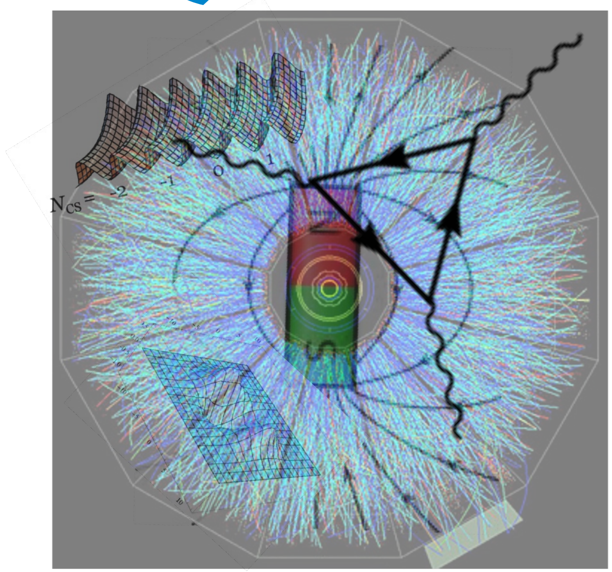
Jul. 27, 2026

Angular Momentum Transport in QCD Matter



Jinfeng Liao

ndiana University, Physics Dept. & CEEM

Plan of the Talk

– ***Introduction***

– ***Angular Momentum Initial Conditions***

– ***Hydrodynamics with Angular Momentum***

– ***Summary***

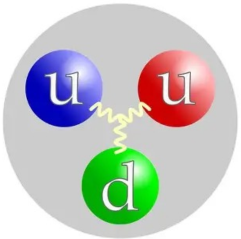
Introduction

From Microscopic Symmetries to Conserved Quantum Numbers

Spacetime translational symmetries →	***Energy momentum conservation*** →	***Energy & momentum (Mass)***	P^μ
Phase symmetry →	***Charge conservation*** →	***Charge***	N
Lorentz symmetry →	***Angular momentum conservation*** →	***Angular momentum***	J

These quantum numbers are essential for characterizing static properties of matter.

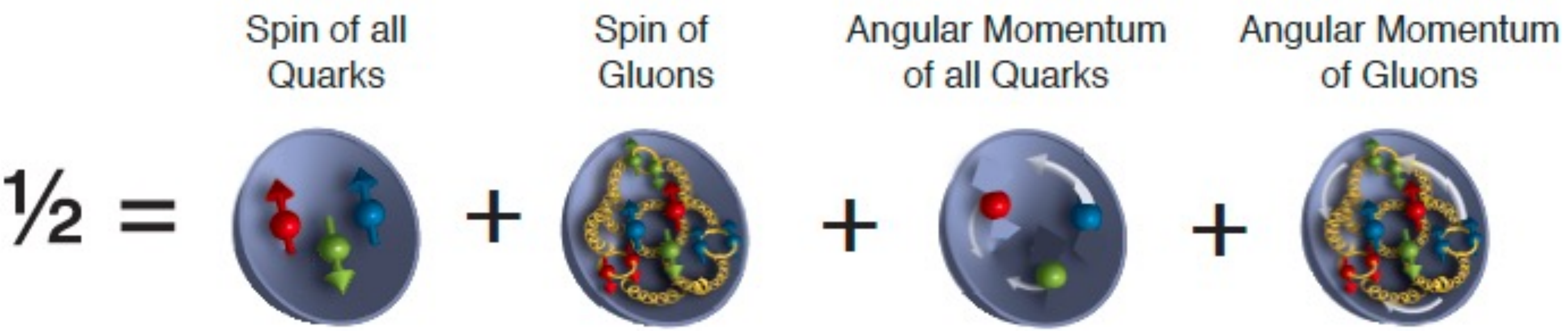
A Proton



A QCD quantum state with M=0.94GeV, Q=+1, and J =1/2

Proton is essentially a many-body quark-gluon system.

Putting quarks/gluons back together into a proton is a lot harder then one would naively expect…



Angular Momentum Decomposition

The definition of angular momentum operator and its decomposition into spin and orbital components depends on the choice of energy momentum tensor (issue of pseudo-gauge)

$$M^{\mu\nu\rho}(x) = x^{\nu} T^{\mu\rho}(x) - x^{\rho} T^{\mu\nu}(x), \qquad J^{\nu\rho} = \int d^3x \, M^{0\nu\rho}(x).$$

$T^{\mu\nu}$ is understood to be the Belinfante–Rosenfeld improved, symmetric, and gauge-invariant

Ambiguities in the decomposition into quarks/gluons

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g \qquad \text{?}$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q^{kin} + J_g \qquad \text{?}$$

……

The Takeaways

The interplay of angular momentum and QCD is highly nontrivial even for a static state like a proton.

The decomposition of angular momentum into spin and orbital components could be very subtle.

The interplay of angular momentum and QCD in a highly dynamical system (e.g. heavy-ion collision) could be really interesting.

From Microscopic Symmetries to Macroscopic Matter

Spacetime translational symmetries ⟶ ***Energy momentum conservation*** ⟶ ***Energy momentum transport***

$$P^{\mu} \rightarrow \beta_{\mu} \rightarrow \partial_{\mu} T^{\mu\nu} = 0$$

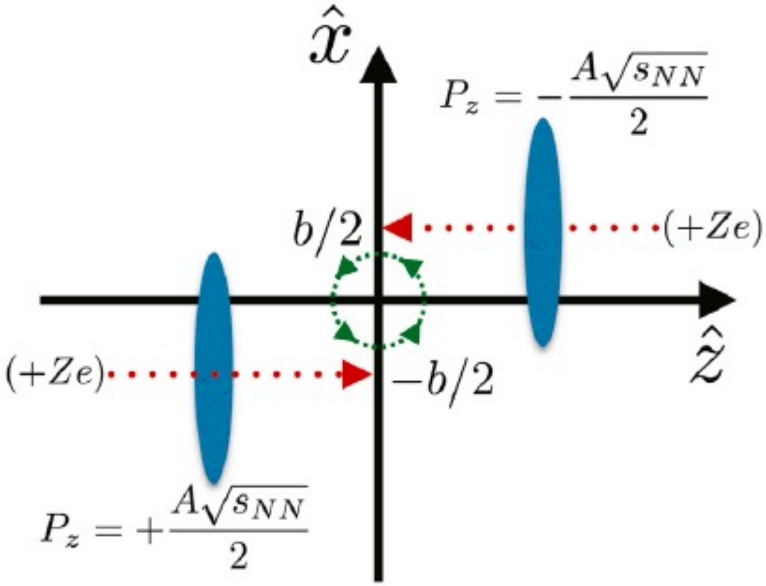
Phase symmetry ⟶ ***Charge conservation*** ⟶ ***Charge transport***

$$N \rightarrow \mu_{N} \rightarrow \partial_{\mu} J^{\mu} = 0$$

Lorentz symmetry ⟶ ***Angular momentum conservation*** ⟶ ***Angular momentum transport***

$$J^{\nu\rho} \rightarrow \omega^{\nu\rho} \rightarrow \partial_{\mu} \Sigma^{\mu\nu\rho} = 0$$

Angular Momentum in Heavy Ion Collisions



Huge angular momentum for the system in non-central collisions

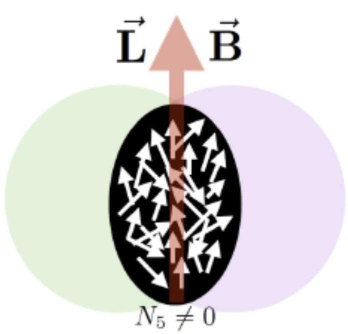
$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim5}\hbar$$

Liang & Wang ~ 2004: ***[nucl-th/0410079, PRL2005]***

Large orbital angular momentum carried by colliding ions

—> spin-orbital coupling —> spin polarization and alignment

The beginning of a new direction:
Angular momentum transport
in heavy ion collisions

