第一届强相互作用自旋物理会议

The transverse-spin asymmetry $A_{UUT}^{\sin\phi_{S_\Lambda}}$ in Λ hyperon production in SIDIS within the collinear framework

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7.26-7.31,山东青岛

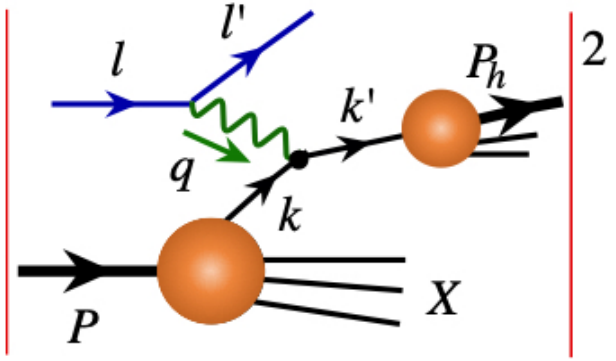
Outline

- Motivation
- Model information for the twist-3,T-odd FF $\widetilde{D}_T$
- Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS
- Conclusion

Motivation:

- Fragmentation functions (FFs) describe parton hadronization;
- **Hadron production in SIDIS:**

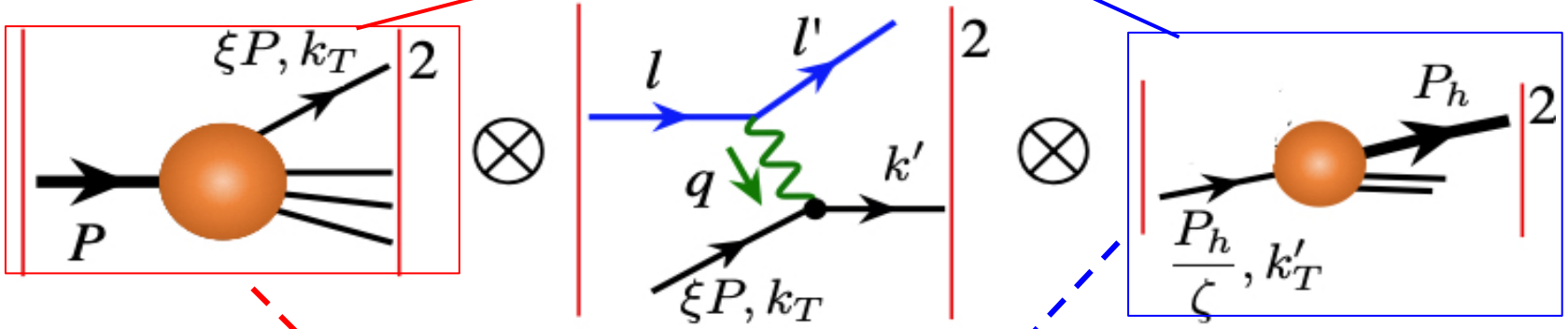
$$\sigma_{\mathrm{SIDIS}} \propto$$



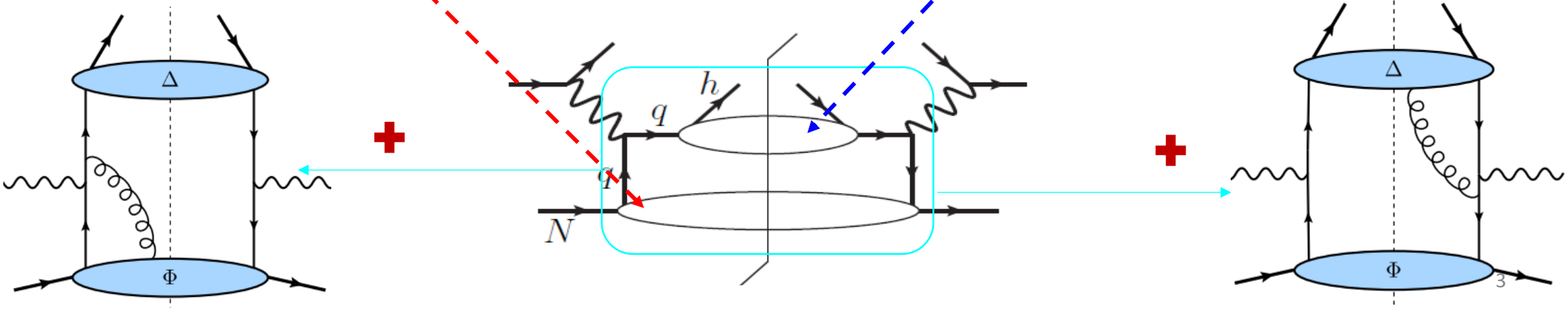
$h = \pi, K, p, \Lambda \ldots$

$$d\sigma \sim d\hat{\sigma} \otimes PDF \otimes FF$$

$$\approx$$

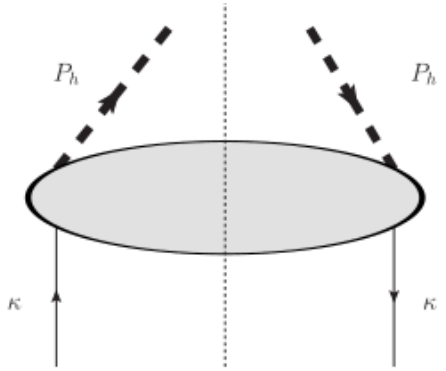


TMD handbook, 2304.03302



Motivation:

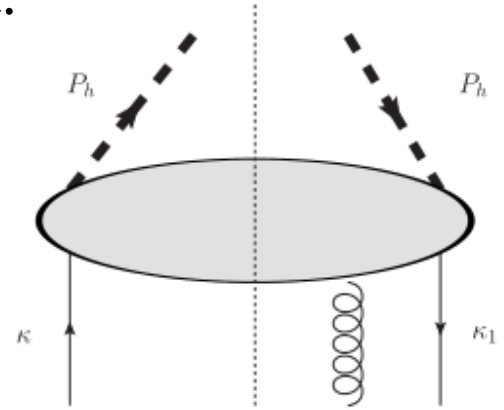
qq correlator:



Probabilistic interpretation of leading-twist fragmentation functions

- Compared with D_1、$D_{1T}^{\perp}$ and Collins function, twist-3 q-g-q correlation fragmentation functions remain poorly constrained.

q-g-q correlator:

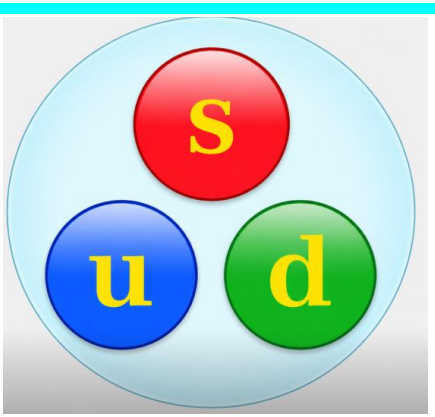


Leading Quark TMDFFs — Hadron Spin, Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = ⊙ Unpolarized		$H_1^{\perp}$ = ⊙ − ⊙ Collins
Polarized Hadrons	L		G_1 = → − → Helicity	$H_{1L}^{\perp}$ = → − →
	T	$D_{1T}^{\perp}$ = ⊙ − ⊙ Polarizing FF	$G_{1T}^{\perp}$ = → − →	H_1 = ⊙ − ⊙ Transversity; $H_{1T}^{\perp}$ = → − →

Motivation:

- Λ hyperon: $|uds\rangle$.



- In SIDIS, one structure function is dominated by $f_1^q(x)\tilde{D}_T^{q\to\Lambda}(z)$.

$$F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z) = x\sum_a e_a^2 \frac{2M_\Lambda}{Q} f_1(x) \frac{\tilde{D}_T(z)}{z},$$

Y.Yang, Z. Lu; Phys. Rev. D 95, 074026 (2017)

- $\tilde{D}_T$ arises from the twist-3 spin-dependent quark–gluon–quark correlator $\tilde{\Delta}_{A\alpha}$, and induces a transverse-spin-dependent asymmetry in transversely polarized **Λ** production:

$$A_{UUT}^{\sin\phi_{S_\Lambda}}$$

- This asymmetry can simultaneously probe the twist-3 fragmentation mechanism $(\tilde{D}_T)$ and the s-quark structure of the proton (f_1^s).

Model calculation for the twist-3,T-odd FF $\tilde{D}_T$

- quark–gluon–quark fragmentation correlator $\tilde{\Delta}_{A\alpha}$:

$$\tilde{\Delta}^{\alpha}_{A}(z,k_T;S_\Lambda) = \sum\!\!\!\!\!\!\int_X \frac{1}{2z}\int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\int e^{ik\cdot\xi}\langle 0|\int_{\pm\infty^+}^{\xi^+} d\eta^+ \mathcal{U}^{\boldsymbol{\xi}_T}_{(\infty^+,\eta^+)}$$

$$\times\, gF_\perp^{-\alpha}(\eta)\mathcal{U}^{\boldsymbol{\xi}_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_\Lambda,S_\Lambda;X\rangle\langle P_\Lambda,S_\Lambda;X|\bar{\psi}(0)\mathcal{U}^{\mathbf{0}_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(\mathbf{0}_T,\boldsymbol{\xi}_T)}|0\rangle\Big|_{\substack{\eta^-=\xi^-=0\\ \eta_T=\xi_T}}$$

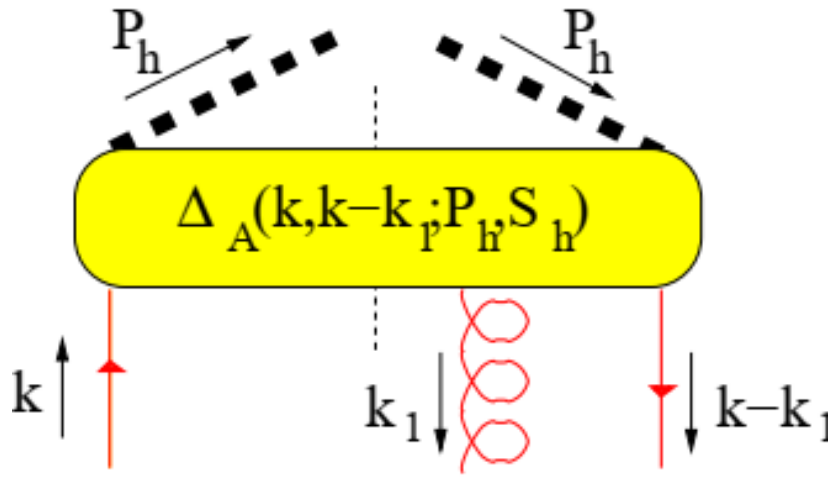
- $\tilde{D}_T$ relation: ⟹ $\tilde{D}_T = \tilde{D}'_T - \frac{k_T^2}{2M_\Lambda^2}\tilde{D}_T^\perp$

- $\tilde{D}'_T(z,k_T^2)$ and $\tilde{D}_T^\perp(z,k_T^2)$ are obtained from:

$$\frac{z}{2M_\Lambda}\mathrm{Tr}[\tilde{\Delta}_{A\alpha}(g_T^{a\alpha} - i\epsilon_T^{a\alpha}\gamma_5)\gamma^-] = \epsilon^{aS_T}(\tilde{D}'_T + i\tilde{G}'_T) - \frac{k_T\cdot S_T}{M_\Lambda^2}\epsilon^{ak_T}(\tilde{D}_T^\perp + i\tilde{G}_T^\perp)$$

