

Quark helicity PDFs of proton from lattice QCD

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in collaboration with

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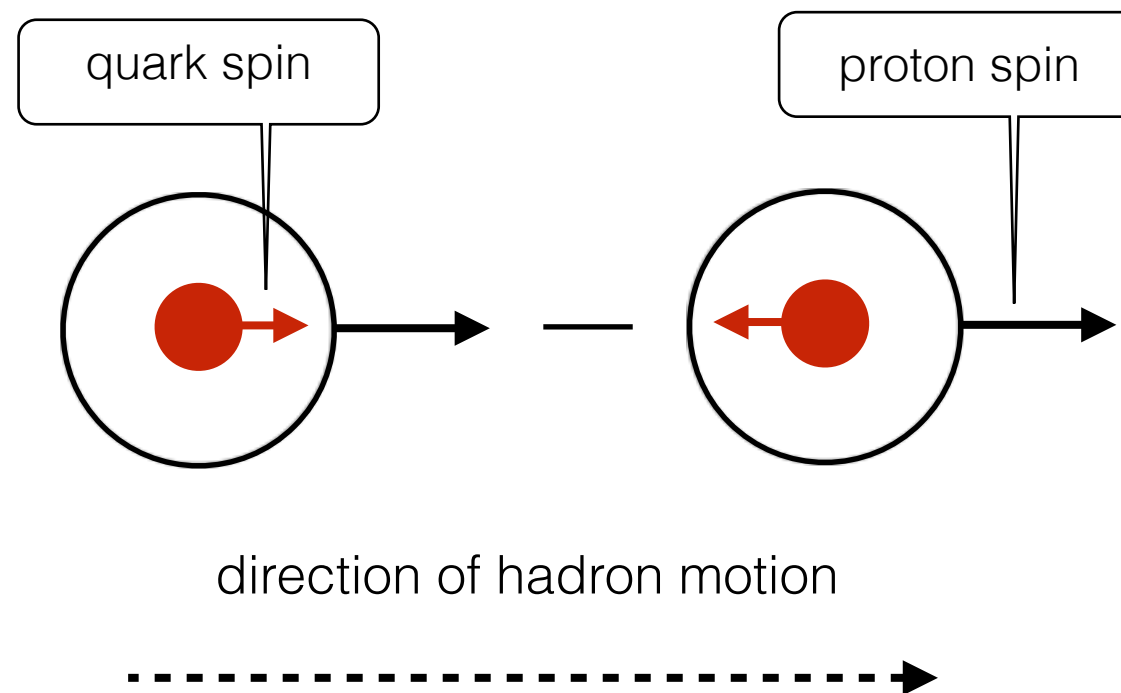
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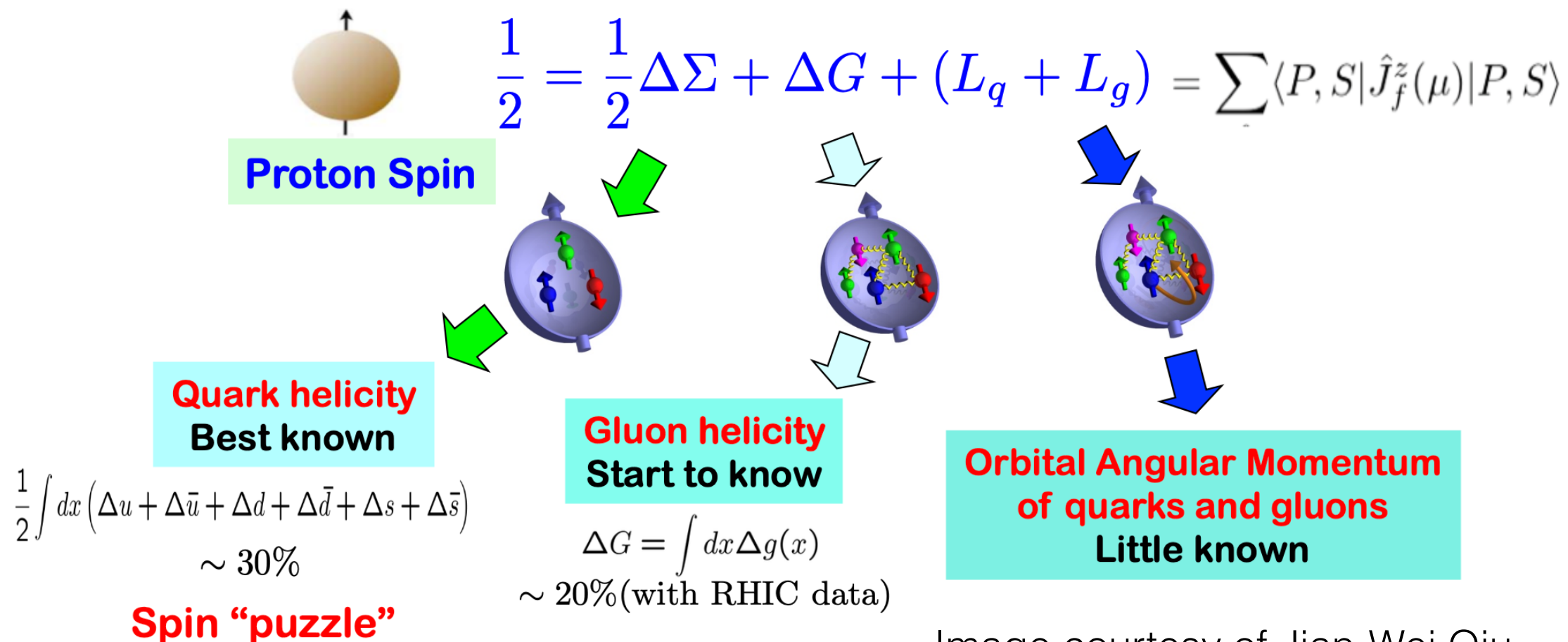
What is quark helicity PDF?

The probability of finding a parton (constituent quark) with momentum fraction x in a longitudinally polarized fast-moving proton



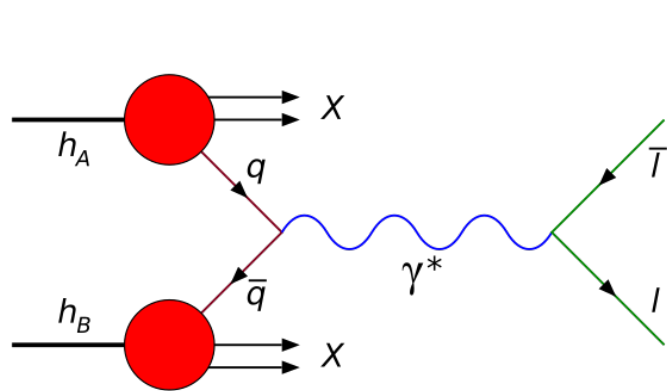
Why is it important?

Spin sum rule: [Jaffe and Manohar, NPB 337, 509 (1990)]

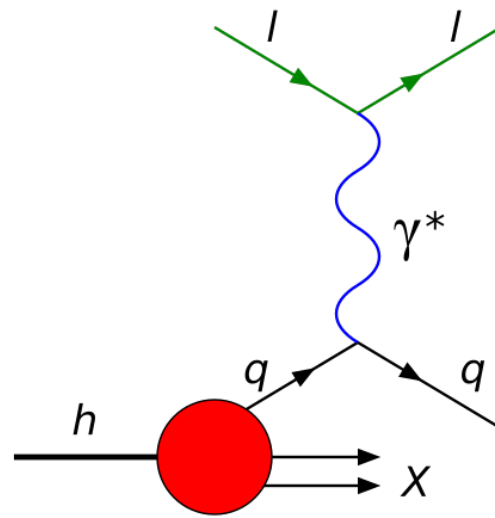


Quark model assumes spin of proton (1/2) equals spin of u+u+d,
but European Muon Collaboration (EMC) found quarks only contribute < 30%

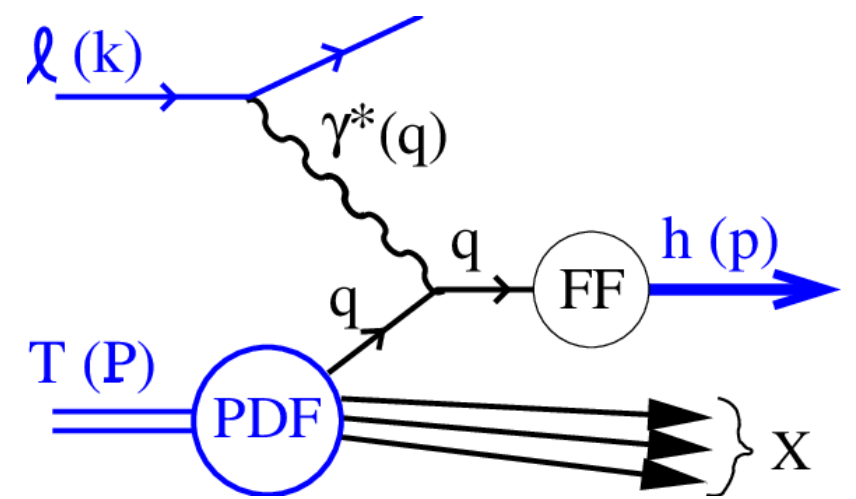
How to access it?



Drell-Yan



DIS



SIDIS

Measure structure functions (via longitudinal spin asymmetries) in processes like polarized lepton/hadron scattering off a polarized target proton

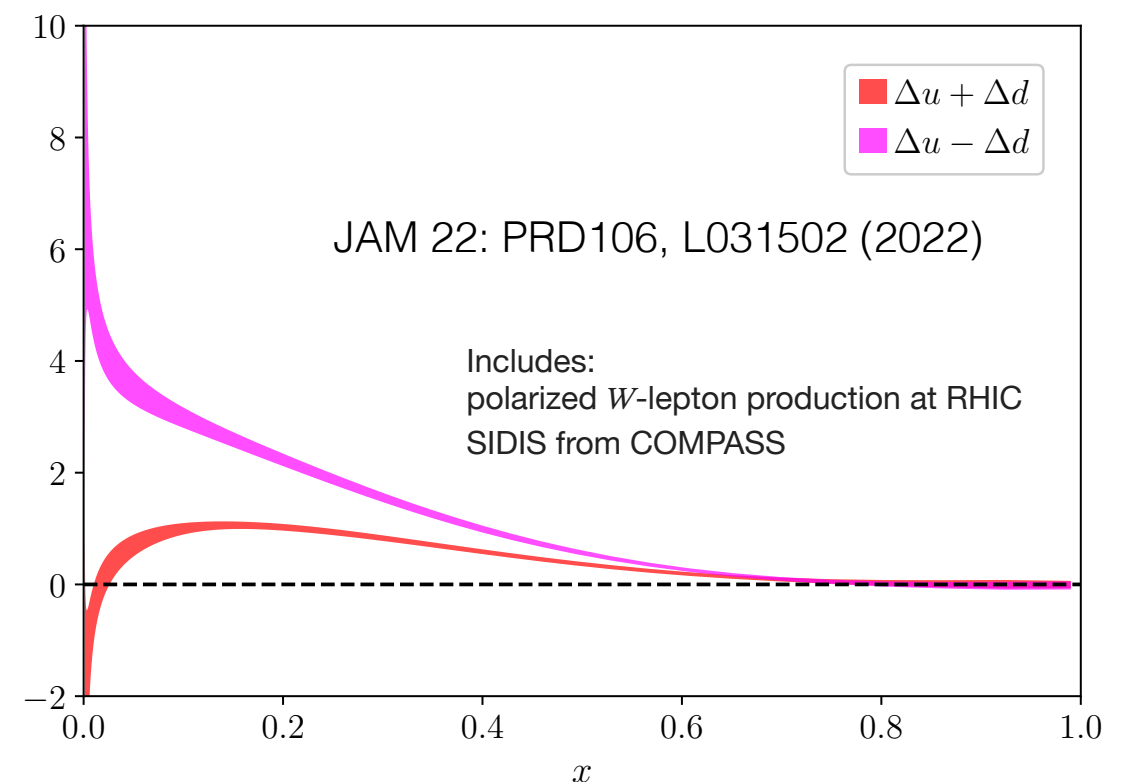
Existing experimental efforts in last 3 decades:

p-p collision at RHIC;