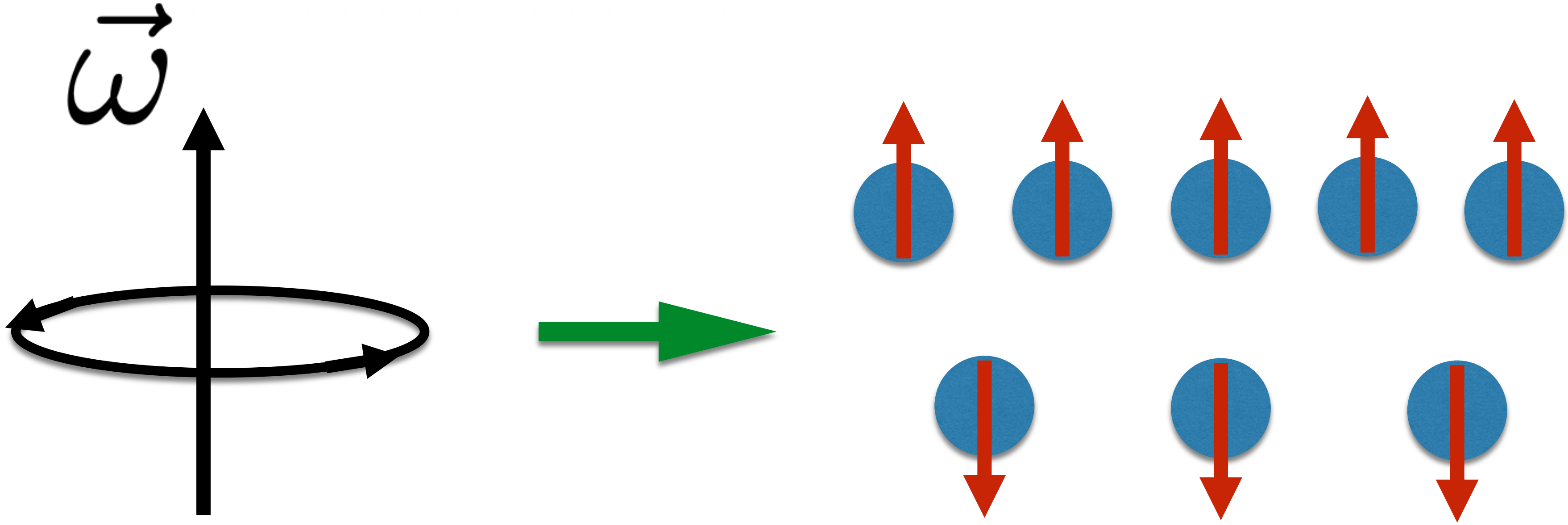


Rotational Polarization

Angular momentum —> fluid vorticity —> spin polarization

Becattini, et al ~ 2008, 2013: A fluid dynamical scenario

$$S^{\mu} = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_{\nu}\varpi_{\rho\sigma} \qquad \varpi_{\mu\nu} = \frac{1}{2}\left[\partial_{\nu}\left(\frac{1}{T}u_{\mu}\right) - \partial_{\mu}\left(\frac{1}{T}u_{\nu}\right)\right]$$

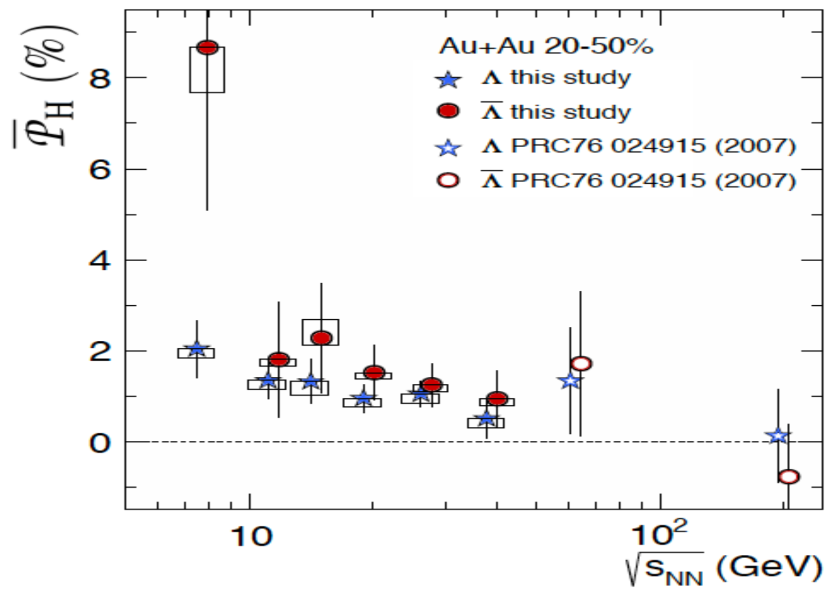


Macroscopic rotation; Global angular momentum

Microscopic spin polarization

The Most Vortical Fluid

An exciting discovery from STAR Collaboration at RHIC: The most vortical fluid!



LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*

Angular Momentum Initial Conditions

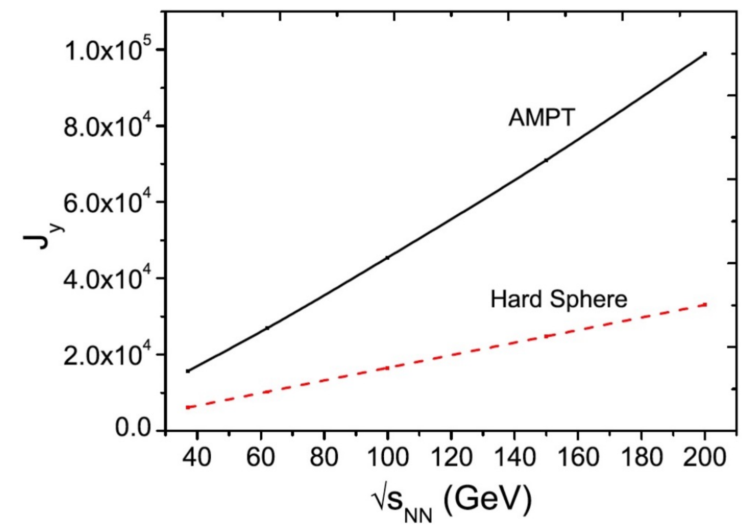
“Rotating” Quark-Gluon Plasma

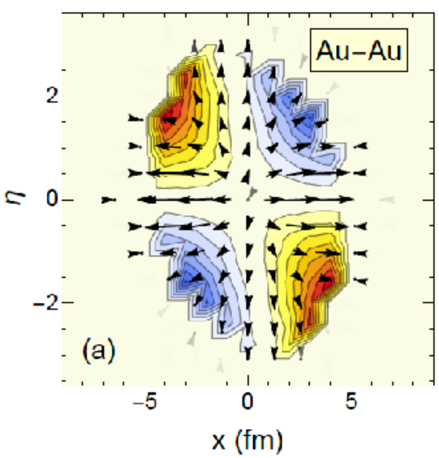
$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim5}\hbar$$

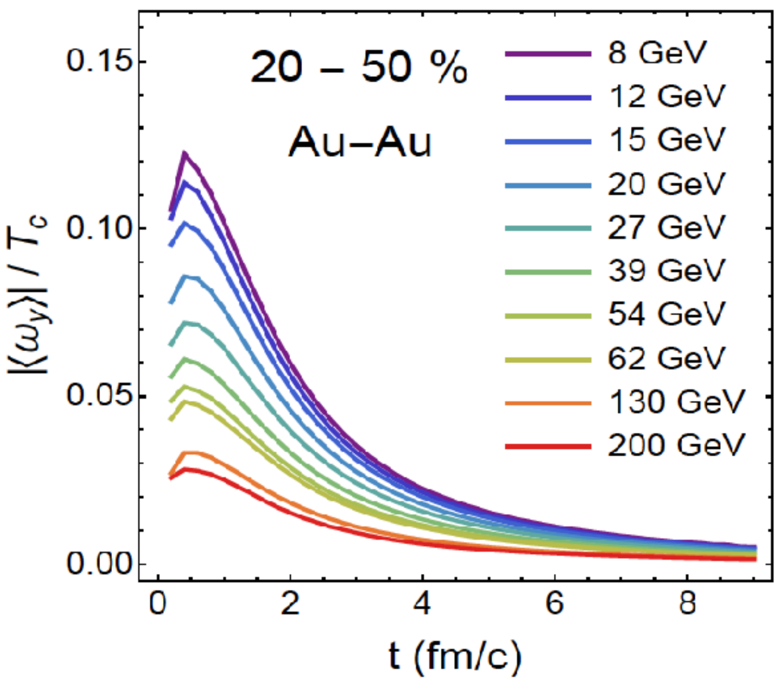
PHYSICAL REVIEW C 94, 044910 (2016)

Rotating quark-gluon plasma in relativistic heavy-ion collisions

Yin Jiang,[1] Zi-Wei Lin,[2] and Jinfeng Liao[1,3]
[1]*Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 North Milo B. Sampson Lane, Bloomington, Indiana 47408, USA*
[2]*Department of Physics, East Carolina University, Greenville, North Carolina 27858, USA*
[3]*RIKEN BNL Research Center, Building 510A, Brookhaven National Laboratory, Upton, New York 11973, USA*



