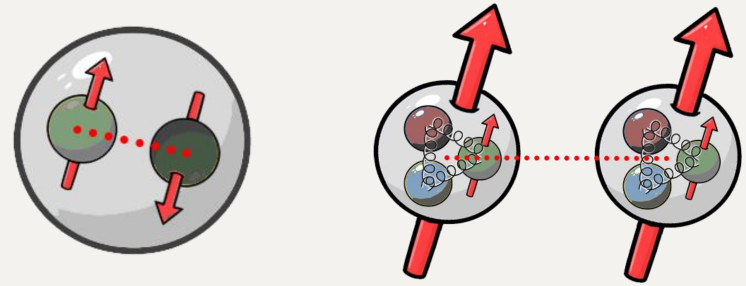


从强场涨落到
矢量介子自旋排列与超子自旋关联

Xin-Li Sheng
盛欣力

第一届强相互作用自旋物理会议
青岛
2026年7月27-30日

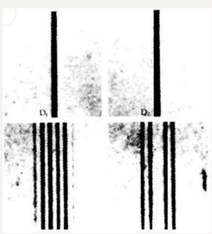
Early studies on spin

- Spin is the intrinsic angular momentum of a particle

  Spin ≠ Rotating around an axis

反常塞曼效应
Thomas Preston

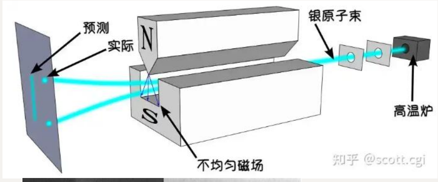
1897



Splitting of atomic spectra in an external magnetic field (spin-magnetic coupling)

Stern-Gerlach实验
Otto Stern, Walther Gerlach

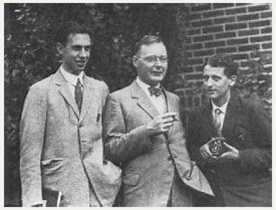
1922



1943

提出电子自旋
George Uhlenbeck, Samuel Goudsmit

1925



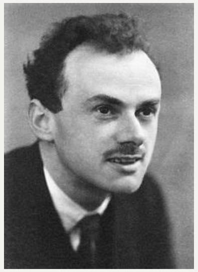
自旋泡利矩阵
Wolfgang Pauli

1927

1945

相对论性电子的
狄拉克旋量
Paul Dirac

1928



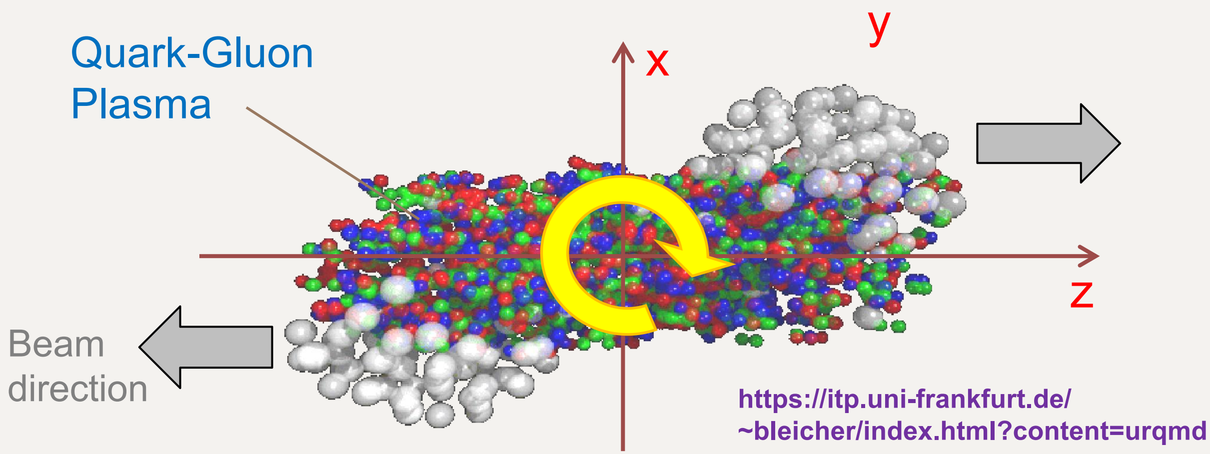
1933

Heavy-ion collisions

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]

Strongly interacting matter with vorticity and magnetic fields



Initial orbital angular momentum → Vorticity field, Magnetic field, Strong field → Polarized quark/gluon → Spin polarization for baryons, Λ, Σ^0, Δ^{++}, Ω^-, …

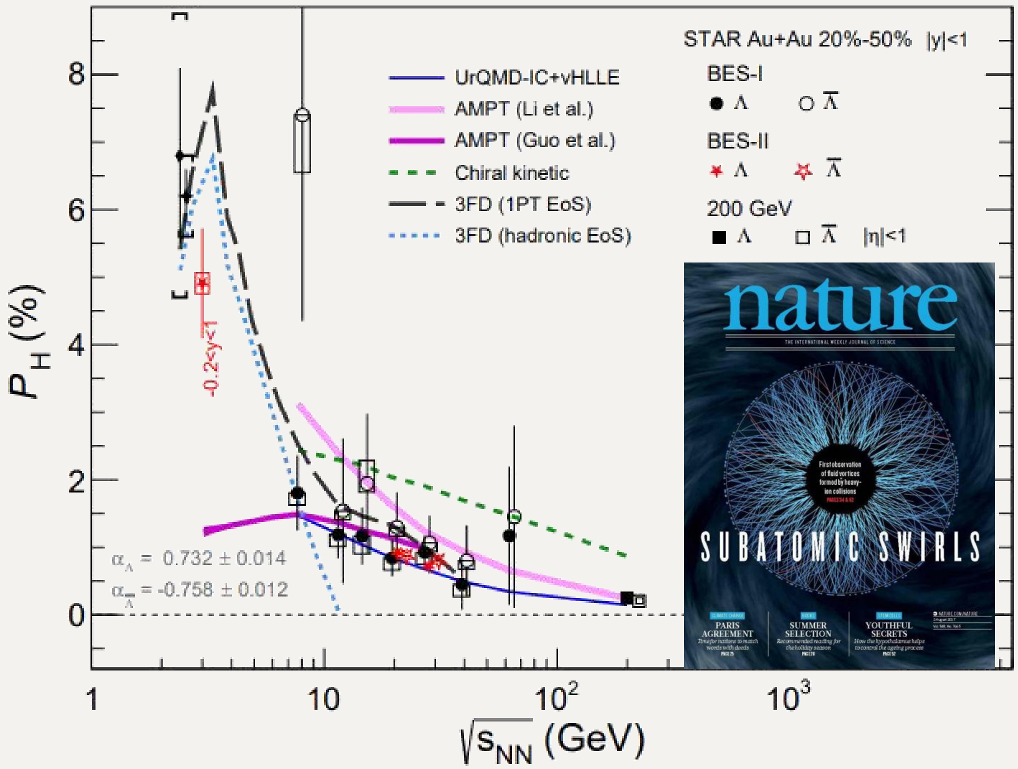
Spin alignment for vector mesons, ϕ, K^{*0}, ρ^0, J/ψ, …

Z.-T. Liang, X.-N. Wang, PRL 94, 102301 (2005); PLB 629, 20 (2005)

S. A. Voloshin, nucl-th/0410089

F. Becattini, F. Piccinini, J. Rizzo, PRC 76, 044901 (2007)

Fermion spin polarization



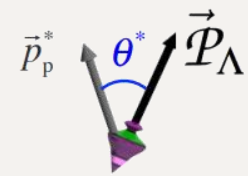
STAR, Nature 548, 62 (2017); Phys. Rev. C 104, L061901 (2021); Phys. Rev. C 98, 014910 (2018);Phys. Rev. C 108, 014910 (2023)

F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A.-H. Tang, Int. J. Mod. Phys. E 33, 2430006 (2024).

