Deeply Virtual π Production through Two-Loop Order

Inverse DVMP via Exclusive Meson–Induced Drell–Yan

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DVMP: arXiv:2604.28164v2 (W. Chen, F. Feng, Y. Jia, Q.-T. Song, G. Tang, Z.-Y. Wang)
Inverse DVMP: arXiv:2607.15214 (Y. Jia, B. Pire, Q.-T. Song, G. Tang, Z.-Y. Wang).

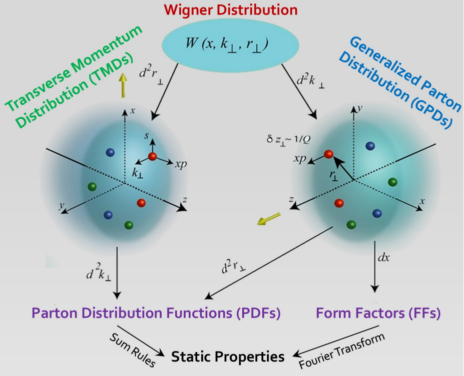
Outline

Generalized Parton Distributions (GPDs)

- Definition of GPDs:

$$F_{i/H}(x,\xi,t) = \frac{1}{2\bar{P}^+}\int \frac{\mathrm{d}x}{2\pi} \mathrm{e}^{-ix\bar{P}^+z^-} \langle P' | \mathcal{O}_i(z^-, -z^-) | P\rangle$$
$$= \frac{1}{2\bar{P}^+} \bar{u}(P') \left(H_i(x,\xi,t)\gamma^+ + E_i(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M} \right) u(P)$$

- GPDs encode the 3D structure of hadrons.
- Closely connected with Impact parameter distribution, PDFs and Form Factors (FFs).
- Access to electromagnetic FFs or axial/pseudoscalar FFs (first moment) and proton spin decomposition or gravitational FFs (second moment). Ji Sum rules(Ji 1996).



Exclusive reactions giving access to GPDs

- Deeply Virtual Compton Scattering (DVCS):$\gamma^* N \to \gamma N$
- Timelike Compton scattering (TCS):$\gamma N \to \gamma^* N$ (see Qin-tao Song's report)
- Double Deeply Virtual Compton Scattering (DDVCS):$\gamma^* N \to \gamma^* N$
- Deeply Meson Virtual Production (DVMP): $\gamma^* N \to N\ M$
  - light mesons production allows one to separate quark flavor GPDs.
- Exclusive Meson-Induced Drell–Yan (Inverse DVMP): $MN \to \gamma^* N$
- Current Experiments: (see Zhihong Ye's report)
  - fixed target experiments: JLab, HERMES, COMPASS.
  - collider experiments: H1, ZEUS.
- Upcoming precision era: EIC, EicC $\Rightarrow$ need high-precision theory.

Need for Higher-Order QCD Corrections

DVMP NLO

- DVπ^+P NLO accuracy (Belitsky et al. 2001).
- DVV_LP NLO accuracy (Ivanov, Szymanowski, et al. 2004, Diehl et al. 2007)
- NLO accuracy of light mesons (Müller et al. 2013)
- DVηP NLO accuracy (Duplančić et al. 2017)
- NLO accuracy of heavy vector quarkonium (Ivanov, Schäfer, et al. 2004, Flett et al. 2021)

NLO corrections are large; NNLO effects expected to be significant. **NNLO**

- DVCS and DDVCS coefficient functions known to NNLO (Braun et al. 2022, Yao Ji et al. 2024, Braun et al. 2024).
- **This work:** First calculation of NNLO QCD corrections to DVπP ($\gamma^* N \to \pi N$) at leading twist.

For DVVP, see the next report by Zhe-yu Wang.

Goal: Provide accurate theory for GPD extraction from pion production data.

Deeply Virtual Pion Production (Leading-twist)

- Process: $\gamma_L^*(q, \lambda = 0) + N(p, s) \to \pi(k) + N(p', s')$
- Transverse polarized photon is high-twist contribution.
- Differential cross section:

$$\frac{d\sigma_L}{dt} = \frac{1}{16\pi(W^2 - m_N^2)\kappa}\left[|M_{++}|^2 + |M_{-+}|^2\right]$$

- $$\kappa = \sqrt{\Lambda(W^2, -Q^2, m_N^2)}$$

- Nucleon helicity (s', s) conserving $(+,+)$ and helicity flip $(-,+)$ amplitudes.

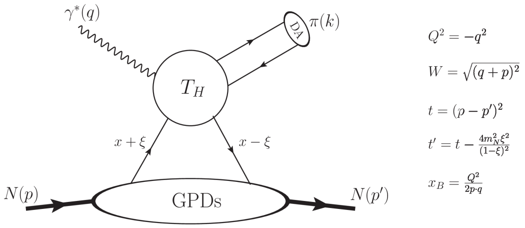


Figure: Factorization of the amplitude for DVπP.