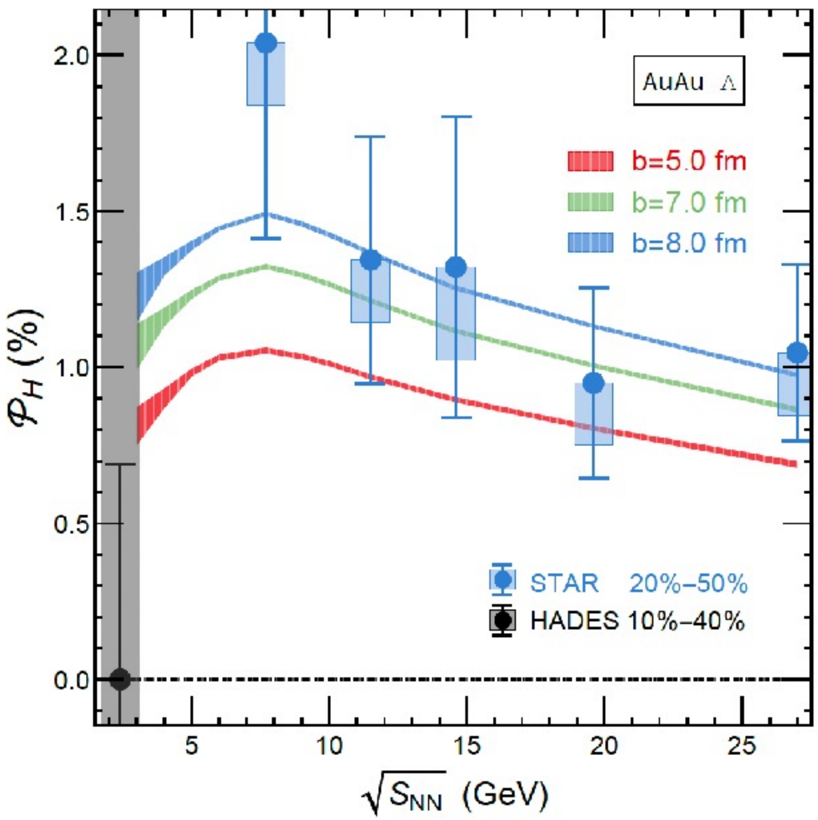




Vorticity @ O(10) GeV >> Vorticity @ O(100) GeV

Trend of Global Polarization toward O(1) GeV

The Question: Trend for global hyperon polarization @ O(1~10) GeV ???

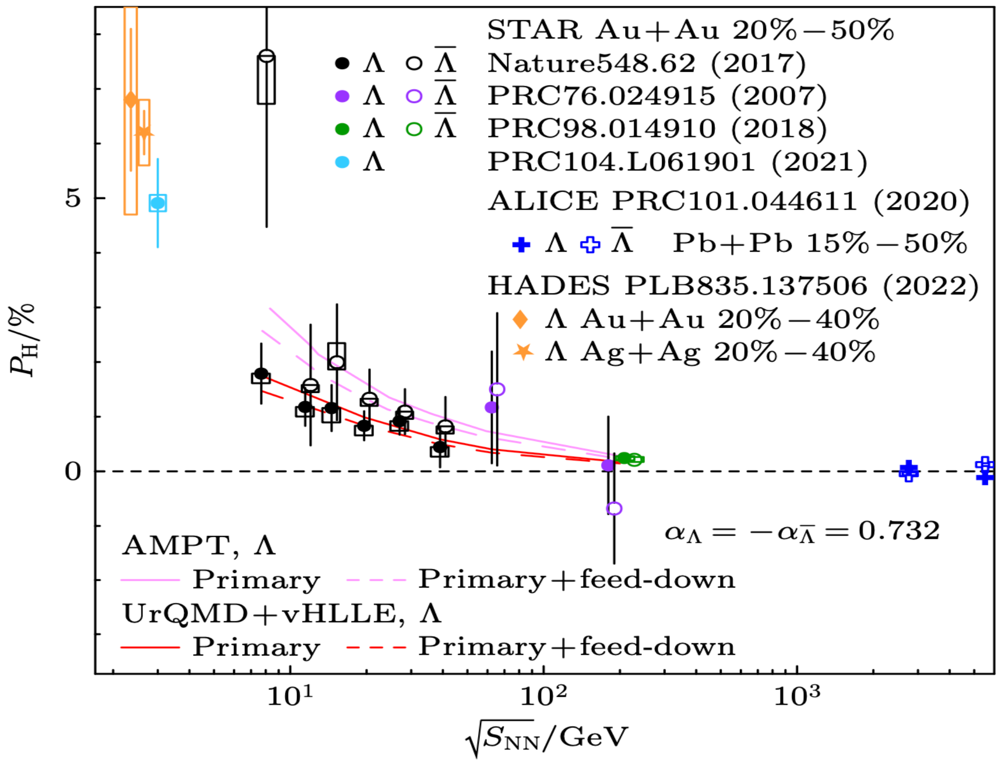


Yu Guo, et al, PRC2021
arXiv:2105.13481

AMPT calculations predict non-monotonic behavior in the dependence of global polarization on beam energy —> maximum around 7.7 GeV

See also results for differential dependence and local polarization in the paper.

Dependence on Beam Energy



Surprisingly large signal even very close to threshold?!

$$L_y = \frac{1}{2} A b \sqrt{s} \sqrt{1 - (2M/\sqrt{s})^2}$$

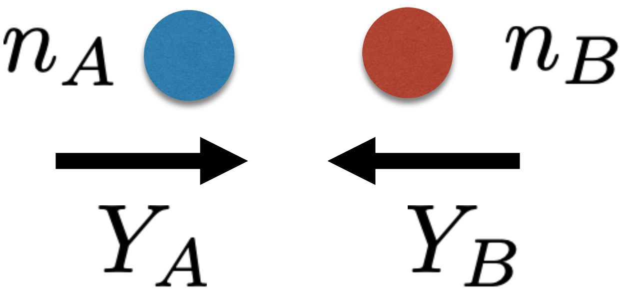
From: Xu Sun, et al, Acta Phys. Sin. 72 (2023)072401

What is relevant to measurements is the amount of angular momentum being stopped in mid rapidity.

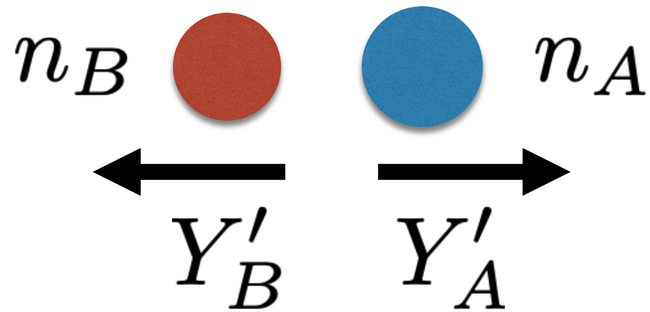
This is quantitatively related to the baryon stoping and can be calibrated with baryon number measurements.

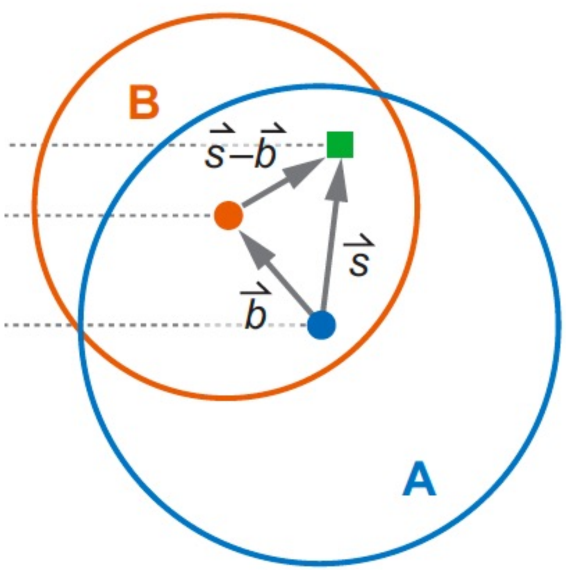
Nucleon Stopping & Angular Momentum

The key is to understand the rapidity loss in the initial collision.