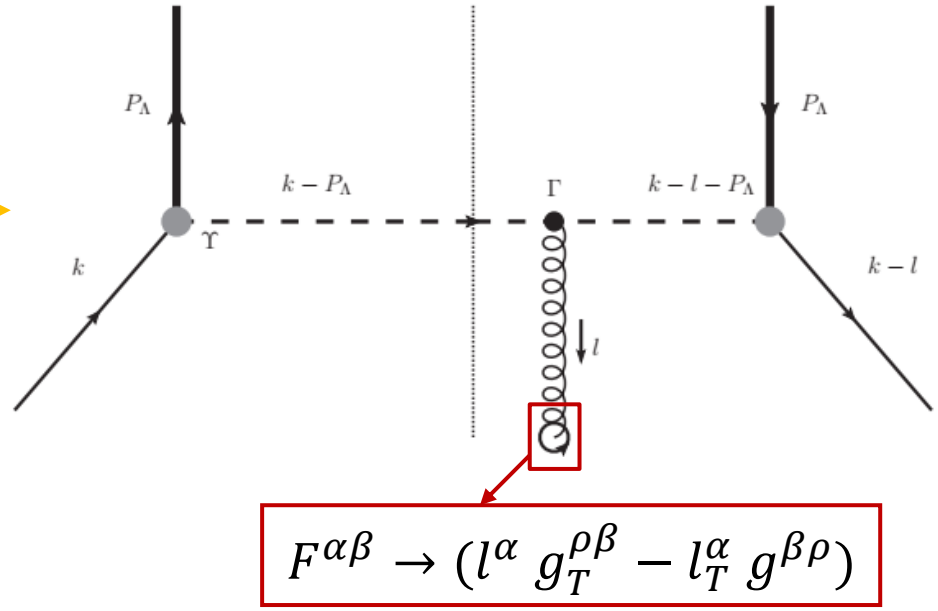
Diquark spectator model calculation for the FF $\tilde{D}_T$

➢ **Diagram:**



(qgq **fragmentation correlator**)

Feynman diagram



➢ **Matrix element:**

$$\langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)|0\rangle = \begin{cases} \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_s \frac{i}{k-m_q} & \text{scalar diquark,} \\ \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_v^\mu \frac{i}{k-m_q}\varepsilon_\mu & \text{axial-vector diquark} \end{cases}$$

J. J. Yang;Phys. Rev. D65, 094035 (2002)

Diquark spectator model

- Scalar and axial-vector coupling vertices:

$$\begin{cases} \mathcal{Y}_s = g_s(k^2) \\ \mathcal{Y}_v^\mu = \frac{g_v(k^2)}{\sqrt{3}}\gamma_5(\gamma^\mu + \frac{P_\Lambda^\mu}{M_\Lambda}) \end{cases}$$

R. Jakob, P. J. Mulders, and J. Rodrigues, Nucl. Phys. A626,937 (1997).

- Polarization sum: $d^{\mu\nu} = \sum_{\lambda_a} \epsilon^\mu_{\lambda_a} \epsilon^\nu_{\lambda_a} = -g^{\mu\nu} + \frac{P_\Lambda^\mu P_\Lambda^\nu}{M_\Lambda^2}$

- Gluon–diquark coupling vertex Γ: A. Bacchetta, F. Conti and M. Radici, Phys. Rev. D 78, 074010 (2008)

$$\Gamma^\rho = i\,(2k - 2P_\Lambda - l)^\rho;$$
$$\Gamma^{\rho\mu\nu} = -\,i\,[(2k - 2P_\Lambda - l)^\rho\, g^{\mu\nu} - (k - P_\Lambda - l)^\mu g^{\nu\rho} - (k - P_\Lambda)^\nu g^{\rho\mu}$$

- Expression:

$$\tilde{\Delta}_A^\alpha = \frac{C_F \alpha_S}{4(2\pi)^2 2(1-z)P_\Lambda^-} \frac{1}{k^2 - m^2} \int \frac{d^4 l}{(2\pi)^4} \frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k} - \not{l} + m)\bar{\mathcal{Y}}^\nu(\not{P}_\Lambda + M_\Lambda)(\gamma_5 \not{S}_\Lambda)\mathcal{Y}^\mu(\not{k} + m)}{((k-l)^2 - m^2)(l^2 - i\epsilon)((k - l - P_\Lambda)^2 - m_s^2)(-l^- \pm i\epsilon)} d^{i\mu} d^{j\nu} \bar{\Gamma}_{ij\rho}$$

The model calculation of the FF $\widetilde{D}_T(z, k_T^2)$

T-odd FF $\widetilde{D}_T(z, k_T^2)$ originates from the imaginary part of the scattering amplitude:

- Imaginary part of the function ⟵ Cutkosky cut：

$$\frac{1}{l^2 + i\varepsilon} \to -2\pi i\delta(l^2),$$

$$\frac{1}{(k-l)^2 - m^2 + i\varepsilon} \to -2\pi i\delta((k-l)^2 - m^2)$$

- SU(6) spin-flavor symmetry

$$D^{u\to\Lambda} = D^{d\to\Lambda} = \frac{1}{4}D^{(S)} + \frac{3}{4}D^{(V)}, \quad D^{s\to\Lambda} = D^{(S)}$$

- **Model result:**

$$\tilde{D}_T^S(z, \boldsymbol{k}_T^2) = \frac{C_F\alpha_S}{4\,M_\Lambda(2\pi)^2(1-z)}\frac{1}{k^2 - m^2}\tilde{D}_{TS}(z, \boldsymbol{k}_T^2)$$

Scalar part:

$$\tilde{D}_{TS}(z, \boldsymbol{k}_T^2) = 2g_s^2(z\,m + M_\Lambda)\{\mathcal{A}[M_\Lambda^2(z-1) - zm_D^2] + \mathcal{B}M_\Lambda^2(z-1) - \boldsymbol{k}_T^2 z[\mathcal{C}k^-(1-z) + \mathcal{I}_2]\}$$

