北京航空航天大学
BEIHANG UNIVERSITY

核子极化部分子分布的第一性原理研究：从自旋不对称度到三维动量成像

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Lattice Parton Collaboration

第一届强相互作用自旋物理会议，青岛

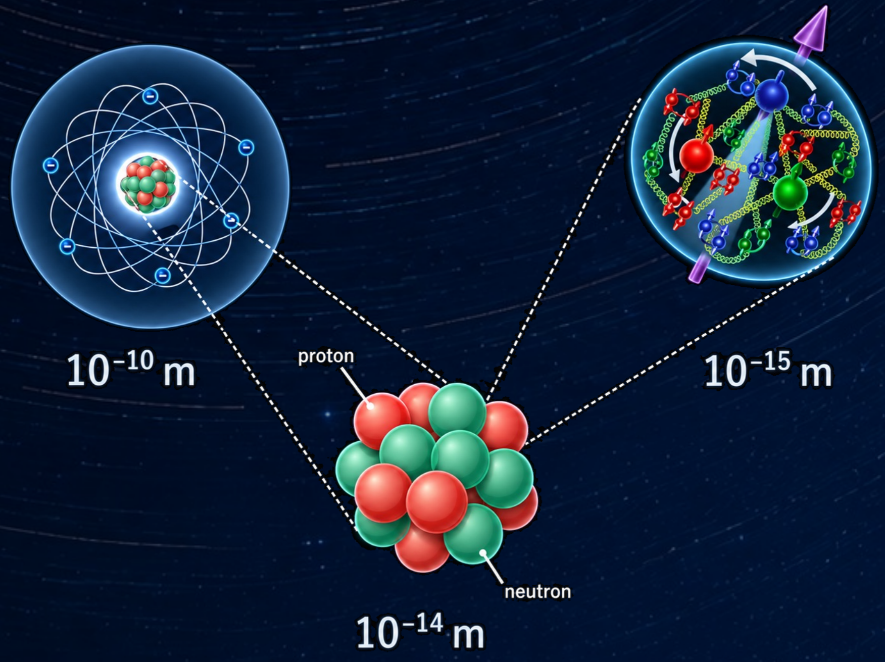
2026/07/28

OUTLINE

- Motivation
- Polarized PDF Ratio: Spin Alignment of Quarks
- Quark TMDPDFs: 3D Momentum Tomography of Nucleon
- Summary and Outlook

Nucleons: Building Blocks of the Visible Universe

Despite more than half a century of exploration, the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe



- How does quarks & gluons form the nucleon, how do they distribute?
- How do these distributions and partonic orbital motion generate the nucleon spin?

— 1st of the 3 central science questions driving the EIC program.

Adapted from the White Paper of EIC, EPJA52, 268 (2016).

Status of the 1D & 3D Nucleon Structure

UNPOLARIZED | POLARIZED

3D

Unpolarized 3D structure
- Limited experimental precision
- Strong model dependence in phenomenology
- Lattice studies remain exploratory

Polarized 3D structure
- Very limited experimental data
- Few reliable global extraction yet
- Lattice studies remain exploratory

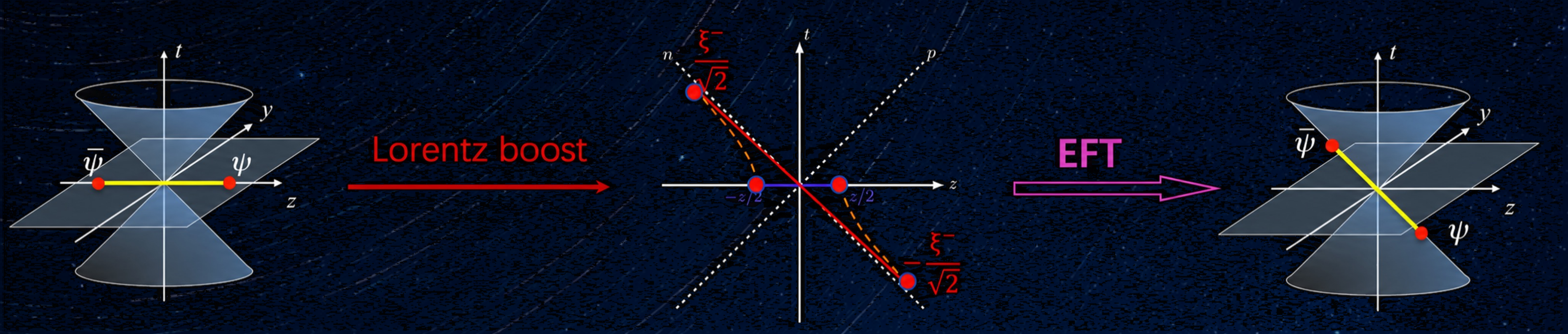
1D

Unpolarized 1D structure
- ✓ Well constrained by experiment
- ✓ Mature global analyses
- ✓ Lattice QCD provides complementary constraints

Polarized 1D structure
- Limited experimental precision
- Large-x behavior remains poorly constrained
- Lattice constraints are not yet precise enough for direct phenomenology

LaMET: A New Horizon for Nucleon Structures from LQCD

Large-momentum effective theory (LaMET): connecting Euclidean lattice and physical observables



Ji, PRL110, 262002 (2013), see Rev.Mod.Phys.93, 035005 (2021) for review

- Equal-time distributions can be directly computed in lattice QCD:

$$\tilde{q}(x, P^z) = \int \frac{dz}{2\pi} e^{-izP^z} \langle P^z | \bar{\psi}(z) \Gamma \mathcal{U}(z,0) \psi(0) | P^z \rangle$$

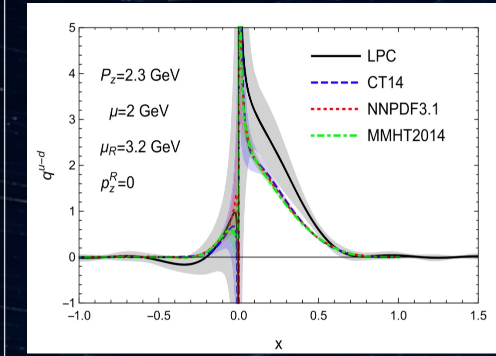
- Recovering light-cone distributions in the large-momentum limit:

$$q(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{q}(x, P^z) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(yP^z, (1-y)P^z)^2}\right)$$