Polarization along direction of global OAM

Evidence of vorticity field !

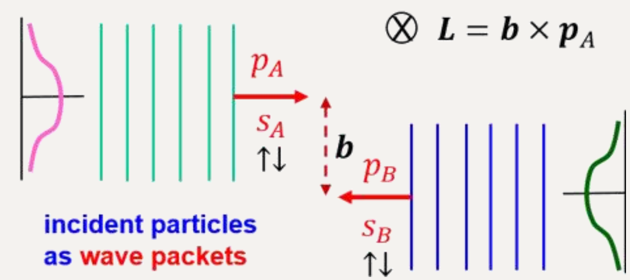
Measured by pairty-violating weak decay



$$\Lambda \to p + \pi^-$$

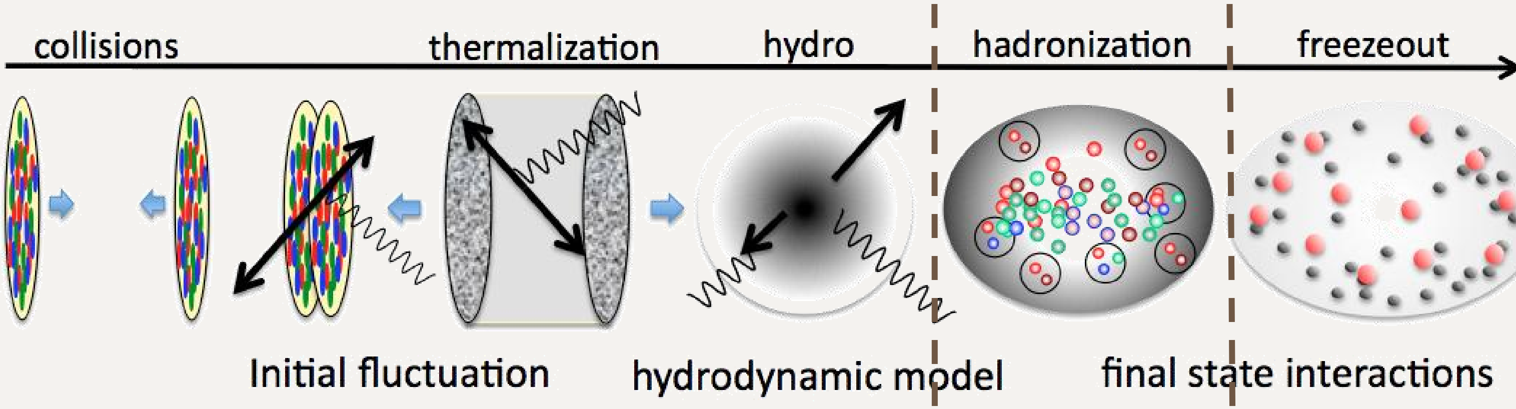
$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*\right)$$

Microscopically, OAM converts to spin through non-local scatterings between partons



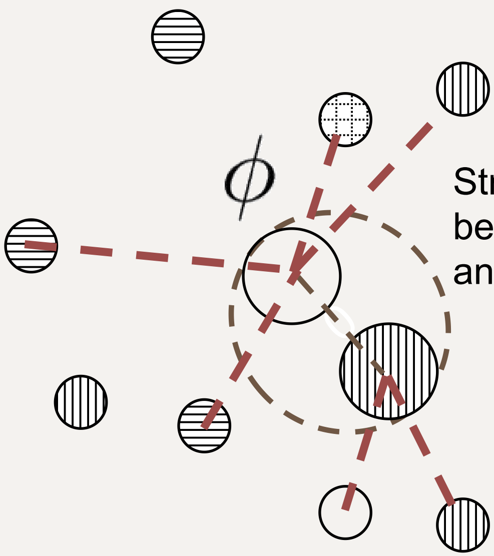
J.-J. Zhang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 100, 064904 (2019)

Strong field



Decreasing T

Theories for strong interaction:	pQCD	Quark-meson model	Hadron gas model



Strong interaction between medium and constituent $q/\bar{q}$

Quark effective mass

$$\mathcal{L}_{\rm eff}(x) = \bar{\psi}(x)\left[i\partial\cdot\gamma - \boxed{(m_0 + g_\sigma\sigma)} - g_V\gamma\cdot V\right]\psi(x) + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{4}V^{\mu\nu}V_{\mu\nu}$$

Vector meson field

$$\begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \boxed{\phi} \end{pmatrix}$$

Nine species of vector mesons

Short wave-length: quantum fields (particles)

Long wave-length: classical fields

Quark polarization

- Polarizations of $s/\bar{s}$ in a local thermal equilibrium system

$$P^{\mu}_{s/\bar{s}}(x,p) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}p_\nu\left[\omega_{\rho\sigma} \pm \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} \pm \frac{g_\phi}{(u\cdot p)T}F^{\phi}_{\rho\sigma}\right] + \cdots$$

Thermal vorticity field (rotation and acceleration)

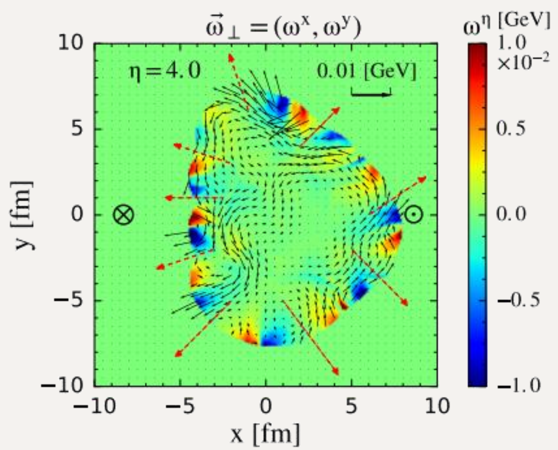
Classical electromagnetic field

Vector ϕ field

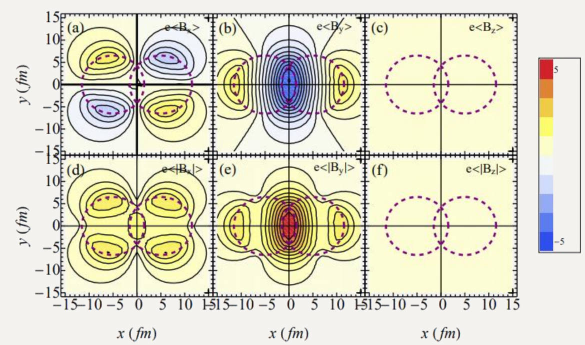
$$\partial_\mu F^{\mu\nu}_{\phi} + m^2_\phi \phi^\nu = J^\nu_s$$

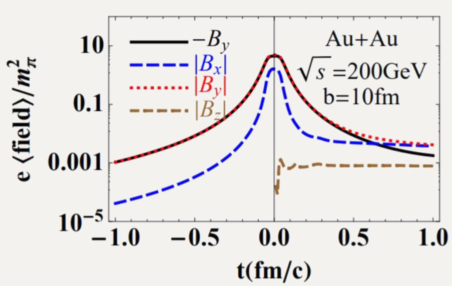
$$\phi^\mu \approx -(g_\phi/m^2_\phi)J^\mu_s$$

Current-field identity



L.-G. Pang, H. Petersen, Q. Wang, X.-N. Wang, PRL 117, 192301





W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012).