The model calculation of the FF $\widetilde{D}_T(z, k_T^2)$

$$\tilde{D}_T^V(z,\boldsymbol{k}_T^2) = \frac{z\,C_F\alpha_S}{4\,M_\Lambda(2\pi)^2(1-z)}\frac{1}{k^2-m^2}\{\tilde{D}_{TV}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV0}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV1}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV2}(z,\boldsymbol{k}_T^2)\}$$

axial-vector part:

$$\tilde{D}_{TV} = -\frac{2g_v^2}{3z}(z\,m+M_\Lambda)\{\mathcal{A}[M_\Lambda^2(z-1)-zm_D^2]+\mathcal{B}M_\Lambda^2(z-1)-\boldsymbol{k}_T^2 z[\mathcal{C}k^-(1-z)+\mathcal{I}_2]\}$$

$$\begin{aligned}\tilde{D}_{TV0} = \frac{2g_v^2}{3M_\Lambda}\{&(k\cdot P_\Lambda+mM_\Lambda)[(\mathcal{A}\mathcal{A}+z\mathcal{A}\mathcal{B})k\cdot P_\Lambda+M_\Lambda^2(\mathcal{A}\mathcal{B}+z\mathcal{B}\mathcal{B})+z\mathcal{W}_1]+\boldsymbol{k}_T^2M_\Lambda^2(\mathcal{A}\mathcal{A}+z\mathcal{A}\mathcal{B})\\&-(\mathcal{A}+z\mathcal{B})[2mM_\Lambda k\cdot P_\Lambda+(k\cdot P_\Lambda)^2+M_\Lambda^2(m^2+2\boldsymbol{k}_T^2)]\}\end{aligned}$$

$$\begin{aligned}\tilde{D}_{TV1} = \frac{g_v^2}{3M_\Lambda}\Big\{&M_\Lambda\{z[-\boldsymbol{k}_T^2(m\mathcal{A}\mathcal{A}+2M_\Lambda\mathcal{A}\mathcal{B}-4m\mathcal{A}-2M_\Lambda\mathcal{B})+(M_\Lambda\mathcal{B}-m\mathcal{I}_2)(k^2+3m^2)+2m\mathcal{W}_1]\\&+2M_\Lambda[-\mathcal{A}\mathcal{A}\boldsymbol{k}_T^2+3\mathcal{A}\boldsymbol{k}_T^2+M_\Lambda(M_\Lambda\mathcal{B}\mathcal{B}-m\mathcal{B})+3\mathcal{W}_1]\}\\&+k\cdot P_\Lambda\{z[-\mathcal{A}\mathcal{A}\boldsymbol{k}_T^2+(\mathcal{A}-\mathcal{I}_2)(k^2+3m^2)-2\mathcal{W}_1]+2k\cdot P_\Lambda(\mathcal{A}\mathcal{A}-3\mathcal{A}+2\mathcal{I}_2)\\&+2M_\Lambda(2\mathcal{A}\mathcal{B}M_\Lambda-\mathcal{A}m-3\mathcal{B}M_\Lambda+2\mathcal{I}_2m)\}\Big\}\end{aligned}$$

$$\begin{aligned}\tilde{D}_{TV2} = -\frac{g_v^2}{3M_\Lambda^2}\Big\{&\frac{z^2}{2}\{-\boldsymbol{k}_T^2[\mathcal{C}\mathcal{C}k^-(k^2-m^2)(m+M_\Lambda)+2mM_\Lambda(\mathcal{C}\mathcal{D}k^-M_\Lambda-2\mathcal{C}k^-m)-m\frac{(k^2-m^2)}{2k^2}\mathcal{I}_1]\\&+\mathcal{W}_2k^-(k^2-m^2)(m+M_\Lambda)+m\frac{k^2-m^2}{2}\mathcal{I}_1\}\\&+M_\Lambda^2z[-\boldsymbol{k}_T^2(2\mathcal{C}\mathcal{C}k^-(zm+M_\Lambda)+\mathcal{C}\mathcal{D}k^-M_\Lambda z)+4\mathcal{W}_2k^-(zm+M_\Lambda)]\\&+z\{-\boldsymbol{k}_T^2\mathcal{C}\mathcal{E}k^-k^-z(M_\Lambda-zm)-k\cdot P_\Lambda[4\mathcal{W}_2k^-(M_\Lambda+zm)+\boldsymbol{k}_T^2(2\mathcal{C}k^-zm-\mathcal{C}\mathcal{C}k^-(3zm+M_\Lambda))]\}\Big\}\end{aligned}$$

Fit model parameters

➢ **Using $D_1(z)$ and $G_1(z)$ model results:** Y. Yang, Z. Lu and I. Schmidt, Phys. Rev. D 96(3), 034010 (2017)

$$D_1^{\Lambda}(z) = \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Big\{z(1-z)((m_q+M_\Lambda)^2-m_D^2) \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) + ((1-z)\Lambda^2 - 2((m_q+M_\Lambda)^2-m_D^2)) \times \frac{z^2L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Big\},$$

