Global Spin Alignment of $^{4}\mathrm{Li}$ in Heavy-Ion Collisions

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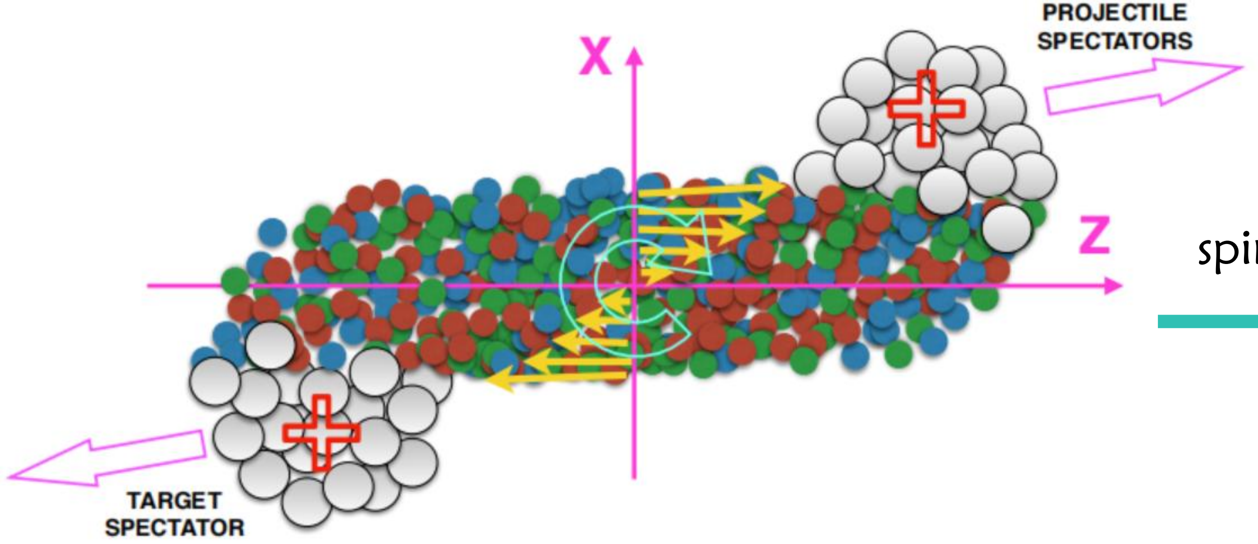
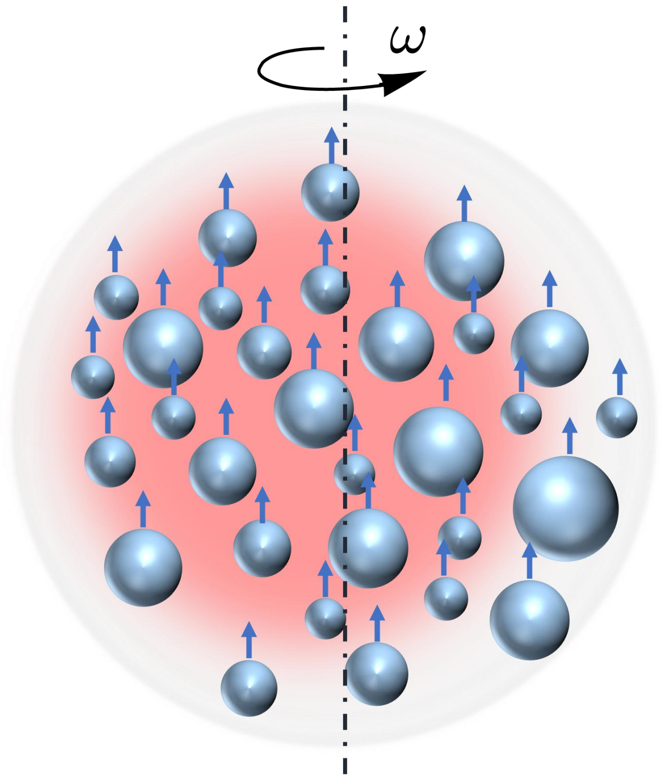
In Collaborated with Dai-Neng Liu, Lie-Wen Chen, Jin-Hui Chen, Che Ming Ko, Yu-Gang Ma, Kai-Jia Sun, Jun Xu and Bo Zhou

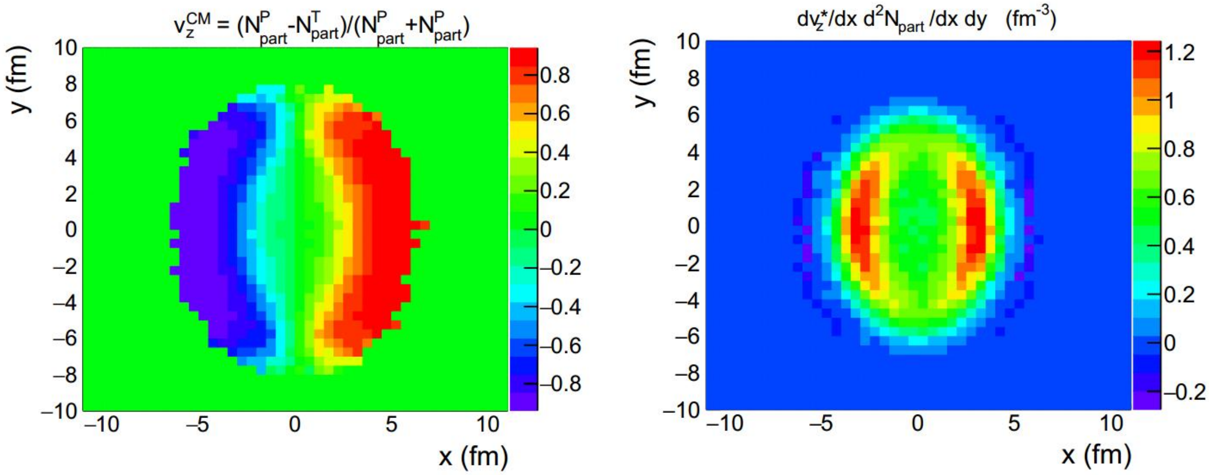
Qingdao, July. 28, 2026

Globally polarized QGP in Heavy-ion Collisions



spin-orbital coupling

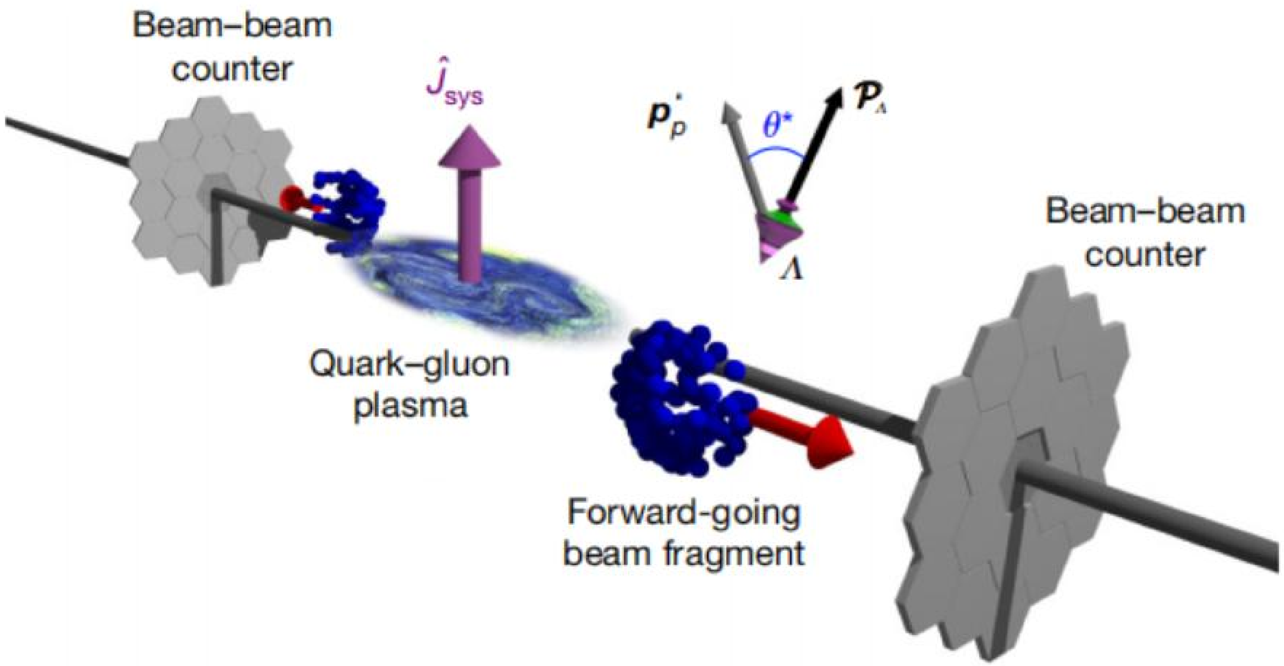




Niida and S. A. Voloshin, Int. J. Mod. Phys. E 33, 2430010(2024)