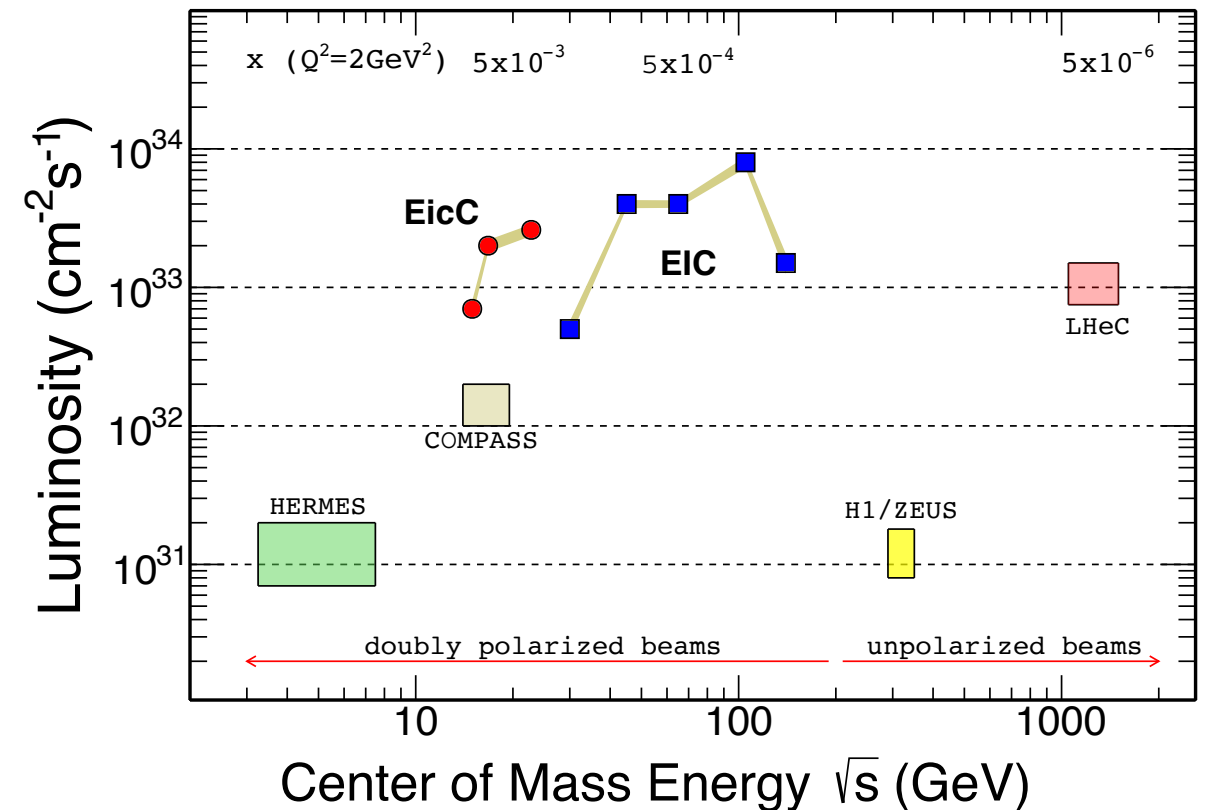
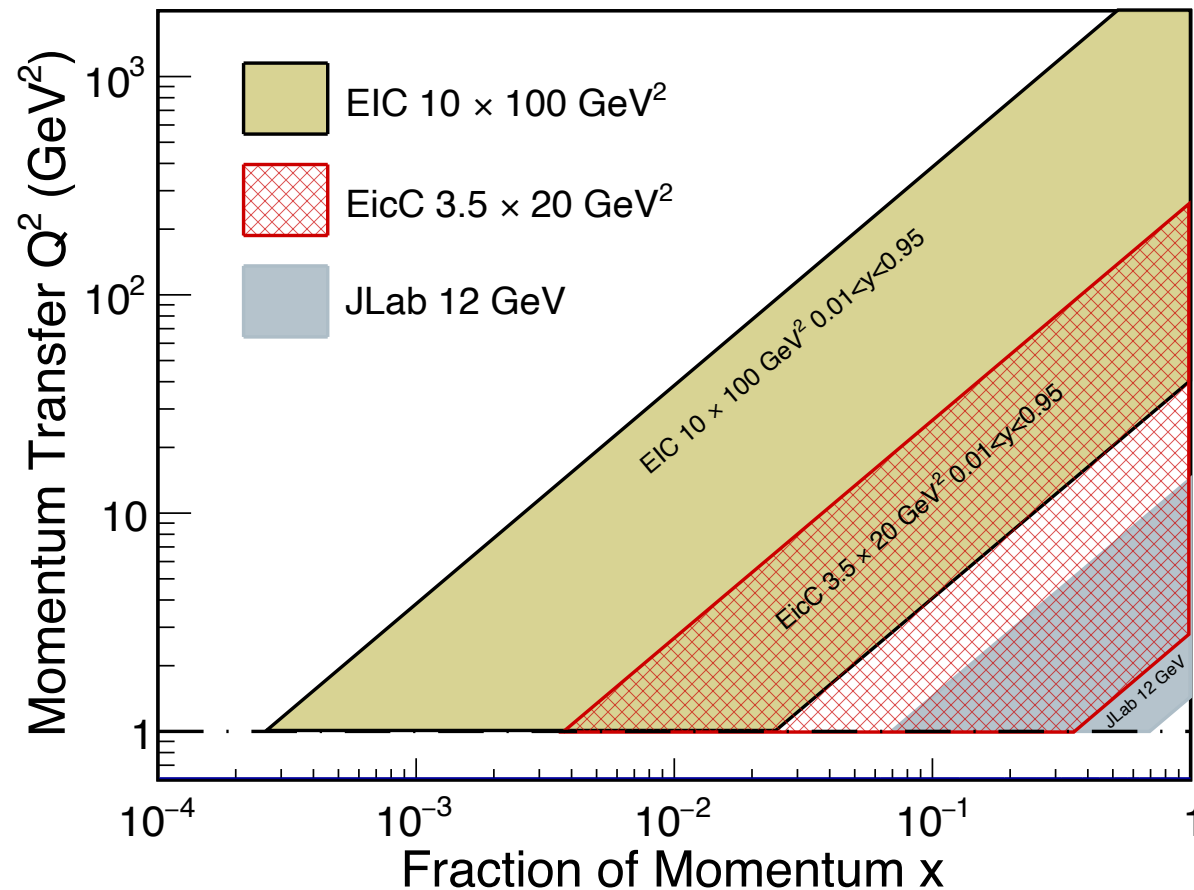
Open-charm muon production at COMPASS;

Neg./pos. pion/kaon production at HERMES...

Global analysis of the experimental data



Why EIC and EICc?



EicC white paper: Frontiers of Physics, Volume 16 Issue (6):64701, 2021

- Unprecedented low- x reach for polarized experiments
- High luminosity provides more precise statistical measurements
- High polarization capacities of electron and nucleon give more access to the spin-dependent observables

Status of LQCD determinations

LQCD provides *ab initio* determination from first-principle!

	m_π [MeV]	a [fm]	P_z [GeV]
LP ³ Coll. 18'	physical	0.09	3.0
Gao, <i>et al.</i> , PRD20'	310	0.042	2.77
HadStruc Coll. 22'	358	0.094	2.5
Holligan and Lin, 24'	315	0	1.75
This work	physical	0.076	1.53

- Having both continuum limit and physical mass limit is still challenging
- This work: physical quark masses, small lattice spacing, NNLO&NNLL matching

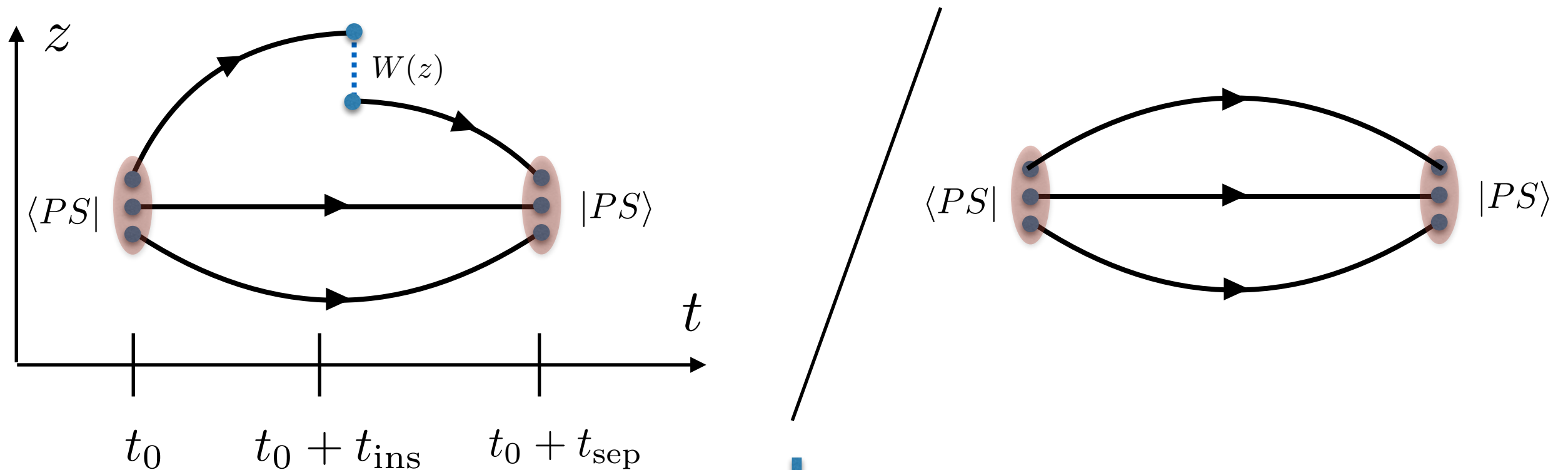
Lattice setup

β	a [fm]	m_π [GeV]	N_σ	N_τ	# conf.
7.13	0.076	0.14	64	64	350

$$m_\pi L = 3.45$$

- Physical light & strange quark mass
- Volume effects under control
- $P_z = \{0, 0.25, 1.02, 1.53\}$ GeV
- Momentum smearing to improve SNR

Ingredient in Lattice QCD: matrix element



$$0 \ll t_{\text{ins}} \ll t_{\text{sep}}$$

Parton spin in z direction

$$\tilde{h}_1^B(z, P_z, a) \equiv \frac{1}{2E} \langle P, S | \bar{\psi}(z) \gamma_5 \gamma_z W(z, 0) \psi(0) | P, S \rangle$$

$$f = u - d$$

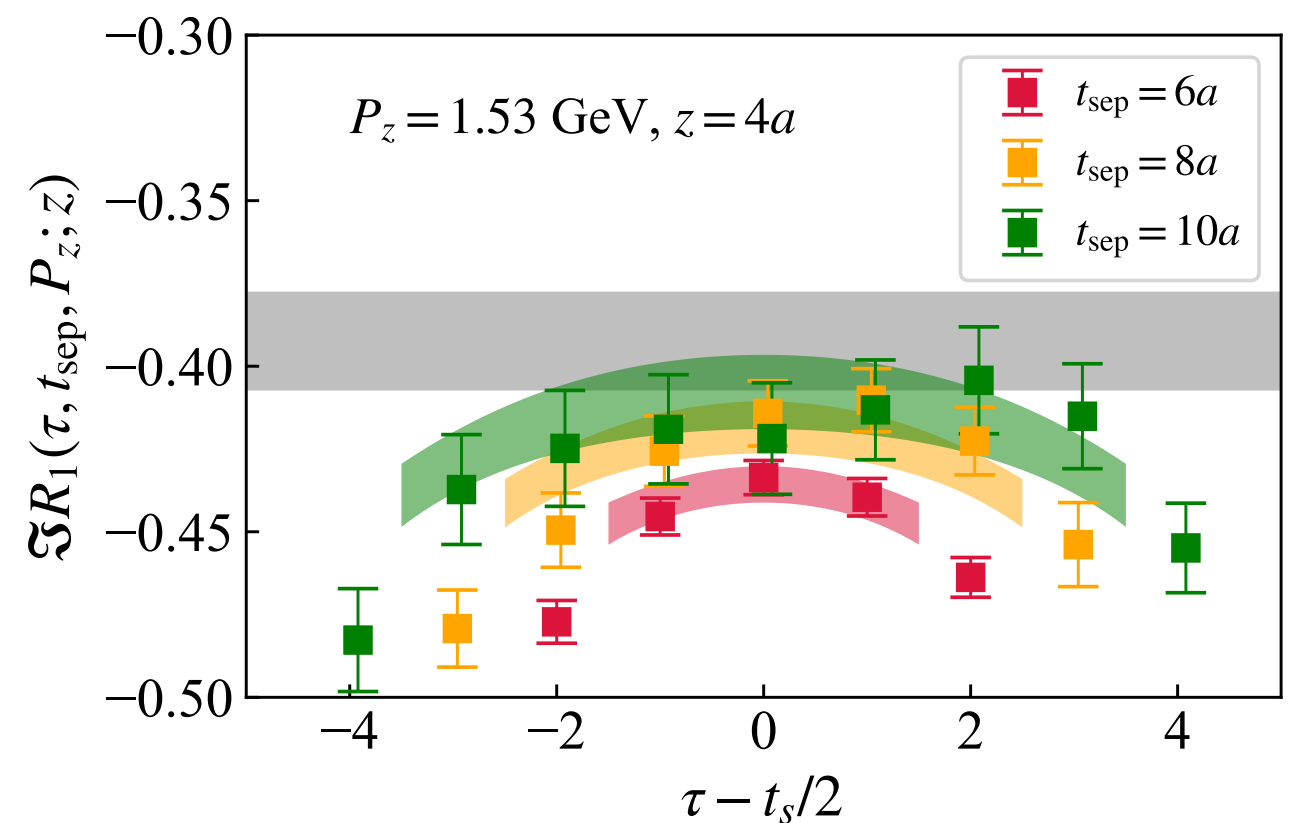
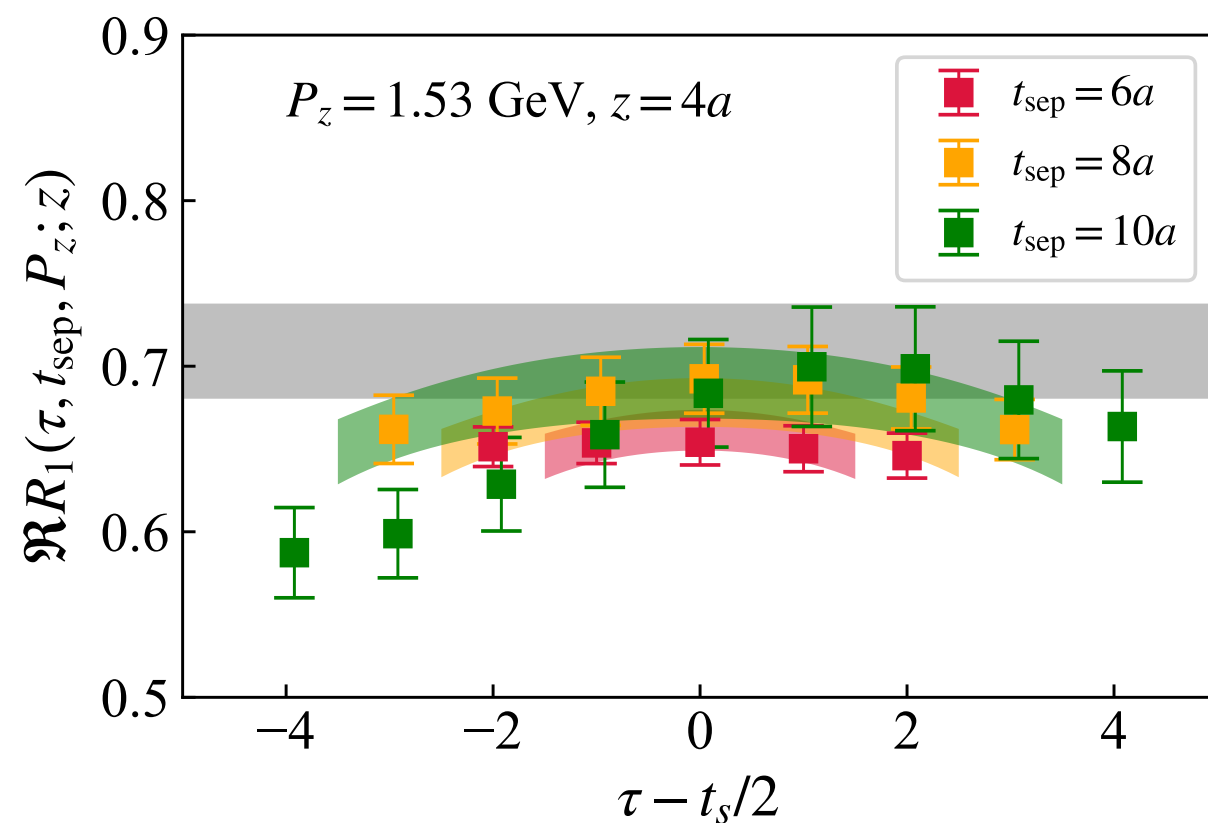
isovector sector: free of disconnected contributions