$$e = \frac{Y'_B - Y'_A}{Y_B - Y_A}$$



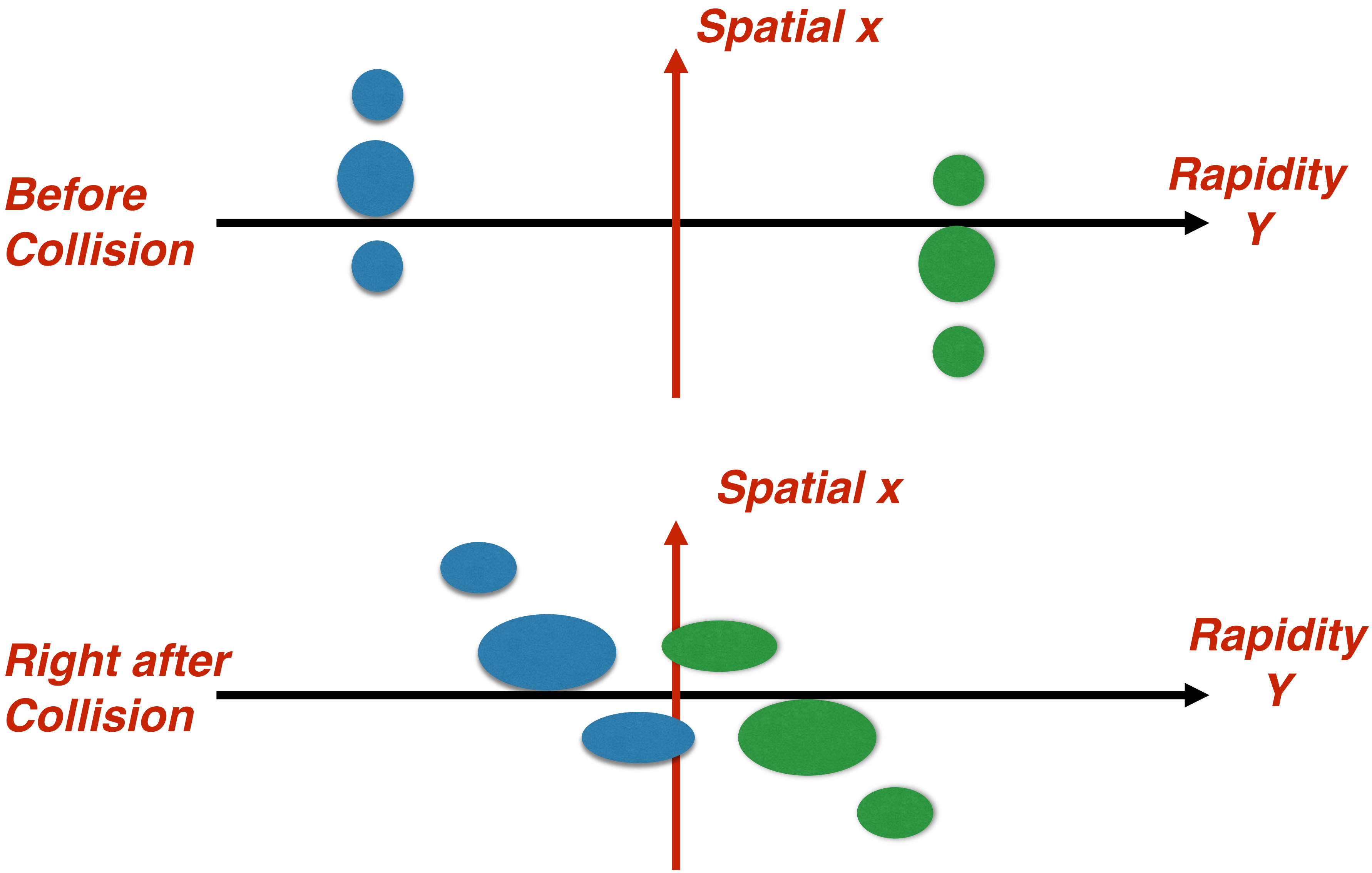


Various spots on the overlapping zone —> "spread-out" (i.e. distribution) along rapidity

Fluctuations at each spot —> Additional "spread-out" along rapidity

Both net baryon and angular momentum come from this "spread-out"

The “Glauber+” Picture



A. Akridge, D. Gallimore, H. Morales, JL, arXiv:2504.02192

Rapidity Loss

"Wounded nucleon"

$$(\lambda m_{\rm N})^2 \equiv m'^2 + p_\perp^2$$

$$e = \frac{Y_B' - Y_A'}{Y_B - Y_A} \qquad Y_A = -Y_B \qquad Y_B' = Y_A' - 2eY_A$$

Energy momentum conservation

$$(n_A - n_B)\sinh(Y_A) =$$
$$\lambda[(n_A + n_B\cosh(2eY_A))\sinh(Y_A') - n_B\sinh(2eY_A)\cosh(Y_A')]$$
$$(n_A + n_B)\cosh(Y_A) =$$
$$\lambda[(n_A + n_B\cosh(2eY_A))\cosh(Y_A') - n_B\sinh(2eY_A)\sinh(Y_A')]$$

$$\rightarrow$$

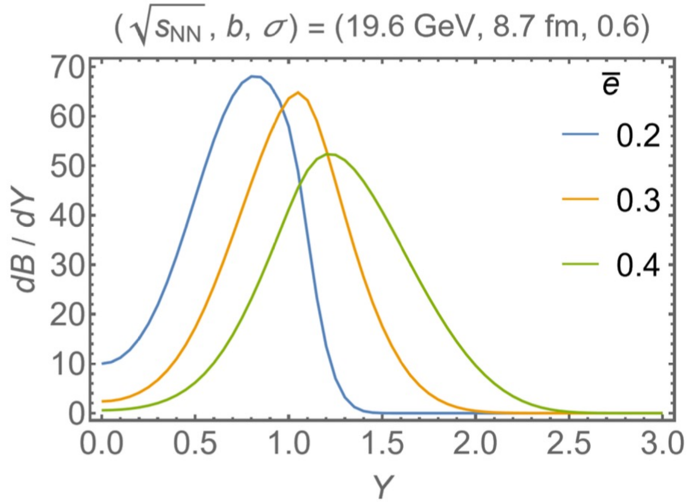
$$Y_A' = \tanh^{-1}\left(\frac{n_A - n_B}{n_A + n_B}\tanh(Y_A)\right) + u$$
$$= \tanh^{-1}\left(\frac{(n_A/n_B) - 1}{(n_A/n_B) + 1}\tanh(Y_A)\right) + \tanh^{-1}\left(\frac{\sinh(2eY_A)}{(n_A/n_B) + \cosh(2eY_A)}\right)$$

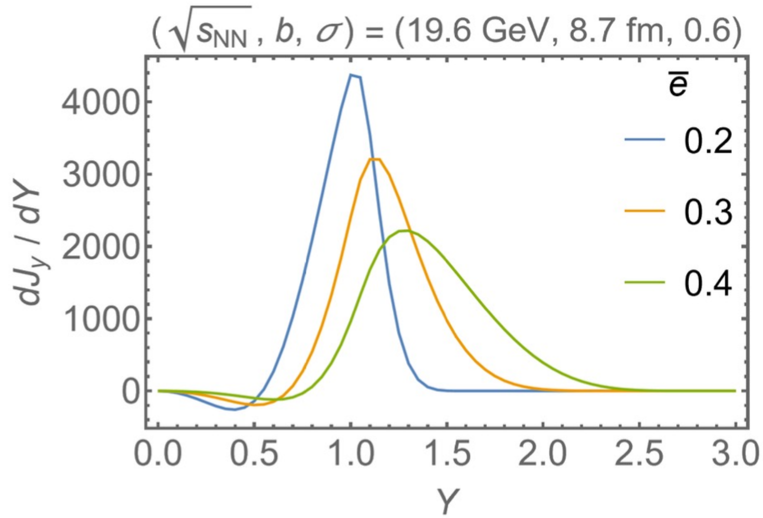
$$\lambda^2 = \frac{\cosh(Y_A - w)\cosh(Y_A + w)}{\cosh(eY_A - w)\cosh(eY_A + w)} \qquad w = \tanh^{-1}\left(\frac{n_A - n_B}{n_A + n_B}\right) = \frac{1}{2}\ln\left(\frac{n_A}{n_B}\right)$$

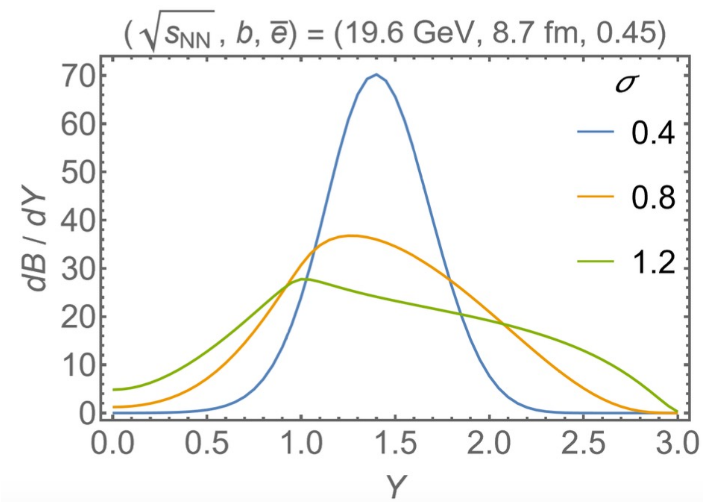
Results depend on three quantities: n_A/n_B, e, and $Y_{\rm beam}$