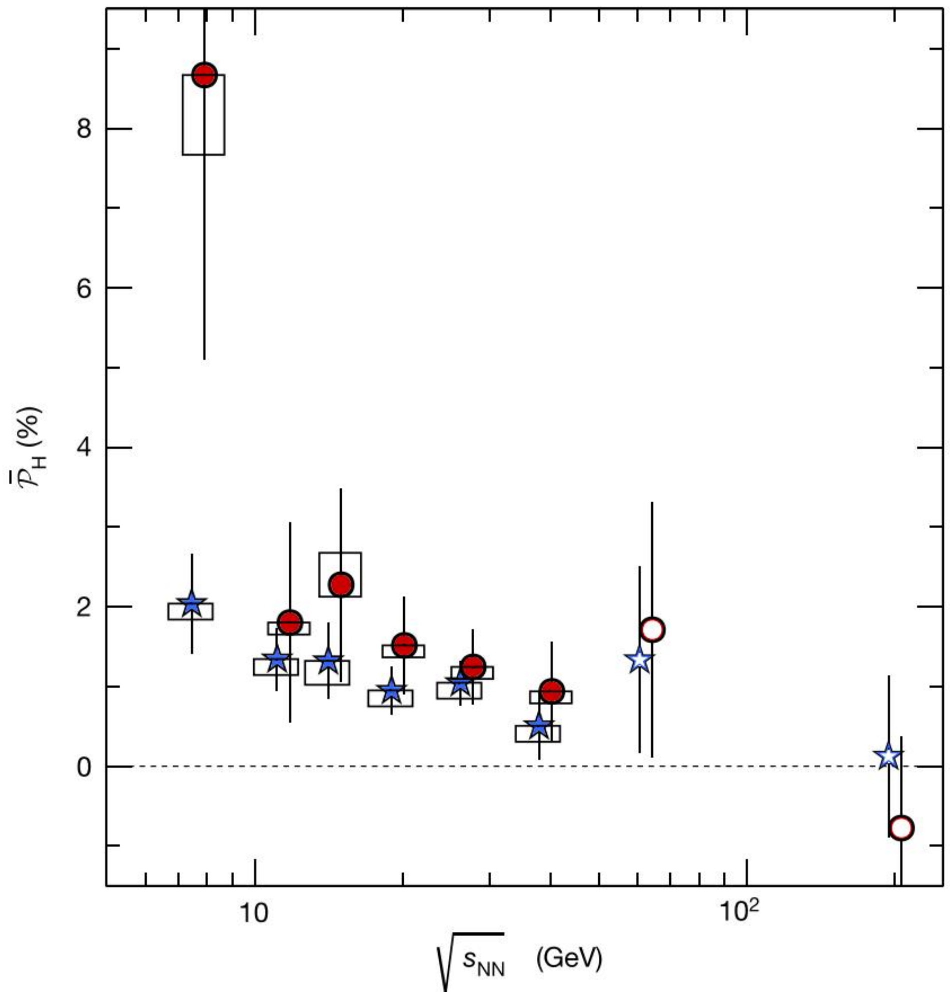
Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005)
Z.-T. Liang and X.-N. Wang, Phys. Lett. B 629, 20 (2005)

Global Λ hyperon polarization



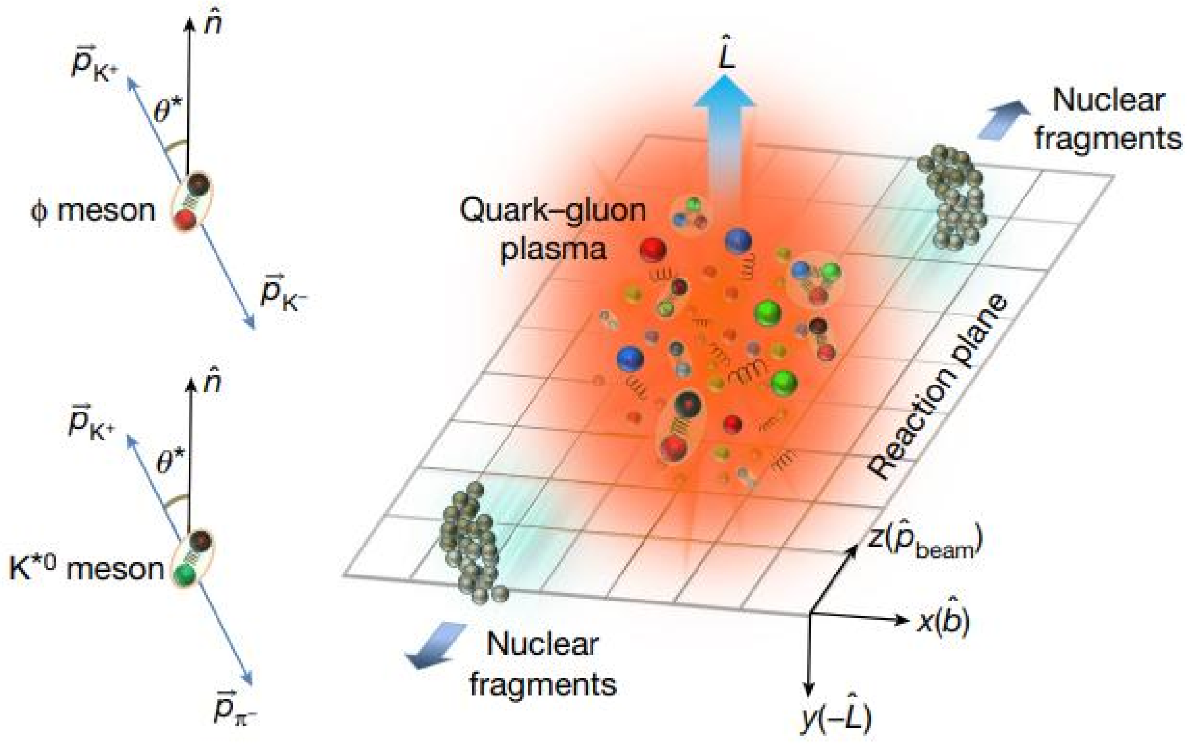
$$\Lambda \to p + \pi^-$$

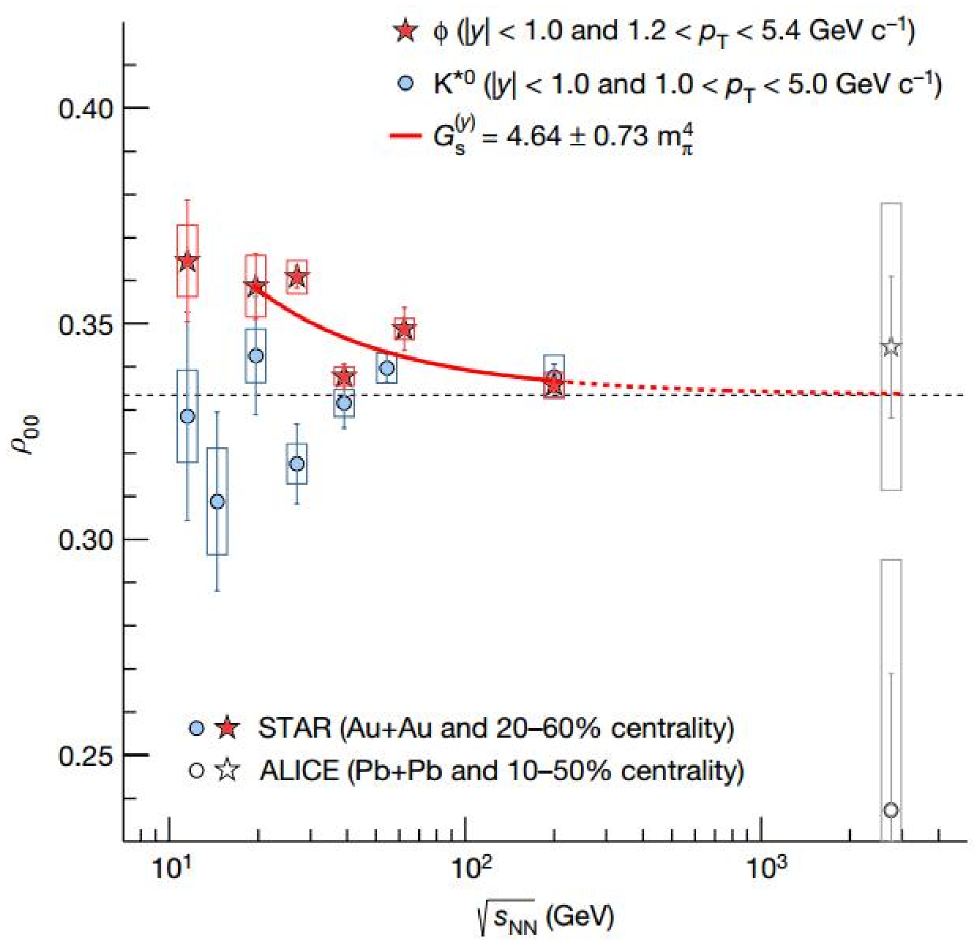
$$\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{4\pi}(1 + \alpha_\Lambda |P_\Lambda| \cos\theta^*)$$