LaMET: A New Horizon for Nucleon Structure from Lattice

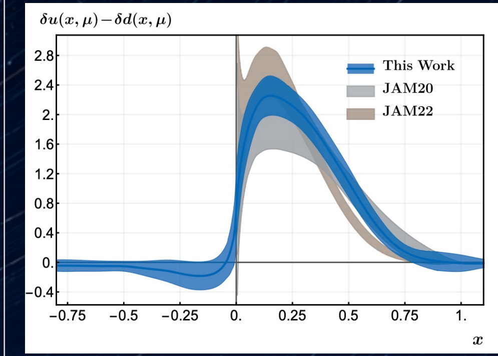
➢ Significant progress has been made on PDFs, LCDAs, TMDs, and more:

Unpolarized PDF



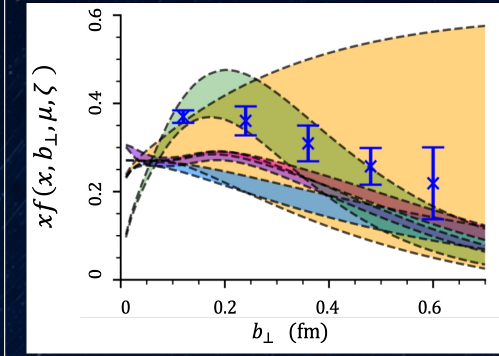
LPC, 2020

Transversity PDF



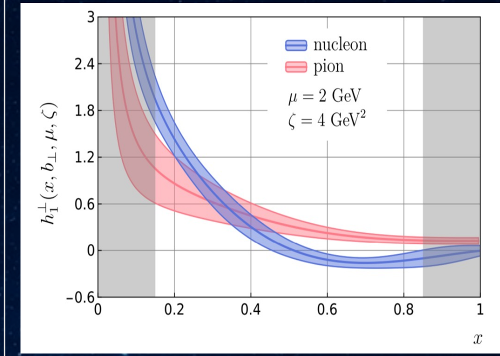
LPC, 2024

Unpolarized TMDPDF



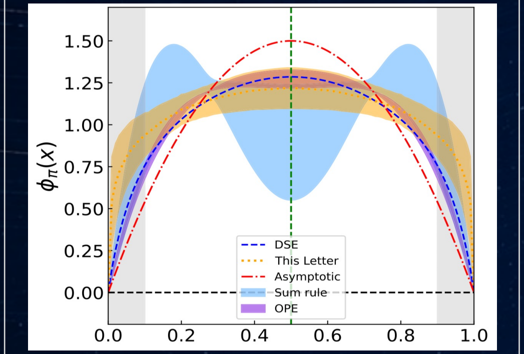
LPC, 2024

Boer-Mulders Function



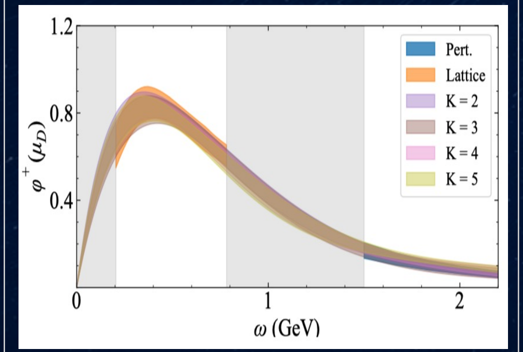
LPC, 2025

Light Meson LCDA



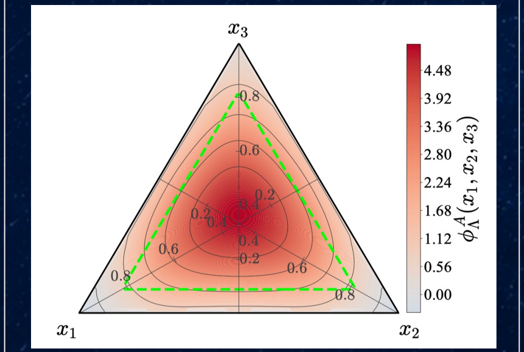
LPC, 2022

Heavy Meson LCDA



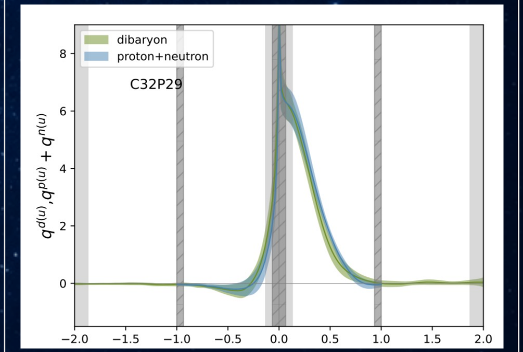
LPC, 2026

Baryon LCDA



LPC, 2026

Deuteron PDF



LPC, 2025

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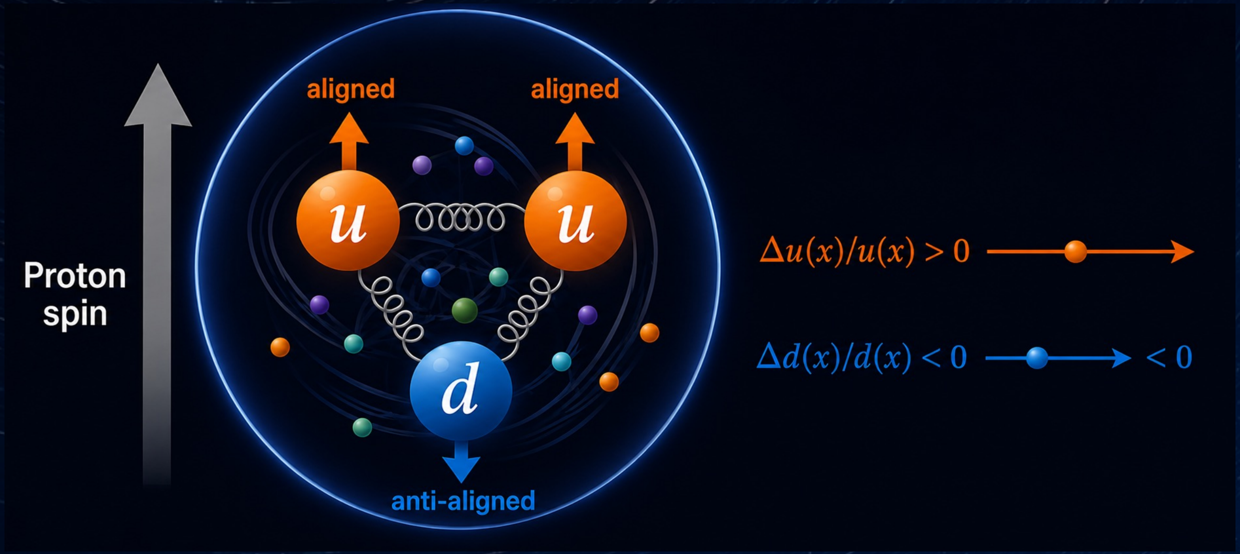
Ratios of Polarized PDFs