Vector ϕ field could be generated by fluctuating strangeness current

J. J. Sakurai, Annals Phys. 11, 1 (1960)

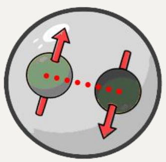
N. M. Kroll, T. D. Lee, B. Zumino, Phys. Rev. 157, 1376 (1967)

L. P. Csernai, J. I. Kapusta, T. Wellwe, PRC 99, 2 (2019)

XLS, L. Oliva, Q. Wang, PRD 101, 9 (2020)

Vector meson spin alignment

- A meson's spin is determined by its constituent q & $\bar{q}$
  Z.-T. Liang & X.-N. Wang, PLB 629, 20 (2005)



q-$\bar{q}$ spin correlation

- Spin density matrix for a vector meson ($J^P = 1^-$)

$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

$$= \frac{1}{3} + \frac{1}{2} P_i \Sigma_i + T_{ij} \Sigma_{ij}$$

Spin alignment
=1/3 for unpolarized meson

Vector polarization
(3 components, not measurable, parity odd)

Tensor polarization
(5 components, measurable, pairty even)

Parity-conserving decays: distribution determined by angular momentum conservation

Processes	Examples	Polar angle distribution $W(\theta)$	Spin is converted to
Strong p-wave decay	$K^{*0} \to K^+ + \pi^-$ $\phi \to K^+ + K^-$	$\frac{3}{4}\left[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta\right]$	OAM
Dilepton decay	$J/\psi \to \mu^+ + \mu^-$	$\frac{3}{8}\left[1 + \rho_{00} + (1 - 3\rho_{00})\cos^2\theta\right]$	Spin

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)

ϕ meson spin alignment

- Spin alignment of the ϕ meson in its rest frame measuring along the direction of ϵ_0

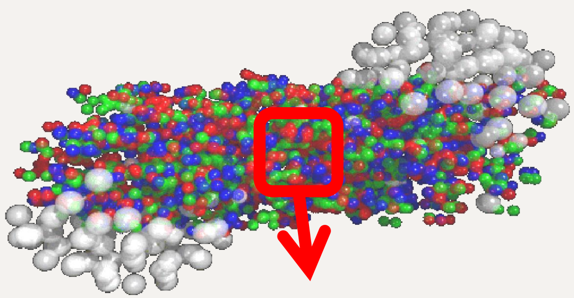
$$\left.\begin{aligned}\rho_{00} \approx\ & \frac{1}{3} - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0 \cdot \mathbf{B}'_\phi)^2\right] \\ & - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_2 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0 \cdot \mathbf{E}'_\phi)^2\right]\end{aligned}\right\}$$

Temperature at hadronization time

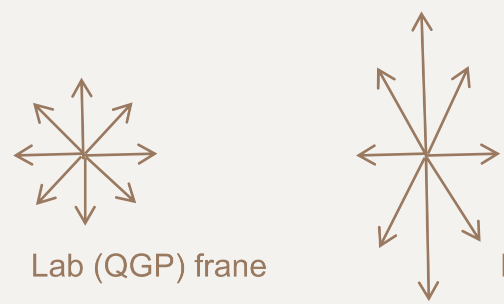
Vector ϕ field:
mean value is zero, but can incorporate large fluctuations

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)}.$$

- Spin alignment measures anisotropy of fluctuations in meson's rest frame



$$\left\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_\mathrm{h}^2 \right\rangle = \left\langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_\mathrm{h}^2 \right\rangle$$
$$= \underbrace{F^2 \delta^{ij}}_{\text{Isotropic}} + \underbrace{\Delta \hat{\mathbf{a}}^i \hat{\mathbf{a}}^j}_{\text{Anisotropy of QGP}}$$

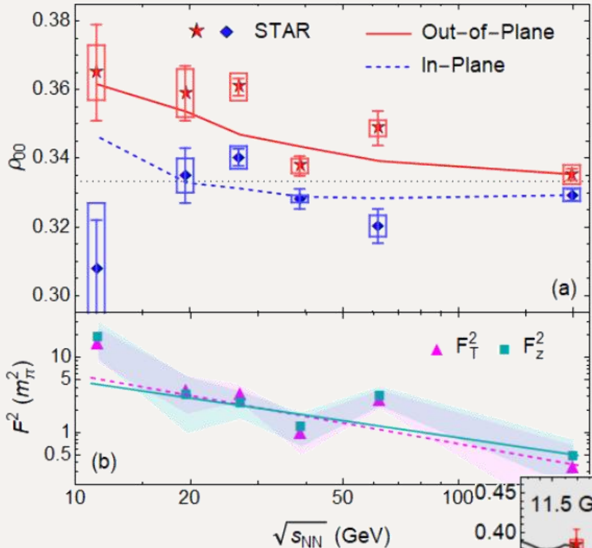


Motion-induced anisotropy

$$\rho_{00}^y - \frac{1}{3} \propto \frac{1}{3}\mathbf{p}\cdot\mathbf{p} - p_y^2$$

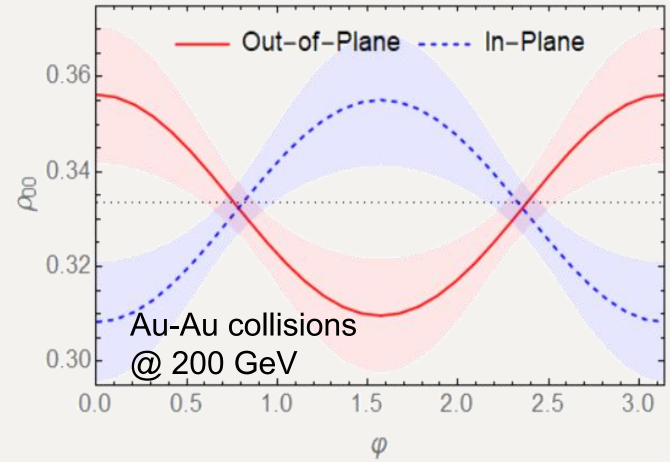
XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024).