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

Spin alignment of vector meson

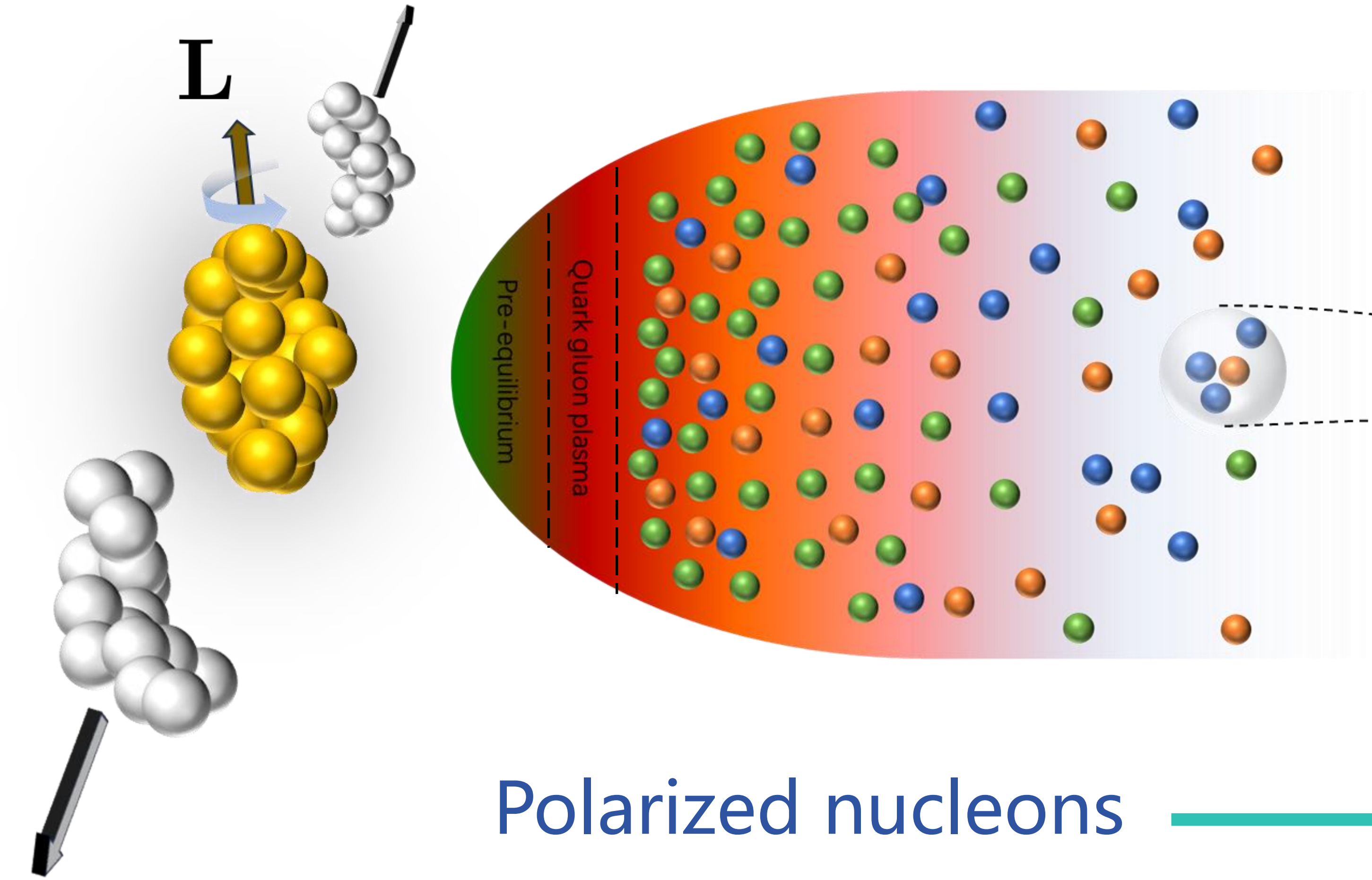


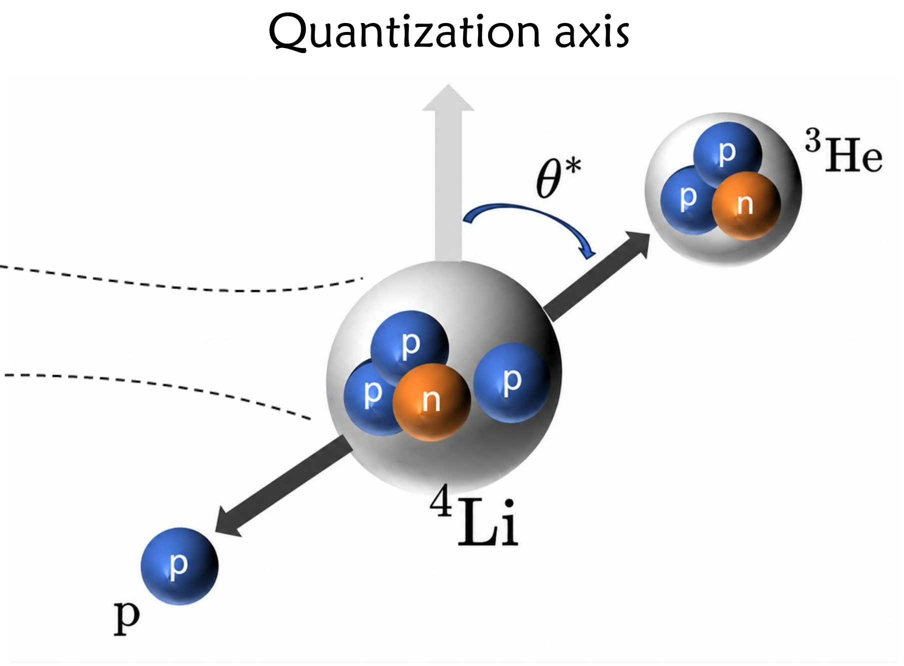


$$\frac{\mathrm{d}N}{\mathrm{d}(\cos\theta^*)} \propto (1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*$$

STAR Collaboration, M. S. Abdallah et al., Nature 614, 244 (2023)

Polarized nuclei





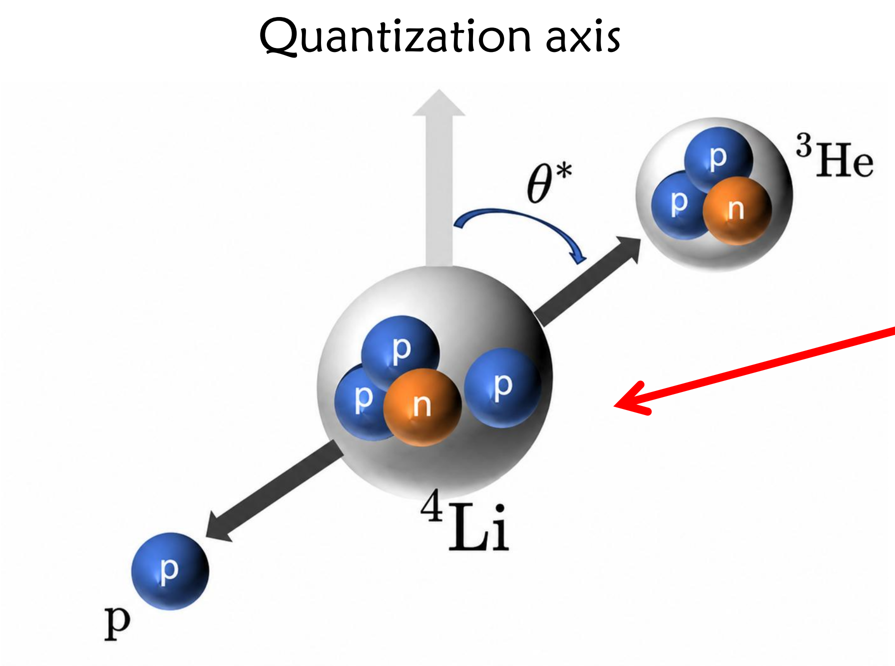
Polarized nucleons → Polarized nuclei

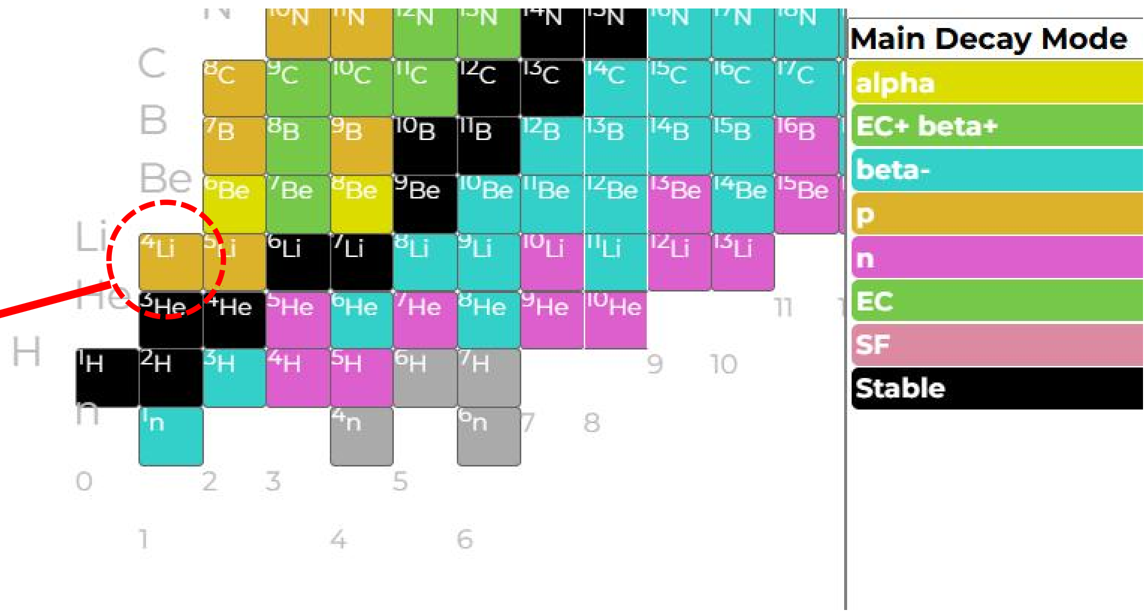
R.-J. Liu and J. Xu, Phys. Rev. C 109, 014615 (2024)