$P_\mu = (P_0, 0, 0, P_z)$ Hadron momentum in z direction

$S_\mu = (P_z, 0, 0, \mp P_0)$ Hadron spin in +z/-z direction

Extraction of the bare matrix element

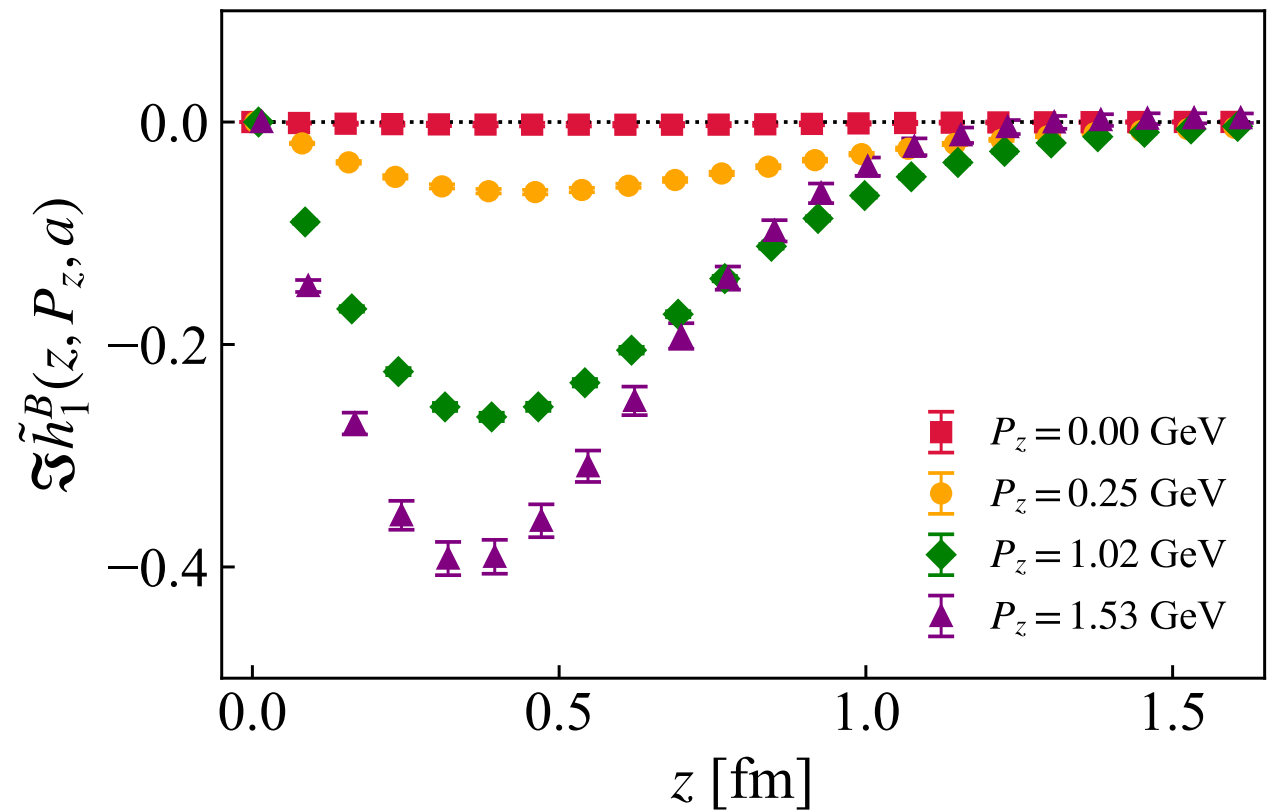
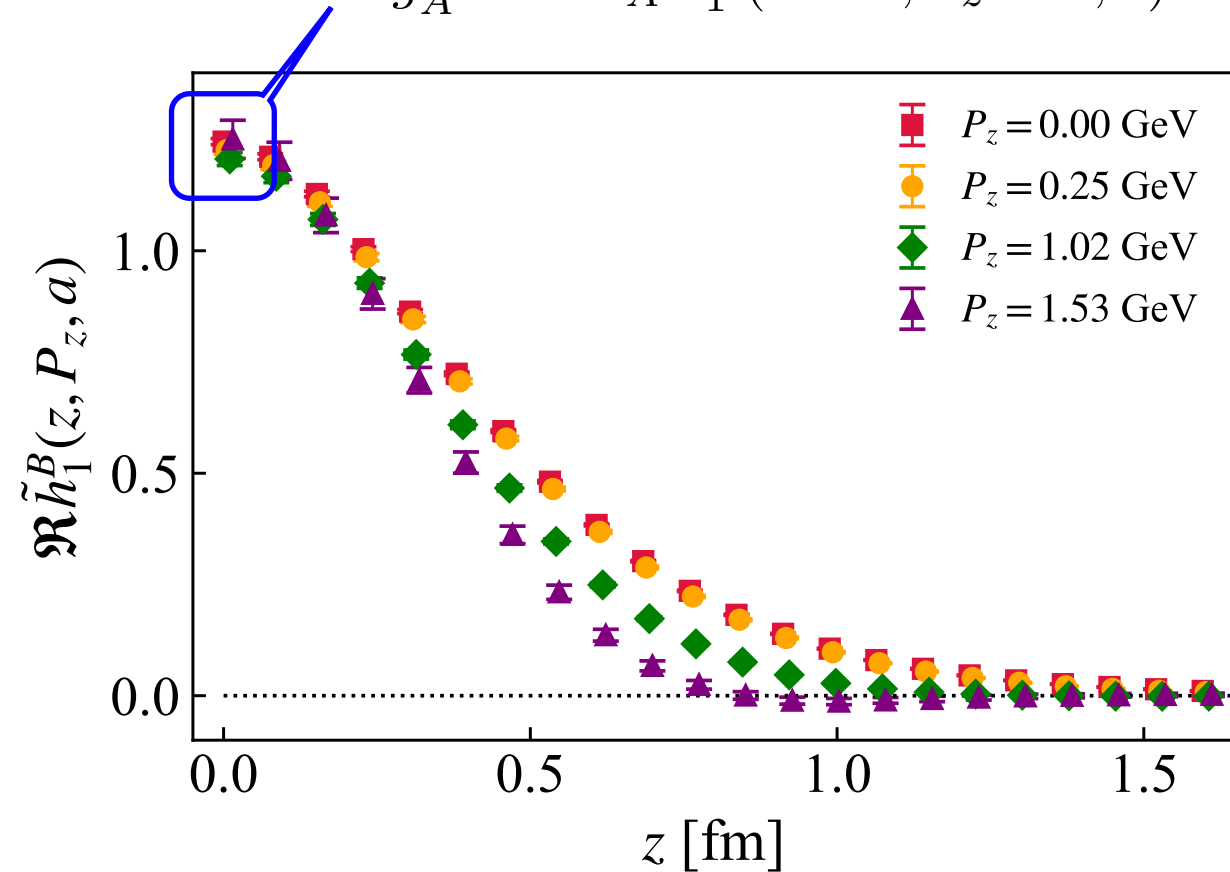
$$R(\tau, t_{\text{sep}}, P_z; z) \equiv \frac{C_{3\text{pt}}^\Gamma(P_z, t_{\text{sep}}, \tau, z)}{C_{2\text{pt}}(P_z, t_{\text{sep}})} = \tilde{h}_1^B(z, P_z, a) \frac{1 + c_1(z, P_z) (e^{-\Delta E t_{\text{ins}}} + e^{-\Delta E(t_{\text{sep}} - t_{\text{ins}})})}{1 + c_2 e^{-\Delta E t_{\text{sep}}}} \xrightarrow{t_{\text{sep}} \gg \tau \gg 0} \tilde{h}_1^B$$



- Good description of the data by the the two-state ansatz
- Sizable excited-state contaminations are under control

z and P_z dependence of bare matrix element

$$g_A^{u-d} = Z_A \tilde{h}_1^B(z=0, P_z=0, a) = 1.211(12)$$

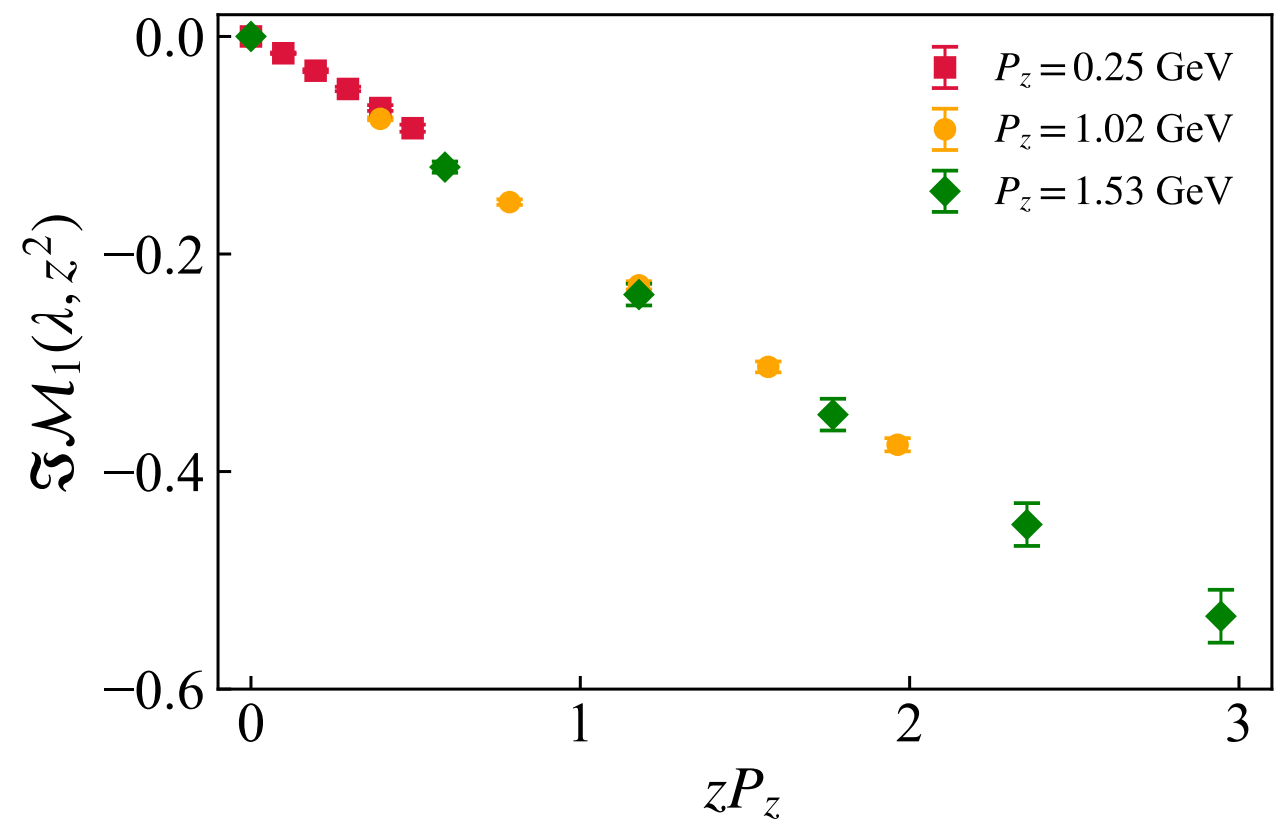
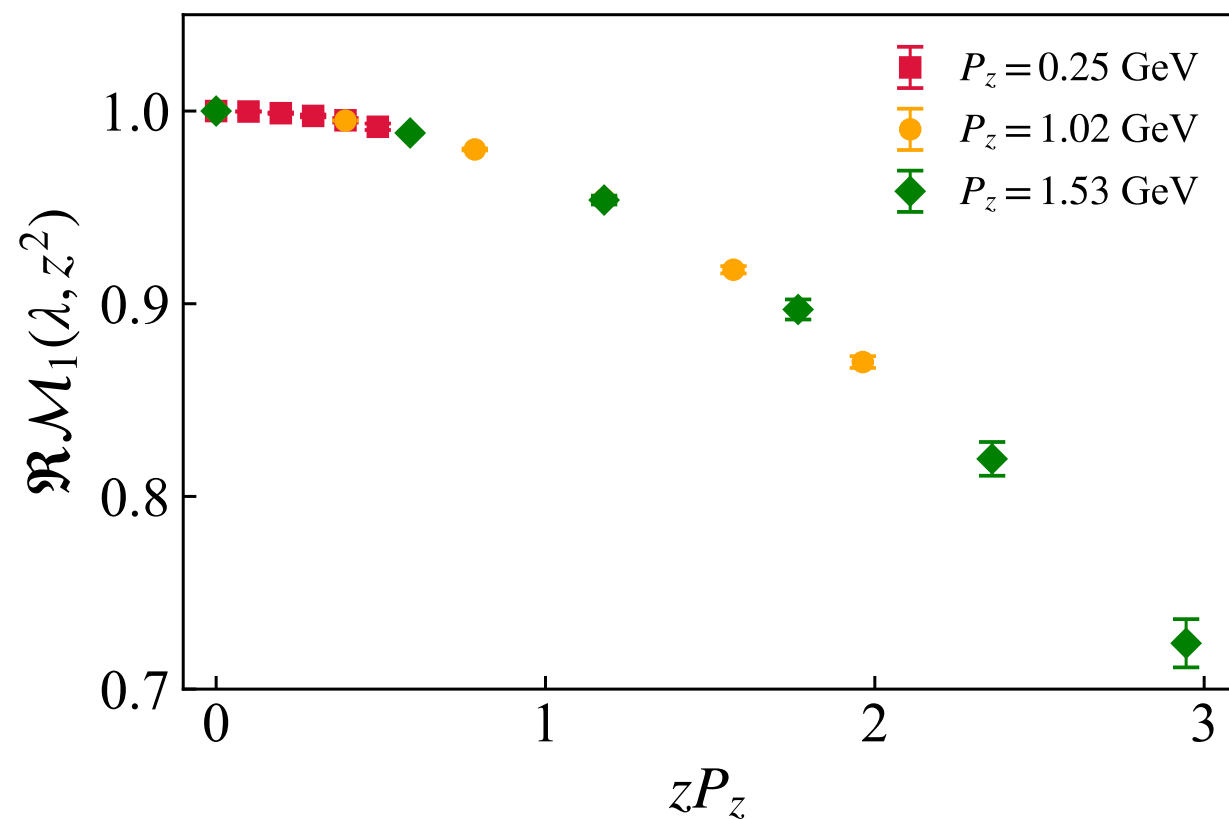