- Quark helicity PDF:

$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

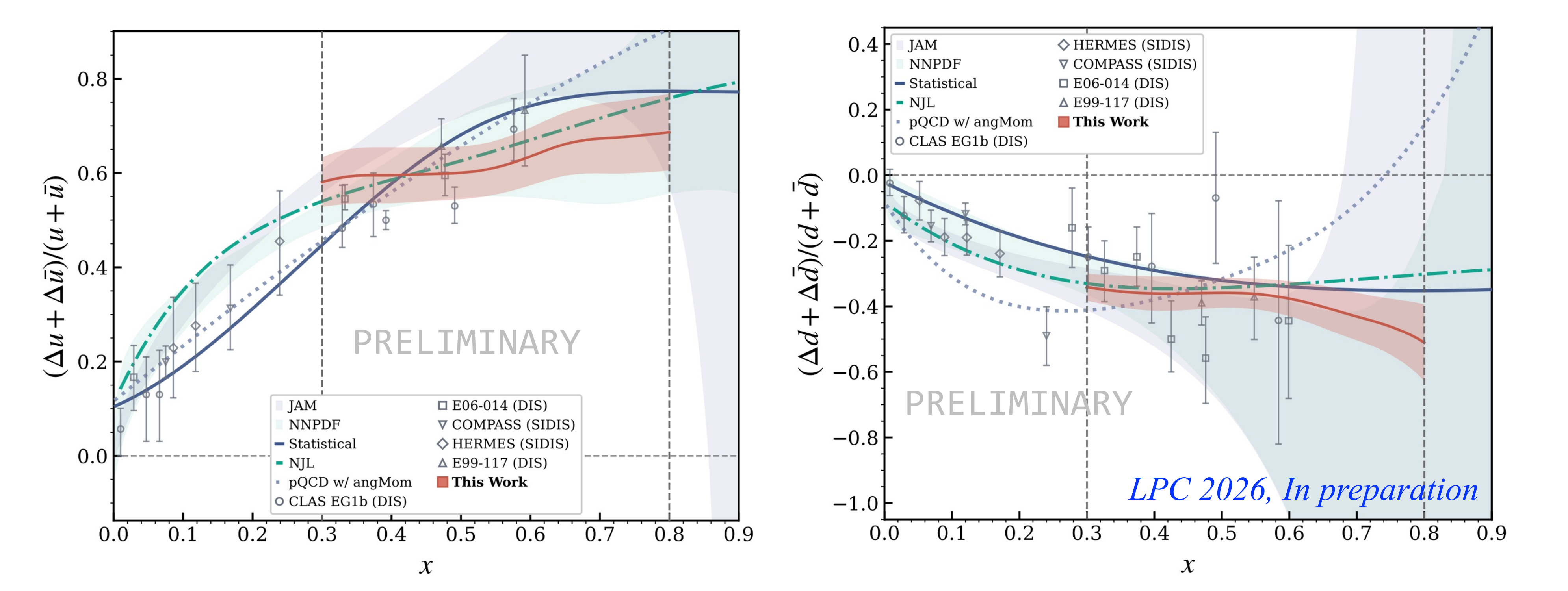
- Quark unpolarized PDF:

$$q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$$

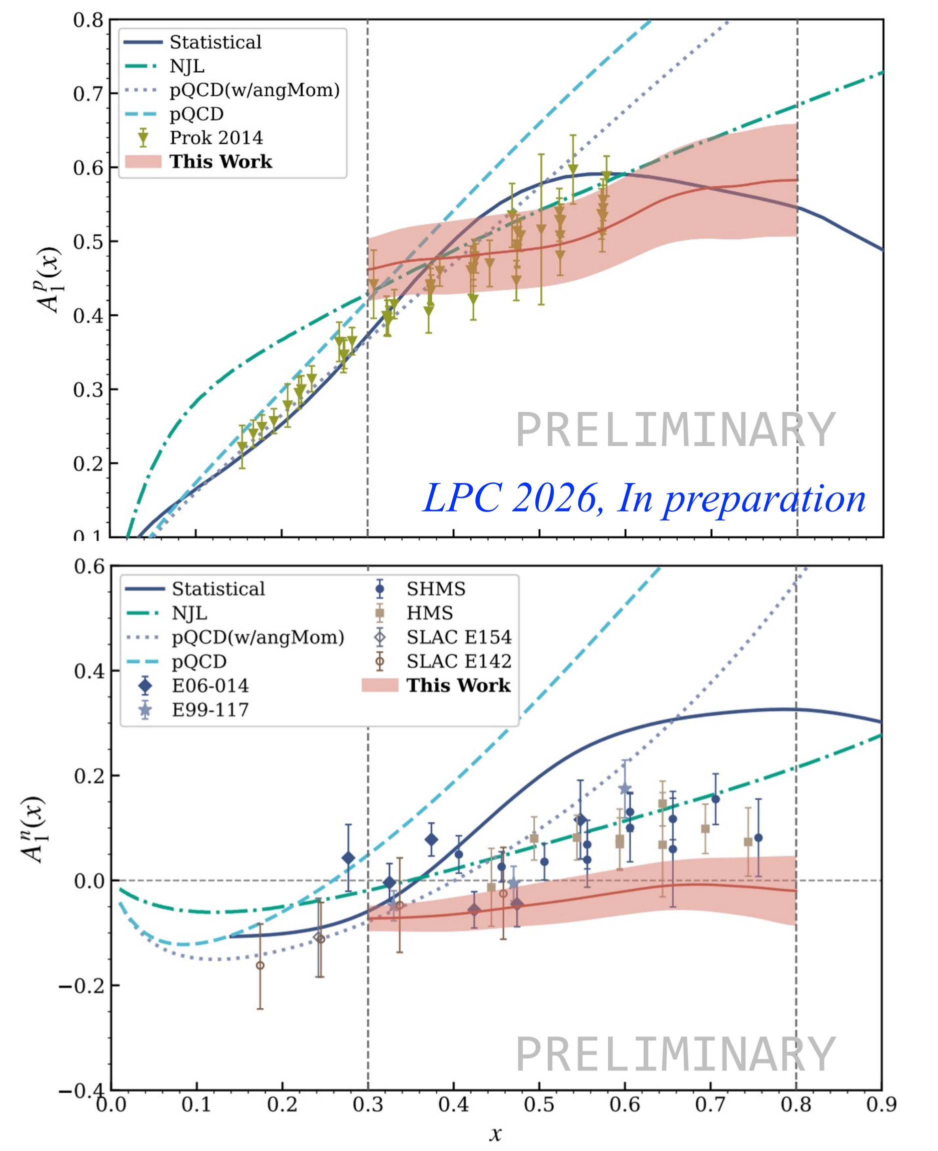
$\Delta q(x)/q(x)$ encodes the longitudinal spin alignment of quarks inside the proton, providing a clean probe of its spin-flavor structure