Kai-Jia Sun et al., Phys. Rev. Lett. 134, 022301 (2025)

Yun-Peng Zheng et al., arXiv:2509. 15286 (2025)

Unstable nucleus ^{4}Li





$A=4$	E_x(MeV)	J^π	Decay channels
^{4}Li	g.s.	2^-	p(100%)
	0.32	1^-	p(100%)
	2.08	0^-	p(100%)
	2.85	1^-	p(100%)

Decay angular distribution :

$$\frac{dN}{d\Omega^*} = \frac{\mathrm{Tr}[\hat{T}^\dagger \hat{\rho} \hat{T}]}{\int d\Omega^* \ \mathrm{Tr}[\hat{T}^\dagger \hat{\rho} \hat{T}]}$$

T_{ij} : initial spin state i $\longrightarrow$ final spin state j

V. Vovchenko, B. D¨onigus, B. Kardan, M. Lorenz, and H. Stoecker, Phys. Lett. B, 135746 (2020)

Productions of the ground states with defferent J_z

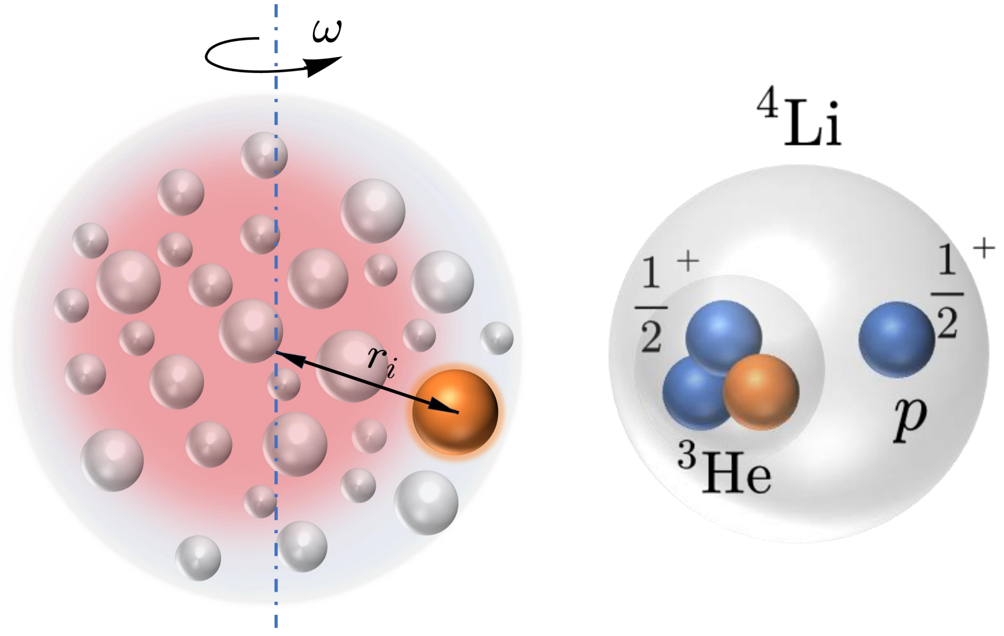
$$E\frac{\mathrm{d}^3N_{^4\mathrm{Li},J_z=m}}{\mathrm{d}\mathbf{K}^3} = E\int\left[\prod_{i=1,2,3,4} k_i^\mu \mathrm{d}^3\sigma_\mu \frac{\mathrm{d}^3k_i}{E_i}\bar{f}_i(\mathbf{x}_i,\mathbf{k}_i)\right]$$
$$\times\ \mathrm{Tr}_s[\hat{W}(\mathbf{r}_1',\mathbf{k}_1',\mathbf{r}_2',\mathbf{k}_2',\mathbf{r}_3',\mathbf{k}_3';J_z=m)$$
$$\times\ \hat{\sigma}_{p_1p_2p_3n}]\times\delta(\mathbf{K}-\sum_i\mathbf{k}_i).$$

$$W^{m_1m_2m_3m_4}_{\tilde{m}_1\tilde{m}_2\tilde{m}_3\tilde{m}_4} = \int d\eta_1' d\eta_2' d\eta_3' e^{-i(\eta_1'\mathbf{k}_1'+\eta_2'\mathbf{k}_2'+\eta_3'\mathbf{k}_3')}$$
$$\times\left\langle \mathbf{r}_1'+\frac{\eta_1'}{2},\mathbf{r}_2'+\frac{\eta_2'}{2},\mathbf{r}_3'+\frac{\eta_3'}{2};\tilde{m}_1\ldots\tilde{m}_4|2,m\right\rangle_{\mathrm{rel}}$$
$$\times\left\langle 2,m|\ \mathbf{r}_1'-\frac{\eta_1'}{2},\mathbf{r}_2'-\frac{\eta_2'}{2},\mathbf{r}_3'-\frac{\eta_3'}{2};m_1\ldots m_4\right\rangle_{\mathrm{rel}}$$

(without hadron spin correlation)

$$\hat{\sigma}_{p_1p_2p_3n} = \hat{\sigma}_{p_1}\otimes\hat{\sigma}_{p_2}\otimes\hat{\sigma}_{p_3}\otimes\hat{\sigma}_n$$

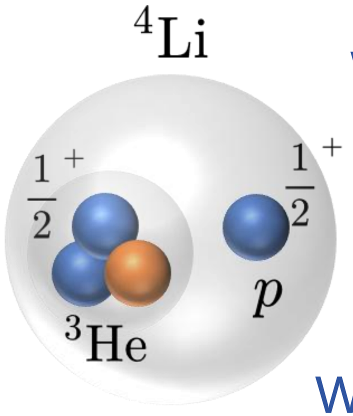
$$\sigma_j = \mathrm{diag}[\frac{1+\mathcal{P}_j}{2},\frac{1-\mathcal{P}_j}{2}]$$



Single particle distribution in a vortical fluid

$$f(\mathbf{r}_i,\mathbf{k}_i) = \frac{\xi_i'}{(2\pi)^3}e^{-\frac{(\mathbf{k}_i-m\omega\times\mathbf{r}_i)^2}{2mT}}e^{\frac{m\omega^2\mathbf{r}_i^2}{2T}}$$

Wavefunctions of $^4\mathrm{Li}$ ground state in shell model



Harmonic oscillator potential

$$V = \frac{1}{2}m_1\omega^2 r_1^2 + \frac{1}{2}m_2\omega^2 r_2^2 + \frac{1}{2}m_3\omega^2 r_3^2 + \frac{1}{2}m_4\omega^2 r_4^2$$
$$= \frac{1}{2}M\omega^2 R^2 + \frac{1}{2}\mu_1\omega^2 {r'_1}^2 + \frac{1}{2}\mu_2\omega^2 {r'_2}^2 + \frac{1}{2}\mu_3\omega^2 {r'_3}^2$$

Wavefunction in shell model

$$\langle \mathbf{r_1},\ldots,\mathbf{r_4}|2,2\rangle = \sqrt{\frac{1}{3!}}\begin{bmatrix} \phi_{M=1}^{L=1}(\mathbf{r_1})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1} & \phi_{M=1}^{L=1}(\mathbf{r_2})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2} & \phi_{M=1}^{L=1}(\mathbf{r_3})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3} \\ \phi_{M=0}^{L=0}(\mathbf{r_1})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1} & \phi_{M=0}^{L=0}(\mathbf{r_2})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2} & \phi_{M=0}^{L=0}(\mathbf{r_3})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3} \\ \phi_{M=0}^{L=0}(\mathbf{r_1})\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p1} & \phi_{M=0}^{L=0}(\mathbf{r_2})\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p2} & \phi_{M=0}^{L=0}(\mathbf{r_3})\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p3} \end{bmatrix} \phi_{M=0}^{L=0}(\mathbf{r_4})\left|\frac{1}{2},\frac{1}{2}\right\rangle_n$$

$$= \sqrt{\frac{1}{6}}(\phi_{M=1}^{L=1}(\mathbf{r_1})\phi_{M=0}^{L=0}(\mathbf{r_2})\phi_{M=0}^{L=0}(\mathbf{r_3})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p3}$$
$$-\phi_{M=1}^{L=1}(\mathbf{r_1})\phi_{M=0}^{L=0}(\mathbf{r_2})\phi_{M=0}^{L=0}(\mathbf{r_3})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p2}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3}$$
$$+\phi_{M=1}^{L=1}(\mathbf{r_3})\phi_{M=0}^{L=0}(\mathbf{r_1})\phi_{M=0}^{L=0}(\mathbf{r_2})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p2}$$
$$-\phi_{M=1}^{L=1}(\mathbf{r_2})\phi_{M=0}^{L=0}(\mathbf{r_1})\phi_{M=0}^{L=0}(\mathbf{r_3})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p1}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p3}$$
$$-\phi_{M=1}^{L=1}(\mathbf{r_3})\phi_{M=0}^{L=0}(\mathbf{r_2})\phi_{M=0}^{L=0}(\mathbf{r_1})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p1}$$
$$+\phi_{M=1}^{L=1}(\mathbf{r_2})\phi_{M=0}^{L=0}(\mathbf{r_3})\phi_{M=0}^{L=0}(\mathbf{r_1})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p2}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p3}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p1})\phi_{M=0}^{L=0}(\mathbf{r_n})\left|\frac{1}{2},\frac{1}{2}\right\rangle_n$$