Helicity Amplitudes and Transition Form Factors

Helicity amplitudes expressed via transition form factors:

$$M_{++} = \frac{4\pi e f_\pi}{N_c Q}\sqrt{1-\xi^2}\left[\tilde{\mathcal{H}}_\pi - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_\pi\right]$$

$$M_{-+} = \frac{4\pi e f_\pi}{N_c Q}\frac{\sqrt{-t'}}{2m}\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_\pi$$

Collinear Factorization (At leading-twist holds for any order perturbation (Collins et al. 1996))

In the general Bjorken limit $Q^2 \gg |t|, \Lambda^2_{QCD}$

$$\tilde{\mathcal{F}}_\pi = \int_{-1}^{1} dx \int_0^1 du\, \phi_\pi(u)\, T(u,x,\xi)\, \tilde{F}(x,\xi,t)$$

T = hard-scattering kernel, ϕ_π = pion DA, $\tilde{F}$ = axial-vector GPD ($\tilde{H}$ or $\tilde{E}$).

$$T(u,x,\xi) = C_F\alpha_s\left[T^{(0)} + \frac{\alpha_s}{\pi}T^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 T^{(2)} + \cdots\right]$$

Transverse Single-Spin Asymmetry A_{UT}

- Transverse Single-Spin Asymmetry (TSSA):

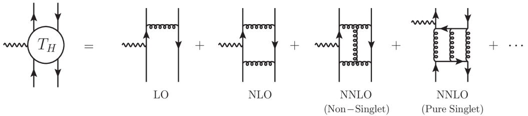
$$A_{UT}(\phi,\phi_S)=\frac{1}{|P_T|}\frac{\mathrm{d}\sigma^{\uparrow}(\phi,\phi_S)-\mathrm{d}\sigma^{\downarrow}(\phi,\phi_S)}{\mathrm{d}\sigma^{\uparrow}(\phi,\phi_S)+\mathrm{d}\sigma^{\downarrow}(\phi,\phi_S)}$$

- The Fourier amplitude $A_{UT}^{\sin(\phi-\phi_S)}$ in case of production by longitudinal photons, can be calculated in our work.

$$A_{UT}^{\sin(\phi-\phi_S)}=\frac{\xi\sqrt{1-\xi^2}\,\mathrm{Im}(\tilde{\mathcal{H}}_\pi\tilde{\mathcal{E}}_\pi^*)}{(1-\xi^2)|\tilde{\mathcal{H}}_\pi|^2-\frac{t}{4m^2}\xi^2|\tilde{\mathcal{E}}_\pi|^2-2\xi^2\,\mathrm{Re}(\tilde{\mathcal{H}}_\pi\tilde{\mathcal{E}}_\pi^*)}\left(-\frac{\sqrt{-t'}}{m}\right)$$

- Sensitive to relative phase of $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$, offering unique sensitivity to the poorly constrained GPD $\tilde{E}(x,\xi,t)$.

Hard-Scattering Kernel and Pion EMFF Connection



For π^+: $T_{\pi^+}(u,x,\xi) = e_u T(\bar{u},-x,\xi) - e_d T(u,x,\xi)$

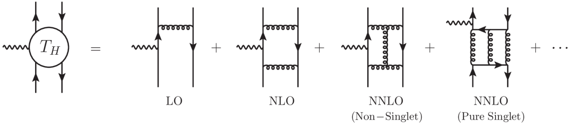
Relation to Pion EMFF

$$T(u,x,\xi) = \frac{1}{2\xi} T_{\text{EMFF}}\left(u, \frac{x+(\xi-i\varepsilon)}{2(\xi-i\varepsilon)}\right)$$

$\Rightarrow$ NNLO kernels obtained from recent two-loop pion EMFF calculations (Chen et al. 2023, Ji et al. 2024).

- Matching equation:$\mathcal{F}^{(n)}(u,x) = \sum_{i=0}^{n}\sum_{j=0}^{n-i} \phi^{(i)}(z|u) \underset{z}{\otimes} T^{(j)}(z,y,\xi) \underset{y}{\otimes} F^{(n-i-j)}(y|x)$.
- The same partonic subprocess $\gamma^* q\bar{q} \to q\bar{q}$ as in pion electromagnetic form factor(EMFF).
- ERBL region: $\mathcal{F}^{(n)} = \mathcal{F}^{(n)}_{\text{EMFF}}$, $F^{(i)} = \phi^{(i)}$.
- DGLAP region: analytic continuation $\xi \to \xi - i\epsilon$. $T^{(0)} = \frac{1}{u(x+\xi-i\epsilon)}$

Neutral Pion Production and Pure Singlet Contribution



For π^0:

$$\tilde{\mathcal{F}}_{\pi^0} = \int dx \int du\, \phi_\pi(u) \Big[T^{\rm NS}_{\pi^0}(u,x,\xi)\tilde{F}^{\pi^0}(x,\xi,t) + \frac{1}{\sqrt{2}} T^{\rm pS}_{\pi^0}(u,x,\xi)\tilde{F}^{S}(x,\xi,t)\Big]$$

