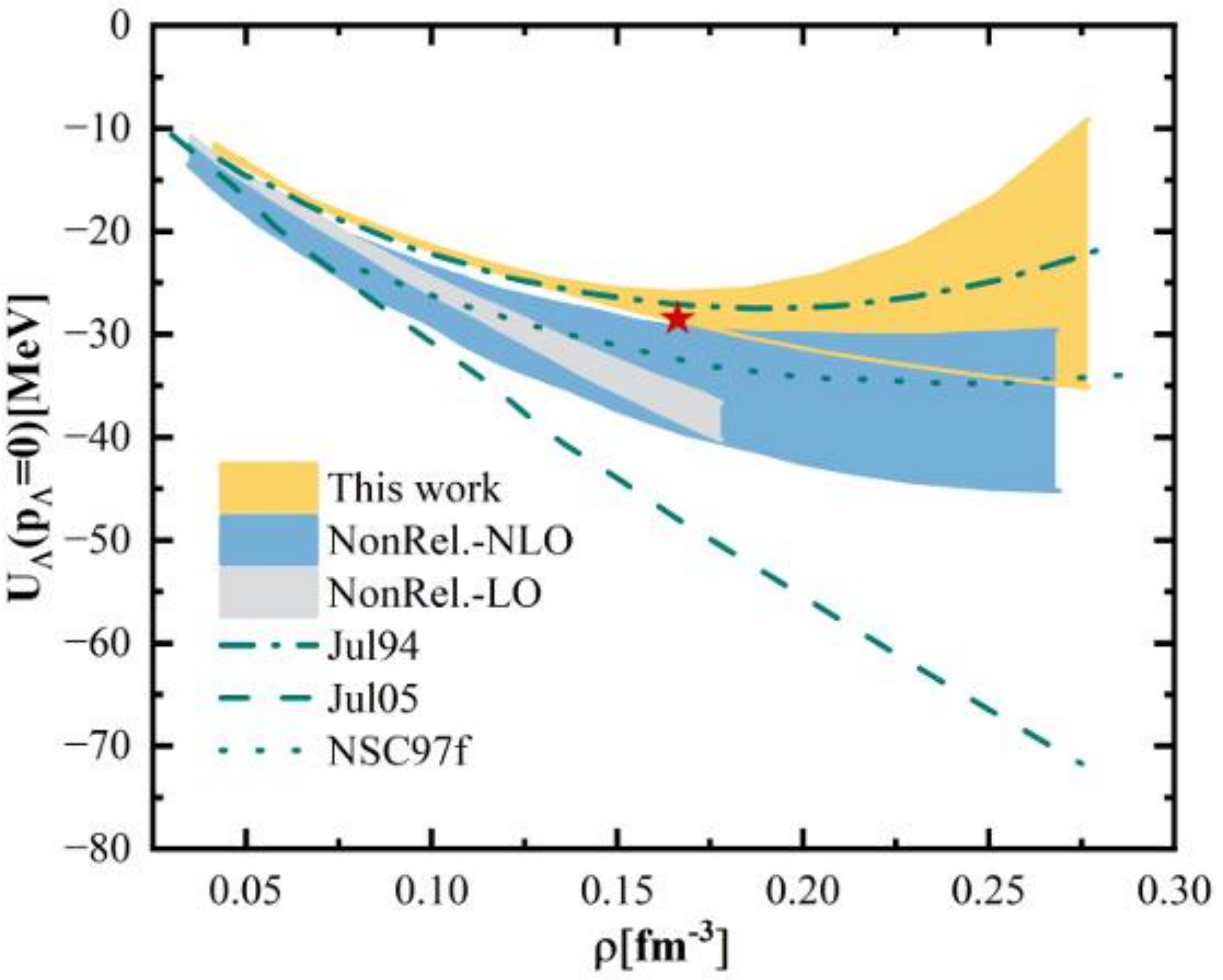
Relativistic chiral hyperon-nuclear force