One dimension harmonic oscillator wavefunction

$$\phi_0 = \frac{1}{\sqrt{b\sqrt{\pi}}}\exp\left[-\frac{x^2}{2b^2}\right]$$

$$\phi_1 = \sqrt{\frac{2}{b\sqrt{\pi}}}\frac{x}{b}\exp\left[-\frac{x^2}{2b^2}\right]$$

Construct three dimension harmonic oscillator wavefunction

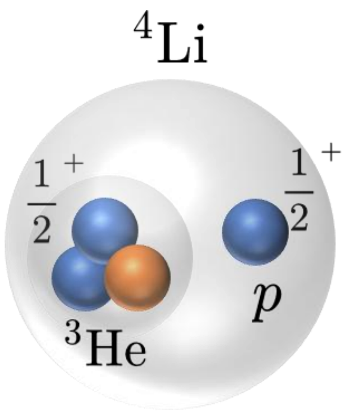
$$\phi_{M=1}^{L=1}(\mathbf{r}) = \frac{1}{\sqrt{2}}[\phi_1(x)\phi_0(y) + i\phi_0(x)\phi_1(y)]\phi_0(z)$$

$$\phi_{M=-1}^{L=1}(\mathbf{r}) = \phi_{M=1}^{L=1}(\mathbf{r})^*$$

$$\phi_{M=0}^{L=1}(\mathbf{r}) = \phi_0(x)\phi_0(y)\phi_1(z)$$

A. J. Baltz and C. Dover, Phys. Rev. C 53, 362 (1996)

^{4}Li internal wavefunction of ground state



$$\begin{aligned}\langle \mathbf{r_1},\ldots,\mathbf{r_4}\,|2,2\rangle &= \langle \mathbf{R},\mathbf{r}'_1,\mathbf{r}'_2,\mathbf{r}'_3\,|2,2\rangle \\ &= 4^{\frac{3}{4}}(B^2\pi)^{-\frac{3}{4}}\exp\left[-\frac{R^2}{2B^2}\right] \\ &\times\sqrt{\frac{1}{6}}(b_1'^2 b_2'^2 b_3'^2\pi^3)^{-\frac{3}{4}}\exp\left[-\frac{r_1'^2}{2b_1'^2}-\frac{r_2'^2}{2b_2'^2}-\frac{r_3'^2}{2b_3'^2}\right]\times\left(\sqrt{2}\frac{x_1'+iy_1'}{b_1'}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n}\right. \\ &+(-\frac{\sqrt{2}}{2}\frac{x_1'+iy_1'}{b_1'}-\frac{\sqrt{6}}{2}\frac{x_2'+iy_2'}{b_2'})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n} \\ &\left.+(-\frac{\sqrt{2}}{2}\frac{x_1'+iy_1'}{b_1'}+\frac{\sqrt{6}}{2}\frac{x_2'+iy_2'}{b_2'})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n}\right).\end{aligned}$$

Under trasformation

$$\mathbf{R}=\frac{\mathbf{r_1}+\mathbf{r_2}+\mathbf{r_3}+\mathbf{r_4}}{4},\quad \mathbf{r}'_1=\frac{\mathbf{r_1}-\mathbf{r_2}}{\sqrt{2}},\quad \mathbf{r}'_2=\sqrt{\frac{2}{3}}\frac{\mathbf{r_1}+\mathbf{r_2}-2\mathbf{r_3}}{2},\quad \mathbf{r}'_3=\sqrt{\frac{3}{4}}\frac{\mathbf{r_1}+\mathbf{r_2}+\mathbf{r_3}-3\mathbf{r_4}}{3},$$

$$\mathbf{K}=\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}+\mathbf{k_4},\quad \mathbf{k}'_1=\frac{\mathbf{k_1}-\mathbf{k_2}}{\sqrt{2}},\quad \mathbf{k}'_2=\sqrt{\frac{2}{3}}\frac{\mathbf{k_1}+\mathbf{k_2}-2\mathbf{k_3}}{2},\quad \mathbf{k}'_3=\sqrt{\frac{3}{4}}\frac{\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}-3\mathbf{k_4}}{3},$$

Internal wavefunction

$$\begin{aligned}\langle \mathbf{r}'_1,\mathbf{r}'_2,\mathbf{r}'_3|2,2\rangle_{\mathrm{rel}} &= \sqrt{\frac{1}{6}}(b_1'^2 b_2'^2 b_3'^2\pi^3)^{-\frac{3}{4}}\exp\left[-\frac{r_1'^2}{2b_1'^2}-\frac{r_2'^2}{2b_2'^2}-\frac{r_3'^2}{2b_3'^2}\right] \\ &\times\left(\sqrt{2}\frac{x_1'+iy_1'}{b_1'}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n}\right. \\ &+(-\frac{\sqrt{2}}{2}\frac{x_1'+iy_1'}{b_1'}-\frac{\sqrt{6}}{2}\frac{x_2'+iy_2'}{b_2'})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n} \\ &\left.+(-\frac{\sqrt{2}}{2}\frac{x_1'+iy_1'}{b_1'}+\frac{\sqrt{6}}{2}\frac{x_2'+iy_2'}{b_2'})\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_1}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{p_2}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{p_3}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{n}\right).\end{aligned}$$

^{4}Li production with Moyal star product

Trace of Wigner matrix

$$\mathbf{Tr}_s[\hat{W}(\mathbf{r}_1',\mathbf{k}_1',\mathbf{r}_2',\mathbf{k}_2',\mathbf{r}_3',\mathbf{k}_3';J_z=2)\times\hat{\sigma}_{p_1p_2p_3n}]$$
$$=8^3\,\frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\exp\left[-\frac{r_1'^{\,2}}{b_1'^{\,2}}-b_1'^{\,2}k_1'^{\,2}-\frac{r_2'^{\,2}}{b_2'^{\,2}}-b_2'^{\,2}k_2'^{\,2}-\frac{r_3'^{\,2}}{b_3'^{\,2}}-b_3'^{\,2}k_3'^{\,2}\right]$$
$$\times\frac{1}{2}\left[\left(\frac{x_1'^{\,2}+y_1'^{\,2}}{b_1'^{\,2}}-1+b_1'^{\,2}(k_{x_1}'^{\,2}+k_{y_1}'^{\,2})+2(k_{y_1}'x_1'-k_{x_1}'y_1')\right)+\left(\frac{x_2'^{\,2}+y_2'^{\,2}}{b_2'^{\,2}}-1+b_2'^{\,2}(k_{x_2}'^{\,2}+k_{y_2}'^{\,2})+2(k_{y_2}'x_2'-k_{x_2}'y_2')\right)\right]$$

^{4}Li production with Moyal star product

$$N_{^4Li,J_z=2}\approx 2\sqrt{2}\,\frac{N^2}{V^2}\left(\frac{2\pi}{mT}\right)^3\frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\int d^3r_1'd^3k_1'\exp\left(-\frac{k_1'^{\,2}}{mT}\right)$$
$$\times 8\exp\left(-\frac{r'^{\,2}}{b^2}-b^2k_1'^{\,2}\right)\left[\frac{x_1'^{\,2}+y_1'^{\,2}}{b^2}-1+b^2(k_x^2+k_y^2)+2(k_{y1}'x_1'-k_{x1}'y_1')\right]$$
$$\boxed{\times\exp_\star\left(\frac{\omega_z(k_{y1}'x_1'-k_{x1}'y_1')}{T}\right)}$$
$$\approx 2\sqrt{2}\,\frac{N^2}{V^2}\left(\frac{2\pi}{mT}\right)^3\frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\int d^3r_1'd^3k_1'\exp\left(-\frac{k_1'^{\,2}}{mT}\right)$$
$$\times 8\exp\left(-\frac{r'^{\,2}}{b^2}-b^2k_1'^{\,2}\right)\left[\frac{x_1'^{\,2}+y_1'^{\,2}}{b^2}-1+b^2(k_x^2+k_y^2)+2(k_{y1}'x_1'-k_{x1}'y_1')\right]$$
$$\boxed{\times\left(1+\frac{\omega_z(k_{y1}'x_1'-k_{x1}'y_1')}{T}+\frac{\omega_z^2(k_{y1}'x_1'-k_{x1}'y_1')^2}{2T^2}-\frac{\omega_z^2}{4T^2}\right)}$$