Ratios of Polarized PDFs from Lattice QCD

- u-quark polarization ratio is positive and rises with x
- d-quark polarization ratio remains negative throughout the accessible x region



Predictions for Nucleon Spin Asymmetries



- A_1 is the ratio between polarized structure function g_1 and unpolarized structure function F_1 :

$$A_1^N(x, Q^2) \simeq \frac{g_1^N(x, Q^2)}{F_1^N(x, Q^2)}$$

- Neglecting the strange quark, sea quark flavor separation and higher-twist corrections, the proton and neutron asymmetries:

$$A_1^p(x) \simeq \frac{4\Delta u(x) + \Delta d(x)}{4u(x) + d(x)},$$

$$A_1^n(x) \simeq \frac{\Delta u(x) + 4\Delta d(x)}{u(x) + 4d(x)}.$$

Y. Prok et al. (CLAS), PRC 90, 025212 (2014).
D. Flay et al. (JLab Hall A), PRD 94, 052003 (2016).
X. Zheng et al. (JLab Hall A), PRL 92, 012004 (2004);
X. Zheng et al., PRC 70, 065207 (2004).

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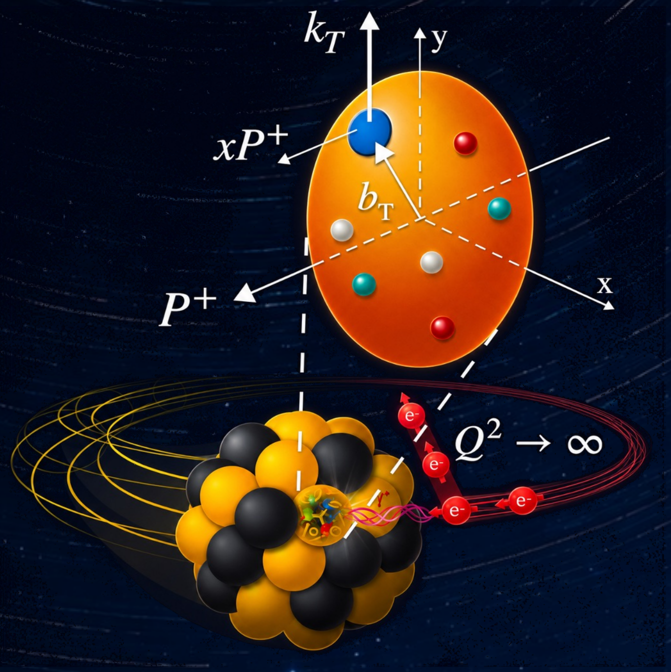
TMDs: 3D Momentum Tomography of Nucleon



TMDs: Spin-dependent 3D momentum space images from semi-inclusive scattering

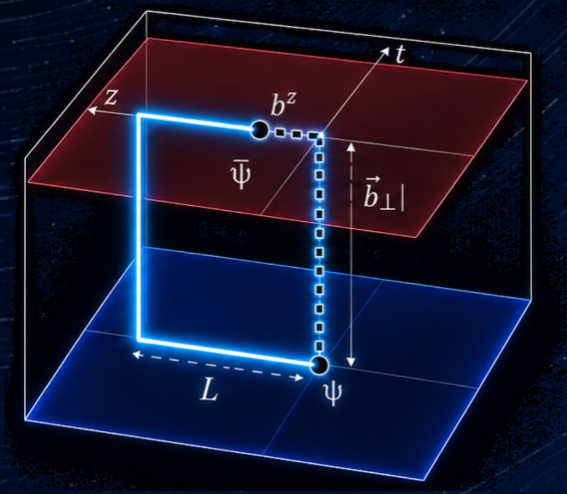
Leading Quark TMDPDFs

Nucleon Spin — Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
	L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
	T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

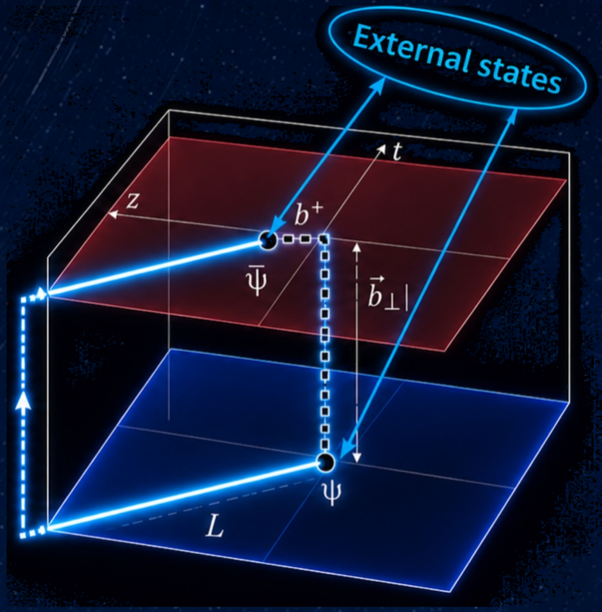
TMDs from LQCD within LaMET

Quasi-TMDs: Equal-time correlators with staple-shaped Wilson lines, lattice calculable



Lorentz boost

$L \to \infty$
$P^z \to \infty$



Factorization in LaMET:

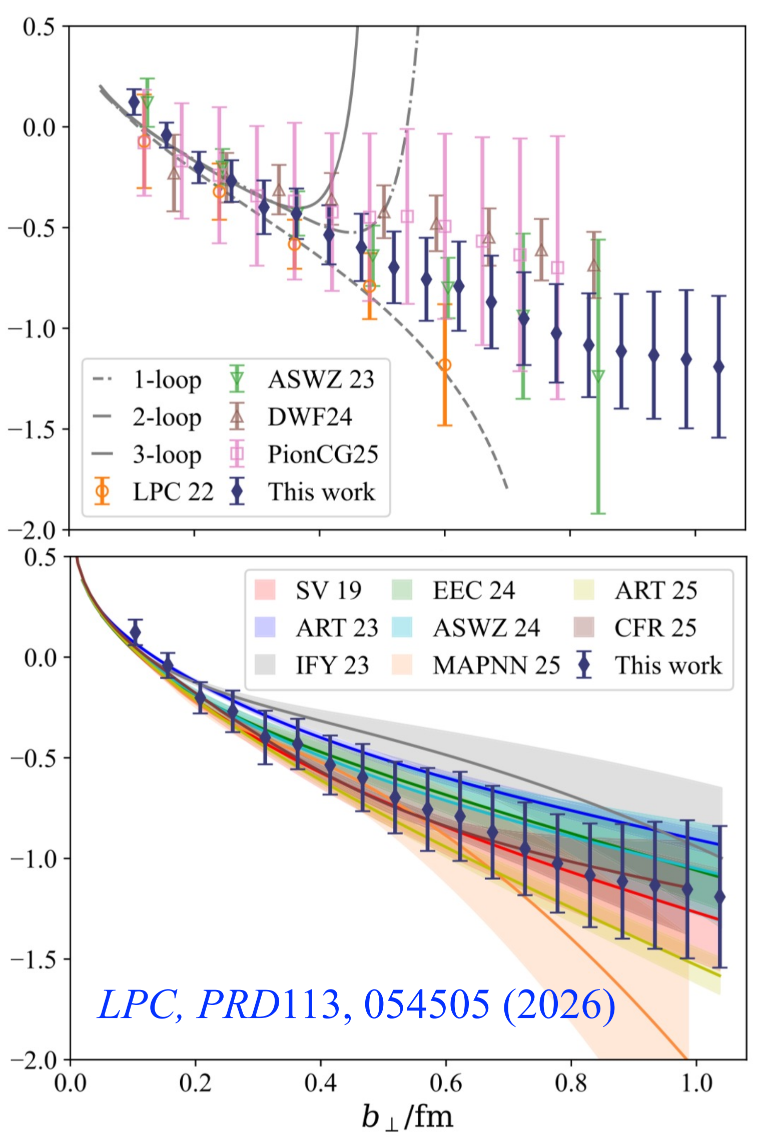
PLB811, 135946 (2020); JHEP04, 178, (2022)

Perturbative matching kernel

$$\boxed{\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z)}\boxed{\sqrt{S_I(b_\perp, \mu)}} = \boxed{H_\Gamma(\zeta^z, \mu)}\exp\left[\frac{1}{2}\boxed{K(b_\perp, \mu)}\ln\frac{\zeta^z}{\zeta}\right]\boxed{f(x, b_\perp, \mu, \zeta)} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta^z}\right)$$

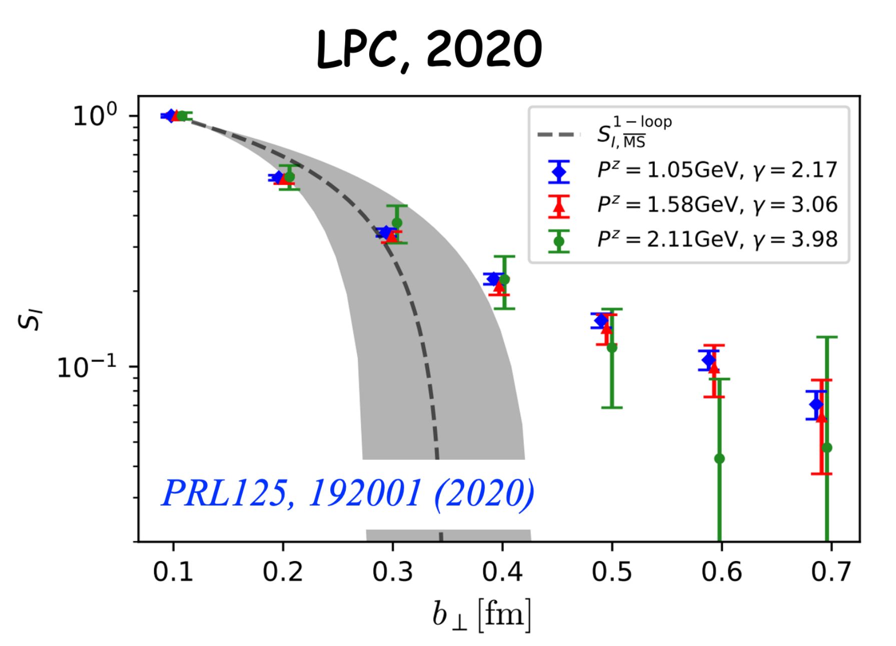
Quasi TMDs Intrinsic Soft Function Collins-Soper kernel Light-cone TMDs

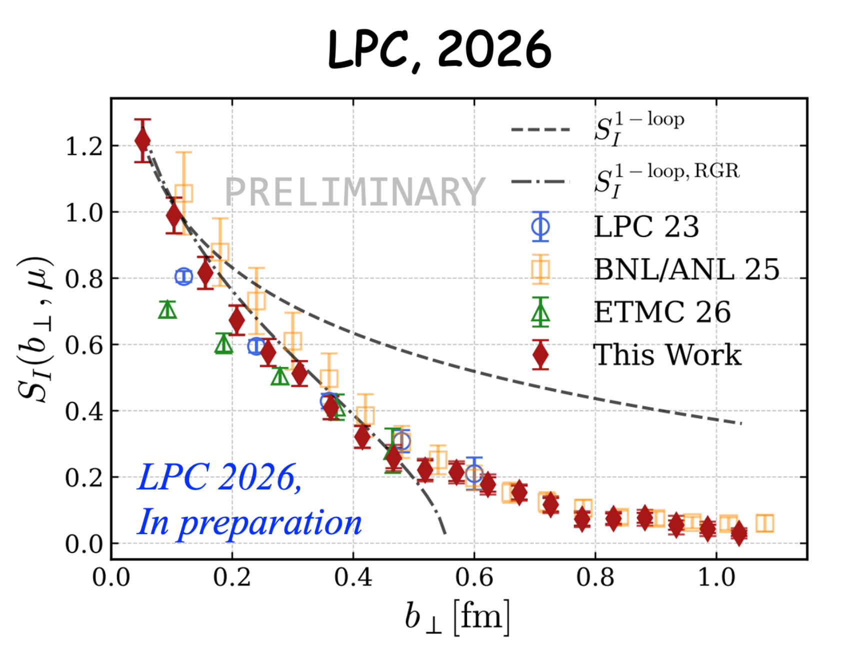
Collins-Soper Kernel from Lattice QCD



- A benchmark quantity for lattice TMD studies
  - ✓ Continuum, physical mass, and infinite-momentum limits are now being addressed
  - ✓ Lattice QCD provides first-principles constraints up to b_T ~ 1fm
  - ✓ Agreement with global fits provides mutual validation
- A precise CS kernel is a key step toward first-principles predictions of TMD observables