- $T^{\rm NS}_{\pi^0}(u,x,\xi) = [T(\bar{u},-x,\xi) - T(u,x,\xi)]$,
- $T^{\rm pS}$ (pure singlet) vanishes at LO and NLO (color, isospin and C-parity), and first non-zero contribution at NNLO.
- $\tilde{F}^{\pi^0}(x,\xi,t) = \frac{1}{\sqrt{2}}(e_u\tilde{F}^{u(-)}(x,\xi,t) - e_d\tilde{F}^{d(-)}(x,\xi,t))$, $\tilde{F}^{S}(x,\xi,t) = \tilde{F}^{u(-)}(x,\xi,t) + \tilde{F}^{d(-)}(x,\xi,t)$
- π^0 probes C-odd, axial-vector GPD combinations:

$$\tilde{F}^{q(-)}(x,\xi,t) = \tilde{F}^{q}(x,\xi,t) - \tilde{F}^{q}(-x,\xi,t)$$

Two-Loop Calculation

- Partonic process $\gamma^* + u(vl) + \bar{u}(\bar{v}l) \to d(uk) + \bar{d}(\bar{u}k)$, $l = k - q$, $\mathcal{A} = \epsilon^\mu \mathcal{A}_\mu$.
- Directly using covariant projection technology to evaluate trace will break Ward-Takahashi identity.
- To preserve the identity, we give up to evaluate the trace and retain all spinor structures and reduce them to a set of total antisymmetric bases.
- We calculated 430 Feynman diagrams and obtained

$$
\begin{aligned}
q^\mu \mathcal{A}_\mu^{(2)} = \mathcal{C}_1^{(2)} & \left\{\gamma^{[k]} \otimes \gamma^{[l]} - l\cdot k\, \gamma^{[\mu]} \otimes \gamma^{[\mu]}\right\} \\
& + \mathcal{C}_2^{(2)} \left\{3\gamma^{[k\mu_1\mu_2]} \otimes \gamma^{[l\mu_1\mu_2]} - l\cdot k\, \gamma^{[\mu_0\mu_1\mu_2]} \otimes \gamma^{[\mu_0\mu_1\mu_2]}\right\} \\
& + \mathcal{C}_3^{(2)} \left\{5\gamma^{[k\mu_1\mu_2\mu_3\mu_4]} \otimes \gamma^{[l\mu_1\mu_2\mu_3\mu_4]} - l\cdot k\, \gamma^{[\mu_0\mu_1\mu_2\mu_3\mu_4]} \otimes \gamma^{[\mu_0\mu_1\mu_2\mu_3\mu_4]}\right\} .
\end{aligned}
$$

$$
\gamma^{[\mu_0\cdots\mu_m]} \otimes \gamma^{[\mu_0\cdots\mu_m]} \equiv u_\alpha(vl)\bar{v}_\beta(\bar{v}l)\bar{u}_\rho(uk)v_\sigma(\bar{u}k)\left(\gamma^{[\mu_0\cdots\mu_m]}\right)_{\beta\alpha}\left(\gamma^{[\mu_0\cdots\mu_m]}\right)_{\rho\sigma}
$$

- Applying this replacement

$$
\gamma^{[\mu_0\mu_1\cdots\mu_{2n}]}\otimes\gamma^{[\mu_0\mu_1\cdots\mu_{2n}]} = \frac{2n+1}{k\cdot l}\gamma^{[k\mu_1\cdots\mu_{2n}]} \otimes \gamma^{[l\mu_1\cdots\mu_{2n}]} ,
$$

According to the drawer principle, we can demonstrate that this equation holds in four dimensions. To satisfy Ward-Takahashi identity, we extend it to the d-dimension.

Two-Loop Calculation

- IBP reduction⇒271master integrals (MIs) for pure singlet contribution.
- Method of differential equations (Kotikov, Remiddi, Gehrmann).
- Boundary conditions from asymptotic expansion around $x = u = 0$ using *method of regions* (Beneke, Smirnov).
- Calculation performed with `AmpRed` package (Chen et al. 2024, 2025).
- The result takes the following form:

$$(k+l)^{\mu}\mathcal{A}_{\mu}^{(2)} = \mathcal{C}_1\gamma^{[k]}\otimes\gamma^{[l]} + \mathcal{C}_2\gamma^{[k\mu_1\mu_2]}\otimes\gamma^{[l\mu_1\mu_2]} + \mathcal{C}_3\gamma^{[k\mu_1\mu_2\mu_3\mu_4]}\otimes\gamma^{[l\mu_1\mu_2\mu_3\mu_4]}\,.$$

- All of the divergence are in $\mathcal{C}_1$, and $\gamma^{[k]}\otimes\gamma^{[l]} = 0$, $\gamma^{[k\mu_1\mu_2\mu_3\mu_4]}\otimes\gamma^{[l\mu_1\mu_2\mu_3\mu_4]} = \mathcal{O}(d-4)$, which means only UV and IR finite term $\mathcal{C}_2$ remains. Then we can safely use covariant projection technology in 4 dimensions.