- Consistent results at $z = 0$ from different boosts
- Pronounced momentum dependence for the real&imag parts

Renormalization-group invariant ratios

RI scheme ratios free of linear and logarithmic UV divergences:

$$\mathcal{M}_1(\lambda, z^2) = \frac{\tilde{h}_1^B(z, P_z)}{\tilde{h}_1^B(z, P_z^0)} \frac{\tilde{h}_1^B(0, P_z^0)}{\tilde{h}_1^B(0, P_z)} = \frac{\tilde{h}_1^{\overline{\text{MS}}}(\lambda, z^2 \mu^2)}{\tilde{h}_1^{\overline{\text{MS}}}(\lambda^0, z^2 \mu^2)}$$



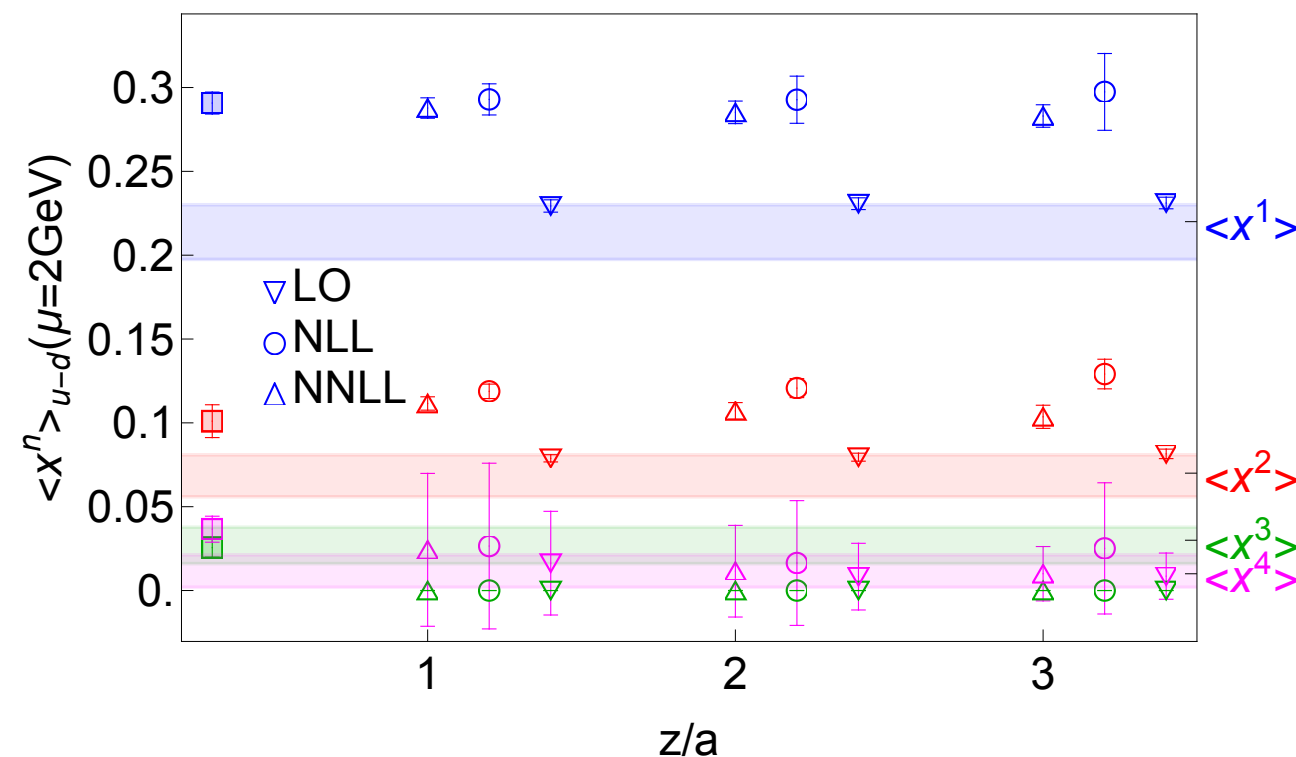
- Normalized to 1 for the real part and 0 for the imaginary part
- Substantial reduction of momentum dependence in the ratio

Mellin moments from reduced ITD

Operator product expansion (OPE) approximation in $\overline{\text{MS}}$ scheme:

$$\mathcal{M}_1(\lambda, z^2) = \frac{\tilde{h}_1^{\overline{\text{MS}}}(\lambda, z^2 \mu^2)}{\tilde{h}_1^{\overline{\text{MS}}}(\lambda^0, z^2 \mu^2)} = \frac{\sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} \langle x^n \rangle(\mu)}{\sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-i\lambda_0)^n}{n!} \langle x^n \rangle(\mu)} + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)$$

$\langle x^n \rangle$ as fit parameters



Points: $\langle x^n \rangle$ from fits of different LQCD data sets

Bands: $\langle x^n \rangle$ from global analysis JAM 22

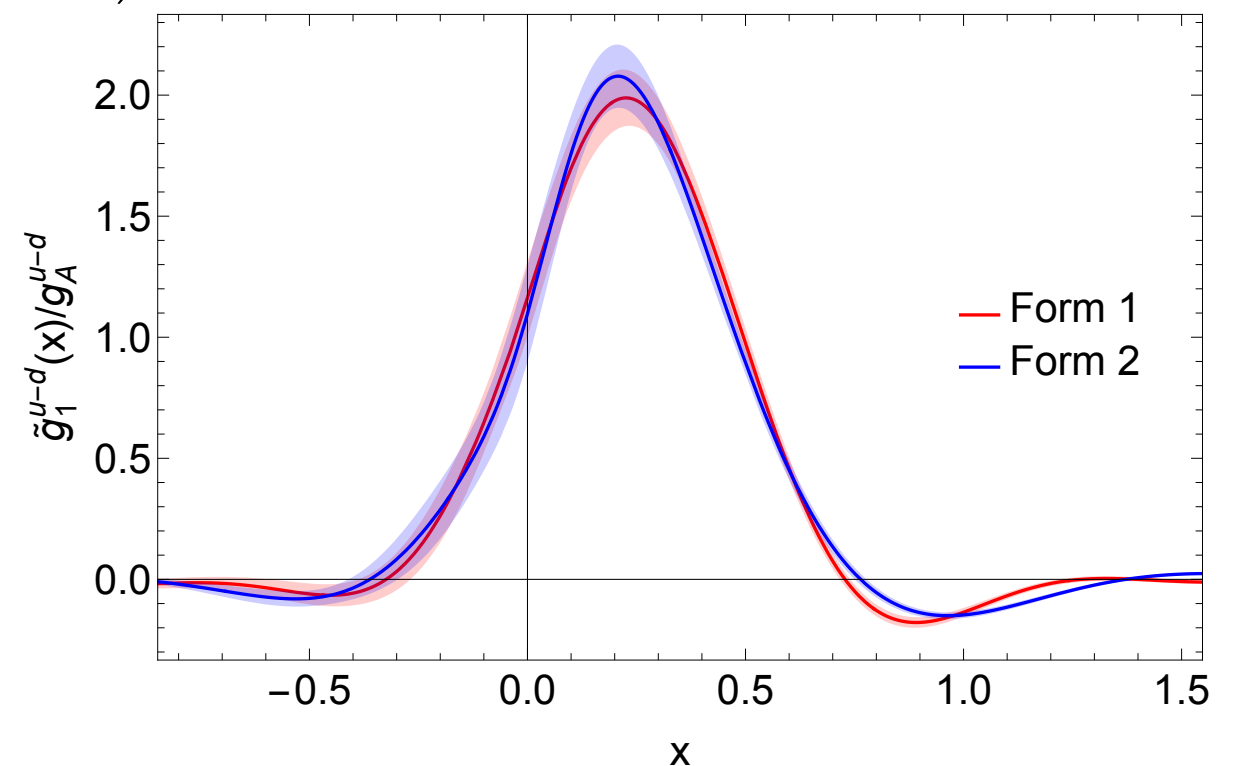
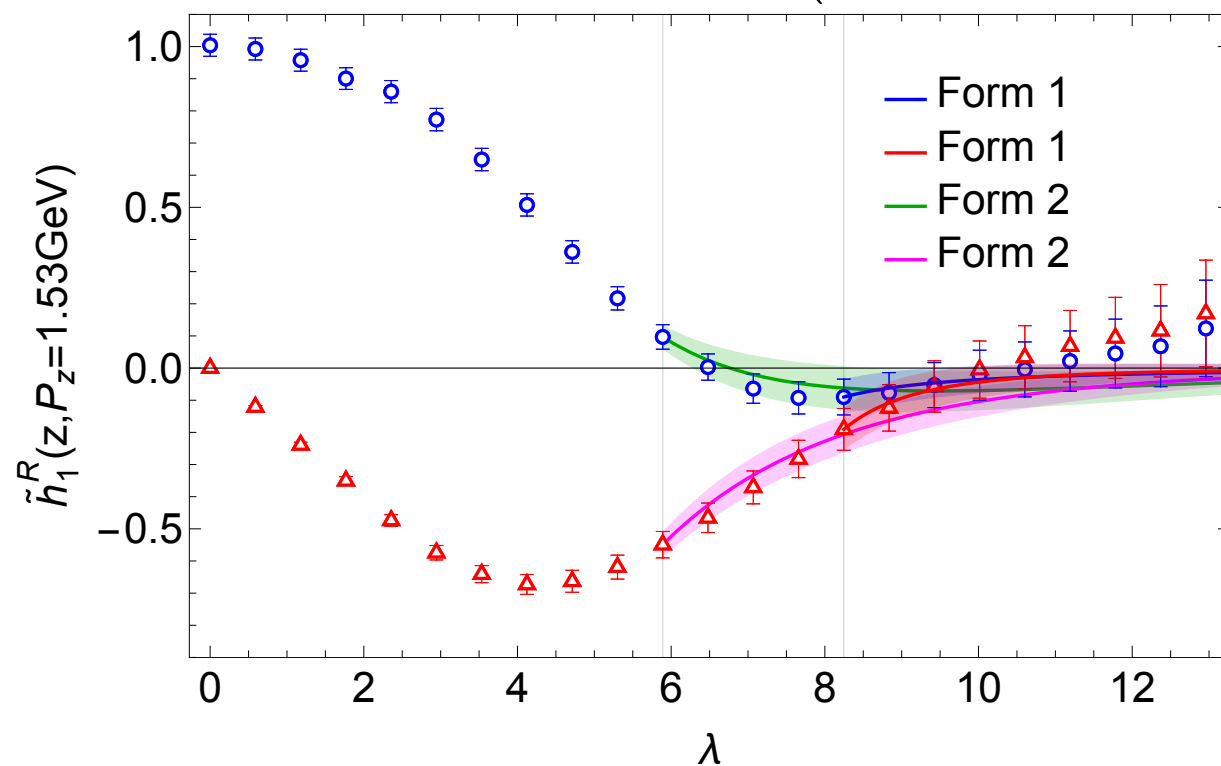
Quasi-PDF from Hybrid renormalization

$$\tilde{h}_R(z, P_z) = \begin{cases} \frac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z, 0, a)}, & |z| \leq z_s \quad \text{Ratio scheme renormalization} \\ \frac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z_s, 0, a)} e^{(\delta m + m_0)(z - z_s)}, & |z| \geq z_s \quad \text{Self renormalization} \end{cases}$$

Large λ extrapolation:

Form 1: $\tilde{h}_1^{\text{asy}}(\lambda, P_z) = \frac{A e^{-m_{\text{eff}} \lambda / P_z}}{|\lambda|^d}$ (dispersive analysis and Regge behavior)

Form 2: $\tilde{h}_1^{\text{asy}}(z, P_z) = \left(A e^{i\phi \text{sgn}(z)} + \frac{A' e^{i\phi' \text{sgn}(z)}}{|z|} \right) e^{-|z|/\Lambda}$ (next-to-leading asymptotics)