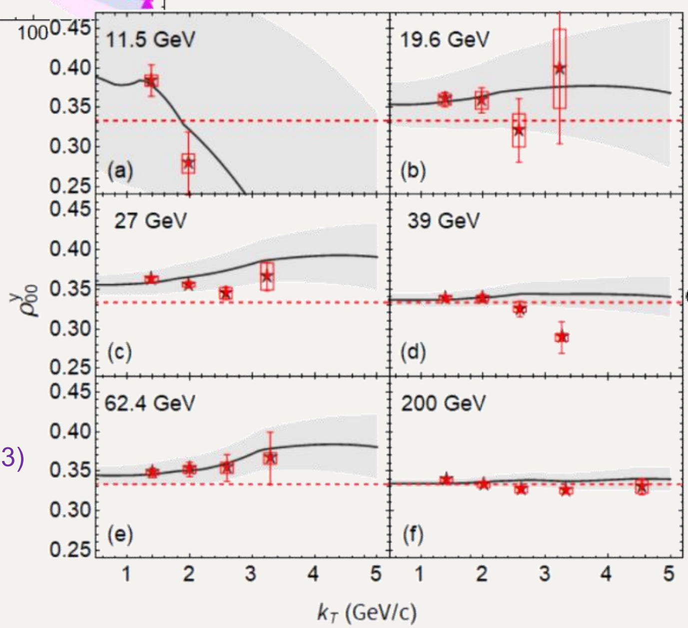
Vector meson spin alignment

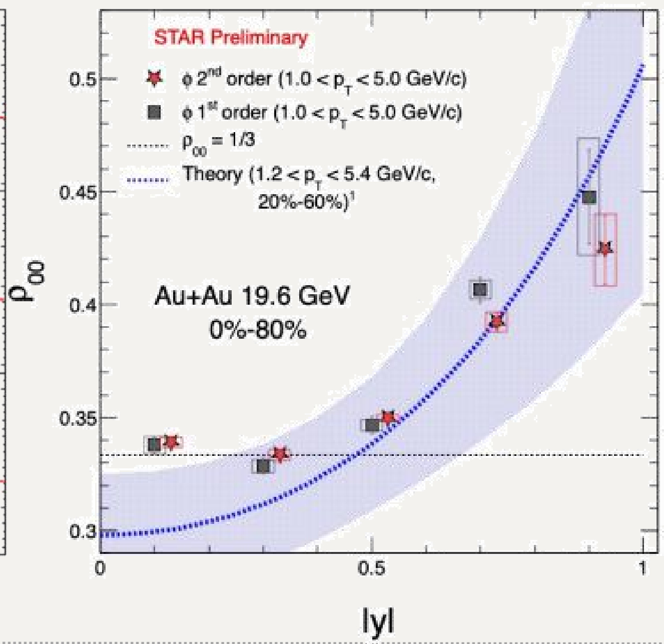


Extracting transverse and longitudinal fluctuations by fitting STAR data

Predictions for differential measurements







XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023)

XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023).

Tensor polarization

- For a massive particle, all possible Lorentz transforms which leave rest-frame momentum $p^{\mu}_{\text{rest}} = (m, 0, 0, 0)$ invariant form Wigner little group [$SO(3)$]

- Transform of spin density matrix

$$\rho^{\mathbf{n}}(p) \to \rho^{\mathbf{n}'}(p') = D[R(\Lambda, p)]\rho^{\mathbf{n}}(p)D^{\dagger}[R(\Lambda, p)]$$

Linear operator on spin-1 representation space of $SO(3)$

Irreducible decomposition

$$1 \otimes 1 = 0 \oplus 1 \oplus 2$$

$$P_i^{\mathbf{n}} \to P_i^{\mathbf{n}'} = R_{ij} P_j^{\mathbf{n}}$$

Vector polarization

$$\mathcal{T}_{ij}^{\mathbf{n}} \to \mathcal{T}_{ij}^{\mathbf{n}'} = R_{ik}R_{jl}\mathcal{T}_{kl}^{\mathbf{n}}$$

Tensor polarization (symmetric and traceless)

- Tensor polarization as a response to source field
  - Vector field: $V_i V_j - \frac{1}{3}\delta_{ij}\mathbf{V}^2$
  - Tensor field: T_{ij}
  - Two vector fields
    $\frac{1}{2}(V_i^{(1)}V_j^{(2)} + V_j^{(1)}V_i^{(2)}) - \frac{1}{3}\delta_{ij}\mathbf{V}^{(1)}\cdot\mathbf{V}^{(2)}$
  - One vector field and one tensor field
    $\epsilon_{ikl}V_k T_{lj} + \epsilon_{jkl}V_k T_{li}$

Type	Examples
Scalar	$T,\ \mu_B,\ \epsilon,\ P,\ \partial_\mu u^\mu,\ G^a_{\mu\nu}G^{a\mu\nu}$
Pseudoscalar	$\mu_5,\ \mathbf{V}\cdot\mathbf{A}$
Polar vector	$\mathbf{u},\ \boldsymbol{\nabla}T,\ \boldsymbol{\nabla}\mu,\ \mathbf{a},\ \mathbf{E},\ \mathbf{E}^a$
Axial vector	$\boldsymbol{\varpi},\ \mathbf{B},\ \mathbf{B}^a$
Pairty-even tensor	$\sigma_{(ij)},\ V_{(i}^{(1)}V_{j)}^{(2)},\ A_{(i}^{(1)}A_{j)}^{(2)}$
Pairty-odd tensor	$V_{(i}A_{j)}$

* Fields in meson's rest frame

XLS, S. Lin, in preparation

Off-diagonal elements

- Tensor polarization of vector meson gives more information about anisotropy of strong field fluctuations (B_ϕ-field as an example)

$$\rho_{00}^{\mathbf{z}} - \frac{1}{3} \propto \left\langle B_z'^2 - \frac{B_x'^2 + B_y'^2}{2} \right\rangle$$

Difference between z-z correlation and x-x (y-y) correlation

$$\mathrm{Re}\rho_{1,-1}^{\mathbf{z}} \propto \langle B_x'^2 - B_y'^2 \rangle$$

Difference between x-x correlation and y-y correlation

$$-\mathrm{Im}\rho_{1,-1}^{\mathbf{z}} \propto \langle B_x' B_y' \rangle$$

x-y correlation

$$\mathrm{Re}(\rho_{10}^{\mathbf{z}} - \rho_{0,-1}^{\mathbf{z}}) \propto \langle B_x' B_z' \rangle$$

x-z correlation

$$-\mathrm{Im}(\rho_{10}^{\mathbf{z}} - \rho_{0,-1}^{\mathbf{z}}) \propto \langle B_y' B_z' \rangle$$

y-z correlation

- Instead of extracting off-diagonal elements from $W(\theta, \phi)$, one can measure ρ_{00} in different directions and make linear combinations