$$T_{\pi^0}^{\mathrm{pS}} = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{8\xi Q^2}\mathcal{C}_2\mathrm{Tr}[\not{l}\gamma^5\gamma^{[k\mu_1\mu_2]}]\mathrm{Tr}[\not{k}\gamma^5\gamma^{[l\mu_1\mu_2]}]\,.$$

Convolution with GPDs and DA

Pion DA expanded in Gegenbauer polynomials:

$$\phi_\pi(u,\mu_D) = \sum_{n=0}{}' a_n(\mu_D)\, 6u\bar{u}C_n^{3/2}(2u-1)$$

Analytical convolution with Gegenbauer expansion of pion DA using `PolyLogTools`.

$$\mathcal{T}_n^M(x,\xi) = \int_0^1 \mathrm{d}u\, 6u\bar{u}C_n^{3/2}(2u-1)\, T_M(u,x,\xi).$$

numeral convolution with GPDs:

$$\tilde{\mathcal{F}}_M = \sum_n{}' a_n(\mu_D) \int_{-1}^1 dx\, \mathcal{T}_n^M(x,\xi,\mu)\, \tilde{F}(x,\xi,t,\mu_G)$$

GPD Models

We have selected two GPD models, GK model and GUMP1.0 global fit, for numerical analysis.

GK Model (Goloskokov, Kroll 2009-2012)

- Double distribution (DD)

$$\{\tilde{H}, \tilde{E}\}^q(x, \xi, t) = \int d\alpha d\beta \delta(x - \beta - \xi\alpha)\{\tilde{h}, \tilde{e}\}^q(\beta, \alpha, t)$$

- Pion-pole contribution

$$\tilde{E}^u_{pole} = -\tilde{E}^d_{pole} = \Theta(|x| \leq \xi)\frac{F_P(t)}{4\xi}\phi_\pi(\frac{x+\xi}{2\xi}), F_P(t) = \frac{2\sqrt{2}m_p f_\pi g_A}{m_\pi^2 - t}\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t'}$$

GUMP (Guo, Ji et al. 2025)

- Conformal moment expansion and Mellin-Barnes integral

$$F(x, \xi, t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} \mathrm{dj} \frac{p_j(x, \xi)}{\sin(\pi[1+j])}\mathcal{F}_j(\xi, t)$$

- Global extraction of GPDs with DVCS, DVρP, PDFs and Lattice result.

LaMET Lattice Result of GPDs

LaMET Lattice (Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013))
GPDs Results

- Bhattacharya et al. 2023-2024
- Xiang Gao et al. 2025
- Min-Huan Chu et al. 2025
- Due to the limited results of lattice results at non-zero skewness, which cannot cover the required parameter range, we have not yet adopted it.

Pion DA Models

Pion DA:

- RQCD lattice results ($a_2 = 0.116^{+0.019}_{-0.020}$ at 2 GeV) (According to the previous results of pion EMFF, which is more in line with the experiments (Chen et al. 2023))

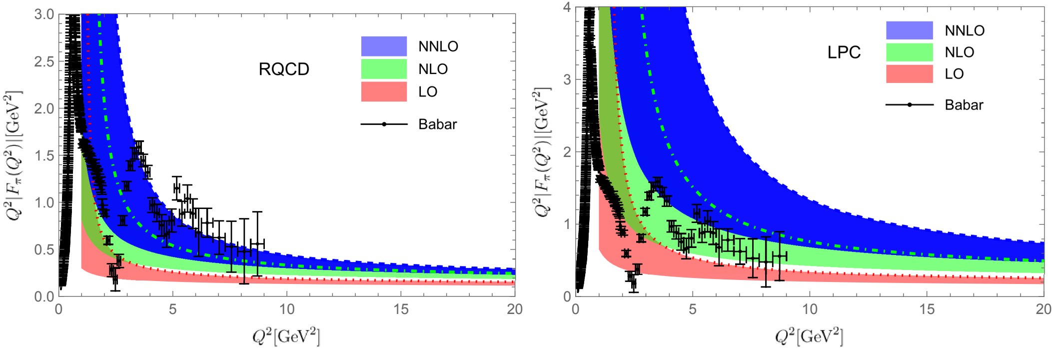


Figure: Comparison of Results from Different DA Models in Calculating Pion EMFF (Chen et al. 2023)

Evolution and Numerical Setup

Evolution:

- GPDs evolved using two-loop evolution program `tiktaalik` (Jianwei Qiu et al. 2024).
- DA moments a_n evolved with three-loop ERBL equations (Braun et al. 2017).
- Strong coupling $\alpha_s(\mu)$ from `FAPT` (Bakulev et al. 2012).