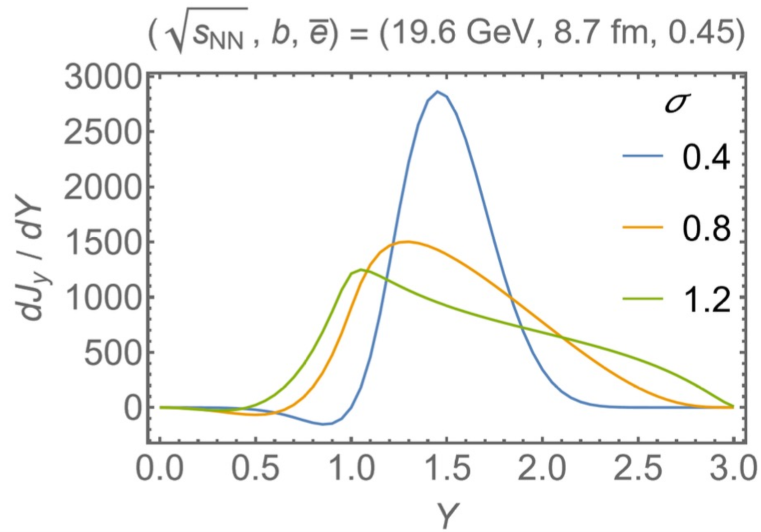
Rapidity Distributions

Net baryon | ***Angular momentum***









A. Akridge, D. Gallimore, H. Morales, JL, arXiv:2504.02192

Rapidity Loss

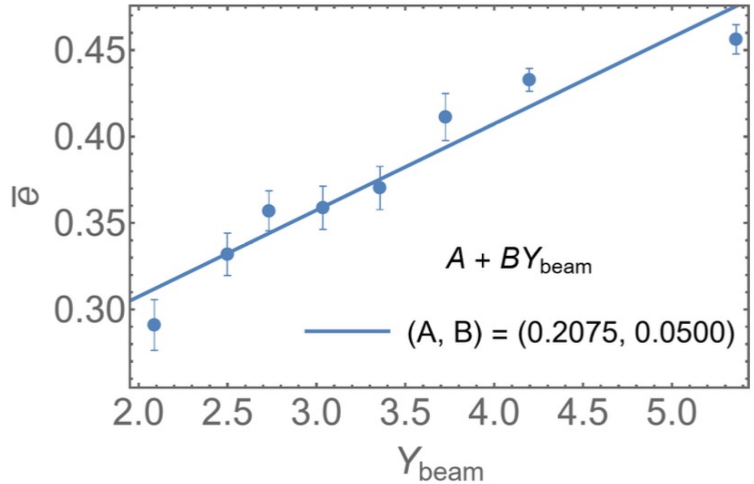
Rapidity distributions of B and J:

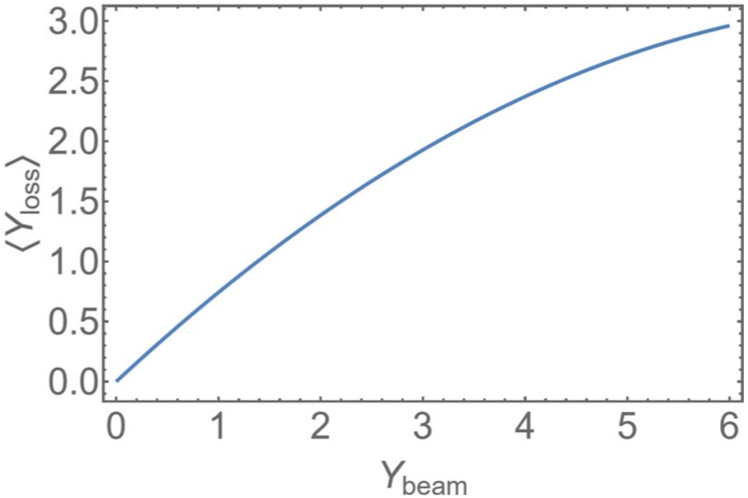
$$\frac{dB}{dY} = \int dxdy\,[n_A\delta(Y - Y'_A) + n_B\delta(Y - Y'_B)]$$

$$\frac{dJ_y}{dY} = -\int dxdy\; x\lambda m_{\mathrm{N}}\,[n_A \sinh(Y'_A)\delta(Y - Y'_A) + n_B \sinh(Y'_B)\delta(Y - Y'_B)]$$

Fluctuations in e is also implemented.

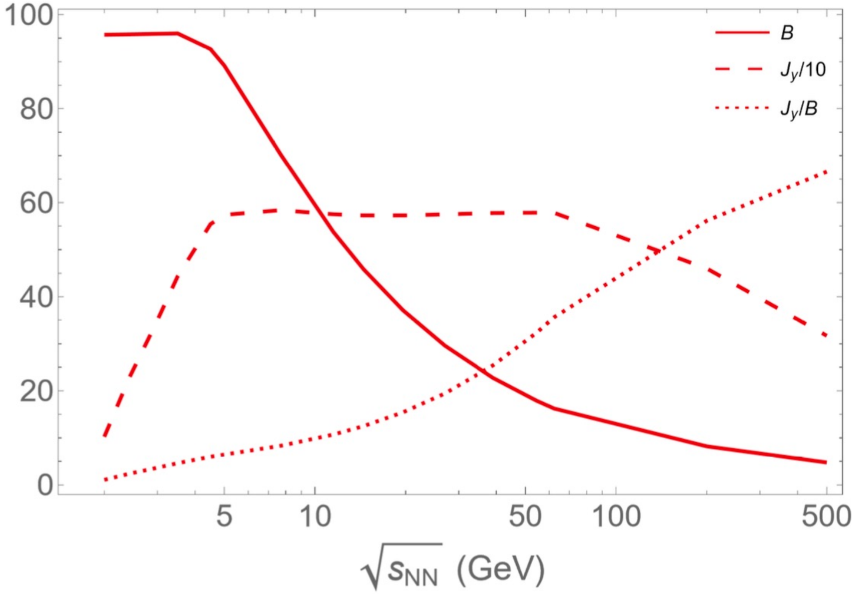
Data for mid-rapidity net baryon is used to calibrate the parameter e at each energy.

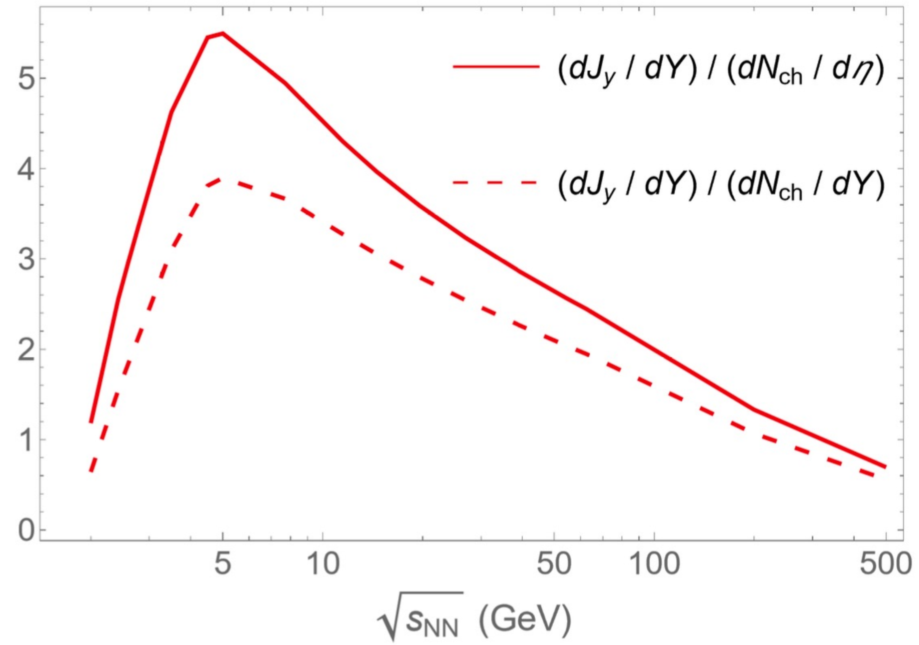




Normalized Initial Angular Momentum

A. Akridge, D. Gallimore, H. Morales, JL, arXiv:2504.02192





Key message: angular momentum driven phenomena unlikely keep increasing toward extremely low energy.

Hydrodynamics with Angular Momentum

Hydrodynamics with Angular Momentum

Phenomenological issues?
How to incorporate the angular momentum into the hydrodynamic framework?
In particular, how to include spin degrees of freedom?

Florkowski, Ryblewski, Kumar, …;
Becattini, Tinti, Buzzegoli, …;
Hattori, Hongo, Huang, …;
Fukushima, Pu;
Shi, Gale, Jeon;
Weickgenannt, Speranza, Sheng, Wang, Rischke;
Liu, Yin, …;
Gallegos, Gursoy, Yarom;
Li, Stephanov, Yee;
……

Relativistic Viscous Hydro with Ang. Mom.