$$\boxed{\rho^{\mathbf{n}'} = D^\dagger(R) \rho^{\mathbf{n}} D(R)}$$

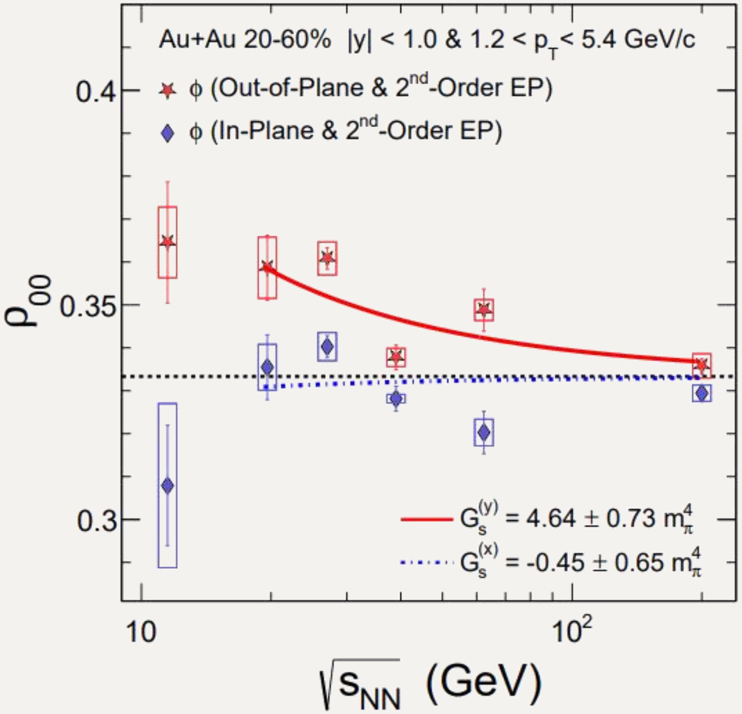
$$\boxed{\begin{aligned}
\mathrm{Re}\rho_{1,-1}^{\mathbf{z}} &= \frac{1}{2}\rho_{00}^{\mathbf{z}} + \rho_{00}^{\mathbf{y}} - \frac{1}{2} \\
-2\mathrm{Im}\rho_{1,-1}^{\mathbf{z}} &= 1 - 2\rho_{00}^{(\mathbf{x}+\mathbf{y})/\sqrt{2}} - \rho_{00}^{\mathbf{z}} \\
\sqrt{2}\mathrm{Re}(\rho_{10}^{\mathbf{z}} - \rho_{0,-1}^{\mathbf{z}}) &= 1 - 2\rho_{00}^{(\mathbf{x}+\mathbf{z})/\sqrt{2}} - \rho_{00}^{\mathbf{y}} \\
-\sqrt{2}\mathrm{Im}(\rho_{10}^{\mathbf{z}} - \rho_{0,-1}^{\mathbf{z}}) &= 1 - 2\rho_{00}^{(\mathbf{y}+\mathbf{z})/\sqrt{2}} - \rho_{00}^{\mathbf{x}}
\end{aligned}}$$

XLS, S. Lin, in preparation

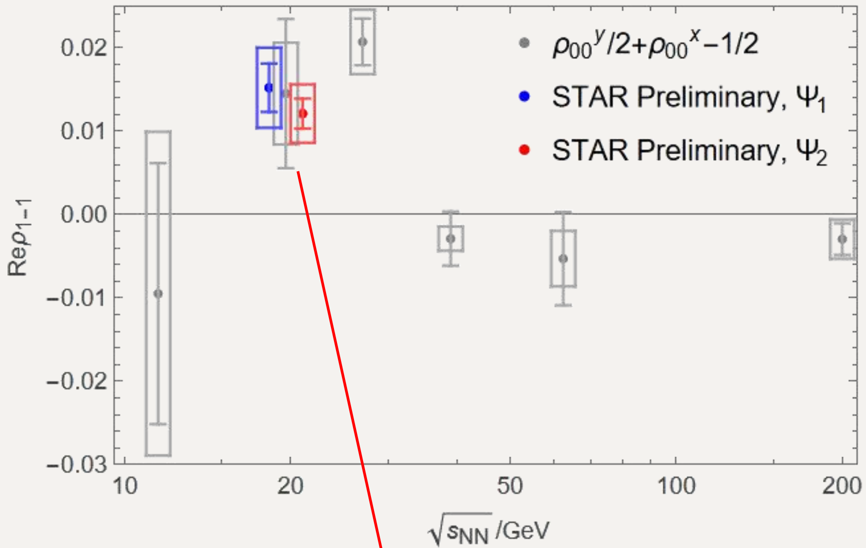
Off-diagonal elements

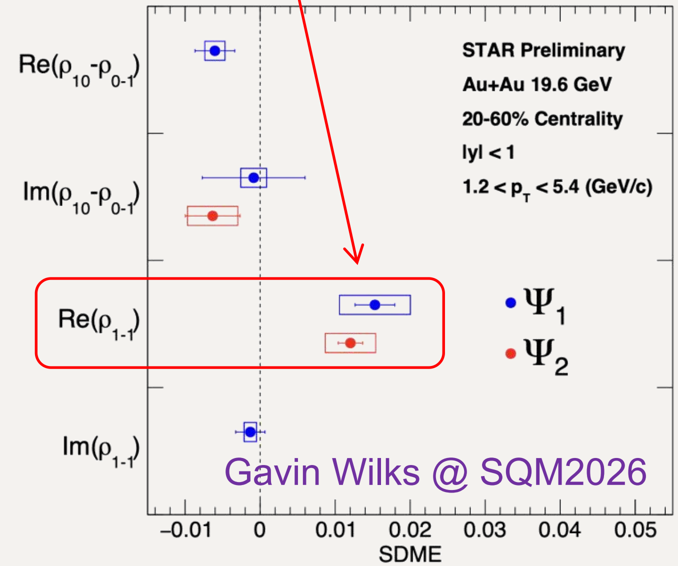
- Deriving off-diagonal element and spin alignment in beam direction using present data

$$\mathrm{Re}\rho^{\hat{\mathbf{y}}}_{1,-1} = \frac{1}{2}\rho^{\hat{\mathbf{y}}}_{00} + \rho^{\hat{\mathbf{x}}}_{00} - \frac{1}{2}$$



STAR, Nature 614, 244 (2023)





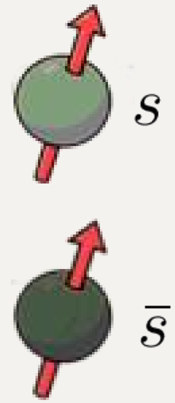
Gavin Wilks @ SQM2026

Hyperon spin correlation

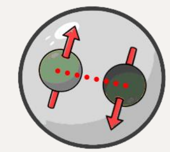
- Vector meson spin alignment vs. hyperon spin correlation

Produced from the same g or γ^* or vacumm spin-triplet state (pp collisions)

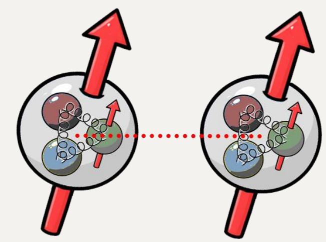
Interaction with QGP (AA collisions)



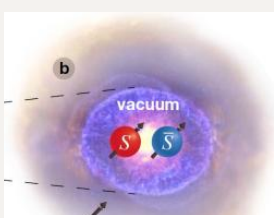
Short range ($\lesssim$ meson size)



Long/short range



- Hyperon spin correlation in
  STAR: p+p collisions
  CMS: p+p (p+Pb) collisions