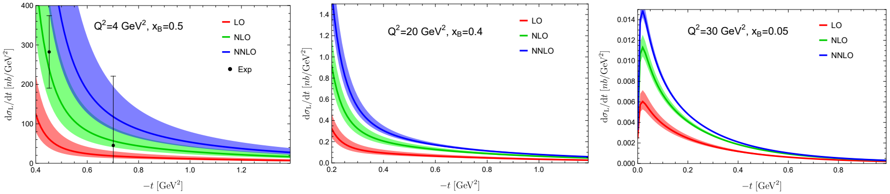
Scale choices:

- $\mu_R = \mu_G = \mu_D = \mu$
- Central scale $\mu = Q$
- Uncertainty band: $\mu \in [Q/\sqrt{2}, \sqrt{2}Q]$

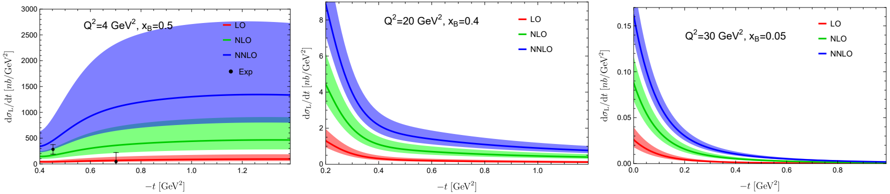
Kinematics:

- JLab 6 GeV π^+: $Q^2 = 4$ GeV^2, $x_B = 0.5$
- JLab 12 GeV π^0: $Q^2 = 4.44$ GeV^2, $x_B = 0.36$
- EicC: $Q^2 = 20$ GeV^2, $x_B = 0.4$
- EIC: $Q^2 = 30$ GeV^2, $x_B = 0.05$

Longitudinal Cross Section: π^+

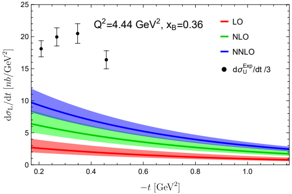


(a) JLab (b) EicC (c) EIC

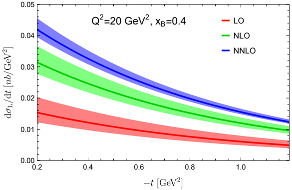


- NNLO corrections are large ($> 100\%$ in some regions).

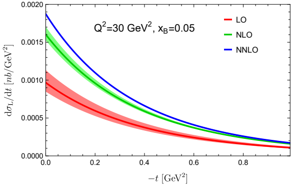
Longitudinal Cross Section: π^0



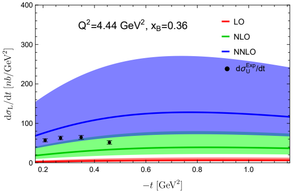
(a) JLab

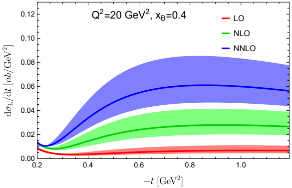


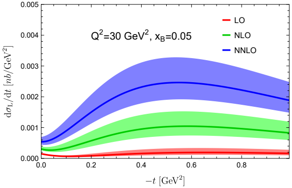
(b) EicC



(c) EIC

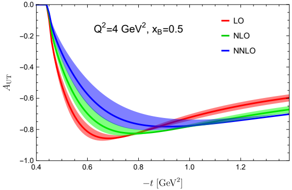




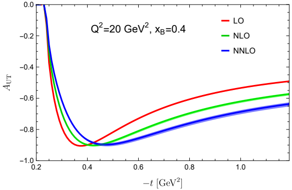


- Data points (Hall A 2020) are unseparated ($\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$), not pure longitudinal polarization.
- Smaller cross section due to absence of pion-pole in GK.
- NLO and NNLO corrections comparable to LO ones.

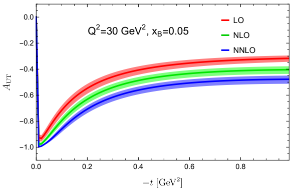
TSSA for π^+ Production



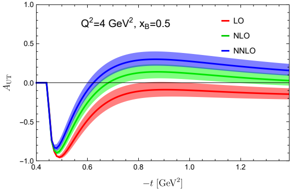
(a) JLab

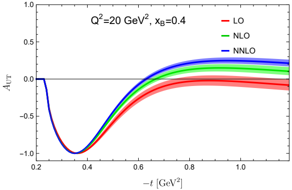


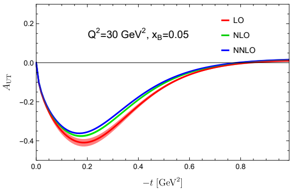
(b) EicC



(c) EIC

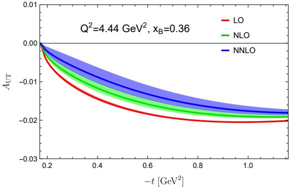




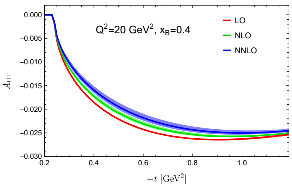


NNLO corrections to A_{UT} are small, consistent with NLO findings (Diehl et al. 2007).