$$L^2 = \frac{1-z}{z^2}M_\Lambda^2 + m_q^2 + \frac{m_D^2-m_q^2}{z}$$

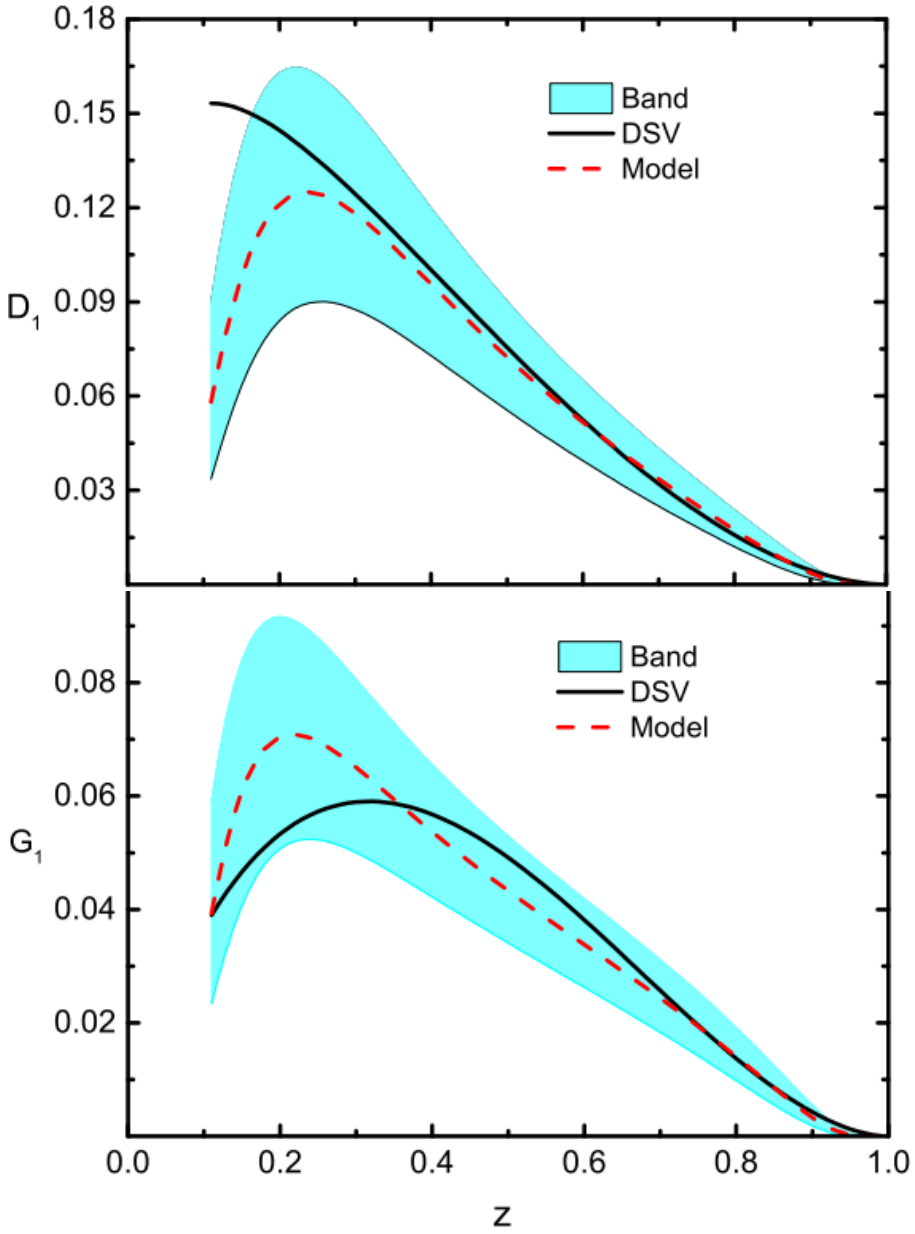
$$G_1^{\Lambda}(z) = \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Big\{(1-z)[M_\Lambda^2(2-z)+2zm_qM_\Lambda + z(m_D^2+m_q^2(2z-1))]\exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) - z[2M_\Lambda^2(2-z)+4zm_qM_\Lambda + z((1-z)\Lambda^2+2(m_D^2+m_q^2(2z-1)))] \times \frac{L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Big\}.$$

where $\Lambda^2 = \lambda^2 z^{\alpha}(1-z)^{\beta}$, $g_s = g_v$, m_D is diquark mass

➢ **Fitted to the LO-DSV parameterization at the initial scale: $\mu_{LO}^2 = 0.23\ GeV^2$**

D. de Florian, M. Stratmann, and W. Vogelsang, Phys. Rev. D 57, 5811 (1998)

Fit model parameters



➢ For D_1^{Λ}:

$$D_1^{u\to\Lambda} = D_1^{d\to\Lambda} = D_1^{s\to\Lambda} \equiv D_1^{\Lambda}.$$

➢ Only $G_1^{s\to\Lambda}$: (DSV scenario 1)

$$G_{1L}^{u\to\Lambda}(z,k_T^2) = 0, \qquad G_{1L}^{d\to\Lambda}(z,k_T^2) = 0,$$

$$G_{1L}^{s\to\Lambda}(z,k_T^2) = G_{1L}^{(s)}(z,k_T^2).$$

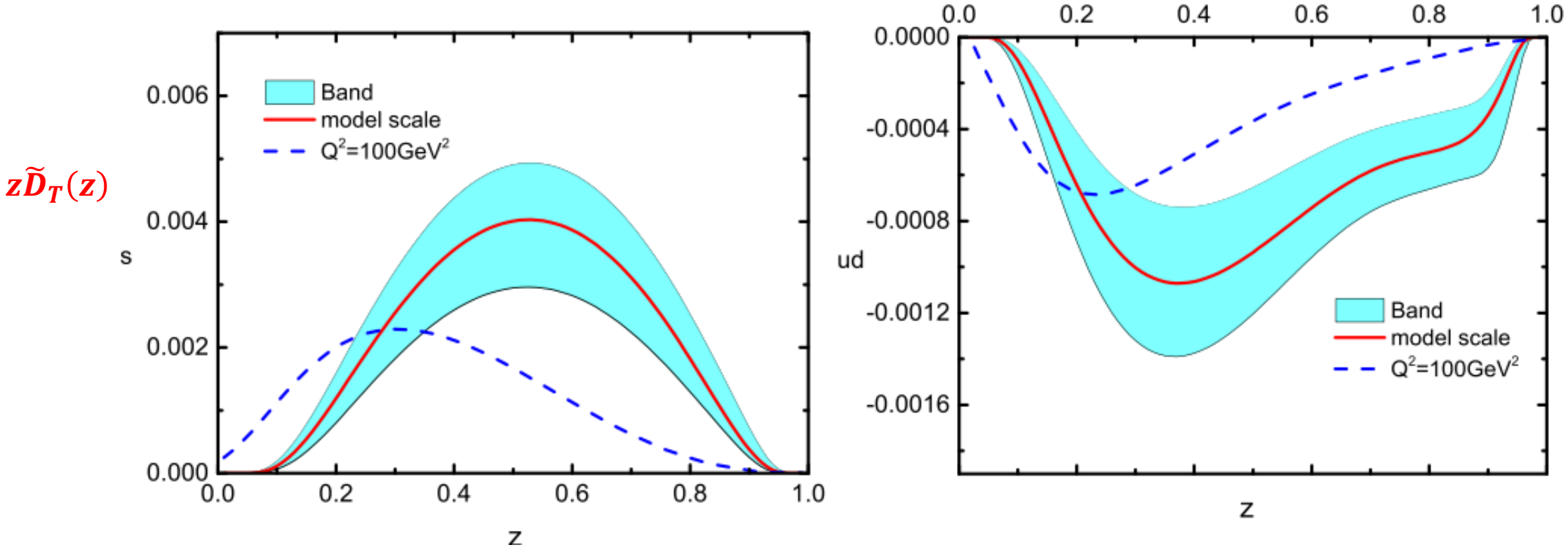
- masses of u, d, s: $m = 0.36\ GeV$;
- Λ hyperon mass to be $1.116\ GeV$;