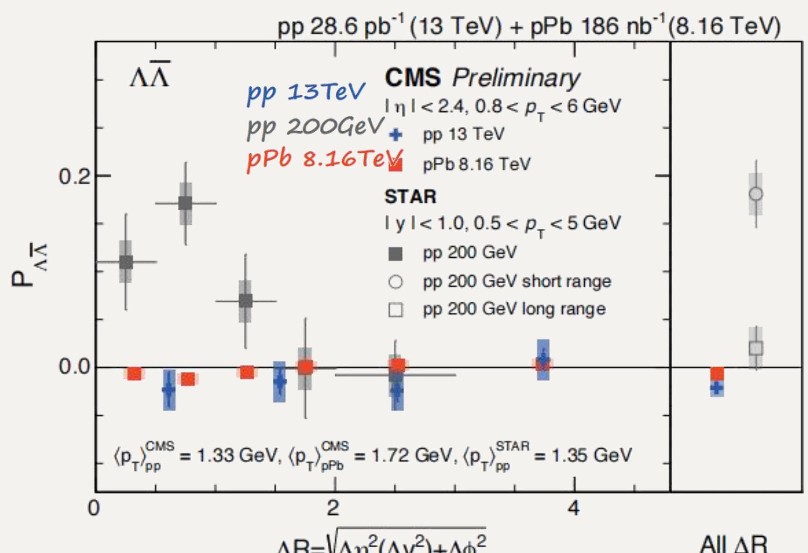


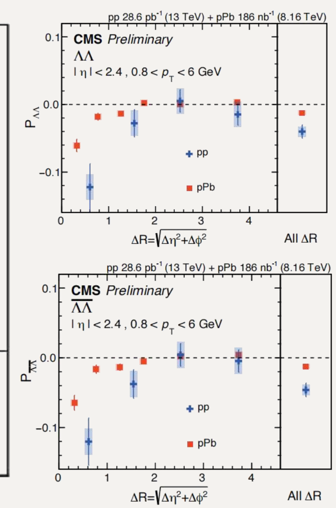
$J^P = 0^+ \to L = S = 1$

$$\langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \rangle = \frac{1}{2} \langle (\mathbf{S}_q + \mathbf{S}_{\bar{q}})^2 - \mathbf{S}_q^2 - \mathbf{S}_{\bar{q}}^2 \rangle > 0$$

Initial correlation

STAR, Nature 650, 44-45 (2026)





Hyperon spin correlation

- Hyperon spin correlation:

$$\boxed{C_{12}^{\mu\nu}(p_1,p_2) \equiv \langle P_1^{\mu}(x_1,p_1) P_2^{\nu}(x_2,p_2)\rangle}$$

4 × 4 Lorentz tensor

$$\boxed{c_{12}^{ab}(p_1,p_2) \equiv n_{1,a}^{\mu}(p_1) n_{2,b}^{\nu}(p_2) C_{\mu\nu}^{12}(p_1,p_2), \quad a,b = x,y,z.}$$

Correlation between spins in two particle's respective rest frames

$$\boxed{\frac{9}{\alpha_1\alpha_2}\langle \cos\theta_a^* \cos\theta_b^*\rangle}$$

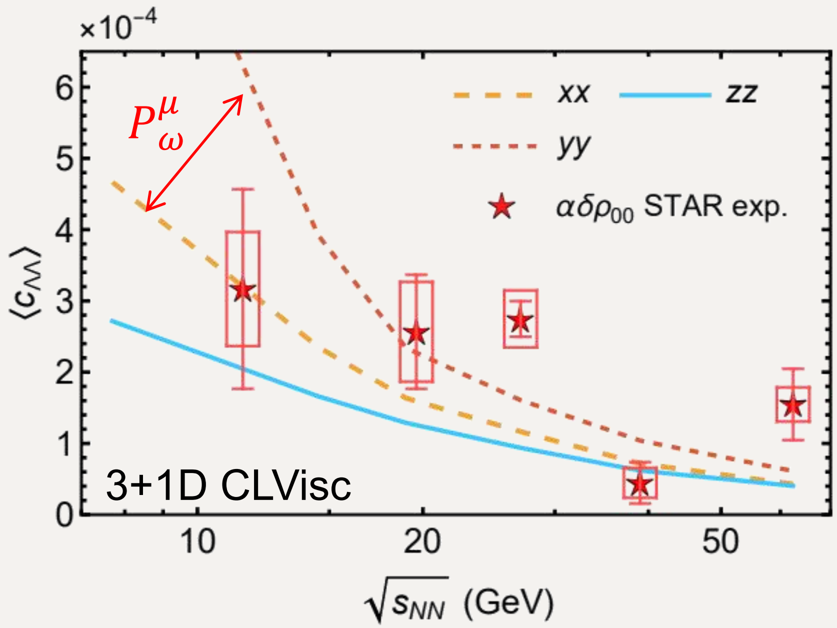
D. Shen, J. Chen, A. Tang, arXiv: 2407.21291

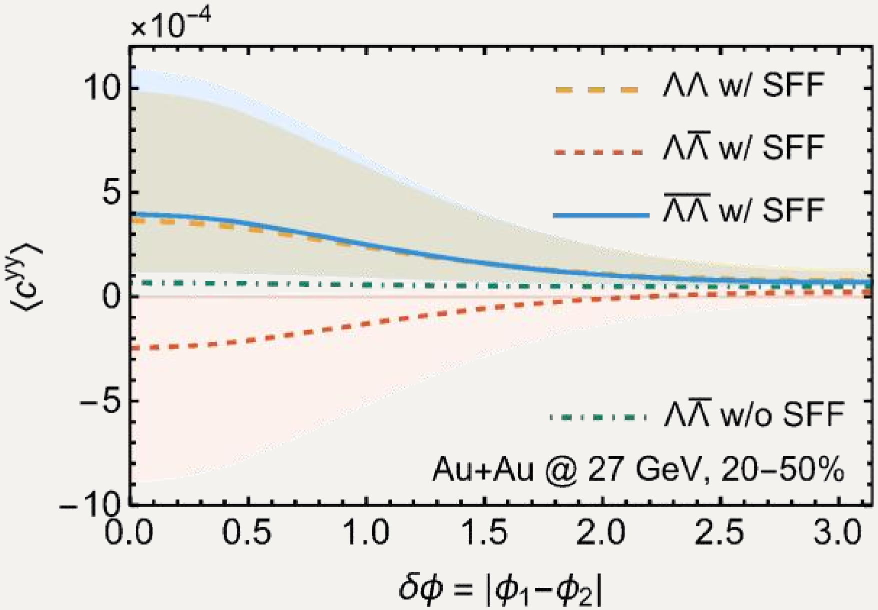
- Relating hyperon polarization to quark polarization

$$P_{\Lambda}^{\mu}(x,p) \approx P_s^{\mu}(x, R_s p)$$

$$P_{s/\bar{s}}^{\mu}(x,p) = \underbrace{P_{\omega}^{\mu} + P_{\text{shear}}^{\mu} + P_{\text{SHE}}^{\mu}}_{\text{Hydrodynamic effects}} + \boxed{P_{\phi}^{\mu}}$$

Strong force field (fluctuation extracted from ϕ spin alignment)