Ru-You Zheng, Zhi-Wei Liu, LSG*, Jin-Niu Hu, Sibo Wang,
Phys.Lett.B 864 (2025) 139416



The Λ single-particle potential

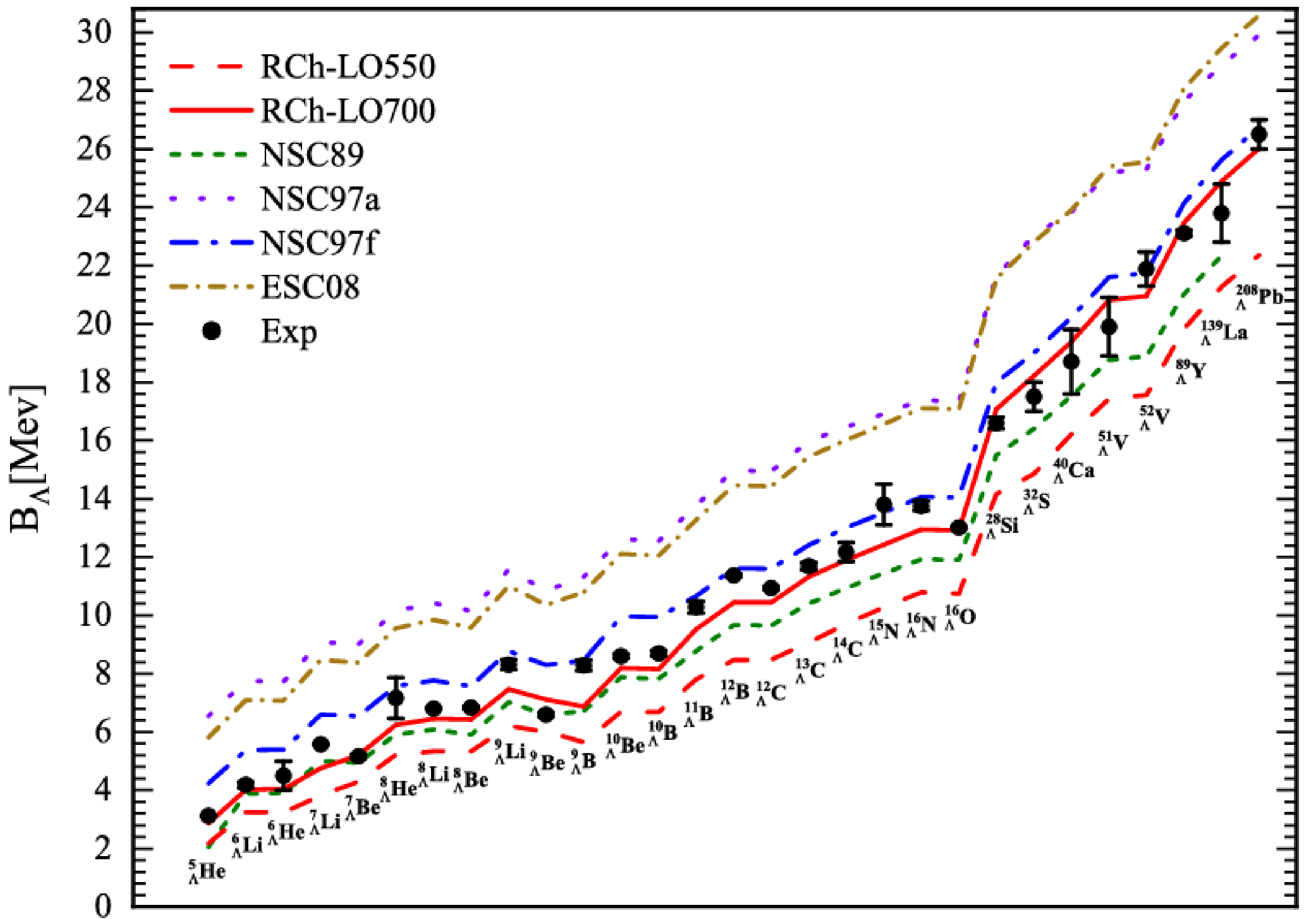
Ru-You Zheng, Zhi-Wei Liu, LSG*, Jin-Niu Hu, Sibo Wang, Phys.Lett.B 864 (2025) 139416