Intrinsic Soft Function from Lattice QCD



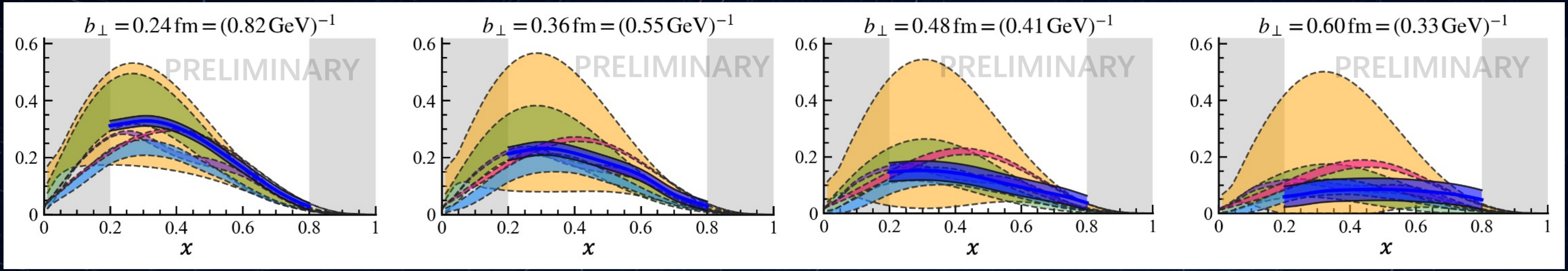


- ✓ Continuum, physical mass and infinite-momentum limits
- ✓ Precise first-principles prediction up to $b_T \sim 1$fm
- ✓ Essential input for the lattice calculation of TMDPDFs

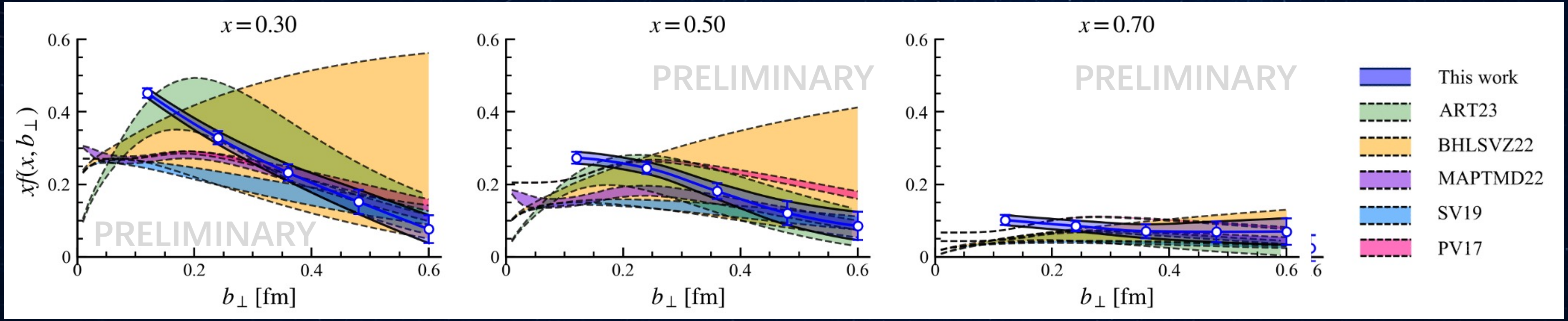
Unpolarized TMDPDFs from Lattice QCD

- x-dependence

LPC (2026), in preparation



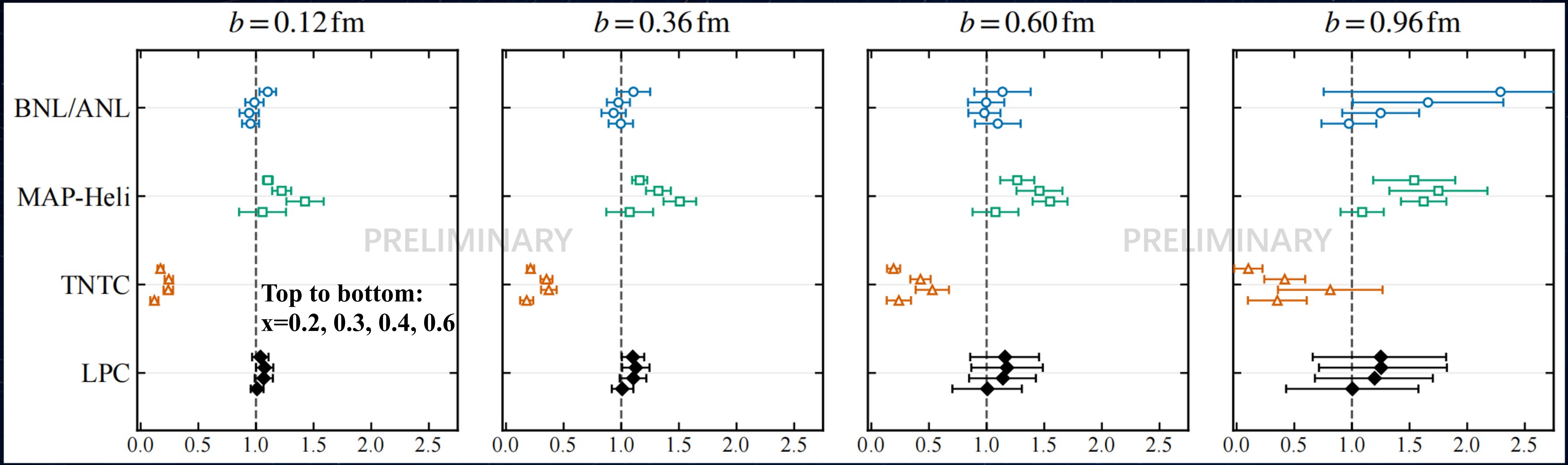
- b_T-dependence



Helicity-to-Unpolarized TMD Ratio

- Preliminary results @ continuum, chiral and infinite-momentum limit

LPC (2026), in preparation



- Systematic uncertainties are still absent;
- More systematic analyses are in progress ……