Moyal star product restores the $\mathcal{P}^2$ term

$$A(x,p)\star B(x,p)=A(x,p)\exp\frac{i\hbar}{2}(\overleftarrow{\partial}_x\overrightarrow{\partial}_p-\overleftarrow{\partial}_p\overrightarrow{\partial}_x)B(x,p)$$

$$\exp_\star\left(\frac{\omega}{T}L_z\right)=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\omega}{T}\right)^n\underbrace{L_z\star L_z\star\cdots\star L_z}_{n\uparrow L_z}$$

$$L_z\star L_z=L_z^2-\frac{\hbar^2}{2}$$

correspond to $\mathcal{P}^2$ term

Decay angular distribution for $^4\mathrm{Li}(2^-)$ state

Productions of the ground states with defferent J_z

$$N_{^4Li,J_z=2} \approx 8\frac{N_p^3 N_n}{V^3}\left(\frac{2\pi}{mT}\right)^{\frac{9}{2}} \frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\left(1+2\mathcal{P}_L+2\mathcal{P}_L^2\right).$$

$$N_{^4\mathrm{Li},J_z=1} \approx 8\frac{N_p^3 N_n}{V^3}\left(\frac{2\pi}{mT}\right)^{\frac{9}{2}}\left(\frac{1}{2}\frac{(1+\mathcal{P}_N)^2(1-\mathcal{P}_N)^2}{16}\left(1+2\mathcal{P}_L+2\mathcal{P}_L^2\right) + \frac{1}{2}\frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\right)$$

$$N_{^4Li,J_z=0} \approx 8\frac{N_p^3 N_n}{V^3}\left(\frac{2\pi}{mT}\right)^{\frac{9}{2}}\left(\frac{1}{6}\frac{(1+\mathcal{P}_N)^3(1-\mathcal{P}_N)}{16}\left(1-2\mathcal{P}_L+2\mathcal{P}_L^2\right) + \frac{2}{3}\frac{(1+\mathcal{P}_N)^2(1-\mathcal{P}_N)^2}{16} + \frac{1}{6}\frac{(1+\mathcal{P}_N)(1-\mathcal{P}_N)^3}{16}\left(1+2\mathcal{P}_L+2\mathcal{P}_L^2\right)\right)$$

$$N_{^4\mathrm{Li},J_z=-1} \approx 8\frac{N_p^3 N_n}{V^3}\left(\frac{2\pi}{mT}\right)^{\frac{9}{2}}\left(\frac{1}{2}\frac{(1+\mathcal{P}_N)^2(1-\mathcal{P}_N)^2}{16}\left(1-2\mathcal{P}_L+2\mathcal{P}_L^2\right) + \frac{1}{2}\frac{(1+\mathcal{P}_N)(1-\mathcal{P}_N)^3}{16}\right)$$

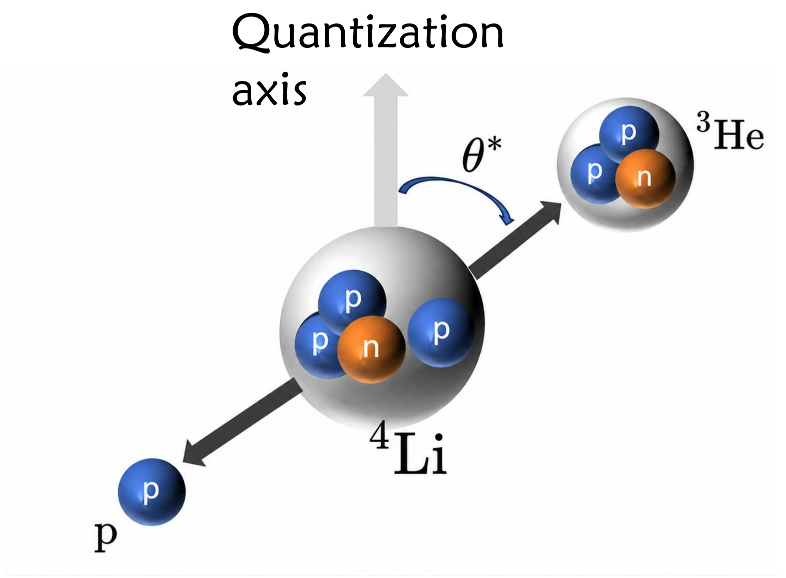
$$N_{^4Li,J_z=-2} \approx 8\frac{N_p^3 N_n}{V^3}\left(\frac{2\pi}{mT}\right)^{\frac{9}{2}} \frac{(1+\mathcal{P}_N)(1-\mathcal{P}_N)^3}{16}\left(1-2\mathcal{P}_L+2\mathcal{P}_L^2\right)$$

polarization of orbital motion $\mathcal{P}_L = \frac{\omega}{2T}$

$$\hat{\rho}\left(^4\mathrm{Li}(2^-)\right) = \mathrm{diag}\left[\frac{3(\mathcal{P}_N+1)^2(2\mathcal{P}_L(\mathcal{P}_L+1)+1)}{4\left(2\mathcal{P}_N^2+5\right)\mathcal{P}_L^2+20\mathcal{P}_N\mathcal{P}_L+5\left(\mathcal{P}_N^2+3\right)}, \frac{3\left(-\mathcal{P}_L(\mathcal{P}_L+1)\mathcal{P}_N^2+\mathcal{P}_N+\mathcal{P}_L^2+\mathcal{P}_L+1\right)}{4\left(2\mathcal{P}_N^2+5\right)\mathcal{P}_L^2+20\mathcal{P}_N\mathcal{P}_L+5\left(\mathcal{P}_N^2+3\right)}, \frac{-\mathcal{P}_N^2-4\mathcal{P}_L\mathcal{P}_N+2\left(\mathcal{P}_N^2+1\right)\mathcal{P}_L^2+3}{4\left(2\mathcal{P}_N^2+5\right)\mathcal{P}_L^2+20\mathcal{P}_N\mathcal{P}_L+5\left(\mathcal{P}_N^2+3\right)}, -\frac{3(\mathcal{P}_N-1)((\mathcal{P}_N+1)(\mathcal{P}_L-1)\mathcal{P}_L+1)}{4\left(2\mathcal{P}_N^2+5\right)\mathcal{P}_L^2+20\mathcal{P}_N\mathcal{P}_L+5\left(\mathcal{P}_N^2+3\right)}, \frac{3(\mathcal{P}_N-1)^2(2(\mathcal{P}_L-1)\mathcal{P}_L+1)}{4\left(2\mathcal{P}_N^2+5\right)\mathcal{P}_L^2+20\mathcal{P}_N\mathcal{P}_L+5\left(\mathcal{P}_N^2+3\right)}\right]$$

$$T = \sqrt{\frac{3}{8\pi}}\begin{bmatrix} -\sin\theta^* e^{i\phi^*} & 0 & 0 & 0 \\ \cos\theta^* & -\frac{1}{2}\sin\theta^* e^{i\phi^*} & -\frac{1}{2}\sin\theta^* e^{i\phi^*} & 0 \\ \sqrt{\frac{1}{6}}\sin\theta^* e^{-i\phi^*} & \sqrt{\frac{2}{3}}\cos\theta^* & \sqrt{\frac{2}{3}}\cos\theta^* & -\sqrt{\frac{1}{6}}\sin\theta^* e^{i\phi^*} \\ 0 & \frac{1}{2}\sin\theta^* e^{-i\phi^*} & \frac{1}{2}\sin\theta^* e^{-i\phi^*} & \cos\theta^* \\ 0 & 0 & 0 & \sin\theta^* e^{-i\phi^*} \end{bmatrix},$$

$$\frac{dN}{\sin\theta^* d\theta^*} = \frac{1}{2} + \left(\frac{3}{8}\hat{\rho}_{1,1} + \frac{3}{8}\hat{\rho}_{-1,-1} + \frac{1}{2}\hat{\rho}_{0,0} - \frac{1}{4}\right)\left(3\cos^2\theta^* - 1\right)$$
$$\approx \frac{1}{2}\left[1 - \frac{7}{30}\left(\mathcal{P}_N^2 + 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)\left(3\cos^2\theta^* - 1\right)\right]$$



Decay angular distribution for ^{4}Li excited states

Decay angular distribution for $^4\mathrm{Li}(^3P_1)$ state

$$\hat{\rho}(^4\mathrm{Li}(^3P_1)) = [\frac{-\mathcal{P}_L(\mathcal{P}_L+1)\mathcal{P}_N^2+\mathcal{P}_N+\mathcal{P}_L^2+\mathcal{P}_L+1}{(\mathcal{P}_N-2\mathcal{P}_L)^2+3}, \frac{\mathcal{P}_N^2-4\mathcal{P}_L\mathcal{P}_N+2\left(\mathcal{P}_N^2+1\right)\mathcal{P}_L^2+1}{(\mathcal{P}_N-2\mathcal{P}_L)^2+3}, -\frac{(\mathcal{P}_N-1)((\mathcal{P}_N+1)(\mathcal{P}_L-1)\mathcal{P}_L+1)}{(\mathcal{P}_N-2\mathcal{P}_L)^2+3}].$$

$$\hat{T}(^4\mathrm{Li}(^3P_1)\to{}^3\mathrm{He}+p) = \sqrt{\frac{3}{8\pi}}\begin{bmatrix} -\cos\theta^* & -\frac{1}{2}\sin\theta^* e^{i\phi^*} & -\frac{1}{2}\sin\theta^* e^{i\phi^*} & 0 \\ -\sqrt{\frac{1}{2}}\sin\theta^* e^{-i\phi^*} & 0 & 0 & -\sqrt{\frac{1}{2}}\sin\theta^* e^{i\phi^*} \\ 0 & -\frac{1}{2}\sin\theta^* e^{-i\phi^*} & -\frac{1}{2}\sin\theta^* e^{-i\phi^*} & \cos\theta^* \end{bmatrix}$$

$$\frac{dN}{\sin\theta^* d\theta^*} = \frac{1}{2} - \frac{3}{8}(\hat{\rho}_{0,0}-\frac{1}{3})(3\cos^2\theta^*-1)$$
$$\approx \frac{1}{2}\left(1-\frac{1}{6}(\mathcal{P}_N^2-4\mathcal{P}_N\mathcal{P}_L+\mathcal{P}_L^2)(3\cos^2\theta^*-1)\right),$$