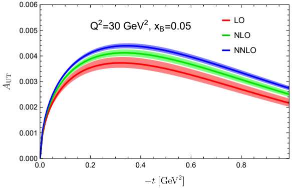
TSSA for π^0 Production



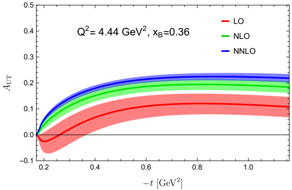
(a) JLab

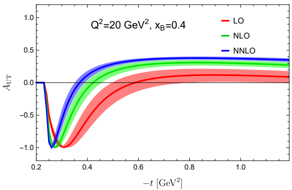


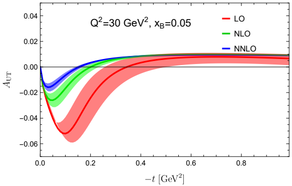
(b) EicC



(c) EIC







GK model (no pion-pole) gives small asymmetry; GUMP shows sizeable A_{UT} at intermediate x_B.

Pure Singlet Contribution

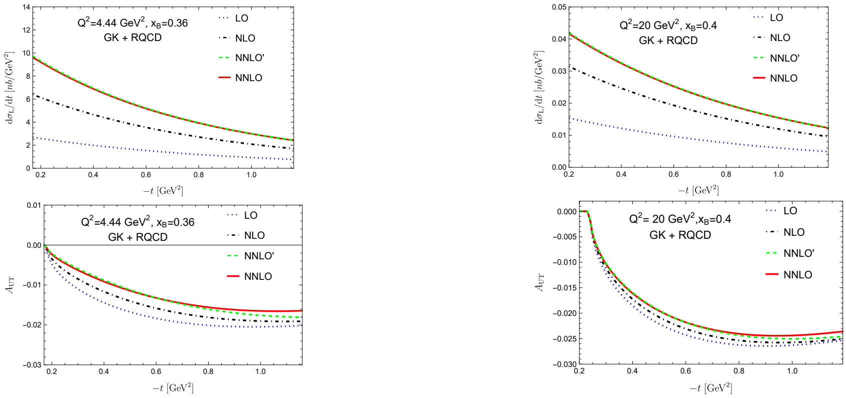


Figure: The contributon of pS portion.

- In phenomenology, the pure singlet portion of quarks provides a tiny correction for $\frac{d\sigma_L}{dt}$ ($\sim$1%) and A_{UT} (<10%).

Inverse DVMP: Crossing to the Timelike Regime

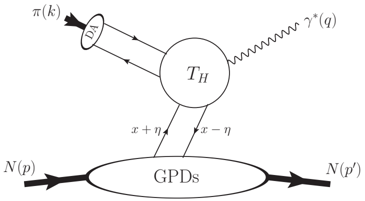
Spacelike DVMP

$$\gamma_L^*(q) + N(p) \to M(k) + N(p'), \qquad q^2 = -Q^2$$

Timelike inverse process

$$M(k) + N(p) \to \gamma_L^*(q) + N(p') \to \ell^+\ell^- + N(p'),$$
$$q^2 = Q^2$$

- Crossing exchanges the incoming photon and outgoing meson.
- The same meson DA and axial-vector GPDs enter both channels.
- Timelike data provide an independent test of factorization and GPD universality. (Zhihong Ye's report, Wen-Chen Chang et al. LOI of J-PARC)



$Q^2 = q^2$

$\tau = \frac{Q^2}{2p\cdot k}$

$t = (p-p')^2$

$t' = t + \frac{4m_N^2\eta^2}{1-\eta^2}$

$s = (k+p)^2$

Kinematics and skewness

$$\bar{q} = \frac{1}{2}(k+q), \qquad P = p + p', \qquad \Delta = p - p'.$$

For both channels: $\quad \xi = -\dfrac{\bar{q}^{\,2}}{P\cdot\bar{q}}, \qquad \eta = -\dfrac{\Delta\cdot\bar{q}}{P\cdot\bar{q}}.$

DVMP: $\quad \xi = \eta, \qquad q^2 = -Q^2,$

Inverse DVMP: $\quad \xi = -\eta, \qquad q^2 = +Q^2,$

$$\eta \simeq -\xi \simeq \frac{\Delta^+}{P^+} = \frac{(p-p')^+}{(p+p')^+}.$$

At J-PARC, moderate timelike virtualities ($Q^2 \sim 2$–$6\ \mathrm{GeV}^2$) make higher-order QCD corrections indispensable.