Two extrapolations are consistent in most regions of interest

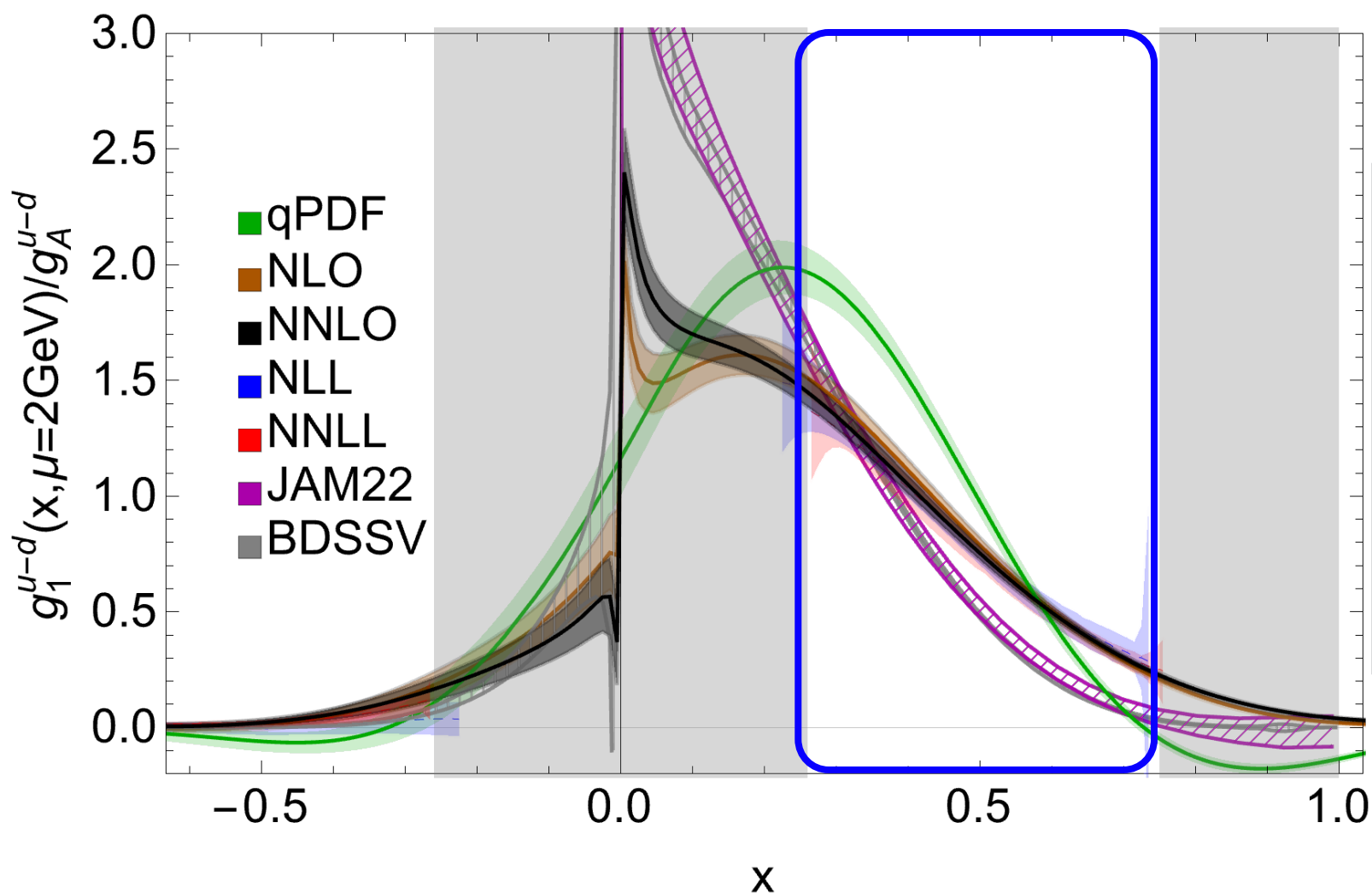
Quasi PDF to light-cone PDF

Perturbative matching:
$$\Delta q(x, \mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} \mathcal{C}^{-1} \left(\frac{x}{y}, \mu, z_s, P_z \right) \Delta \tilde{q}(y, \mu) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)P_z)^2} \right)$$

Perturbative expansion:
$$\mathcal{C} \left(\frac{x}{y}, \mu, z_s, P_z \right) = \delta \left(\frac{x}{y} - 1 \right) + \sum_{n=1}^{\infty} \alpha_s^n \mathcal{C}^{(n)} \left(\frac{x}{y}, \mu, z_s, P_z \right)$$

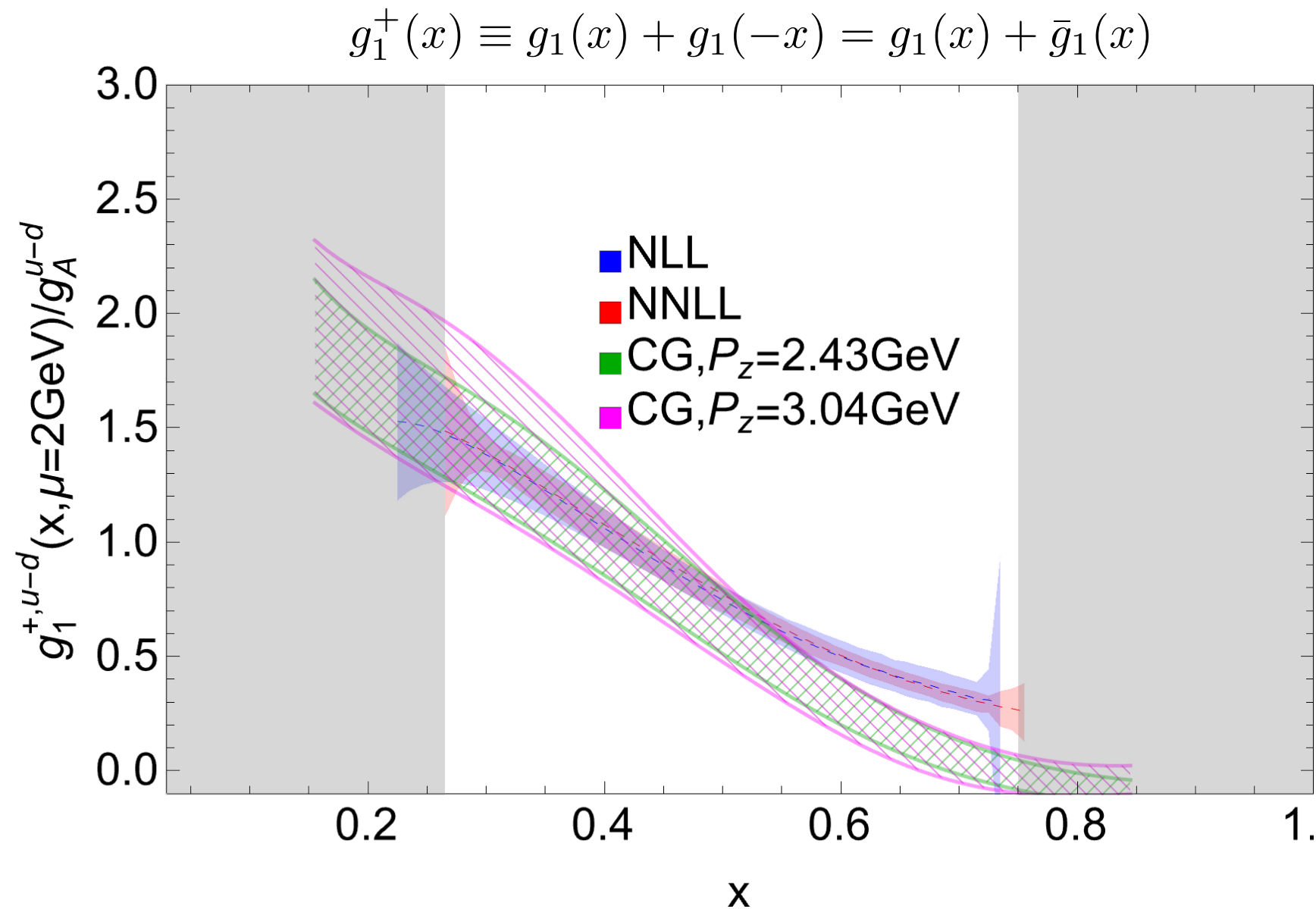
NNLO matching coefficient: PRL. 126, 072001 (2021); PRL. 126, 072002 (2021).

NNLO hybrid scheme corrections: Nucl. Phys. B 991, 116201 (2023).



- RG Resummation is needed when $2xP_z$ becomes small for $x \rightarrow 0$
- Threshold Resummation is needed when $2(1-x)P_z$ becomes small for $x \rightarrow 1$
- Both resummations applied up to NLL/NNLL
- Both scales are large + scale variations are small $\rightarrow$ reliable region $x \in [0.25, 0.75]$

Comparison with the Coulomb gauge method



Results from Coulomb gauge method: [arXiv:2602.11283](https://arxiv.org/abs/2602.11283)

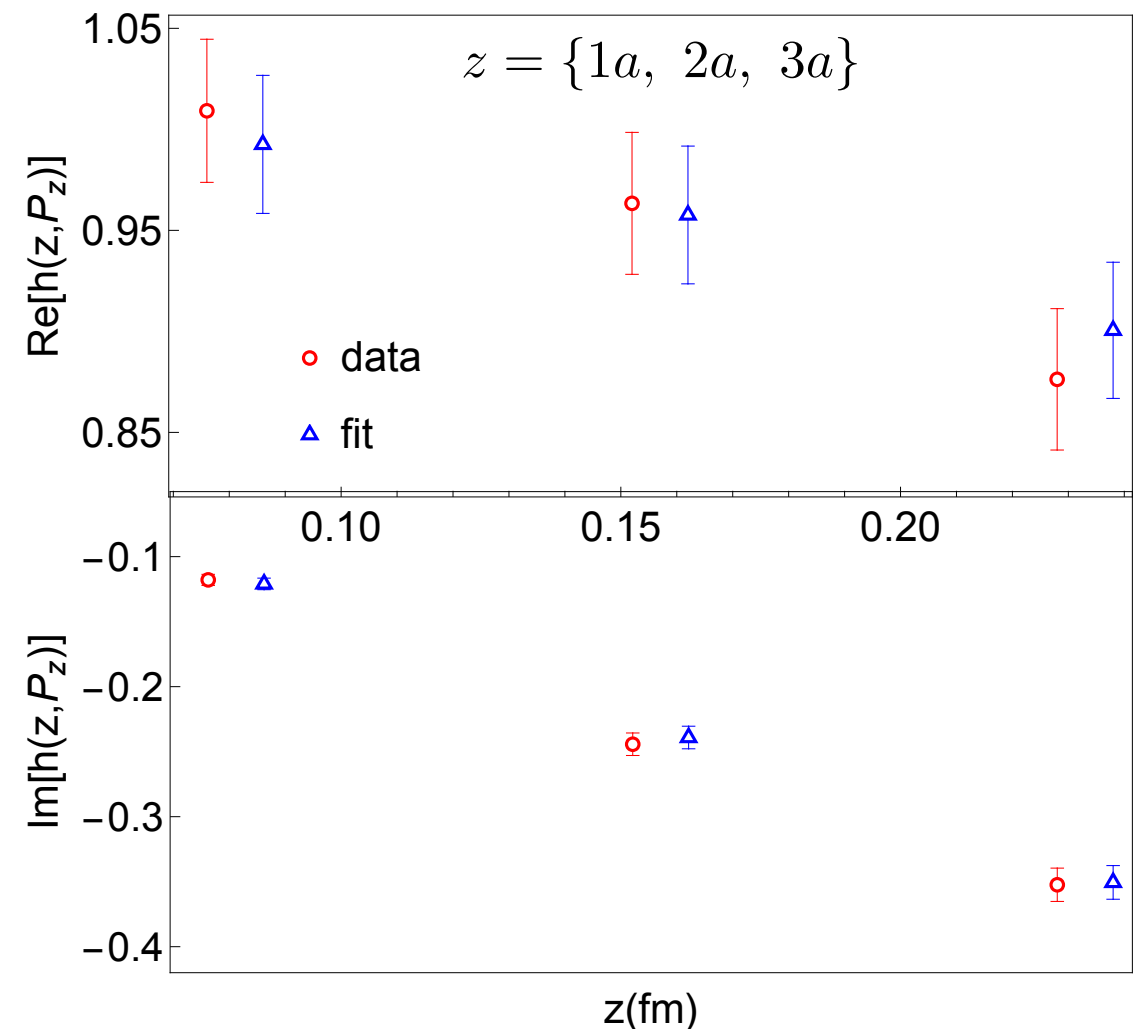
- Good consistency for $x < 0.6$
- Noticeable discrepancies for $x > 0.6$, potentially from the power corrections due to the small momentum $P_z = 1.5 \text{ GeV}$

Constrain the endpoint regions

- Constrain the endpoint regions via short-distance factorization (SDF)
- Fit the small- z matrix elements convoluted with perturbative matching
- Power-law ansatz on both sides from Regge behavior and power counting rules