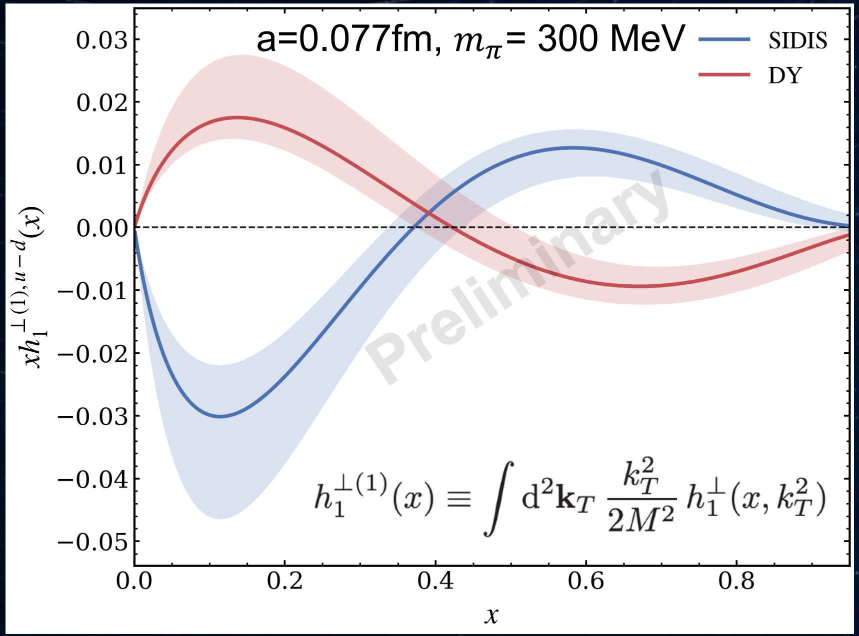
BNL/ANL (LQCD): PRL135, 20 (2025)
MAP (Global fit): PRL134, 121901 (2025)
TNTC (Global fit): PRL134, 121902 (2025)

Quark Spin-Transverse-Momentum Correlation

- First transverse moment of quasi Boer-Mulders function and its matching

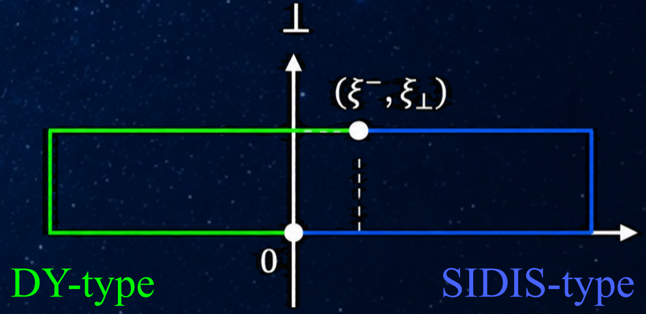
$$\tilde{h}_1^{\perp(1)}(x, b_\perp^2, P^z, \mu) = \frac{1}{iM\epsilon_\perp^{ij} b_{\perp j}} \frac{E}{P^z} \int \frac{dz}{2\pi} e^{ixP^z z} \frac{\sum_S \mathrm{Tr}\left[P_+ u(P,S) \langle N(P,S)| O^{i\sigma^{1z}\gamma_5}(z, b_\perp) |N(P,S)\rangle \bar{u}(P,S)\right]_R}{\sum_S \mathrm{Tr}\left[P_+ u(P,S)\bar{u}(P,S)\right]_R}$$

$$\tilde{h}_1^{\perp(1)}(x, b_\perp^2, P^z, \mu)\sqrt{S_I(b_\perp,\mu)} = \exp\left[\frac{1}{2}\ln\frac{\zeta_z}{\zeta} K(b_\perp,\mu)\right] H\left(\frac{x}{y}, \frac{\mu^2}{\zeta_z}\right) h_1^{\perp(1)}(y, b_\perp^2, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta_z}\right)$$



- Gauge links encode initial and final state interactions, leading to the predicted SIDIS–Drell–Yan sign reversal.

$$\mathcal{U}_n(a,b) = \exp\left[-ig\int_a^b d\lambda\, n\cdot A^\alpha(\lambda n) t^\alpha\right]$$



LPC (2026), in preparation

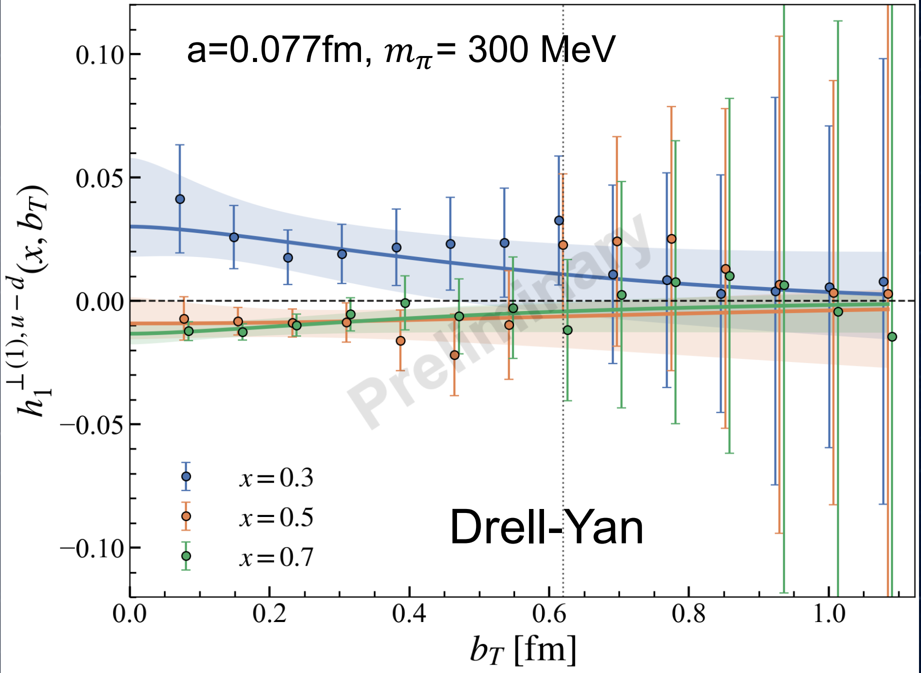
Quark Spin-Transverse-Momentum Correlation