➢ **Fitted model parameters:**

g_s	λ (GeV)	m_D (GeV)	m (GeV)	α	β
$2.83^{+0.12}_{-0.1}$	$3.567^{+0.23}_{-0.21}$	$0.397^{+0.03}_{-0.03}$	0.36 (fixed)	$0.2^{+0.04}_{-0.04}$	$0.52^{+0.08}_{-0.08}$

Model calculation for the twist-3,T-odd FF $\widetilde{D}_T$

Collinear function $\widetilde{D}_T(z) = z^2 \int d^2\boldsymbol{k}_T \, \widetilde{D}_{\mathrm{T}}(z, \boldsymbol{k}_T^2)$



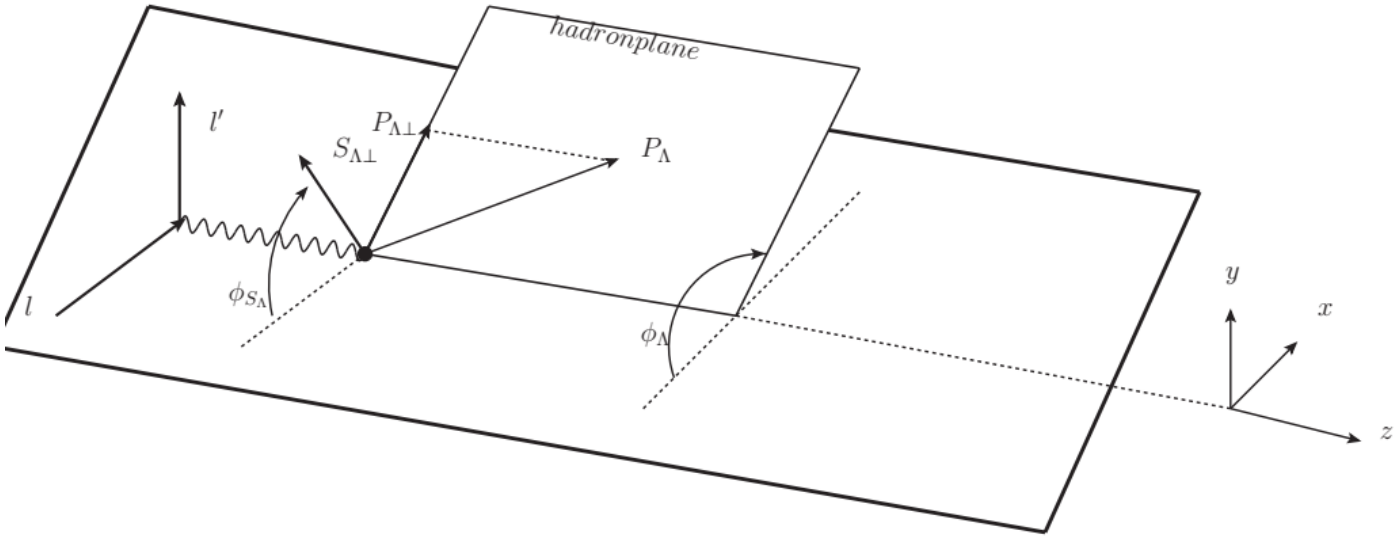
$\alpha_s = 0.817$ in initial scale

- $u(d) \to \Lambda$: negative and small, with scalar and axial-vector contributions;
- $s \to \Lambda$: positive, with a magnitude larger than $u(d)$, with only the scalar contribution;
- QCD evolution from $Q^2 = 0.23 \to 100\ GeV^2$, using the same evolution kernel as D_1;

Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS

Transversely polarized Λ hyperon production in SIDIS:

$$\ell + N \rightarrow \ell' + \Lambda^{\uparrow} + X$$



- The azimuthal angle ϕ_{S_Λ} is the angle between the lepton scattering plane and the transverse-spin direction of the Λ hyperon.

$$\sin\phi_{S_\Lambda} = -\frac{l_\mu S_{\Lambda\nu}\epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_{\Lambda\perp}^2}}.$$

- Variables in the scattering cross section

$$x = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot P_h}{P\cdot q},$$

$$\gamma = \frac{2Mx}{Q}, \qquad W^2 = (P+q)^2, \qquad s = (P+l)^2$$

Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS

➢ **z- and x-dependent $sin\phi_{S_\Lambda}$ asymmetries:**

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}.$$

$$A_{UUT}^{\sin\phi_{S_\Lambda}}(z) = \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1+\varepsilon)}F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UUU}(x,z)}$$

$$A_{UUT}^{\sin\phi_{S_\Lambda}}(x) = \frac{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1+\varepsilon)}F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UUU}(x,z)},$$

➢ **The corresponding structure functions are**

$$F_{UUU}(x,z) = x\sum_q e_q^2 f_1^q(x)D_1^q(z),$$

$$F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z) = x\sum_q e_q^2 \frac{2M_\Lambda}{Q} f_1^q(x)\frac{\tilde{D}_T^q(z)}{z},$$

- Both numerator and denominator contain the PDF: $f_1^q(x)$.