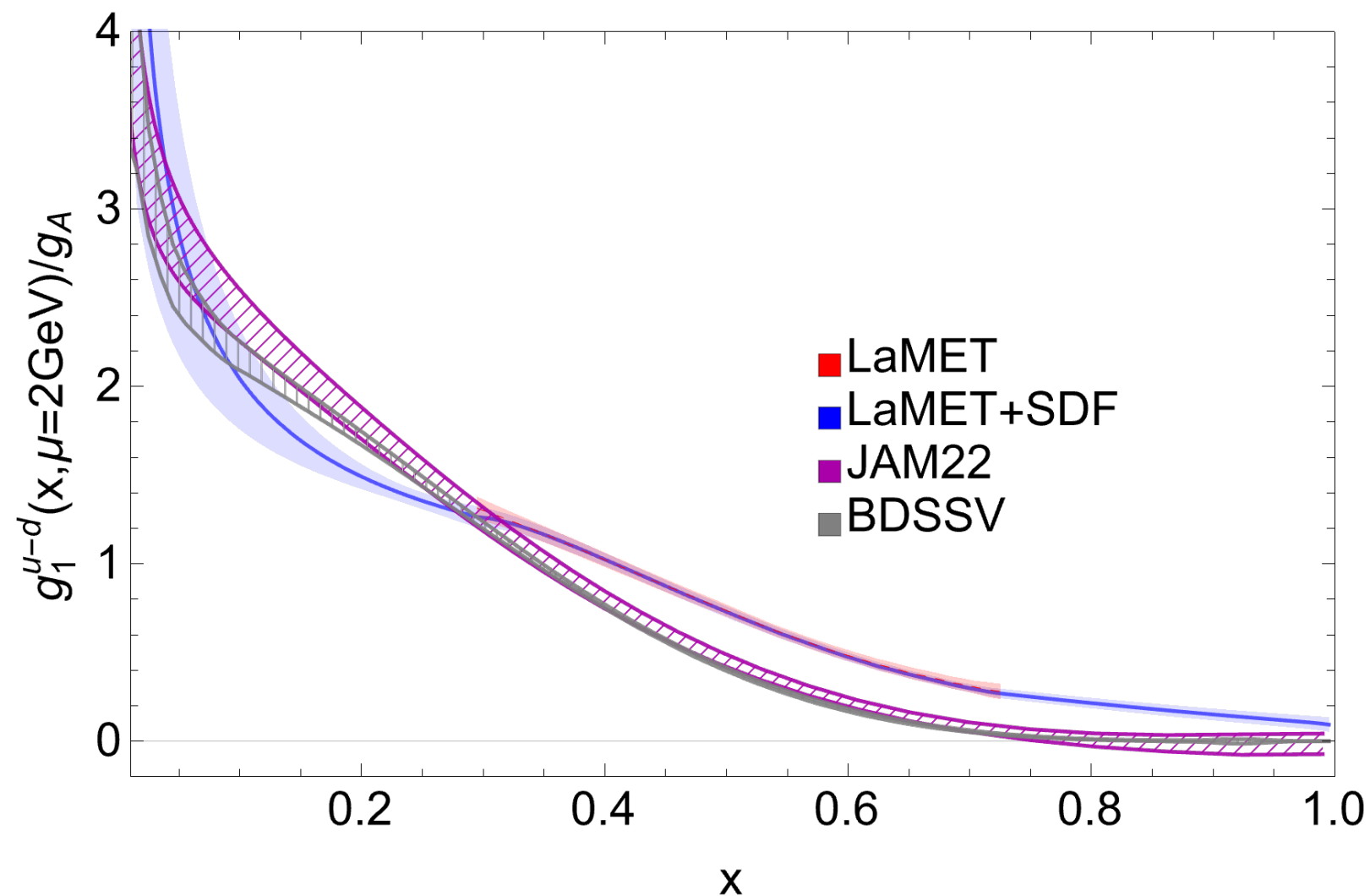
$$g_1(x) = \begin{cases} f(x_0) \frac{x^a}{x_0^a}, & x < x_0, \\ f(x), & x_0 < x < 1 - x_0, \\ f(1 - x_0) \frac{(1 - x)^b}{(1 - x_0)^b}, & x > 1 - x_0. \end{cases}$$

$$h(z^2, z \cdot P) = \int_{-1}^1 K(x\omega, z^2, \mu) g_1(x, \mu) dx$$



Good consistency between the fits and the lattice data

Complete comparison with global fits

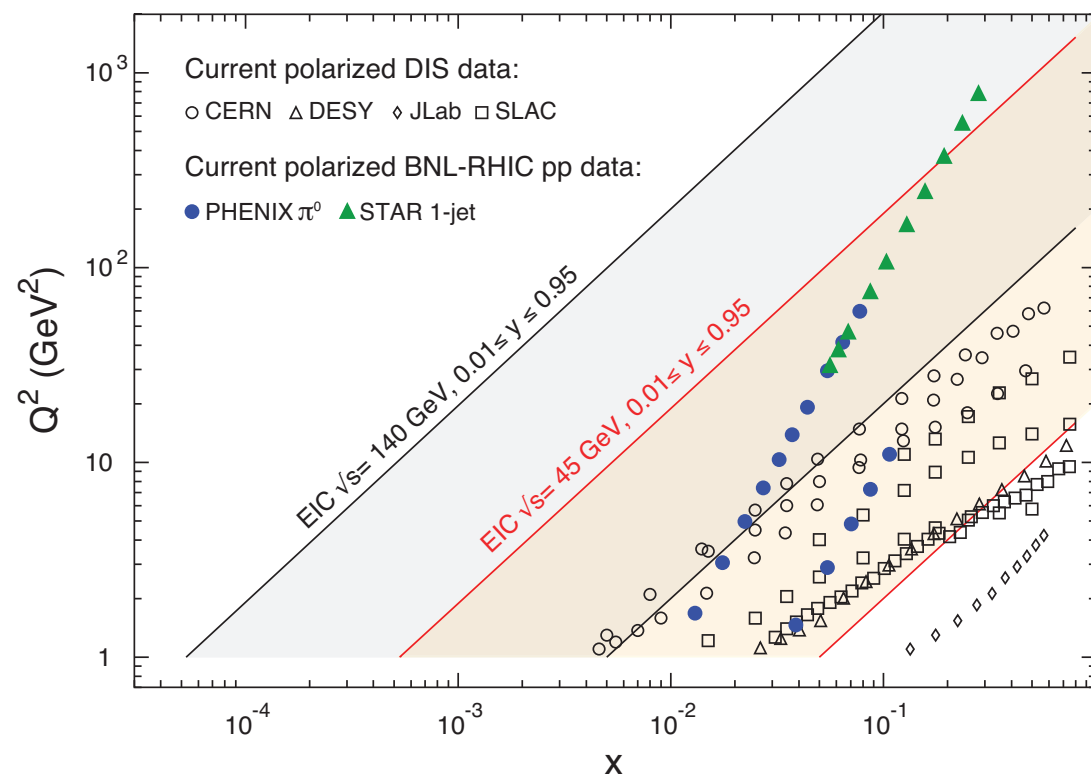


- Smooth connecting between the LaMET determinations and the SDF modeling
- Elevated PDF than the global fitting PDF in mid- x region
- Extra model uncertainties due to finite cutoff et al not quantified & shown
- Further studies on lattice systematics and with higher hadron momentum needed

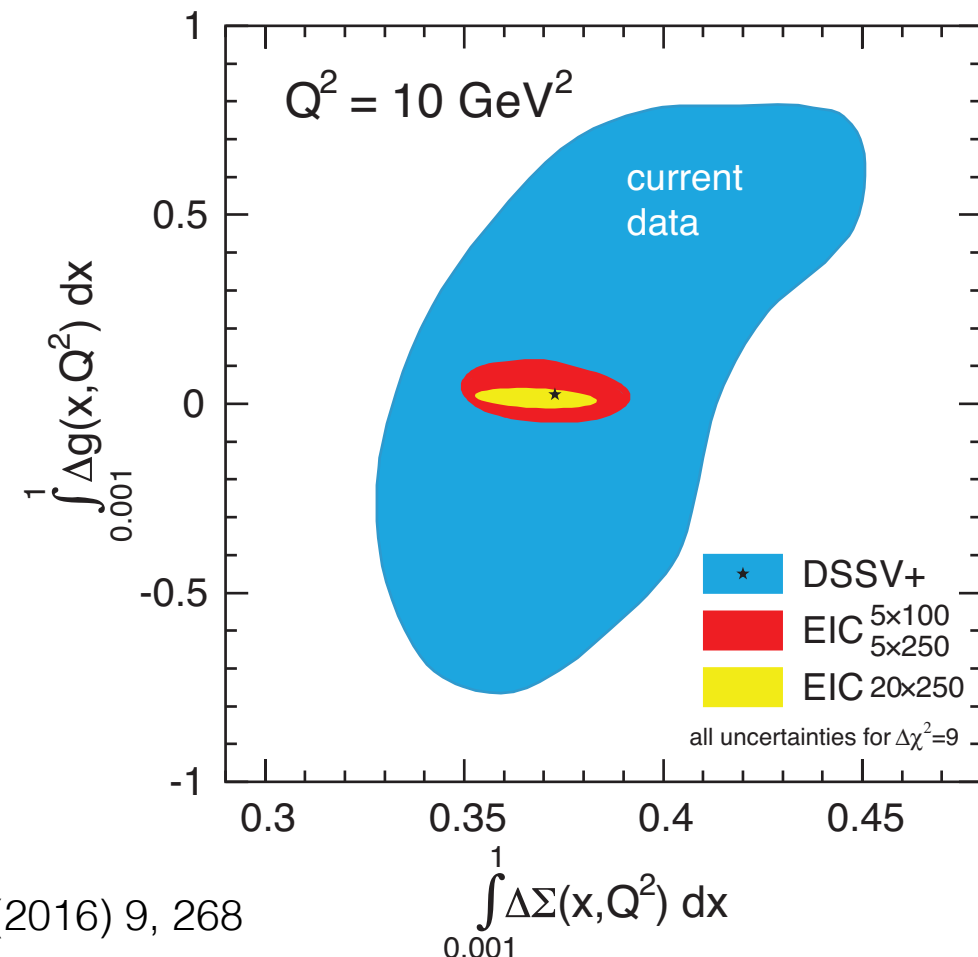
Conclusion

- ☒ Computed quark helicity matrix elements for the isovector sectors
- ☒ Computed the Mellin moments
- ☒ Computed the light cone PDF using the quasi-PDF approach with NNLO matching including NNLL resummations
- ☒ Constrain the endpoint regions using complementary approach
- ☐ Push to larger momentum to suppress the power corrections
- ☐ Add more lattice spacings for the continuum limit

Backup: Why EIC?

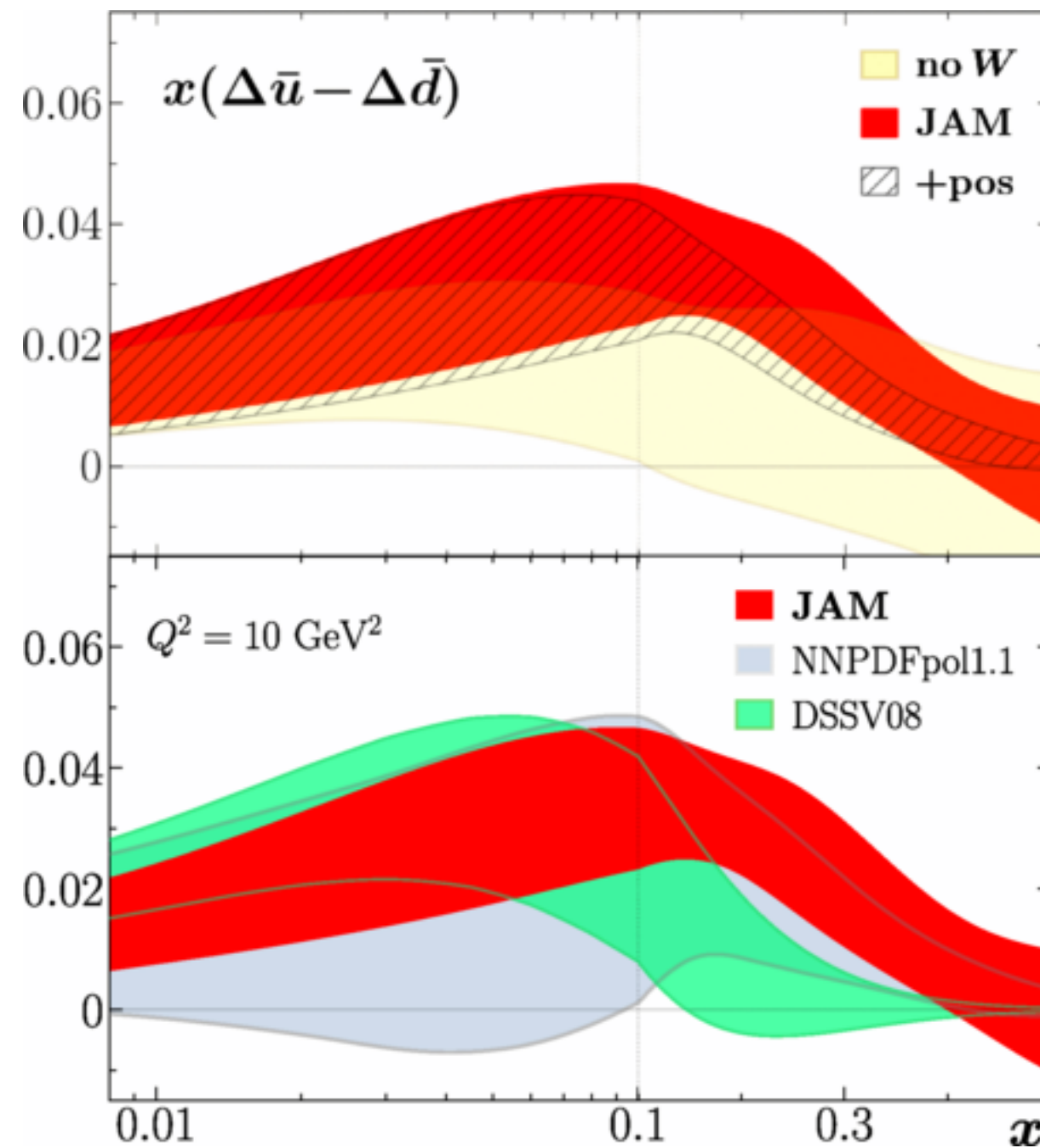


EIC White Paper: EPJA 52 (2016) 9, 268



- High luminosity provides more precise statistical measurements
- High polarization capacities of electron and nucleon give more access to the spin-dependent observables
- Unprecedented low- x reach for polarized DIS experiment

Backup: global helicity PDF



JAM 22: PRD106, L031502 (2022)

Non-zero contribution to spin
from antiquarks

Backup: ratio ansatz

$$R = \frac{C_{3\text{pt}}^{\mathcal{P},\Gamma,f}(\vec{p}, t_{\text{sep}}, t_{\text{ins}}, z; \vec{x}, t_0)}{C_{2\text{pt}}^{\mathcal{P}}(\vec{p}, t_{\text{sep}}; \vec{x}, t_0)} = \frac{\sum_{\vec{y}, \vec{z}_0} e^{-i\vec{p}\cdot(\vec{y}-\vec{x})} \mathcal{P} \left\langle N(\vec{y}, t_{\text{sep}} + t_0) \mathcal{O}_{\Gamma}^f(\vec{z}_0 + z\hat{z}, t_{\text{ins}} + t_0) \bar{N}(\vec{x}, t_0) \right\rangle}{\sum_{\vec{y}} e^{-i\vec{p}\cdot(\vec{y}-\vec{x})} \mathcal{P} \left\langle N(\vec{y}, t_{\text{sep}} + t_0) \bar{N}(\vec{x}, t_0) \right\rangle}$$