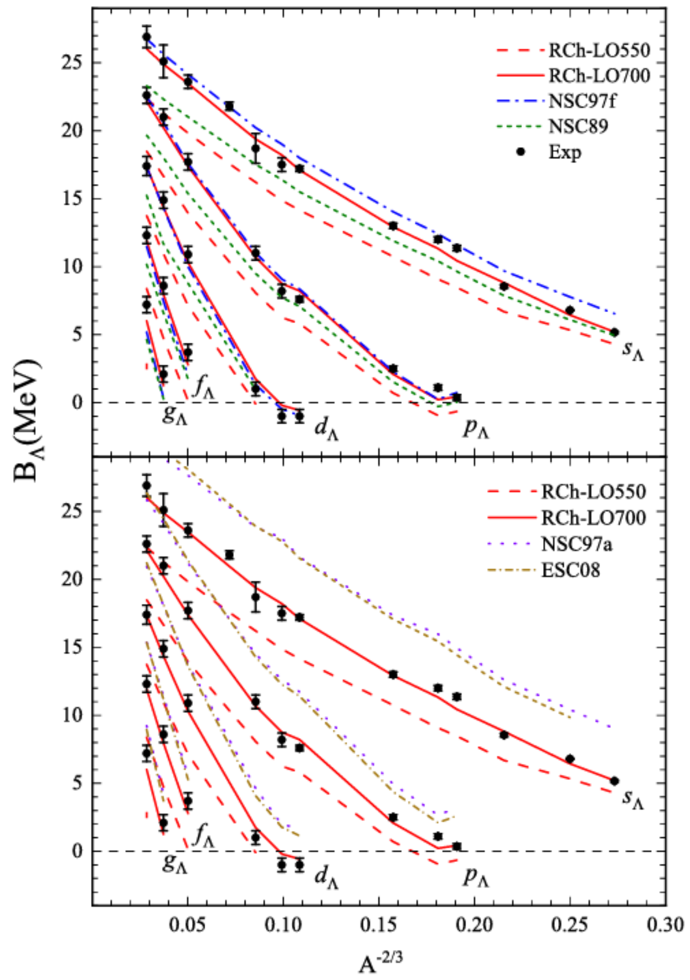


Hypernuclear structure

Xuan Zeng, Ru-You Zheng, Zhi-Wei Liu, LSG*, Xian-Rong Zhou,
PRC 113 (2026) 014310

Λ hyperon binding energies





Energy levels of the Λ single-particle major shells

Summary and Outlook

- □ We have developed a high-precision covariant chiral nuclear force up to the NNLO.
- □ We are now working on N3LO covariant NN forces and covariant three-body forces, and have worked out all the technical complexities
- □ We have applied the leading order NN and YN forces to study symmetric nuclear matter, finite nuclei, as well as hypernuclear systems. The results are very encouraging.
- □ More to come. Remain tuned.

Collaborators

- **Beihang University:** Jun-Xu Lu, Yang-Xiao, Zhi-Wei Liu, Qing-Yu Zhai, Ye-bin Wen, Ru-You Zheng, Shi-Hang Shen
- **Peking University:** Jie Meng, Peng-Wei Zhao, Wei-Jiang Zou
- **Institute of Modern Physics:** Ju-jun Xie
- **Chinese Academy for Atomic Energy:** Rui-Rui Xu
- **University of Tokyo:** Hao-Zhao Liang
- **East China Normal University:** Xian-Rong Zhou

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Ministry of Education of the People's Republic of China

北京航空航天大学
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