H.L. Lai, M. Guzzi, J. Huston,et al.,Phys.Rev.D 82 (2010) 074024

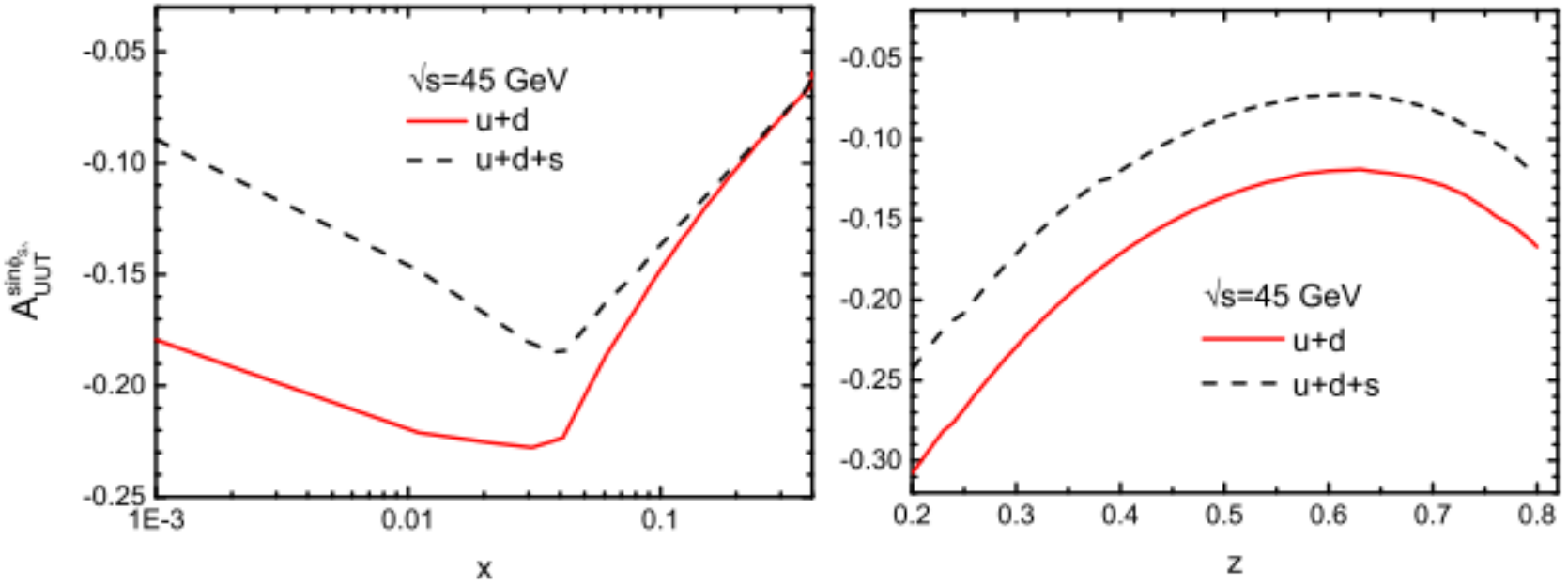
- twist-3 structure function contains the $\frac{2M_\Lambda}{Q}$ factor.

Prediction at EIC

EIC,kinematic region:

$$0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 45\ \text{GeV}, \quad W > 5\ \text{GeV}.$$

Numerical results for the x- and z-dependent asymmetries:



- In the x and z distributions, the asymmetry is negative and sizable.
- Because the s-quark channel has an opposite $\widetilde{D}_T$ sign, including the s-quark contribution reduces the absolute asymmetry.

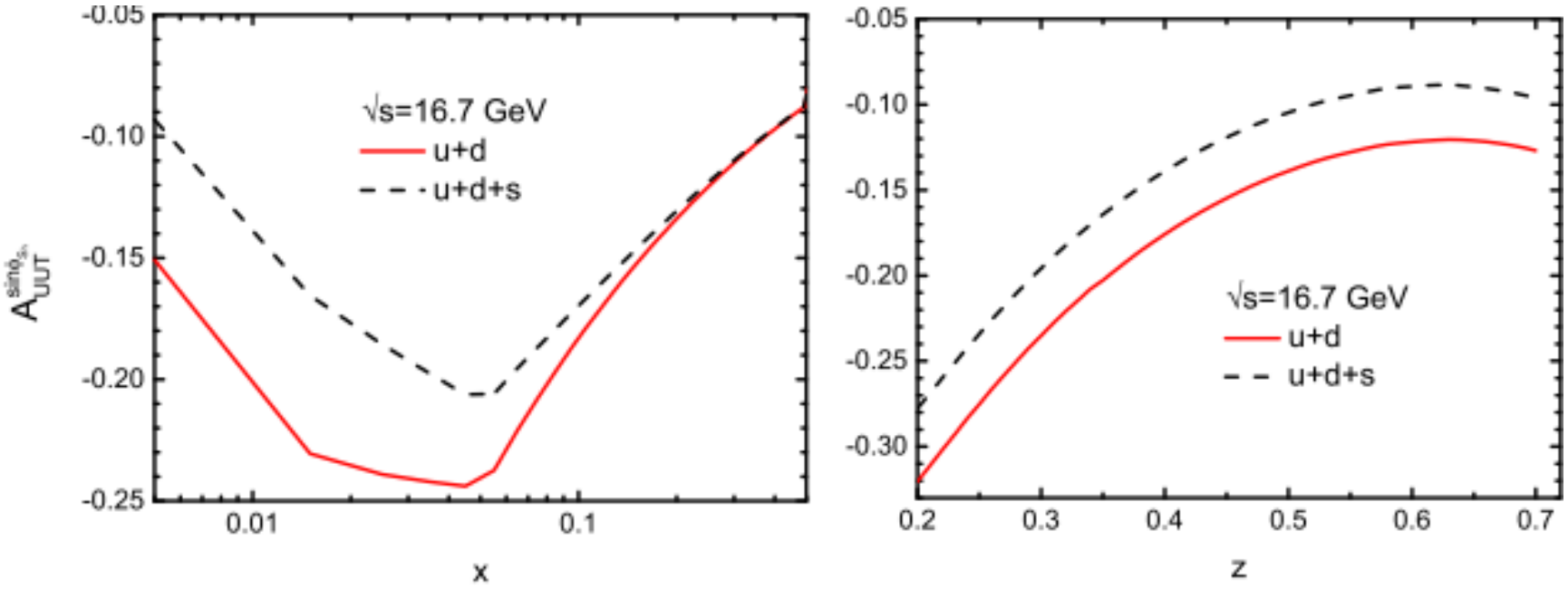
The asymmetry versus x peaks near $x \sim 0.04$ and then decreases as x increases.