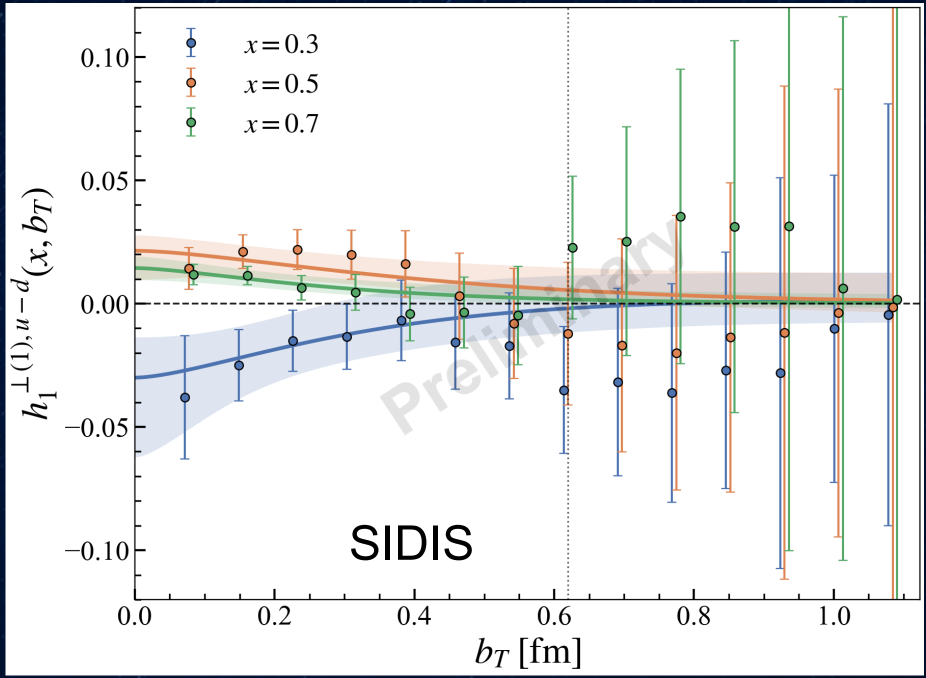
- Rebuild the full distribution of $h_1^{\perp}(x, k_\perp^2)$ from model parametrization

Bacchetta, Conti, Radici, PRD78, 074010 (2008)

$$h_1^{\perp,u-d}(x,k_\perp^2) = \frac{M_N^2}{2\pi k_\perp}\int db_\perp b_\perp^2 J_1(k_\perp b_\perp) h_1^{\perp(1)}(x,b_\perp^2)$$

$$\equiv H_S(x)\frac{4M^2}{\pi L_S^4(x)}\left(1+\frac{k_\perp^2}{L_S^2(x)}\right)^{-3} - H_A\frac{4M^2}{\pi L_A^4(x)}\left(1+\frac{k_\perp^2}{L_A^2(x)}\right)^{-3}$$

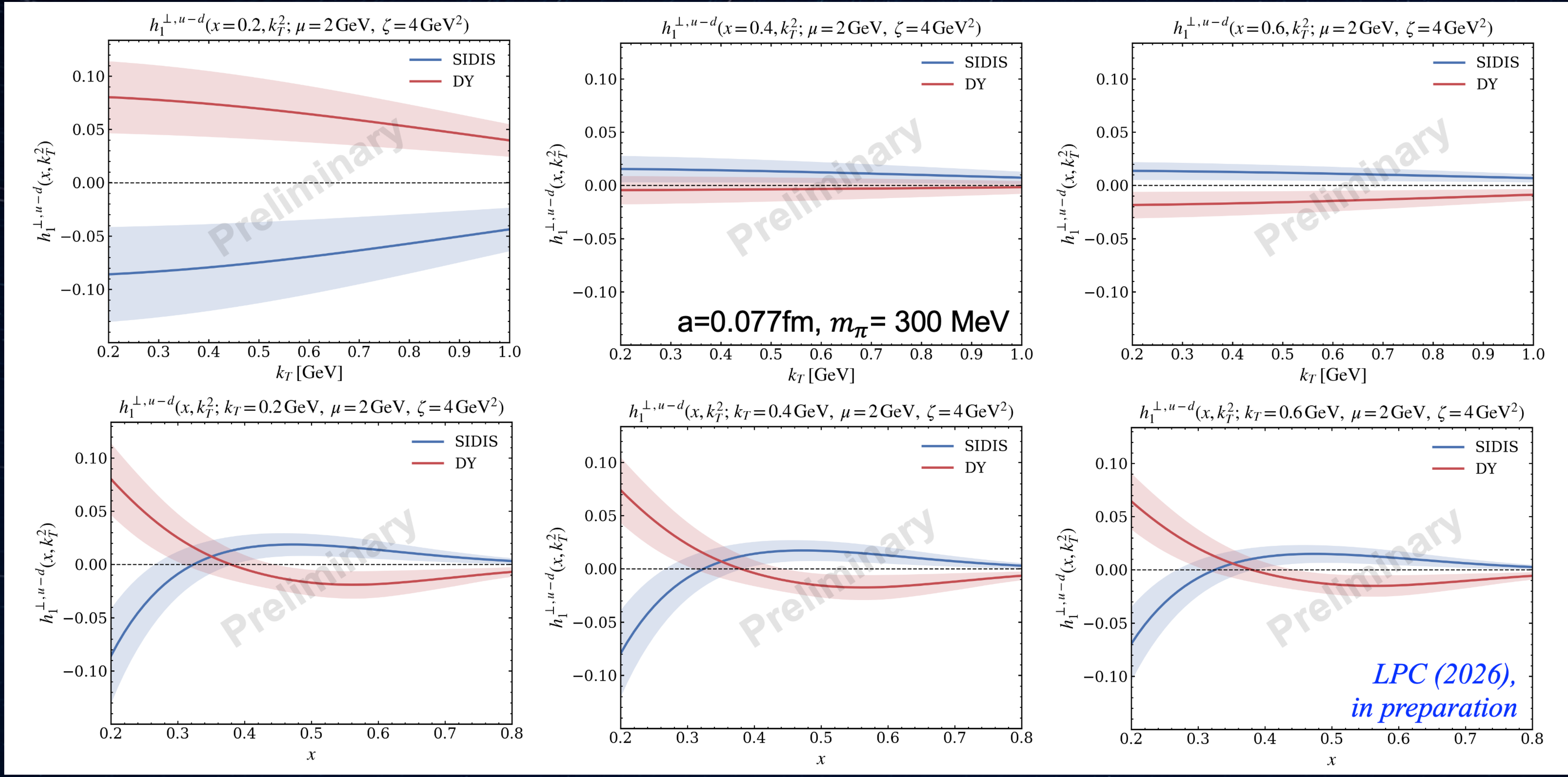




LPC (2026), in preparation

Quark Spin-Transverse-Momentum Correlation

- Rebuild the full $k_\perp$-dependent distribution from model parametrization



Summary and Outlook

Polarized PDFs reveal how quark spins align with the nucleon spin, and TMDs promote this one-dimensional picture to a three-dimensional momentum image, exposing fundamental spin-momentum correlations.

Lattice QCD + LaMET is establishing a first-principles pathway from Euclidean correlation functions to physical partonic observables.

Together with global analyses and future EIC and EicC measurements, precision lattice calculations will open a new era of multidimensional nucleon imaging.