Courtesy of Jun Hua

To obtain the g_V and g_A from the 3pt correlation function, we should do the reduction by inserting,

$$1 = \int \frac{d^3 \vec{P}_q}{(2\pi)^3 2E_q} |q\rangle \langle q| \quad (24)$$

thus

$$C_{3,\mathcal{O}}^{N\vec{\rightarrow}P}(q^2, t, t_{\text{seq}}) = \int d^3 \vec{x} d^3 \vec{y} \int \frac{d^3 \vec{P}_N}{(2\pi)^3 2E_N} \frac{d^3 \vec{P}_P}{(2\pi)^3 2E_P} T^{\gamma\gamma'} \left\langle 0 \left| \psi_{\gamma}^{(P)}(\vec{x}, t_{\text{seq}}) \right| P \right\rangle \langle P | \mathcal{O}^{\mu}(\vec{y}, t) | N \rangle \left\langle N \left| \bar{\psi}_{\gamma'}^{(N)}(\vec{0}, 0) \right| 0 \right\rangle. \quad (25)$$

Consider the reduction for 2pt correlation function:

$$\begin{aligned} C_2^P &= \int d^3 \vec{x} T'^{\gamma\gamma'} \left\langle \psi_{\gamma}^{(P)}(\vec{x}, t) \bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0) \right\rangle \\ &= \int d^3 \vec{x} T'^{\gamma\gamma'} \int \frac{d^3 \vec{P}_q}{(2\pi)^3 2E_q} \left\langle \psi_{\gamma}^{(P)}(\vec{x}, t) | q \rangle \langle q | \bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0) \right\rangle. \end{aligned} \quad (26)$$

then the $1/2E \cdot 1/2E$ becomes one $1/2E$. due to the “same” condition impose a delta function

The u, d quarks do not differ in chroma, so the 2pt of protons and neutrons is the same. We can easily extract the g_V and g_A by taking the ratio of:

$$R_{V/A}(T, \mu) = \frac{C_3^{V/A}(q^2, t, t_{\text{seq}})}{C_2^P(t_{\text{seq}})}. \quad (27)$$

Backup: RI-MOM

Euclidean state with momentum squared $-p^2 \gg \Lambda_{\text{QCD}}^2$ in a fixed gauge, and then defines $\overline{\text{MS}}$ operators as,

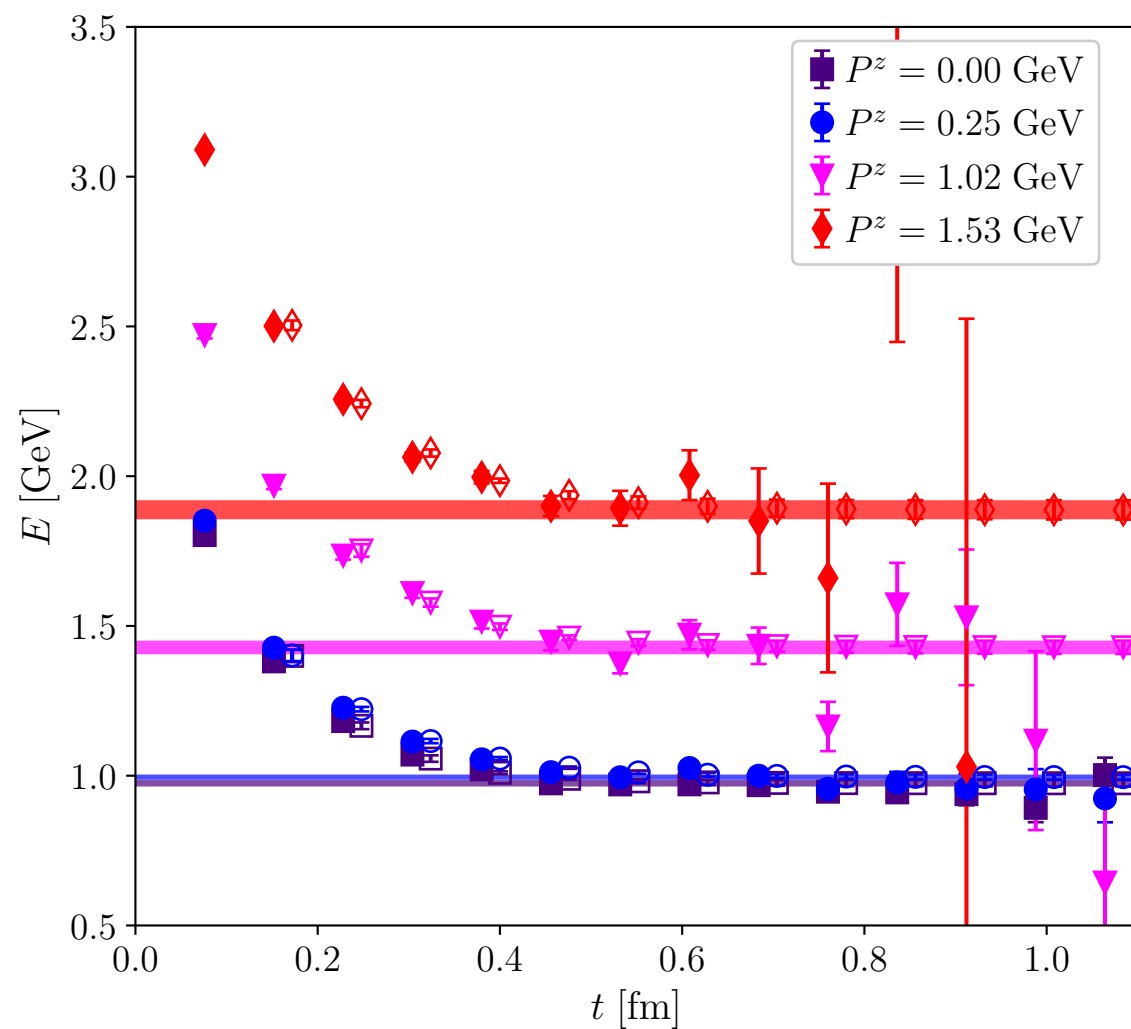
$$O_{\overline{\text{MS}}}(z, \mu) \equiv Z_{\overline{\text{MS}}}(z, -p^2, \mu) \frac{O(z, a)}{Z(z, -p^2, a)}, \quad (13)$$

where $Z_{\overline{\text{MS}}}$ converts the RI/MOM renormalized result to the $\overline{\text{MS}}$ scheme. The gauge and $-p^2$ dependences cancel between two Z -factors. The r.h.s. has a proper continuum limit $a \rightarrow 0$ without divergences.

However, while the RI/MOM approach is justified for local operators, it has potential problems when applied to nonlocal ones. For instance, when z becomes large, $Z_{\overline{\text{MS}}}(z, -p^2, \mu^2)$ contains IR logarithms of z and the perturbative calculation of z -dependence is not reliable. Moreover, although the RI/MOM factor $Z(z, -p^2, a)$ helps to cancel the lattice UV divergences, the composite operator at large- z contains non-perturbative physics as well. Therefore, both Z -factors contain non-cancelling non-perturbative effects which alter the IR properties of $O(z)$. Thus, the RI/MOM renormalization scheme is not reliable at large- z . Moreover, when gluon distributions are involved, it requires external off-shell gluon states which bring in potential mixing with gauge-variant operators and make things much more complicated [67].

arXiv:2008.03886

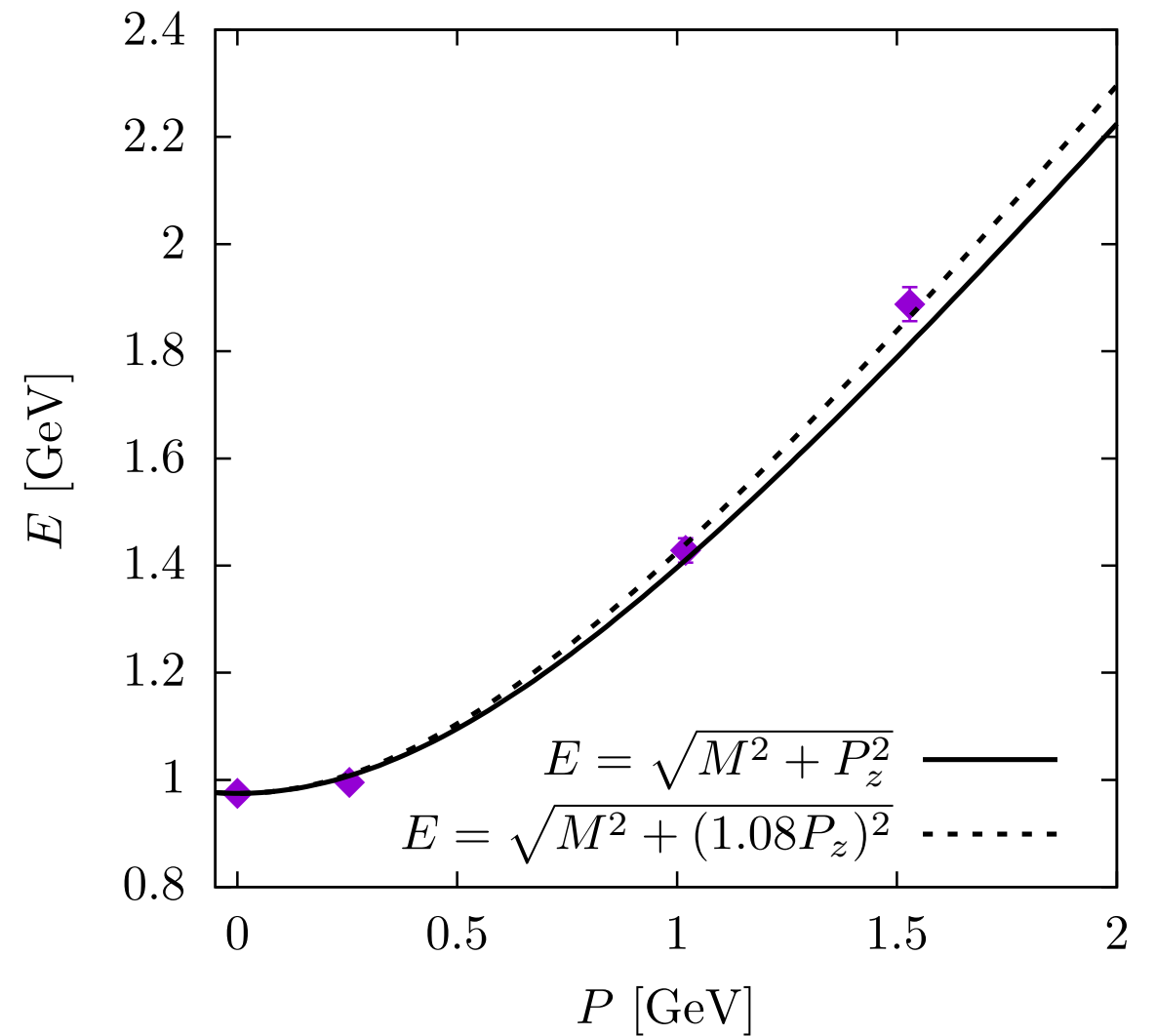
Sanity check of the joint fit



Filled points: effective mass from 2pt

Open points: effective mass from 2pt reconstructed from the joint-fits

Bands: ground-state mass from joint fit of R and 2pt

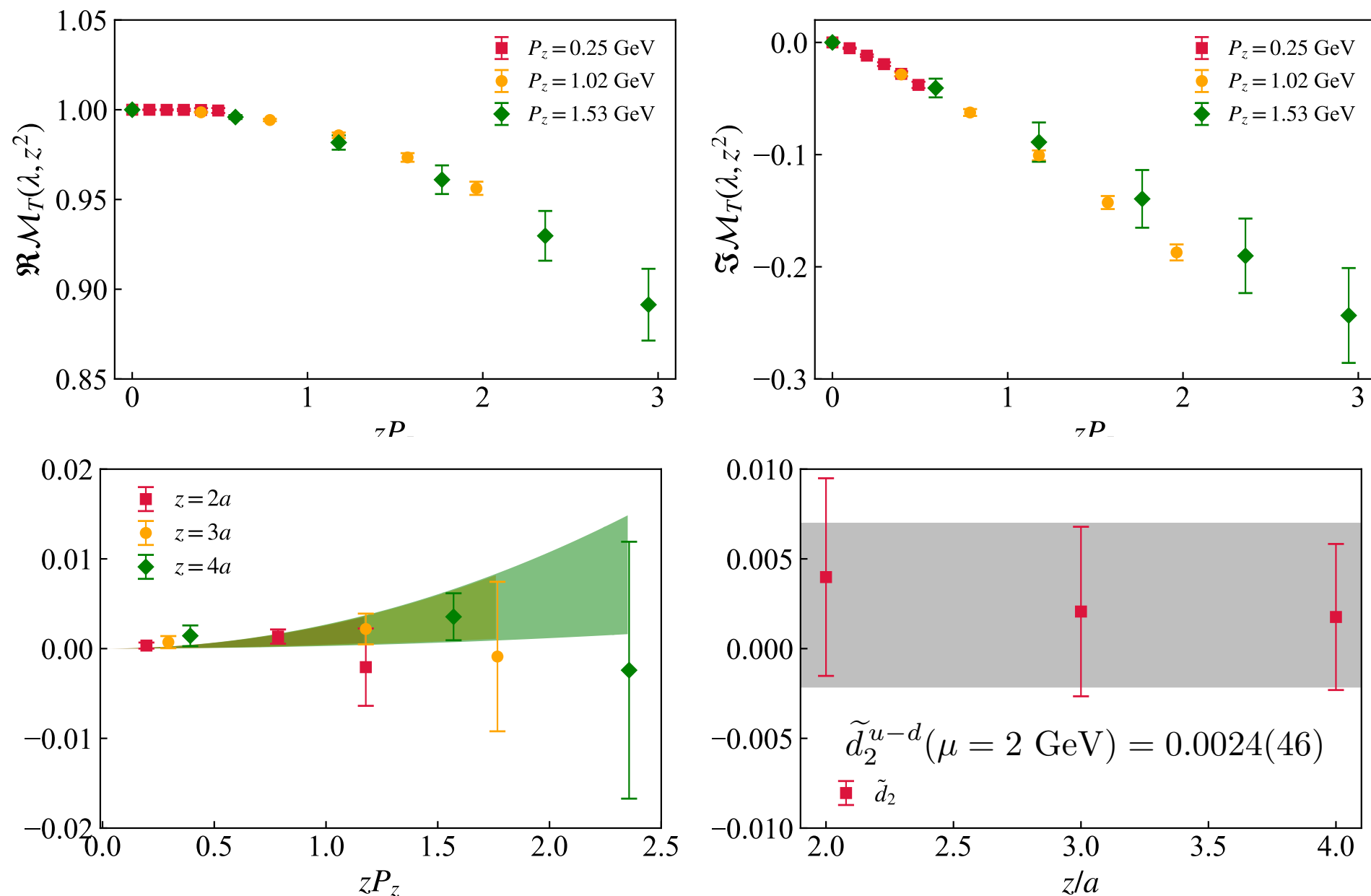


Good control of lattice discretization effects

Backup: $\tilde{d}_2$ -term

$\tilde{d}_2$ -term: genuine twist-3 quark–gluon correlation inside the polarized proton, probing how the quark is coupled to the gluon

$$\tilde{d}_2 = \int_0^1 dx x^2 [3g_T(x) - g_1(x)] \quad \Gamma = \gamma_5 \gamma_x$$