Prediction at EicC

EicC,kinematic region:

$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$Q^2 > 1\mathrm{GeV}^2, \quad \sqrt{s} = 16.7\ \mathrm{GeV}, \quad W > 2\mathrm{GeV}.$$

Numerical results for the x- and z-dependent asymmetries:

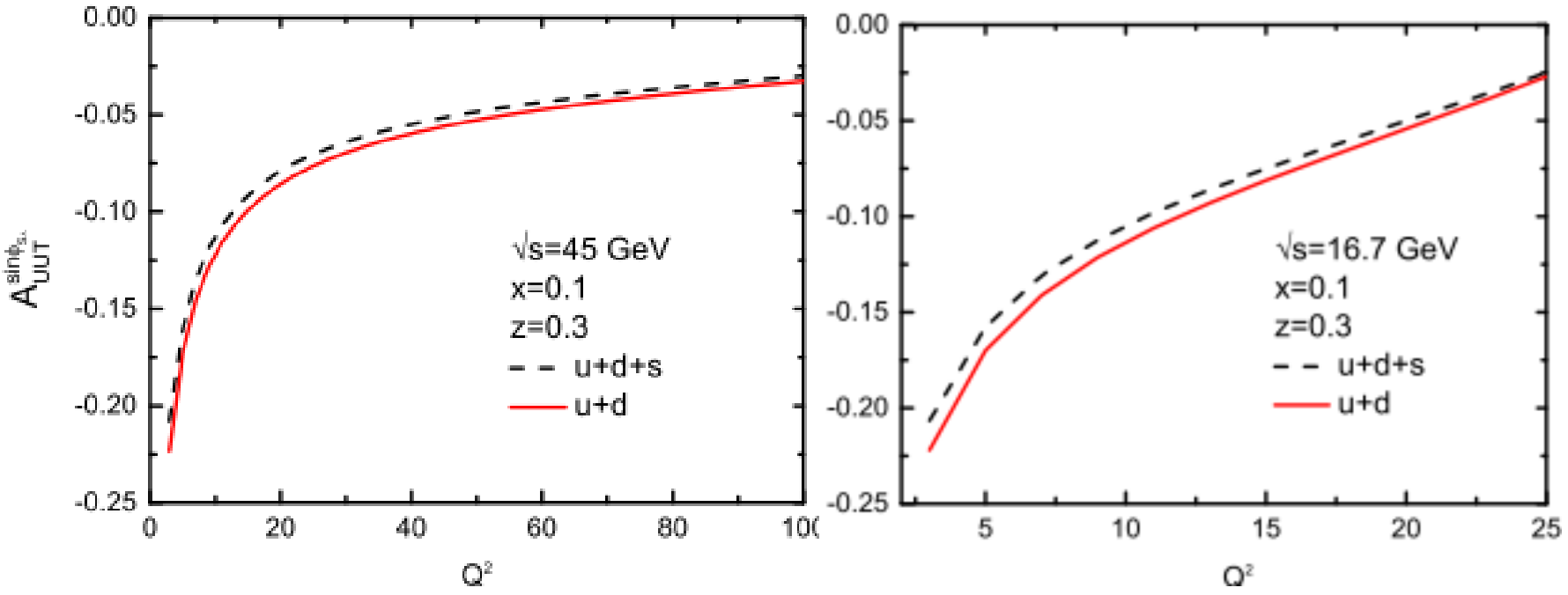


- Its sign and overall trend are consistent with the EIC case.

Although the s-quark PDF f_1 is small, the s-quark contribution still has a noticeable impact.

Q2 dependence

Fixed $x = 0.1$ and $z = 0.3$; Q^2-dependent $A_{UUT}^{\sin\phi_{S_\Lambda}}$



- As Q^2 increases, the absolute asymmetry decreases rapidly.
- This behavior is mainly driven by the kinematic factor $\frac{2M_\Lambda}{Q}$.

➢ From $Q^2 = xys$; because EicC has a lower center-of-mass energy, its accessible Q^2 range is limited to about 25 GeV^2。

Results of disscusion

➢ Asymmetry magnitude

- The loop term scales with the strong coupling αs; at the model scale, $\mu_0^2 = 0.23 GeV^2$ and $\boldsymbol{\alpha_s(\mu_0{}^2) = 0.817}$.
- For the Λ hyperon (mass approximately $1.116\ GeV$), twist-3 kinematics enhance the asymmetry at low and intermediate Q.

➢ s-quark impact

- Flavor weighting includes the s-quark contribution $f_1^s \widetilde{D}_T^{s\to\Lambda}$.
- The $s \to \Lambda$ channel partly cancels the negative u, d contribution, reducing the asymmetry magnitude.

Summary

- Using a diquark spectator model, we obtain a nonzero twist-3 naive-T-odd qgq FF $\widetilde{D}_T$ for the Λ hyperon, with the (u(d))-quark and (s)-quark contributions having opposite signs.
- The parameters were fitted to D_1, G_1, and the QCD evolution effects of $\widetilde{D}_T$ were investigated by approximating its evolution with the same kernel as that of D_1.
- In the considered x, z, and Q^2 regions , the predicted $A_{UUT}^{sin\,\phi_{S_\Lambda}}$ asymmetries at both the EIC and EicC are negative and can reach sizable magnitudes. Their precise values are subject to uncertainties arising from the model assumptions and the evolution scheme.
- The s- quark channel noticeably modifies the asymmetry and provides complementary sensitivity to the strange-quark distribution function f_1^s in the proton.

THANKS!