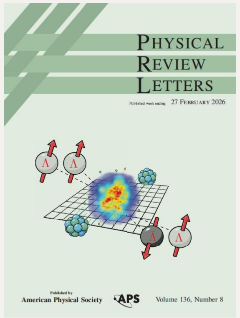


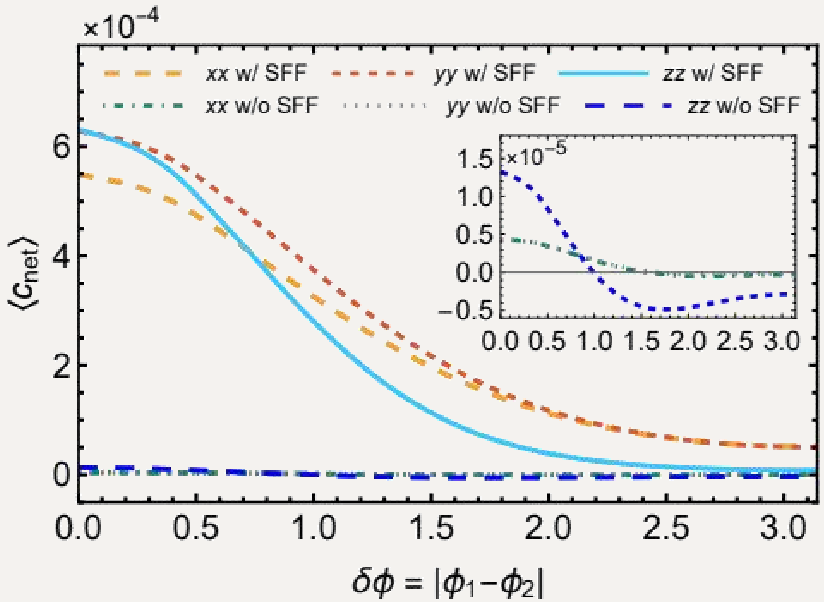
Hyperon spin correlation

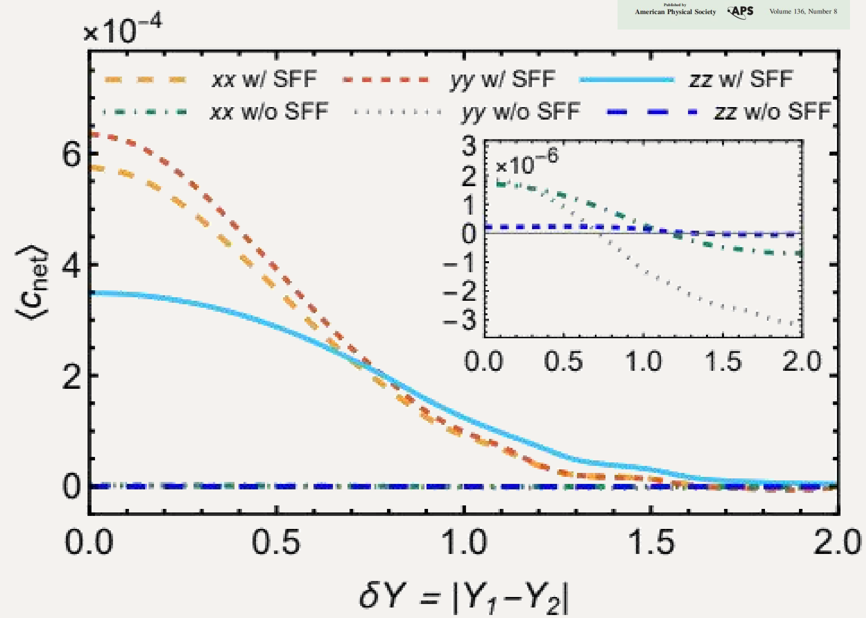
- Net spin correlation:
  difference between "same-sign" and "opposite-sign" pairs

$$c^{ab}_{\rm net} \equiv \frac{1}{2}(c^{ab}_{\Lambda\Lambda} + c^{ab}_{\bar{\Lambda}\bar{\Lambda}}) - c^{ab}_{\Lambda\bar{\Lambda}}$$

XLS, X.-Y. Wu, X.-N. Wang, PRL 136, 8 (2026)







- Spin correlation is small ($\sim 10^{-4}$)
- Converge to zeros at large $\delta\phi$ or δY (short-range)
- Hydrodynamic effect is suppressed by 1-2 orders of magnitude

Summary

- Vector meson spin alignment

$\Leftrightarrow$ anisotropy of short-range quark spin correlation

$\Leftrightarrow$ strong-field fluctuations (vector meson field or color field)

$\Uparrow$

anisotropy induced by motion of meson relative to QGP
or intrinsic anisotropy of field fluctuations

- Vector meson tensor polarization (off-diagonal elements)

$\Leftrightarrow$ anisotropy of field fluctuation or quark spin correlation

$$\mathrm{Re}\rho^{\hat{\mathbf{y}}}_{1,-1} = \frac{1}{2}\rho^{\hat{\mathbf{y}}}_{00} + \rho^{\hat{\mathbf{x}}}_{00} - \frac{1}{2}$$

$$-2\mathrm{Im}\rho^{\hat{\mathbf{y}}}_{1,-1} = 1 - 2\rho^{(\hat{\mathbf{x}}+\hat{\mathbf{z}})/\sqrt{2}}_{00} - \rho^{\hat{\mathbf{y}}}_{00}$$

$$-\sqrt{2}\mathrm{Im}(\rho^{\hat{\mathbf{y}}}_{10} - \rho^{\hat{\mathbf{y}}}_{0,-1}) = 1 - 2\rho^{(\hat{\mathbf{x}}+\hat{\mathbf{y}})/\sqrt{2}}_{00} - \rho^{\hat{\mathbf{z}}}_{00}$$

$$\sqrt{2}\mathrm{Re}(\rho^{\hat{\mathbf{y}}}_{10} - \rho^{\hat{\mathbf{y}}}_{0,-1}) = 1 - 2\rho^{(\hat{\mathbf{y}}+\hat{\mathbf{z}})/\sqrt{2}}_{00} - \rho^{\hat{\mathbf{x}}}_{00}$$

Summary

- Identify different mechanisms for hyperon spin correlation

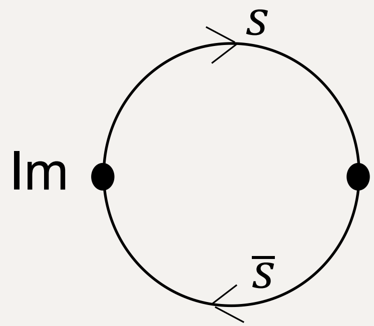
		$\Lambda\Lambda$ & $\bar{\Lambda}\bar{\Lambda}$	$\Lambda\bar{\Lambda}$	Net spin correlation
Initial	Excited vacuum 0^+ $q\bar{q}$	~ 0	$+$	$-$
	Partonic scattering / pair creation $gg \to q\bar{q},\ qq \to qq,\ q\bar{q} \to q\bar{q}$	$+$	$-$ (s-channel) $+$ (t-channel) $+$ (total)	$+$
	Initial net spin polarization	$+$	?	?
Field-induced	Hydrodynamic effects	$+$	$+$	~ 0
	Strong-force field	$+$	$-$	$+$

Thanks for your attention!!