Leading-Twist Factorization and Kinematics

$$\pi^-(k) + p(p) \to \gamma_L^*(q) + n(p') \to e^- e^+ + n(p'), \qquad Q^2 = q^2, \quad t = (p - p')^2, \quad \tau = \frac{Q^2}{2p \cdot k}.$$

Longitudinal cross section

$$\frac{d\sigma}{dQ^2\, dt} = \frac{\alpha\tau^2}{48\pi^2 Q^6} \left(|\mathcal{M}_{++}|^2 + |\mathcal{M}_{-+}|^2\right).$$

$$\mathcal{M}_{++} = \frac{4\pi e f_\pi}{N_c Q} \sqrt{1-\eta^2} \left[\widetilde{\mathcal{H}} - \frac{\eta^2}{1-\eta^2} \widetilde{\mathcal{E}}\right],$$

$$\mathcal{M}_{-+} = \frac{4\pi e f_\pi}{N_c Q} \frac{\sqrt{|t'|}}{2m_N} \eta\, \widetilde{\mathcal{E}}.$$

$$T^0(u, x, \xi) = C_F \alpha_s \left[\frac{e_d}{u(\xi - x - i\epsilon)} - \frac{e_u}{\bar{u}(\xi + x - i\epsilon)}\right]$$

(Berger, Diehl and Pire, 2001)

Universal nonperturbative inputs

$$\widetilde{\mathcal{H}} = \int du\, dx\, \phi_\pi(u)\, T(u, x, \xi)\, \widetilde{H}^{du}(x, \eta, t),$$

$$\widetilde{\mathcal{E}} = \int du\, dx\, \phi_\pi(u)\, T(u, x, \xi)\, \widetilde{E}^{du}(x, \eta, t).$$

- Generalized Bjorken limit: $Q^2 \gg |t|, \Lambda_{\rm QCD}^2$.
- The leading-twist term produces longitudinal photons.
- Lepton-pair angular distributions can isolate the longitudinal contribution.

One NNLO Kernel, Two Crossed Processes

Analytic continuation of the hard kernel

$$T(u,x,\xi) = \pm\frac{1}{2\xi}\sum_{\ell=0}\sum_{i=0}^{\ell}\left(\frac{\alpha_s}{2\pi}\right)^{\ell}\log^{i}\left(\frac{\mu^2}{\pm Q^2}\right)\left[e_u C_\ell^i\left(\bar{u},\frac{\mp x+\xi}{2\xi}\right) - e_d C_\ell^i\left(u,\frac{\pm x+\xi}{2\xi}\right)\right].$$

Upper sign: DVπP; lower sign: π-induced exclusive Drell–Yan.

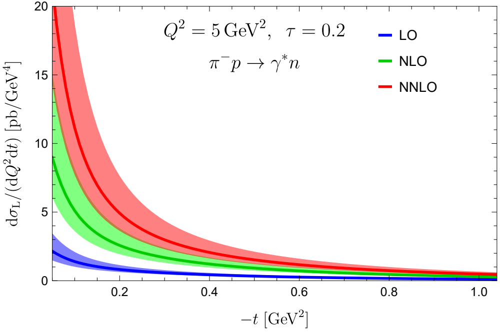
- Timelike continuation generates $\log(\mu^2/(-Q^2)) = \log(\mu^2/Q^2) + i\pi$.
- The causal prescription at $x = \pm\xi$ fixes the complex phase.
- No independent two-loop calculation is required.

All-order TFF relation

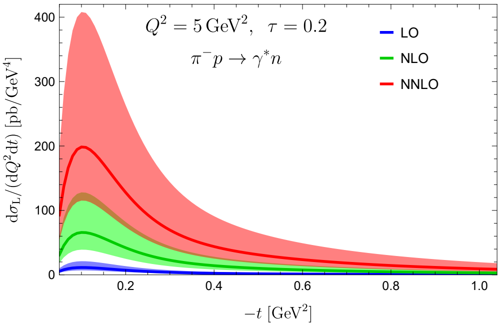
$$\mathcal{F}_{\rm exDY} = \mathcal{F}^*_{\rm DVMP} + \sum_{n=1}^{\infty}\frac{1}{n!}\left(-i\pi\frac{d}{d\log Q^2}\right)^n \mathcal{F}^*_{\rm DVMP},$$
$$\mathcal{F} \in \left\{\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\right\}.$$

The DVMP NNLO result directly unlocks NNLO predictions in the timelike channel.

Pion-Induced Cross Section: NNLO Effects Are Essential



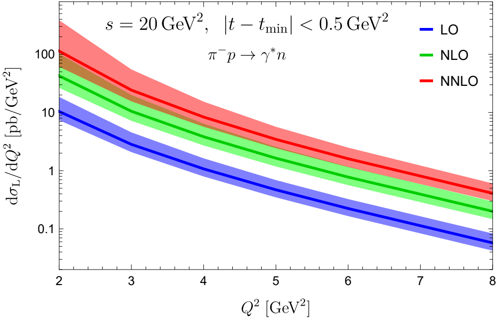
(a) RQCD pion DA + GK GPDs



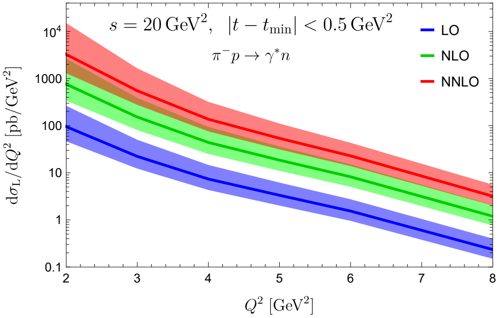
(b) RQCD pion DA + GUMP GPDs

- At $Q^2 = 5\ \mathrm{GeV}^2$ and $\tau = 0.2$, NLO and NNLO corrections strongly enhance $d\sigma/(dQ^2\,dt)$.
- GUMP predicts a cross section roughly one order of magnitude above GK and a different small-$|t|$ shape; the pion-pole treatment drives much of the contrast.