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Perspective

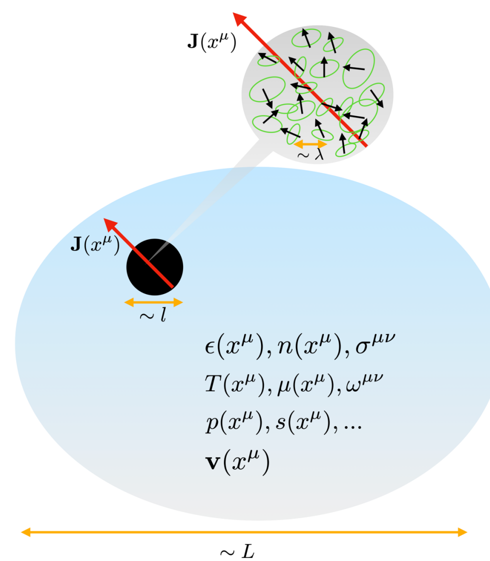
Relativistic viscous hydrodynamics with angular momentum

Duan She [a,b,c,1], Anping Huang [b,d,1], Defu Hou [a,*], Jinfeng Liao [b,*]

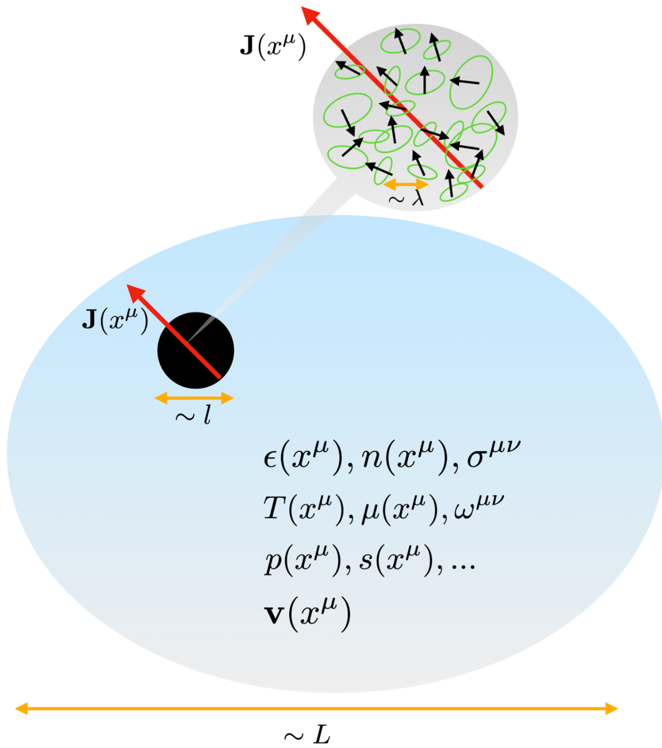
[a] Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
[b] Physics Department and Center for Exploration of Energy and Matter, Indiana University, Bloomington IN 47408, USA
[c] Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China
[d] School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

[arXiv:2105.04060]

A key challenge: the issue of decomposition into orbital and spin components



Goal: Navier-Stokes Program for Ang. Mom.



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0,$$

$$\partial_\mu J^{\mu\alpha\beta} = 0,$$

$$J^{\mu\alpha\beta} = \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}\right) + \Sigma^{\mu\alpha\beta}.$$

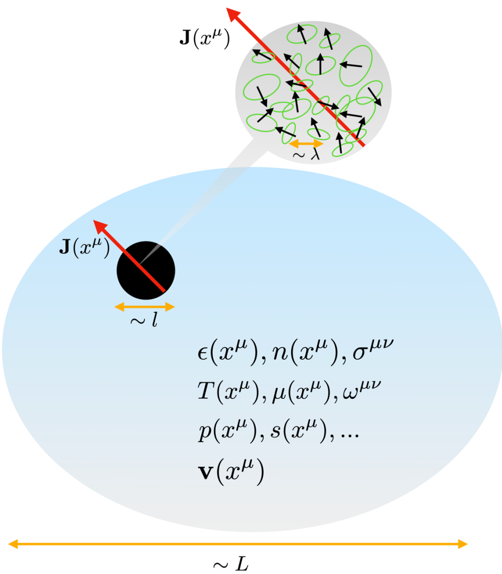
We choose to deal with only the conserved quantities, i.e. angular momentum.

Local angular momentum current $\Sigma^{\mu\alpha\beta}$

Local angular momentum density $\sigma^{\alpha\beta}(x^\mu)$

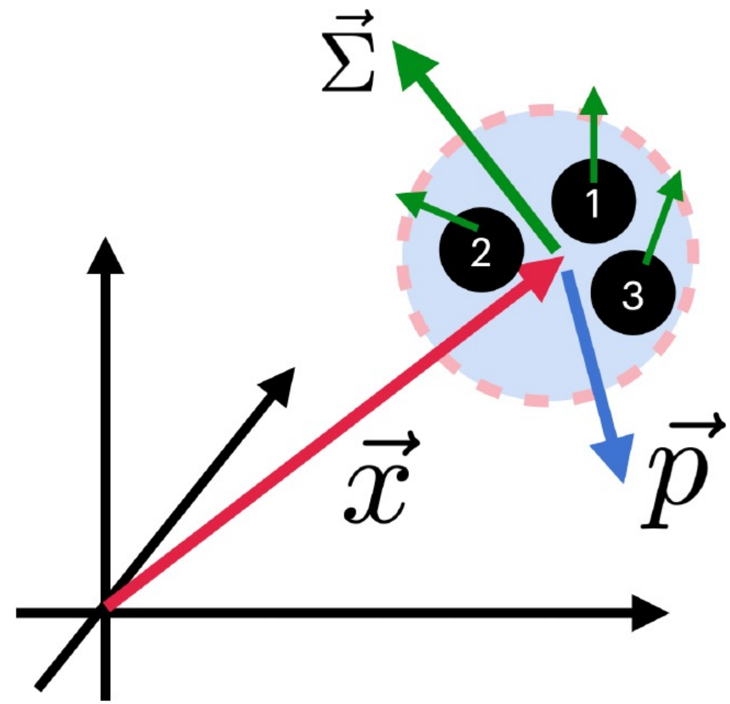
Local angular momentum chemical potential $\omega_{\alpha\beta}(x^\mu)$

Decomposition of Fluid Cell Angular Momentum



$\lambda \ll l \ll L$, a coarse-graining process

$$J^{\mu\alpha\beta} = \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}\right) + \Sigma^{\mu\alpha\beta}\,.$$



$$\vec{\mathbf{J}} = \vec{\mathbf{x}} \times \vec{\mathbf{p}} + \sum_{i=1,2,3} \left(\vec{\mathbf{x}}'_i \times \vec{\mathbf{p}}'_i + \vec{\mathbf{s}}_i\right).$$

$$\vec{\mathbf{\Sigma}} = \sum_{i=1,2,3} \left(\vec{\mathbf{x}}'_i \times \vec{\mathbf{p}}'_i + \vec{\mathbf{s}}_i\right)$$

Conceptually, it may NOT be feasible to further separate the spin part out of the local angular momentum of a coarse-grained fluid cell.

Ideal Hydro with Ang. Mom.

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},\ N^\mu_{(0)} = n u^\mu.$$

$$\Sigma^{\mu\alpha\beta}_{(0)} = \sigma^{\alpha\beta} u^\mu.$$

$$\partial_\mu J^{\mu\alpha\beta}_{(0)} = \sigma^{\alpha\beta}\theta + D\sigma^{\alpha\beta} = 0.$$

Generalized thermodynamics:

$$\epsilon = -p + Ts + \mu n + \omega_{\alpha\beta}\sigma^{\alpha\beta}$$

It is straightforward to verify: no entropy generation

$$\partial_\mu S^\mu_{(0)} = \partial_\mu \left(s u^\mu \right) = 0.$$

Viscous Hydro with Ang. Mom.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \widetilde{T}^{\mu\nu} \,,$$
$$N^\mu = n u^\mu + \widetilde{N}^\mu \,,$$
$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + \widetilde{\Sigma}^{\mu\alpha\beta} \,,$$
$$S^\mu = s u^\mu + \widetilde{S}^\mu .$$

Let us first focus on entropy current:

$$S^\mu = p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}$$

Leading order:

$$\partial_\mu S^\mu_{(0)} = 2\beta\omega_{\alpha\beta}\widetilde{T}^{\alpha\beta}_{(a)} . \quad \Rightarrow \quad \widetilde{T}^{\alpha\beta}_{(a)} = 0.$$

Next order (2nd-gradient):

$$\begin{aligned} \partial_\mu S^\mu &= \partial_\mu \left(p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta} \right) \\ &= \widetilde{T}^{\mu\nu}\partial_\mu\beta_\nu - \widetilde{N}^\mu \partial_\mu \alpha - \widetilde{\Sigma}^{\mu\alpha\beta}\partial_\mu \left(\beta\omega_{\alpha\beta} \right) , \end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

Viscous Hydro with Ang. Mom.: Eckart Frame

$$u_E^\mu = \frac{N^\mu}{\sqrt{N^\nu N_\nu}},$$

Write down all allowed Lorentz structures to the correct order of gradient expansion

$$\begin{aligned} T^{\mu\nu} &= \epsilon u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + 2u^{(\mu}q^{\nu)} + \pi^{\mu\nu}, \\ N^\mu &= n u^\mu, \\ \Sigma^{\mu\alpha\beta} &= u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha}\Delta^{\mu\beta]}\Phi \\ &\qquad + 2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}. \end{aligned}$$

Plug these into the entropy current divergence and look for conditions of positivity:

$$\begin{aligned} \partial_\mu S^\mu &= \partial_\mu\left(p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}\right) \\ &= \widetilde{T}^{\mu\nu}\partial_\mu\beta_\nu - \widetilde{N}^\mu\partial_\mu\alpha - \widetilde{\Sigma}^{\mu\alpha\beta}\partial_\mu\left(\beta\omega_{\alpha\beta}\right), \end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

Viscous Hydro with Ang. Mom.: Eckart Frame

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + 2u^{(\mu} q^{\nu)} + \pi^{\mu\nu},$$
$$N^\mu = n u^\mu,$$
$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha}\Delta^{\mu\beta]}\Phi + 2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.$$

$$\partial_\mu S^\mu \geq 0.$$

$$\Pi = -\zeta\theta,$$
$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle},$$
$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right) = -\frac{\lambda n T^2}{\epsilon+p}\left[\nabla^\mu\left(\frac{\mu}{T}\right) + \frac{\sigma^{\alpha\beta}}{n}\nabla^\mu\left(\frac{\omega_{\alpha\beta}}{T}\right)\right]$$