Decay angular distribution for $^4\mathrm{Li}(^1P_1)$ state

$$\hat{\rho}(^4\mathrm{Li}(^1P_1)) = \begin{bmatrix} \frac{2\mathcal{P}_L(\mathcal{P}_L+1)+1}{4\mathcal{P}_L^2+3} & 0 & 0 \\ 0 & \frac{1}{4\mathcal{P}_L^2+3} & 0 \\ 0 & 0 & \frac{2(\mathcal{P}_L-1)\mathcal{P}_L+1}{4\mathcal{P}_L^2+3} \end{bmatrix}$$

$$\hat{T}(^4\mathrm{Li}(^1P_1)\to{}^3\mathrm{He}+p) = \sqrt{\frac{3}{8\pi}}\begin{bmatrix} 0 & -\sqrt{\frac{1}{2}}\sin\theta^* e^{i\phi^*} & \sqrt{\frac{1}{2}}\sin\theta^* e^{i\phi^*} & 0 \\ 0 & \cos\theta^* & -\cos\theta^* & 0 \\ 0 & \sqrt{\frac{1}{2}}\sin\theta^* e^{-i\phi^*} & -\sqrt{\frac{1}{2}}\sin\theta^* e^{-i\phi^*} & 0 \end{bmatrix}$$

$$\frac{dN}{\sin\theta^* d\theta^*} = \frac{1}{2} + \frac{3}{4}(\hat{\rho}_{0,0}-\frac{1}{3})(3\cos^2\theta^*-1)$$
$$\approx \frac{1}{2}\left(1-\frac{2}{3}\mathcal{P}_L^2(3\cos^2\theta^*-1)\right)$$

State	E (MeV)	Structure	L	Decay mode	Γ(MeV)	$\frac{dN}{\sin\theta^* d\theta^*}$
$^4\mathrm{Li}(^3P_2)$	g.s.	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	6.03	$\frac{1}{2}\left(1 - \frac{7}{30}\left(\mathcal{P}_N^2 + 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)(3\cos^2\theta^* - 1)\right)$
$^4\mathrm{Li}(^3P_1)$	0.32	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	7.35	$\frac{1}{2}\left(1 - \frac{1}{6}\left(\mathcal{P}_N^2 - 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)(3\cos^2\theta^* - 1)\right)$
$^4\mathrm{Li}(^3P_0)$	2.08	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	9.35	$\frac{1}{2}$
$^4\mathrm{Li}(^1P_1)$	2.85	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	13.51	$\frac{1}{2}\left(1 - \frac{2}{3}\mathcal{P}_L^2(3\cos^2\theta^* - 1)\right)$

Averaged decay angular distribution : $\frac{dN}{\sin\theta d\theta} = \frac{1}{2}\left(1 - \frac{1}{36}\left(5\mathcal{P}_N^2 + 8\mathcal{P}_N\mathcal{P}_L + 11\mathcal{P}_L^2\right)(3\cos^2\theta - 1)\right)$

^{4}Li decay angular distribution within thermal model

$$\hat{\rho}(^4\mathrm{Li}(2^-)) = \frac{1}{1 + 2\cosh(\frac{\omega}{T}) + 2\cosh(\frac{2\omega}{T})}\begin{bmatrix} e^{\frac{2\omega}{T}} & 0 & 0 & 0 & 0 \\ 0 & e^{\frac{\omega}{T}} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e^{-\frac{\omega}{T}} & 0 \\ 0 & 0 & 0 & 0 & e^{-\frac{2\omega}{T}} \end{bmatrix}, \hat{\rho}(^4\mathrm{Li}(1^-)) = \frac{1}{1 + 2\cosh(\frac{\omega}{T})}\begin{bmatrix} e^{\frac{\omega}{T}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\frac{\omega}{T}} \end{bmatrix}, \hat{\rho}(^4\mathrm{Li}(0^-)) = 1.$$

Let $\mathcal{P} = \frac{\omega}{2T}$,

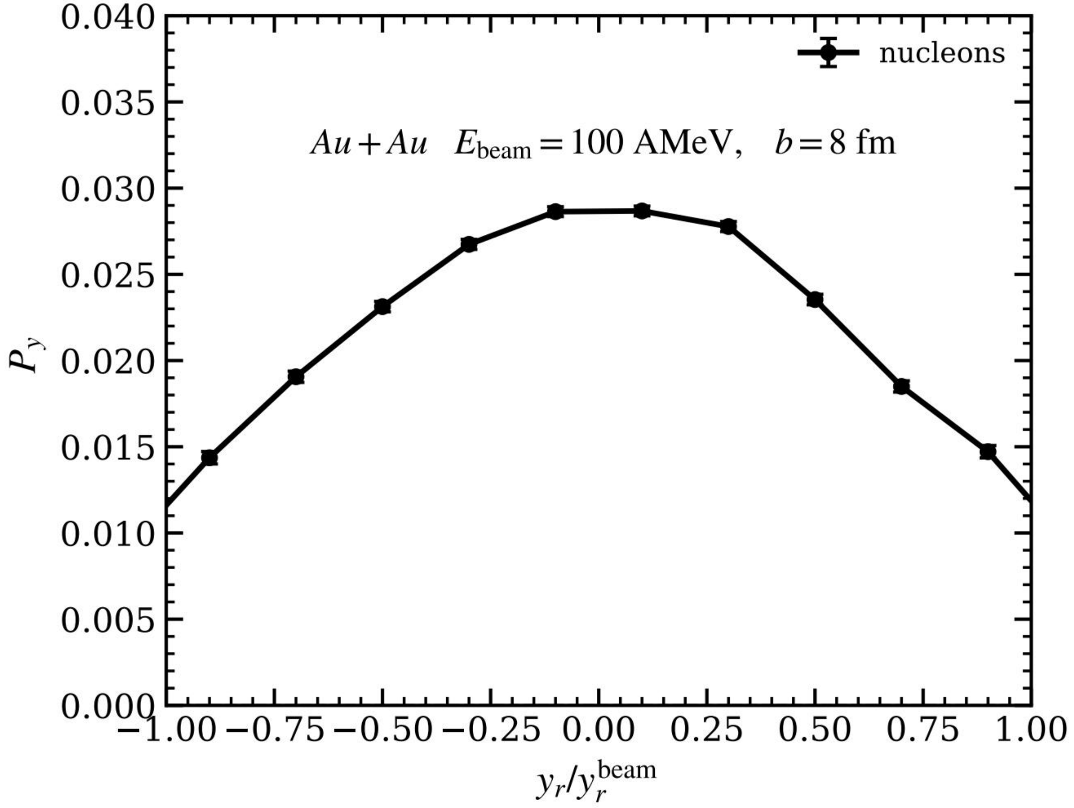
Averaged decay angular distribution from thermal model

$$\frac{dN}{\sin\theta d\theta} = \frac{1}{2}\left(1 - \frac{2}{3}\mathcal{P}^2(3\cos^2\theta - 1)\right)$$

$\mathcal{P}_L = \mathcal{P}_N = \mathcal{P}$

Y.P. Zheng et al., Phys. Rev. C 113, 054906(2026)

Spin dependent coalescence of $^4\mathrm{Li}(2^-)$state



$$\frac{dN}{\sin\theta^* d\theta^*} = \frac{1}{2} + \left(\frac{3}{8}\hat{\rho}_{1,1} + \frac{3}{8}\hat{\rho}_{-1,-1} + \frac{1}{2}\hat{\rho}_{0,0} - \frac{1}{4}\right)(3\cos^2\theta^* - 1)$$
$$\approx \frac{1}{2}\left[1 - \frac{7}{30}\left(\mathcal{P}_N^2 + 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)(3\cos^2\theta^* - 1)\right]$$