Backup

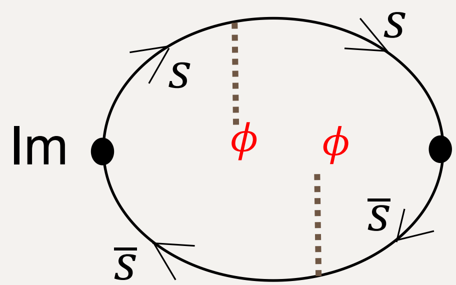
Strong field

- Production of ϕ meson from $s\bar{s}$ coalescence

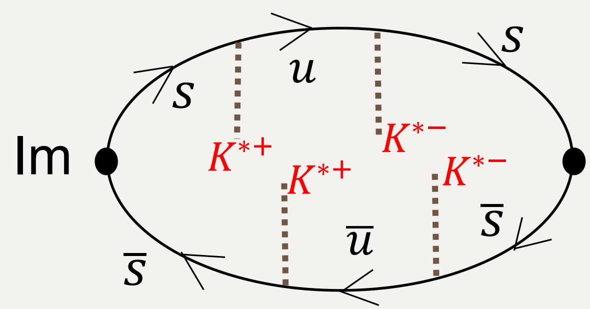


$$\begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix}$$

- Spin alignment induced by vector meson field



$$\left\langle F^{\phi}_{\mu\nu} F^{\phi}_{\alpha\beta} \right\rangle$$



$$\left\langle F^{K^{*+}}_{\mu\nu} F^{K^{*+}}_{\alpha\beta} F^{K^{*-}}_{\rho\sigma} F^{K^{*-}}_{\lambda\tau} \right\rangle$$

Higher order correlation

Off-diagonal elements

- Off-diagonal elements of vector meson density matrix (spin quantization direction is set to z-direction)

$$\rho_{00}^{\hat{\mathbf{z}}}(\mathbf{x},\mathbf{p}) \approx \frac{1}{3} - \frac{2}{3}\left\langle P_q^z P_{\bar{q}}^z - \frac{1}{3}\mathbf{P}_q \cdot \mathbf{P}_{\bar{q}} \right\rangle_V$$

Difference between z-z correlation and x-x (y-y) correlation

$$\mathrm{Re}\rho_{1,-1}^{\hat{\mathbf{z}}}(\mathbf{x},\mathbf{p}) \approx \frac{1}{3}\langle P_q^x P_{\bar{q}}^x - P_q^y P_{\bar{q}}^y \rangle_V$$

Difference between x-x correlation and y-y correlation

$$-\mathrm{Im}\rho_{1,-1}^{\hat{\mathbf{z}}}(\mathbf{x},\mathbf{p}) \approx \frac{1}{3}\langle P_q^x P_{\bar{q}}^y + P_q^y P_{\bar{q}}^x \rangle_V$$

x-y correlation

$$\mathrm{Re}\left(\rho_{10}^{\hat{\mathbf{z}}} - \rho_{0,-1}^{\hat{\mathbf{z}}}\right)(\mathbf{x},\mathbf{p}) \approx \frac{\sqrt{2}}{3}\langle P_q^x P_{\bar{q}}^z + P_q^z P_{\bar{q}}^x \rangle_V$$

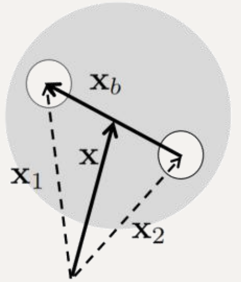
x-z correlation

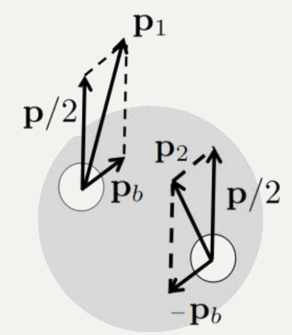
$$-\mathrm{Im}\left(\rho_{10}^{\hat{\mathbf{z}}} - \rho_{0,-1}^{\hat{\mathbf{z}}}\right)(\mathbf{x},\mathbf{p}) \approx \frac{\sqrt{2}}{3}\langle P_q^y P_{\bar{q}}^z + P_q^z P_{\bar{q}}^y \rangle_V$$

y-z correlation

- Average over constituent $q/\bar{q}$'s relative position and relative momentum

$$\left\langle P_q^i P_{\bar{q}}^j \right\rangle_V \equiv \frac{1}{\pi^3}\int d^3\mathbf{x}_b d^3\mathbf{p}_b \exp\left(-\frac{\mathbf{p}_b^2}{a_V^2} - a_V^2\mathbf{x}_b^2\right) \times P_q^i(\mathbf{x}_1,\mathbf{p}_1)P_{\bar{q}}^j(\mathbf{x}_2,\mathbf{p}_2)$$





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XLS, Q. Wang, X.-N. Wang, Phys.Rev.D 102 (2020) 5, 056013
J.-H. Chen, Z.-T. Liang, Y.-G. Ma, XLS, Q. Wang, arXiv:2407.06480

How to measure?

- Vector polarization

$$\boxed{\begin{aligned} P_1^{\mathbf{n}} &= \sqrt{2}\mathrm{Re}(\rho_{10}^{\mathbf{n}} + \rho_{0,-1}^{\mathbf{n}}) \\ P_2^{\mathbf{n}} &= -\sqrt{2}\mathrm{Im}(\rho_{10}^{\mathbf{n}} + \rho_{0,-1}^{\mathbf{n}}) \\ P_3^{\mathbf{n}} &= \rho_{11}^{\mathbf{n}} - \rho_{-1,-1}^{\mathbf{n}} \end{aligned}}$$

- Tensor polarization

$$\boxed{\begin{aligned} \mathcal{T}_{12}^{\mathbf{n}} &= -2\mathrm{Im}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{23}^{\mathbf{n}} &= -\sqrt{2}\mathrm{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \\ \mathcal{T}_{31}^{\mathbf{n}} &= \sqrt{2}\mathrm{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \\ \mathcal{T}_{11}^{\mathbf{n}} &= \rho_{00}^{\mathbf{n}} - \frac{1}{3} + 2\mathrm{Re}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{22}^{\mathbf{n}} &= \rho_{00}^{\mathbf{n}} - \frac{1}{3} - 2\mathrm{Re}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{33}^{\mathbf{n}} &= -2(\rho_{00}^{\mathbf{n}} - \frac{1}{3}) \end{aligned}}$$

$$\mathcal{T}_{11}^{\mathbf{n}} + \mathcal{T}_{22}^{\mathbf{n}} + \mathcal{T}_{33}^{\mathbf{n}} = 0$$

- Strong p-wave decay

$$K^{*0} \rightarrow K^+ + \pi^-$$
$$\phi \rightarrow K^+ + K^-$$

$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{8\pi}\Big[&(1-\rho_{00}^{\mathbf{n}}) + (3\rho_{00}^{\mathbf{n}} - 1)\cos^2\theta \\ &-2\mathrm{Re}\rho_{1,-1}^{\mathbf{n}}\sin^2\theta\cos 2\phi + 2\mathrm{Im}\rho_{1,-1}^{\mathbf{n}}\sin^2\theta\sin 2\phi \\ &-\sqrt{2}\mathrm{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}})\sin 2\theta\cos\phi \\ &+\sqrt{2}\mathrm{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}})\sin 2\theta\sin\phi\Big] \end{aligned}$$

Direction of **n** matters!

- Dilepton decay

$$J/\psi \rightarrow \mu^+ + \mu^-$$

$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{16\pi}\Big[&(1+\rho_{00}^{\mathbf{n}}) + (1 - 3\rho_{00}^{\mathbf{n}})\cos^2\theta \\ &+2\mathrm{Re}\rho_{1,-1}^{\mathbf{n}}\sin^2\theta\cos 2\phi - 2\mathrm{Im}\rho_{1,-1}^{\mathbf{n}}\sin^2\theta\sin 2\phi \\ &+\sqrt{2}\mathrm{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}})\sin 2\theta\cos\phi \\ &-\sqrt{2}\mathrm{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}})\sin 2\theta\sin\phi\Big] \end{aligned}$$

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