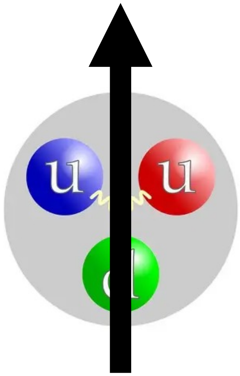
$$\boxed{\begin{aligned}
\Phi &= -\chi_1 u^\alpha \nabla^\beta\left(\frac{\omega_{\alpha\beta}}{T}\right), && (25)\\
\tau_{(s)}^{\mu\beta} &= -\chi_2 u^\alpha \Big[\left(\Delta^{\beta\rho}\Delta^{\mu\gamma} + \Delta^{\mu\rho}\Delta^{\beta\gamma}\right) - \frac{2}{3}\Delta^{\mu\beta}g^{\rho\gamma}\Big]\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right) && (26)\\
\tau_{(a)}^{\mu\beta} &= -\chi_3 u^\alpha\left(\Delta^{\beta\rho}\Delta^{\mu\gamma} - \Delta^{\mu\rho}\Delta^{\beta\gamma}\right)\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right), && (27)\\
\Theta^{\mu\alpha\beta} &= -\chi_4\left(u^\beta u^\rho \Delta^{\alpha\delta} - u^\alpha u^\rho \Delta^{\beta\delta}\right)\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right) + \chi_5 \Delta^{\alpha\delta}\Delta^{\beta\rho}\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right). && (28)
\end{aligned}}$$

Five new positive angular momentum transport coefficients:

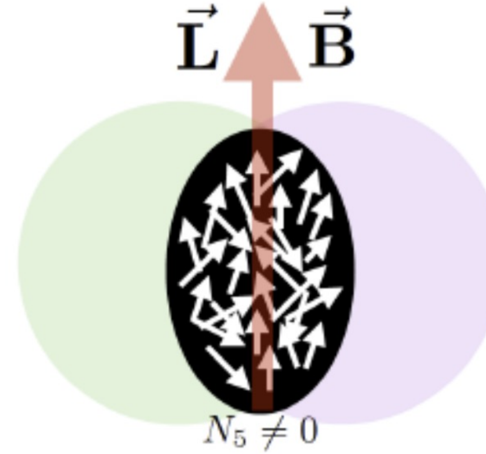
$\chi_1, \chi_2, \chi_3, \chi_4$ and χ_5

Summary

Summary

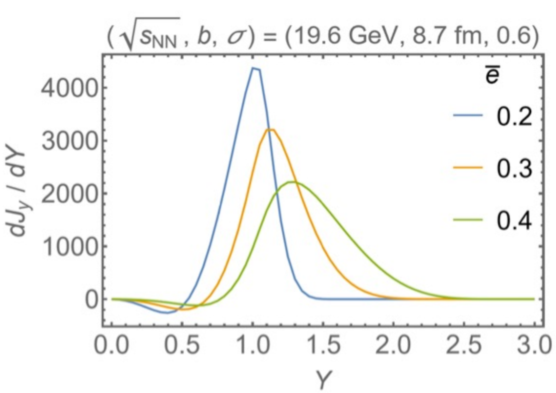


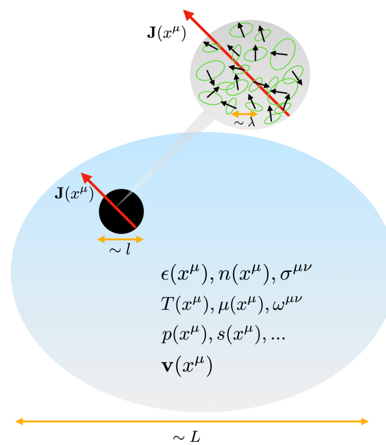
Nontrivial interplay between angular momentum and QCD: Common challenges from proton to QGP!

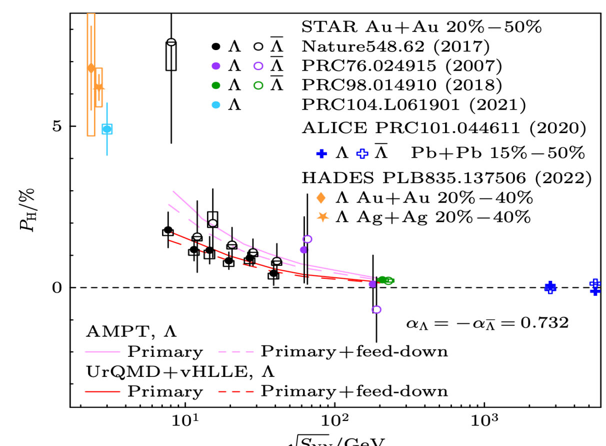


Angular momentum transport in heavy-ion collisions:

Initial conditions → **Hydrodynamic evolution** → **Final state observables**

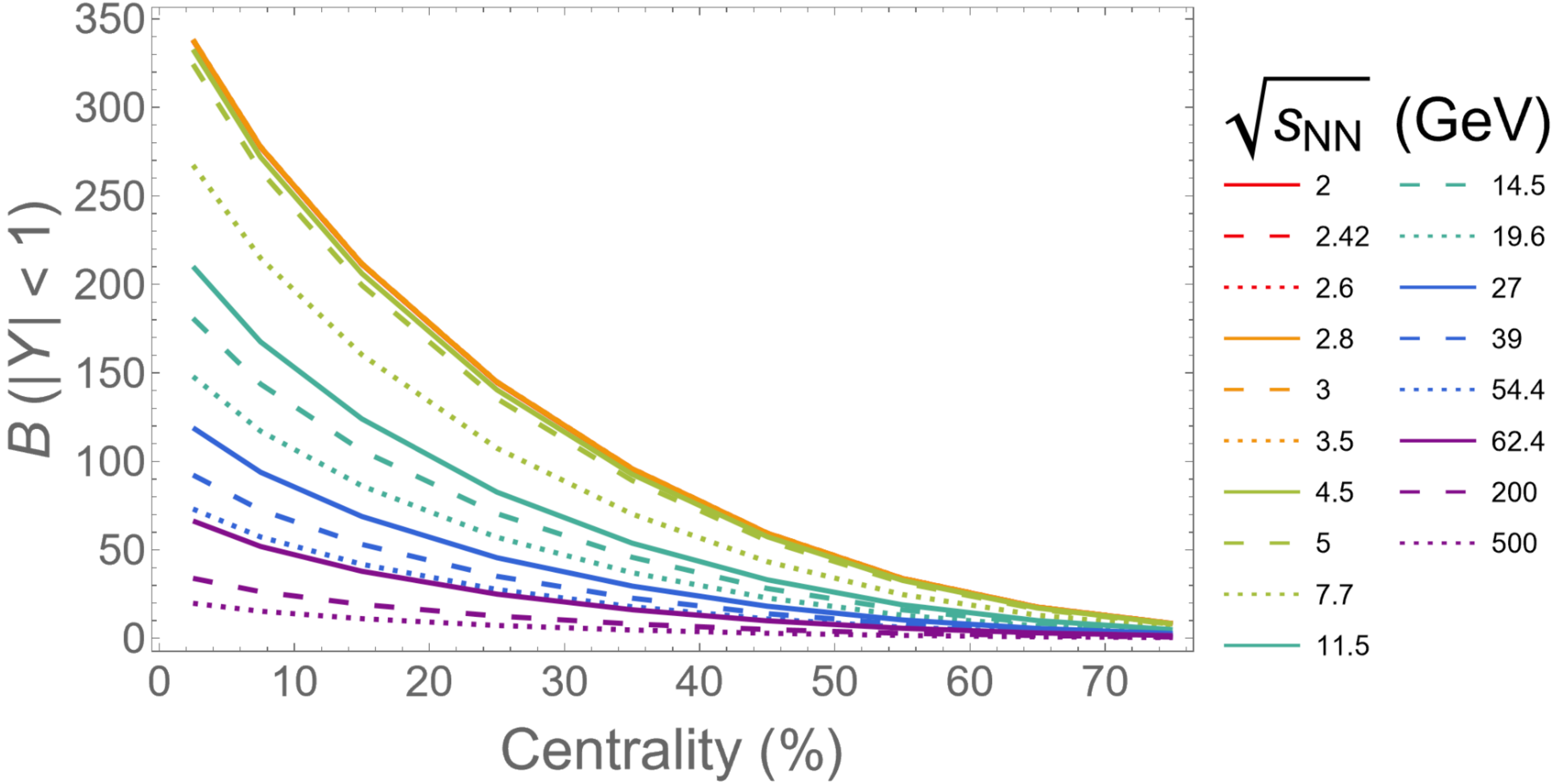






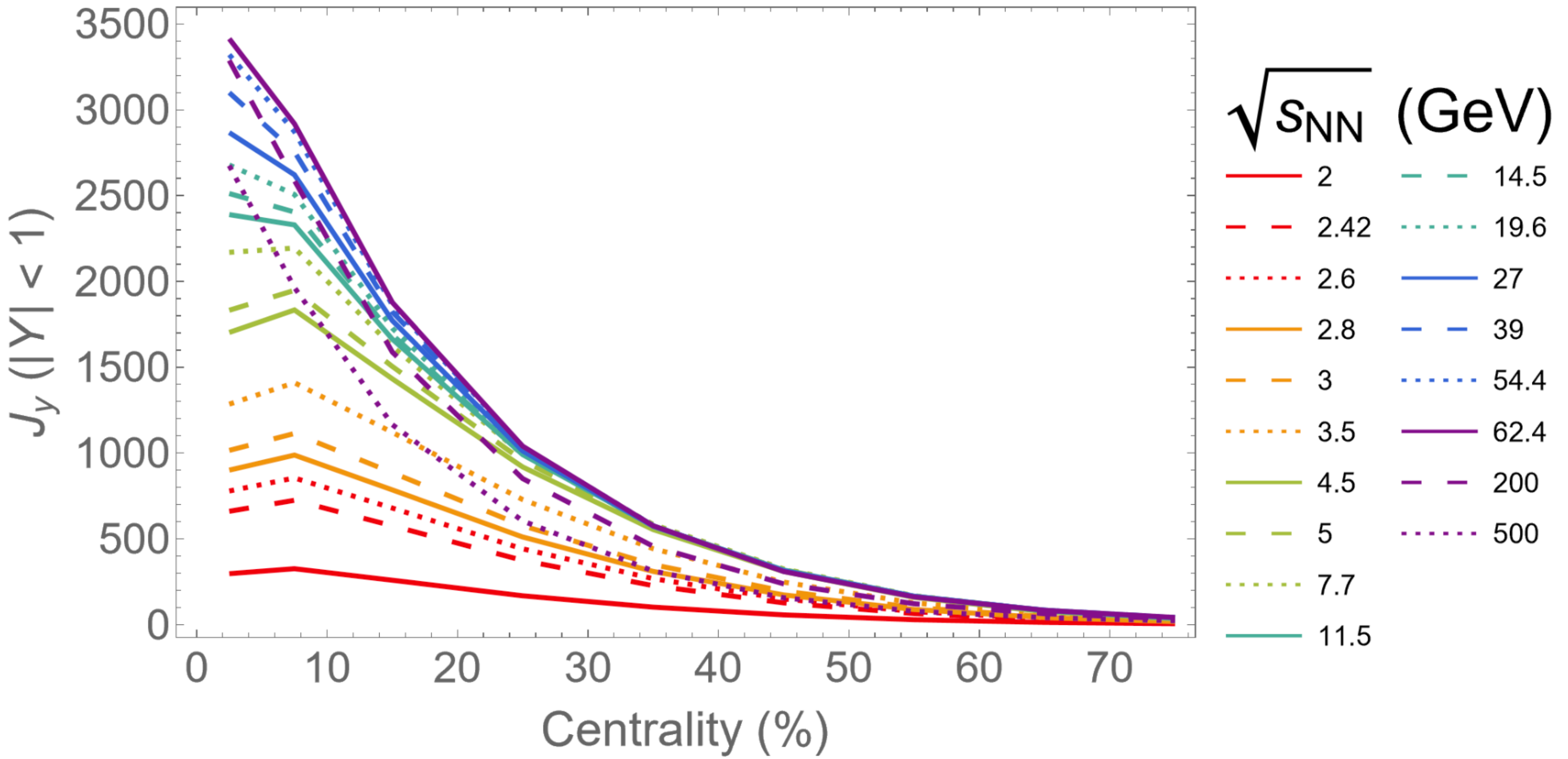
Backup Slides

Initial Net Baryon Number



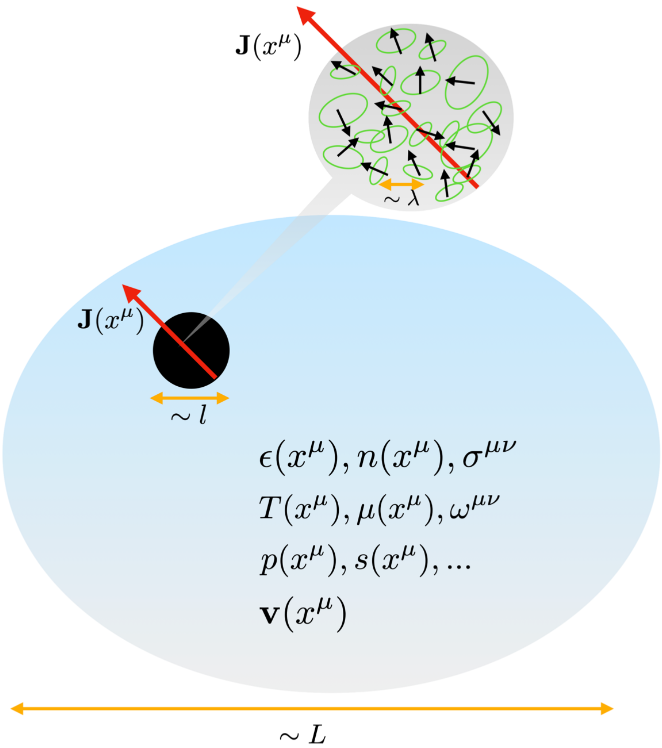
A. Akridge, D. Gallimore, H. Morales, JL, arXiv:2504.02192

Initial Angular Momentum



A. Akridge, D. Gallimore, H. Morales, JL, arXiv:2504.02192

Ideal Hydrodynamics



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0,$$

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},\ N^\mu_{(0)} = n u^\mu.$$

$$\epsilon = -p + Ts + \mu n$$

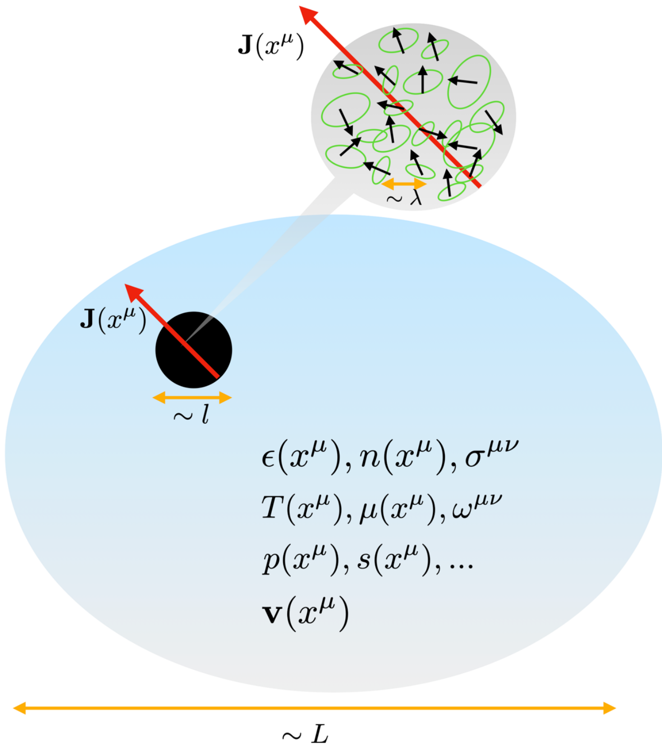
$$S^\mu_{(0)} = s u^\mu$$

$$\partial_\mu S^\mu_{(0)} = \partial_\mu \left(s u^\mu\right) = 0$$

Microscopic physics enters via thermodynamic relations (i.e. EOS).

See e.g. Landau and Lifshitz

Navier-Stokes Viscous Hydrodynamics



$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0,$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \widetilde{T}^{\mu\nu},$$
$$N^\mu = n u^\mu + \widetilde{N}^\mu,$$
$$S^\mu = s u^\mu + \widetilde{S}^\mu.$$

$$\partial_\mu S^\mu \geq 0.$$

$\lambda \ll l \ll L$, a coarse-graining process

See e.g. Landau and Lifshitz

$$\Pi = -\zeta\theta,$$
$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle},$$
$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right)$$

Microscopic physics also enters via transport coefficients in viscous terms.

Viscous Hydro with Ang. Mom.: Landau Frame

$$u_L^\mu = \frac{T^\mu_\nu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$$

$$T^{\mu\nu} = \epsilon u_L^\mu u_L^\nu - (p+\Pi)\,\Delta_L^{\mu\nu} + \pi^{\mu\nu},$$

$$N^\mu = n u_L^\mu - n\frac{q^\mu}{\epsilon + p},$$

$$\Sigma^{\mu\alpha\beta} = u_L^\mu \sigma^{\alpha\beta} - \frac{q^\mu}{\epsilon+p}\sigma^{\alpha\beta} + 2u_L^{[\alpha}\Delta_L^{\mu\beta]}\Phi$$
$$+2u_L^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u_L^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.$$

Following the same procedure, one obtains essentially the same consistent results.