Predicts a angular anisotropy coefficient on the order of 10^{-4}

Two step coalescence

$$p_1 + p_2 + n_3 \longrightarrow {}^3\mathrm{He}$$

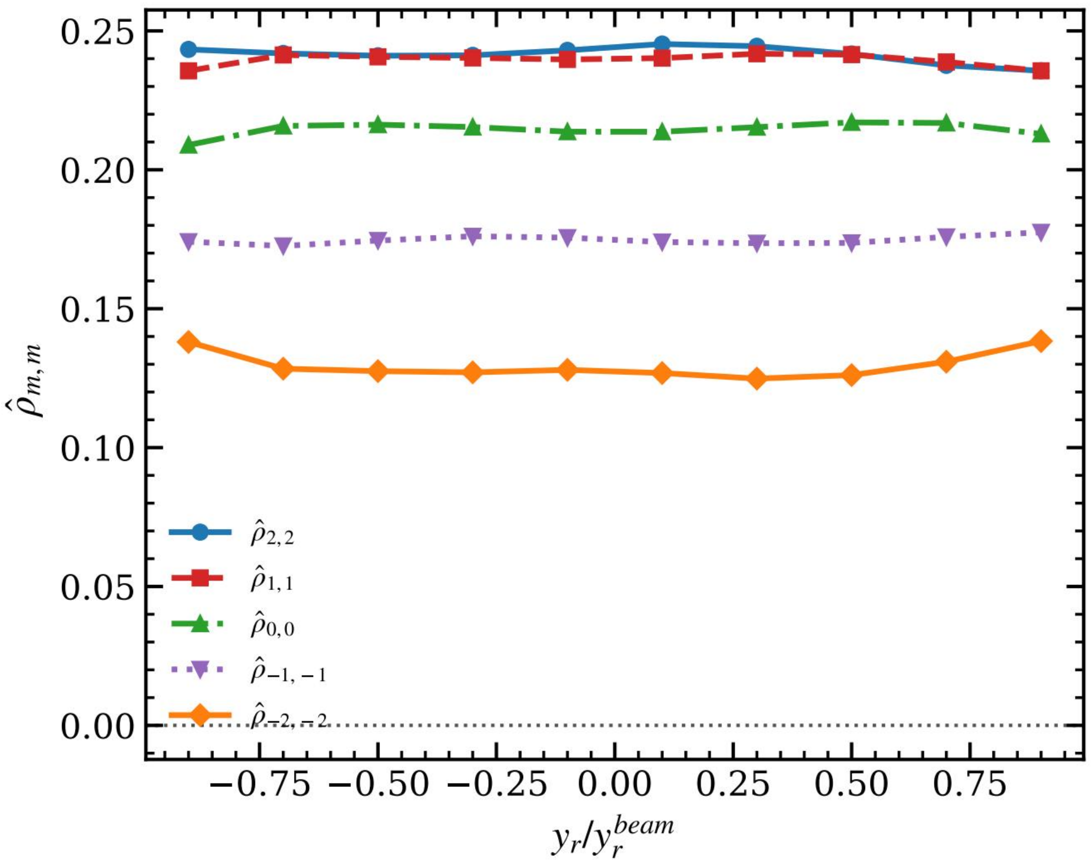
$$f_{^3\mathrm{He}} = 8^2 g_{^3\mathrm{He}} \exp\left(-\frac{\rho^2+\lambda^2}{\sigma^2_{^3\mathrm{He}}} - \left(p_\rho^2 + p_\lambda^2\right)\sigma^2_{^3\mathrm{He}}\right)$$

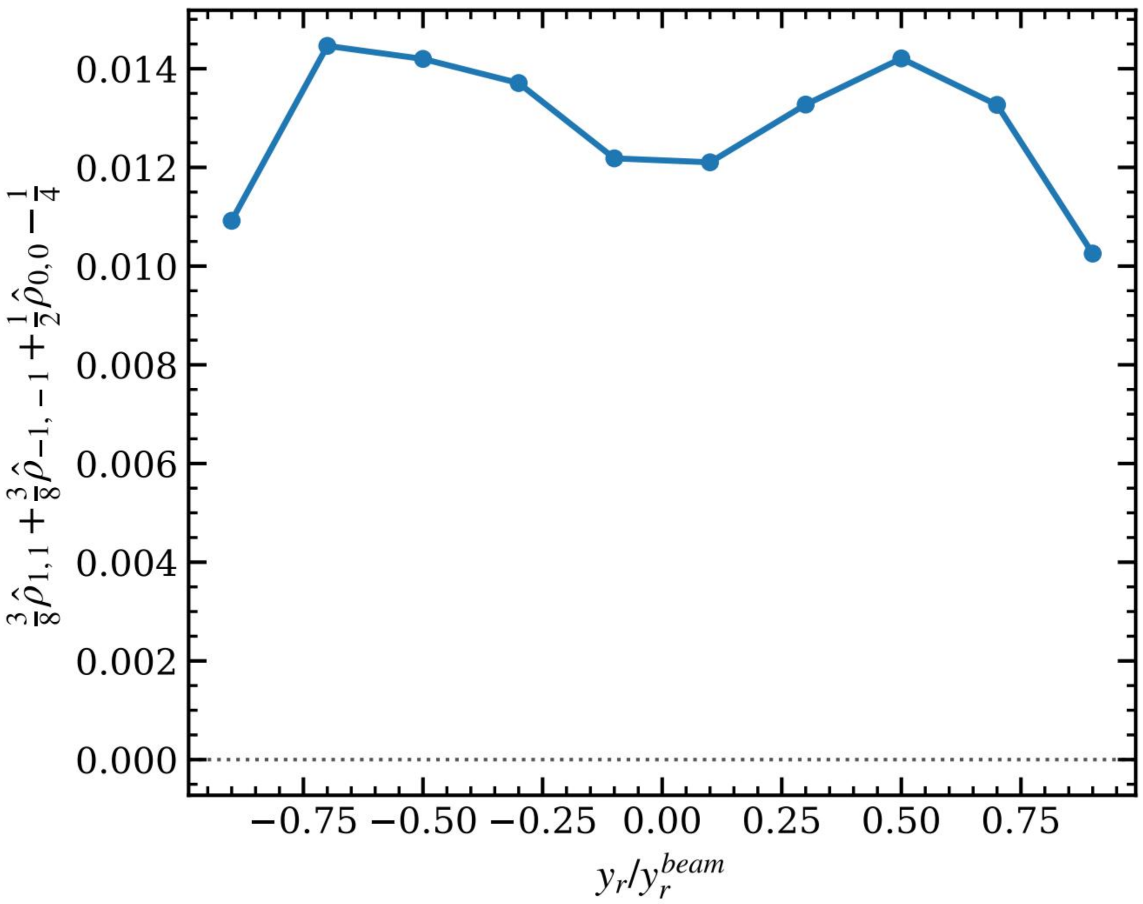
$${}^3\mathrm{He} + p_4 \longrightarrow {}^4\mathrm{Li}$$

$$f_{^4\mathrm{Li}}^{J_z=2} = f^{L_z=1} g_{^4\mathrm{Li}}^{S_z=1},$$
$$f_{^4\mathrm{Li}}^{J_z=1} = \frac{1}{2} f^{L_z=1} g_{^4\mathrm{Li}}^{S_z=0} + \frac{1}{2} f^{L_z=0} g_{^4\mathrm{Li}}^{S_z=1},$$
$$f_{^4\mathrm{Li}}^{J_z=0} = \frac{1}{6} f^{L_z=1} g_{^4\mathrm{Li}}^{S_z=-1} + \frac{2}{3} f^{L_z=0} g_{^4\mathrm{Li}}^{S_z=0} + \frac{1}{6} f^{L_z=-1} g_{^4\mathrm{Li}}^{S_z=1},$$
$$f_{^4\mathrm{Li}}^{J_z=-1} = \frac{1}{2} f^{L_z=0} g_{^4\mathrm{Li}}^{S_z=-1} + \frac{1}{2} f^{L_z=-1} g_{^4\mathrm{Li}}^{S_z=0},$$
$$f_{^4\mathrm{Li}}^{J_z=-2} = f^{L_z=-1} g_{^4\mathrm{Li}}^{S_z=-1}.$$

R.-J. Liu and J. Xu, Phys. Rev. C 109, 014615 (2024)

Decay angular distribution of $^4\mathrm{Li}(2^-)$ state





A angular anisotropy coefficient on the order of 10^{-2} !

Summary

1. The coalescence of polarized nucleons results in polarized light nuclei, the polarization of unstable nuclei can be measured.
2. We calculate the decay angular distribution of $^4\mathrm{Li}$ which can be measured experimentally.

State	E (MeV)	Structure	L	Decay mode	Γ(MeV)	$\frac{dN}{\sin\theta^* d\theta^*}$
$^4\mathrm{Li}(^3P_2)$	g.s.	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	6.03	$\frac{1}{2}\left(1 - \frac{7}{30}\left(\mathcal{P}_N^2 + 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)(3\cos^2\theta^* - 1)\right)$
$^4\mathrm{Li}(^3P_1)$	0.32	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	7.35	$\frac{1}{2}\left(1 - \frac{1}{6}\left(\mathcal{P}_N^2 - 4\mathcal{P}_N\mathcal{P}_L + \mathcal{P}_L^2\right)(3\cos^2\theta^* - 1)\right)$
$^4\mathrm{Li}(^1P_0)$	2.08	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	9.35	$\frac{1}{2}$
$^4\mathrm{Li}(^1P_1)$	2.85	$^3\mathrm{He}(\frac{1}{2}^+) - p(\frac{1}{2}^+)$	1	$^4\mathrm{Li} \to {}^3\mathrm{He} + p$	13.51	$\frac{1}{2}\left(1 - \frac{2}{3}\mathcal{P}_L^2(3\cos^2\theta^* - 1)\right)$

3. For nuclei with nonzero internal orbital angular momentum, there may be a large spin alignment induced by the phase-space anisotropy.

Thanks for your attention！