Q^2 Dependence at J-PARC Kinematics



(a) RQCD pion DA + GK GPDs

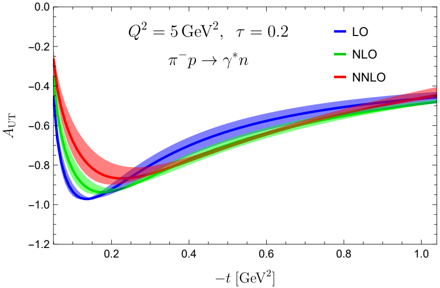


(b) RQCD pion DA + GUMP GPDs

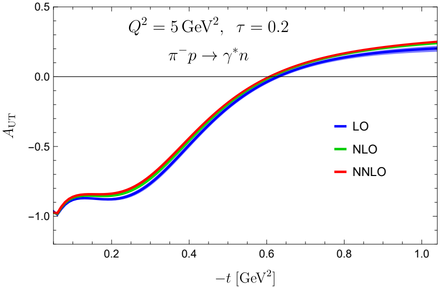
- The cross section integrated over $|t - t_{\min}| < 0.5\ \mathrm{GeV}^2$ falls steeply with Q^2 at $s = 20\ \mathrm{GeV}^2$.
- Both GPD models show the same qualitative scaling and a persistent positive radiative enhancement from LO to NLO to NNLO.

Reliable GPD extraction across the accessible range requires NNLO accuracy.

TSSA Is Perturbatively Stable but GPD-Sensitive



(a) RQCD pion DA + GK GPDs



(b) RQCD pion DA + GUMP GPDs

- $A_{UT} \propto \mathrm{Im}\left(\widetilde{\mathcal{H}}^* \widetilde{\mathcal{E}}\right)$ is sizable in the planned kinematic region and changes only mildly at NLO and NNLO.
- Its small-$|t|$ behavior clearly separates the GK and GUMP descriptions, providing direct leverage on the poorly constrained $\widetilde{E}$ GPD.

Kaon Beams Extend the Flavor Reach

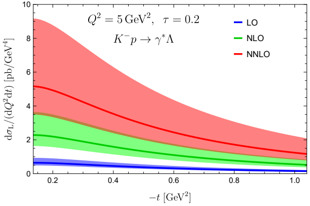
Channel and transition GPD

$$K^- p \to \gamma_L^* \Lambda \to e^- e^+ \Lambda,$$

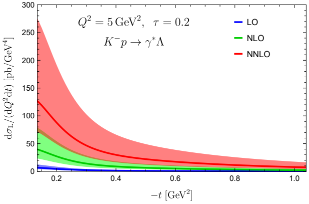
$$F^{us}_{p\to\Lambda} = -\frac{1}{\sqrt{6}}\left(2F^u - F^d - F^s\right),$$

$$F = \widetilde{H}, \widetilde{E}.$$

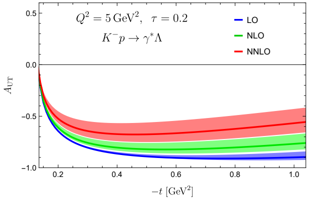
- The asymmetric kaon DA requires its full Gegenbauer expansion.
- Higher-order corrections enhance the longitudinal cross section by more than 100% in representative regions.



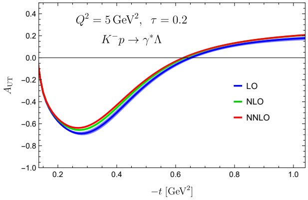
(a) Cross section: LPC + GK



(b) Cross section: LPC + GUMP



(c) A_{UT}: LPC + GK



(d) A_{UT}: LPC + GUMP

Summary and Outlook

What the two crossed channels establish

1. The first NNLO predictions are available for both deeply virtual π production and its timelike inverse at leading twist.
2. A common hard-scattering kernel, obtained from the pion EMFF and continued between spacelike and timelike kinematics, connects the two processes.
3. NNLO corrections to longitudinal cross sections are positive and large; they are essential for quantitative GPD extraction.
4. The transverse single-spin asymmetry A_{UT} is comparatively stable under higher-order corrections but remains highly sensitive to the GPD model and pole contributions.

Outlook

- Combine J-PARC timelike measurements with EIC/EicC spacelike data to test factorization and GPD universality.
- Extend the NNLO program to vector mesons (see the next report by Zhe-yu Wang) and joint global analyses.

Thank you for your attention!