Backup: determination of renormalon

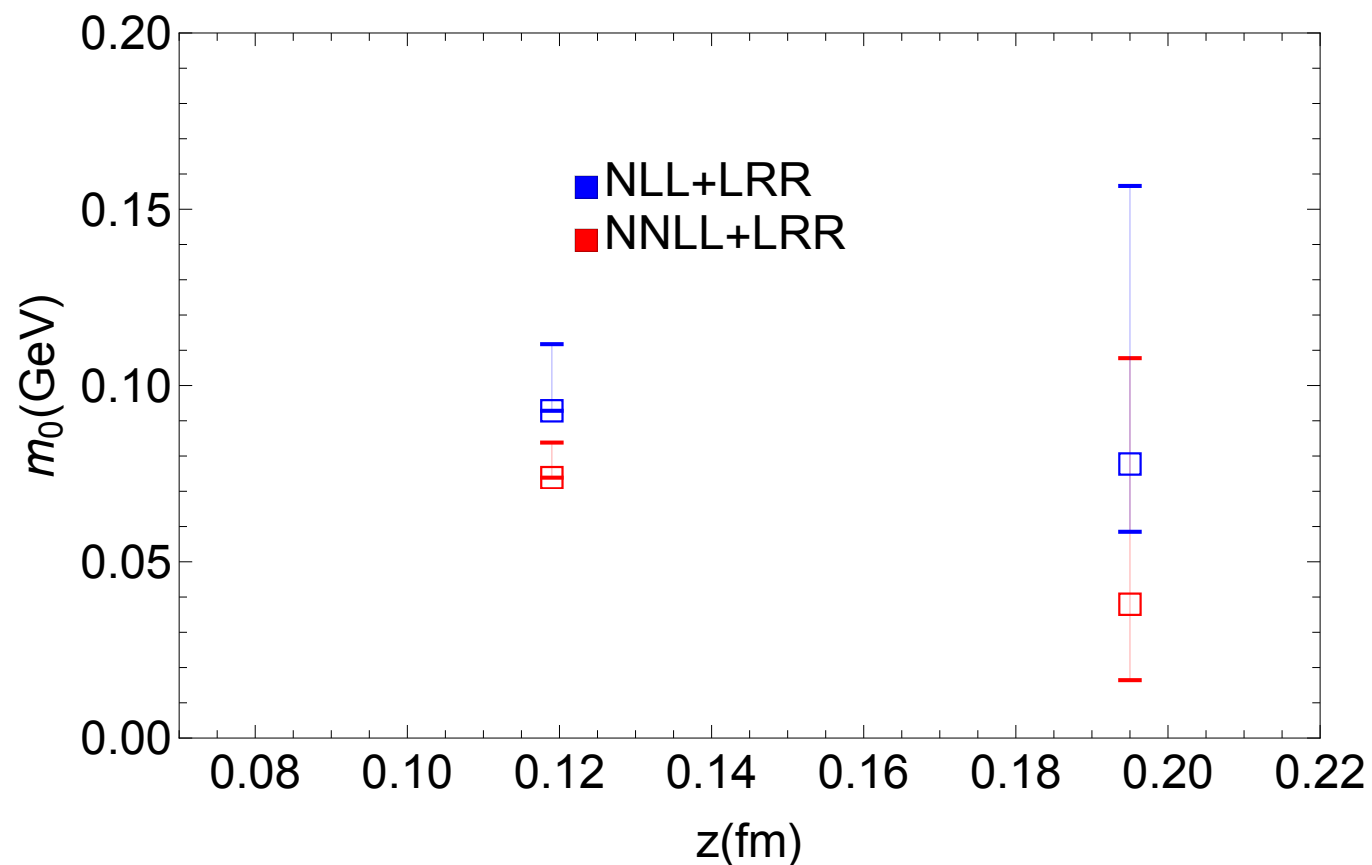
$$\frac{\tilde{h}_1^B(z, 0, a)}{\tilde{h}_1^B(z', 0, a)} e^{|z-z'|[\delta m(a)+m_0]} = \frac{C_{0,\text{PV}}^{\text{F.O.}+\text{LRR}}(z, \mu_0) e^{-\mathcal{I}(\mu_0)}}{C_{0,\text{PV}}^{\text{F.O.}+\text{LRR}}(z', \mu'_0) e^{-\mathcal{I}(\mu'_0)}}$$

Linear divergence from
WL self-energy

$a\delta m = 0.1597(16)$

$$C_{0,\text{PV}}^{\text{F.O.}+\text{LRR}}(\mu_0) \equiv C_{0,\text{PV}}^{\text{LRR}}(\mu_0) + \left[C_0^{\text{F.O.}}(\mu_0) - C_{0,\text{PV}}^{\text{LRR}}(\mu_0) \Big|_{\text{F.O.}} \right]$$

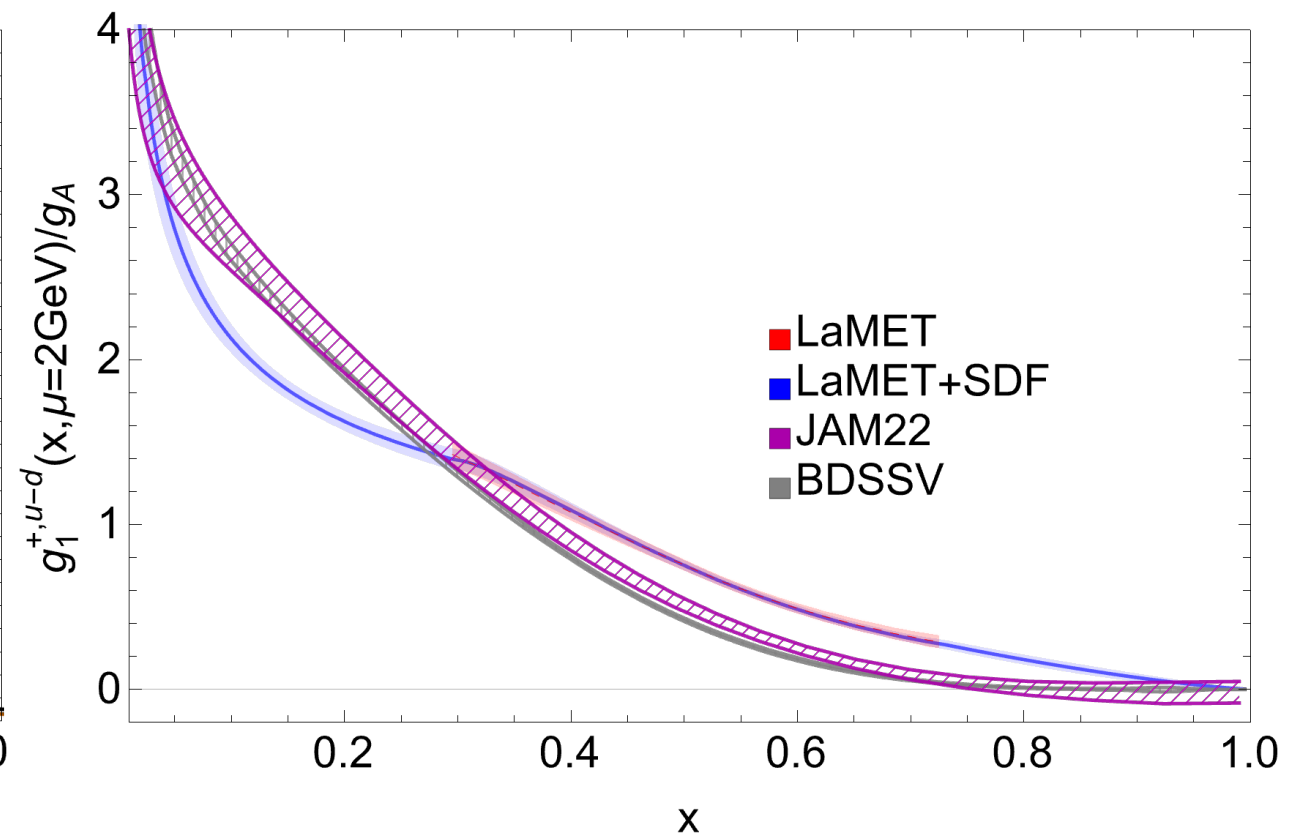
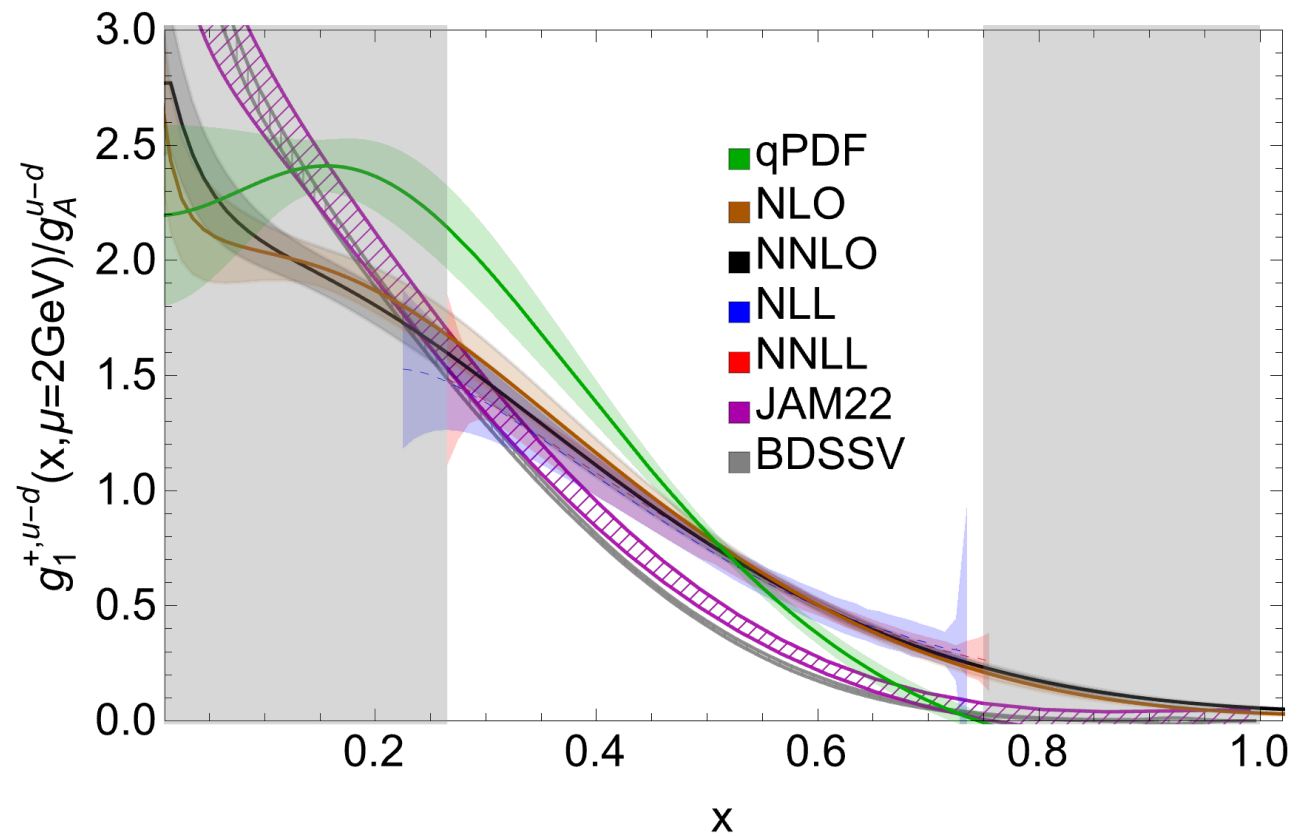
$$\mu_0 = \frac{2\kappa e^{-\gamma_E}}{z}, \quad \mu'_0 = \frac{2\kappa e^{-\gamma_E}}{z'}, \quad \mathcal{I}(\mu) = \int d\alpha \frac{\gamma(\alpha)}{\beta(\alpha)} \Big|_{\alpha=\alpha_s(\mu)}$$



The size of z -dependence and perturbative convergence is small

Backup: full distributions g_+ (valence dist. g_-)

$$g_1^+(x) \equiv g_1(x) + g_1(-x) = g_1(x) + \bar{g}_1(x)$$



Backup: iso-scalar u+d

